16 (CS 775) GRTH

2019

GRAPH THEORY

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

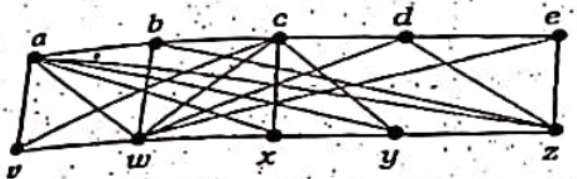
*Answer **any ten** questions.*

1. (a) Define a graph. Calculate the total number of graphs with n vertices. When a graph is said to be a complete graph and what is the number of edges in such a graph ? 5

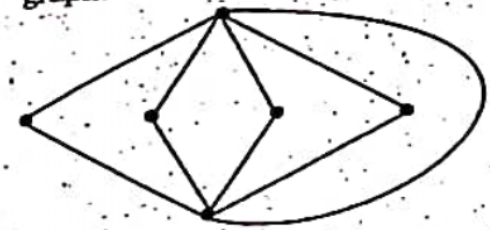
(b) Draw the graph with the vertex set $S = \{2, 3, 4, 7, 11, 13\}$ where two vertices i and j are adjacent if either $i+j$ or $|i-j|$ is in S. Calculate the diameter. 5

3. (a) Define a matching in a graph. When is a matching called a perfect one? Give an example of a connected non-Hamiltonian graph having two disjoint perfect matchings. 5

(b) State Hall's theorem and verify it for the following graph: 5



(c) Define a clique and an independent set. Determine the clique number and the independence number for the following graph. 5



(d) A certain graph has order 14 and size 27. The degree of each vertex is 3, 4 or 5. There are six vertices of degree 4. How many vertices have degree 3 and how many have degree 5? 5

4. (a) When is a graph Hamiltonian? Show that if a graph is Hamiltonian then for any subset S of vertices $\omega(G-S) \leq |S|$. 5

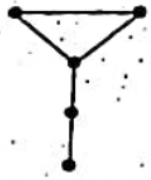
(b) Define an Eulerian graph. Only one graph of order 5 has the property that addition of any edge produces an Eulerian graph. What is it? 5

5. (a) Show that in a party with six persons either there are three mutual friends or three mutual strangers. 5

(b) Define a bipartite graph. Show that a graph is bipartite if and only if it has no odd cycle. 5

6. (a) When is a sequence said to be graphical? Check whether the sequence 5, 3, 3, 3, 3, 2, 2, 2, 1 is graphical. If so construct the corresponding graph. 5

(b) Find the adjacency matrix A of the following graph and then compute A^2 and A^3 without computing matrix multiplication. 5

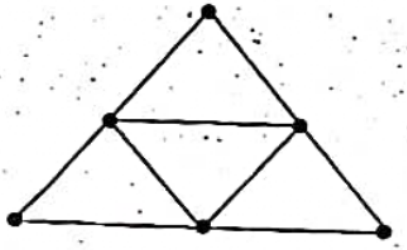


7. (a) Give an example of *two* graphs having same order, same size and same degree sequence such that the graphs are not isomorphic. 5

(b) Count the number of distinct trees with n vertices. 5

8. (a) Show that every n vertex graph with m edges has at least $m-n+1$ cycles. 5

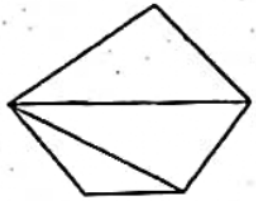
(b) Find all the spanning trees of the following graph: 5



9. (a) A connected r-regular graph of order 12 is embedded in the plane resulting in eight regions. What is r? Explain. 5

(b) Define the chromatic number of a graph. What is the chromatic number of (i) a tree, (ii) the Petersen graph? 5

10. (a) Find the chromatic polynomial of the following graph: 5



(b) What is a planar graph and what is its dual? Describe how to obtain the dual of a planar graph. 5

11. (a) Define a binary tree. Determine the maximum and minimum height of a binary tree with 41 vertices. 5

(b) Describe an algorithm to determine the connectedness of a graph. 5

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