Total No. of printed pages = 4

**MA 181102**

Roll No. of candidate | | | | | | | | | | |

**2021**

**B.Tech. 1st Semester End-Term Examination**

**MATHEMATICS-I**

**New Regulation (w.e.f 2017-18)**

**New Syllabus – Group A (w.e.f. 2018-19)**

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. Choose the appropriate answer : (10 × 1 = 10)

(i) The value of $\int_0^{\frac{\pi}{2}} \cos^6 x\, dx$ is equal to

(a) $\frac{15}{96}\pi$

(b) $\frac{96}{15}\pi$

(c) $\frac{1}{96}\pi$

(d) $15\pi$

(ii) The total volume of the solid of revolution generated by the curve $y = f(x)$ bounded between $x = x_0$ and $x = x_1$ is

(a) $\int_{x_0}^{x_1} x^2\, dx$

(b) $\int_{x_0}^{x_1} x^2\, dy$

(c) $\int_{x_0}^{x_1} y^2\, dx$

(d) $\pi \int_{x_0}^{x_1} y^2\, dx$

(iii) $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ is equal to

(a) $\frac{\pi}{2}$

(b) $\pi$

(c) $-\pi$

(d) 0

[Turn over

$y_1 =$

(iv) If $y = \log x$ then $n^{th}$ derivative of $y$ is

(a) $\dfrac{(-1)^n n!}{x^n}$ (b) $\dfrac{(-1)^{n-1}(n-1)!}{x^n}$

(c) $\dfrac{(-1)^n (n-1)!}{x^{n-1}}$ (d) None of these

(v) $f(x,y) = \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ is a homogeneous function of degree

(a) 1 (b) $\frac{1}{2}$

$1 - \frac{1}{2}$

$\frac{2-1}{2}$ $\frac{1}{2}$

(c) 2 (d) None of the above

(vi) If $f(x,y) = 0$ then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{\partial f}{\partial x} \Big/ \dfrac{\partial f}{\partial y}$ (b) $-\dfrac{\partial f}{\partial x} \Big/ \dfrac{\partial f}{\partial y}$

(c) $\dfrac{\partial f}{\partial y} \Big/ \dfrac{\partial f}{\partial x}$ (d) $-\dfrac{\partial f}{\partial y} \Big/ \dfrac{\partial f}{\partial x}$

(vii) $\int_a^b \int_c^d \int_e^f dz\,dy\,dx$ is equal to

(a) $a+b+c+d+e+f$ (b) $(b-a)(d-c)(f-e)$

(c) $abcdef$ (d) $(a+b)(c+d)(e+f)$

(viii) If $A$ is a non-singular $n \times n$ matrix then

(a) $\rho(A) = 0$ (b) $\rho(A) = n$

(c) $\rho(A) = -n$ (d) None of these

(ix) If $\lambda$ be an eigen value of a matrix $A$ then $\lambda^{-1}$ is an eigen value of

(a) $A$ (b) $\dfrac{|A|}{\lambda}$

(c) $A^{-1}$ (d) $\dfrac{\lambda}{|A|}$

(x) A system of homogeneous linear equation $AX = 0$ is always

(a) Consistent (b) Inconsistent

(c) Has a unique solution (d) Has an infinite number of solutions

2. (a) Find the reduction formula for $\int \sin^n x \, dx$. (5)

(b) If the cardioide $r = a(1-\cos\theta)$ is rotated about the initial line, find the area of the solid of revolution generated. (5)

(c) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$. (5)

3. (a) Find $y_n$ if $y = x^2 e^{ax}$. (3)

(b) If $y = \cos(\log x)$ prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2-1)y_n = 0$. (5)

(c) Evaluate any one (3)

(i) $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$

(ii) $\lim_{x \to 0} \left(\frac{1}{x^2}\right)^{\tan x}$.

(d) Find the radius of curvature of the cycloid (4)

$x = a(\theta + \sin\theta)$

$y = a(1-\cos\theta)$ at the point $\theta$.

4. (a) Find the Fourier series for the function $f(x) = x^2, -\pi \le x \le \pi$. Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \frac{\pi^2}{6}$ (6+2=8)

(b) Discuss the convergence of the series $\sum_{n=0}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$. (3)

(c) If $u = x^y$, show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial z}$. (4)

5. (a) If $u = f(r)$, where $r = x^2 + y^2$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. (5)

(b) If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (5)

(c) Expand $f(x) = x$ in a half range sine series in $0 < x < 2$. (3)

(d) If $A$ is an orthogonal matrix prove $|A| = \pm 1$. (2)

6. (a) Prove that the inverse of an orthogonal matrix is orthogonal and its transpose is also orthogonal. (2+2=4)

(b) Find the inverse of $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by using elementary row transformations. (4)

(c) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$. (4)

(d) If $u = x\varphi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x\varphi\left(\frac{y}{x}\right)$. (3)

7. (a) Examine for minimum and maximum values: $\sin x + \sin y + \sin(x+y)$. (5)

(b) Evaluate $\iint_R y\,dx\,dy$, where $R$ is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. (5)

(c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. (5)

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