Total No. of printed pages = 4

CSE 181401

Roll No. of candidate | | | | | | | | | | |

2023

B.Tech. 4th Semester End-Term Examination

DISCRETE MATHEMATICS

New Regulation (w.e.f. 2017-18) and

New Syllabus (w.e.f. 2018-19)

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and any *four* from the rest.

1. Choose the appropriate answer : (10 × 1 = 10)

(i) Which of the following statement is not true regarding two sets A and B

(a) $A - B \subseteq A$ (b) $A - \varnothing = A$

(c) $(A - B) \cup A = A$ (d) $A - (A \cap B) = A \cap B$

(ii) The relation '$<$' on the set of all integers is

(a) Reflexive (b) Symmetric

(c) Transitive (d) None of these

(iii) $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 2x + 3$ is a bijective mapping

(a) Yes (b) No

(iv) Which one of the following is not countable?

(a) The set of integers

(b) The interval $[a, a]$

(c) The set of natural numbers

(d) The interval $[a, b]$ where a and b are distinct real numbers

(v) The number of generators of the cyclic group $G = (1, -1, i, -i)$ with respect to multiplication is

(a) 1 (b) 2

(c) 3 (d) none

[Turn over

(vi) In the group $(G, *)$ the value of $\left(a^{-1} b\right)^{-1}$ is

(a) ab^{-1} (b) $a^{-1} b$

(c) $b^{-1} a$ (d) none of these

(vii) If A and B are two subrings of a ring C then

(a) $A \cup B$ is a subring (b) $C - A$ is a subring

(c) $C - A \cap B$ is a subring (d) $A \cap B$

(viii) The proposition $\neg (p \wedge q)$ is equivalent to

(a) $p \vee q$ (b) $\neg p \wedge \neg q)$

(c) $\neg p \vee \neg q)$ (d) None of the above

(ix) Three persons entered into a railway compartment. If there are 5 vacant seat, in how many ways can they take these seats?

(a) 60 (b) 20

(c) 15 (d) None of these

(x) Suppose that a person deposits Rs.10,000 in a saving account at a bank yielding 11% interest per year compounded annually. How much amount will the person have in the account after n years?

(a) $a_n = 0.11\, a_{n-1}$ (b) $a_n = 1.11\, a_{n-1}$

(c) $a_n = 11.1\, a_{n-1}$ (d) $a_n = 1.10\, a_{n-1}$

2. (a) Prove that for any three sets A, B, C (3+3=6)

(i) $(A \cap B)^c = A^c \cup B^c$

(ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(b) (i) Let R and S are two relations from A to B then show that (3+3=6)

$(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

(ii) Let $\mathbb{Z}$ be the set of integers and a relation R is defined in $\mathbb{Z}$ as $R = \{(x, y) : x \equiv y \pmod{m}, m \text{ is positive integer}\}$. Prove that R is an equivalence relation.

(c) Let $f : \mathbf{R} \to \mathbf{R}$, $g : \mathbf{R} \to \mathbf{R}$ are defined by $f(x) = 3x + 4$, $g(x) = x^2 - 2$. Find the formulae which defines $f \circ g$ and $g \circ f \circ g$. (3)

3. (a) Apply Principle of Mathematical Induction to prove that $1.2+2.3+3.4+\ldots\ldots+n(n+1)=\frac{n(n+1)(n+2)}{3}$. (5)

(b) Among the first 1000 positive integers determine (3+3=6)

(i) the number of integers which are not divisible by 5, nor by 7, nor by 9.

(ii) the number of integers divisible by 5, but not by 7, not by 9.

(c) State Pigeonhole principle and apply the same to prove that at least two people out of 13 people assembled in a room must have their birthdays in the same month. (1+3=4)

4. (a) Prove that the statement $(p \to q) \leftrightarrow (\neg q \to \neg p)$ is a tautology. (5)

(b) Find equivalent formula for $p \wedge (q \leftrightarrow r)$ which contains neither conditional nor biconditional. Also find its dual. (4+1=5)

(c) Obtain principal disjunctive normal form of (5)

$(\neg p \vee \neg q) \to (\neg p \wedge r)$.

5. (a) What is a monoid? Give an example of a semigroup which is not a monoid. (1+2=3)

(b) Show that $G = Q - \{1\}$ where Q is the set of rational numbers forms a group under the operation $*$ defined by $a * b = a + b - ab$, $a, b \in G$. (4)

(c) Prove that intersection of two subgroup is a subgroup. Also give an example to show that union of two subgroup may not be a subgroup. (4+1=5)

(d) If $f : G \to G'$ is a homomorphism then prove that $f(e) = e'$, where e and e' are identity element of G and G'. (3)

6. (a) Prove that order of each subgroup of a finite group is a divisor of the order of the group. (5)

(b) Write all the postulates of the algebraic structure Ring. Also prove that in a ring $(R, +, \circ)$. (4+2=6)

(i) $a \circ 0 = 0 \circ a = 0$ for all a in R

(ii) $(-a) \circ (b) = a \circ (-b) = -(a \circ b)$

(c) Let R and R' are two rings and $f : R \to R'$ is a homomorphism. Then show that $Ker f$ is an ideal of R. (4)

7. (a) Let the set of all factors of 70, $D_{70} = \{1, 2, 5, 7, 10, 14, 35, 70\}$ is Boolean Algebra with respect to operations $\vee$, $\wedge$, $'$ defined by $a \vee b = LCM$ of a, b; $\wedge b = HCF$ of a, b ; $a' = \frac{70}{a}$. (4)

(b) Discuss the validity of the following arguments: (6)

All educated person are well behaved.

Ram is educated.

No well behaved person is quarrelsome.

Therefore, Ram is not quarrelsome.

(c) Show that every field is an integral domain. Show by an example that converse is not true. (4+1=5)