Total No. of printed pages = 6

MA 181301B

Roll No. of candidate

2021

B.Tech. 3rd Semester End-Term Examination

ECE, ETE, CSE

MATHEMATICS III – B

(New Regulation)

(w.e.f. 2017–2018)

(New Syllabus)

(w.e.f. 2018–2019)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

(Answer question No. 1 and any *four* from the rest.)

1. Choose the correct answer: (10 × 1 = 10)

(i) An integer is chosen from 2 to 15. What is the probability that it is prime?

(a) $\frac{4}{7}$

(b) $\frac{3}{7}$

(c) $\frac{2}{7}$

(d) $\frac{1}{7}$

[Turn over

(ii) Let A and B be two events such that $P(B)=1$, then $P(A/B)=$ ________

(a) $P(A)$

(b) $P(B)$

(c) $P(A\cap B)$

(d) $P(A\cup B)$

(iii) For a random variable X which of the following is false?

(a) $0\le F_X(x)\le 1$

(b) $F_X(\infty)=1$

(c) $P(a<X\le b)=F_X(b)-F_X(a)$

(d) $F_X(x)=P(X\ge x)$

(iv) If X is a continuous random variable with probability density function

$f_X(x)=\begin{cases} Kx^2 & for\ \ 0<x<3 \\ 0 & otherwise \end{cases}$. Then the value of K is ________

(a) $\frac{2}{9}$

(b) $\frac{1}{9}$

(c) $\frac{4}{9}$

(d) $\frac{5}{9}$

(v) If X is a continuous random variable with probability density function

$f_X(x)=\begin{cases} \frac{1}{2}x & for\ \ 0<x<2 \\ 0 & otherwise \end{cases}$. Then $E(X)$ is ________

(a) 1

(b) 0

(c) 2

(d) 3

(vi) The frequency curve which is symmetrical about its mean is known as

(a) Platykurtic

(b) Mesokurtic

(c) Leptokurtic

(d) None of these

(vii) Which of the vector is a probability vectors?

(a) $\left(\frac{1}{4}, \frac{3}{2}, -\frac{1}{4}, \frac{1}{2}\right)$

(b) $\left(\frac{5}{2}, 0, \frac{8}{3}, \frac{1}{6}, \frac{1}{6}\right)$

(c) $\left(\frac{1}{12}, \frac{1}{2}, \frac{1}{6}, 0, \frac{1}{4}\right)$

(d) $\left(\frac{3}{13}, \frac{2}{13}, -\frac{1}{6}, 0, \frac{1}{5}\right)$

(viii) The joint probability mass function of two random variables X any Y is

$P_{X,Y}(x, y) = \begin{cases} \frac{1}{21}(x+y) \text{ for } x = 1,2 \text{ and } y = 1, 2, 3 \\ 0 \quad \text{otherwise} \end{cases}$. The $P_X(1) =$ ______

(a) $\frac{3}{8}$

(b) $\frac{3}{7}$

(c) $\frac{5}{6}$

(d) $\frac{1}{4}$

(ix) 2% of the items produced by a firm are defective. If a box contains 100 items, then the variance is ______

(a) 2

(b) 3

(c) 1

(d) 4

(vi) The frequency curve which is symmetrical about its mean is known as

(a) Platykurtic

(b) Mesokurtic

(c) Leptokurtic

(d) None of these

(vii) Which of the vector is a probability vectors?

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(b) $\left(\frac{5}{2}, 0, \frac{8}{3}, \frac{1}{6}, \frac{1}{6}\right)$

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(d) $\frac{1}{4}$

(ix) 2% of the items produced by a firm are defective. If a box contains 100 items, then the variance is ______

(a) 2

(b) 3

(c) 1

(d) 4

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(x) If θ be the angle between the lines of regression of the variables X and Y, then the lines of regression are perpendicular to each other if ————

(a) $\tan\theta = \frac{\pi}{2}$

(b) $\sin\theta = \frac{\pi}{2}$

(c) $\tan\theta = \infty$

(d) $\sin\theta = 0$

2. Answer the following:

(a) A bag contains 6 white, 3 red and 9 black balls. Three balls are drawn one by one with replacement. What is the probability that at least one is white? (5)

(b) State and prove Baye's Theorem. (1+4=5)

(c) The probability density function of a random variable X is $f_X(x) = \frac{1}{2}e^{-|x|}$ for $-\infty < x < \infty$. Find the cumulative distribution function of X. (5)

3. Answer the following :

(a) The probability mass function of a random variable X is

$$p_X(x) = \begin{cases} \frac{1}{K^x} & for\ x = 1, 2, \dots \\ 0, & otherwise \end{cases}, \text{ where K is a constant.}$$

Find moment generating function of X. Hence evaluate mean of X. (3+2=5)

(b) How many tosses of a fair coin are needed so that the probability of getting at least one head is 87.5%? (5)

(c) Using the least square method fit a straight line to the four points (–1.0,1.000), (–0.1,1.099), (0.2,0.808), (1.0,1.000). (5)

4. Answer the following :

(a) In a normal distribution, 7% of the items are under 35 and 89% of the items are under 63. What is the mean and standard deviation of the distribution? (5)

(b) The first four moments of a distribution about the value 4 of the variable are –1.5, 17, –30 and 108. Calculate measure of skewness and measure of kurtosis, and comment upon the nature of the frequency distribution. (5)

(c) If the random variables Y, X_1 and X_2 are defined as $Y = aX_1 + bX_2$, where a and b are constants, find variance of Y. (5)

5. Answer the following :

(a) The joint probability mass function of two random variables X and Y is

$$P_{X,Y}(x, y) = \begin{cases} \frac{1}{42}(2x + y), & for\ x = 0, 1, 2\ and\ y = 0, 1, 2, 3 \\ 0, & otherwise \end{cases}$$

Find $p_y(y/2)$. Hence, find $P(Y = 1/X = 2)$. (3+2=5)

(b) Find the unique fixed probability vector t of $P = \begin{bmatrix} 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (5)

(c) Show that the Poisson distribution is the limiting form of the Binomial distribution. (5)

6. Answer the following :

(a) The joint probability density function of X and Y is

$$f_{X,Y}(x, y) = \begin{cases} 4xye^{-(x^2+y^2)}, & for\ 0 \le x < \infty\ and\ 0 \le y < \infty. \\ 0, & otherwise \end{cases}$$

Show that X and Y are independent. (5)

(b) The marks secured by recruits in the selection test (X) and in the proficiency test (Y) are given below:

Sl.No.	1	2	3	4	5	6	7	8	9
X	10	15	12	17	13	16	24	14	22
Y	30	42	45	46	33	34	40	35	39

Calculate the rank correlation coefficient. (5)

(c) Let two dice be thrown at random. Let X be the discrete random variable that assigns to each point (a, b) the maximum of its numbers. Find the cumulative distribution function of X. (5)

7. Answer the following :

(a) The theory predicts the proportion of beans in four groups G_1, G_2, G_3 and G_4 should be in the ratio 9:3:3:1. In an experiment with 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory? (5)

(b) Find the regression line of Y on X if $n = 5$, $\Sigma x = \Sigma y = 15$, $\Sigma x^2 = \Sigma y^2 = 49$ and $\Sigma xy = 44$. (5)

(c) Consider a two-state Markov chain with the transition probability matrix $P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$ for $0 < a < 1$ and $0 < b < 1$. Find the n-step transition probability matrix P^n. (5)

Total No. of printed pages = 4

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MATHEMATICS – III B

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. (a) Fill in the blanks : $(8 \times 1 = 8)$

(i) Kurtosis measures the ———— of a distribution.

(ii) If the two regression coefficients are –0.4 and –0.9 respectively, then the correlation coefficient is ————

(iii) If $f(x) = \frac{1}{2}(x+1)$ for $-1 < x < 1$ and 0 otherwise, represents the density function of a random variable x, then $E(x)$= ————.

(iv) If x is a Poisson variable such that $P(x = 1) = 0.3$ and $P(x = 2) = 0.2$, then $P(0)$= ————.

(v) If A and B are events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, $P(\overline{A}) = \frac{2}{3}$, then $P(A)$= ————.

(vi) If the two regression lines are $3x + 2y = 26$ and $6x + y = 31$, then $\overline{x}$ and $\overline{y}$ are respectively.

(vii) A group of 100 items have a mean of 60. If the mean of 60 of these items be 51, then the mean of the other 40 items is ————.

(viii) The first three moments of a distribution about the value 3 are –1, 10, –28. The third moment about the mean is ————.

[Turn over

(b) Choose the correct answer : (2 × 1 = 2)

(i) Which of the vectors is a probability vector?

(1) $\left(\frac{1}{4}, \frac{1}{2}, 0, \frac{1}{4}\right)$ (2) $\left(\frac{1}{3}, \frac{2}{3}, -\frac{1}{4}, \frac{1}{4}\right)$

(3) $\left(\frac{1}{2}, \frac{1}{3}, -\frac{1}{5}, \frac{1}{7}\right)$ (4) $(3, 4, 5, 0)$

(ii) If $f(x) = x + \frac{2}{k}$, $x = 1, 2, 3, 4, 5$ is the probability function of a discrete random variable, then k=?

(1) $\frac{5}{7}$ (2) $\frac{7}{5}$

(3) $-\frac{5}{7}$ (4) $-\frac{7}{5}$

2. (a) The scores of two golfers for 10 rounds each are given below :

A:	58	59	60	54	65	66	52	75	69	52
B:	84	56	92	65	86	78	44	54	78	68

Which may be regarded as the more consistent player? (6)

(b) Find the coefficient of correlation for the following data and discuss the nature of correlation. (4+1=5)

x:	1	2	3	4	5
y:	10	12	16	27	25

(c) A committee consists of 8 students two of which are from 1st Year, three from 2nd year and three from 3rd year. What is the chance that

(i) the three students belong to different classes?

(ii) two belong to the same class and the 3rd to different class? (2+2=4)

3. (a) A letter is known to come either from London or from Clifton. On the post only the consecutive letters on are legible. Find the probability that it came from London. (4)

(b) Fit a least square straight line to the following data: (4+1=5)

x:	2	7	9	1	5	12
y:	13	21	23	14	15	21

Hence find $y(10)$.

(c) Calculate the moment coefficient of skewness and kurtosis of the following data :

Class:	0-4	4-8	8-12	12-16	16-20
Frequency:	4	10	6	12	8

Hence comment on their nature. (4+2=6)

4. (a) From the following data obtain the two regression lines and hence find the correlation coefficient. (2+2+2=6)

x:	100	98	78	85	110	93	80
y:	85	90	70	72	95	81	74

(b) A machine produces an average of 20% defective bolts. A batch is accepted if a sample of 5 bolts taken from that batch contains no defective and rejected if it contains 3 or more defective. In other cases a second sample is taken. What is the probability that the second sample is required? (4)

(c) Find the mean and standard deviation of a normal distribution in which 5% of the items are under 30 and 80% are under 50. (5)

5. (a) Under what conditions Poisson distribution is a limiting case of bionomial distribution? Verify it. (4)

(b) A sample of six fathers and their eldest sons gave the following data about their heights in inches.

Father (X)	65	63	67	64	68	62
Son (Y)	68	60	68	65	69	61

Calculate the coefficient of rank correlation. (5)

(c) Write one use of t-test. A filling machine is expected to fill 5 kg of powder into bags. A sample of 10 bags gave the following weights 4.7, 4.9, 5.0, 5.1, 5.4, 5.2, 4.6, 5.1, 4.6 and 4.7. Test whether the machine is working properly or not. (1+5=6)

6. (a) Define null hypothesis in test of significance. A sample analysis of examination results of 600 students, it was found that 280 students have failed, 170 have secured a 3rd class, 90 have secured a second class and the rest a first class. Do this data supports the general belief that above categories are in the ratio 4:3:2:1 respectively. (1+4=5)

(b) Two dice are thrown simultaneously. Let X be the random variable denoting the sum of the two faces obtained. Write the distribution of X and find the mean. (3+1=4)

(c) Determine

(i) marginal distributions of x and y

(ii) $E(x)$, $E(y)$ and $E(x, y)$ and

(iii) $h(x/y = 1)$ for the following joint probability distribution. (2+3+1=6)

x/y	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

7. (a) Find the unique fixed probability vector for the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$. (4)

(b) Define regular stochastic matrix. Test whether the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$ is regular stochastic or not. (1+4=5)

(c) The diameter of an electric cable is assumed to be a continuous random variable x with probability density function $f(x) = 6x(1-x)$, $0 \le x \le 1$. Determine b such that $P[x < b] = P[x > b]$. (4)

(d) Write the transition matrix for the following diagram. (2)