EE 113 - Control Systems Module

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Key elements of control

Trajectory optimization is the process of designing a trajectory that minimizes (or maximizes) some measure of performance while satisfying a set of constraints

Control Theory

- Regulation & Tracking (often needs 'feedback')
- Trajectory Optimization (often pre-calculated)

Mathematical aspects get applied in engineering

Electrical, Mechanical, Aerospace, Mathematics, Chemical Engineering Depts

Energy Engineering, Financial Engineering,

Involves analysis (mathematical modelling, and response analysis) and then synthesis (controller design, shaping)

Dynamical systems

Dynamic System is time dependent. f(t).

- Systems have delays/dynamics/memory
- Generators: when load increases, speed decreases (in some time)
- Heat: effect on temperature
- Vehicle control: steering/brakes: some delay before it has an effect
- Filters: input/output relation is 'dynamic'
- Economic systems: cash-reserve-ratio: effect on markets/liquidity in some time
- Economic systems: interest rates (FD/lending): effect on economy: delays
- Traffic congestion: lights-timings depends on queue (feedback)
- Internet Traffic congestion: packet-drop probability depending on router congestion (feedback)

Overview: read from Polderman & Willems:
Intro to mathematical systems theory: a behavioral approach

Control Theory - Regulation

- Plant: system to be controlled
- Regulation: keep certain to-be-controlled variables at desired values (set-point)
- In spite of external disturbances
- In spite of changes in plant properties

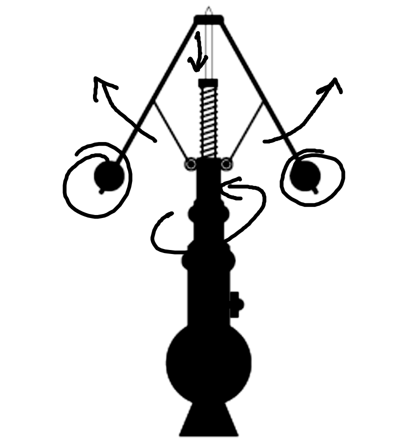
- Examples:
  - Temperature control at home/office
  - Suspension (passive/active) of an automobile (absorbs the irregularities of the road to improve the comfort and safety)
- Regulation: for efficiency, quality control, safety, and reliability.

Control inventions

History

- Christiaan Huygens (1629-1695) invented a flywheel device for speed control of windmills
- Main idea used later: centrifugal fly-ball governor (by James Watt, 1736-1819, the inventor of the steam engine)
- Tuning centrifugal governors that achieved ‘fast regulation’, but avoided ‘hunting’ (James Clerk Maxwell)

Fly ball Governor



(Source: Polderman/Willems 1998 book)

Two key recent control inventions

- About a century ago: two main inventions drove control theory: regulation

1. Proportional-Integral-Differential (PID) controller
2. In 1930s, the ‘negative feedback amplifier’ by Harold Black (of Bells)

Control Inventions: feedback amplifier

- Had far-reaching applications to telephone technology and other areas of communication.
- Long-distance communication used to be hampered due to the drifting of the gains of the amplifiers used in repeater stations.
- Impressive technological development: it permitted signals to be amplified in a reliable way,
- Now: insensitive to the parameter changes inherent in vacuum-tube (and also solid-state) amplifiers.

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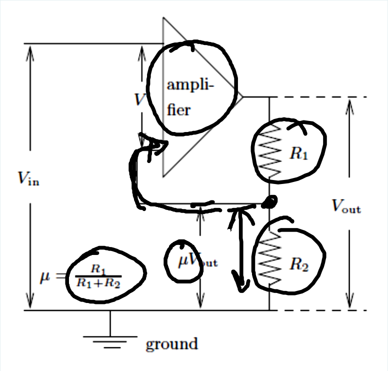
Feedback-Amplifier ($V = V_{diff}$)



Black's Feedback Amplifier

Control Inventions - Feedback Amplifier

- Assume that we have an electronic amplifier that amplifies its input voltage to output voltage with a gain K .

$$V_{out} = K \; V_{diff} \tag{1}$$

- Use a voltage divider and 'feed back' the voltage: μV_{out} to the amplifier input.
- Basic calculations give:

$$V = V_{in} - \mu V_{out} \tag{2}$$

- Combining these two gives a crucial relation

$$V_{out} = \frac{V_{in}}{\mu + \frac{1}{K}}$$

$$V_{out} = \frac{1}{\mu + \frac{1}{K}} V_{in} \tag{3}$$

Feedback amplifier

What's the big deal with this formula?

- Value of K of an electronic amplifier is typically large, but also very unstable: due to: sensitivity to aging, temperature, loading
- The voltage divider: can be implemented by means of passive resistors: gives a very stable value for μ.
- Now, for large values of K (although varying values of K):

$$\frac{1}{\mu + \frac{1}{K}} \approx \frac{1}{\mu} \tag{4}$$

and so Black's feedback amplifier gives:
an amplifier with a stable amplification gain $(1/\mu)$
based on an amplifier that has an inherent uncertain gain K.

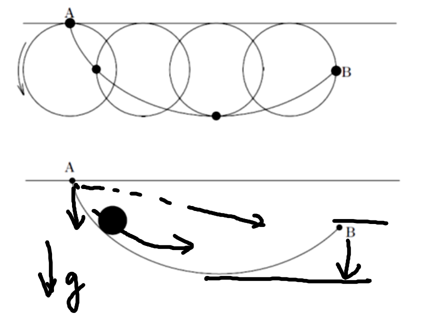
Control theory - trajectory optimization

- Trajectory transfer (for a dynamical system): find a path from a given initial state to a given terminal state.
- Most common example: satellite: go from one periodic orbit to another: with least power/energy
- Path with maximum distance from obstacles
- Classic example: Brachystochrone problem (Johann Bernoulli - 1696)
- Find path/curve between two points A and B such that a body falling-under-its-own-weight reaches B in least time.

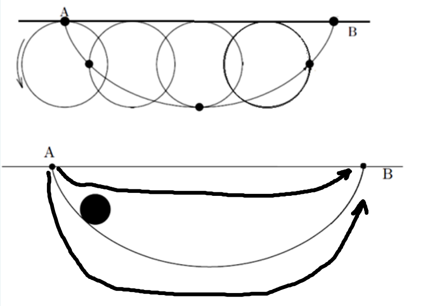
Control theory - trajectory optimization

cycloid

Case 1

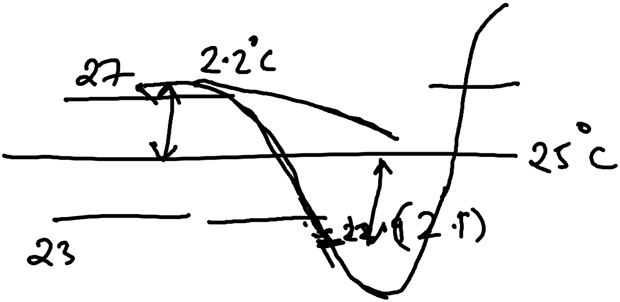


Case 2



(Source: Polderman/Willems 1998 book)


R
2πR


27
2·2°C
25°C
23
(2·5)

Example of Air Conditioning Mechanism

- Air Conditioner with fixed time interval on/off: cheaper implementation
  - For example AC is on for 20 minutes and off for 10 minutes.
  - Timings are preset to save energy consumption.
  - Actual room conditions unconsidered (blind control)
- Air Conditioner with sensor based control (on/off): expensive implementation
  - AC with sensor based control takes the actual parameters,
  - analyse them with reference values, adjust the conditioning to the required level.
  - There will be a slight band at reference value, where no control action is taken.
  - Feedback: can make closed loop 'unstable' (even if open-loop was well-behaved)

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  - (Impatience (or 'over-demanding' could cause instability)

Closed Loop Control System

Automatic control ≡ closed-loop ≡ feedback-control

- Advantages of Closed Loop Control System
  - Closed loop control systems are usually more accurate
  - Error is corrected due to presence of feedback signal
  - Wide bandwidth (range of frequencies for which system responds desirably)
  - Facilitates automation
  - The sensitivity of system may be made small to make system more stable
  - This system is less affected by noise/modelling/system-uncertainties: robust control
- Disadvantages of Closed Loop Control System
  - They are costlier
  - They are complicated to design/implement
  - They are less reliable (complex design means more scope for breakdowns/degradation)
  - Require more maintenance
  - Feedback could lead to oscillatory/unstable response (unless we all learn control-theory well)

Exercise:

$$A = \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix} \qquad cI \qquad c \neq 0$$

- Find example from our daily experience that involves
- open loop control
- closed loop control (involves feedback)

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + 3 \times -2 \\ \\ \end{bmatrix} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$$

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

Exercise:

- Find example from our daily experience that involves
- open loop control
- closed loop control (involves feedback)

Suggest/think of closed-loop desirable properties and optimality/performance criteria

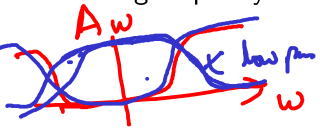
$A\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}-16\\-14\end{bmatrix}$ $\qquad A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}-3\\-3\end{bmatrix}$ $\quad -3$

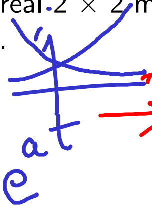
$Av = -3v.$

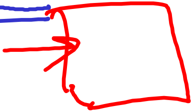
- Consider: $A = \begin{bmatrix} 7 & -10 \\ 5 & -8 \end{bmatrix}$
- In general, when A acts on a vector v (i.e. Av), the vector v gets scaled (lengthened/shortened/flipped) and also rotated.
- Vectors: $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\qquad \begin{bmatrix} 7 & -10 \\ 5 & -8 \end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}4\\2\end{bmatrix}$
- Check that vector v_1 gets purely scaled (when A acts on v_1): what is the scaling?
- Check that vector v_2 also gets purely ....
- Suggest a square real 2×2 matrix in which no real vector gets purely scaled.

$A_w \sin(wt+\phi)$

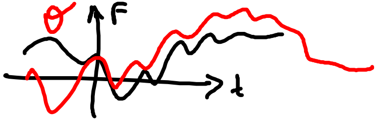
$A \sin$

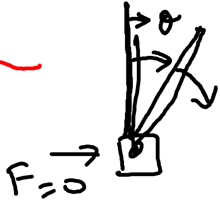






Positive/negative feedback

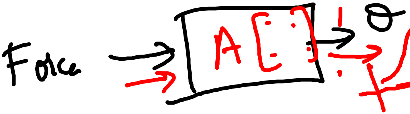


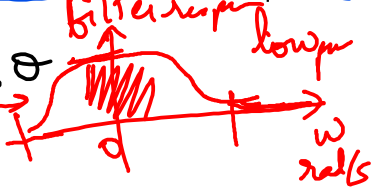


- $\frac{d}{dt}x = u$
- Suppose $u = +5x$ (positive feedback)
- Suppose $u = -5x$ (negative feedback)
- Solve both differential equations (exponentially growing/decaying)

?

Find what is discrete-time impulse δ_d and continuous-time impulse δ_c





LTI systems and exponential signals

- Linear Time Invariant (LTI) systems:
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LTI systems and exponential signals

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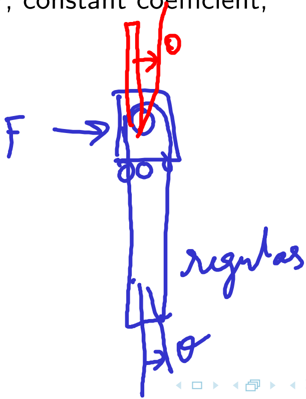

LTI systems and exponential signals

Regular pendulum

$F = M\ddot{\theta} + \theta$ deviat. from down

Inverted pendul

$F = M\ddot{\theta} - \theta$ devi from up point

theta

- Linear Time Invariant (LTI) systems:
  systems governed by linear, constant coefficient, ordinary differential equations
- Signals: exponential functions e^{at}: function of time t
- a need not be real: can consider complex-valued functions of real-variable time t

$\frac{d^2}{dt^2}\theta = \ddot{\theta}$

$\frac{d}{dt} f = \dot{f}$

$\frac{d^2}{dt^2} f = \ddot{f}$

$F \rightarrow \square \rightarrow \theta$

$M\ddot{\theta} + \theta = F$

$\boxed{M\ddot{\theta} = F}$

Exponential signals are 'eigenfunctions'

- For (square) matrices, we speak of eigenvectors
- Consider: $A = \begin{bmatrix} 7 & -10 \\ 5 & -8 \end{bmatrix}$
- Vectors: $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$
- For some real square matrices, need to allow eigenvectors to be complex

LTI systems and exponential signals

- LTI systems have all exponential signals as eigen-'functions'
- Scaling: depends on the system. 'Transfer function'
- Transfer function also rings a bell (resonates?) with 'resonance'!
- Poles and zeros for input to output map
- Convolution is how input gets mapped to output