Total No. of printed pages = 3

CSE 181401

Roll No. of candidate ____________

2022

B.Tech 4th Semester End-Term Examination

DISCRETE MATHEMATICS

(New regulation & New Syllabus)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any Four from the rest.

1. Answer the following : (Choose the most suitable answer) (10 × 1 = 10)

(i) If $P = \{1,2,3\}$ and $Q = \{2,3,4\}$, then the set $\{(x,y):(x,y) \in P \times Q \, and \, x \geq y\}$ is

(a) $\{(1,2),(2,3),(3,4)\}$ (b) $\{(4,3),(3,2),(2,1)\}$

(c) $\{(2,2),(3,3),(3,2)\}$ (d) None of these

(ii) Which one of the following is not countable

(a) The set of integers.

(b) The interval [a, a]

(c) The set of natural numbers

(d) The interval [a, b] where a and b are distinct.

(iii) Which of the following statements is not correct

(a) Every cyclic group is abelian.

(b) Every subgroup of a cyclic group is cyclic.

(c) Every abelian group is cyclic.

(d) Every subgroup of an abelian group is abelian.

(iv) Which one of the following statements is true

(a) Paris is in France and 2+2 = 4

(b) Paris is in France and 2+2 = 5

(c) Paris is in England and 2+2 = 4

(d) Paris is in England and 2+2 = 5

[Turn over

(v) The dual of $(U \cap A) \cup (B \cap A) = A$ is

(a) $(U \cap A) \cup (B \cap A) = A$ (b) $(\phi \cap A) \cup (B \cap A) = A$

(c) $(\phi \cup A) \cap (B \cup A) = A$ (d) $(\phi \cap B) \cup (B \cap A) = B$

(vi) The number of distinct permutations that can be formed from all the letters of each word "UNUSUAL" is

(a) 5040 (b) 2520

(c) 1680 (d) 840

(vii) If $o(H) = 5$ and $o(G) = 10$, H is a subgroup of G then

(a) G is a normal subgroup of H

(b) H is a normal subgroup of G.

(c) H is not a normal subgroup of G

(d) None of these

(viii) Converse of Lagrange's theorem holds in case of

(a) Ring (b) Abelian group

(c) Finite abelian group (d) None of these

(ix) If $A = \{x : -1 \le x \le 6\}$ and $B = \{x : x > 3\}$ then $A \cup B =$

(a) $\{x : x > -1\}$ (b) $\{x : x \ge -1\}$

(c) $\{x : x = -1\}$ (d) None of these

(x) If R is a ring where the cancellation law holds then

(a) R is an Integral Domain (b) R is without Zero Divisor

(c) R is with Zero Divisor (d) None of these

2. (a) Prove the following proposition by using Principle of Mathematical Induction: (6)

$p(n): 1 + 2 + 2^2 + 2^3 + \ldots + 2^n = 2^{(n+1)} - 1$, $\forall\ n \ge 0$ $2^{n+1} - 1$

(b) Show that the mapping $f : R \rightarrow R$ defined $f(x) = 3x + 5, x \in R$ is bijective. Determine f^{-1}. (6)

(c) Define Euclidean Algorithm (3)

3. (a) The *60000 fans* who attended the homecoming football game brought up all the *paraphernalia* for their cars. Altogether 20000 bumper stickers, 36000 windows decals, and 12000 key rings were sold. We know that 52000 fans bought at least 1 item and no one bought more than 1 of the given items. Also 6000 fans bought both decals and key rings, 9000 bought both decals and bumper stickers and 5000 bought both key rings and bumper stickers.

(i) How many fans bought all 3 items?

(ii) How many fans bought exactly 1 item? (6)

(b) Out of 12 employees, a group of 4 trainees is to be sent for "Software Testing and Quality Assessment" training of one month. Then find (6)

(i) in how many ways 4 employees can be selected?

(ii) in how many ways 4 employees can be selected if two employees refuse to go together?

(c) State "The Pigeonhole Principle" (3)

4. (a) Prove that: (3 + 3 = 6)

(i) $\neg(p \vee q) \equiv \neg p \wedge \neg q$,

(ii) $\neg\neg p \equiv p$

(b) Check for validity of the argument using truth table: (6)

Statement 1: Either it is below freezing or raining now.

Statement 2: It is not below freezing.

Conclusion: It is raining now.

(c) Check for validity of that statement using truth table: (3)

"If p implies q and q implies r, then p implies r".

5. (a) Prove that (R-{1},*) is an abelian group where * is defined by a*b=a+b–ab. (6)

(b) Show that the set of all permutations on n-symbols forms a group under operation multiplication of permutations. (6)

(c) Prove that the centre of a group is a subgroup of the group. (3)

6. (a) Check whether the set R={0,1,2,3,4,5} is a commutative ring with respect to '$+_6$' and 'X_6' as two ring compositions. (6)

(b) If R is the additive group of real numbers and R^+ is the multiplicative group of positive real numbers, prove that the mapping $f: R \to R^+$ defined by $f(x) = e^x \forall x \in R$ is an isomorphism. (6)

(c) Prove the Every field is an Integral Domain. (3)

7. (a) Prove that the intersection of any two normal groups of a group is itself a normal subgroup. (6)

(b) Find the dnf and cnf of $p \leftrightarrow (\bar{p} \vee \bar{q})$ (6)

(c) Define disjunctive and conjunctive normal forms (3)