

Problem Sheet – 4: Digital Electronics

Topics: Combinatorial Circuits – K-map minimization, truth tables, Combinatorial circuit design using Multiplexers

Part A– K-map minimization

1. Minimize the following function using K-map and write the minimized function in sum-of-products form.

$$F(A, B, C, D) = \sum (0, 1, 6, 7, 8, 10, 15)$$

2. Minimize the following function using K-map and write the minimized function in sum-of-products form.

$$F(A, B, C, D) = \sum (0, 3, 4, 5, 8, 14, 15)$$

3. Minimize the following functions using K-map and write the minimized function in sum-of-products form.

$$a) F(A, B, C, D) = \sum (0, 4, 5, 8, 10, 11, 12, 13, 14)$$

$$b) F(A, B, C, D) = \sum (1, 2, 4, 6, 9, 11, 13, 15)$$

4. Minimize the following functions using K-map and write the minimized functions.

$$F(A, B, C, D) = \sum (1, 3, 5, 7, 9, 10, 13), \text{ Don't care conditions, } d(A, B, C, D) = \sum (4, 8)$$

5. Problems from “Digital Design 5e – Morris Mano”, Chap.3 Problem set.

5.1 Simplify the following Boolean functions, using three-variable *Karnaugh* maps:

$$(a) F(x, y, z) = \sum (0, 2, 4, 5) \quad (b) F(x, y, z) = \sum (0, 2, 4, 5, 6)$$

$$(c) F(x, y, z) = \sum (0, 1, 2, 3, 5) \quad (d) F(x, y, z) = \sum (1, 2, 3, 7)$$

5.2 Simplify the following Boolean functions, using three-variable maps:

$$(a) F(x, y, z) = \sum (0, 1, 5, 7)$$

$$(b) F(x, y, z) = \sum (1, 2, 3, 6, 7)$$

$$(c) F(x, y, z) = \sum (2, 3, 4, 5)$$

$$(d) F(x, y, z) = \sum (1, 2, 3, 5, 6, 7)$$

$$(e) F(x, y, z) = \sum (0, 2, 4, 6)$$

$$(f) F(x, y, z) = \sum (3, 4, 5, 6, 7)$$

5.3 Simplify the following Boolean expressions, using three-variable maps:

$$(a) xy + \bar{x}\bar{y}z + \bar{x}y\bar{z}$$

$$(b) \bar{x}\bar{y} + yz + \bar{x}y\bar{z}$$

$$(c) F(x, y, z) = \bar{x}y + y\bar{z} + \bar{y}\bar{z}$$

$$(d) F(x, y, z) = \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z$$

5.4 Simplify the following Boolean functions, using *Karnaugh* maps:

$$(a) F(x, y, z) = \sum (2, 3, 6, 7)$$

$$(b) F(A, B, C, D) = \sum (4, 6, 7, 15)$$

$$(c) F(A, B, C, D) = \sum (3, 7, 11, 13, 14, 15)$$

$$(d) F(w, x, y, z) = \sum (2, 3, 12, 13, 14, 15)$$

(e)  $F(w, x, y, z) = \sum (11, 12, 13, 14, 15)$

(f)  $F(w, x, y, z) = \sum (8, 10, 12, 13, 14)$

**5.5** Simplify the following Boolean functions, using four-variable maps:

(a)  $F(w, x, y, z) = \sum (1, 4, 5, 6, 12, 14, 15)$

(b)  $F(A, B, C, D) = \sum (2, 3, 6, 7, 12, 13, 14)$

(c)  $F(w, x, y, z) = \sum (1, 3, 4, 5, 6, 7, 9, 11, 13, 15)$

(d)  $F(A, B, C, D) = \sum (0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$

**5.6** Simplify the following Boolean expressions, using four-variable maps:

(a)  $\bar{A}\bar{B}\bar{C}\bar{D} + A\bar{C}\bar{D} + \bar{B}C\bar{D} + \bar{A}BCD + B\bar{C}D$

(b)  $\bar{x}z + \bar{w}x\bar{y} + w(\bar{x}y + x\bar{y})$

(c)  $\bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}D + \bar{A}B\bar{C} + ABCD + A\bar{B}C$

(d)  $\bar{A}\bar{B}\bar{C}\bar{D} + B\bar{C}D + \bar{A}\bar{C}D + \bar{A}BCD + AC\bar{D}$

**5.7** Simplify the following Boolean expressions, using four-variable maps:

(a)  $\bar{w}z + xz + \bar{x}y + w\bar{x}z$

(b)  $A\bar{D} + \bar{B}\bar{C}D + BC\bar{D} + B\bar{C}D$

(c)  $A\bar{B}C + \bar{B}\bar{C}\bar{D} + BCD + AC\bar{D} + \bar{A}\bar{B}C + \bar{A}B\bar{C}D$

(d)  $wxy + xz + w\bar{x}z + \bar{w}x$

## Part B – Logic design using Multiplexers

6. Implement  $F(x, y, z)$  using an 8-to-1 MUX. Given:  $F(x, y, z) = \sum (0, 2, 4, 5, 6)$ .

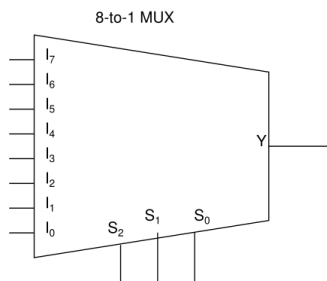


Fig.P6 8-to-1 MUX

7. Implement  $F(A, B, C, D)$  using a 16-to-1 MUX. Given:  $F(A, B, C, D) = \sum (0, 4, 5, 8, 10, 11, 12, 13, 14)$ .

8. Implement  $F(x, y, z)$  using a 4-to-1 MUX. Given:  $F(x, y, z) = \sum (0, 2, 4, 5, 6)$ .

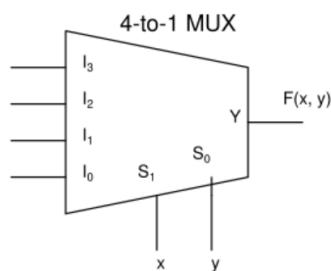


Fig.P8 4-to-1 MUX

9. Implement  $F(A, B, C, D)$  using an 8-to-1 MUX. Given:  $F(A, B, C, D) = \sum(0, 4, 5, 8, 10, 11, 12, 13, 14)$ .
10. The implementation of a combinatorial circuit using a 4-to-1 MUX is shown in Fig.P10. Write the corresponding Boolean function  $F(x, y)$ .

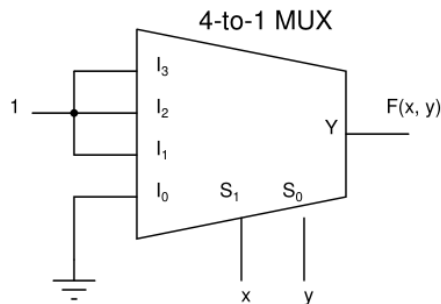


Fig. P10

### Part C – Decoders

11. In order to display numerals 0 to 9 corresponding to their BCD codes, we need a BCD-to-7 segment decoder circuit with 7 outputs and a 7-segment LED display. Each LED of the 7-segment LED display is connected to the corresponding output (**a** to **g**) of the BCD-to-7-segment decoder. Segment identification and numerical displays of the 7-segment LED display are shown in Fig.5B2. Assume that for an LED segment to light up, the corresponding decoder output should be '1'. When a 4-bit BCD code (**PQXY**) of numerals 0 to 9 is given to the inputs of the decoder, the appropriate outputs will be '1'. As shown in Fig.P11, all the decoder outputs will be '0' for BCD codes (**PQXY**): **1010 to 1111**.

- Write the truth table for the decoder output **b** (**P,Q,X,Y**). Note that **P** is the MSB.
- Implement the above function **b** (**P,Q,X,Y**) using an 8-to-1 multiplexer. Block diagram of the 8-to-1 MUX to be used for your implementation is shown in Fig. P6.



Fig. 11 Seven-segment LED display segment identification, numerical designations and resultant displays

12. The pin diagram of a very popular decoder, viz. 74LS138 3-to-8 decoder is shown below. This IC is commonly used as an address decoder for RAM and ROM ICS (for their Chip Select inputs) in microprocessor applications.

Summary of its working:

#### Inputs:

**Address inputs (Select):** A, B, C (Note: C is the MSB and A is the LSB. Hence, when C=1, B=0, A=0, the Y4 output will get selected; similarly is C=0, B=0, A=1, the Y1 output will get selected)

**Enable inputs:** G2A, G2B and G1 (Note: G2A and G2B are active-low inputs, whereas G1 is an active-high input. For the IC to be enabled, the following need to be satisfied: G2A = G2B = 0, and G1= '1').

#### Outputs

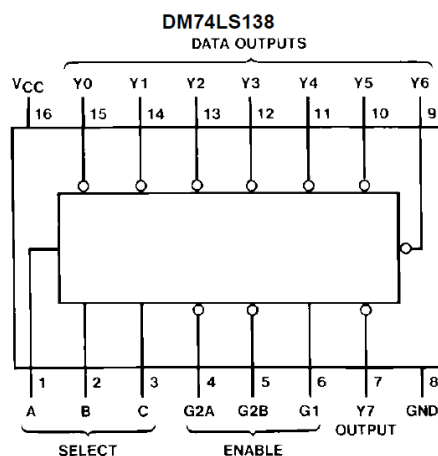
**Decoder Outputs:** Y0, Y1,...Y7. All are active-low outputs, meaning when an output gets selected, it becomes low. At a time only one output will be low.

For an output to get selected (i.e. to become low), two conditions must be satisfied:

- The CBA inputs should give the address of the output.
- The outputs should be enabled. i.e. the enable inputs should be G2A = G2B = 0, and G1= '1'.

For example, if C=0, B=0, A=1, the Y1 output will be at '0' provided, G2A = G2B = 0, and G1= '1'.

If any of the enable inputs do not have the above condition, then all the outputs will be at '1'.



Design problem statement:

You are given four LEDs: Red, Yellow, Green and Blue.

**Red** should turn ON if the decimal equivalent of 'CBA' input is 2

**Yellow** should turn ON if the decimal equivalent of 'CBA' input is 3

**Green** should turn ON if the decimal equivalent of 'CBA' input is 5

**Blue** should turn ON if the decimal equivalent of 'CBA' input is 7

Assume that  $G2A = G2B = 0$ , and  $G1 = '1'$ . Show the circuit connections. Assume that for an LED to turn ON, its current should be around 10 mA. Assume LED forward voltage to be 1.75 volts. You should not use any other IC or gate other than the 74LS138 decoder. You may use any resistor values.

Specifications of the Y0 to Y7 outputs:  $V_{OL} \leq 0.4 \text{ V}$ ,  $I_{OL} = 16 \text{ mA}$ .  $V_{OH} \geq 2.4 \text{ V}$ ,  $I_{OH} = 0.4 \text{ mA}$ .