

Total No. of printed pages = 3

CSE 181405

Roll No. of candidate

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2023

B.Tech. 4<sup>th</sup> Semester End-Term Examination

GRAPH THEORY

(New Regulation (w.e.f. 2017-18)) & (New Syllabus (w.e.f. 2018-19))

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and any four from the rest.

1. Choose the most appropriate choice to answer the following : (10 × 1 = 10)

(i) A graph is a null graph if and only if it has

- (a) No cycle (b) No vertex  
(c) No edge (d) None of the above is correct answer

(ii) Adding one edge to a tree defines

- (a) No cycle (b) Exactly one cycle  
(c) At least one cycle (d) At most one cycle

(iii) Number of edges in a simple tree with N number of vertices is

- (a) N+1 (b) N  
(c) N-1 (d) None above

(iv) The maximum number of edges possible in a bipartite graph having 12 vertices is :

- (a) 24 (b) 144  
(c) 36 (d) 12

(v) A regular graph is a graph where each node has

- (a) degree one (b) zero degree  
(c) self loop (d) same degree

[Turn over

(vi) The complete graph with four vertices ( $K_4$ ) contains

- (a) 4 vertices and 4 edges      (b) 4 vertices and 16 edges  
(c) 4 vertices and 6 edges      (d) 4 vertices and 12 edges

(vii) A cycle having  $n$  vertices is a planar graph with chromatic number

- (a) 1      (b) 2  
(c) 3      (d)  $N$

(viii) The Number of odd degree vertices in a simple connected graph is

- (a) even      (b) odd  
(c) zero      (d) either odd or even

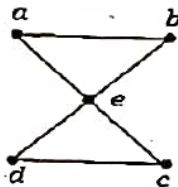
(ix) Number of spanning trees that can be formed from a complete graph with 4 vertices :

- (a) 12      (b) 8  
(c) 9      (d) 16

(x) The chromatic number of a tree with  $n$  ( $n > 2$ ) vertices is

- (a) 0      (b) 1  
(c) 2      (d)  $n$

2. Consider the following graph and answer the following questions :



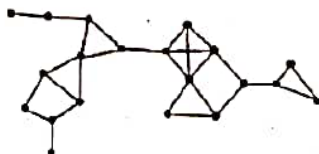
(a) Compute degree of each vertex. What is the longest path in the graph? Also represent the graph using adjacency list representation. (3+1+6=10)

(b) What is a directed graph? How do you define degree of vertices in case of a directed graph? (2+3=5)

3. (a) What is a bipartite graph? Give example. Show that a graph is bipartite if and only if it has no odd cycles. (4+6=10)

(b) "Each tree is a bipartite graph". Justify whether the statement is true or false with proof. (5)

4. Consider the following graph and answer each of the following questions with justification.



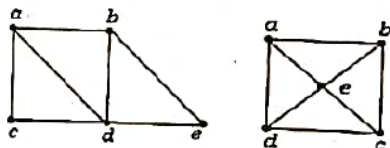
- (a) What is a cut vertex and what is a bridge? Find the total number of cut vertices in the graph (show all such cut vertices). (4+6=10)
- (b) "An edge 'e' of a connected graph G is a bridge if and only if 'e' is not on any cycle of G" Justify if the statement is true or false with proof. (5)

5. (a) Define a tree and a forest with examples. Prove that every two vertices of a tree have a unique path between them. (4+6=10)

- (b) State and explain briefly the Mengers theorem for disjoint paths in finite graph. (5)

6. (a) Define Eulerian graph and Hamiltonian graph. Prove that a connected multigraph has a Euler circuit if and only if each of its vertices has an even degree. (4+6=10)

- (b) Do the following graphs have Hamiltonian circuit? Justify. (5)



7. (a) What is a planar graph? State the four-color theorem of planar graph. Also state the Euler's Formula on connected planar graph. (2+3+2=7)
- (b) State briefly the Szemerédi's regularity lemma, in graph theory. (5)
- (c) Define matching for an undirected graph with an example. (3)