Total number of printed pages–8

16 (MA 103) MATH-I

2017

MATHEMATICS-I

(New Syllabus)

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Group-A

(Differential Calculus)

Answer ***any ten*** *questions from this* **Group** *:*

$10 \times 5 = 50$

1. Find the nth derivative of $2\frac{1}{2}+2\frac{1}{2}=5$

   (i) $\sin(ax+b)$

   (ii) $\cos x \sin x$

Contd.

2. (a) If $y = \frac{1}{x^2 + b^2}$, find y_n 5

Or

(b) Find the nth derivative of $x^{n-1} \log x$. 5

3. If $y = e^{a \sin^{-1} x}$, show that,

$$(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 + a^2) y_n = 0$$

5

4. Expand e^x in the power of $(x - 1)$ upto four terms. Hence evaluate e^2 upto 3 decimal places. 3+2=5

5. Show that the sum of the intercepts on the axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = a$ is a constant. 5

6. Show that in the exponential curves $y = be^{x/a}$, the subtangent is of constant length and that the subnormal varies as the square of the ordinate. 5

7. *(a)* If ϕ be the angle between the radius vector and the tangent at any point $P(r,\theta)$ of the curve $r=f(\theta)$, then show that, $tan\phi=r\dfrac{d\theta}{dr}$ 5

OR

(b) For the curve $y=f(x)$, show that

$$\frac{ds}{dx}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$ 5

8. *(i)* What do you mean by polar subtangent and polar subnormal ?

(ii) For the cardioid $r=a(1-cos\theta)$, show that $\phi=\theta/2$.

2+3=5

9. Find the asymptotes of the curve,

$$(x+y)^2(x+2y)+2(x+y)^2-(x+9y)-2=0$$

5

10. *(a)* *(i)* Define radius of curvature of a curve $y=f(x)$, at $P(x,y)$.

(ii) Find the radius of curvature at the origin for the curve,

$$5x^3+7y^2x^2+4x^2y+xy^2+2x^2+3xy+y^2+4x=0$$

2+3=5

OR

(b) If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2=4ax$, then show that,

$$\rho_1^{-2/3}+\rho_2^{-2/3}=(2a)^{-2/3}$$ 5

11. If u is a homogeneous function of x and y of degree 'n', show that,

$$x^2\frac{\partial^2 y}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(n-1)\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right)$$

5

12. If $u=f(r,s,t)$ and $r=x/y$, $s=y/z$, $z=z/x$, show that, $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$ 5

13. (a) If $f(y-z,z-x,z-y)$, what is the value of $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$? 5

OR

(b) If $z = f(x,y)$ and

$x = e^{u} + e^{-v}$, $y = e^{-u} - e^{v}$, then prove

that, $\frac{\partial z}{\partial u} - x\frac{\partial z}{\partial x} = \frac{\partial z}{\partial v} - y\frac{\partial z}{\partial y}$. 5

14. If $u = x^2 - y^2, v = 2xy$ and .

$x = r\cos\theta, y = r\sin\theta$ then show that

$$\frac{\partial(u,v)}{\partial(r,\theta)} = 4r^3$$ 5

15. Examine the following function for extreme values, 5

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$$

Group-B

(Integral Calculus)

*Answer **any ten** questions from this **Group**.*

10×5=50

16. If $I_n = \int_0^{\pi/4} tan^n x \, dx$, prove that, 3+2=5

(i) $I_n + I_{n-2} = \frac{1}{n-1}$

(ii) $n(I_{n+1} + I_{n-1}) = 1$

17. If $U_n = \int_0^{\pi/2} x^n \sin^x dx$, prove that,

$$U_n + n(n-1)U_{n-2} = n(\pi/2)^{n-1}$$ 5

18. Evaluate : 3+2=5

(i) $\int_0^a \frac{x^4}{\sqrt{a^2 - x^2}} dx$

(ii) $\int_0^{\pi/2} \sin^8 x \cos^{10} x\, dx$

19. Prove that the length of the arc of the parabola, $y^2 = 4ax$ cut off by the latus rectum is, $2a\left[\sqrt{2} + \log\left(\sqrt{2}+1\right)\right]$ 5

20. Find the area bounded by the cardioid $r = a(1+\cos\theta)$ 5

21. Prove that the volume of the ellipsoid formed by the revolution of the ellipse $x^2/a^2 + y^2/b^2 = 1$ round its major axis is $4/3\pi ab^2$. 5

22. Find the surface area of the solid generated by revolving the cardiod, $r = a(1-\cos\theta)$ about the initial line. 5

23. Evaluate, $\int_0^1 dx \int_0^x e^{y/x}\, dy$ 5

24. (a) Evaluate, $\int_0^1 \int_0^{1-x} \int_0^{2-x} xyz\, dz\, dy\, dx$ 5

OR

(b) Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)\, dx\, dy\, dz$

5

25. (a) Evaluate $\iint_R (x+y)^2\, dx\, dy,$ where R is the parallelogram in the xy-plane with vertices (1,0), (3,1), (2,2), (0,1) using the transformation $u=x+y$ and $v=x-2y$. 5

OR

(b) Evaluate $\int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)} dx\, dy$ by changing to polar co-ordinates. 5

26. Find the area lying inside the circle $r=a\sin\theta$ and outside the cardioid $r=a(1-\cos\theta)$. 5

Contd.

27. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes, $y + z = 4$ and $z = 0$ by double integration. 5

28. Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x/a + y/b + z/c = 1$, $a, b, c \geq 0$. 5

29. *(a)* Show that,

$$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$$ and

use it to calculate $\int_0^{\pi/2} \sin^6\theta \, d\theta$.

4+1=5

OR

(b) Prove that, $\beta(m,n) = \dfrac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$ 5

30. Express the following integrals in terms of gamma functions, 2½+2½=5

(i) $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$

(ii) $\int_0^{\pi/2} \sqrt{\tan\theta} \, d\theta$