Total No. of printed pages = 4

MA 181202

Roll No. of candidate | | | | | | | | | | |

2022

B.Tech. 2nd Semester End-Term Examination

MATHEMATICS — II

(New Regulation & New Syllabus (Group B))

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. Choose the appropriate answers : (10 × 1 = 10)

(i) The directional derivative of $f = xy + yz + zx$ in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$ at the point $(1, 2, 0)$ is

(a) $\frac{10}{3}$ (b) $\frac{1}{2}$

(c) $-\frac{10}{3}$ (d) None of these

(ii) The tangent plane to the surface $xz^2 + x^2y - z + 1 = 0$ at the point $(1, -3, 2)$ is

(a) $2x + y + 3z + 1 = 0$ (b) $2x - y - 3z + 1 = 0$

(c) $-2x + y + 3z + 1 = 0$ (d) None of these

(iii) The magnitude of the vector drawn in a perpendicular to the surface $x^2 + 2y^2 + z^2 = 7$ at the point $(1, -1, 2)$ is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) 6 (d) None of these

(iv) The general solution of the differential equation $py = p^2(x - b) + a$ is

(a) $y^2 = 4a(x - b)$ (b) $cy = c^2(x - b) + a$

(c) $y = (x - b) + a$ (d) None of these

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(v) The integrating factor of the differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$ is

(a) $\tan x$ (b) $\cos x$

(c) $\sin x$ (d) $\sec x$

(vi) A particular solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$ when $x=0,\ y=3$

(a) $y = Ae^x + Be^{-2x}$ (b) $y = 2e^x + e^{-2z}$

(c) $y = ce^x + (3-c)e^{-2x}$ (d) None of these

(vii) The integral $\int x J_0(x)\, dx$ is equal to

(a) $x J_1(x) - J_0(x)$ (b) $x J_1(x)$

(c) $J_1(x)$ (d) None of these

(viii) The function $f(z) = \bar{z}$ is

(a) analytic at the origin.

(b) analytic at all points in the complex plane

(c) not analytic at any point in the complex plane

(d) analytic at finite number of point in the complex plane

(ix) The Cauchy-Riemann equation in polar form is given by

(a) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta},\ \frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$

(b) $\frac{\partial u}{\partial r} = -\frac{1}{r}\frac{\partial v}{\partial \theta},\ \frac{\partial u}{\partial \theta} = r\frac{\partial v}{\partial r}$

(c) $\frac{\partial u}{\partial r} = r\frac{\partial v}{\partial \theta},\ \frac{\partial u}{\partial \theta} = -\frac{1}{r}\frac{\partial v}{\partial r}$

(d) None of these

(x) Let $f(z)$ is analytic at all points interior to a rectifiable non-intersecting closed curve C except at the points at $z=1$ and $z=3$, the residues of $f(z)$ at these are -1 and 2 then the value of $\oint_C f(z)\, dz$ is equal to

(a) $4\pi i$ (b) $2\pi i$

(c) $8\pi i$ (d) 2π

2. (a) A particle moves along the curve $x=2t^2$, $y=t^2-4t$ and $z=3t-5$ where t is the time. Find the components of velocity and acceleration at time $t=1$ in the direction of $\hat{i}-3\hat{j}+2\hat{k}$. (4)

(b) Find a unit vector normal to the surface $x^3+y^3+3xyz=3$ at the point $(1,2,-1)$. (4)

(c) Find the work done in moving a particle in the force field $\vec{F}=3x^2\hat{i}+(2xz-y)\hat{j}+2\hat{k}$ along the curve defined by $x^2=4y$ and $3x^3=8z$ from $x=0$ to $x=2$. (4)

(d) Show that the vector $\bar{v}=(x+3y)\hat{i}+(y-3z)\hat{j}+(x-2z)\hat{k}$ is solenoidal. (3)

3. (a) Find the differential equation of all circles touching the axis of y at the origin and centres on the axis of x. (3)

(b) Solve any 3 (three) of the following equations : (3 × 4 = 12)

(i) $x(1-x^2)\dfrac{dy}{dx}+(2x^2-1)y=x^3$

(ii) $x\dfrac{dy}{dx}+y=x^3y^6$

(iii) $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\dfrac{x}{y}\right)dy=0$

(iv) $y=2px+y^2p^3$

(v) $(px-y)(py+x)=2p$.

4. (a) Evaluate $\int_S \vec{A}.\hat{n}\,ds$ where $\vec{A}=(x+y^2)\hat{i}-2x\hat{j}+2yz\hat{k}$ and S is the surface of the plane $2x+y+2z=6$ included in the first octant. (6)

(b) Use divergence theorem of Gauss to evaluate $\int_S \vec{F}.\hat{n}\,ds$ where $\vec{F}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$. (6)

(c) Solve the differential equation

$y=2px-p^2$ (3)

5. (a) Show that the function $f(z)=\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ when $z\neq 0$ and $f(0)=0$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied at the origin. (5)

(b) Prove that $u=x^3-3xy^2$ is a harmonic function. Determine its harmonic conjugate and then find the corresponding analytic function $f(z)$ in terms of z. (5)

(c) Under the transformation $w=\dfrac{1}{z}$, find the image of $|z-2i|=2$. (5)

6. (a) State Cauchy's Integral formula and hence evaluate $\oint_C \dfrac{e^{2x}}{(z+1)^4}$ where C is the circle $|z|=2$. (1+4=5)

(b) Expand $f(z)$ using Laurent's series valid for the following regions where

$$f(z)=\frac{1}{(z+1)(z+3)}$$

(i) $|z|>3$ (ii) $0<|1+z|<2$ (5)

(c) Solve the differential equation $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}+6y=e^x\cos 2x$. (5)

7. (a) Find the power series solution of $(1-x^2)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=0$ about $x=0$. (7)

(b) Evaluate any *two* of the following integrals: (2 × 4 = 8)

(i) $\oint_C \dfrac{e^x}{\cos \pi z}\,dz$ where C is the circle $|z|=1$

(ii) $\displaystyle\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$

(iii) $\displaystyle\int_0^{\infty}\frac{dx}{x^4+1}$.