$\dot{y} + 3y = 2u$, $h(t) = 0$ for $t < 0$.

$(s+3)\hat{y}(s) = 2\hat{u}(s)$

$\frac{\hat{y}(s)}{\hat{u}(s)} = \frac{2}{s+3}$ $\quad h(s) = \frac{2}{s+3}$.

impulse response $= h(t) = \mathcal{L}^{-1}\left(\frac{2}{s+3}\right) = 2e^{-3t}$.

b) $u(t) = 5e^{at}$ (unforced, open loop control),

$\dot{y} + 3y = 10e^{at}$

$\hat{y}(s)(s+3) = \frac{10}{s-a}$.

$\hat{y}(s) = \frac{10}{(s-a)(s+3)}$ $\quad \mathcal{L}^{-1}\left(\frac{10((s-a)-(s+3))}{-(a+3)(s-a)(s+3)}\right)$

$= \frac{-10}{a+3}\left(\mathcal{L}^{-1}\left(\frac{1}{s+3}\right) - \mathcal{L}^{-1}\left(\frac{1}{s-a}\right)\right)$

$= \frac{-10}{a+3}\left(e^{-3t} - e^{at}\right)$.

c) $y(t) = \int_0^t u(\tau)h(t-\tau)d\tau$

$= \int_0^t 5e^{a\tau} \times 2e^{-3(t-\tau)} d\tau$

$= \int_0^t 10\, e^{a\tau - 3t + 3\tau} d\tau$

$= 10e^{-3t} \int_0^t e^{(a+3)\tau} d\tau$

$y(t) = \frac{10e^{-3t}}{(a+3)} \left[e^{(a+3)\tau}\right]_0^t$

$\frac{10e^{-3t}\left(e^{t(a+3)} - 1\right)}{a+3}$

$= \frac{10\left(e^{at} - e^{-3t}\right)}{a+3}$.

$= \frac{-10\left(e^{-3t} - e^{at}\right)}{a+3}$

d) $\cancel{y(t) = \frac{10}{a+3}\left(e^{at} - e^{3t}\right)}$

Scaled for only e^{at} eigen function.

$\cancel{\frac{1}{5} \times \frac{10}{a+3}\left(1 - e^{(3-a)t}\right) = \frac{2}{a+3}\left(1 - e^{(3-a)t}\right)}$

Coe. of exponent e^{at} : $\frac{2}{a+3}$ in output/input

$\cancel{G(s)}\Big|_{\cancel{s=a}}$,

$y(t) = \frac{10\left(e^{at} - e^{-3t}\right)}{a+3}$

$\boxed{G(s) = \frac{2}{s+3}}$

$G(s) = \frac{2}{s+3}$

$\frac{10}{a+3}$ $\quad \frac{2}{a+3}\left(1 - e^{(a+3)t}\right)$

e^{at} eigen function

Q2) a) —

b) $\int_0^t C_0 e^{5\tau} d\tau = \frac{C_0}{5}\left(e^{5t} - 1\right).$

Q-3) $u \to \boxed{S_1} \to \boxed{S_2} \to y$, m

$m = u * h_1$

$y = u * h_1 * h_2$

$y = (h_1 * h_2) * u.$

$h_{net} = h_1 * h_2$

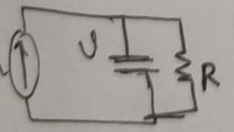
$h_{net} = \mathcal{L}^{-1}\left(\hat{h}_1(s)\hat{h}_2(s)\right)$

$= \mathcal{L}^{-1}\left(\frac{C_0}{s} \times \frac{1}{s-5}\right)$

$= \frac{C_0}{5}\mathcal{L}^{-1}\left(\frac{s-(s-5)}{s(s-5)}\right)$

$h_{net} = \frac{C_0}{5}\left(e^{5t} - 1\right) = \frac{C_0}{5}\left(e^{5t} - 1\right).$

Q4)



$C = 1F, \; R = 10^3 \Omega$

$i \to \boxed{} \to v$

a) $\frac{dv}{dt} + \frac{v}{R} = i(t)$

$\left(s + \frac{1}{R}\right)\hat{v}(s) = \hat{i}(s)$

$\frac{R}{1+sR} = \frac{\hat{v}(s)}{\hat{i}(s)}$

$\boxed{G(s) = \frac{R}{1+sR}}.$

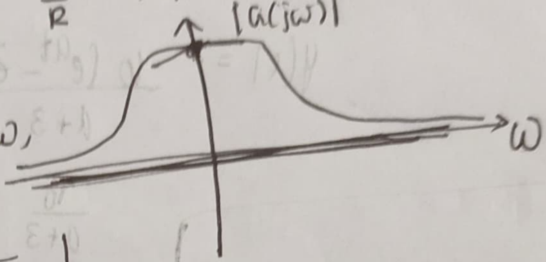
b) Pole at -5,

$1+sR$, $s=-\frac{1}{R}$, root.

$-\frac{1}{R} = -5$, $R = 0.2\,\Omega$

Pole at -1, $-\frac{1}{R} = -1$, $R = 1\,\Omega$

c) $|G(j\omega)|$ vs ω,



$|G(j\omega)| = \left|\frac{R}{1+2\pi f R j}\right|$

$\boxed{\text{low pass filter}}$

$\frac{R}{\sqrt{1+(2\pi f R)^2}} = \frac{R}{\sqrt{1+(\omega R)^2}}$

e) $|G(j\omega)| = 1/\sqrt{2}$, $R^2\,2 = 1+\omega^2 R^2$

$R^2 = \frac{1}{(2-\omega^2)}$

$\sqrt{\frac{1}{2-\omega^2}} = 10^3$

$10^6 = \frac{1}{2-\omega^2}$

$\boxed{\sqrt{2-10^{-6}} = \omega}$

Q5) $F = \ddot{\theta} - \theta$

$\hat{F}(s) = (s^2-1)\hat{\theta}(s)$

$\frac{\hat{\theta}(s)}{\hat{F}(\theta)} = \frac{1}{s^2-1}$

(a) $s = \pm 1$.

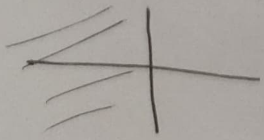
One root in RHP, inverted pendulum.

(b) $F = K_p\theta$

$\ddot{\theta} - \theta = K_p\theta$

$s = \pm\sqrt{1+K_p}$,

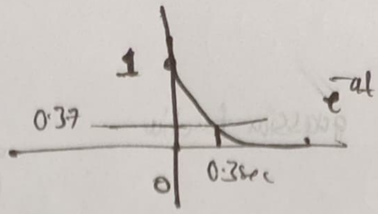
$(s^2 - (1+K_p))\hat{\theta}(s) = 0$,

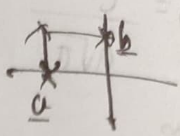


Root in open LHP → Not possible

Closed LHP → $\cancel{K_p}$. $K_p \leq -1$.

(c) $F = K_p\theta + K_d\dot{\theta}$





$\ddot{\theta} - (K_p+1)\theta - K_d\dot{\theta} = 0$

$e^{-a \times 0.3} = e^{-1}$

$(s^2 - (K_p+1) - K_d s)\,\hat{\theta}(s) = 0$

$a = 10/3$

$s^2 - K_d s - (K_p+1) = 0,$

$a = 10/3, \; b = 2$

time constant $= 0.3$ sec,

oscillation freq $= 2$ rad/sec

$-\frac{10}{3} \pm 2j$

$$\frac{K_d \pm \sqrt{K_d^2 - 4(K_p+1)}}{2}$$

$$K_d \pm \sqrt{K_d^2 - 4(K_p+1)} = -\frac{20}{3} \pm 4j$$

$K_d = \frac{-20}{3}, \qquad \frac{400}{9} - 4(K_p+1) = -16,$

a6) $(s^3 + 4s^2 + s - 6)\,\hat{y}(s) = \hat{u}(s)$

$(s^3 + 4s^2 + s - 6 - K)\,\hat{y}(s) = 0.$

closed loop is stable,

$\boxed{K_0 = K_1 K_2}$

$(-6-K) = 4$

$\boxed{K = -10}$ → Max of $|K|$

$\boxed{-10 \le K \le -6}$

$3s^2 + 8s + 1$

$s^2 + s^3(s+a)$

$s^3 + as^2 + s\underline{s}^2 + \underline{s^2 a}$

$\frac{1}{(s+2)(s-1)(s+3)}$