Total No. of printed pages = 4

MA 181202

Roll No. of candidate | | | | | | | | | | |

2021

B.Tech. 2nd Semester End-Term Examination

MATHEMATICS — II

Full Marks – 50 Time – Two and half hours

The figures in the margin indicate full marks for the questions.

Question 1 is compulsory and answer any *four* questions from the rest of the questions.

1. Choose the correct alternative from the following : (10 × 1 = 10)

(i) The value of $e^{\pm i2n\pi}$ is

(a) 1

(b) 0

(c) i

(d) –1

(e) None of these

(ii) The complementary function of $\frac{d^2y}{dx^2}+4y=0$ is

(a) $A\cosh x+B\sinh x$

(b) $Ae^{2x}+Be^{-2x}$

(c) $A\cos x+B\sin x$

(d) $A\cos 2x+B\sin 2x$

(e) None of these

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(iii) The value of $\int_c \frac{1}{z}\cos z\,dz$ where c is the ellipse $9x^2+4y^2=1$ is

(a) $2\pi i$

(b) 2π

(c) 0

(d) $3\pi i$

(e) none of these

(iv) If $\overline{A}$ be a constant vector and $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then grad $(\vec{A}\cdot\vec{r})=$

(a) $\overline{A}$

(b) $2\vec{A}$

(c) $\vec{r}$

(d) $3\vec{A}$

(e) none of these

(v) If $f(z)$ is analytic in a closed curve C except a finite number of poles with C then $\int_C f(z)\,dz=$

(a) $2\pi i$ (sum of residues at the poles within C)

(b) 2π (sum of residues at the poles within C)

(c) $2\pi i$

(d) $-2\pi i$ (sum of residues at the poles within C)

(e) none of these

(vi) If $J_0(x)$ and $J_1(x)$ are Bessel function then $J_1(x)$ is given by

(a) $J_0(x)-\frac{1}{x}J_1(x)$

(b) $-J_0(x)+J_1(x)$

(c) $J_0(x)+\frac{1}{x}J_1(x)$

(d) $J_0(x)-\frac{1}{x^2}J_1(x)$

(e) none of these

(vii) The value of $div(curl\ \vec{A})$ is

(a) 1

(b) 2

(c) 3

(d) 0

(e) none of these

(viii) The value of $\cos iz$ is

(a) $\dfrac{e^{iz}+e^{-iz}}{2}$

(b) $\dfrac{e^{-z}+e^{z}}{2}$

(c) $\dfrac{e^{z}-e^{-z}}{z}$

(d) $\dfrac{e^{iz}-e^{-iz}}{2}$

(e) none of these

(ix) The first order differential equation $M(x, y)dx+N(x, y)dy=0$ is exact if

(a) $\dfrac{\partial M}{\partial y}+\dfrac{\partial N}{\partial x}=0$

(b) $\dfrac{\partial M}{\partial x}+\dfrac{\partial N}{\partial y}=0$

(c) $\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}=0$

(d) $\dfrac{\partial M}{\partial x}-\dfrac{\partial N}{\partial y}=0$

(e) none of these

(x) The integrating factor of $\dfrac{dy}{dx}+\dfrac{y}{x}=x^3$ is

(a) x

(b) $-x$

(c) $\log x$

(d) $\log\dfrac{1}{x}$

(e) none of these

2. (a) Find the directional derivative of $\phi=x^2yz+4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\hat{i}-j-2\hat{k}$. (3)

(b) Find the value of $P_n(1)$. (1)

(c) Show that $u=e^{-2xy}\sin(x^2-y^2)$ is harmonic. Pr 1044 (3)

(d) State residue theorem and use it to evaluate $\int_c \dfrac{(2z-1)}{z(z+2)(2z+1)}dz$ where C is $|Z|=1$. (3)

3. (a) If $\vec{F} = 2y\hat{i} - 2\hat{j} + x\hat{k}$ evaluate $\oint_c F\,d\vec{r}$ along the curve $x = \cos t,\ y = \sin t,$ $z = 2\cos t$ from $t = 0$ to $t = \frac{\pi}{2}$. (3)

(b) Solve $y^2 p^3 - y + 2px = 0$. – P-127. (2)

(c) Solve : $(D^2 + D)\,y = x^2$. (2)

(d) Find and plot the image of the triangular region with vertices at $(0, 0), (1, 0), (0, 1)$ under the transformation $w = (1 - i)\,Z + 3$. (3)

4. (a) Define a solenoidal vector, show that the vector $3y^4z^2\hat{i} + 4x^3z^2\hat{j} + 3x^2y^2\hat{k}$ is solenoidal. (3)

(b) Solve : $\frac{dy}{dx} + y\cot x = 2\cos x$. (2)

(c) Prove that $x J_n' = n J_n - x J_{n+1}$. (2)

(d) Find the analytic function whose imaginary part is $V = \log(x^2 + y^2) + x - 2y$. (3)

5. (a) If $r = |\vec{r}|$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ prove that $\nabla r = \frac{1}{r}\vec{r}$. (2)

(b) Test whether the following equation is exact or not. If not find an integrating factor to make it exact and hence solve it. (1+2+1=4)

$(2xy^2 + y)\,dx + (x + 2x^2y - x^4y^3)\,dy = 0$

(c) (i) Define a regular singularity of the differential equation. (4)

(ii) Find the series solution of

$2x^2y'' + (2x^2 - x)\,y' + y = 0$.

6. (a) Using Green's theorem, evaluate $\int_c (x^2y\,dx + x^2dy)$ where C is the boundary described counter clock wise of the triangle with vertices (0, 0), (1, 0), (1, 1). (3)

(b) (i) Prove that $xP_n' - P_{n-1}' = nP_n$. (2)

(ii) Find $P_3(x)$. (2)

(c) Apply the calculus of residue to evaluate :

$$\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2 + a^2)(x^2 + b^2)}.$$ (3)

7. (a) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of legendre polynomial. (3)

(b) (i) Find e^z and $|e^z|$ if z equals to $4\pi(2 + i)$. (2+2)

(ii) Find $\sin(iz)$.

(c) Evaluate $\int_c (x^2 + y^2)\,dz$ from $z = 0$ to $z = 2 + 4i$. (3)