2

Total number of printed pages-7

4 SEM BE (R) DM

2018

(May-June)

DISCRETE MATHEMATICS

Full Marks : 100

Pass Marks : 35

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer question no. **1** *and* ***any four*** *from the rest.*

1. $10 \times 2 = 20$

(i) If $P = \{1, 2, 3\}$, $Q = \{2, 3, 4\}$ then find the set $\{(x,y) : (x,y) \in P \times Q \text{ and } x \geq y\}$.

(ii) The relation R defined on set of integers Z as $R = \{(a,b) : a,b \in Z, ab > 0\}$. Show that R is an equivalence relation.

Contd.

(iii) Show that $f : R \to R$ defined by $f(x) = 2x + 3$ is a bijection.

(iv) Show that the operation $*$ defined by $a * b = \dfrac{a+b}{5}$ is not a binary relation on the set of negative integers.

(v) Write all the subgroups of the multiplicative group $\{1, -1, i, -i\}$.

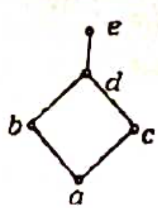
(vi) If x is an element of a group G and $O(x) = 5$ then what is the order of x^{15} ?

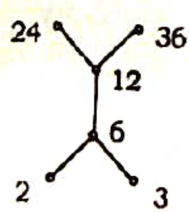
(vii) Find the inverse of the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$$

(viii) Prove that ring of integers $(Z, +, \cdot)$ is not a field.

(ix) Which of the following is not a lattice?





(x) Translate the following statement into symbolic form :

"The crop will be destroyed if there is flood."

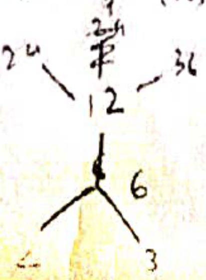
2. (a) Prove that : $3\times2=6$

(i) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

(ii) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

(b) Let $R=\{(x,y): x, y\in Z$ and $x-y$ is divisible by $5\}$, Z is the set of integers. Show that R is an equivalence relation. Also find all the distinct equivalence classes of R. $4+2=6$

(c) Let $X=\{2, 3, 6, 12, 24, 36\}$ and $x\leq y$ if x divides y for all $x\in X$. Show that $(X,\leq)$ is a partial ordered set. Find least upper bound and greatest lower bound of the set $A=\{2, 3, 6\}$. Also draw Hasse diagram for $(X,\leq)$. $3+2+1=6$



(d) Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=x^2+3$, $g(x)=x+6$. Find the composite function $f\circ g$. 2

3. (a) Show that the set of all non-zero real numbers namely $R-\{0\}$ forms an abelian group with respect to operation $*$ defined by $a*b=\frac{ab}{2}\ \forall\, a,b\in R-\{0\}$. INV 6

(b) Show that identity element of a group is unique. 4

(c) Let $(G,*)$ be a group and $a,b\in G$. Show that

(i) $(a^{-1})^{-1}=a$

(ii) $(a*b)^{-1}=b^{-1}*a^{-1}$

3+3=6

(d) Give an example of a finite non cyclic abelian group. 4

4. (a) Prove that the order of each subgroup of a finite group is a divisor of the order of the group. 5

(b) Show that intersection of two subgroups is again a subgroup. Give a counter example to show that union of two subgroups may not be a group. 4+1=5

(c) Prove that every subgroup of an abelian group is normal subgroup. 4

(d) If ϕ is a homomorphism of a group G into a group G', then show that

(i) $\phi(e) = e'$

(ii) $\phi(a^{-1}) = [\phi(a)]^{-1}$

where e and e' are identities of G and G' respectively. 3+3=6

5. (a) Show that the set $R = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring with respect to $+_6$ and $\times_6$ as the ring operations. Verify that R is a ring with zero divisor. 6+1=7

(b) Define an integral domain. Show that every field is an integral domain. 1+4=5

(c) If $(R, +, \cdot)$ is a ring such that $a^2 = a \,\forall\, a \in R$, prove that

(i) $a + a = 0 \quad \forall\, a \in R$

(ii) $a + b = 0 \Rightarrow a = b$

(iii) R is a commutative ring. 2+1+2=5

(d) Write all the axioms of a Boolean algebra. 3

6. (a) Construct truth table for $(p \wedge q) \wedge r$. 4

(b) Define tautology and contradiction. Show that $(p \wedge q) \wedge \sim (p \vee q)$ is a contradiction. 1+4=5

(c) Show that 5

(i) $\sim (p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) \Leftrightarrow (\sim p \vee q)$

(ii) $(p \vee q) \wedge (\sim p \wedge (\sim p \wedge q)) \Leftrightarrow (\sim p \wedge q)$

(d) Find the principal conjuctive and principal disjunctive normal form of $(\sim p \rightarrow r) \vee (q \leftrightarrow p)$. 6