chapter one

MAGNETIC CIRCUITS

This book is concerned primarily with the study of devices that convert electrical energy into mechanical energy or the reverse. Rotating electrical machines, such as dc machines, induction machines, and synchronous machines, are the most important ones used to perform this energy conversion. The transformer, although not an electromechanical converter, plays an important role in the conversion process. Other devices, such as actuators, solenoids, and relays, are concerned with linear motion. In all these devices, magnetic materials are used to shape and direct the magnetic fields that act as a medium in the energy conversion process. A major advantage of using magnetic material in electrical machines is the fact that high flux density can be obtained in the machine, which results in large torque or large machine output per unit machine volume. In other words, the size of the machine is greatly reduced by the use of magnetic materials.

In view of the fact that magnetic materials form a major part in the construction of electric machines, in this chapter properties of magnetic materials are discussed and some methods for analyzing the magnetic circuits are outlined.

1.1 MAGNETIC CIRCUITS

In electrical machines, the magnetic circuits may be formed by ferromagnetic materials only (as in transformers) or by ferromagnetic materials in conjunction with an air medium (as in rotating machines). In most electrical machines, except permanent magnet machines, the magnetic field (or flux) is produced by passing an electrical current through coils wound on ferromagnetic materials.

1.1.1 i–H RELATION

We shall first study how the current in a coil is related to the magnetic field intensity (or flux) it produces. When a conductor carries current, a magnetic field is produced around it, as shown in Fig. 1.1. The direction of flux lines or magnetic field intensity H can be determined by what is known as the *thumb rule*, which states that if the conductor is held with the right

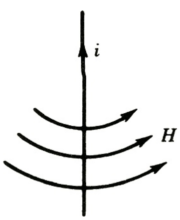


FIGURE 1.1 Magnetic field around a current-carrying conductor.

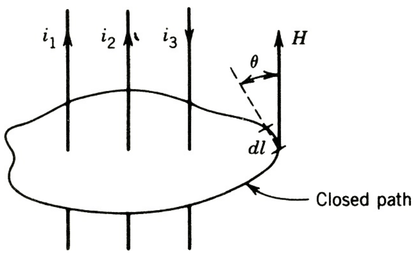


FIGURE 1.2 Illustration of Ampère's circuit law.

hand with the thumb indicating the direction of current in the conductor, then the fingertips will indicate the direction of magnetic field intensity. The relationship between current and field intensity can be obtained by using *Ampère's circuit law*, which states that the line integral of the magnetic field intensity H around a closed path is equal to the total current linked by the contour.

Referring to Fig. 1.2,

$$\oint H \cdot dl = \sum i = i_1 + i_2 - i_3 \tag{1.1}$$

where H is the magnetic field intensity at a point on the contour and dl is the incremental length at that point. If θ is the angle between vectors H and dl, then

$$\oint H \cdot dl \cos\theta = \sum i \tag{1.2}$$

Now, consider a conductor carrying current i as shown in Fig. 1.3. To obtain an expression for the magnetic field intensity H at a distance r from the conductor, draw a circle of radius r. At each point on this circular contour, H and dl are in the same direction, that is, $\theta = 0$. Because of symmetry, H will be the same at all points on this contour. Therefore, from Eq. 1.2,

$$\oint H \cdot dl = i$$

$$H\,2\pi r = i$$

$$H = \frac{i}{2\pi r} \tag{1.2a}$$

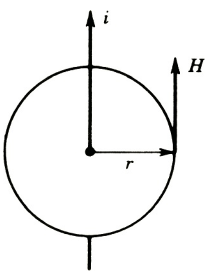


FIGURE 1.3 Determination of magnetic field intensity H due to a current-carrying conductor.

1.1.2 *B–H* RELATION

The magnetic field intensity H produces a magnetic flux density B everywhere it exists. These quantities are functionally related by

$$B = \mu H \text{ weber/m}^2 \quad \text{or} \quad \text{tesla} \tag{1.3}$$

$$B = \mu_r \mu_0 H \text{ Wb/m}^2 \quad \text{or} \quad \text{T} \tag{1.4}$$

where μ is a characteristic of the medium and is called the *permeability* of the medium
μ_0 is the permeability of free space and is $4\pi 10^{-7}$ henry/meter
μ_r is the *relative permeability* of the medium

For free space or electrical conductors (such as aluminum or copper) or insulators, the value of μ_r is unity. However, for ferromagnetic materials such as iron, cobalt, and nickel, the value of μ_r varies from several hundred to several thousand. For materials used in electrical machines, μ_r varies in the range of 2000 to 6000. A large value of μ_r implies that a small current can produce a large flux density in the machine.

1.1.3 MAGNETIC EQUIVALENT CIRCUIT

Figure 1.4 shows a simple magnetic circuit having a ring-shaped magnetic core, called a *toroid*, and a coil that extends around the entire circumference. When current i flows through the coil of N turns, magnetic flux is mostly confined in the core material. The flux outside the toroid, called *leakage flux*, is so small that for all practical purposes it can be neglected.

Consider a path at a radius r. The magnetic intensity on this path is H and, from Ampère's circuit law,

$$\oint H \cdot dl = Ni \tag{1.5}$$

$$Hl = Ni \tag{1.5a}$$

$$H\, 2\pi r = Ni \tag{1.6}$$

The quantity Ni is called the *magnetomotive force* (*mmf*) F, and its unit is ampere-turn.

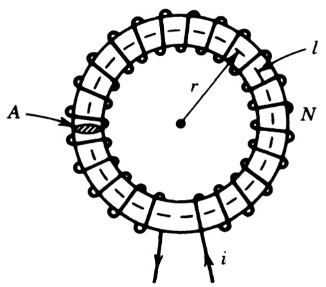


FIGURE 1.4 Toroid magnetic circuit.

$$Hl = Ni = F \tag{1.7}$$

$$H = \frac{N}{l} i \text{ At/m} \tag{1.8}$$

From Eqs. 1.3 and 1.8

$$B = \frac{\mu Ni}{l} \text{ T} \tag{1.9}$$

If we assume that all the fluxes are confined in the toroid—that is, there is no magnetic leakage—the flux crossing the cross section of the toroid is

$$\Phi = \int B\,dA \tag{1.10}$$

$$\Phi = BA \text{ Wb} \tag{1.11}$$

where B is the average flux density in the core and A is the area of cross section of the toroid. The average flux density may correspond to the path at the mean radius of the toroid. If H is the magnetic intensity for this path, then from Eqs. 1.9 and 1.11,

$$\begin{aligned}\Phi &= \frac{\mu Ni}{l} A = \frac{Ni}{l/\mu A} \\ &= \frac{Ni}{\mathscr{R}} \end{aligned} \tag{1.12}$$

$$= \frac{F}{\mathscr{R}} \tag{1.13}$$

where

$$\mathscr{R} = \frac{l}{\mu A} = \frac{1}{P} \tag{1.14}$$

is called the *reluctance* of the magnetic path, and P is called the *permeance* of the magnetic path. Equations 1.12 and 1.13 suggest that the driving force in the magnetic circuit of Fig. 1.4 is the magnetomotive force F ($=Ni$), which produces a flux Φ against a magnetic reluctance $\mathscr{R}$. The magnetic circuit of the toroid can therefore be represented by a magnetic equivalent circuit as shown in Fig. 1.5*a*. Also note that Eq. 1.13 has the form of Ohm's law for an electric

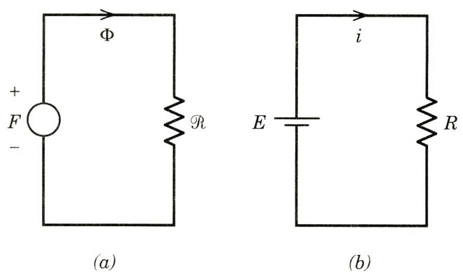


FIGURE 1.5 Analogy between (a) magnetic circuit and (b) electric circuit.

TABLE 1.1 Electrical versus Magnetic Circuits

	Electric Circuit	Magnetic Circuit
Driving force	Emf (E)	Mmf (F)
Produces	Current ($i = E/R$)	Flux ($\Phi = F/\mathcal{R}$)
Limited by	Resistance ($R = l/\sigma A$)[a]	Reluctance ($\mathcal{R} = l/\mu A$)[a]

[a] σ, Conductivity; μ, permeability.

circuit ($i = E/R$). The analogous electrical circuit is shown in Fig. 1.5*b*. A magnetic circuit is often looked upon as analogous to an electric circuit. The analogy is illustrated in Table 1.1.

1.1.4 MAGNETIZATION CURVE

If the magnetic intensity in the core of Fig. 1.4 is increased by increasing current, the flux density in the core changes in the way shown in Fig. 1.6. The flux density B increases almost linearly in the region of low values of the magnetic intensity H. However, at higher values of H, the change of B is nonlinear. The magnetic material shows the effect of saturation. The $B-H$ curve, shown in Fig. 1.6, is called the *magnetization curve*. The reluctance of the magnetic path is dependent on the flux density. It is low when B is low, and high when B is high. The magnetic circuit differs from the electric circuit in this respect; resistance is normally independent of current in an electric circuit, whereas reluctance depends on the flux density in the magnetic circuit.

The $B-H$ characteristics of three different types of magnetic cores—cast iron, cast steel, and silicon sheet steel—are shown in Fig. 1.7. Note that to establish a certain level of flux density B^* in the various magnetic materials, the values of current required are different.

1.1.5 MAGNETIC CIRCUIT WITH AIR GAP

In electric machines, the rotor is physically isolated from the stator by the air gap. A cross-sectional view of a dc machine is shown in Fig. 1.8. Practically the same flux is present in the poles (made of magnetic core) and the air gap. To maintain the same flux density, the air gap will require much more mmf than the core. If the flux density is high, the core portion of the magnetic circuit may exhibit a saturation effect. However, the air gap remains unsaturated, since the $B-H$ curve for the air medium is linear (μ is constant).

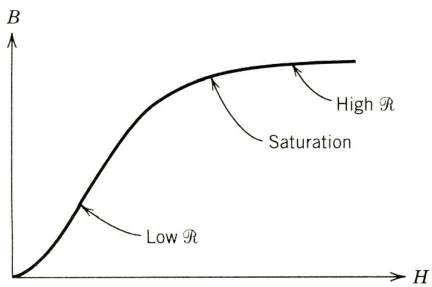


FIGURE 1.6 $B-H$ characteristic (magnetization curve).

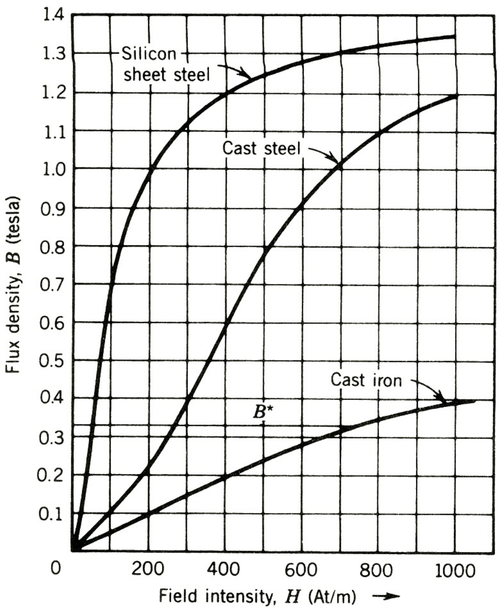


FIGURE 1.7 Magnetization curves.

A magnetic circuit having two or more media—such as the magnetic core and air gap in Fig. 1.8—is known as a *composite structure*. For the purpose of analysis, a magnetic equivalent circuit can be derived for the composite structure.

Let us consider the simple composite structure of Fig. 1.9*a*. The driving force in this magnetic circuit is the mmf, $F = Ni$, and the core medium and the air gap medium can be represented by their corresponding reluctances. The equivalent magnetic circuit is shown in Fig. 1.9*b*.

$$\mathscr{R}_c = \frac{l_c}{\mu_c A_c} \tag{1.15}$$

$$\mathscr{R}_g = \frac{l_g}{\mu_0 A_g} \tag{1.16}$$

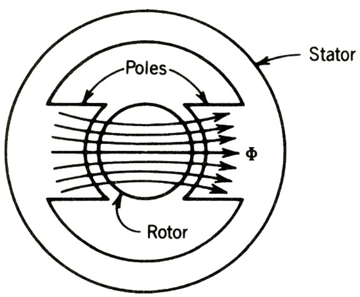


FIGURE 1.8 Cross section of a rotating machine.

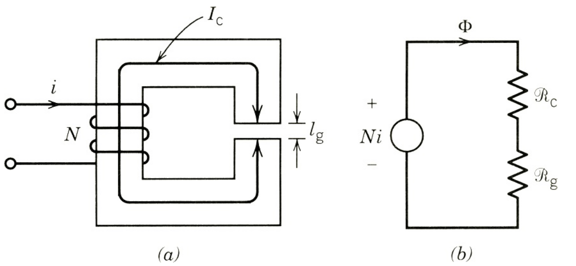


FIGURE 1.9 Composite structure. (*a*) Magnetic core with air gap. (*b*) Magnetic equivalent circuit.

$$\Phi = \frac{Ni}{\mathcal{R}_c + \mathcal{R}_g} \tag{1.17}$$

$$Ni = H_c l_c + H_g l_g \tag{1.18}$$

where l_c is the mean length of the core
l_g is the length of the air gap

The flux densities are

$$B_c = \frac{\Phi_c}{A_c} \tag{1.19}$$

$$B_g = \frac{\Phi_g}{A_g} \tag{1.20}$$

In the air gap the magnetic flux lines bulge outward somewhat, as shown in Fig. 1.10; this is known as *fringing* of the flux. The effect of the fringing is to increase the cross-sectional area of the air gap. For small air gaps the fringing effect can be neglected. If the fringing effect is neglected, the cross-sectional areas of the core and the air gap are the same and therefore

$$A_g = A_c$$

$$B_g = B_c = \frac{\Phi}{A_c}$$

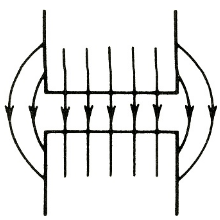


FIGURE 1.10 Fringing flux.

EXAMPLE 1.1

Figure E1.1 represents the magnetic circuit of a primitive relay. The coil has 500 turns and the mean core path is $l_c = 360$ mm. When the air gap lengths are 1.5 mm each, a flux density of 0.8 tesla is required to actuate the relay. The core is cast steel.

(a) Find the current in the coil.

(b) Compute the values of permeability and relative permeability of the core.

(c) If the air gap is zero, find the current in the coil for the same flux density (0.8 T) in the core.

Solution

(a) The air gap is small, so fringing can be neglected. Hence the flux density is the same in both air gap and core. From the $B-H$ curve of the cast steel core (Fig. 1.7).

For

$$B_c = 0.8 \text{ T}, \quad H_c = 510 \text{ At/m}$$

$$\text{mmf } F_c = H_c l_c = 510 \times 0.36 = 184 \text{ At}$$

For the air gap,

$$\text{mmf } F_g = H_g 2l_g = \frac{B_g}{\mu_0} 2l_g = \frac{0.8}{4\pi 10^{-7}} \times 2 \times 1.5 \times 10^{-3}$$

$$= 1910 \text{ At}$$

Total mmf required:

$$F = F_c + F_g = 184 + 1910 = 2094 \text{ At}$$

Current required:

$$i = \frac{F}{N} = \frac{2094}{500} = 4.19 \text{ amps}$$

Note that although the air gap is very small compared to the length of the core ($l_g = 1.5$ mm, $l_c = 360$ mm), most of the mmf is used at the air gap.

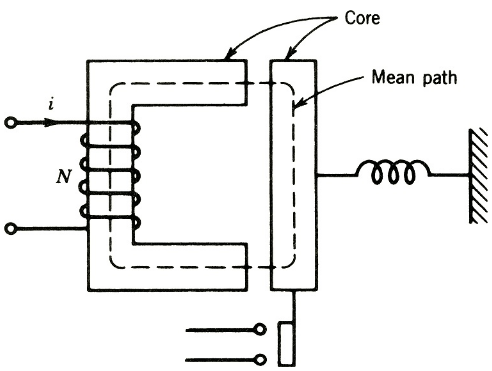


FIGURE E1.1 $N = 500$ turns, $l_c = 36$ cm.

(b) Permeability of core:

$$\mu_c = \frac{B_c}{H_c} = \frac{0.8}{510} = 1.57 \times 10^{-3}$$

(c) Relative permeability of core:

$$\mu_r = \frac{\mu_c}{\mu_0} = \frac{1.57 \times 10^{-3}}{4\pi 10^{-7}} = 1250$$

$$F = H_c l_c = 510 \times 0.36 = 184 \text{ At}$$

$$i = \frac{184}{500} = 0.368 \text{ A}$$

Note that if the air gap is not present, a much smaller current is required to establish the same flux density in the magnetic circuit. ■

EXAMPLE 1.2

Consider the magnetic system of Example 1.1. If the coil current is 4 amps when each air gap length is 1 mm, find the flux density in the air gap.

Solution

In Example 1.1, the flux density was given and so it was easy to find the magnetic intensity and finally the mmf. In this example, current (or mmf) is given and we have to find the flux density. The $B-H$ characteristic for the air gap is linear, whereas that of the core is nonlinear. We need nonlinear magnetic circuit analysis to find out the flux density. Two methods will be discussed.

1. *Load line method*. For a magnetic circuit with core length l_c and air gap length l_g,

$$Ni = H_g l_g + H_c l_c = \frac{B_g}{\mu_0} l_g + H_c l_c$$

Rearranging,

$$B_g = B_c = -\mu_0 \frac{l_c}{l_g} H_c + \frac{Ni\mu_0}{l_g} \tag{1.21}$$

This is in the form $y = mx + c$, which represents a straight line. This straight line (also called the *load line*) can be plotted on the $B-H$ curve of the core. The slope is

$$m = -\mu_0 \frac{l_c}{l_g} = -4\pi 10^{-7} \frac{360}{2} = -2.26 \times 10^{-4}$$

The intersection on the B axis is

$$c = \frac{Ni\mu_0}{l_g} = \frac{500 \times 4 \times 4\pi 10^{-7}}{2 \times 10^{-3}} = 1.256 \text{ tesla}$$

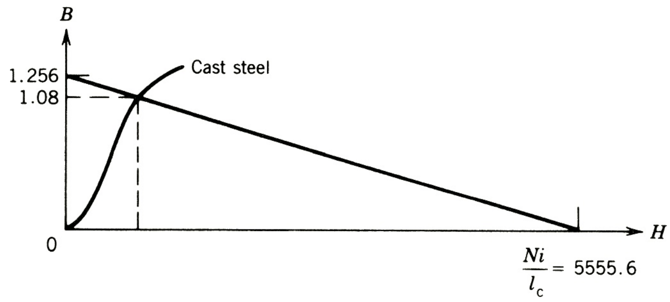


FIGURE E1.2

The load line intersects the $B-H$ curve (Fig. E1.2) at $B = 1.08$ tesla.

Another method of constructing the load line is as follows: If all mmf acts on the air gap (i.e., $H_c = 0$) the air gap flux density is

$$B_g = \frac{Ni}{l_g}\mu_0 = 1.256 \text{ T}$$

This value of B_g is the intersection of the load line on the B axis.

If all mmf acts on the core (i.e., $B_g = 0$),

$$H_c = \frac{Ni}{l_c} = \frac{500 \times 4}{36 \times 10^{-2}} = 5556 \text{ At/m}$$

This value of H_c is the intersection of the load line on the H axis.

2. *Trial-and-error method*. The procedure in this method is as follows.

(a) Assume a flux density.

(b) Calculate H_c (from the $B-H$ curve) and H_g $(=B_g/\mu_0)$.

(c) Calculate $F_c(=H_c l_c)$, $F_g(=H_g l_g)$, and $F(=F_c + F_g)$.

(d) Calculate $i = F/N$.

(e) If i is different from the given current, assume another judicious value of the flux density. Continue this trial-and-error method until the calculated value of i is close to 4 amps.

If all mmf acts on the air gap, the flux density is

$$B = \frac{Ni}{l_g}\mu_0 = 1.256 \text{ T}$$

Obviously, the flux density will be less than this value. The procedure is illustrated in the following table.

B	H_c	H_g	F_c	F_g	F	i
1.1	800	8.7535×10^5	288	1750.7	2038.7	4.08
1.08	785	8.59435×10^5	282	1718.87	2000.87	4.0

■

EXAMPLE 1.3

In the magnetic circuit of Fig. El.3*a*, the relative permeability of the ferromagnetic material is 1200. Neglect magnetic leakage and fringing. All dimensions are in centimeters, and the magnetic material has a square cross-sectional area. Determine the air gap flux, the air gap flux density, and the magnetic field intensity in the air gap.

Solution

The mean magnetic paths of the fluxes are shown by dashed lines in Fig. El.3*a*. The equivalent magnetic circuit is shown in Fig. El.3*b*.

$$F_1 = N_1 I_1 = 500 \times 10 = 5000 \text{ At}$$

$$F_2 = N_2 I_2 = 500 \times 10 = 5000 \text{ At}$$

$$\mu_c = 1200\mu_0 = 1200 \times 4\pi 10^{-7}$$

$$\mathcal{R}_{\text{bafe}} = \frac{l_{\text{bafe}}}{\mu_c A_c}$$

$$= \frac{3 \times 52 \times 10^{-2}}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}}$$

$$= 2.58 \times 10^6 \text{ At/Wb}$$

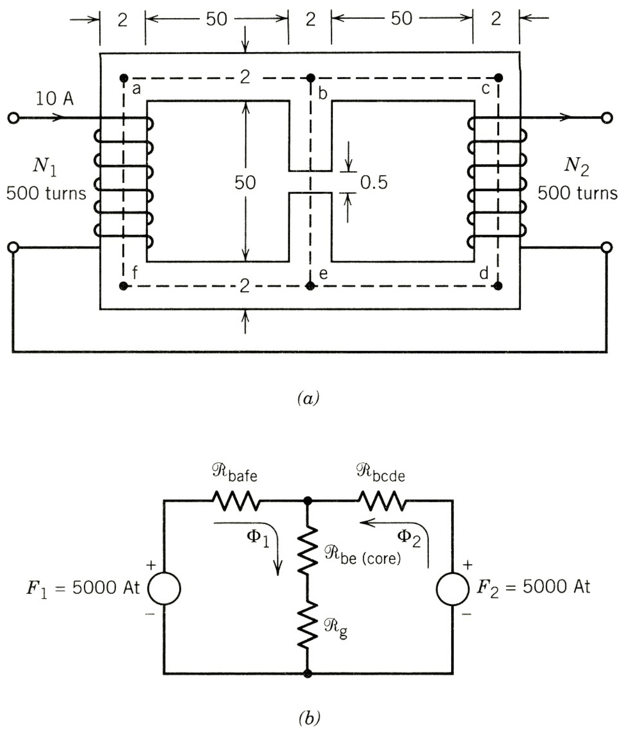


FIGURE E1.3

From symmetry

$$\mathscr{R}_{\text{bcde}} = \mathscr{R}_{\text{bafe}}$$

$$\mathscr{R}\text{g} = \frac{l_g}{\mu_0 A_g}$$

$$= \frac{5 \times 10^{-3}}{4\pi 10^{-7} \times 2 \times 2 \times 10^{-4}}$$

$$= 9.94 \times 10^6 \text{ At/Wb}$$

$$\mathscr{R}_{\text{be(core)}} = \frac{l_{\text{be(core)}}}{\mu_{\text{c}} A_{\text{c}}}$$

$$= \frac{51.5 \times 10^{-2}}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}}$$

$$= 0.82 \times 10^6 \text{ At/Wb}$$

The loop equations are

$$\Phi_1(\mathscr{R}_{\text{bafe}} + \mathscr{R}_{\text{be}} + \mathscr{R}_{\text{g}}) + \Phi_2(\mathscr{R}_{\text{be}} + \mathscr{R}_{\text{g}}) = F_1$$

$$\Phi_1(\mathscr{R}_{\text{be}} + \mathscr{R}_{\text{g}}) + \Phi_2(\mathscr{R}_{\text{bcde}} + \mathscr{R}_{\text{be}} + \mathscr{R}_{\text{g}}) = F_2$$

or

$$\Phi_1(13.34 \times 10^6) + \Phi_2(10.76 \times 10^6) = 5000$$

$$\Phi_1(10.76 \times 10^6) + \Phi_2(13.34 \times 10^6) = 5000$$

or

$$\Phi_1 = \Phi_2 = 2.067 \times 10^{-4} \text{ Wb}$$

The air gap flux is

$$\Phi_{\text{g}} = \Phi_1 + \Phi_2 = 4.134 \times 10^{-4} \text{ Wb}$$

The air gap flux density is

$$B_{\text{g}} = \frac{\Phi_{\text{g}}}{A_{\text{g}}} = \frac{4.134 \times 10^{-4}}{4 \times 10^{-4}} = 1.034 \text{ T}$$

The magnetic intensity in the air gap is

$$H_{\text{g}} = \frac{B_{\text{g}}}{\mu_0} = \frac{1.034}{4\pi 10^{-7}} = 0.822 \times 10^6 \text{ At/m} \qquad \blacksquare$$

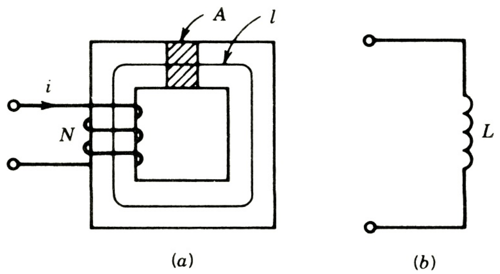


FIGURE 1.11 Inductance of a coil–core assembly. (*a*) Coil–core assembly. (*b*) Equivalent inductance.

1.1.6 INDUCTANCE

A coil wound on a magnetic core, such as that shown in Fig. 1.11*a*, is frequently used in electric circuits. This coil may be represented by an ideal circuit element, called *inductance*, which is defined as the flux linkage of the coil per ampere of its current.

$$\text{Flux linkage } \lambda = N\Phi \tag{1.22}$$

$$\text{Inductance } L = \frac{\lambda}{i} \tag{1.23}$$

From Eqs. 1.3, 1.11, 1.14, 1.22, and 1.23,

$$L = \frac{N\Phi}{i} = \frac{NBA}{i} = \frac{N\mu HA}{i}$$

$$= \frac{N\mu HA}{Hl/N} = \frac{N^2}{l/\mu A} \tag{1.24}$$

$$L = \frac{N^2}{\mathscr{R}} \tag{1.25}$$

Equation 1.24 defines inductance in terms of physical dimensions, such as cross-sectional area and length of core, whereas Eq. 1.25 defines inductance in terms of the reluctance of the magnetic path. Note that inductance varies as the square of the number of turns. The coil–core assembly of Fig. 1.1*a* is represented in an electric circuit by an ideal inductance as shown in Fig. 1.11*b*.

EXAMPLE 1.4

For the magnetic circuit of Fig. 1.9, $N = 400$ turns.
Mean core length $l_c = 50$ cm.
Air gap length $l_g = 1.0$ mm.
Cross-sectional area $A_c = A_g = 15\,\text{cm}^2$.
Relative permeability of core $\mu_r = 3000$.
$i = 1.0$ A.

Find

(a) Flux and flux density in the air gap.
(b) Inductance of the coil.

Solution

(a)

$$\mathscr{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{50 \times 10^{-2}}{3000 \times 4\pi 10^{-7} \times 15 \times 10^{-4}}$$

$$= 88.42 \times 10^3 \text{ AT/Wb}$$

$$\mathscr{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{1 \times 10^{-3}}{4\pi 10^{-7} \times 15 \times 10^{-4}}$$

$$= 530.515 \times 10^3 \text{ At/Wb}$$

$$\Phi = \frac{Ni}{R_c + R_g}$$

$$= \frac{400 \times 1.0}{(88.42 + 530.515)10^3}$$

$$B = \frac{\Phi}{A_g} = \frac{0.6463 \times 10^{-4}}{15 \times 10^{-4}} = 0.4309 \text{ T}$$

(b)

$$L = \frac{N^2}{\mathscr{R}_c + \mathscr{R}_g} = \frac{400^2}{(88.42 + 530.515)10^3}$$

$$= 258.52 \times 10^{-3} \text{ H}$$

or

$$L = \frac{\lambda}{i} = \frac{N\Phi}{i} = \frac{400 \times 0.6463 \times 10^{-3}}{1.0}$$

$$= 258.52 \times 10^{-3} \text{ H} \quad \blacksquare$$

EXAMPLE 1.5

The coil in Fig. 1.4 has 250 turns and is wound on a silicon sheet steel. The inner and outer radii are 20 and 25 cm, respectively, and the toroidal core has a circular cross section. For a coil current of 2.5 A, find

(a) The magnetic flux density at the mean radius of the toroid.
(b) The inductance of the coil, assuming that the flux density within the core is uniform and equal to that at the mean radius.

Solution

(a) Mean radius is $\frac{1}{2}(25+20) = 22.5$ cm

$$H = \frac{Ni}{l} = \frac{250 \times 2.5}{2\pi 22.5 \times 10^{-2}} = 442.3 \text{ At/m}$$

From the $B-H$ curve for silicon sheet steel (Fig. 1.7),

$$B = 1.225 \text{ T}$$

(b) The cross-sectional area is

$$A = \pi(\text{radius of core})^2$$
$$= \pi\left(\frac{25-20}{2}\right)^2 \times 10^{-4}$$
$$= \pi 6.25 \times 10^{-4} \text{ m}^2$$
$$\Phi = BA$$
$$= 1.225 \times \pi 6.25 \times 10^{-4}$$
$$= 24.04 \times 10^{-4} \text{ Wb}$$
$$\lambda = 250 \times 24.04 \times 10^{-4}$$
$$= 0.601 \text{ Wb} \cdot \text{turn}$$
$$L = \frac{\lambda}{i} = \frac{0.601}{2.5} = 0.2404 \text{ H}$$
$$= 240.4 \text{ mH}$$

Inductance can also be calculated using Eq. 1.25:

$$\mu \text{ of core} = \frac{B}{H} = \frac{1.225}{442.3}$$

$$\mathscr{R}_{\text{core}} = \frac{l}{\mu A} = \frac{2\pi 22.5 \times 10^{-2}}{(1.225/442.3) \times \pi 6.25 \times 10^{-4}}$$
$$= 2599.64 \times 10^2 \text{ At/Wb}$$
$$= \frac{N^2}{\mathscr{R}} L = \frac{250^2}{2599.64 \times 10^2} = 0.2404 \text{ H}$$
$$= 240.4 \text{ mM} \quad \blacksquare$$

1.2 In the magnetic system of Fig. P1.2 two sides are thicker than the other two sides. The depth of the core is 10 cm, the relative permeability of the core $\mu_r = 2000$, the number of turns $N = 300$, and the current flowing through the coil is $i = 1$ A.

(a) Determine the flux in the core.

(b) Determine the flux densities in the parts of the core.

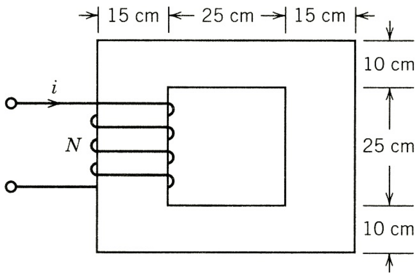


FIGURE P1.2

1.3 For the magnetic system of Problem 1.2, find the current i in the coil to produce a flux $\Phi = 0.012$ Wb.

1.4 Two coils are wound on a toroidal core as shown in Fig. P1.4. The core is made of silicon sheet steel and has a square cross section. The coil currents are $i_1 = 0.28$ A and $i_2 = 0.56$ A.

(a) Determine the flux density at the mean radius of the core.

(b) Assuming constant flux density (same as at the mean radius) over the cross section of the core, determine the flux in the core.

(c) Determine the relative permeability, μ_r, of the core.

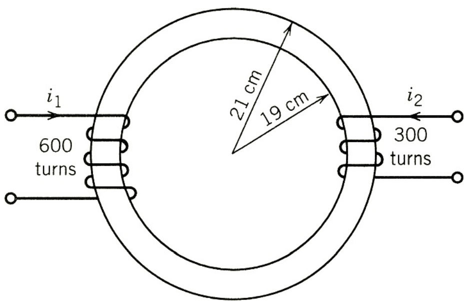


FIGURE P1.4

1.5 The magnetic circuit of Fig. P1.5 provides flux in the two air gaps. The coils ($N_1 = 700$, $N_2 = 200$) are connected in series and carry a current of 0.5 ampere. Neglect leakage flux, reluctance of the iron (i.e., infinite permeability), and fringing at the air gaps. Determine the flux and flux density in the air gaps.

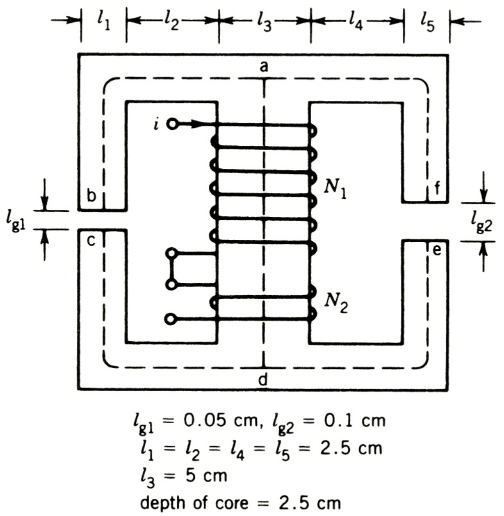


FIGURE P1.5

1.6 A two-pole generator, as shown in Fig. P1.6, has a magnetic circuit with the following dimensions:

Each pole (cast steel):

magnetic length = 10 cm

cross section = 400 cm^2

Each air gap:

length = 0.1 cm

cross section = 400 cm^2

Armature (Si-steel):

average length = 20 cm

average cross section = 400 cm^2

Yoke (cast steel):

mean circumference = 160 cm

average cross section = 200 cm^2

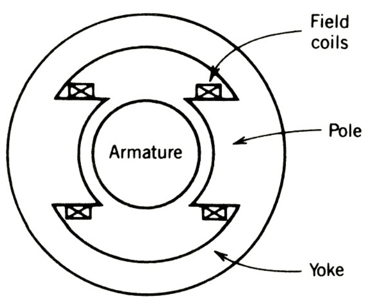


FIGURE P1.6

Half the exciting ampere-turns are placed on each of the two poles.

(a) Draw the magnetic equivalent circuit.

(b) How many ampere-turns per pole are required to produce a flux density of 1.1 tesla in the magnetic circuit. (Use the magnetization curves for the respective materials.)

(c) Calculate the armature flux.

1.7 A two-pole synchronous machine, as shown in Fig. P1.7, has the following dimensions:

Each air gap length, $l_g = 2.5$ mm

Cross-sectional area of pole face, $A_g = 500\ \text{cm}^2$

$N = 500$ turns

$I = 5$ A

$\mu_c =$ infinity

(a) Draw the magnetic equivalent circuit.

(b) Find the flux density in the air gap.

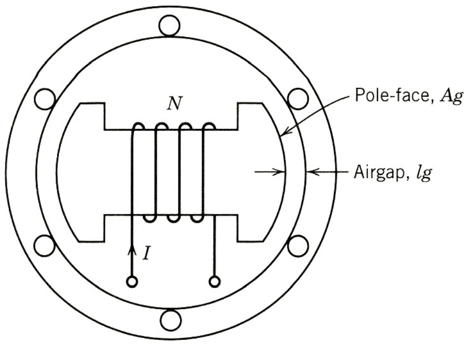


FIGURE P1.7

1.8 The electromagnet shown in Fig. P1.8 can be used to lift a length of steel strip. The coil has 500 turns and can carry a current of 20 amps without overheating. The magnetic material has negligible reluctance at flux densities up to 1.4 tesla. Determine the maximum air gap for which a flux

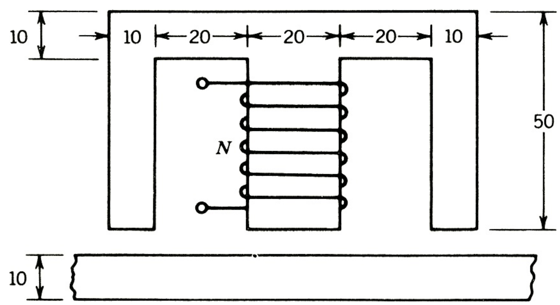


FIGURE P1.8

density of 1.4 tesla can be established with a coil current of 20 amps. Neglect magnetic leakage and fringing of flux at the air gap.

1.9 The toroidal (circular cross section) core shown in Fig. P1.9 is made from cast steel.

(a) Calculate the coil current required to produce a core flux density of 1.2 tesla at the mean radius of the toroid.

(b) What is the core flux, in webers? Assume uniform flux density in the core.

(c) If a 2-mm-wide air gap is made in the toroid (across A–A'), determine the new coil current required to maintain a core flux density of 1.2 tesla.

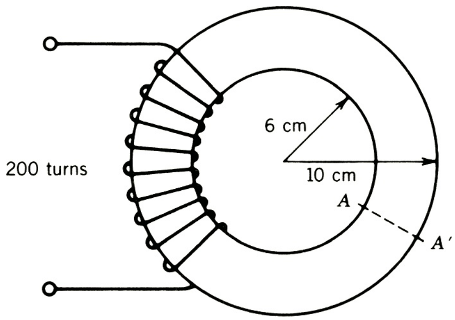


FIGURE P1.9

1.10 In the toroidal core coil system of Fig. P1.9, the coil current is 2.0 A and the relative permeability of the core is 2000. The core has a square cross section.

(a) Determine the maximum and minimum values of the flux density in the core.

(b) Determine the magnetic flux in the core.

(c) Determine the flux density at the mean radius of the toroid, and compare it with the average flux density across the core.

1.11 The magnetic circuit of Fig. P1.11 has a core of relative permeability $\mu_r = 2000$. The depth of the core is 5 cm. The coil has 400 turns and carries a current of 1.5 A.

(a) Draw the magnetic equivalent circuit.

(b) Find the flux and the flux density in the core.

(c) Determine the inductance of the coil.

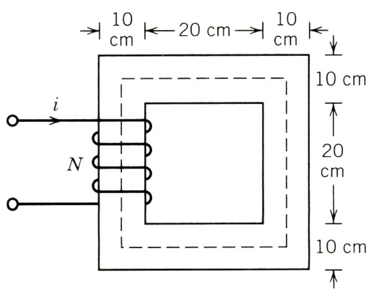


FIGURE P1.11

1.12 Repeat Problem 1.11 for a 1.0-cm-wide air gap in the core. Assume a 10% increase in the effective cross-sectional area of the air gap due to fringing in the air gap.

1.13 The magnetic circuit of Fig. 1.9 has the following parameters:

$$N = 100 \text{ turns}$$

$$A_c = A_g = 5 \text{ cm}^2$$

$$\mu_{core} = \text{infinity}$$

Determine the air gap length, l_g, to provide a coil inductance of 10 mH.

1.14 An inductor is made of two coils, A and B, having 350 and 150 turns, respectively. The coils are wound on a cast steel core and in directions as shown in Fig. P1.14. The two coils are connected *in series* to a dc voltage.

(a) Determine the two possible values of current required in the coils to establish a flux density of 0.5 T in the air gap.

(b) Determine the self-inductances L_A and L_B of the two coils. Neglect magnetic leakage and fringing.

(c) If coil B is now disconnected and the current in coil A is adjusted to 2.0 A, determine the mean flux density in the air gap.

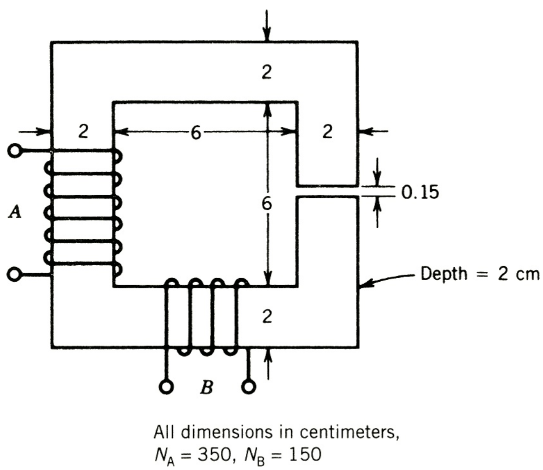


FIGURE P1.14

1.15 The magnetic circuit for a saturable reactor is shown in Fig. P1.15. The $B-H$ curve for the core material can be approximated as two straight lines as in Fig. P1.15.

(a) If $I_1 = 2.0$ A, calculate the value of I_2 required to produce a flux density of 0.6 T in the vertical limbs.

(b) If $I_1 = 0.5$ A and $I_2 = 1.96$ A, calculate the total flux in the core.

Neglect magnetic leakage.

(*Hint: Trial-and-error method*.)

chapter four

DC MACHINES

Applications such as light bulbs and heaters require energy in electrical form. In other applications, such as fans and rolling mills, energy is required in mechanical form. One form of energy can be obtained from the other form with the help of converters. Converters that are used to continuously translate electrical input to mechanical output or vice versa are called *electric machines*. The process of translation is known as *electromechanical energy conversion*. An electric machine is therefore a link between an electrical system and a mechanical system, as shown in Fig. 4.1. In these machines the conversion is reversible. If the conversion is from mechanical to electrical, the machine is said to act as a *generator*. If the conversion is from electrical to mechanical, the machine is said to act as a *motor*. Hence, the same electric machine can be made to operate as a generator as well as a motor. Machines are called ac machines (generators or motors) if the electrical system is ac and dc machines (generators or motors) if the electrical system is dc.

Note that the two systems in Fig. 4.1, electrical and mechanical, are different in nature. In the electrical system the primary quantities involved are *voltage* and *current*, while the analogous quantities in the mechanical system are *torque* and *speed*. The coupling medium between these different systems is the field, as illustrated in Fig. 4.2.

4.1 ELECTROMAGNETIC CONVERSION

Three electrical machines (dc, induction, and synchronous) are used extensively for electromechanical energy conversion. In these machines, conversion of energy from electrical to mechanical form or vice versa results from the following two electromagnetic phenomena:

1. When a conductor moves in a magnetic field, voltage is induced in the conductor.

2. When a current-carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force.

These two effects occur simultaneously whenever energy conversion takes place from electrical to mechanical or vice versa. In motoring action, the electrical system makes current flow through conductors that are placed in the magnetic field. A force is produced on each conductor. If the conductors are placed on a structure free to rotate, an electromagnetic torque will be produced, tending to make the rotating structure rotate at some speed. If the conductors rotate in a magnetic field, a voltage will also be induced in each conductor. In generating action, the process is reversed. In this case, the rotating structure, the rotor, is driven by a prime mover (such as a steam turbine or a diesel engine). A voltage will be induced in the conductors that are rotating with the rotor. If an electrical load is connected to the winding formed by these conductors, a current i will flow, delivering electrical power to the load. Moreover, the current flowing through the conductor will interact with the magnetic field to produce a reaction torque, which will tend to oppose the torque applied by the prime mover.

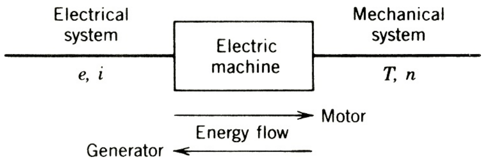


FIGURE 4.1 Electromechanical energy conversion.

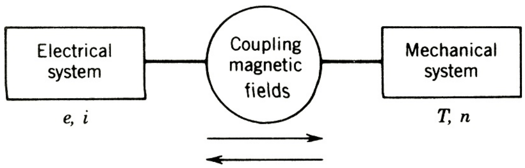


FIGURE 4.2 Coupling field between electrical and mechanical systems.

Note that in both motoring and generating actions, the coupling magnetic field is involved in producing a torque and an induced voltage.

The basic electric machines (dc, induction, and synchronous), which depend on electromagnetic energy conversion, are extensively used in various power ratings. The operation of these machines is discussed in detail in this and other chapters.

Motional Voltage, *e*

An expression can be derived for the voltage induced in a conductor moving in a magnetic field. As shown in Fig. 4.3*a*, if a conductor of length l moves at a linear speed v in a magnetic field B, the induced voltage in the conductor is

$$e = Blv \tag{4.1}$$

where $B, l,$ and v are mutually perpendicular. The polarity of the induced voltage can be determined from the so-called right-hand screw rule.

The three quantities $v, B,$ and e are shown in Fig. 4.3*b* as three mutually perpendicular vectors. Turn the vector v toward the vector B. If a right-hand screw is turned in the same way the motion of the screw will indicate the direction of positive polarity of the induced voltage.

Electromagnetic Force, *f*

For the current-carrying conductor shown in Fig. 4.4*a*, the force (known as Lorentz force) produced on the conductor is

$$f = Bli \tag{4.2}$$

where $B, l,$ and i are mutually perpendicular. The direction of the force can be determined by using the right-hand screw rule, illustrated in Fig. 4.4*b*.

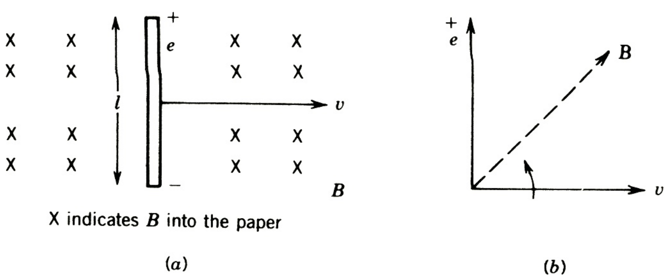


FIGURE 4.3 Motional voltage. (*a*) Conductor moving in the magnetic field. (*b*) Right-hand screw rule.

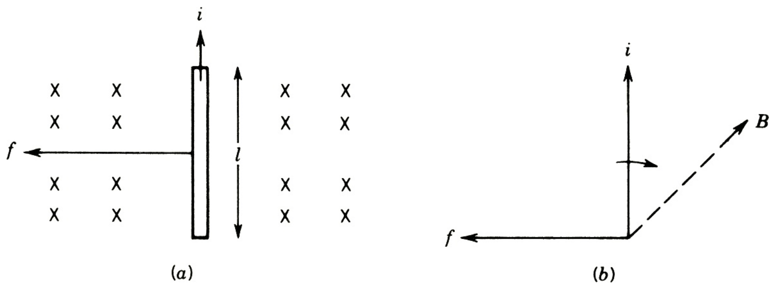


FIGURE 4.4 Electromagnetic force. (*a*) Current-carrying conductor moving in a magnetic field. (*b*) Force direction.

Turn the current vector i toward the flux vector B. If a screw is turned in the same way, the direction in which the screw will move represents the direction of the force f.

Note that in both cases (i.e., determining the polarity of the induced voltage and determining the direction of the force) the moving quantities (v and i) are turned toward B to obtain the screw movement.

Equations 4.1 and 4.2 can be used to determine the induced voltage and the electromagnetic force or torque in an electric machine. There are, of course, other methods by which these quantities (e and f) can be determined.

Basic Structure of Electric Machines

The structure of an electric machine has two major components, stator and rotor, separated by the air gap, as shown in Fig. 4.5.

Stator: This part of the machine does not move and normally is the outer frame of the machine.

Rotor: This part of the machine is free to move and normally is the inner part of the machine.

Both stator and rotor are made of ferromagnetic materials. In most machines, slots are cut on the inner periphery of the stator and outer periphery of the rotor structure, as shown in Fig. 4.5*a*. Conductors are placed in these slots. The iron core is used to maximize the coupling between the coils (formed by conductors) placed on the stator and rotor, to increase the flux density in the machine, and to decrease the size of the machine. If the stator or rotor (or both)

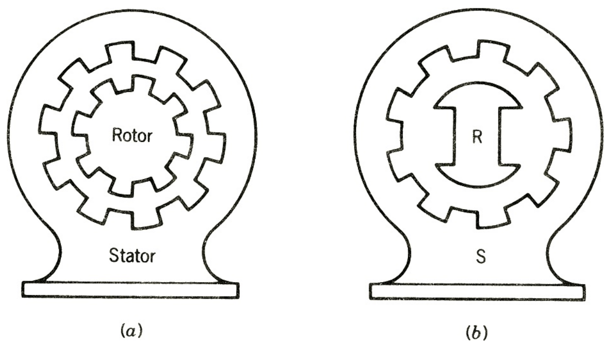
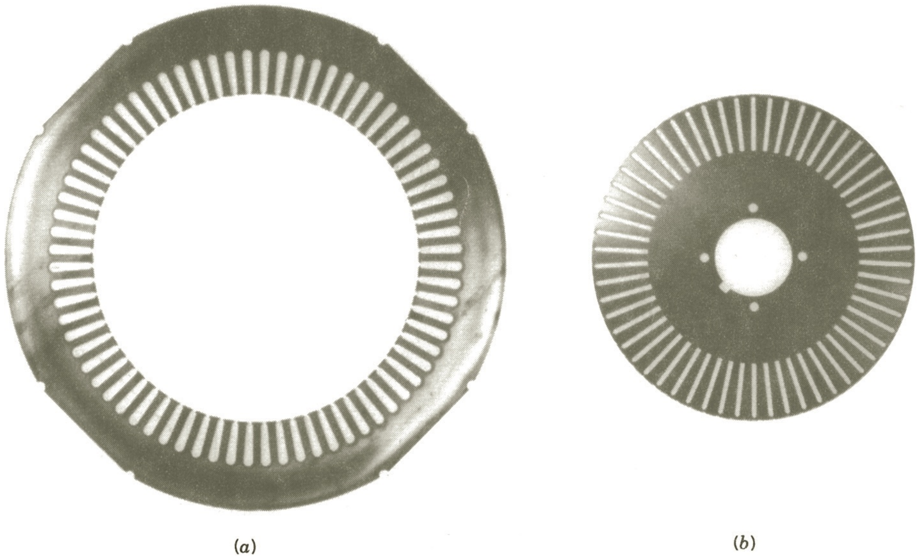


FIGURE 4.5 Structure of electric machines. (*a*) Cylindrical machine (uniform air gap). (*b*) Salient pole machine (nonuniform air gap).



Courtesy of Westinghouse Canada Inc.

FIGURE 4.6 Laminations. (*a*) Stator. (*b*) Rotor.

is subjected to a time-varying magnetic flux, the iron core is laminated to reduce eddy current losses. The thin laminations of the iron core with provisions for slots are shown in Fig. 4.6.

The conductors placed in the slots of the stator or rotor are interconnected to form windings. The winding in which voltage is induced is called the *armature winding*. The winding through which a current is passed to produce the primary source of flux in the machine is called the *field winding*. Permanent magnets are used in some machines to provide the major source of flux in the machine.

Rotating electrical machines take many forms and are known by many names. The three basic and common ones are dc machines, induction machines, and synchronous machines. There are other machines, such as permanent magnet machines, hysteresis machines, and stepper machines.

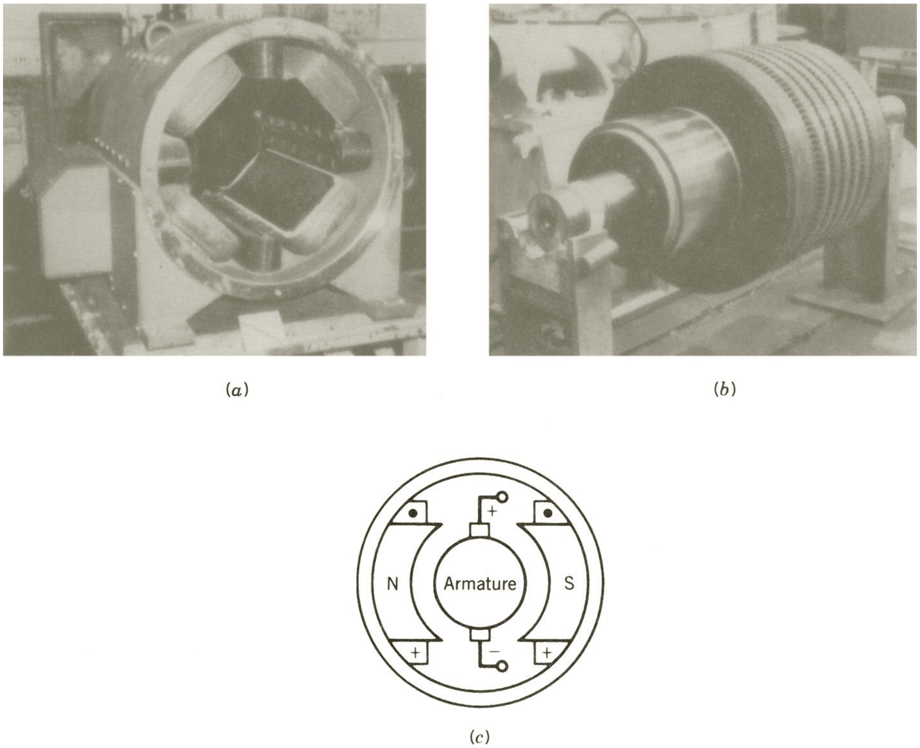
DC Machine

In the dc machine, the field winding is placed on the stator and the armature winding on the rotor. These windings are shown in Fig. 4.7. A dc current is passed through the field winding to produce flux in the machine. Voltage induced in the armature winding is alternating. A mechanical commutator and a brush assembly function as a rectifier or inverter, making the armature terminal voltage unidirectional.

Induction Machine

In this machine the stator windings serve as both armature windings and field windings. When the stator windings are connected to an ac supply, flux is produced in the air gap and revolves at a fixed speed known as *synchronous speed*. This revolving flux induces voltage in the stator windings as well as in the rotor windings. If the rotor circuit is closed, current flows in the rotor

Courtesy of General Electric Canada Inc.



Courtesy of General Electric Canada Inc.

FIGURE 4.7 DC machine. (*a*) Stator. (*b*) Rotor. (*c*) Schematic cross-sectional view for a 2-pole machine.

winding and reacts with the revolving flux to produce torque. The steady-state speed of the rotor is very close to the synchronous speed. The rotor can have a winding similar to the stator or a cage-type winding. The latter is formed by placing aluminum or copper bars in the rotor slots and shorting them at the ends by means of rings. Figure 4.8 shows the structure of the induction machine.

Synchronous Machine

In this machine, the rotor carries the field winding and the stator carries the armature winding. The structure of the synchronous machine is shown in Fig. 4.9. The field winding is excited by direct current to produce flux in the air gap. When the rotor rotates, voltage is induced in the armature winding placed on the stator. The armature current produces a revolving flux in the air gap whose speed is the same as the speed of the rotor—hence the name synchronous machine.

These three major machine types, although they differ in physical construction and appear to be quite different from each other, are in fact governed by the same basic laws. Their behavior can be explained by considering the same fundamental principles of voltage and torque production. Various analytical techniques can be used for the machines, and various forms of torque or voltage equations can be derived for them, but the forms of the equations will differ merely to reflect the difference in construction of the machines. For example, analysis will

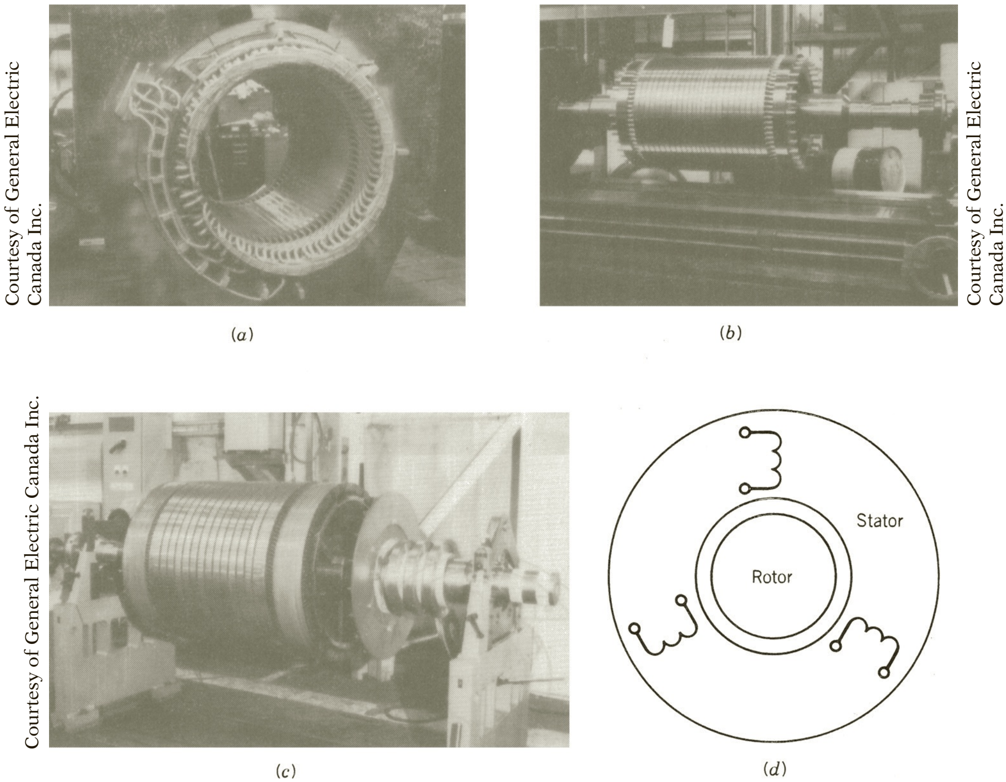


FIGURE 4.8 Induction machine. (*a*) Stator. (*b*) Rotor—cage type. (*c*) Rotor—wound type. (*d*) Schematic cross-sectional view.

show that in dc machines, the stator and rotor flux distributions are fixed in space, and a torque is produced because of the tendency of these two fluxes to align. The induction machine is an ac machine and differs in many ways from the dc machine, but works on the same principle. Analysis will indicate that the stator flux and the rotor flux rotate in synchronism in the air gap, and the two flux distributions are displaced from each other by a torque-producing displacement angle. The torque is produced because of the tendency of the two flux distributions to align with each other. It must be emphasized at the outset that ac machines are not fundamentally different from dc machines. Their construction details are different, but the same fundamental principles underlie their operation.

The three basic and commonly used machines—dc, induction, and synchronous—are described, analyzed, and discussed in separate chapters. In this chapter the various aspects of the steady-state operation of the dc machine are studied in detail.

4.2 DC MACHINES

The dc machines are versatile and extensively used in industry. A wide variety of volt–ampere or torque–speed characteristics can be obtained from various connections of the field windings.

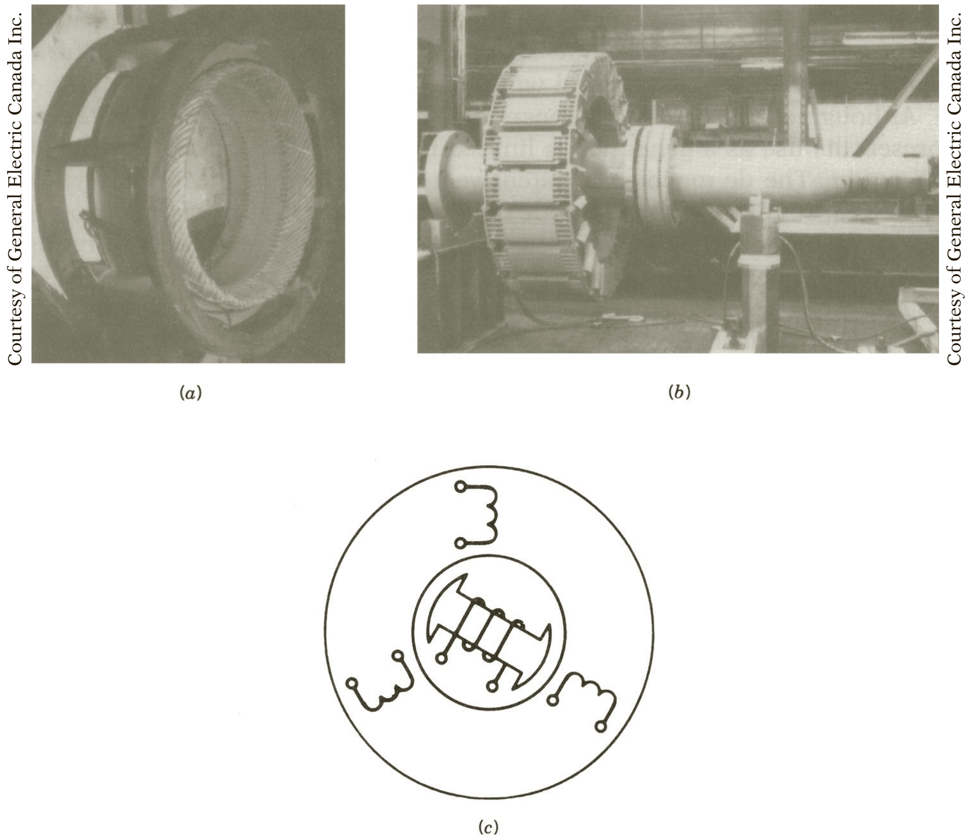


FIGURE 4.9 Synchronous machine. (*a*) Stator. (*b*) Rotor. (*c*) Schematic cross-sectional view for a 2-pole machine.

Although a dc machine can operate as either a generator or a motor, at present its use as a generator is limited because of the widespread use of ac power. The dc machine is extensively used as a motor in industry. Its speed can be controlled over a wide range with relative ease. Large dc motors (in tens or hundreds of horsepower) are used in machine tools, printing presses, conveyors, fans, pumps, hoists, cranes, paper mills, textile mills, rolling mills, and so forth. Additionally, dc motors still dominate as traction motors used in transit cars and locomotives. Small dc machines (in fractional horsepower rating) are used primarily as control devices—such as tachogenerators for speed sensing and servomotors for positioning and tracking. The dc machine definitely plays an important role in industry.

4.2.1 CONSTRUCTION

In a dc machine, the armature winding is placed on the rotor and the field windings are placed on the stator. The essential features of a two-pole dc machine are shown in Fig. 4.10. The stator has salient poles that are excited by one or more field windings, called *shunt field windings* and

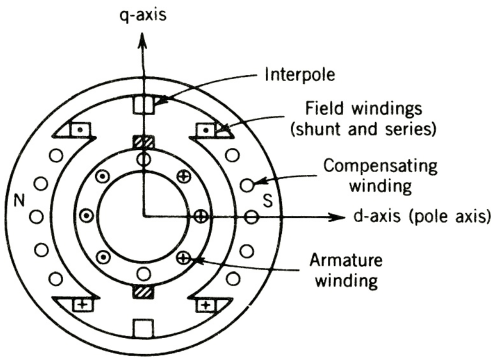


FIGURE 4.10 Schematic diagram of a dc machine.

series field windings. The field windings produce an air gap flux distribution that is symmetrical about the *pole axis* (also called the *field axis*, *direct axis*, or *d-axis*).

The voltage induced in the turns of the armature winding is alternating. A commutator–brush combination is used as a mechanical rectifier to make the armature terminal voltage unidirectional and also to make the mmf wave, due to the armature current fixed in space. The brushes are so placed that when the sides of an armature turn (or coil) pass through the middle of the region between field poles, the current through it changes direction. This makes all the conductors under one pole carry current in one direction. As a consequence, the mmf due to the armature current is along the axis midway between the two adjacent poles, called the *quadrature* (or *q*) *axis*. In the schematic diagram of Fig. 4.10, the brushes are shown placed on the q-axis to indicate that when a turn (or coil) undergoes commutation, its sides are in the q-axis. However, because of the end connection, the actual brush positions will be approximately 90° from the position shown in Fig. 4.10 (see also Fig. 4.17).

Note that because of the commutator and brush assembly, the armature mmf (along the q-axis) is in quadrature with the field mmf (d-axis). This positioning of the mmfs will maximize torque production. The armature mmf axis can be changed by changing the position of the brush assembly as shown in Fig. 4.11. For improved performance, interpoles (in between two main field poles) and compensating windings (on the face of the main field poles) are required. These will be discussed in Sections 4.3.5 and 4.3.1, respectively.

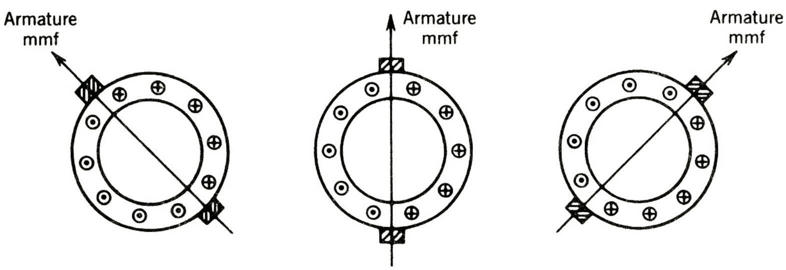


FIGURE 4.11 Shift of brush position.

4.2.2 EVOLUTION OF DC MACHINES

Consider a two-pole dc machine as shown in Fig. 4.12*a*. The air gap flux density distribution of the field poles is shown in Fig. 4.12*b*. Consider a turn a–b placed on diametrically opposite slots of the rotor. The two terminals a and b of the turn are connected to two slip rings. Two stationary brushes pressing against the slip rings provide access to the revolving turn a–b.

The voltage induced in the turn is due primarily to the voltage induced in the two sides of the turn under the poles. Using the concept of "conductor cutting flux" (Eq. 4.1), these two voltages are in series and aid each other. The voltage induced in the turn, e_{ab} (same as the voltage e_{12} across the brushes), is alternating in nature, and its waveform is the same as that of the flux density distribution wave in space.

Let us now replace the two slip rings by two commutator segments (which are copper segments separated by insulating materials) as shown in Fig. 4.13*a*. Segment C_a is connected to terminal a of the turn and segment C_b to terminal b of the turn. For counterclockwise motion of the rotor the terminal under the N pole is positive with respect to the terminal under the S pole. Therefore, brush terminal B_1 is always connected to the positive end of the turn (or coil) and brush terminal B_2 to the negative end of the turn (or coil). Consequently, although the voltage induced in the turn, e_{ab}, is alternating, the voltage at the brush terminals, e_{12}, is unidirectional as shown in Fig. 4.13*b*. This voltage contains a significant amount of ripple. In an actual

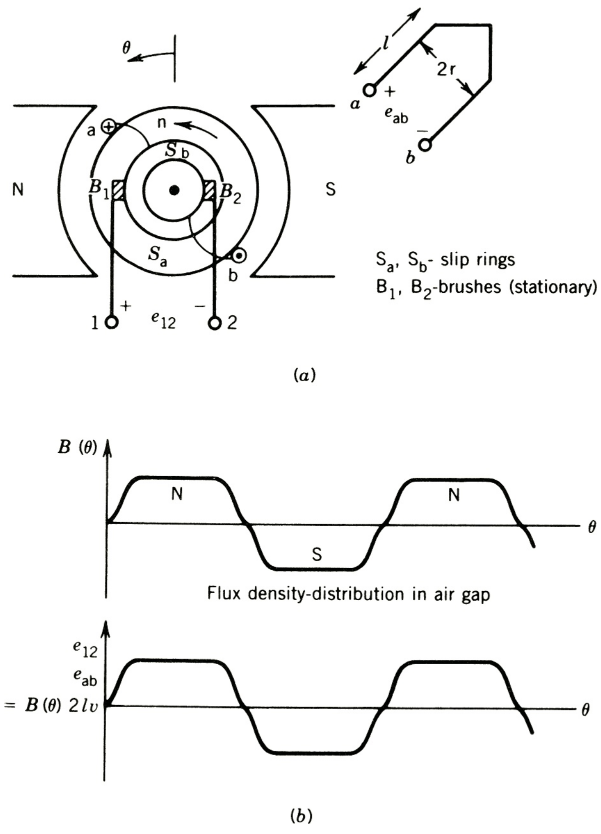


FIGURE 4.12 Induced voltage in a dc machine. (*a*) Two-pole dc machine. (*b*) Induced voltage in a turn.

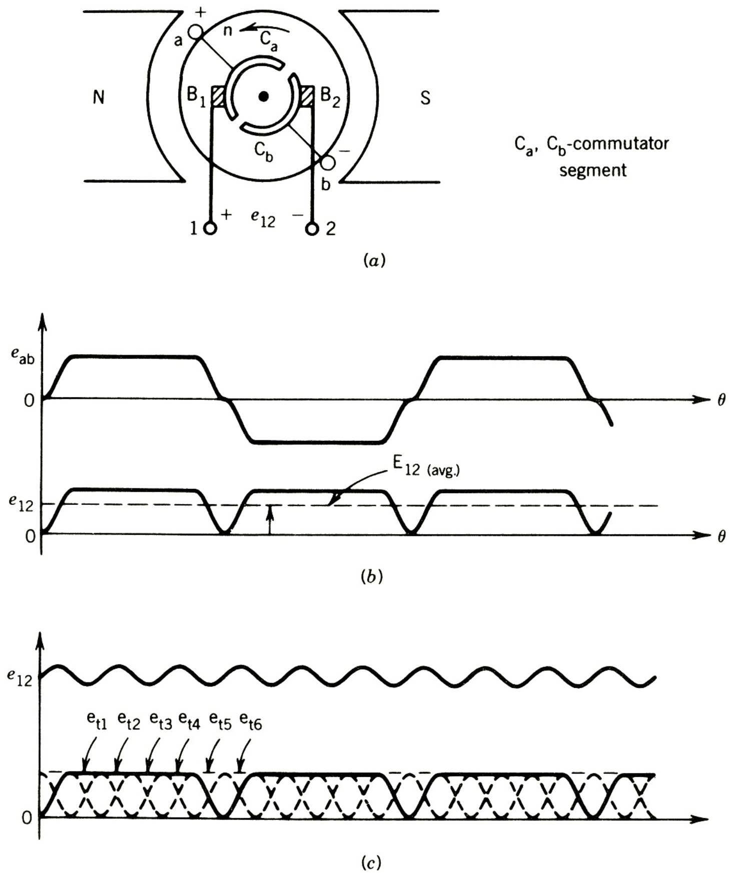


FIGURE 4.13 Voltage rectification by commutators and brushes. (*a*) DC machine with commutator segments. (*b*) Single-turn machine. (*c*) Multiturn machine.

machine, a large number of turns are placed in several slots around the periphery of the rotor. By connecting these in series through the commutator segments (to form an armature winding), a good dc voltage (having a small amount of ripple) can be obtained across the brushes of the rotor armature, as shown in Fig. 4.13*c*.

Note that turn a–b is short-circuited by the brushes when its sides pass midway between the field poles (i.e., the q-axis). In the case of a dc motor, current will be fed into the armature through the brushes. The current in the turn will reverse when the turn passes the interpolar region and the commutator segments touch the other brushes. This phenomenon is illustrated by the three positions of the turn in Fig. 4.14.

4.2.3 ARMATURE WINDINGS

As stated earlier, in the dc machine the field winding is placed on the stator to excite the field poles, and the armature winding is placed on the rotor so that the commutator and brush combination can rectify the voltage. There are various ways to construct an armature winding. Before these are discussed, some basic components of the armature winding and terms related to it are defined.

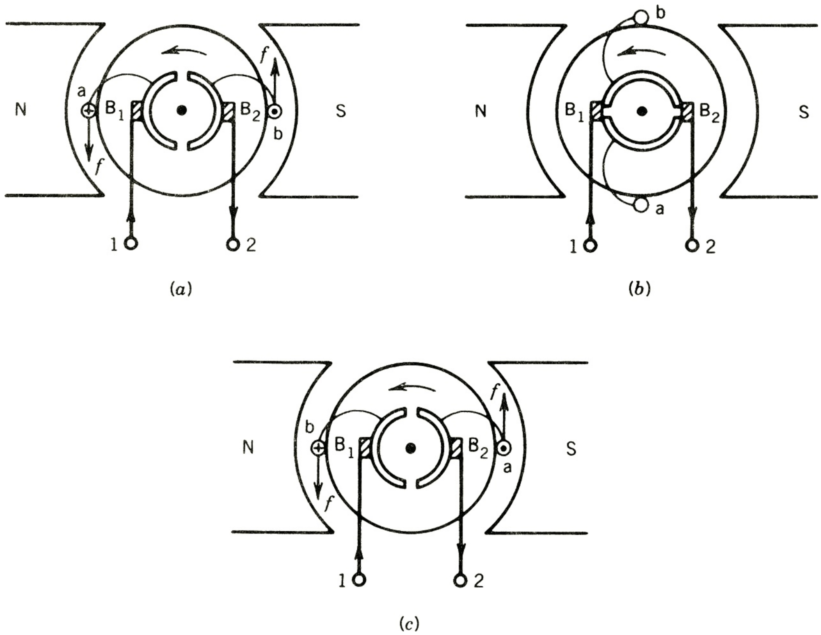


FIGURE 4.14 Current reversal in a turn by commutators and brushes. (*a*) End a touches brush B_1; current flows from a to b. (*b*) The turn is shorted; turn is in interpolar region. (*c*) End a touches brush B_2; current flows from b to a.

A *turn* consists of two conductors connected to one end by an end connector.

A *coil* is formed by connecting several turns in series.

A *winding* is formed by connecting several coils in series.

The turn, coil, and winding are shown schematically in Fig. 4.15. The beginning of the turn, or coil, is identified by the symbol S, and the end of the turn or coil by the symbol F.

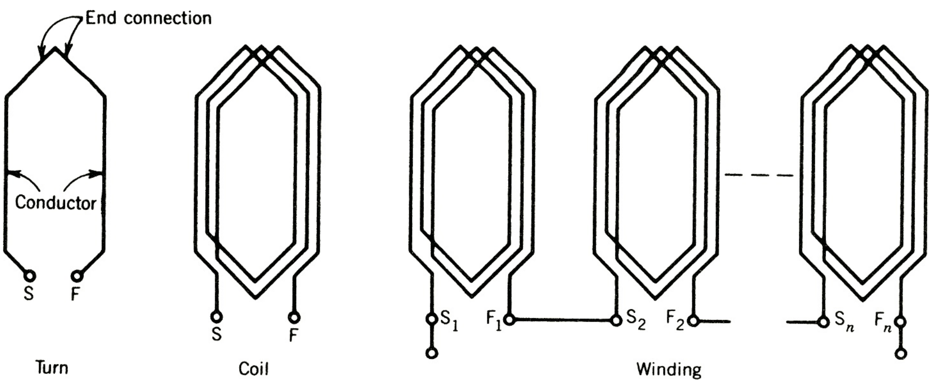


FIGURE 4.15 Turn, coil, and winding.

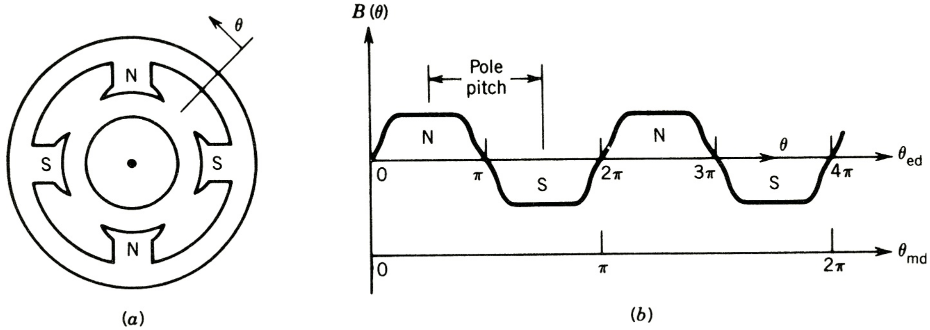


FIGURE 4.16 Mechanical and electrical degrees. (*a*) Four-pole dc machine. (*b*) Flux density distribution.

Most dc machines, particularly larger ones, have more than two poles, so most of the armature conductors can be in the region of high air gap flux density. Figure 4.16*a* shows the stator of a dc machine with four poles. This calls for an armature winding that will also produce four poles on the rotor. The air gap flux density distribution due to the stator poles is shown in Fig. 4.16*b*. Note that for the four-pole machine, in going around the air gap once (i.e., one mechanical cycle), two cycles of variation of the flux density distribution are encountered. If we define

$$\theta_{\text{md}} = \text{mechanical degrees or angular measure in space}$$

$$\theta_{\text{ed}} = \text{electrical degrees or angular measure in cycles}$$

then, for a p-pole machine,

$$\theta_{\text{ed}} = \frac{p}{2}\theta_{\text{md}} \tag{4.3}$$

The distance between the centers of two adjacent poles is known as *pole pitch* or pole span. Obviously,

$$\text{One pole pitch} = 180^{\circ}{}_{\text{ed}} = \frac{360^{\circ}{}_{\text{md}}}{p}$$

The two sides of a coil are placed in two slots on the rotor surface. The distance between the two sides of a coil is called the *coil pitch*. If the coil pitch is one pole pitch, it is called a *full-pitch* coil. If the coil pitch is less than one pole pitch, the coil is known as a *short-pitch* (or *fractional-pitch*) coil. Short-pitch coils are desirable in ac machines for various reasons (see Appendix A). The dc armature winding is mostly made of full-pitch coils.

There are a number of ways in which the coils of the armature windings of a dc machine can be interconnected. Two kinds of interconnection, *lap* and *wave*, are very common. These are illustrated in Figs. 4.17 and 4.18, respectively.

Lap Winding

Figure 4.17 illustrates an unrolled lap winding of a dc armature, along with the commutator segments (bars) and stationary brushes. The brushes are located under the field poles at their centers. Consider the coil shown by dark lines with one end connected to the commutator

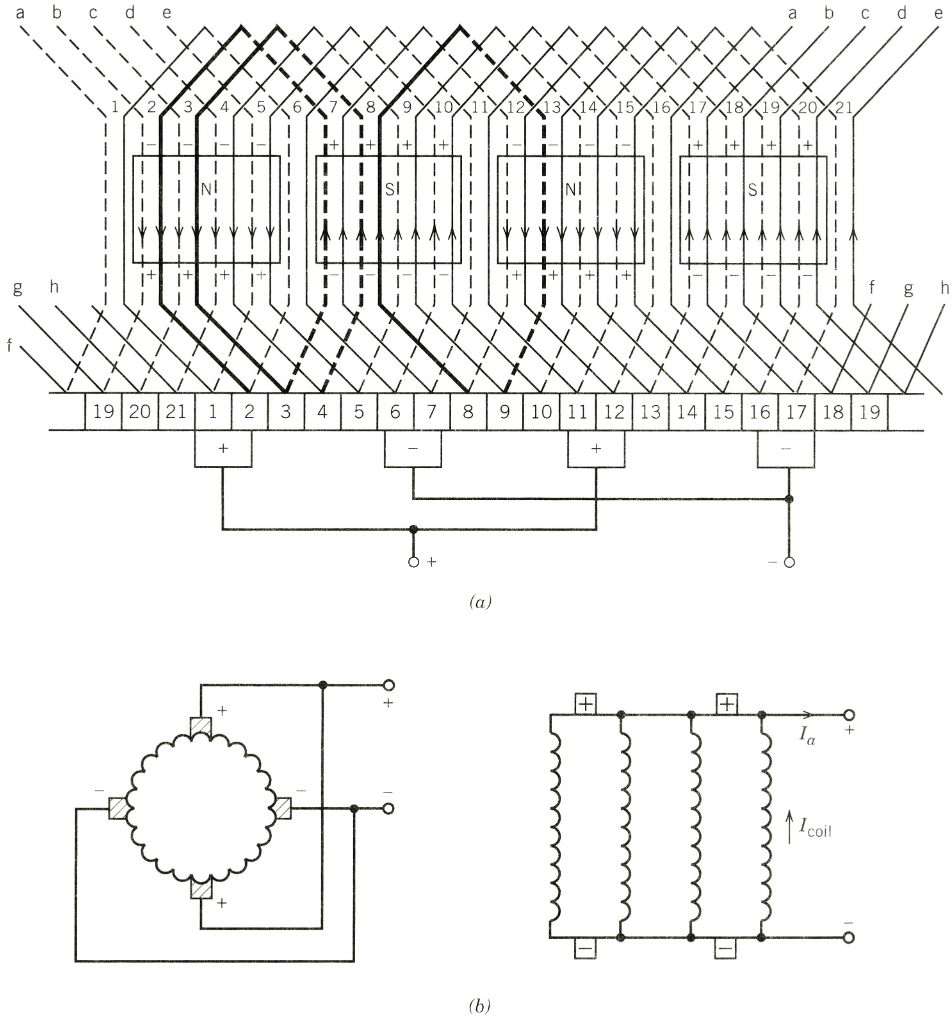


FIGURE 4.17 Lap winding. (*a*) Unrolled winding. (*b*) Equivalent coil representation.

bar numbered 2. The coil is placed in slots 2 and 7 such that the coil sides are placed in similar positions under adjacent poles. The other end of the coil is connected to the commutator bar numbered 3. The second coil starts at commutator 3 and finishes at the next commutator, numbered 4. In this way all the coils are added in series and the pattern is continued until the end of the last coil joins the start of the first coil. This is called a lap winding, because as the winding progresses, the coil *laps* back on itself. It progresses in a continuous loop fashion.

Note that there is one coil between two adjacent commutator bars. Also note that $1/p$ of the total coils of the winding are connected in series between two adjacent brushes and the total voltage induced in these series-connected coils will appear across these two brushes. The brushes making up the positive set are connected together, as are the brushes in the negative set. In a four-pole machine, therefore, there are four parallel paths between the positive and negative terminals of the armature, as shown in Fig. 4.17*b*.

In a lap winding, the number of parallel paths (a) *is always equal to the number of poles* (p), *and also to the number of brushes.*

Wave Winding

The layout of a wave-wound armature winding is shown in Fig. 4.18*a*. The coil arrangement and the end connections are illustrated by the dark lines shown in Fig. 4.18*a* for two coils. One end of the coil starts at commutator bar 2, and the coil sides are placed in slots 7 and 12. The other end of the coil is connected to commutator bar 13. The second coil starts at this commutator bar and is placed in slots 18 and 2, and ends on commutator bar 3. The coil connections are continued in this fashion. The winding is called a wave winding, because the coils are laid down in a wave pattern.

Note that between two adjacent commutator bars there are $p/2$ coils connected in series, as opposed to a single coil in the lap winding. Between two adjacent brushes there are $1/p$ of the total commutator bars. Between two adjacent brushes, therefore, there are $(p/2)(1/p)$, or $\frac{1}{2}$, of all the

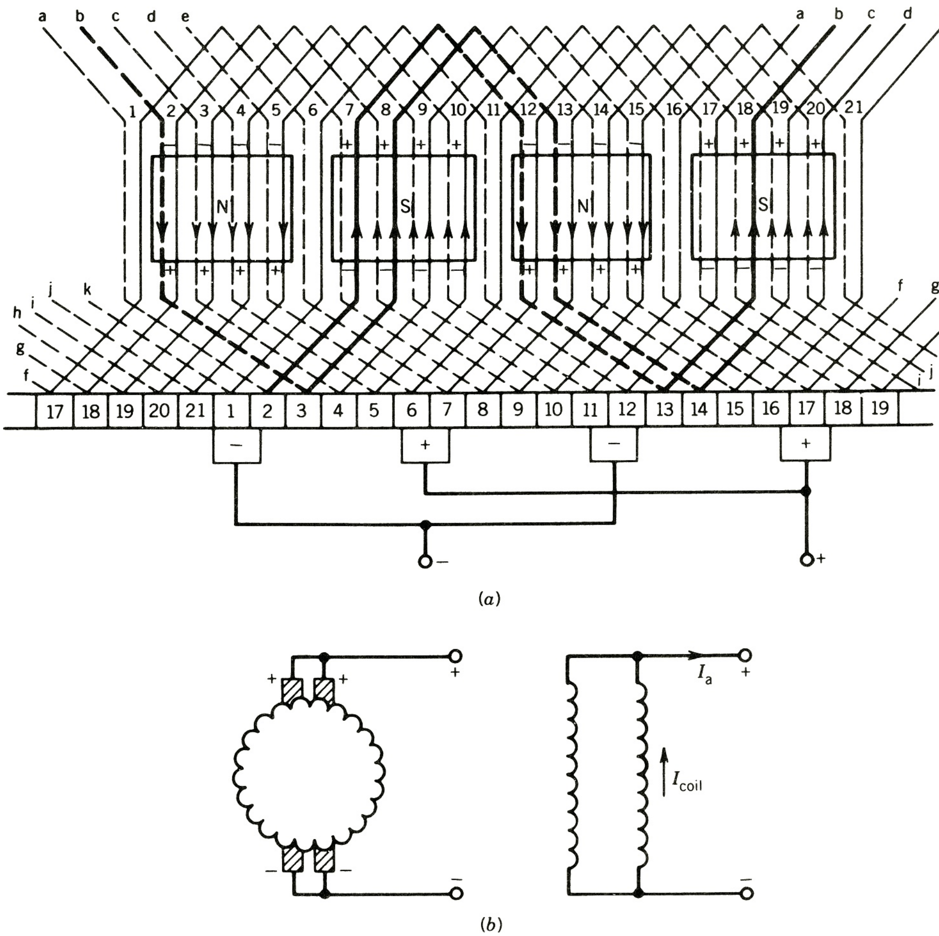


FIGURE 4.18 Wave winding. (*a*) Unrolled winding. (*b*) Equivalent coil representation.

coils. This indicates that in the wave winding, the coils are arranged in two parallel paths, irrespective of the number of poles, as illustrated in Fig. 4.18*b*. Note also in Fig. 4.18*a* that the two brushes of the same polarity are connected essentially to the same point in the winding, except that there is a coil between them. However, between the positive and negative brushes, a large number of coils are connected in series. Although two brush positions are required, one positive and one negative, in a wave winding (and this minimum number is often used in small machines), in large machines more brush positions are used in order to decrease the current density in the brushes.

In wave windings, the number of parallel paths (a) *is always two and there may be two or more brush positions.*

Also note from Figs. 4.17*a* and 4.18*a* that when the coil ends pass the brushes, the current through the coil reverses. This process is known as commutation, and it happens when the coil sides are in the interpolar region. During the time when two adjacent commutator bars make contact with a brush, one coil is shorted by the brush in the lap winding and $p/2$ coils in the wave winding. The effects of these short-circuited coils, undergoing commutation, will be discussed later.

In small dc motors, the armature is machine wound by putting the wire into the slots one turn at a time. In larger motors, the armature winding is composed of prefabricated coils that are placed in the slots.

Because many parallel paths can be provided with a lap winding, it is suitable for high-current, low-voltage dc machines, whereas wave windings having only two parallel paths are suitable for high-voltage, low-current dc machines.

4.2.4 ARMATURE VOLTAGE

As the armature rotates in the magnetic field produced by the stator poles, voltage is induced in the armature winding. In this section an expression will be derived for this induced voltage. We can start by considering the induced voltage in the coils due to change of flux linkage (Faraday's law) or by using the concept of "conductor cutting flux." Both approaches will provide the same expression for the armature voltage.

The waveform of the voltage induced in a turn is shown in Fig. 4.12*b*, and because a turn is made of two conductors, the induced voltage in a turn a–b (Fig. 4.12) from Eq. 4.1 is

$$e_{\mathrm{t}} = 2B(\theta)l\omega_{\mathrm{m}}r \tag{4.4}$$

where l is the length of the conductor in the slot of the armature

ω_{m} is the mechanical speed

r is the distance of the conductor from the center of the armature—that is, the radius of the armature

The average value of the induced voltage in the turn is

$$\bar{e}_{\mathrm{t}} = 2\overline{B(\theta)}l\omega_{\mathrm{m}}r \tag{4.5}$$

Let

$$\Phi = \text{flux per pole}$$

$$\mathrm{A} = \text{area per pole} = \frac{2\pi rl}{p}$$

Then

$$\overline{B(\theta)} = \frac{\Phi}{A} = \frac{\Phi p}{2\pi r l} \tag{4.6}$$

From Eqs. 4.5 and 4.6,

$$\bar{e}_t = \frac{\Phi p}{\pi}\omega_m \tag{4.7}$$

The voltages induced in all the turns connected in series for one parallel path across the positive and negative brushes will contribute to the average terminal voltage E_a. Let

N = total number of turns in the armature winding

a = number of parallel paths

Then

$$E_a = \frac{N}{a}\bar{e}_t \tag{4.8}$$

From Eqs. 4.7 and 4.8,

$$E_a = \frac{Np}{\pi a}\Phi\omega_m$$

$$E_a = K_a\Phi\omega_m \tag{4.9}$$

where K_a is known as the machine (or armature) constant and is given by

$$K_a = \frac{Np}{\pi a} \tag{4.10}$$

or

$$K_a = \frac{Zp}{2\pi a} \tag{4.11}$$

where Z is the total number of conductors in the armature winding. In the MKS system, if Φ is in webers and ω_m in radians per second, then E_a is in volts.

This expression for induced voltage in the armature winding is independent of whether the machine operates as a generator or a motor. In the case of generator operation, it is known as a *generated voltage*, and in motor operation it is known as *back emf (electromotive force)*.

4.2.5 DEVELOPED (OR ELECTROMAGNETIC) TORQUE

There are various methods by which an expression can be derived for the torque developed in the armature (when the armature winding carries current in the magnetic field produced by the stator poles). However, a simple method is to use the concept of Lorentz force, as illustrated by Eq. 4.2.

Consider the turn aa′b′b shown in Fig. 4.19, whose two conductors aa′ and bb′ are placed under two adjacent poles. The force on a conductor (placed on the periphery of the armature) is

$$f_c = B(\theta)li_c = B(\theta)\,l\frac{I_a}{a} \tag{4.12}$$

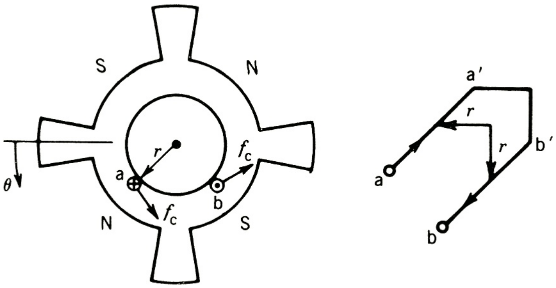


FIGURE 4.19 Torque production in dc machine.

where i_c is the current in the conductor of the armature winding
I_a is the armature terminal current

The torque developed by a conductor is

$$T_c = f_c r \tag{4.13}$$

The average torque developed by a conductor is

$$\overline{T}_c = \overline{B(\theta)}\, l \frac{I_a}{a} r \tag{4.14}$$

From Eqs. 4.6 and 4.14,

$$\overline{T}_c = \frac{\Phi p I_a}{2\pi a} \tag{4.15}$$

All the conductors in the armature winding develop torque in the same direction and thus contribute to the average torque developed by the armature. The total torque developed is

$$T = 2N\overline{T}_c \tag{4.16}$$

From Eqs. 4.15 and 4.16,

$$T = \frac{N\Phi p}{\pi a} I_a = K_a \Phi I_a \tag{4.17}$$

In the case of motor action, the electrical power input ($E_a I_a$) to the magnetic field by the electrical system must be equal to the mechanical power ($T\omega_m$) developed and withdrawn from the field by the mechanical system. The converse is true for generator action. This is confirmed from Eqs. 4.9 and 4.17.

$$\text{Electrical power,} \quad E_a I_a = K_a \Phi \omega_m I_a = T\omega_m, \quad \text{mechanical power} \tag{4.17a}$$

EXAMPLE 4.1

A four-pole dc machine has an armature of radius 12.5 cm and an effective length of 25 cm. The poles cover 75% of the armature periphery. The armature winding consists of 33 coils, each coil having seven turns. The coils are accommodated in 33 slots. The average flux density under each pole is 0.75 T.

1. If the armature is lap-wound,
   (a) Determine the armature constant K_a.
   (b) Determine the induced armature voltage when the armature rotates at 1000 rpm.
   (c) Determine the current in the coil and the electromagnetic torque developed when the armature current is 400 A.
   (d) Determine the power developed by the armature.
2. If the armature is wave-wound, repeat parts (a) to (d) above. The current rating of the coils remains the same as in the lap-wound armature.

Solution

1. Lap-wound dc machine

(a)

$$K_a = \frac{Np}{\pi a} = \frac{Z\,p}{2a\,\pi}$$

$$Z = 2 \times 33 \times 7 = 462, \qquad a = p = 4$$

$$K_a = \frac{462 \times 4}{2 \times 4 \times \pi} = 73.53$$

(b)

$$\text{Pole area, } A_p = \frac{2\pi \times 0.125 \times 0.25 \times 0.75}{4}$$

$$= 36.8 \times 10^{-3}\text{m}^2$$

$$\Phi = A_p \times B = 36.8 \times 10^{-3} \times 0.75$$

$$= 0.0276 \text{ Wb}$$

$$E_a = K_a \Phi \omega_m = 73.53 \times 0.0276 \times \frac{1000}{60} \times 2\pi$$

$$= 212.5 \text{ V}$$

(c)

$$I_{coil} = \frac{I_a}{a} = \frac{400}{4} = 100 \text{ A}$$

$$T = K_a \Phi I_a = 73.53 \times 0.0276 \times 400 = 811.8 \text{ N} \cdot \text{m}$$

(d)

$$P_a = E_a I_a = 212.5 \times 400 = 85.0 \text{ kW}$$

or

$$= T\omega_m = 811.8 \times \frac{1000}{60} \times 2\pi = 85.0 \text{ kW}$$

2. Wave-wound dc machine

$$p = 4, \qquad a = 2, \qquad Z = 462$$

(a)

$$K_a = \frac{462 \times 4}{2 \times 2 \times \pi} = 147.06$$

$$\omega_m = \frac{1000}{60} \times 2\pi = 104.67 \text{ rad/sec}$$

(b)

$$E_a = 147.06 \times 0.0276 \times 104.67 = 425 \text{ V}$$

(c)

$$I_{coil} = 100 \text{ A}$$

$$I_a = 2 \times 100 = 200 \text{ A}$$

$$T = 147.06 \times 0.0276 \times 200 = 811.8 \text{ N} \cdot \text{m}$$

(d)

$$P_a = 425 \times 200 = 85.0 \text{ kW} \quad \blacksquare$$

4.2.6 MAGNETIZATION (OR SATURATION) CURVE OF A DC MACHINE

A dc machine has two distinct circuits, a field circuit and an armature circuit. The mmfs produced by these two circuits are at quadrature—the field mmf is along the direct axis and the armature mmf is along the quadrature axis. A simple schematic representation of the dc machine is shown in Fig. 4.20.

The flux per pole of the machine will depend on the ampere turns F_p provided by one or more field windings on the poles and the reluctance $\mathcal{R}$ of the magnetic path. The magnetic circuit of a two-pole dc machine is shown in Fig. 4.21*a*. The flux passes through the pole, air gap, rotor teeth, rotor core, rotor teeth, air gap, and opposite pole and returns through the yoke of the stator of the machine. The magnetic equivalent circuit is shown in Fig. 4.21*b*, where different sections of the magnetic system in which the flux density can be considered reasonably uniform are represented by separate reluctances.

The magnetic flux Φ that crosses the air gap under each pole depends on the magnetomotive force F_p (hence the field current) of the coils on each pole. At low values of flux Φ, the magnetic

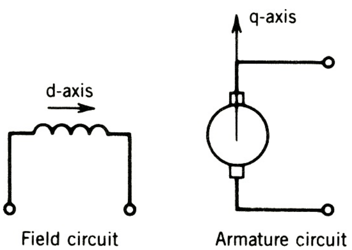


FIGURE 4.20 DC machine representation.

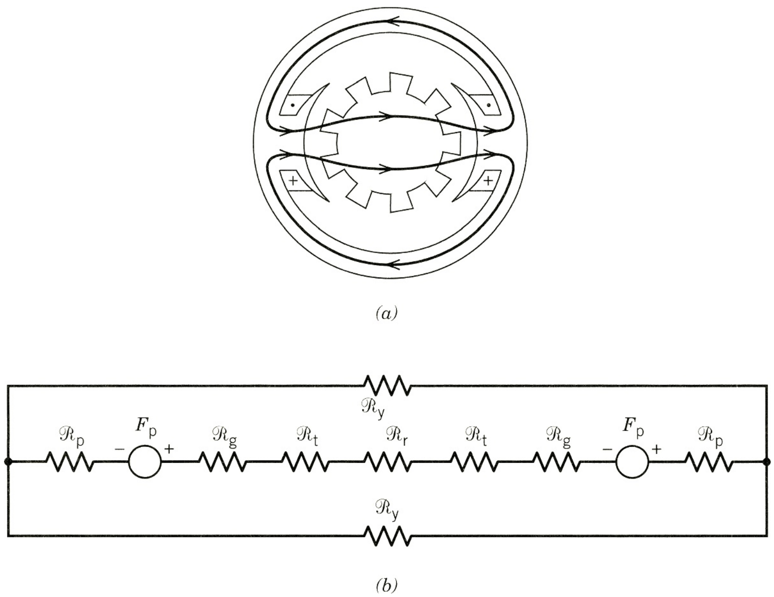


FIGURE 4.21 Magnetic circuit. (*a*) Cross-sectional view. (*b*) Equivalent circuit.

material may be considered to have infinite permeability, making the reluctances for magnetic core sections zero. The magnetic flux in each pole is then

$$\Phi = \frac{2F_p}{2\mathscr{R}_g} = \frac{F_p}{\mathscr{R}_g} \tag{4.18}$$

If F_p is increased, flux Φ will increase and saturation will occur in various parts of the magnetic circuit, particularly in the rotor teeth. The relationship between field excitation mmf F_p and flux Φ in each pole is shown in Fig. 4.22. With no field excitation, the flux in the pole is the residual flux left over from the previous operation. As the field excitation is increased, the flux increases linearly, as long as the reluctance of the iron core is negligible compared with that of the air gap. Further increase in the field excitation will result in saturation of the iron core, and the flux increase will no longer be linear with the field excitation. It is assumed here that the armature mmf has no effect on the pole flux (d-axis flux), because the armature mmf acts along the q-axis. We shall reexamine this assumption later on.

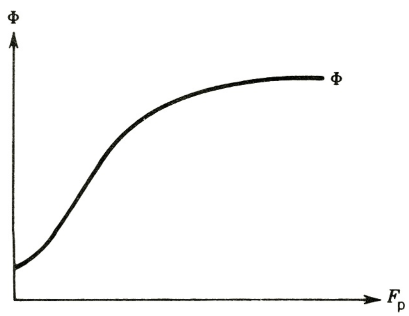


FIGURE 4.22 Flux–mmf relation in a dc machine.

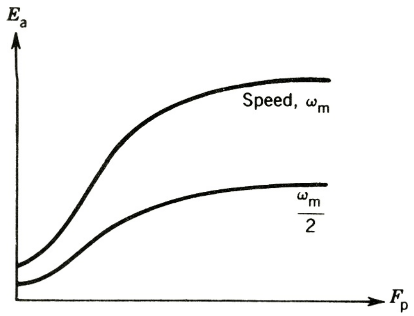


FIGURE 4.23 Magnetization curve.

The induced voltage in the armature winding is proportional to flux times speed (Eq. 4.9). It is more convenient if the magnetization curve is expressed in terms of armature induced voltage E_a at a particular speed. This is shown in Fig. 4.23. This curve can be obtained by performing tests on a dc machine. Figure 4.24 shows the magnetization curve obtained experimentally by rotating the dc machine at 1000 rpm and measuring the open-circuit armature terminal voltage as the current in the field winding is changed. This magnetization curve is of great importance, because it represents the saturation level in the magnetic system of the dc machine for various values of the excitation mmf.

4.2.7 CLASSIFICATION OF DC MACHINES

The field circuit and the armature circuit can be interconnected in various ways to provide a wide variety of performance characteristics—an outstanding advantage of dc machines. Also, the field poles can be excited by two field windings, a *shunt field winding* and a *series field winding*. The shunt winding has a large number of turns and takes only a small current (less than 5% of the rated armature current). A picture of a shunt winding is shown in Fig. 4.25. This winding can be connected across the armature (i.e., parallel with it), hence the name shunt winding. The series winding has fewer turns but carries a large current. It is connected in

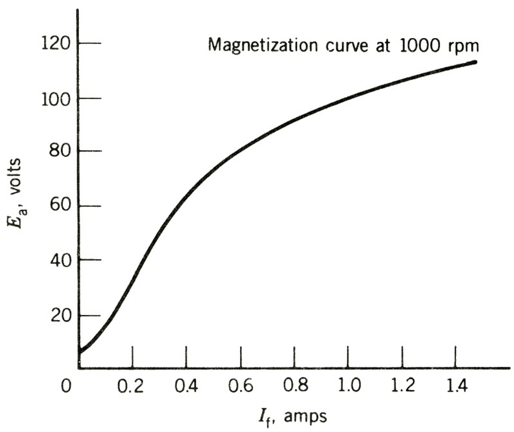


FIGURE 4.24 Test result: magnetization curve.

Courtesy of General Electric Canada Inc.

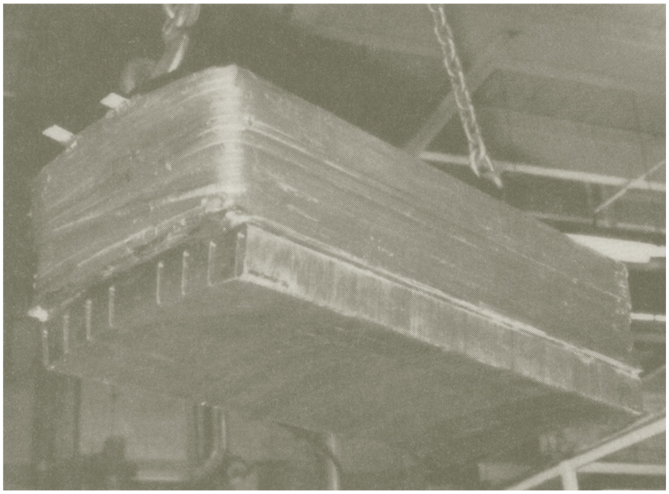


FIGURE 4.25 Shunt field winding.

series with the armature, hence the name series winding. If both shunt and series windings are present, the series winding is wound on top of the shunt winding, as shown in Fig. 4.26.

The various connections of the field circuit and armature circuit are shown in Fig. 4.27. In the *separately excited* dc machine (Fig. 4.27*a*), the field winding is excited from a separate source. In the *self-excited* dc machine, the field winding can be connected in three different ways. The field winding may be connected in series with the armature (Fig. 4.21*b*), resulting in a series dc machine; it may be connected across the armature (i.e., in shunt), resulting in a *shunt machine* (Fig. 4.27*c*); or both shunt and series windings may be used (Fig. 4.27*d*), resulting in a compound machine. If the shunt winding is connected across the armature, it is known as *short-shunt* machine. In an alternative connection, the shunt winding is connected across the series connection of armature and series winding, and the machine is known as *long-shunt* machine. There is no significant difference between these two connections, which are shown in Fig. 4.27*d*. In the compound machine, the series winding mmf may aid or oppose the shunt winding mmf, resulting in different performance characteristics.

Courtesy of General Electric Canada Inc.

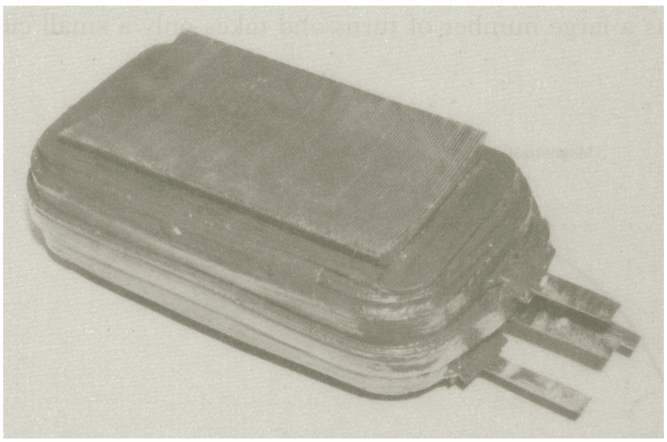


FIGURE 4.26 Series winding on top of shunt winding.

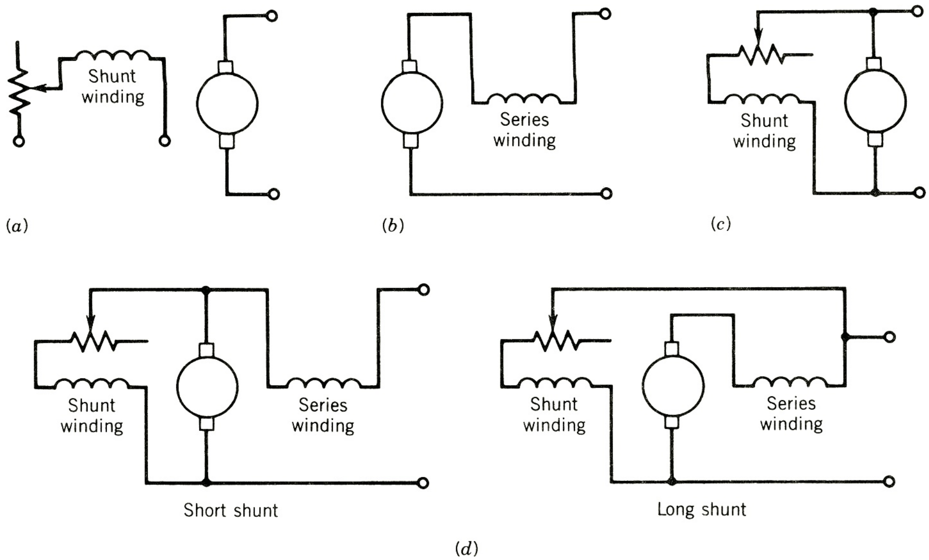


FIGURE 4.27 Different connections of dc machines. (*a*) Separately excited dc machine. (*b*) Series dc machine. (*c*) Shunt dc machine. (*d*) Compound dc machine.

A rheostat is normally included in the circuit of the shunt winding to control the field current and thereby to vary the field mmf.

Field excitation may also be provided by permanent magnets. This may be considered as a form of separately excited machine, the permanent magnet providing the separate but constant excitation.

In the following sections the operation of the various dc machines, first as generators and then as motors, will be studied.

4.3 DC GENERATORS

The dc machine operating as a generator is driven by a prime mover at a constant speed and the armature terminals are connected to a load. In many applications of dc generators, knowledge of the variation of the terminal voltage with load current, known as the *external* or (*terminal*) characteristic, is essential.

4.3.1 SEPARATELY EXCITED DC GENERATOR

As stated in Section 4.2.7, in the separately excited dc generator, the field winding is connected to a separate source of dc power. This source may be another dc generator, a controlled rectifier, or a diode rectifier, or a battery. The steady-state model of the separately excited dc generator is shown in Fig. 4.28. In this model

R_{fw} is the resistance of the field winding.

R_{fc} is the resistance of the control rheostat used in the field circuit.

$R_f = R_{fw} + R_{fc}$ is the total field circuit resistance.

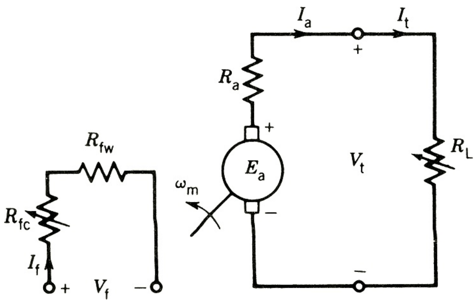


FIGURE 4.28 Steady-state model of a separately excited dc generator.

R_a is the resistance of the armature circuit, including the effects of the brushes. Sometimes R_a is shown as the resistance of the armature winding alone; the brush–contact voltage drop is considered separately and is usually assumed to be about 2 V.

R_L is the resistance of the load.

In the steady-state model, the inductances of the field winding and armature winding are not considered.

The defining equations are the following:

$$V_f = R_f I_f \tag{4.19}$$

$$E_a = V_t + I_a R_a \tag{4.20}$$

$$E_a = K_a \Phi \omega_m \tag{4.21}$$

$$V_t = I_t R_L \tag{4.22}$$

$$I_a = I_t \tag{4.23}$$

From Eq. 4.20,

$$V_t = E_a - R_a I_a \tag{4.24}$$

Equation 4.20 defines the terminal or external characteristic of the separately excited dc generator; the characteristic is shown in Fig. 4.29. As the terminal (i.e., load) current I_t increases,

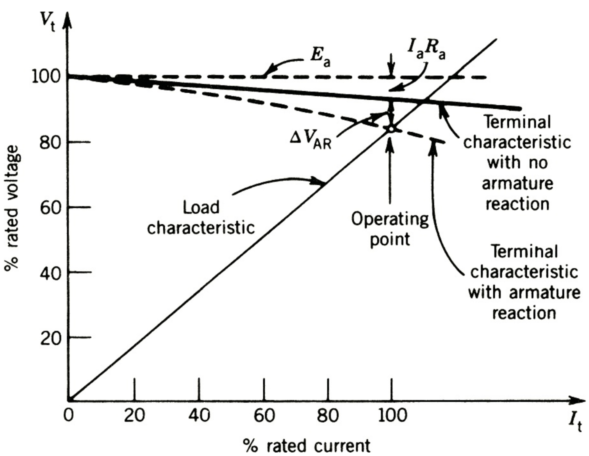


FIGURE 4.29 Terminal characteristic of a separately excited dc generator.

the terminal voltage V_t decreases linearly (assuming E_a remains constant) because of the voltage drop across R_a. This voltage drop $I_a R_a$ is small, because the resistance of the armature circuit R_a is small. A separately excited dc generator maintains an essentially constant terminal voltage.

At high values of the armature current a further voltage drop (ΔV_{AR}) occurs in the terminal voltage; that is known as *armature reaction* (or the *demagnetization effect*) and causes a divergence from the linear relationship. This effect can be neglected for armature currents below the rated current. It will be discussed in the next section.

The load characteristic, defined by Eq. 4.22, is also shown in Fig. 4.29. The point of intersection between the generator external characteristic and the load characteristic determines the operating point—that is, the operating values of the terminal voltage V_t and the terminal current I_t.

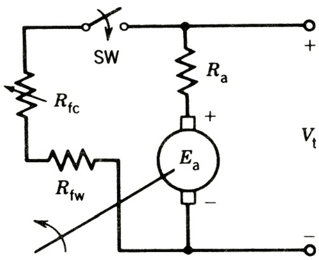


FIGURE 4.34 Schematic of a shunt or self-excited dc machine.

4.3.2 SHUNT (SELF-EXCITED) GENERATOR

In the shunt or self-excited generator the field is connected across the armature so that the armature voltage can supply the field current. Under certain conditions, to be discussed here, this generator will build up a desired terminal voltage.

The circuit for the shunt generator under no-load conditions is shown in Fig. 4.34. If the machine is to operate as a self-excited generator, some residual magnetism must exist in the magnetic circuit of the generator. Figure 4.35 shows the magnetization curve of the dc machine. Also shown in this figure is the *field resistance line*, which is a plot of $R_f I_f$ versus I_f. A simplistic explanation of the voltage buildup process in the self-excited dc generator is as follows.

Assume that the field circuit is initially disconnected from the armature circuit and the armature is driven at a certain speed. A small voltage, E_{ar}, will appear across the armature terminals because of the residual magnetism in the machine. If the switch SW is now closed (Fig. 4.34) and the field circuit is connected to the armature circuit, a current will flow in the field winding. If the mmf of this field current aids the residual magnetism, eventually a current I_{f1} will flow in the field circuit. The buildup of this current will depend on the time constant of the field circuit. With I_{f1} flowing in the field circuit, the generated voltage is E_{a1}—from the magnetization curve—but the terminal voltage is $V_t = I_{f1} R_f < E_{a1}$. The increased armature voltage E_{a1} will eventually increase the field current to the value I_{f2}, which in turn will build

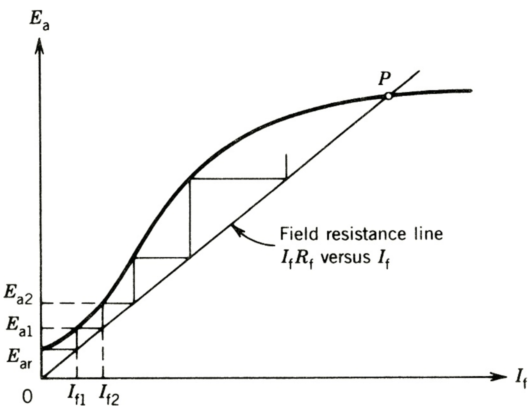


FIGURE 4.35 Voltage buildup in a self-excited dc generator.

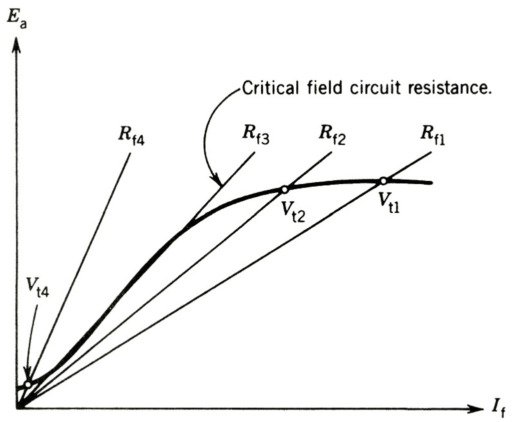


FIGURE 4.36 Effect of field resistance.

up the armature voltage to E_{a2}. This process of voltage buildup continues. If the voltage drop across R_a is neglected (i.e., $R_a \ll R_f$), the voltage builds up to the value given by the crossing point (P in Fig. 4.35) of the magnetization curve and the field resistance line. At this point, $E_a = I_f R_f = V_t$ (assume R_a is neglected), and no excess voltage is available to further increase the field current. In the actual case, the changes in I_f and E_a take place simultaneously and the voltage buildup follows approximately the magnetization curve, instead of climbing the flight of stairs.

Figure 4.36 shows the voltage buildup in the self-excited dc generator for various field circuit resistances. At some resistance value R_{f3}, the resistance line is almost coincident with the linear portion of the magnetization curve. This coincidence condition results in an unstable voltage situation. This resistance is known as the *critical field circuit resistance*. If the resistance is greater than this value, such as R_{f4}, buildup (V_{t4}) will be insignificant. On the other hand, if the resistance is smaller than this value, such as R_{f1} or R_{f2}, the generator will build up higher voltages (V_{t1},V_{t2}). To sum up, three conditions are to be satisfied for voltage buildup in a self-excited dc generator:

1. Residual magnetism must be present in the magnetic system.
2. Field winding mmf should aid the residual magnetism.
3. Field circuit resistance should be less than the critical field circuit resistance.

EXAMPLE 4.3

The dc machine in Example 4.2 is operated as a self-excited (shunt) generator at no load.

(a) Determine the maximum value of the generated voltage.

(b) Determine the value of the field circuit control resistance (R_{fc}) required to generate rated terminal voltage.

(c) Determine the value of the critical field circuit resistance.

Solution

(a) The maximum voltage will be generated at the lowest value of the field circuit resistance, $R_{fc} = 0$. Draw a field resistance line (Fig. E4.3*b*) for $R_f = R_{fw} = 80\ \Omega$. The maximum generated voltage is

$$E_a = 111 \text{ volts}$$

(b)

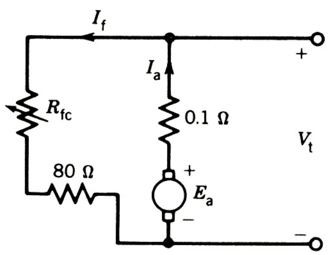
$$\begin{aligned} V_t &= E_a - I_a R_a \\ &\simeq E_a \\ &= 100 \text{ V} \end{aligned}$$

Draw a field resistance line that intersects the magnetization curve at 100 V (Fig. E4.3*b*). For this case,

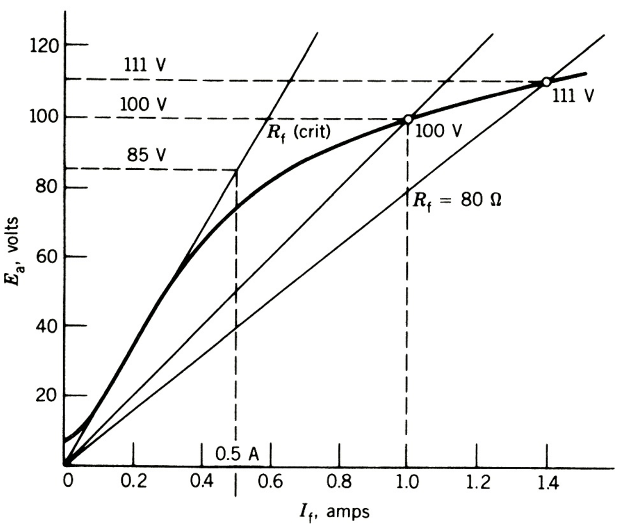
$$I_f = 1 \text{ A}$$

$$R_f = \frac{100}{1} = 100\ \Omega = R_{fw} + R_{fc}$$

$$R_{fc} = 100 - 80 = 20\ \Omega$$



(*a*)



(*b*)

FIGURE E4.3

provides axial flux through the armature. The radial current flowing through the disk armature interacts with the axial flux to produce torque that rotates the rotor, as in any dc motor.

This type of motor has several advantages:

Because of its low rotor inertia, it has a high torque/inertia ratio and thus can provide rapid acceleration and deceleration. The motor can accelerate from 0 to 4000 rpm in 10 milliseconds.

The armature inductance is low, because there is no iron in the rotor. Because of the low inductance, there is little arcing, which leads to longer brush life and high-speed capability. Low armature inductance makes the armature time constant low. Consequently, the armature current can build up very quickly (in less than 1 millisecond), which implies that full torque is available almost instantly, a key to quick motion response and accurate tracking.

The motor has no cogging torque, because the rotor is nonmagnetic.

These motors are particularly suitable for applications requiring high performance characteristics. Examples are high-speed tape readers, $X-Y$ recorders, point-to-point tool positioners, robots, and other servo drives. Typical sizes of these motors are in the fractional horsepower ranges. However, integral horsepower sizes are also available.

PROBLEMS

4.1 Two dc machines of the following rating are required:

DC machine 1: 120 V, 1500 rpm, four poles

DC machine 2: 240 V, 1500 rpm, four poles

Coils are available that are rated at 4 volts and 5 amperes. For the same number of coils to be used for both machines, determine the

(a) Type of armature winding for each machine.

(b) Number of coils required for each machine.

(c) kW rating of each machine.

4.2 A four-pole dc machine has a wave winding of 300 turns. The flux per pole is 0.025 Wb. The dc machine rotates at 1000 rpm.

(a) Determine the generated voltage.

(b) Determine the kW rating if the rated current through the turn is 25 A.

4.3 A dc machine (6 kW, 120 V, 1200 rpm) has the following magnetization characteristics at 1200 rpm.

I_f	(A)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2
E_a	(V)	5	20	40	60	79	93	102	114	120	125

The machine parameters are $R_a = 0.2\ \Omega$, $R_{fw} = 100\ \Omega$. The machine is driven at 1200 rpm and is separately excited. The field current is adjusted at $I_f = 0.8$ A. A load resistance $R_L = 2\ \Omega$ is connected to the armature terminals. Neglect armature reaction effect.

(a) Determine the quantity $K_a\Phi$ for the machine.

(b) Determine E_a and I_a.

(c) Determine torque T and load power P_L.

4.4 Repeat Problem 4.3 if the speed is 800 rpm.

4.5 The dc generator in Problem 4.3 rotates at 1500 rpm, and it delivers rated current at rated terminal voltage. The field winding is connected to a 120 V supply.

(a) Determine the value of the field current.

(b) Determine the value of R_{fc} required.

4.6 The dc machine in Problem 4.3 has a field control resistance whose value can be changed from 0 to 150 Ω. The machine is driven at 1200 rpm. The machine is separately excited, and the field winding is supplied from a 120 V supply.

(a) Determine the maximum and minimum values of the no-load terminal voltage.

(b) The field control resistance (R_{fc}) is adjusted to provide a no-load terminal voltage of 120 V. Determine the value of R_{fc}. Determine the terminal voltage at full load for no armature reaction and also if $I_{f(AR)} = 0.1$ A.

4.7 Repeat Problem 4.6 if the speed is 1500 rpm.

4.8 The dc machine in Problem 4.3 is separately excited. The machine is driven at 1200 rpm and operates as a generator. The rotational loss is 400 W at 1200 rpm, and the rotational loss is proportional to speed.

(a) For a field current of 1.0 A, with the generator delivering rated current, determine the terminal voltage, the output power, and the efficiency.

(b) Repeat part (a) if the generator is driven at 1500 rpm.

4.9 The dc machine in Problem 4.6 is self-excited.

(a) Determine the maximum and minimum values of the no-load terminal voltage.

(b) R_{fc} is adjusted to provide a no-load terminal voltage of 120 V. Determine the value of R_{fc}.

(i) Assume no armature reaction. Determine the terminal voltage at rated armature current. Determine the maximum current the armature can deliver. What is the terminal voltage for this situation?

(ii) Assume that $I_{f(AR)} = 0.1$ A at $I_a = 50$ A and consider armature reaction proportional to armature current. Repeat part (i).

4.10 A dc machine (10 kW, 250 V, 1000 rpm) has $R_a = 0.2$ Ω and $R_{fw} = 133$ Ω. The machine is self-excited and is driven at 1000 rpm. The data for the magnetization curve are

I_f	(A)	0	0.1	0.2	0.3	0.4	0.5	0.75	1.0	1.5	2.0
E_a	(V)	10	40	80	120	150	170	200	220	245	263

(a) Determine the generated voltage with no field current.

(b) Determine the critical field circuit resistance.

(c) Determine the value of the field control resistance (R_{fc}) if the no-load terminal voltage is 250 V.

chapter five

INDUCTION (ASYNCHRONOUS) MACHINES

The induction machine is the most rugged and the most widely used machine in industry. Like the dc machine discussed in the preceding chapter, the induction machine has a stator and a rotor mounted on bearings and separated from the stator by an air gap. However, in the induction machine both stator winding and rotor winding carry alternating currents. The alternating current (ac) is supplied to the stator winding directly and to the rotor winding by induction—hence the name induction machine.

The induction machine can operate both as a motor and as a generator. However, it is seldom used as a generator supplying electrical power to a load. The performance characteristics as a generator are not satisfactory for most applications. The induction machine is extensively used as a motor in many applications.

The induction motor is used in various sizes. Small single-phase induction motors (in fractional horsepower rating; see Chapter 7) are used in many household appliances, such as blenders, lawn mowers, juice mixers, washing machines, refrigerators, and stereo turntables.

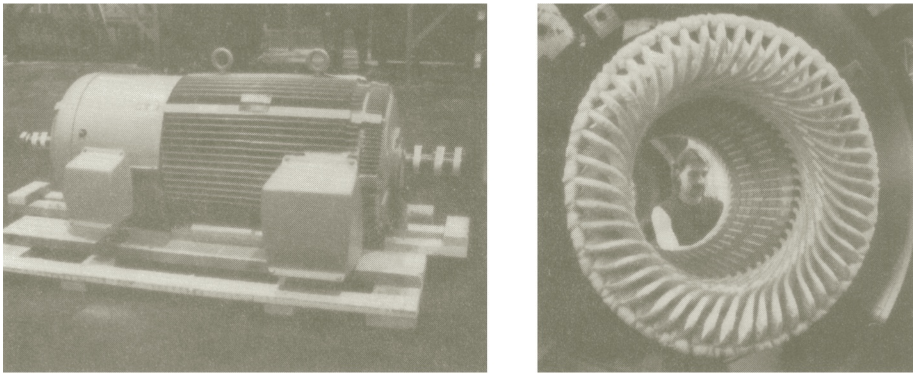
Large three-phase induction motors (in tens or hundreds of horsepower) are used in pumps, fans, compressors, paper mills, textile mills, and so forth.

The linear version of the induction machine has been developed primarily for use in transportation systems.

The induction machine is undoubtedly a very useful electrical machine. Single-phase induction motors are discussed in Chapter 7. Two-phase induction motors are used primarily as servomotors in a control system. These motors are discussed in Chapter 8. Three-phase induction motors are the most important ones, and are most widely used in industry. In this chapter, the operation, characteristic features, and steady-state performance of the three-phase induction machine are studied in detail.

5.1 CONSTRUCTIONAL FEATURES

Unlike dc machines, induction machines have a uniform air gap. A pictorial view of the three-phase induction machine is shown in Fig. 5.1*a*. The stator is composed of laminations of high-grade sheet steel. A three-phase winding is put in slots cut on the inner surface of the stator frame as shown in Fig. 5.1*b*. The rotor also consists of laminated ferromagnetic material, with slots cut on the outer surface. The rotor winding may be either of two types, the *squirrel-cage type* or the *wound-rotor type*. The squirrel-cage winding consists of aluminum or copper bars embedded in the rotor slots and shorted at both ends by aluminum or copper end rings as shown in Fig. 5.2*a*. The wound-rotor winding has the same form as the stator winding. The terminals of the rotor winding are connected to three slip rings, as shown in Fig. 5.2*b*. Using stationary



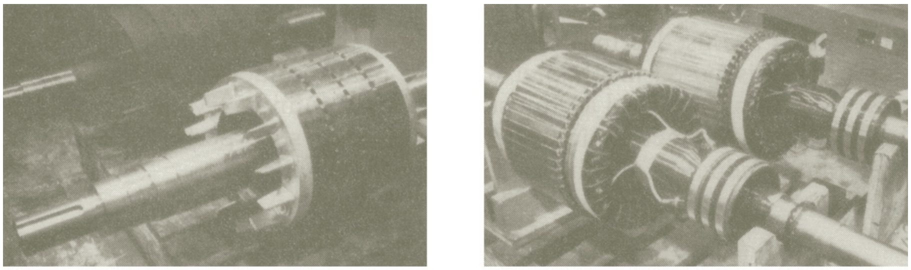
Courtesy of Westinghouse Canada Inc.

(a)

Courtesy of Westinghouse Canada Inc.

(b)

FIGURE 5.1 Three-phase induction machine. (*a*) Induction machine with enclosure. (*b*) Stator with three-phase winding.



Courtesy of Westinghouse Canada Inc.

(a)

Courtesy of Westinghouse Canada Inc.

(b)

FIGURE 5.2 Rotor of an induction machine. (*a*) Squirrel-cage rotor. (*b*) Wound-rotor type.

brushes pressing against the slip rings, the rotor terminals can be connected to an external circuit. In fact, an external three-phase resistor can thus be connected for the purpose of speed control of the induction motor, as shown in Fig. 5.39 and discussed in Section 5.13.6. It is obvious that the squirrel-cage induction machine is simpler, more economical, and more rugged than the wound-rotor induction machine.

The three-phase winding on the stator and on the rotor (in the wound-rotor type) is a distributed winding. Such windings make better use of iron and copper and also improve the mmf waveform and smooth out the torque developed by the machine. The winding of each phase is distributed over several slots. When current flows through a distributed winding, it produces an essentially sinusoidal space distribution of mmf. The properties of a distributed winding are discussed in Appendix A.

Figure 5.3*a* shows a cross-sectional view of a three-phase squirrel-cage induction machine. The three-phase stator winding, which in practice would be a distributed winding, is represented by three concentrated coils for simplicity. The axes of these coils are 120 electrical degrees apart. Coil aa′ represents all the distributed coils assigned to the phase-a winding for one pair of poles. Similarly, coil bb′ represents the phase-b distributed winding, and coil cc′

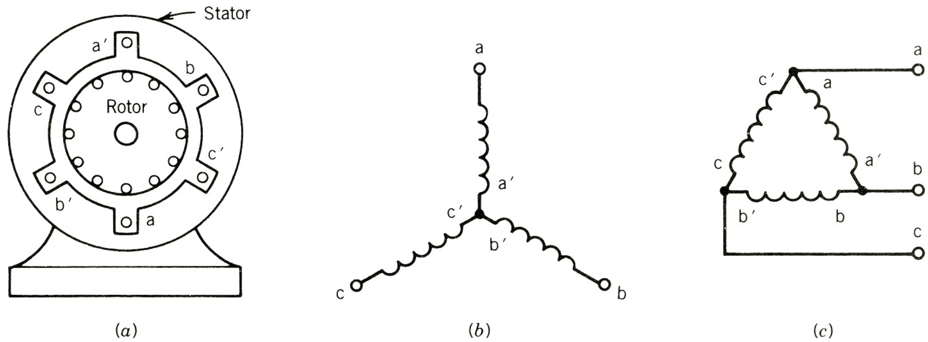


FIGURE 5.3 Three-phase squirrel-cage induction machine. (*a*) Cross-sectional view. (*b*) Y-connected stator winding. (*c*) Δ-connected stator winding.

represents the phase-c distributed winding. The ends of these phase windings can be connected in a wye (Fig. 5.3*b*) or a delta (Fig. 5.3*c*) to form the three-phase connection. As shown in the next section, if balanced three-phase currents flow through these three-phase distributed windings, a rotating magnetic field of constant amplitude and speed will be produced in the air gap and will induce current in the rotor circuit to produce torque.

5.2 ROTATING MAGNETIC FIELD

In this section we study the magnetic field produced by currents flowing in the polyphase windings of an ac machine. In Fig. 5.4*a* the three-phase windings, represented by aa′, bb′, and cc′, are displaced from each other by 120 electrical degrees in space around the inner circumference of the stator. A two-pole machine is considered. The concentrated coils represent the actual distributed windings. When a current flows through a phase coil, it produces a sinusoidally distributed mmf wave centered on the axis of the coil representing the phase winding. If an alternating current flows through the coil, it produces a pulsating mmf wave, whose amplitude and direction depend on the instantaneous value of the current flowing through the winding. Figure 5.4*b* illustrates the mmf distribution in space at various instants due to an alternating current flow in coil aa′. Each phase winding will produce similar sinusoidally distributed mmf waves, but displaced by 120 electrical degrees in space from each other.

Let us now consider a balanced three-phase current flowing through the three-phase windings. The currents are

$$i_{\mathrm{a}} = I_{\mathrm{m}} \cos \omega t \tag{5.1}$$

$$i_{\mathrm{b}} = I_{\mathrm{m}} \cos(\omega t - 120^\circ) \tag{5.2}$$

$$i_{\mathrm{c}} = I_{\mathrm{m}} \cos(\omega t + 120^\circ) \tag{5.3}$$

These instantaneous currents are shown in Fig. 5.4*c*. The reference directions, when positive-phase currents flow through the windings, are shown by dots and crosses in the coil sides in Fig. 5.4*a*. When these currents flow through the respective phase windings, each produces a sinusoidally distributed mmf wave in space, pulsating along its axis and having a peak located

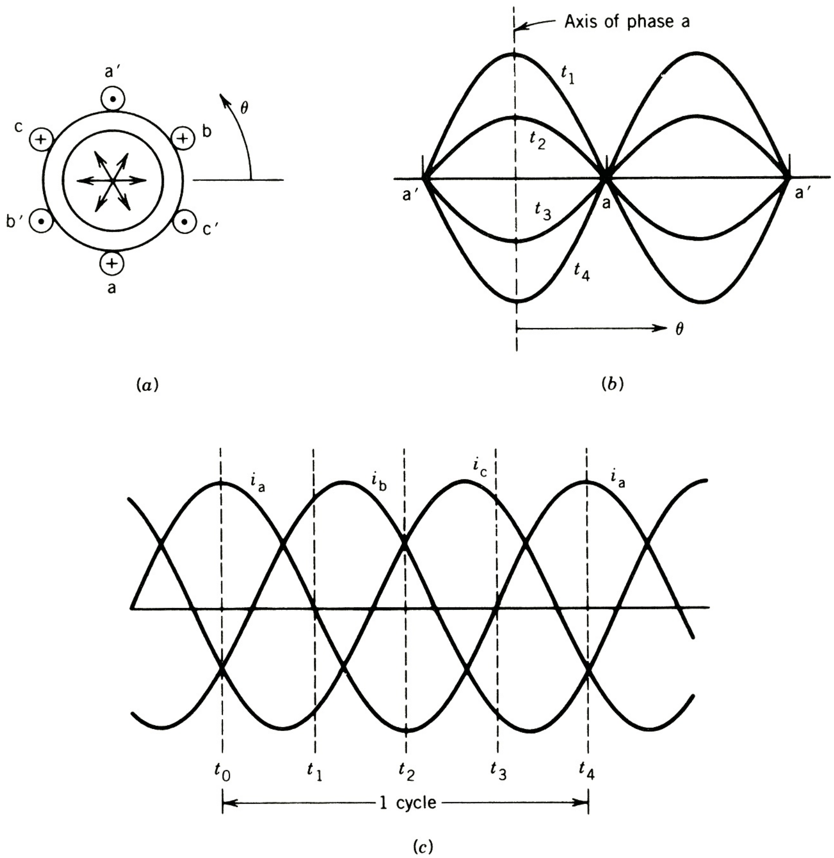


FIGURE 5.4 Pulsating mmf.

along the axis. Each mmf wave can be represented by a space vector along the axis of its phase with magnitude proportional to the instantaneous value of the current. The resultant mmf wave is the net effect of the three component mmf waves, which can be computed either graphically or analytically.

5.2.1 GRAPHICAL METHOD

Let us consider situations at several instants of time and find out the magnitude and direction of the resultant mmf wave. From Fig. 5.4*c*, at instant $t = t_0$, the currents in the phase windings are as follows:

$$i_a = I_m \qquad \text{flowing in phase winding a} \tag{5.4}$$

$$i_b = -\frac{I_m}{2} \qquad \text{flowing in phase winding b} \tag{5.5}$$

$$i_c = -\frac{I_m}{2} \qquad \text{flowing in phase winding c} \tag{5.6}$$

The current directions in the representative coils are shown in Fig. 5.5*a* by dots and crosses. Because the current in the phase-a winding is at its maximum, its mmf has its maximum value and is represented by a vector $F_a = F_{max}$ along the axis of phase a, as shown in Fig. 5.5*a*. The mmfs of phases b and c are shown by vectors F_a and F_c, respectively, each having magnitude $F_{max}/2$ and shown in the negative direction along their respective axes. The resultant of the three vectors is a vector $F = \frac{3}{2}F_{max}$ acting in the positive direction along the phase-a axis. Therefore, at this instant, the resultant mmf wave is a sinusoidally distributed wave, which is the same as that due to phase-a mmf alone, but with $\frac{3}{2}$ the amplitude of the phase-a mmf wave. The component mmf waves and the resultant mmf wave at this instant ($t = t_0$) are shown in Fig. 5.5*b*.

At a later instant of time t_1 (Fig. 5.4*c*), the currents and mmfs are as follows:

$$i_a = 0, \qquad F_a = 0 \tag{5.7}$$

$$i_b = \frac{\sqrt{3}}{2} I_m, \qquad F_b = \frac{\sqrt{3}}{2} F_{max} \tag{5.8}$$

$$i_c = \frac{\sqrt{3}}{2} I_m, \qquad F_c = \frac{\sqrt{3}}{2} F_{max} \tag{5.9}$$

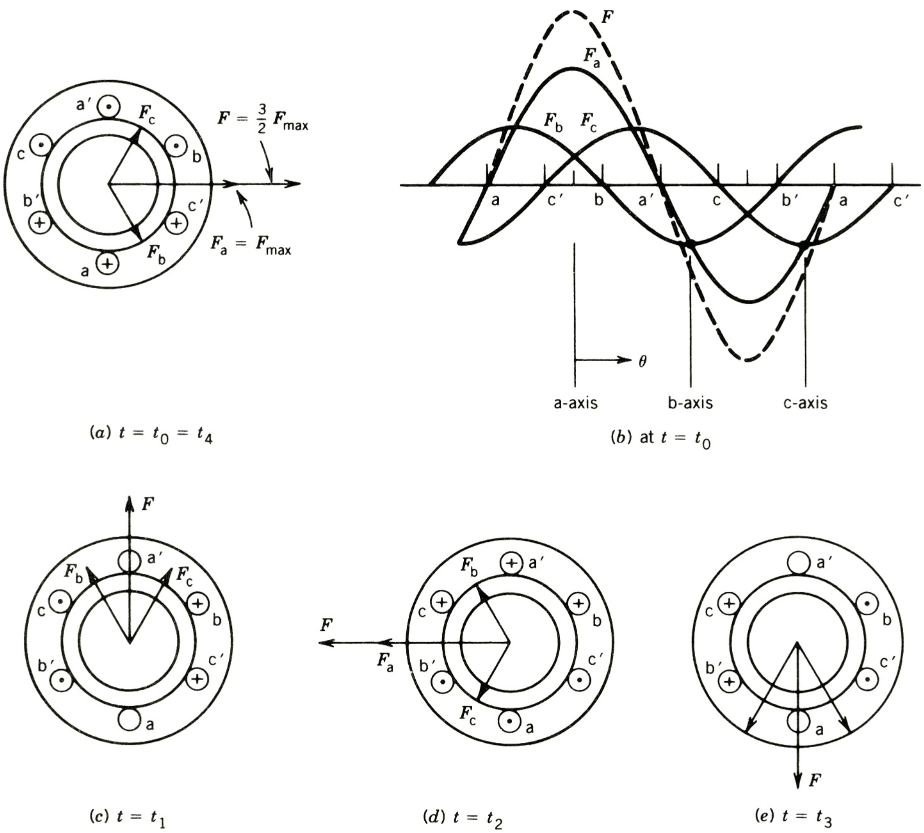


FIGURE 5.5 Rotating magnetic field by graphical method: mmfs at various instants.

The current directions, component mmf vectors, and resultant mmf vector are shown in Fig. 5.5*c*. Note that the resultant mmf vector F has the same amplitude $\frac{3}{2}F_{\max}$ at $t = t_1$ as it had at $t = t_0$, but it has rotated counterclockwise by 90° (electrical degrees) in space.

Currents at other instants $t = t_2$ and $t = t_3$ are also considered, and their effects on the resultant mmf vector are shown in Figs. 5.5*d* and 5.5*e*, respectively. It is obvious that as time passes, the resultant mmf wave retains its sinusoidal distribution in space with constant amplitude, but moves around the air gap. In one cycle of the current variation, the resultant mmf wave comes back to the position of Fig. 5.5*a*. Therefore, the resultant mmf wave makes one revolution per cycle of the current variation in a two-pole machine. In a p-pole machine, one cycle of variation of the current will make the mmf wave rotate by $2/p$ revolutions. The revolutions per minute n (rpm) of the traveling wave in a p-pole machine for a frequency f cycles per second for the currents are

$$n = \frac{2}{p} f 60 = \frac{120f}{p} \tag{5.10}$$

It can be shown that if i_a flows through the phase-a winding but i_b flows through the phase-c winding and i_c flows through the phase-b winding, the traveling mmf wave will rotate in the clockwise direction. Thus, a reversal of the phase sequence of the currents in the windings makes the rotating mmf rotate in the opposite direction.

5.2.2 ANALYTICAL METHOD

Again we shall consider a two-pole machine with three phase windings on the stator. An analytical expression will be obtained for the resultant mmf wave at any point in the air gap, defined by an angle θ. The origin of the angle θ can be chosen to be the axis of phase a, as shown in Fig. 5.6*a*. At any instant of time, all three phases contribute to the air gap mmf along the path defined by θ. The mmf along θ is

$$F(\theta) = F_a(\theta) + F_b(\theta) + F_c(\theta) \tag{5.11}$$

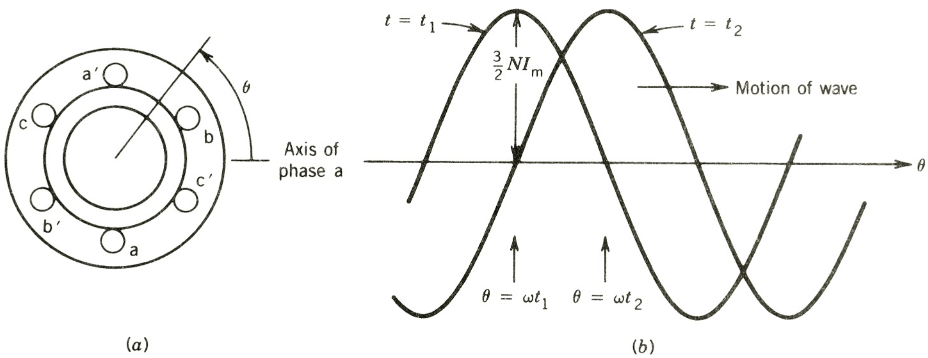


FIGURE 5.6 Motion of the resultant mmf.

At any instant of time, each phase winding produces a sinusoidally distributed mmf wave with its peak along the axis of the phase winding and amplitude proportional to the instantaneous value of the phase current. The contribution from phase a along θ is

$$F_{\mathrm{a}}(\theta) = Ni_{\mathrm{a}} \cos\theta \tag{5.12}$$

where N is the effective number of turns in phase a
i_{a} is the current in phase a

Because the phase axes are shifted from each other by 120 electrical degrees, the contributions from phases b and c are, respectively,

$$F_{\mathrm{b}}(\theta) = Ni_{\mathrm{b}} \cos(\theta - 120^\circ) \tag{5.13}$$

$$F_{\mathrm{c}}(\theta) = Ni_{\mathrm{c}} \cos(\theta + 120^\circ) \tag{5.14}$$

The resultant mmf at point θ is

$$F(\theta) = Ni_{\mathrm{a}} \cos\theta + Ni_{\mathrm{b}} \cos(\theta - 120^\circ) + Ni_{\mathrm{c}} \cos(\theta + 120^\circ) \tag{5.15}$$

The currents i_{a}, i_{b}, and i_{c} are functions of time and are defined by Eqs. 5.1, 5.2, and 5.3, and thus

$$\begin{aligned} F(\theta, t) = {} & NI_{\mathrm{m}} \cos\omega t \cos\theta \\ & + NI_{\mathrm{m}} \cos(\omega t - 120^\circ) \cos(\theta - 120^\circ) \\ & + NI_{\mathrm{m}} \cos(\omega t + 120^\circ) \cos(\theta + 120^\circ) \end{aligned} \tag{5.16}$$

Using the trigonometric identity

$$\cos A \cos B = \tfrac{1}{2}\cos(A - B) + \tfrac{1}{2}\cos(A + B) \tag{5.17}$$

each term on the right-hand side of Eq. 5.16 can be expressed as the sum of two cosine functions, one involving the difference and the other the sum of the two angles. Therefore,

$$\begin{aligned} F(\theta, t) = {} & \tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t - \theta) + \tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t + \theta) \\ & + \tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t - \theta) + \tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t + \theta - 240^\circ) \\ & \underbrace{+ \tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t - \theta)}_{\text{Forward-rotating components}} + \underbrace{\tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t + \theta + 240^\circ)}_{\text{Backward-rotating components}} \end{aligned} \tag{5.18}$$

$$= \tfrac{3}{2} NI_{\mathrm{m}} \cos(\omega t - \theta) \tag{5.19}$$

The expression of Eq. 5.19 represents the resultant mmf wave in the air gap. This represents an mmf rotating at the constant angular velocity $\omega\,(=2\pi f)$. At any instant of time, say t_1, the wave is distributed sinusoidally around the air gap (Fig. 5.6*b*) with the positive peak acting along $\theta = \omega t_1$. At a later instant, say t_2, the positive peak of the sinusoidally distributed wave is along $\theta = \omega t_2$; that is, the wave has moved by $\omega(t_2 - t_1)$ around the air gap.

The angular velocity of the rotating mmf wave is $\omega = 2\pi f$ radians per second and its rpm for a p-pole machine is given by Eq. 5.10.

It can be shown in general that an m-phase distributed winding excited by balanced m-phase currents will produce a sinusoidally distributed rotating field of constant amplitude when the phase windings are wound $2\pi/m$ electrical degrees apart in space. Note that a rotating magnetic field is produced without physically rotating any magnet. All that is necessary is to pass a polyphase current (ac) through the polyphase windings of the machine.

5.3 INDUCED VOLTAGES

In the preceding section it was shown that when balanced polyphase currents flow through a polyphase distributed winding, a sinusoidally distributed magnetic field rotates in the air gap of the machine. This effect can be visualized as one produced by a pair of magnets, for a two-pole machine, rotating in the air gap, the magnetic field (i.e., flux density) being sinusoidally distributed with the peak along the center of the magnetic poles. The result is illustrated in Fig. 5.7. The rotating field will induce voltages in the phase coils aa′, bb′, and cc′. Expressions for the induced voltages can be obtained by using Faraday's laws of induction.

The flux density distribution in the air gap can be expressed as

$$B(\theta) = B_{\max}\cos\theta \tag{5.20}$$

The air gap flux per pole, Φ_p, is

$$\Phi_p = \int_{-\pi/2}^{\pi/2} B(\theta) lr\, d\theta = 2B_{\max}\, lr \tag{5.21}$$

where l is the axial length of the stator
r is the radius of the stator at the air gap

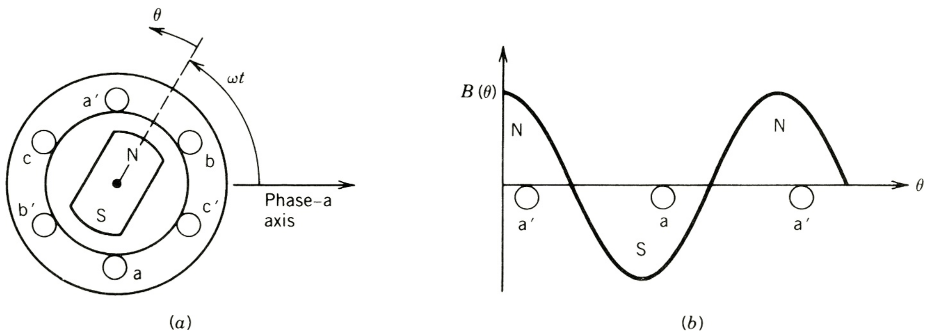


FIGURE 5.7 Air gap flux density distribution.

Let us consider that the phase coils are full-pitch coils of N turns (the coil sides of each phase are 180 electrical degrees apart as shown in Fig. 5.7). It is obvious that as the rotating field moves (or the magnetic poles rotate) the flux linkage of a coil will vary. The flux linkage for coil aa′ will be maximum ($=N\Phi_p$) at $\omega t = 0°$ (Fig. 5.7*a*) and zero at $\omega t = 90°$. The flux linkage $\lambda_a(\omega t)$ will vary as the cosine of the angle ωt. Hence,

$$\lambda_a(\omega t) = N\Phi_p \cos \omega t \tag{5.22}$$

Therefore, the voltage induced in phase coil aa′ is obtained from Faraday's law as

$$e_a = -\frac{d\lambda_a}{dt} = \omega N\Phi_p \sin \omega t \tag{5.23}$$

$$= E_{max} \sin \omega t \tag{5.23a}$$

The voltages induced in the other phase coils are also sinusoidal, but phase-shifted from each other by 120 electrical degrees. Thus,

$$e_b = E_{max} \sin(\omega t - 120°) \tag{5.24}$$

$$e_c = E_{max} \sin(\omega t + 120°) \tag{5.25}$$

From Eq. 5.23, the rms value of the induced voltage is

$$E_{rms} = \frac{\omega N\Phi_p}{\sqrt{2}} = \frac{2\pi f}{\sqrt{2}} N\Phi_p$$

$$= 4.44 f N\Phi_p \tag{5.26}$$

where f is the frequency in hertz. Equation 5.26 has the same form as that for the induced voltage in transformers (Eq. 1.40). However, Φ_p in Eq. 5.26 represents the flux per pole of the machine.

Equation 5.26 shows the rms voltage per phase. The N is the total number of series turns per phase, with the turns forming a concentrated full-pitch winding. In an actual ac machine, each phase winding is distributed in a number of slots (see Appendix A) for better use of the iron and copper and to improve the waveform. For such a distributed winding, the emfs induced in various coils placed in different slots are not in time phase, and therefore the phasor sum of the emfs is less than their numerical sum when they are connected in series for the phase winding. A reduction factor K_W, called the winding factor, must therefore be applied. For most three-phase machine windings, K_W is about 0.85 to 0.95. Therefore, for a distributed phase winding, the rms voltage per phase is

$$E_{rms} = 4.44 f N_{ph} \Phi_p K_W \tag{5.27}$$

where N_{ph} is the number of turns in series per phase.

5.4 POLYPHASE INDUCTION MACHINE

We shall now consider the various modes of operation of a polyphase induction machine. We shall first consider the standstill behavior of the machine, and then study its behavior in the running condition.

5.4.1 STANDSTILL OPERATION

Let us consider a three-phase wound-rotor induction machine with the rotor circuit left open-circuited. If the three-phase stator windings are connected to a three-phase supply, a rotating magnetic field will be produced in the air gap. The field rotates at synchronous speed n_s given by Eq. 5.10. This rotating field induces voltages in both stator and rotor windings at the same frequency f_1. The magnitudes of these voltages, from Eq. 5.27, are

$$E_1 = 4.44 f_1 N_1 \Phi_p K_{W1} \tag{5.28}$$

$$E_2 = 4.44 f_1 N_2 \Phi_p K_{W2} \tag{5.29}$$

Therefore,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \frac{K_{W1}}{K_{W2}} \tag{5.30}$$

The winding factors K_{W1} and K_{W2} for the stator and rotor windings are normally the same. Thus,

$$\frac{E_1}{E_2} \simeq \frac{N_1}{N_2} = \text{turns ratio} \tag{5.31}$$

5.4.2 PHASE SHIFTER

Notice that the rotor can be held in such a position that the axes of the corresponding phase windings in the stator and the rotor make an angle β (Fig. 5.8*a*). In such a case, the induced

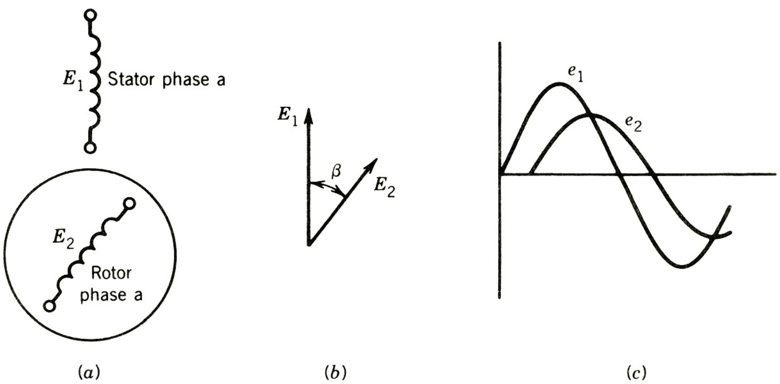


FIGURE 5.8 Induction machine as phase shifter.

voltage in the rotor winding will be phase-shifted from that of the stator winding by the angle β (Figs. 5.8*b* and 5.8*c*). Thus, a stationary wound-rotor induction machine can be used as a phase shifter. By turning the rotor mechanically, a phase shift range of 360° can be achieved.

5.4.3 INDUCTION REGULATOR

The stationary polyphase induction machine can also be used as a source of variable polyphase voltage if it is connected as an induction regulator as shown in Fig. 5.9*a*. The phasor diagram is shown in Fig. 5.9*b* to illustrate the principle. As the rotor is rotated through 360°, the output voltage V_o follows a circular locus of variable magnitude. If the induced voltages E_1 and E_2 are of the same magnitude (i.e., identical stator and rotor windings), the output voltage may be adjusted from zero to twice the supply voltage.

The induction regulator has the following advantages over a variable autotransformer:

- A continuous stepless variation of the output voltage is possible.
- No sliding electrical connections are necessary.

However, the induction regulator suffers from the disadvantages of higher leakage inductances, higher magnetizing current, and higher costs.

5.4.4 RUNNING OPERATION

If the stator windings are connected to a three-phase supply and the rotor circuit is closed, the induced voltages in the rotor windings produce rotor currents that interact with the air gap field to produce torque. The rotor, if free to do so, will then start rotating. According to Lenz's law, the rotor rotates in the direction of the rotating field such that the relative speed between the rotating field and the rotor winding decreases. The rotor will eventually reach a steady-state speed n that is less than the synchronous speed n_s at which the stator rotating field rotates in the air gap. It is obvious that at $n = n_s$ there will be no induced voltage and current in the rotor circuit, and hence no torque.

The difference between the rotor speed n and the synchronous speed n_s of the rotating field is called the *slip* s and is defined as

$$s = \frac{n_s - n}{n_s} \tag{5.32}$$

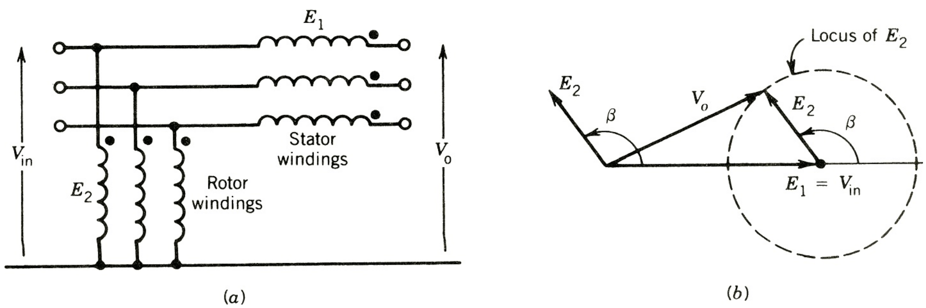


FIGURE 5.9 Induction regulator.

If you were sitting on the rotor, you would find that the rotor was slipping behind the rotating field by the *slip rpm* $= n_s - n = sn_s$. The frequency f_2 of the induced voltage and current in the rotor circuit will correspond to this slip rpm, because this is the relative speed between the rotating field and the rotor winding. Thus, from Eq. 5.10,

$$\begin{aligned} f_2 &= \frac{p}{120}(n_s - n) \\ &= \frac{p}{120} s n_s \\ &= s f_1 \end{aligned} \tag{5.33}$$

This rotor circuit frequency f_2 is also called *slip frequency*. The voltage induced in the rotor circuit at slip s is

$$\begin{aligned} E_{2s} &= 4.44 f_2 N_2 \Phi_p K_{W2} \\ &= 4.44 s f_1 N_2 \Phi_p K_{W2} \\ &= s E_2 \end{aligned} \tag{5.34}$$

where E_2 is the induced voltage in the rotor circuit at standstill—that is, at the stator frequency f_1.

The induced currents in the three-phase rotor windings also produce a rotating field. Its speed (rpm) n_2 with respect to the rotor is

$$\begin{aligned} n_2 &= \frac{120 f_2}{p} \\ &= \frac{120 s f_1}{p} \\ &= s n_s \end{aligned} \tag{5.35}$$

Because the rotor itself is rotating at n rpm, the induced rotor field rotates in the air gap at speed $n + n_2 = (1 - s)n_s + sn_s = n_s$ rpm. Therefore, both the stator field and the induced rotor field rotate in the air gap at the same synchronous speed n_s. The stator magnetic field and the rotor magnetic field are therefore stationary with respect to each other. The interaction between these two fields can be considered to produce the torque. As the magnetic fields tend to align, the stator magnetic field can be visualized as dragging the rotor magnetic field.

EXAMPLE 5.1

A 3ϕ, 460 V, 100 hp, 60 Hz, four-pole induction machine delivers rated output power at a slip of 0.05. Determine the

(a) Synchronous speed and motor speed.

(b) Speed of the rotating air gap field.

(c) Frequency of the rotor circuit.

(d) Slip rpm.

(e) Speed of the rotor field relative to the

(i) rotor structure.

(ii) stator structure.

(iii) stator rotating field.

(f) Rotor induced voltage at the operating speed, if the stator-to-rotor turns ratio is 1:0.5.

Solution

(a)

$$n_s = \frac{120f}{p} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$n = (1 - s)n_s = (1 - 0.05)1800 = 1710 \text{ rpm}$$

(b) 1800 rpm (same as synchronous speed)

(c) $f_2 = sf_1 = 0.05 \times 60 = 3$ Hz

(d) slip rpm $= sn_s = 0.05 \times 1800 = 90$ rpm

(e) **(i)** 90 rpm

(ii) 1800 rpm

(iii) 0 rpm

(f) Assume that the induced voltage in the stator winding is the same as the applied voltage. Now,

$$\begin{aligned} E_{2s} &= sE_2 \\ &= s\frac{N_2}{N_1}E_1 \\ &= 0.05 \times 0.5 \times \frac{460 \text{ V}}{\sqrt{3}} \\ &= 6.64 \text{ V/phase} \quad \blacksquare \end{aligned}$$

5.5 THREE MODES OF OPERATION

The induction machine can be operated in three modes: motoring, generating, and plugging. To illustrate these three modes of operation, consider an induction machine mechanically coupled to a dc machine, as shown in Fig. 5.10*a*.

5.5.1 MOTORING

If the stator terminals are connected to a three-phase supply, the rotor will rotate in the direction of the stator rotating magnetic field. This is the natural (or motoring) mode of

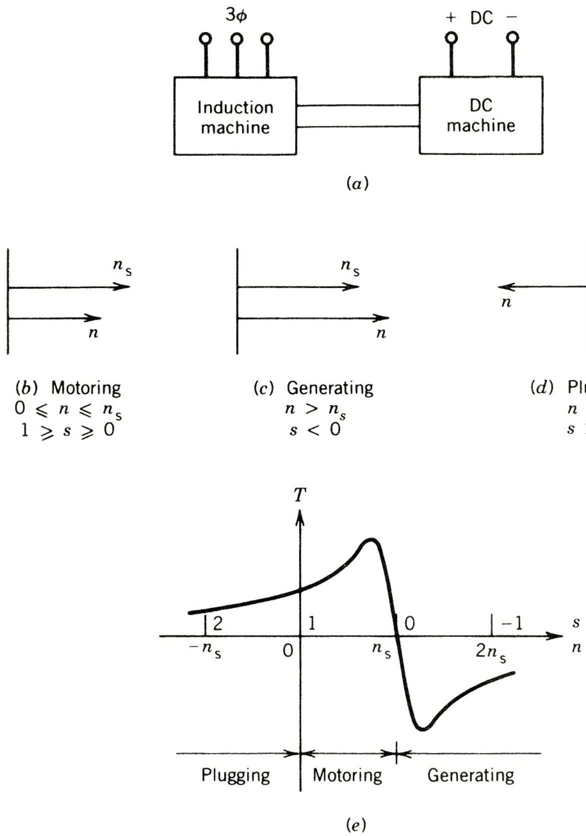


FIGURE 5.10 Three modes of operation of an induction machine.

operation of the induction machine. The steady-state speed n is less than the synchronous speed n_s, as shown in Fig. 5.10*b*.

5.5.2 GENERATING

The dc motor can be adjusted so that the speed of the system is higher than the synchronous speed and the system rotates in the same direction as the stator rotating field as shown in Fig. 5.10*c*. The induction machine will produce a generating torque—that is, a torque acting opposite to the rotation of the rotor (or acting opposite to the stator rotating magnetic field). The generating mode of operation is utilized in some drive applications to provide regenerative braking. For example, suppose an induction machine is fed from a variable-frequency supply to control the speed of a drive system. To stop the drive system, the frequency of the supply is gradually reduced. In the process, the instantaneous speed of the drive system is higher than the instantaneous synchronous speed because of the inertia of the drive system. As a result, the generating action of the induction machine will cause the power flow to reverse and the kinetic energy of the drive system will be fed back to the supply. The process is known as regenerative braking.

5.5.3 PLUGGING

If the dc motor of Fig. 5.10*a* is adjusted so that the system rotates in a direction opposite to the stator rotating magnetic field (Fig. 5.10*d*), the torque will be in the direction of the rotating field but will oppose the motion of the rotor. This torque is a braking torque.

This mode of operation is sometimes utilized in drive applications where the drive system is required to stop very quickly. Suppose an induction motor is running at a steady-state speed. If its terminal phase sequence is changed suddenly, the stator rotating field will rotate opposite to the rotation of the rotor, producing the plugging operation. The motor will come to zero speed rapidly and will accelerate in the opposite direction, unless the supply is disconnected at zero speed.

The three modes of operation and the typical torque profile of the induction machine in the various speed ranges are illustrated in Fig. 5.10*e*.

PROBLEMS

5.1 A 3ϕ, 460 V, 60 Hz, 4-pole, Y-connected wound-rotor induction machine has 230 V between the slip rings at standstill when the stator is connected to a 3ϕ, 460 V, 60 Hz power supply. The rotor is mechanically connected to a dc motor whose speed can be changed. Determine the magnitude and frequency of the voltage between the slip rings when the induction machine is driven at the following speeds:

(a) 1620 rpm in the same direction as the rotating field.

(b) 1620 rpm in the opposite direction to the rotating field.

(c) 1800 rpm in the same direction as the rotating field.

(d) 1800 rpm in the opposite direction to the rotating field.

(e) 3600 rpm in the same direction as the rotating field.

5.2 A three-phase, 5 hp, 208 V, 60 Hz induction motor runs at 1746 rpm when it delivers rated output power.

(a) Determine the number of poles of the machine.

(b) Determine the slip at full load.

(c) Determine the frequency of the rotor current.

(d) Determine the speed of the rotor field with respect to the

(i) Stator.

(ii) Stator rotating field.

5.3 A 3ϕ, 460 V, 100 hp, 60 Hz, six-pole induction machine operates at 3% slip (positive) at full load.

(a) Determine the speeds of the motor and its direction relative to the rotating field.

(b) Determine the rotor frequency.

(c) Determine the speed of the stator field.

(d) Determine the speed of the air gap field.

(e) Determine the speed of the rotor field relative to

(i) the rotor structure.

(ii) the stator structure.

(iii) the stator rotating field.

5.4 Repeat Problem 5.3 if the induction machine is operated at 3% slip (negative).

5.5 Repeat Problem 5.3 if the induction machine operates at 150% slip (positive).

5.6 A 3ϕ, 10 hp, 208 V, six-pole, 60 Hz, wound-rotor induction machine has a stator-to-rotor turns ratio of 1 : 0.5 and both stator and rotor windings are connected in star.

(a) The stator of the induction machine is connected to a 3ϕ, 208 V, 60 Hz supply, and the motor runs at 1140 rpm.

(i) Determine the operating slip.

(ii) Determine the voltage induced in the rotor per phase and frequency of the induced voltage.

(iii) Determine the rpm of the rotor field with respect to the rotor and with respect to the stator.

chapter six

SYNCHRONOUS MACHINES

A synchronous machine rotates at a constant speed in the steady state. Unlike in induction machines, the rotating air gap field and the rotor in the synchronous machine rotate at the same speed, called the synchronous speed. Synchronous machines are used primarily as generators of electrical power. In this case they are called *synchronous generators* or *alternators*. They are usually large machines generating electrical power at hydro, nuclear, or thermal power stations. Synchronous generators with power ratings of several hundred MVA (mega-volt-amperes) are quite common in generating stations. It is anticipated that machines of several thousand MVA ratings will be used in the twenty first century. Synchronous generators are the primary energy conversion devices of the world's electric power systems today. In spite of continuing research for more direct energy conversion techniques, it is conceded that synchronous generators will continue to be used well into the next century.

Like most rotating machines, a synchronous machine can also operate as both a generator and a motor. In large sizes (several hundred or thousand kilowatts) synchronous motors are used for pumps in generating stations, and in small sizes (fractional horsepower) they are used in electric clocks, timers, record turntables, and so forth where constant speed is desired. Most industrial drives run at variable speeds. In industry, synchronous motors are used mainly where a constant speed is desired. In industrial drives, therefore, synchronous motors are not as widely used as induction or dc motors. A linear version of the synchronous motor (LSM) is being considered for high-speed transportation systems of the future.

An important feature of a synchronous motor is that it can draw either lagging or leading reactive current from the ac supply system. A synchronous machine is a doubly excited machine. Its rotor poles are excited by a dc current and its stator windings are connected to the ac supply (Fig. 6.1). The air gap flux is therefore the resultant of the fluxes due to both rotor current and stator current. In induction machines, the only source of excitation is the stator current, because rotor currents are induced currents. Therefore, induction motors always operate at a lagging power factor, because lagging reactive current is required to establish flux in the machine. On the other hand, in a synchronous motor, if the rotor field winding provides just the necessary excitation, the stator will draw no reactive current; that is, the motor will operate at a unity power factor. If the rotor excitation current is decreased, lagging reactive current will be drawn from the ac source to aid magnetization by the rotor field current, and the machine will operate at a lagging power factor. If the rotor field current is increased, leading reactive current will be drawn from the ac source to oppose magnetization by the rotor field current, and the machine will operate at a leading power factor. Thus, by changing the field current, the power factor of the synchronous motor can be controlled. If the motor is not loaded, but is simply floating on the ac supply system, it will thus behave as a variable inductor or capacitor as its rotor field current is changed. A synchronous machine with no load is called a *synchronous condenser*. It may be used in power transmission systems to regulate line voltage. In industry, synchronous motors are sometimes used with other induction motors and

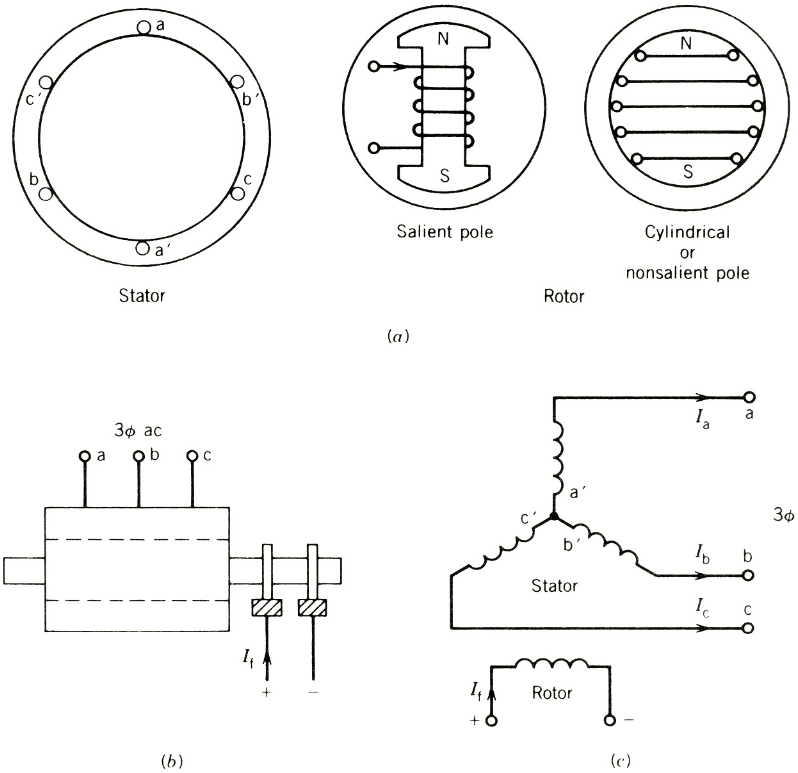


FIGURE 6.1 Basic structure of the three-phase synchronous machine.

operated in an overexcited mode so that they draw leading current to compensate the lagging current drawn by the induction motors, thereby improving the overall plant power factor. Example 6.1 illustrates the use of synchronous motors for power factor improvement. The power factor characteristics of synchronous motors will be further discussed in a later section.

EXAMPLE 6.1

In a factory a 3ϕ, 4 kV, 400 kVA synchronous machine is installed along with other induction motors. The following are the loads on the machines:

Induction motors: 500 kVA at 0.8 PF lagging.

Synchronous motor: 300 kVA at 1.0 PF.

(a) Compute the overall power factor of the factory loads.

(b) To improve the factory power factor, the synchronous machine is overexcited (to draw leading current) without any change in its load. Without overloading the motor, to what extent can the factory power factor be improved? Find the current and power factor of the synchronous motor for this condition.

Solution

(a) Induction motors:

$$\text{Power} = 500 \times 0.8 = 400 \ \text{kW}$$

$$\text{Reactive power} = 500 \times 0.6 = 300 \ \text{kVAR}$$

Synchronous motor:

$$\text{Power} = 300 \ \text{kW}$$

$$\text{Reactive power} = 0.0$$

Factory:

$$\text{Power} = 700 \ \text{kW}$$

$$\text{Reactive power} = 300 \ \text{kVAR}$$

$$\text{Complex power} = \sqrt{700^2 + 300^2} = 762 \ \text{kVA}$$

$$\text{Power factor} = \frac{700}{762} = 0.92 \ \text{lagging}$$

(b) The maximum leading kVAR that the synchronous motor can draw without exceeding its rating is

$$\sqrt{400^2 - 300^2} = 264.58 \ \text{kVAR}$$

$$\text{Factory kVAR} = j300 - j264.48$$

$$= j35.42 \ \text{(i.e., lagging)}$$

$$\text{New factory kVA} = \sqrt{700^2 + 35.42^2}$$

$$= 700.9 \ \text{kVA}$$

$$\text{Improved factory power factor} = \frac{700}{700.9}$$

$$= 0.996$$

Synchronous motor current:

$$I_{SM} = \frac{400 \ \text{kVA}}{\sqrt{3} \times 4 \ \text{kV}} = 57.74 \ \text{A}$$

Synchronous motor power factor:

$$PF_{SM} = \frac{300 \ \text{kW}}{400 \ \text{kVA}} = 0.75 \ \text{lead}$$

6.1 CONSTRUCTION OF THREE-PHASE SYNCHRONOUS MACHINES

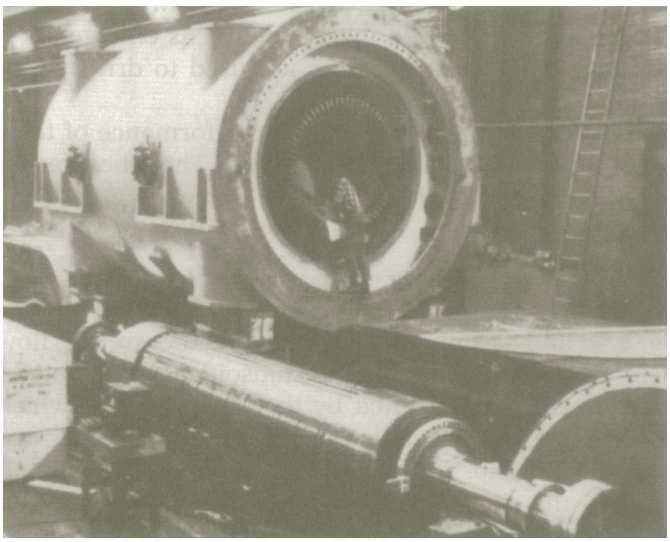
The stator of the three-phase synchronous machine has a three-phase distributed winding similar to that of the three-phase induction machine. Unlike the dc machine, the stator winding, which is connected to the ac supply system, is sometimes called the *armature* winding. It is designed for high voltage and current.

The rotor has a winding called the *field* winding, which carries direct current. The field winding on the rotating structure is normally fed from an external dc source through slip rings and brushes. The basic structure of the synchronous machine is illustrated in Fig. 6.1.

Synchronous machines can be broadly divided into two groups:

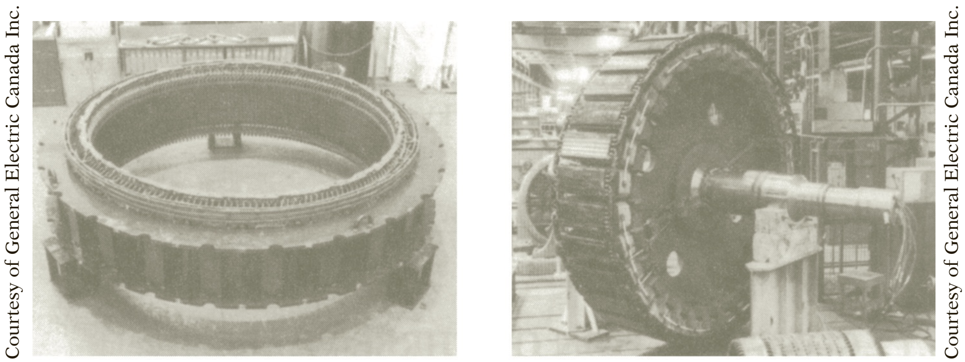
1. High-speed machines with cylindrical (or nonsalient pole) rotors.
2. Low-speed machines with salient pole rotors.

The cylindrical or nonsalient pole rotor has one distributed winding and an essentially uniform air gap. These motors are used in large generators (several hundred megawatts) with two or sometimes four poles and are usually driven by steam turbines. The rotors are long and have a small diameter, as shown in Fig. 6.2. On the other hand, salient pole rotors have concentrated windings on the poles and a nonuniform air gap. Salient pole generators have a large number of poles, sometimes as many as 50, and operate at lower speeds. The synchronous generators in hydroelectric power stations are of the salient pole type and are driven by water turbines. These generators are rated for tens or hundreds of megawatts. The rotors are shorter but have a large diameter as shown in Fig. 6.3. Smaller salient pole synchronous machines in the range of 50 kW to 5 MW are also used. Such synchronous generators are used independently as emergency power supplies. Salient pole synchronous motors are used to drive pumps, cement mixers, and some other industrial drives.



Courtesy of General Electric Canada Inc.

FIGURE 6.2 High-speed cylindrical-rotor synchronous generator.



(a) (b)

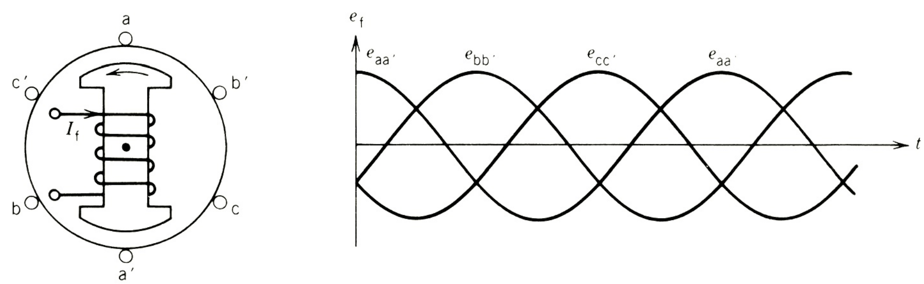
FIGURE 6.3 Low-speed salient pole synchronous generator. (*a*) Stator. (*b*) Rotor.

In the following sections the steady-state performance of the cylindrical-rotor synchronous machine will be studied first. Then the effects of saliency in the rotor poles will be considered.

6.2 SYNCHRONOUS GENERATORS

Refer to Fig. 6.4*a* and assume that when the field current I_f flows through the rotor field winding, it establishes a sinusoidally distributed flux in the air gap. If the rotor is now rotated by the prime mover (which can be a turbine or diesel engine or dc motor or induction motor), a revolving field is produced in the air gap. This field is called the excitation field, because it is produced by the excitation current I_f. The rotating flux so produced will change the flux linkage of the armature windings aa′, bb′, and cc′ and will induce voltages in these stator windings. These induced voltages, shown in Fig. 6.4*b*, have the same magnitudes but are phase-shifted by 120 electrical degrees. They are called *excitation voltages* E_f. The rotor speed and frequency of the induced voltage are related by

$$n = \frac{120f}{p} \tag{6.1}$$



(a) (b)

FIGURE 6.4 Excitation voltage in synchronous machines.

or

$$f = \frac{np}{120} \tag{6.2}$$

where n is the rotor speed in rpm
p is the number of poles

The excitation voltage in rms from Eq. 5.27 is

$$E_f = 4.44 f \Phi_f N K_w \tag{6.3}$$

Where Φ_f is the flux per pole due to the excitation current I_f
N is the number of turns in each phase
K_w is the winding factor

From Eqs. 6.2 and 6.3,

$$E_f \propto n\Phi_f \tag{6.4}$$

The excitation voltage is proportional to the machine speed and excitation flux, and the latter in turn depends on the excitation current I_f. The variation of the excitation voltage with the field current is shown in Fig. 6.5. The induced voltage at $I_f = 0$ is due to the residual magnetism. Initially the voltage rises linearly with the field current, but as the field current is further increased, the flux Φ_f does not increase linearly with I_f because of saturation of the magnetic circuit, and therefore E_f levels off. If the machine terminals are kept open, the excitation voltage is the same as the terminal voltage and can be measured using a voltmeter. The curve shown in Fig. 6.5 is known as the *open-circuit characteristic (OCC)* or *magnetization characteristic* of the synchronous machine.

If the stator terminals of the machine (Fig. 6.1*c*) are connected to a 3ϕ load, stator current I_a will flow. The frequency of I_a will be the same as that of the excitation voltage E_f. The stator currents flowing in the 3ϕ windings will also establish a rotating field in the air gap. The net air gap flux is the resultant of the fluxes produced by rotor current I_f and stator current I_a.

Let Φ_f be the flux due to I_f and Φ_a be the flux due to I_a, known as the *armature reaction* flux. Then,

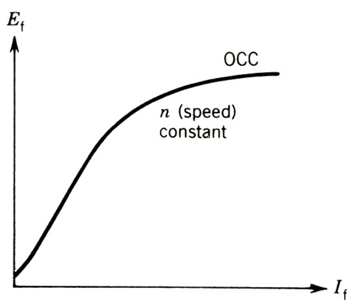


FIGURE 6.5 Open circuit characteristic (OCC) or magnetization characteristic of a synchronous machine.

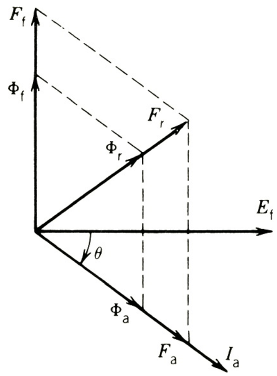


FIGURE 6.6 Space phasor diagram.

$$\Phi_r = \Phi_f + \Phi_a = \text{resultant air gap flux, assuming no saturation}$$

It may be noted that the resultant and the component fluxes rotate in the air gap at the same speed, governed by Eq. 6.1. The space phasor diagram for these fluxes is shown in Fig. 6.6. The rotor field mmf F_f (due to I_f) and the flux Φ_f produced by the mmf F_f are represented along the same line. The induced voltage E_f lags the flux Φ_f by 90°. Assume that the stator current I_a lags E_f by an angle θ. The mmf F_a (due to the current I_a) and the flux Φ_a produced by the mmf F_a are along the same axis as the current I_a. The resultant mmf F_r is the vector sum of the mmfs F_f and F_a. Assuming no saturation, the resultant flux Φ_r is also the vector sum of the fluxes Φ_f and Φ_a. The space phasor relationship of mmfs and fluxes will be discussed further in later sections.

6.2.1 THE INFINITE BUS

Synchronous generators are rarely used to supply individual loads. These generators, in general, are connected to a power supply system known as an *infinite bus* or *grid*. Because a large number of synchronous generators of large sizes are connected together, the voltage and frequency of the infinite bus hardly change. Loads are tapped from the infinite bus at various load centers. A typical infinite bus or grid system is shown in Fig. 6.7. Transmission of power is normally at higher voltage levels (in hundreds of kilovolts) to achieve higher efficiency of power transmission. However, generation of electrical energy by the synchronous generators or alternators is at relatively lower voltage levels (20 – 30 kV). A transformer is used to step up the alternator voltage to the infinite bus voltage. At the load centers, the infinite bus (or grid) voltage is stepped down through several stages to bring the voltage down to the domestic voltage level (115/230 V) or industrial voltage levels such as 4.16 kV, 600 V, or 480 V.

In a power plant the synchronous generators are connected to or disconnected from the infinite bus, depending on the power demand on the grid system. The operation of connecting a synchronous generator to the infinite bus is known as *paralleling with the infinite bus*. Before the alternator can be connected to the infinite bus, the incoming alternator and the infinite bus must have the same

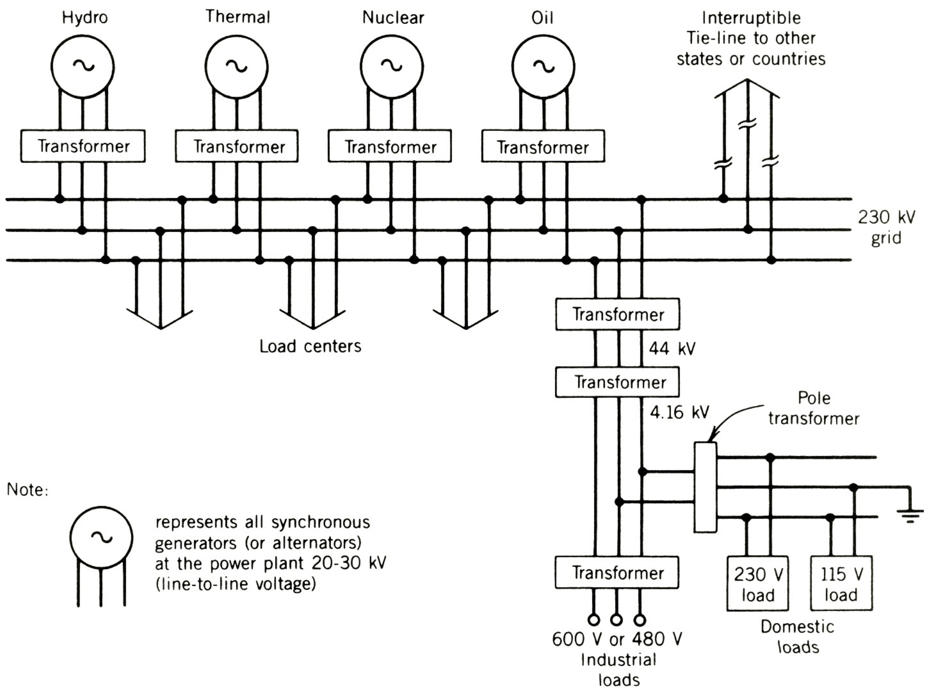
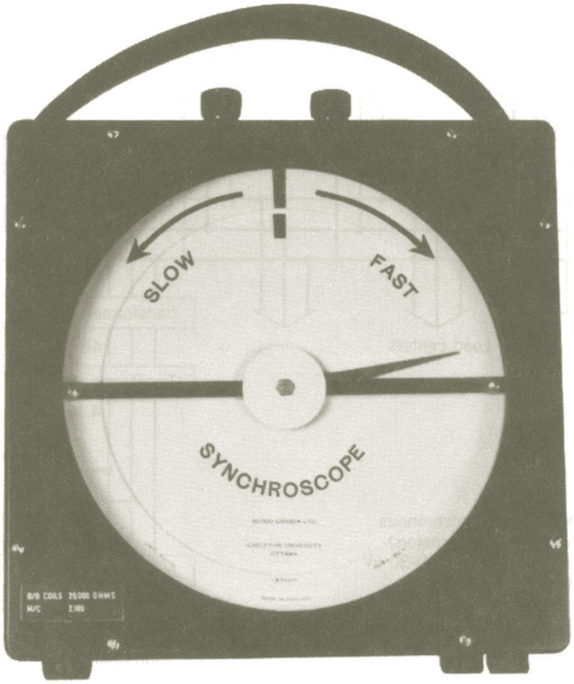


FIGURE 6.7 Infinite bus (or grid) system.

1. Voltage
2. Frequency
3. Phase sequence
4. Phase

In the power plant the satisfaction of these conditions is checked by an instrument known as a *synchroscope*, shown in Fig. 6.8. The position of the indicator indicates the phase difference between the voltages of the incoming machine and the infinite bus. The direction of motion of the indicator shows whether the incoming machine is running too fast or too slow—that is, whether the frequency of the incoming machine is higher or lower than that of the infinite bus. The phase sequence is predetermined, because if phase sequence is not correct, it will produce a disastrous situation. When the indicator moves very slowly (i.e., frequencies almost the same) and passes through the zero phase point (vertical up position), the circuit breaker is closed and the alternator is connected to the infinite bus.

A set of *synchronizing lamps* can be used to check that the conditions for paralleling the incoming machine with the infinite bus are satisfied. In a laboratory, such a set of lamps can be used to demonstrate what happens if the conditions are not satisfied. Figure 6.9 shows the schematic of the laboratory setup for this purpose. The prime mover can be a dc motor or an induction motor. It can be adjusted to a speed such that the frequency of the synchronous machine is the same as that of the infinite bus. For example, if the synchronous machine has four poles, the prime mover can be adjusted for 1800 rpm so that the frequency is 60 cycles—the same as that of the infinite bus. The field current I_f can then be adjusted so that the two



Courtesy of Dr. P. C. Sen

FIGURE 6.8 Synchroscope.

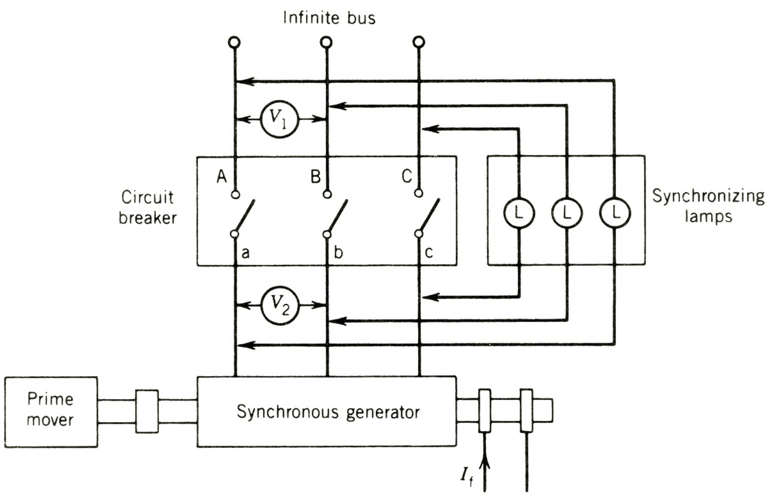


FIGURE 6.9 Schematic diagram for paralleling a synchronous generator with the infinite bus using synchronizing lamps.

voltmeters (V_1 and V_2) read the same. If the phase sequence is correct, all the lamps will have the same brightness, and if the frequencies are not exactly the same, the lamps will brighten and darken in step.

Let us examine what we expect to observe in the lamps if the conditions are not satisfied. The phenomena can be explained by drawing phasor diagrams for the voltages of the incoming machine and the infinite bus. Let

E_A, E_B, E_C	represent the phasor voltages of the infinite bus.
E_a, E_b, E_c	represent the phasor voltages of the incoming machine.
E_{Aa}, E_{Bb}, E_{Cc}	represent the phasor voltages of the synchronizing lamps. The magnitude of these will represent the brightness of the corresponding lamps.

1. *Voltages are not the same, but frequency and phase sequence are the same.*

Referring to Fig. 6.10a, one sees that the two sets of phasor voltages (E_A, E_B, E_C and E_a, E_b, E_c) rotate at the same speed. The lamp voltages E_{Aa}, E_{Bb}, and E_{Cc} have equal magnitudes, and therefore all the three lamps will glow with the same intensity. To make the voltages equal, the field current I_f must be adjusted.

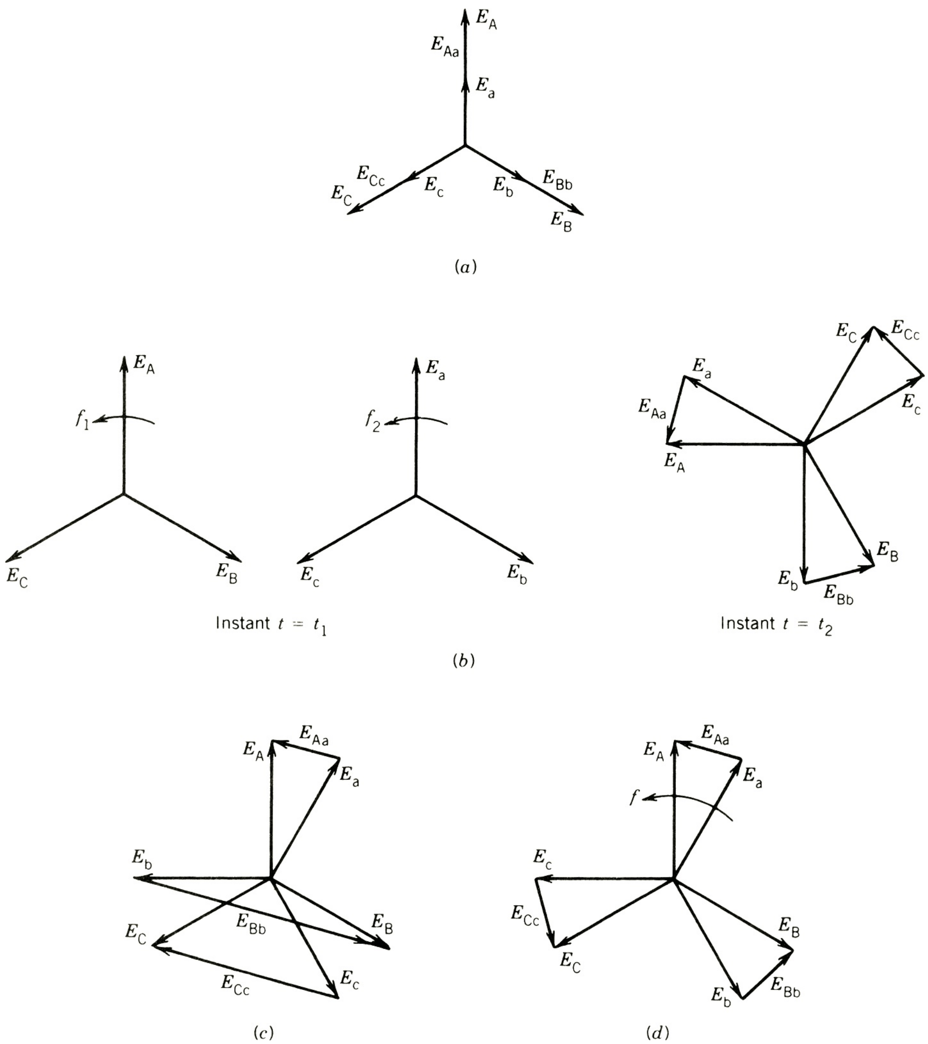


FIGURE 6.10 Phasor voltages of the incoming machine and infinite bus.

2. *Frequencies are not the same, but voltages and phase sequences are the same.*

The two sets of phasor voltages rotate at different speeds, depending on the frequencies. Assume that the phase voltages are in phase at an instant $t = t_1$ (Fig. 6.10*b*). At this instant, the voltages across the lamps are zero, and therefore they are all dark. If $f_1 > f_2$ at a later instant $t = t_2$, phasors E_A, E_B, and E_C will move ahead of phasors E_a, E_b, and E_c. Equal voltages will appear across the three lamps, and they will glow with the same intensity. It is therefore evident that if the frequencies are different, the lamps will darken and brighten in step.

To make the frequencies the same, the speed has to be adjusted until the lamps brighten and darken very slowly in step. It may be noted that as the speed of the incoming machine is adjusted, its voltages will change. Therefore, simultaneous adjustment of the field current I_f will also be necessary to keep the voltages the same.

3. *Phase sequences are not the same, but voltages and frequencies are the same.*

Let the phase sequence of the voltages of the infinite bus be E_A, E_B, E_C, and of the incoming bus be E_a, E_c, E_b, as shown in Fig. 6.10*c*. The voltages across the lamps are of different magnitudes, and therefore the lamps will glow with different intensities. If the frequencies are slightly different, one set of phasor voltages will pass the other set of phasor voltages, and the lamps will darken and brighten out of step.

To make the phase sequence the same, interchange connections to two terminals; for instance, connect a to B and b to A (Fig. 6.9).

4. *Phase is not the same, but voltage, frequency, and phase sequence are the same.*

The two sets of phasor voltages will maintain a steady phase difference (as shown in Fig. 6.10*d*), and the lamps will glow with the same intensity. To make the phase the same or the phase difference zero, the frequency of the incoming machine is slightly altered. At zero phase difference, all the lamps will be dark, and if the circuit breaker is closed, the incoming machine will be connected to the infinite bus. Once the synchronous machine is connected to the infinite bus, its speed cannot be changed further. However, the real power transfer from the machine to the infinite bus can be controlled by adjusting the prime mover power. The reactive power (and hence the machine power factor) can be controlled by adjusting the field current. Real and reactive power control will be discussed in detail in later sections.

6.3 SYNCHRONOUS MOTORS

When a synchronous machine is used as a motor, one should be able to connect it directly to the power supply like other motors, such as dc motors or induction motors. However, a synchronous motor is not self-starting. If the rotor field poles are excited by the field current and the stator terminals are connected to the ac supply, the motor will not start; instead, it vibrates. This can be explained as follows.

Let us consider a two-pole synchronous machine. If it is connected to a 3ϕ, 60 Hz ac supply, stator currents will produce a rotating field that will rotate at 3600 rpm in the air gap. Let us represent this rotating field by two stator poles rotating at 3600 rpm, as shown in Fig. 6.11*a*. At start ($t = 0$), let the rotor poles be at the position shown in Fig. 6.11*a*. The rotor will therefore experience a clockwise torque, making it rotate in the direction of the stator rotating poles.

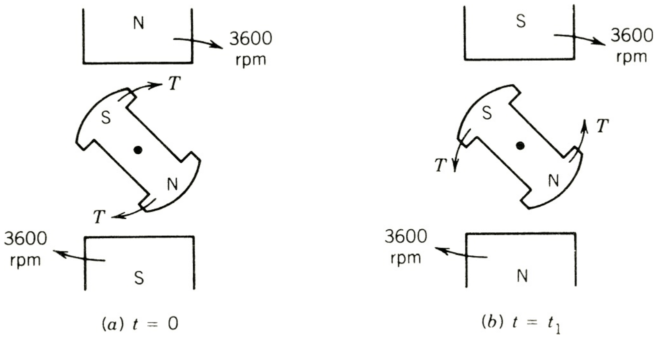


FIGURE 6.11 Torque on rotor at start.

At $t = t_1$, let the stator poles move by half a revolution, shown in Fig. 6.11b. The rotor poles have hardly moved, because of the high inertia of the rotor. Therefore, at this instant the rotor experiences a counterclockwise torque, tending to make it rotate in the direction opposite to that of the stator poles. The net torque on the rotor in one revolution will be zero, and therefore the motor will not develop any starting torque. The stator field is rotating so fast that the rotor poles cannot catch up or lock onto it. The motor will not speed up, but will vibrate.

Two methods are normally used to start a synchronous motor: (a) use a variable-frequency supply or (b) start the machine as an induction motor. These methods will now be described.

Start with Variable-Frequency Supply

By using a frequency converter, a synchronous motor can be brought from standstill to its desired speed. The arrangement is shown schematically in Fig. 6.12. The motor is started with a low-frequency supply. This will make the stator field rotate slowly so that the rotor poles can follow the stator poles. Afterward, the frequency is gradually increased and the motor brought to its desired speed.

The frequency converter is a costly power conditioning unit, and therefore this method is expensive. However, if the synchronous motor has to run at variable speeds, this method may be used.

Start as an Induction Motor

If the frequency converter is not available, or if the synchronous motor does not have to run at various speeds, it can be started as an induction motor. For this purpose an additional winding, which resembles the cage of an induction motor, is mounted on the rotor. This cage-type winding is known as a *damper* or *amortisseur winding* and is shown in Fig. 6.13.

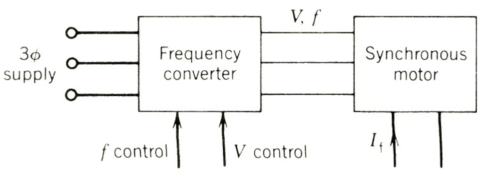


FIGURE 6.12 Starting of a synchronous motor using a variable-frequency supply.

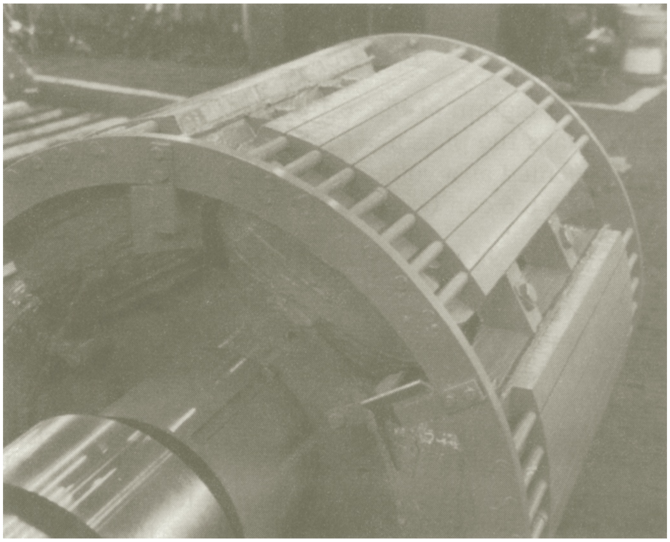


FIGURE 6.13 Cage-type damper (or amortisseur) winding in a synchronous machine.

To start the motor, the field winding is left unexcited; often it is shunted by a resistance. If the motor terminals are now connected to the ac supply, the motor will start as an induction motor, because currents will be induced in the damper winding to produce torque. The motor will speed up and will approach synchronous speed. The rotor is then closely following the stator field poles, which are rotating at the synchronous speed. Now if the rotor poles are excited by a field current from a dc source, the rotor poles, closely following the stator poles, will be locked to them. The rotor will then run at synchronous speed.

If the machine runs at synchronous speed, no current will be induced in the damper winding. The damper winding is therefore operative for starting. Note that if the rotor speed is different from the synchronous speed because of sudden load change or other transients, currents will be induced in the damper winding to produce a torque to restore the synchronous speed. The presence of this restorative torque is the reason for the name "damper" winding. Also note that a damper winding is not required to start a synchronous generator and parallel it with the infinite bus. However, both synchronous generators and motors have damper windings to damp out transient oscillations.

6.4 EQUIVALENT CIRCUIT MODEL

In the preceding sections the qualitative behavior of the synchronous machine as both a generator and a motor has been discussed to provide a "feel" for the machine behavior. We can now develop an equivalent circuit model that can be used to study the performance characteristics with sufficient accuracy. Since the steady-state behavior will be studied, the circuit time constants of the field and damper windings need not be considered. The equivalent circuit will be derived on a per-phase basis.

The current I_f in the field winding produces a flux Φ_f in the air gap. The current I_a in the stator winding produces flux Φ_a. Part of it, Φ_{al}, known as the *leakage flux*, links with the stator

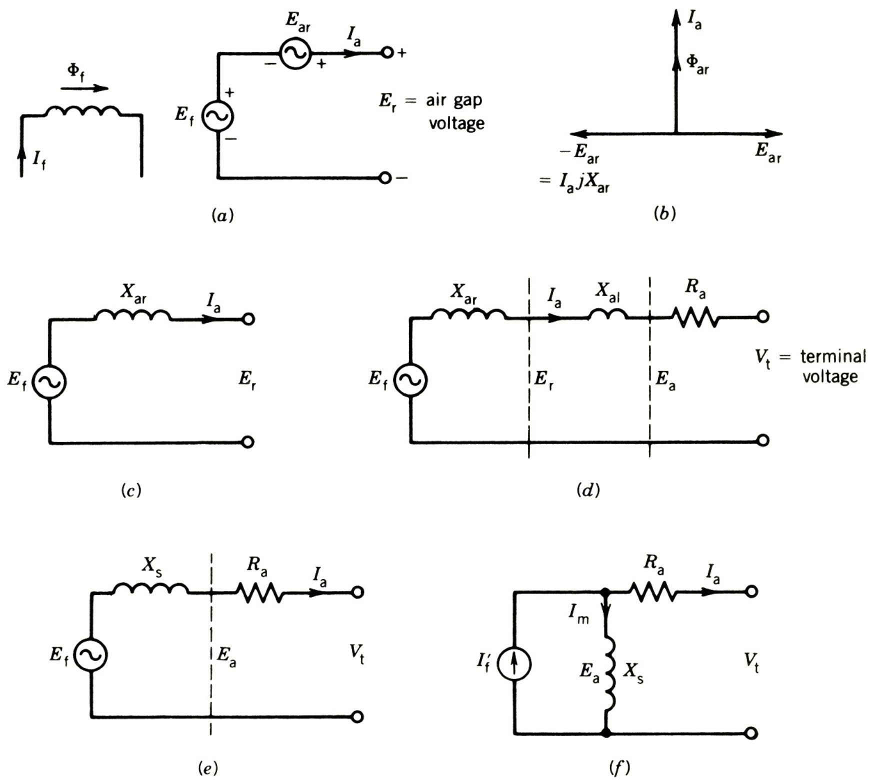


FIGURE 6.14 Equivalent circuit of a synchronous machine.

winding only and does not link with the field winding. A major part, Φ_{ar}, known as the *armature reaction flux*, is established in the air gap and links with the field winding. The resultant air gap flux Φ_r is therefore due to the two component fluxes, Φ_f and Φ_{ar}. Each component flux induces a component voltage in the stator winding. In Fig. 6.14*a*, E_f is induced by Φ_f, E_{ar} by Φ_{ar}, and the resultant voltage E_r by the resultant flux Φ_r. The excitation voltage E_f can be found from the open-circuit curve of Fig. 6.5. However, the voltage E_{ar}, known as the *armature reaction voltage*, depends on Φ_{ar} (and hence on I_a). From Fig. 6.14*a*,

$$E_r = E_{ar} + E_f \tag{6.5}$$

or

$$E_f = -E_{ar} + E_r \tag{6.6}$$

From the phasor diagram of Fig. 6.14*b*, the voltage E_{ar} lags Φ_{ar} (or I_a) by 90°. Therefore, I_a lags the phasor $-E_{ar}$ by 90°. In Eq. 6.6, the voltage $-E_{ar}$ can thus be represented as a voltage drop across a reactance X_{ar} due to the current I_a. Equation 6.6 can be written as

$$E_f = I_a jX_{ar} + E_r \tag{6.7}$$

This reactance X_{ar} is known as the *reactance of armature reaction* or the *magnetizing reactance* and is shown in Fig. 6.14*c*. If the stator winding resistance R_a and the leakage reactance X_{al} (which accounts for the leakage flux Φ_{al}) are included, the per-phase equivalent circuit is represented by the circuit of Fig. 6.14*d*. The resistance R_a is the *effective resistance* and is approximately 1.6 times the dc resistance of the stator winding. The effective resistance includes the effects of the operating temperature and the skin effect caused by the alternating current flowing through the armature winding.

If the two reactances X_{ar} and X_{al} are combined into one reactance, the equivalent circuit model reduces to the form shown in Fig. 6.14*e*, where

$$X_s = X_{ar} + X_{al} \qquad \text{(called } \textit{synchronous reactance}\text{)}$$

$$Z_s = R_a + jX_s \qquad \text{(called } \textit{synchronous impedance}\text{)}$$

The synchronous reactance X_s takes into account all the flux, magnetizing as well as leakage, produced by the armature (stator) current.

The values of these machine parameters depend on the size of the machine. Table 6.1 shows their order of magnitude. The per-unit system is described in Chapter 2. A 0.1 pu impedance means that if the rated current flows, the impedance will produce a voltage drop of 0.1 (or 10%) of the rated value. In general, as the machine size increases, the per-unit resistance decreases but the per-unit synchronous reactance increases.

In an alternative form of the equivalent circuit the excitation voltage E_f and the synchronous reactance X_s can be replaced by a Norton equivalent circuit, as shown in Fig. 6.14*f*, where

$$I'_f = \frac{E_f}{X_s} \tag{6.7a}$$

It can be shown[1] that

$$|I'_f| = \frac{X_{ar}}{X_s} nI_f \tag{6.7b}$$

where

$$n = \frac{\sqrt{2}}{3}\frac{N_{re}}{N_{se}} \tag{6.7c}$$

TABLE 6.1 Synchronous Machine Parameters

	Smaller Machines (tens of kVA)	**Larger Machines (tens of MVA)**
R_a	0.05−0.02	0.01−0.005
X_{al}	0.05−0.08	0.1−0.15
X_s	0.5−0.8	1.0−1.5

[1] G. R. Slemon and A. Straughen, *Electric Machines*, Addison–Wesley, Reading, Mass., 1980.

N_{re} is the effective field winding turns

N_{se} is the effective stator phase winding turns

The equivalent circuit of Fig. 6.14*f* is useful for determining the actual field current I_f and also for assessing the performance of a synchronous motor if it is fed from a current source power supply.

6.4.1 DETERMINATION OF THE SYNCHRONOUS REACTANCE X_s

The synchronous reactance is an important parameter in the equivalent circuit of the synchronous machine. This reactance can be determined by performing two tests, an open-circuit test and a short-circuit test.

Open-Circuit Test

The synchronous machine is driven at the synchronous speed, and the open-circuit terminal voltage $V_t(= E_f)$ is measured as the field current I_f is varied (see Fig. 6.15*a*). The curve showing the variation of E_f with I_f is known as the *open-circuit characteristic* (OCC, shown in Fig. 6.15*c*). Because the terminals are open, this curve shows the variation of the excitation voltage E_f with the field current I_f. Note that as the field current is increased, the magnetic circuit shows saturation effects. The line passing through the linear part of the OCC is called the *air gap line*. The excitation voltage would have changed along this line if there were no magnetic saturation effects in the machine.

Short-Circuit Test

The circuit arrangement for this test is shown in Fig. 6.15*b*. Ammeters are connected to each phase, and the terminals are then shorted. The synchronous machine is driven at synchronous

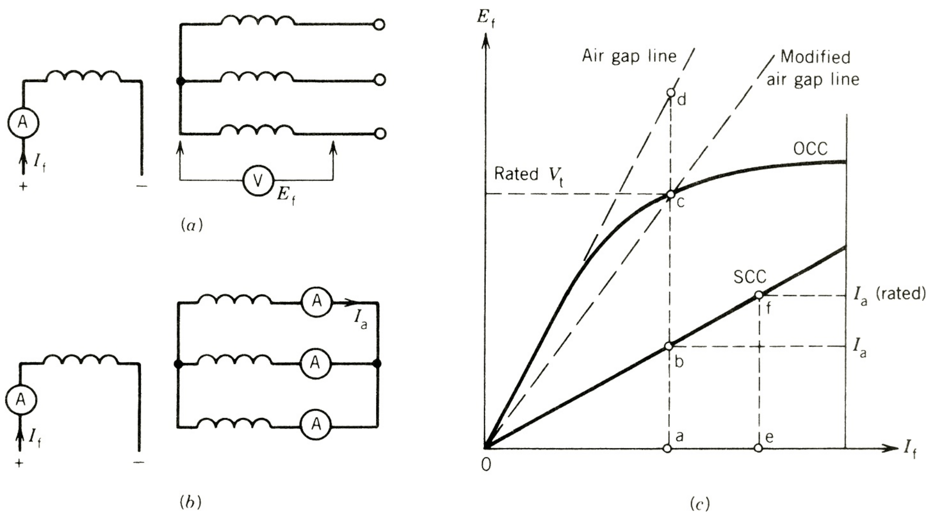


FIGURE 6.15 Open-circuit and short-circuit characteristics. (*a*) Circuit for open-circuit test. (*b*) Circuit for short-circuit test. (*c*) Characteristics.

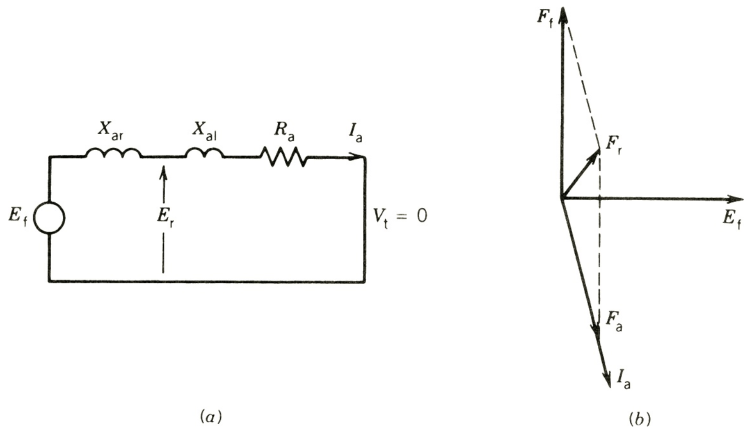


FIGURE 6.16 Short-circuit operation of a synchronous generator.

speed. The field current I_f is now varied, and the average of the three armature currents is measured. The variation of the armature current with the field current is shown in Fig. 6.15*c* and is known as the *short-circuit characteristic* (SCC). Note that the SCC is a straight line. This is due to the fact that under short-circuit conditions, the magnetic circuit does not saturate because the air gap flux remains at a low level. This fact can be explained as follows.

The equivalent circuit under short-circuit conditions is shown in Fig. 6.16*a*. Because $R_a \ll X_s$ (see Table 6.1), the armature current I_a lags the excitation voltage E_f by almost 90°. The armature reaction mmf F_a therefore opposes the field mmf F_f, and the resultant mmf F_r is very small, as can be seen from Fig. 6.16*b*. The magnetic circuit therefore remains unsaturated even if both I_f and I_a are large.

Also note from the equivalent circuit of Fig. 6.16*a* that the air gap voltage is $E_r = I_a(R_a + jX_{al})$. Because both R_a and X_{al} are small (see Table 6.1) at rated current, the air gap voltage will be less than 20 percent of the rated voltage (signifying unsaturated magnetic conditions at short-circuited operation). If the machine stays unsaturated, the excitation voltage E_f will increase linearly with the excitation current I_f along the air gap line, and therefore the armature current will increase linearly with the field current.

Unsaturated Synchronous Reactance

This can be obtained from the air gap line voltage and the short-circuit current of the machine for a particular value of the field current. From Fig. 6.15*c*,

$$Z_{s(unsat)} = \frac{E_{da}}{I_{ba}} = R_a + jX_{s(unsat)} \tag{6.8}$$

If R_a is neglected,

$$X_{s(unsat)} \simeq \frac{E_{da}}{I_{ba}} \tag{6.9}$$

Saturated Synchronous Reactance

Recall that prior to connecting a synchronous machine to the infinite bus, its excitation voltage is raised to the rated value. From Fig. 6.15*c*, this voltage is E_{ca}(= rated V_t) and the machine operates at some saturation level. If the machine is connected to the infinite bus, its terminal voltage remains the same at the bus value. If the field current is now changed, the excitation voltage will change, but not along the OCC line. The excitation voltage E_f will change along the line 0c, known as the *modified air gap line*. This line represents the same magnetic saturation level as that corresponding to the operating point c. This can be explained as follows.

From the equivalent circuit of Fig. 6.14*d*,

$$E_r = V_t + I_a(R_a + jX_{al}) \tag{6.10}$$

If the drop across R_a and X_{al} is neglected,

$$E_r \simeq V_t \tag{6.10a}$$

Because V_t is constant, the air gap voltage remains essentially the same, as the field current is changed. This implies that the air gap flux level (i.e., magnetic saturation level) remains practically unchanged, and hence as I_f is changed, E_f will change linearly along the line 0c of Fig. 6.15*c*.

The saturated synchronous reactance at the rated voltage is obtained as follows:

$$Z_{s(sat)} = \frac{E_{ca}}{I_{ba}} = R_a + jX_{s(sat)} \tag{6.11}$$

If R_a is neglected,

$$X_{s(sat)} \simeq \frac{E_{ca}}{I_{ba}} \tag{6.12}$$

6.4.2 PHASOR DIAGRAM

The phasor diagrams showing the relationship between voltages and currents for both synchronous generator and synchronous motor are shown in Fig. 6.17. The diagrams are based on the per-phase equivalent circuit of the synchronous machine. The terminal voltage is taken as the reference phasor in constructing the phasor diagram.

The per-phase equivalent circuit of the synchronous generator is shown in Fig. 6.17*a*. For convenience, the current I_a is shown as flowing out of the machine in the case of a synchronous generator.

$$E_f = V_t + I_aR_a + I_ajX_s = |E_f|\underline{/\delta} \tag{6.13}$$

The phasor for the excitation voltage E_f is obtained by adding the voltage drops I_aR_a and I_ajX_s to the terminal voltage V_t. The synchronous generator is considered to deliver a lagging current to the load or infinite bus represented by V_t.

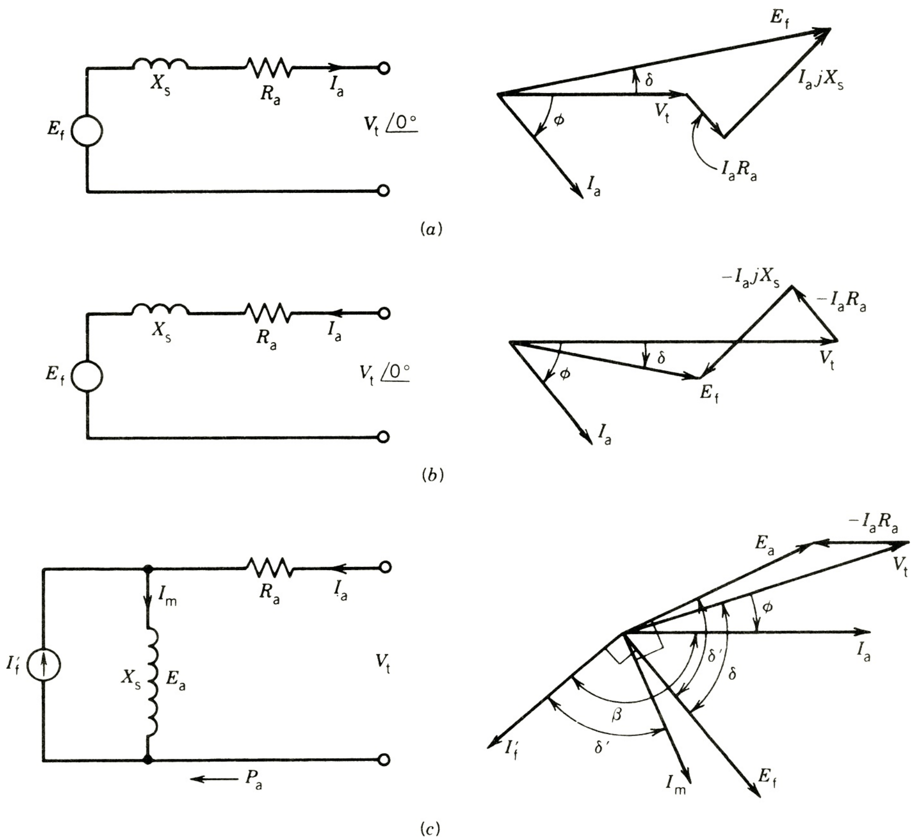


FIGURE 6.17 Phasor diagram for synchronous machines. Assume $V_t I_a$, and ϕ known. (*a*) Synchronous generator. (*b*) Synchronous motor. (*c*) Synchronous motor for current source equivalent circuit.

In the case of a synchronous motor, the current is shown (Fig. 6.17*b*) as flowing into the motor.

$$V_t = E_f + I_a R_a + I_a jX_s \tag{6.14}$$

$$E_f = V_t \underline{/0^\circ} - I_a R_a - I_a jX_s = |E_f| \underline{/-\delta} \tag{6.15}$$

The phasor E_f is constructed by subtracting the voltage drops from the terminal voltage. Here also, the synchronous motor is considered to draw a lagging current from the infinite bus.

It is important to note that the angle δ between V_t and E_f is positive for the generating action and negative for the motoring action. This angle δ (known as the power angle) plays an important role in power transfer and in the stability of synchronous machine operation and will be discussed further in the following sections.

The phasor diagram based on the equivalent current source model of Fig. 6.14*f* is shown in Fig. 6.17*c*. In this case the stator current I_a is taken as the reference for convenience. Note that the angle between E_a and E_f is the same as that between I_m and I'_f. If R_a is neglected, this angle is the power angle δ (angle between the phasors V_t and E_f). The angle between I_a and I'_f is called β.

6.10 SPEED CONTROL OF SYNCHRONOUS MOTORS

The speed of a synchronous motor can be controlled by changing the frequency of the power supply. At any fixed frequency the speed remains constant, even for changing load conditions, unless the motor loses synchronism. The synchronous motor is therefore very suitable for accurate speed control, and also where several motors have to run in synchronism. A synchronous motor can run at high power factor and efficiency (no power losses due to slip as in the induction motor). At present, it is being increasingly considered for use in variable-speed drives.

Two types of speed control methods are normally in use and are discussed here. In one method, the speed is directly controlled by changing the output voltage and frequency of an inverter or cycloconverter. In the other method, the frequency is automatically adjusted by the motor speed, and the motor is called a "self-controlled" synchronous motor.

6.10.1 FREQUENCY CONTROL

Schematic diagrams for open-loop speed control of a synchronous motor by changing the output frequency and voltage of an inverter or a cycloconverter are shown in Fig. 6.31. The inverter circuit (Fig. 6.31*a*) allows variation of frequency (and hence motor speed) over a

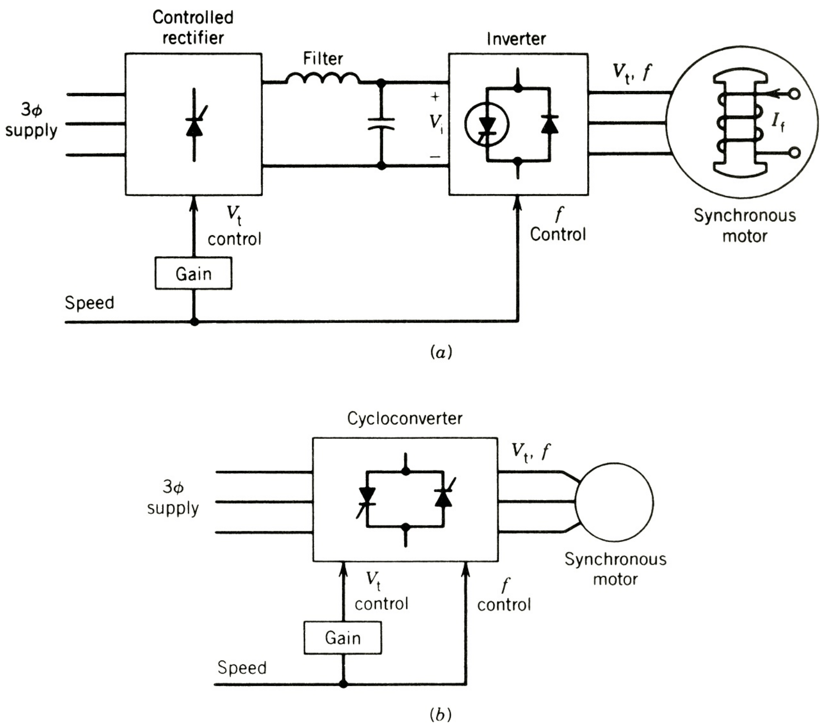


FIGURE 6.31 Open-loop frequency control.

wide range, whereas the cycloconverter circuit (Fig. 6.31*b*) permits variation of frequency below one-third of the supply frequency.

To obtain the same maximum torque over the whole range of speed variation, and also to avoid magnetic saturation in the machine, it is necessary to change the voltage with the frequency. From Eq. 6.24, for a three-phase synchronous machine,

$$P = T\omega_{\mathrm{m}} = \frac{3V_{\mathrm{t}}E_{\mathrm{f}}}{X_{\mathrm{s}}}\sin\delta \tag{6.61}$$

Now

$$\omega_{\mathrm{m}} = \frac{4\pi f}{p} \tag{6.62}$$

Let

$$X_{\mathrm{s}} = 2\pi f L_{\mathrm{s}} \tag{6.63}$$

If the field current I_{f} is kept constant, E_{f} is proportional to speed, and so

$$E_{\mathrm{f}} = K_1 f \tag{6.64}$$

where K_1 is a constant.
From Eqs. 6.61 to 6.64,

$$T = K\frac{V_{\mathrm{t}}}{f}\sin\delta \tag{6.65}$$

where K is a constant. A base speed can be defined for which V_{t} and f are the rated values for the motor. If the ratio V_{t}/f corresponding to this base speed is maintained at lower speeds by changing voltage with frequency, the maximum torque (i.e., pull-out torque, KV_{t}/f) is maintained equal to that at the base speed.

The torque–speed characteristic for variable-voltage, variable-frequency (VVVF) operation of the synchronous motor is shown in Fig. 6.32. For regenerative braking of the synchronous motor, the power flow reverses. Note that in the inverter system (Fig. 6.31*a*), because of diodes

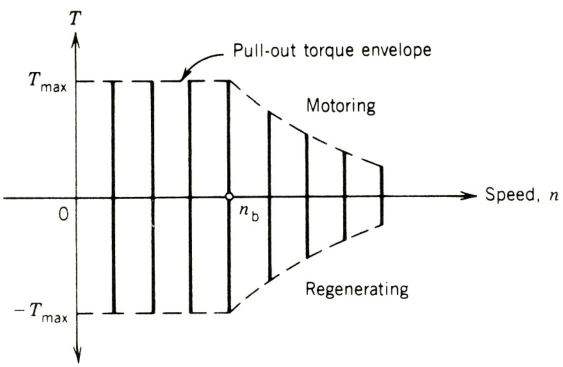


FIGURE 6.32 Torque–speed characteristic of synchronous machines.

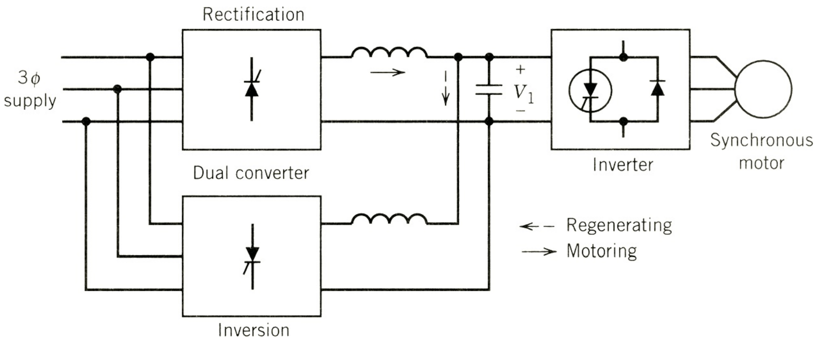


FIGURE 6.33 Controller with reversible power flow.

in the inverter, voltage at the input to the inverter cannot reverse. Therefore, for reverse power flow, current reverses. A dual converter, as shown in Fig. 6.33, is therefore necessary to accept the reversed current flow in the regenerating mode of operation. In a cycloconverter, however, the power flow is reversible. To vary the speed above the base speed, the frequency is increased while maintaining the terminal voltage at the rated value. This, of course, will make the pull-out torque decrease at higher speed range, as shown in Fig. 6.32. In Fig. 6.31*a*, if the controlled rectifier is replaced by a diode-bridge rectifier, the inverter input voltage V_i is constant. The inverter can be a PWM (pulse-width-modulated) inverter so that both output voltage and frequency can be changed in the inverter.

Note that if the frequency is suddenly changed or changed at a high rate, the rotor poles may not be able to follow the stator rotating field, and the motor will lose synchronism. Therefore, the rate at which the frequency is changed must be restricted. A sudden change in load torque may also cause the motor to lose synchronism. The open-loop drive is therefore not suitable for applications in which load may change suddenly.

6.10.2 SELF-CONTROLLED SYNCHRONOUS MOTOR

A synchronous motor tends to lose synchronism on shock loads. In the open-loop speed control system discussed in the preceding section, if a load is suddenly applied, the rotor momentarily slows down, making the torque angle δ increase beyond 90° and leading to loss of synchronism. However, if the rotor position is sensed as the rotor slows down and the information is used to decrease the stator frequency, the motor will stay in synchronism. In such a scheme, the rotor speed will adjust the stator frequency, and the drive system is known as a *self-controlled synchronous motor* drive.

The schematic of a self-controlled synchronous motor drive system is shown in Fig. 6.34. Two controlled rectifiers are used, one at the supply end and the other at the machine end. In the motoring mode of operation, the supply end rectifier operates in the rectification mode and the machine end rectifier in the inversion mode. The roles of the rectifiers reverse for regenerative braking, in which the power flow reverses. The thyristors in the supply end rectifier are commutated by the supply line voltage, and those in the machine end rectifier by the excitation voltage of the synchronous machine.

The rotor position sensor, mounted on the rotor shaft, generates signals having rotor position information. These signals are processed in the control logic circuit and used to fire the

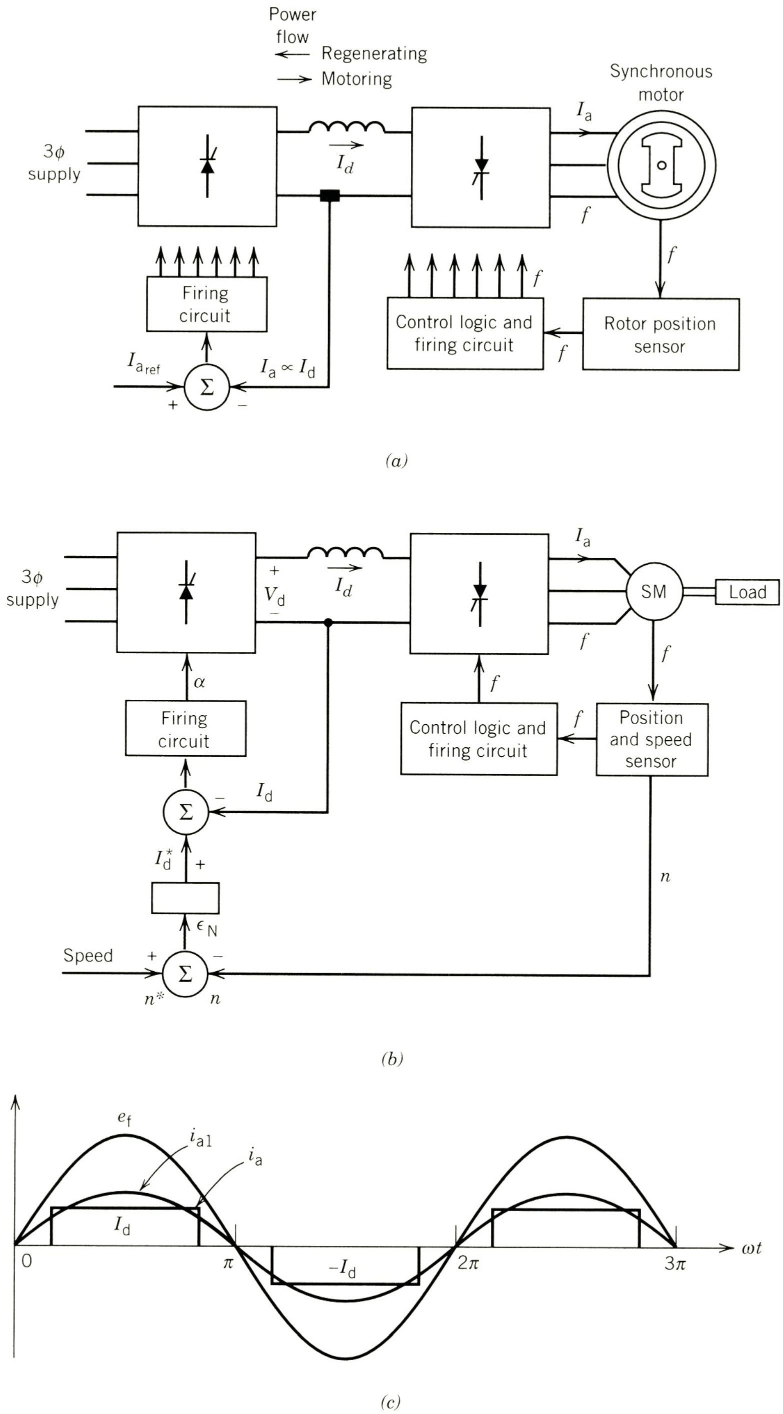


FIGURE 6.34 Self-controlled synchronous motor drive. (*a*) Open-loop control. (*b*) Closed-loop control. (*c*) Waveform of e_f and i_a for operation similar to a dc motor.

thyristors of the machine end rectifier. Therefore, any change in the rotor speed due to change in load will immediately change the frequency of firing of the thyristors, and hence adjust the stator frequency at the correct rate to maintain synchronism.

A current loop is implemented around the supply end rectifier to maintain the machine current at the desired value. The dc link current I_d, being proportional to the machine current I_a, is compared with the reference current, and the error signal adjusts the firing of the supply end rectifier to keep the armature current constant at the reference value.

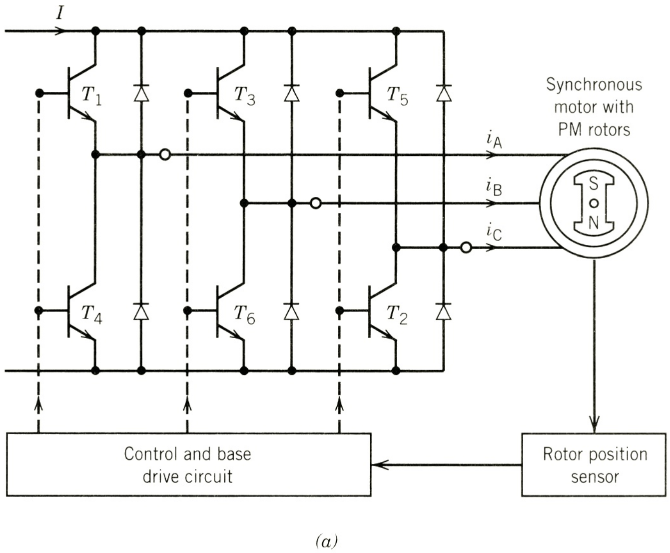
From Eq. 6.29j, the torque depends on the value of the angle β. This angle β can be controlled in the control logic circuit for the machine end rectifier, because the signal from the rotor position sensor defines the position of the field axis, and the firing instant of the thyristors defines the position of the armature field axis. If angle β is regulated at a specified value and the field current is kept constant, the torque (and hence the speed) can be directly controlled by the armature current. This current is controlled by the current control loop of the supply end rectifier.

Both rectifiers are simple and inexpensive circuits and, unlike forced commutated inverters, do not require commutation circuits for commutation of the thyristors. As a result, such drive systems can be designed for very high-power applications—on the order of a megawatt.

dc motor.

6.13 BRUSHLESS DC (BLDC) MOTORS

A synchronous motor with a permanent magnet in the rotor and operated in self-controlled mode—using a rotor position sensor and an inverter to control current in the stator windings—is generally known as a brushless dc motor. The BLDC is an inside-out brushed dc motor, because the armature is in the stator and the magnets are in the rotor, and its operating characteristics are similar to those of the conventional brushed dc motor, discussed in Chapter 4. The position sensor and the solid-state switches in the inverter perform the role of the brushes and the mechanical commutator of the conventional dc motor. The BLDC motor operates in a manner similar to the self-controlled synchronous motor of Fig. 6.34. The thyristors in the inverter controlling the motor current are commutated by the back emf of the motor. At low speed, the back emf may not be sufficient to commute the thyristors. In such a situation, the dc link current is to be reduced to zero by operating the supply-end converter in the inversion mode. However, an inverter can be used that employs self-commutating devices, such as BJTs, IGBTs, MOSFETs, and GTOs. The on–off state of these switches can be controlled by their gate signals, which are obtained from the rotor position sensors. A typical transistor (BJT) inverter configuration fed from a dc current source, as shown in Fig. 6.39, can be used for a BLDC drive system. The rotor



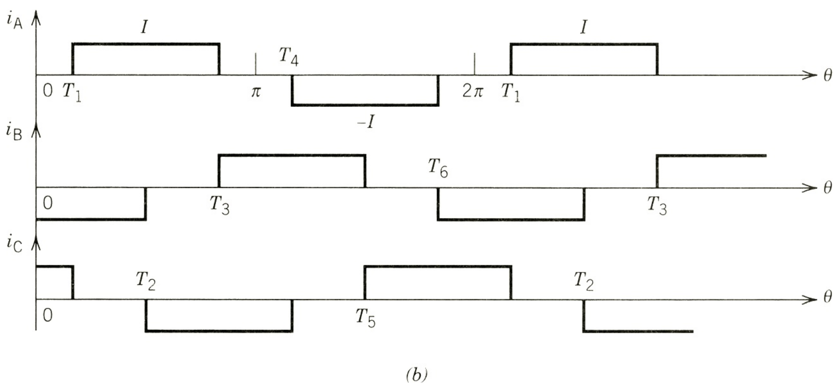


FIGURE 6.39 Brushless dc motor drive. (*a*) Circuit. (*b*) Stator current waveforms.

position sensor will control the turn-on and turn-off instants for the switches such that the angle between the rotor field and the stator field is regulated at 90°, similar to the conventional brushed dc motor. The idealized waveforms of the stator currents are also shown in Fig. 6.39. The switches of the inverter are turned on every 60 electrical degrees, and the switches are numbered in the sequence in which they are turned on. If the inverter is fed from a dc voltage source (voltage source inverter), pulse-width modulation of the individual switches can provide the regulation of the motor current.

There are two basic types of BLDC motors: trapezoidal type and sinusoidal type. In the trapezoidal type the back emf is of trapezoidal shape and, for ripple-free torque operation, the phase current required is a 120° quasi-square wave. In the sinusoidal type, the back emf is sinusoidal and, for ripple-free torque operation, the phase current required is sinusoidal.

A trapezoidal back emf is generated by rotor permanent magnets with a square air gap flux distribution and a concentrated stator winding. The 120° width of the phase current requires a low-resolution position sensor, such as a Hall effect sensor or an electro-optical sensor, to switch devices every 60° for commutation of the phase currents.

A sinusoidal back emf is generated by rotor permanent magnets with an essentially sinusoidal air gap flux distribution and a distributed stator winding. The winding can be short pitched to reduce the effect of the space-harmonic flux distribution. The sinusoidal phase current requires an absolute position sensor or resolver for continuous position sensing to allow accurate synthesis of the sinusoidal current waveform. The current in a phase is a sinusoidal function of the rotor position.

The ideal waveforms of the various quantities in the two types of BLDC motors are shown in Fig. 6.40.

In the trapezoidal-type motor, since the shapes of the back emf (trapezoidal) and the winding current (constant amplitude) are similar to those in a conventional dc machine, the motor is referred to as a brushless dc motor, whereas the sinusoidal type is referred to as a brushless synchronous motor.

The trapezoidal-type motor requires an inexpensive position sensor. Its commutation scheme is simple, and since only two switches are on at any time, the efficiency of the inverter is high. The overall cost will be lower. However, torque ripple and audible noise will be high if the voltage and current waveforms are nonideal.

The sinusoidal-type motor requires a relatively more expensive position sensor. More hardware and software for signal processing are required. The inverter efficiency will be lower because three switches are on at the same time. The overall cost will be higher. However, torque ripple will be lower even with nonideal back emf.

Four types of permanent magnet rotor configurations are commonly used. These types are shown in Fig. 6.41. The four rotor types are characterized by surface-mounted magnets, inset magnets, and embedded or buried magnets, the latter having two variants.

Surface-Mounted Magnets

The radially magnetized permanent magnets are mounted on a solid steel-core rotor structure as shown in Fig. 6.41*a*. Since the relative permeability of the magnet material is near unity, it acts like a large air gap. The effective air gap is therefore large, making L_d low. The structure is magnetically nonsalient, and thus $L_d = L_q$.

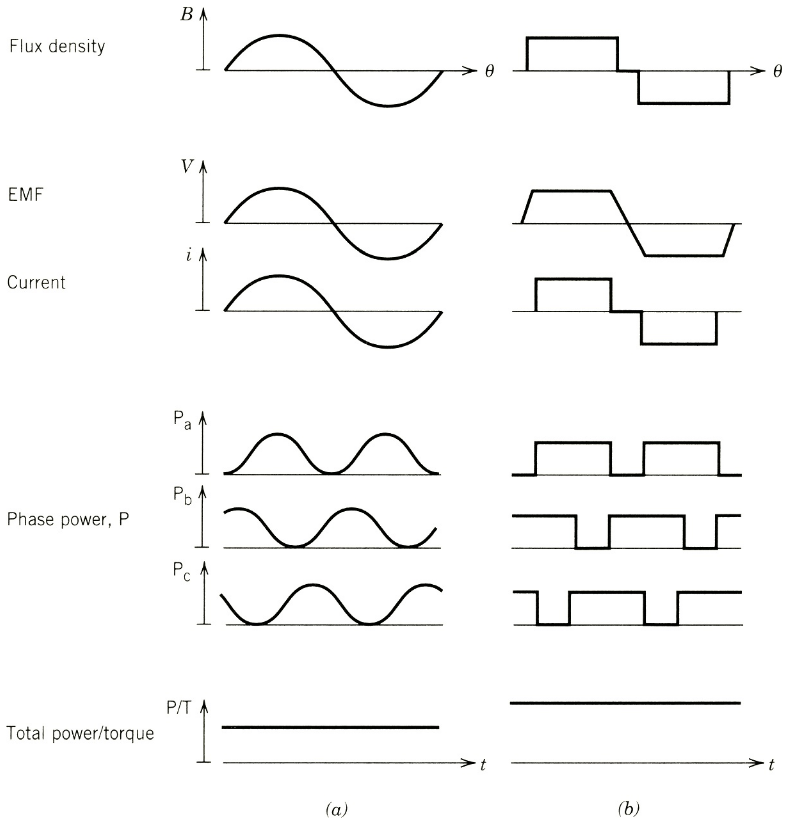


FIGURE 6.40 Idealized waveforms in BLDC motors. (*a*) Sinusoidal type. (*b*) Trapezoidal type.

This type of rotor structure is the most common topology, and is the industrial standard. The surface-mounted permanent magnet (SMPM) motor results in lower cogging torque due to the large effective air gap. SMPM motors are widely used in fans, blowers, robotics, servo drives such as vehicle electric power steering, machine tools, and so on, due to their low inertia. In some high-power applications, the wound stator is placed in the center of the machine, while the magnets are mounted along the inner circumference of the outer rotating rotor. This allows a higher number of poles, as the rotor diameter is larger. In addition, the centrifugal forces exert a pressure on the permanent magnets, leading to more robust and higher-speed operation. This structure is also adapted to wind turbines, as the hub carrying the blades can be fixed directly to the outer rotor.

Inset Magnets

In the inset arrangement, the permanent magnets are inserted in the steel rotor structure as shown in Fig. 6.41*b*. This construction provides a more secure magnet setting and ease of assembly. In this configuration, $L_d < L_q$. For the same magnet size, the peak torque developed with inset magnets is higher than that for the surface-mounted magnets because of the

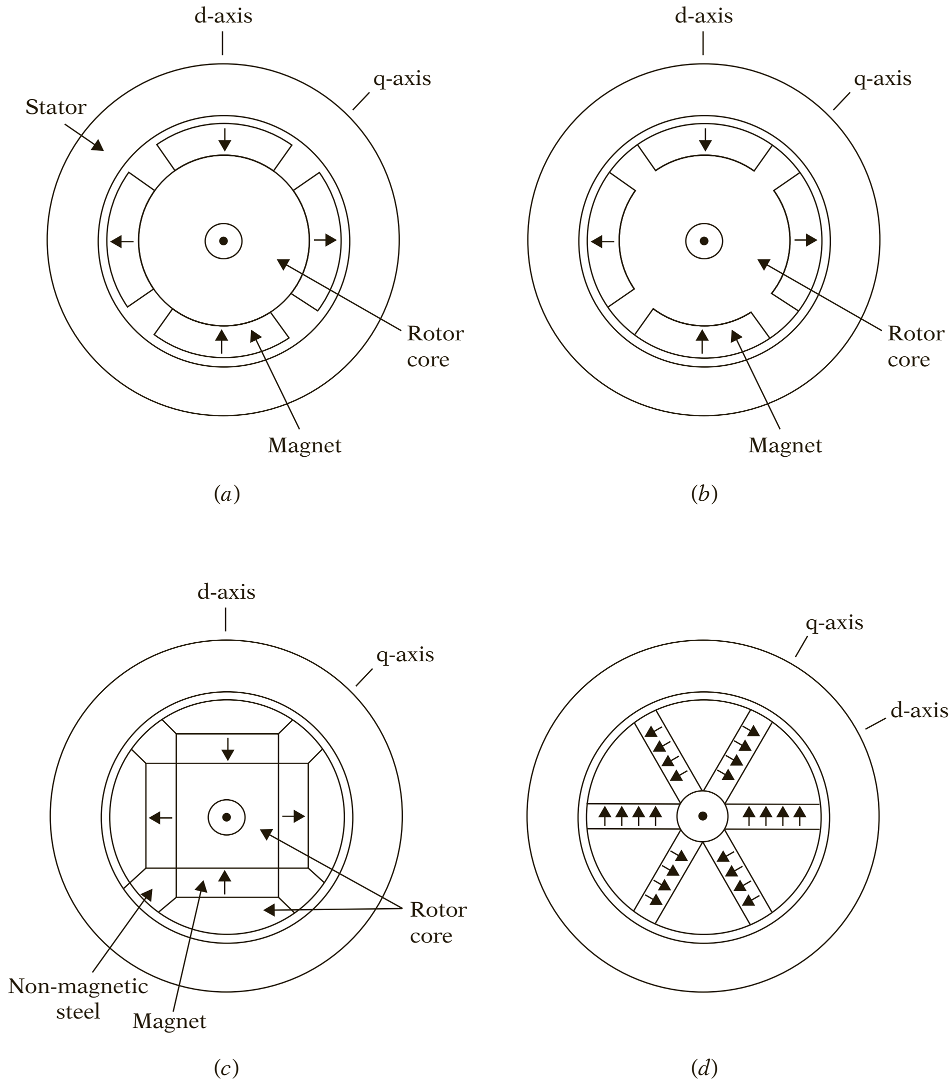


FIGURE 6.41 Permanent magnet rotor configurations. (*a*) Surface-mounted magnets. (*b*) Inset magnets. (*c*) Interior (buried) magnets with radial magnetization. (*d*) Interior (buried) magnets with circumferential magnetization.

reluctance torque developed with the former. To produce the same torque, the thickness required for the inset magnets is smaller and hence L_d is larger.

This type of rotor structure is more robust and has the same applications as the surface-mounted magnet type. They are suitable for high-power and high-speed applications, such as windmills.

Interior (Buried) PM with Radial Magnetization

In this arrangement, the magnets are buried inside the rotor structure, with radial magnetization, as shown in Fig. 6.41*c*. The q-axis inductance is larger than the d-axis inductance ($L_q > L_d$), and both L_d and L_q are larger than their corresponding values in surface-type and inset-type rotors.

Due to the additional reluctance torque capability and secure placement of magnets, IPM-type motors are used in electric vehicles and propulsion applications.

Interior (Buried) PM with Circumferential Magnetization

This arrangement of rotor magnets is shown in Fig. 6.41*d*. Because of the flux-focusing effect, circumferential magnetization yields greater air gap flux than radial magnetization. The d-axis inductance (L_d) is large, and the structure is magnetically salient, having $L_d > L_q$. Ferrite magnets have low flux density. The circumferential magnetization arrangement is particularly advantageous for ferrite magnets, because a substantial increase in flux density can be achieved.

This type of rotor arrangement can be used for low-cost, high-efficiency propulsion motors, machine tools, and appliances.

The surface-mounted magnets, because of constant magnetic gap between the stator and rotor, can provide a square wave flux distribution. This type of magnet is normally used in trapezoidal-type motors. The inset and buried magnet arrangements are used primarily for sinusoidal-type motors, in which the sinusoidal flux distribution can easily be achieved by proper design of the rotor geometry. Because of sine commutation providing smooth and low audible noise, IPM motors are suitable for air compressors, refrigerators, washing machines, dishwashers, and electric power steering.

In BLDC motors, the rotor flux is essentially constant because of the permanent magnet. The flux weakening required to operate the motor beyond the base speed for constant-horsepower operation can be achieved by providing a negative d-axis stator current component ($-i_d$) to oppose the rotor magnet flux. In the surface-mounted magnet rotor, since L_d is low, a very large i_d is required to provide effective field weakening. Thus, a limited speed range (up to $1.2n_b$, where n_b is the base speed) can be achieved with the surface-mounted magnet rotor. This rotor configuration is therefore used primarily for constant-torque operation. With the inset-magnet rotor, the speed range can be extended to $1.5n_b$. For the buried-magnet rotor with radial magnetization, the speed range can be extended to $2n_b$. For the circumferential-magnetization type of rotor, a speed range up to $3n_b$ can be achieved.

BLDC motors have the following advantages compared with conventional brushed dc motors:

- Small rotor size and high-power density, because of the absence of mechanical commutators, brushes, and field windings.
- Lower inertia and faster dynamic response.
- Higher speed and torque capability, due to the absence of brushes and sparking.
- Lower maintenance cost.
- High torque/inertia ratio.
- Better heat dissipation, due to the stationary armature winding.

- Better overall reliability and life.

BLDC motors have the following advantages compared with induction motors:

- High efficiency, because of no slip power losses.
- High power and torque density, because of high air gap flux density.
- Lower operating temperature, because of low losses.

On the negative side, BLDC motor drive systems require rotor position sensors, complex power electronics (inverters), and complex controllers, resulting in a higher initial system cost. There is a possibility that demagnetization of the permanent magnets may occur due to accidental overload or fault. The advantages, generally, outweigh the disadvantages.

Improvements in permanent magnet materials and cost reduction in integrated circuits and electronic power switches have made BLDC motors—in particular the trapezoidal type—major contenders in the field of high-performance drives. These motors are being extensively used in computer disk drives, servo drives, robotics, machine tools, electric vehicles, battery-powered applications, windmills, and many other applications.

6.14 SWITCHED RELUCTANCE MOTORS (SRM)

Switched reluctance motors have saliency in both stator and rotor and are perhaps the simplest electric motors in construction. The stator consists of salient poles with excitation windings on them, and the rotor has salient poles with no windings. The torque is produced by the tendency of the rotor pole to align with the stator pole to maximize the stator flux linkage when the winding of the stator pole is excited by a current.

6.14.1 BASIC OPERATION OF SRM

Figure 6.42 shows a cross-sectional view of a switched reluctance motor with eight stator poles and six rotor poles. These motors have one pair of poles less on the rotor than on the stator. Diametrically opposite stator poles are excited simultaneously, as shown in Fig. 6.42. The SRM

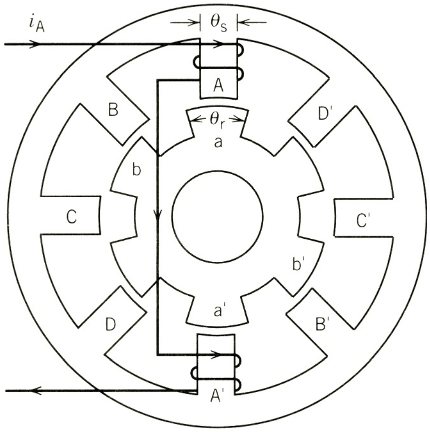


FIGURE 6.42 Cross section of a switched reluctance motor (SRM).

in Fig. 6.42 has four stator phases. If phase A is excited, rotor poles marked a and a' will be aligned with stator poles marked A and A′. If phase B winding now is excited, rotor poles b and b′ will be aligned with poles B and B′ of stator phase B, and therefore the rotor will move clockwise. Thus, if the phases are excited in the sequence A, B, C, D, A . . ., the rotor will move in the clockwise direction. The rotor will move with some synchronism with the stator field, but the motions of the stator field and rotor poles are in opposite directions. Correct timing of the excitation of a phase winding depends on the position of the rotor. Therefore, for proper operation of the motor, a rotor position sensor is required. The excitation must be switched sequentially from phase to phase as the rotor moves, hence the name switched reluctance motor.

6.14.2 MODELING AND TORQUE PRODUCTION

A simplified model of the SRM is obtained on the basis of the following assumptions:

1. There is no mutual flux linkage between phase windings; that is, one phase is excited at a time.
2. The ferromagnetic materials in the machine have a linear $B-H$ characteristic.

With these assumptions, the flux linkage of a winding can be represented by an inductance. As the rotor changes position, the inductance of a phase winding will change. Various positions of the rotor are shown in Fig. 6.43. Figure 6.44*a* shows the inductance profile of phase A. The torque produced by the excitation of the kth phase is

$$T_k = \frac{\partial}{\partial \theta}\left(\tfrac{1}{2} L_k(\theta_k) i_k^2\right)\Big|_{i_k} \tag{6.76}$$

$$= \tfrac{1}{2} i_k^2 \frac{\partial L_{\mathrm{k}}(\theta_k)}{\partial \theta} \tag{6.76a}$$

Since the torque is proportional to the square of current, it is independent of the current direction.

The torque produced for a constant current through the phase winding (Fig. 6.44*b*) is shown in Fig. 6.44*c*. The average torque produced is zero. If the current is applied only when the inductance is increasing (Fig. 6.44*d*), the negative torque is eliminated (Fig. 6.44*e*), and the motor will produce an average positive torque. It is important, therefore, for the production of an average positive torque that the current in a phase winding be applied when the rotor is in a particular position, and switched off at another predetermined rotor position.

In a practical circuit the current in the phase winding will not have a square waveform. When a voltage is applied to a phase winding at rotor position θ_2 (Fig. 6.45*a*), the current builds up with a finite time lag. The current will build up quickly, because the inductance is low. When the voltage is removed at rotor position θ_2, the current will decay at a slower rate, because the inductance is high. The current waveform and the corresponding torque waveform are also shown in Fig. 6.45*a*. If the negative torque is to be eliminated, the voltage from the phase winding would have to be removed earlier, as shown in Fig. 6.45*b*.

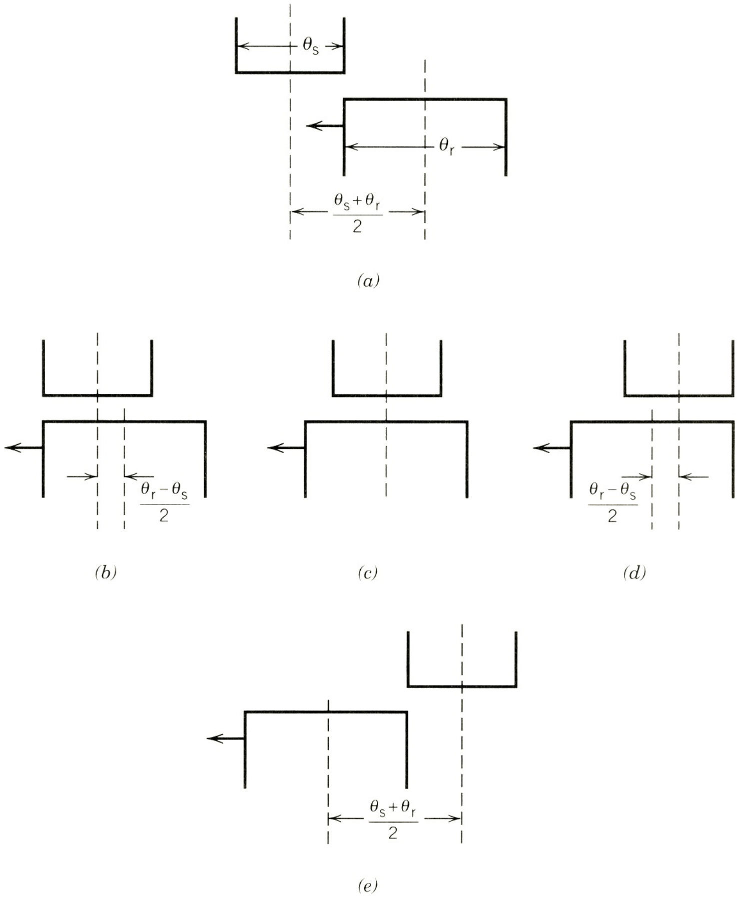


FIGURE 6.43 Rotor positions. (*a*) Inductance linearly increases from this position. (*b–d*) Maximum inductance positions. (*d*) Inductance linearly decreases as rotor moves from this position. (*e*) Minimum inductance position.

The voltage–current relationship of a phase winding can be expressed as follows:

$$V_k = Ri_k + \frac{d}{dt}\lambda_k \tag{6.77}$$

$$= Ri_k + \frac{d}{dt}(L_k(\theta_\kappa)i_k) \tag{6.77a}$$

$$= Ri_k + i_k\frac{dL_k(\theta_\kappa)}{d\theta}\frac{d\theta}{dt} + L_k(\theta_k)\frac{di_k}{dt} \tag{6.77b}$$

$$= \left(R + \frac{dL(\theta_k)}{d\theta}\frac{d\theta}{dt}\right)i_k + L_k(\theta_2)\frac{di_k}{dt} \tag{6.77c}$$

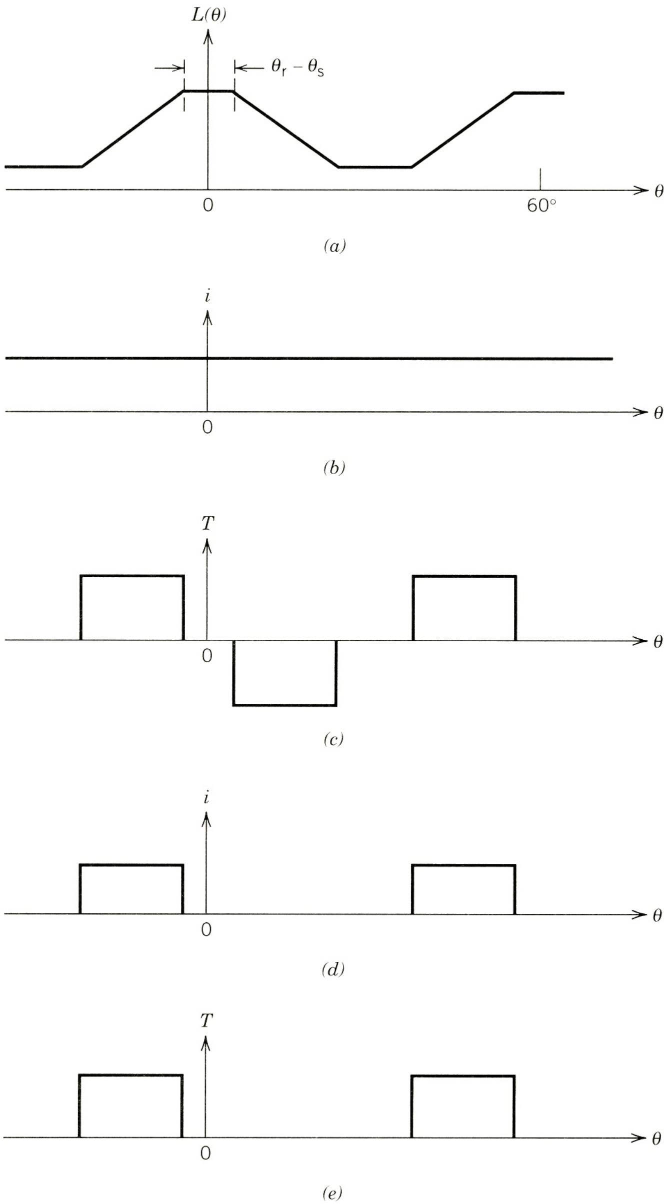


FIGURE 6.44 (a) Variation of inductance with rotor position. (*b*) Constant-current excitation. (*c*) T versus θ. (*d*) Pulsed-current excitation. (*e*) T versus θ.

EXAMPLE 6.9

In a three-phase switched reluctance motor, having eight stator poles and six rotor poles, the pole widths for the stator and rotor poles are the same and $\theta_s = \theta_r = 20°$. The currents in the phase windings are applied when the inductances are increasing. Draw qualitatively the current and torque waveforms of each phase and the total torque developed by the motor for a square wave current having

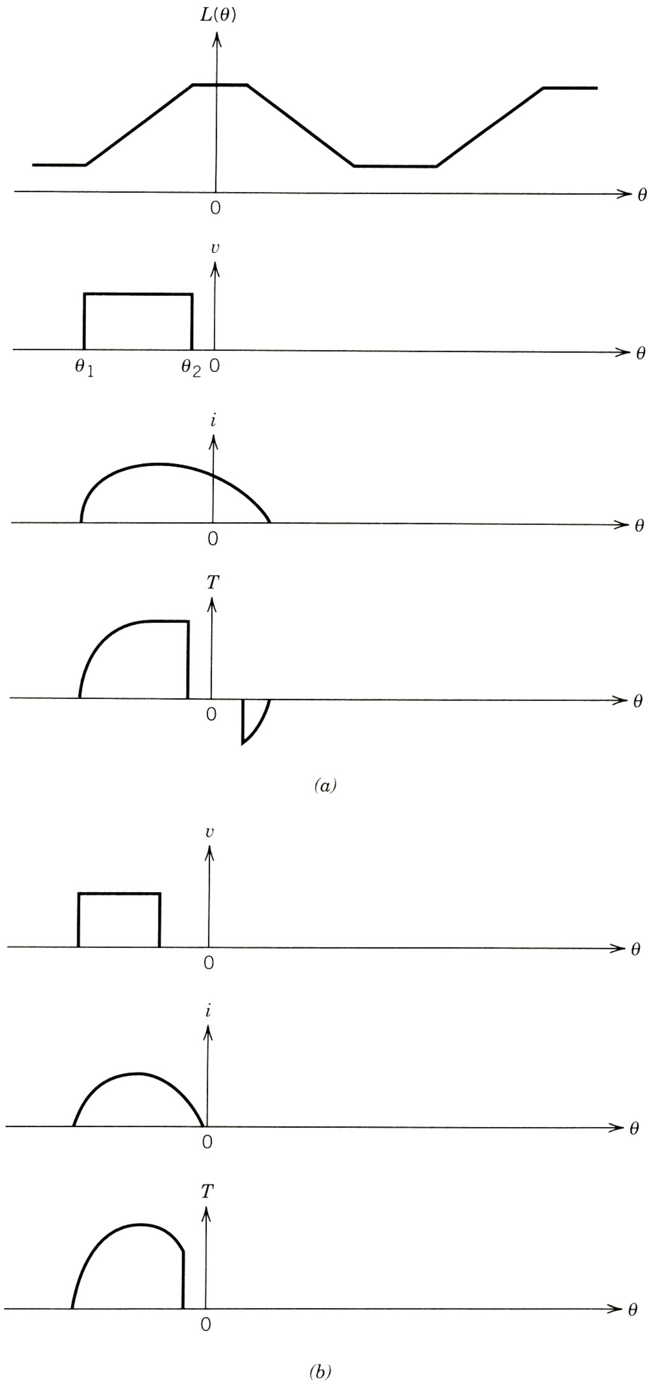


FIGURE 6.45 (*a*) Characteristics when voltage applied during the period of inductance increases. (*b*) Characteristics when voltage is removed earlier than in (*a*).

(a) 20° width.

(b) 15° width.

Solution

The rotor has six poles. Therefore, the inductance variation for a phase will repeat every 60° $(= 300 \div 6)$. Since $\theta_s = \theta_r = 20°$, the inductance will increase for 20° duration and decrease for 20° duration.

There are eight stator poles and four stator phases. Each phase is thus excited for an interval of 45°. The inductance profile and current and torque variations will be the same for each phase, except that these are shifted by 45° with respect to each other.

The inductance profile of the winding of phase a is shown in Fig. E6.9*a*.

(a) The waveforms of current and torque produced by different phases and the total torque, $T = T_a + T_b + T_c$, are shown in Fig. E6.9*b*. The resultant torque has considerable ripple.

(b) The current and torque of each phase, as well as the resultant torque, are shown in Fig. E6.9*c*. For this particular current width, the torque ripple is eliminated. However, the average value of the resultant torque is reduced compared with the 20° width of the current.