Total No. of printed pages = 7

MA 181102

Roll No. of candidate

2019

B.Tech. 1st Semester End-Term Examination

MATHEMATICS – I

(New Regulation (w.e.f. 2017 – 2018)) and

(New Syllabus – (Group – A) (w.e.f. 2018 – 2019))

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. (A) Choose the appropriate answers : $(8 \times 1 = 8)$

(i) $\int_0^{\frac{\pi}{2}} \frac{(n-1)(n-3)(n-5)....4.2}{n(n-2)(n-4)....5.3}$ when n is

(a) an integer

(b) a real number

(c) a positive odd number greater than 1

(d) an even number not equal to zero

[Turn over

(ii) The volume of solid generated by revolution of area bounded by the curve $r = f(\theta)$ and the radii vectors $\theta = \alpha$, $\theta = \beta$ about the line $\theta = \frac{\pi}{2}$ is

(a) $\int_{\alpha}^{\beta} \frac{2}{3} \pi r^3 \sin\theta \, d\theta$

(b) $\int_{\alpha}^{\beta} \frac{2}{3} \pi r^2 \sin\theta \, d\theta$

(c) $\int_{\alpha}^{\beta} \frac{2}{3} \pi r^2 \cos\theta \, d\theta$

(d) $\int_{\alpha}^{\beta} \frac{2}{3} \pi r^3 \cos\theta \, d\theta$

(iii) If $y = e^{ax}$ then n^{th} derivative of y is equal to

(a) $a^n e^{ax}$

(b) a^n

(c) e^{ax}

(d) $\frac{e^{ax}}{a^n}$

(iv) $\lim_{x \to 0} \frac{1 - \cos x}{\sin x}$ is equal to

(a) 0

(b) 1

(c) –1

(d) 2

(v) If $f(x, y) = 0$ then $\frac{dy}{dx}$ is equal to

(a) $\left.\frac{\partial f}{\partial x}\middle/\frac{\partial f}{\partial y}\right.$

(b) $\left.\frac{\partial f}{\partial y}\middle/\frac{\partial f}{\partial x}\right.$

(c) $\left.-\frac{\partial f}{\partial y}\middle/\frac{\partial f}{\partial x}\right.$

(d) $\left.-\frac{\partial f}{\partial x}\middle/\frac{\partial f}{\partial y}\right.$

(vi) Which one of the following matrices is a singular matrix?

(a) $\begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 2 \\ 3 & 6 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$

(vii) If a square matrix has an eigen value λ then the eigen value of $(kA)^T$ where $k \neq 0$ is scalar is

(a) $\frac{\lambda}{k}$

(b) $\frac{k}{\lambda}$

(c) $k\lambda$

(d) λ^k

(viii) $\int_a^b \int_c^d \int_e^f dz\, dy\, dx$ is equal to

(a) $a+b+c+d+e+f$

(b) $abcdef$

(c) $(b-a)(d-c)(f-e)$

(d) $(a+b)(c+d)(e+f)$

(B) Fill in the blanks : (2 × 1 = 2)

(i) The series $1+\frac{1}{2^{7/5}}+\frac{1}{3^{7/5}}+\frac{1}{4^{7/5}}+....$ is __________.

(ii) If n is a positive integer then $\Gamma(n+1)=$ __________.

2. (a) If $f_n = \int_0^{\frac{\pi}{2}} x^n \sin x\, dx$ the show that $f_n + n(n-1)f_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$. Hence, evaluate $\int_0^{\frac{\pi}{2}} x^4 \sin x\, dx$. (4 + 1 = 5)

(b) Find the whole area of the curve $a^2y^2 = x^3(2a - x)$. (5)

(c) Find the surface area of the solid of revolution of the asteroid $x = a\cos^3 t$, $y = a\sin^3 t$ about x-axis. (5)

3. (a) If $y = a\cos(\log x) + b\sin(\log x)$ show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (5)

(b) Evaluate (any one) : (2)

(i) $\lim\limits_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$

(ii) $\lim\limits_{x \to \infty} \dfrac{x^4}{e^x}$.

(c) Find Lagrange form of reminder in the expansion of $e^x \cos x$. (4)

(d) Show that radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$ is $\dfrac{-8\sqrt{2}}{3a}$. (4)

4. (a) Test the convergence of the series $\dfrac{1}{\sqrt{1.2}} + \dfrac{1}{\sqrt{2.3}} + \dfrac{1}{\sqrt{3.4}} + \ldots$ to ∞. (4)

(b) Find the Fourier series for the function $f(x) = x - x^2$, $-\pi < x < \pi$. Hence show that $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \ldots = \dfrac{\pi^2}{12}$. (6 + 2 = 8)

(c) Expand $f(x) = 2x - 1$ in a half range sine series in $0 < x < 1$. (3)

5. (a) State and prove Euler's theorem on homogeneous functions. (1 + 4 = 5)

(b) If $Z = f(x, y)$ where $x = e^u \cos v$ and $y = e^u \sin v$ show that $y \frac{\partial z}{\partial u} + x \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial y}$. (4)

(c) Find the inverse of the matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ by applying elementary row transformation. (4)

(d) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$. (2)

6. (a) Solve the following system of linear equations (4)

$$2x + 3y + 4z = 11$$
$$x + 5y + 7z = 15$$
$$3x + 11y + 13z = 25$$

(b) Show that the vectors $X_1 = (1, 0, 2, 1)$, $X_2 = (3, 1, 2, 1)$, $X_3 = (4, 6, 2, -4)$, $X_4 = (-6, 0, -3, -4)$ are linearly dependent. Also find the relation between them. (4 + 1 = 5)

(c) Evaluate the following integral by changing the order of integration $\int_0^2 \int_y^2 e^{x^2} \, dx \, dy$. (3)

(d) Evaluate : $\int_0^\infty x^4 e^{-x^2} \, dx$. (3)

7. (a) Evaluate : $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ over the region bounded by the co-ordinate planes and the plane $x+y+z=1$. (5)

(b) Find all the stationary points of the function $f(x,y)=x^3+y^3+3xy$. Also examine for the maximum and minimum values of $f(x,y)$. (5)

(c) Verify Cayley – Hamilton theorem for the matrix (5)

$$A=\begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$