Total No. of printed pages = 8

MA 181102

Roll No. of candidate | | | | | | | | | | |

2018

B.Tech. 1st Semester End-Term Examination

MATHEMATICS – I

(w.e.f. 2017-2018) (New Regulation)
(New Syllaabus) (w.e.f. 2018-2019)

(Group – A)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Q.No. 1 and any *four* from the rest.

1. $(10 \times 1 = 10)$

(A) Choose the most appropriate one:

(i) The value of $\int_0^{\pi/2} \sin^7 x \, dx$ is equal to

(a) $\frac{35}{16}$

(b) $\frac{16}{35}$

(c) $\frac{1}{35}$

(d) $\frac{1}{16}$

[Turn over

(ii) The area enclosed by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ is

(a) $\int_a^b y\,dx$

(b) $\int_a^b x\,dy$

(c) $\int_a^b x\,dx$

(d) $\int_a^b y\,dy$

(iii) The value of $\overline{|(1)}$ is equal to

(a) 0

(b) 1

(c) -1

(d) none of these

(iv) Taylor's series expansion of $y = \frac{1}{x}$ about $x = 1$ is equal to

(a) $1-(x-1)+(x-1)^2-(x-1)^3+....$

(b) $1+(x-1)+(x-1)^2+(x-1)^3+....$

(c) $1-(x-1)+\frac{(x-1)^2}{2!}-\frac{(x-1)^3}{3!}+...$

(d) None of these

(v) If a series $\Sigma\mu_n$ is convergent then

(a) $\lim_{n\to\infty} \mu_n = 0$

(b) $\lim_{n\to\infty} \mu_n \neq 0$

(c) $\lim_{n\to\infty} \mu_n = 1\alpha$

(d) none of these

(vi) $f(x,y) = \frac{x+y}{\sqrt{x}+\sqrt{y}}$ is a homage neous function of degree

(a) 1

(b) 1/2

(c) 2

(d) none of these

(vii) For a system of non-homogeneous linear equation $AX = B$ if $\rho[A:B] \neq \rho(A)$ then

(a) the system is consistent

(b) the system is inconsistent

(c) the system has a unique solution

(d) the system has an infinite number of solutions

(viii) For a non-singular square matrix A,

(a) $A^{-1} = \frac{adj\ A}{|A|}$

(b) $A^{-1} = \frac{|A|}{adj\ A}$

(c) $A^{-1} = |A|\ adj\ A$

(d) none of these

(B) Fill in the blanks.

(ix) The expansion of the function $\cos x$ upto four non-zero terms $\cos x = \ldots.$

(x) A square matrix A is orthogonal if $AA' = A'A =$ _____.

2. Answer the following :

(a) Find the reduction formula for $\int \sin^m x \cos^n x \, dx$. (4)

(b) Prove that $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ if $f(x)$ is an even function of x. (2)

(c) Find the volume and the surface area of the solid generated by the revolution of the cardioid $r = a(1-\cos\theta)$ about the initial line. (5)

(d) (i) Evaluate $\int_{0}^{\pi/4} \tan^5 x \, dx$. (4)

Or

(ii) Find the area induced between the curve $xy^2 = a^2(a-x)$ and its asymptote.

3. Answer the following :

(a) If $y = \sin 2x \sin 3x$ find y_n (2)

(b) If $y = \sin\left(m \sin^{-1} x\right)$ then show that $\left(1-x^2\right)y_{x+2} - (2n+1)\,xy_{n+1} - \left(n^2 - m^2\right)y_n = 0$. (4)

(c) Expand $\log x$ in powers of $(x-1)$ by Taylor's theorem. (3)

(d) Evaluate (any one). (3)

(i) $\lim_{x \to 0} \sin x \log x$

(ii) $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$.

(e) (i) Find the radius of curvature of the curve $y = a \log \sec\left(\frac{x}{a}\right)$ at any point (x, y). (3)

Or

(ii) Discuss the convergence of the series $\Sigma \frac{n!}{n^n}$.

4. Answer the following :

(a) Find the Fourier series for the function $f(x) = x^2, -\pi \leq x \leq \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \frac{\pi^2}{6}. \qquad (7+2 = 9)$$

(b) Expand $f(x) = x$ as a half range .

(i) sine series in $0 < x < 2$.

(ii) cosine series in $0 < x < 2$. (6)

Or

(iii) If $u = f(r)$, where $r^2 = x^2 + y^2$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(x) + \frac{1}{r} f'(r).$$

5. Answer any *Three* from the following : $(3 \times 5 = 15)$

(a) If $u = \sin^{-1}(x/y) + \tan^{-1}(y/x)$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

(b) Apply Lagrange's method of multipliers to find the minimum value of $x^2 + y^2 + z^2$ under the condition $x + y + z = 12$.

(c) Evaluate $\iint_R y\,dx\,dy$, where R is the region bounded by the parabola $y^2 = 4x$ and $x^2 = 4y$.

(d) Find the Fourier series for the function $f(x) = x + x^2, \pi < x < \pi$.

6. Answer the following :

(a) Define orthogonal matrix. For any two orthogonal matrices A and B of the same size, show that AB is an orthogonal matrix. (2)

(b) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}. \quad (4)$$

(c) For what value of λ and μ do the system of equations. (4)

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + yz = \mu$

have

(i) No solution

(ii) Unique solution

(iii) More than one solution.

(d) (i) Find the eigen values and eigen vectors of the matrix. (5)

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

Or

(ii) If $u = x\log(xy)$ where $x^3 + y^3 + 3xy = 1$ find $\frac{dy}{dx}$.

7. Answer any THREE from the following :

(a) (i) Obtain a reduction formula for

$$\int (ax^2 + bx + c)^n dx. \quad (3)$$

(ii) Show that $\left|\frac{1}{2} = \sqrt{\pi}\right.$. (2)

(b) (i) Expand the function $\sin x$ in powers of x in infinite series. (3)

(ii) Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$. (2)

(c) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\, dy\, dx}{\sqrt{1-x^2-y^2-z^2}}$. (5)

(d) Verify Cayley -Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence compute A^{-1}. (5)