EE 103 - Control Systems Module

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Outline

$i \rightarrow \boxed{\frac{1}{s}} \rightarrow \omega$

$u \rightarrow \boxed{G(s)} \rightarrow y$ 2

3

- Role of poles (we saw)
- Role of zeros (today)
- High gain feedback: not always advantageous (we saw partially)
- Relative degree: denominator degree - numerator degree
- 'Proper' and 'strictly proper' transfer function $G(s)$

$$G(s) = \frac{n(s)}{d(s)}$$

open/closed loop

left

Integrator: $\frac{1}{s}$ and

$$\omega(t) = \int_0^t i(\tau)\, d\tau$$

Sd

Outline

$i \to \boxed{s} \to v$

Pos $\to \boxed{s} \to$ velocity

- Role of poles (we saw)
- Role of zeros (today)
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- Relative degree: denominator degree - numerator degree
- 'Proper' and 'strictly proper' transfer function $G(s)$

$u \to \boxed{} \to y = \dot{u}$

rel dg = 1

Integrator: $\frac{1}{s}$ and differentiator: s.

proper, strictly proper

improper

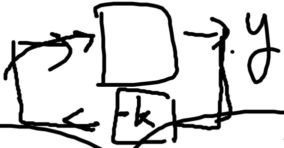
Outline

- Role of poles (we saw)
- Role of zeros (today)
- High gain feedback: not always advantageous (we saw partially)
- Relative degree: denominator degree - numerator degree
- 'Proper' and 'strictly proper' transfer function $G(s)$

Integrator: $\frac{1}{s}$ and differentiator: s.
Improper is possible (causal) physical, in continuous time)

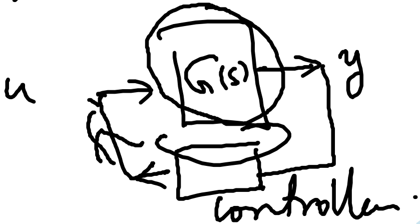
Various phrases so far

~~$u(t) = \sin \omega t$~~



$u = -ky$

Open-loop control, open-loop system
Closed loop system,
Study of:
Open-loop system : $\equiv$ (almost) unforced system: open-loop poles
Intend to place closed loop poles in LHP



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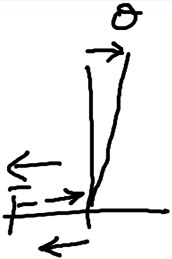
$$F = \pm \frac{d^2}{dt^2}\theta + \theta$$



$$F = -\frac{d^2}{dt^2}\theta - \theta$$

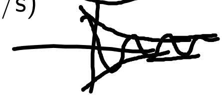
$$F = -\frac{d^2}{dt^2}\theta + \theta$$

$$F = \frac{d^2}{dt^2}\theta - \theta$$

Recap

$e^{at} \to \boxed{G(s)} \to c\, e^{at}$

- Convolution $h * u$: easier to understand/interpret when: h is impulse response, and u is input
- LTI systems: for (always) exponential inputs e^{at}: 'eigenfunctions': gain (i.e. scaling): just $G(a)$ $s = a$
- Reasonable to call transfer function $G(s)$ as the 'gain' of the system
- Gain $G(a)$ depends on the exponent a: some values of a: verrrrry large (poles)
- Some values of a: gain is zero (Zeros of the transfer function $G(s)$)
- Complex exponents: real part: decay/growth rate, and imaginary part: oscillation frequency (in rad/s)



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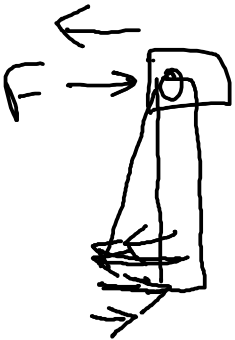
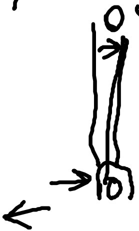


$$\pm F = \ddot{\theta} + \theta$$



$$\ddot{\theta} - \theta$$

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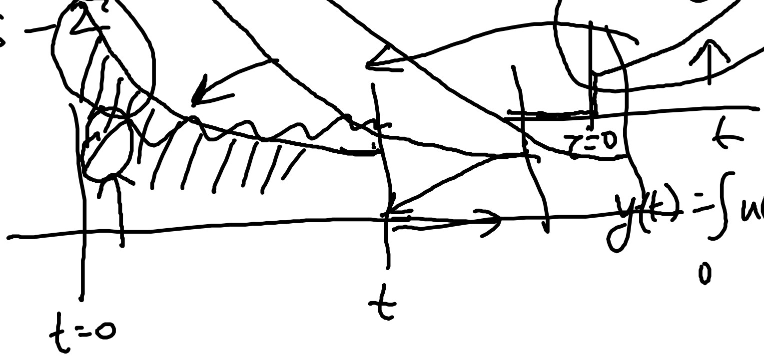
$$y(t) = \int_0^t u(\tau)\, h(t-\tau)\, d\tau$$

$$h(t) = e^{2t}$$

$$\frac{1}{s-2} = G(s)$$

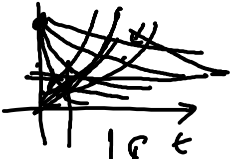


$$y(t) = \int_0 u(\tau)\, d\tau$$

High gain feedback

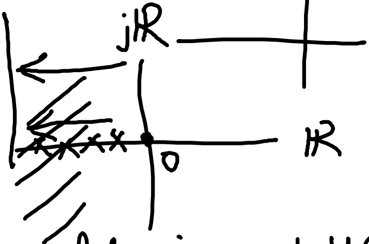
- Relative degree = 3 (or more): check that high gain would cause instability

$$\frac{d^3}{dt^3}y + \frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 6 = u$$



$$u = -ky$$

loosely speaking we want poles closed loop system far left in LHCP



often we get by large k

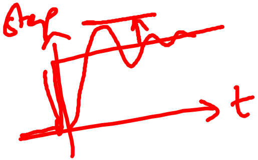
High gain feedback

- Relative degree = 3 (or more): check that high gain would cause closed loop instability
- Open loop $G(s)$ does not change. Only closed loop is affected by feedback gain k
- Open-loop Poles in RHP: need corrective action: $u = -ky$, for appropriate k.
- Corrective action (i.e. u) is proportional to deviation y.
- PD controller: proportional + derivative
- (Similarly PID, etc.:
- more complicated performance objectives need more than P, PD, PID)

Sometimes: need PD control

- Inverted pendulum: proportional feedback cannot ‘dampen’ oscillations
- Make $u = -k_p y - k_d \frac{d}{dt} y$
- PD often used for ‘quickening transients’

Often more requirements: percent over-shoot: not too large
Small rise-time (means: more band-width, quicker response)



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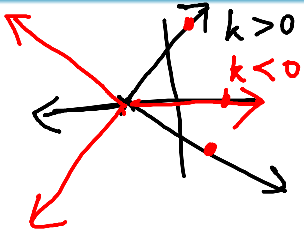
$$(s+1)^3 + k = 0$$



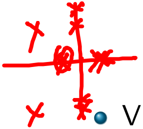
$$s^3 + 6s^2 + 3s + 10 + k = 0$$

$$\frac{\cancel{k}\ 1}{s^3+6s^2+5s+10\cancel{k}}$$

$$u = -ky$$

$$(s^2+\omega^2)(s+a) = 0$$

High Gain feedback



$s^2 + (1+k)$

$\ddot{\theta} + \theta = F$ regular pend

$F = -k\theta$

OLHP

- View k as ‘penalizing policy’
- Ensure k is such that all closed loop poles are in LHP (closed loop stability)
- Small time-constant: deviations decay to zero quickly

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RHP zero

$\frac{s-1}{s+2} = \frac{Y(s)}{U(s)}$

$\frac{d}{dt}y + 2y = \frac{d}{dt}u - u.$

$u = -ky$

$u \rightarrow \boxed{G} \rightarrow y$

- Large gain would cause instability (if there is a RHP zero)
- Take: $G(s) = \frac{s-1}{s+2}$
- Take: $G(s) = \frac{s-1}{s^2+2s+1}$

$u = -ky$

closed loop poly

$\left[(s+2) + k(s-1)\right]Y(s) = 0$

$(1+k)s + (2-k)$

roots $= \frac{(k-2)}{1+k}.$

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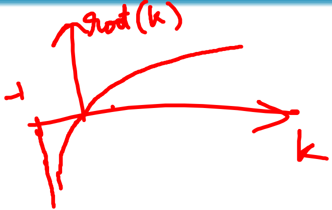

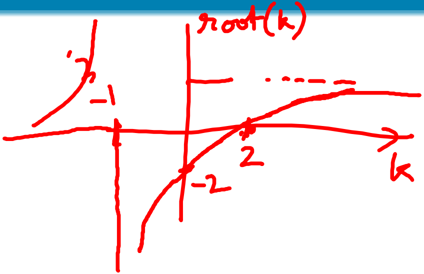

$$\frac{(s+1)}{s^2+2s+1}$$

$$\ddot{y} + 2\dot{y} + y = \dot{u} - u$$

$$u = -ky.$$

$$s^2 Y(s) + 2s Y(s) + Y(s) + sk Y(s) - k Y(s) = 0$$

$$s^2 + (2+k)s$$

$$\left(s^2 + s(2+k) + (1-k)\right) Y(s) = 0$$

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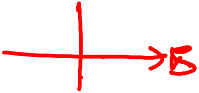
closed loop poles at roots $F(s)$ $s^2 + s(2+k) + (1-k)$

for large k, why roots in RHP??



(at least one)

$$K(s) = \frac{\quad}{s-1}$$

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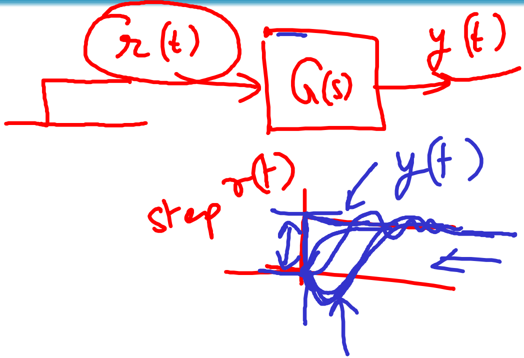

to track r(t)

m

reversing sign in step resp

⇓

RHP zero-chann

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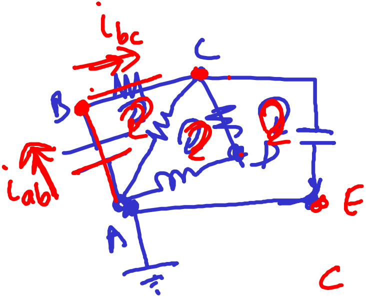
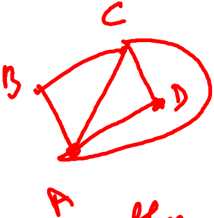


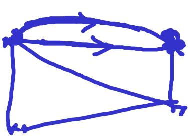
put the nodes



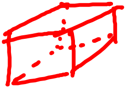
no edge has to jump/cross on another edge

More topics (not for Friday quiz)

graph theory

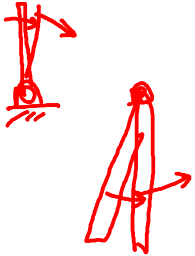


- KCL/KVL laws: how many?
- For n-devices (each of 2-terminals): need $2n$ equations
- Each node: one KCL. Each loop: one KVL
- For planar graphs: link with Euler's faces/vertices/edges rule
- More general link: well-known: 'Hairy Ball theorem'
- Equilibrium point: several types



$F - E + V = 2$

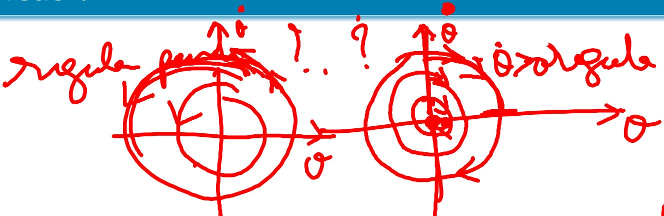
Equilibrium point



When all forces perfectly cancel each other.

- Small perturbations: do they cause oscillations?
- Or bring back all perturbations back
- Or bring back only some but not most others
- Phase portrait (2-D plot of trajectories)

Plot of $\dot{\theta}$ versus θ



If a circle (on a plot of $\frac{d}{dt}\theta$ versus θ:
clockwise or anti-clockwise?
Regular-pendulum? (θ: deviation from vertically down)
Inverted pendulum? (θ: deviation from vertically up)

Euler characteristic

Consider only 2-terminal devices (resistors, capacitors, inductors,

Euler characteristic

Consider only 2-terminal devices (resistors, capacitors, inductors, voltage source, current source)
E = Edges: 2-terminal devices
V = Nodes: KCL
F = Faces: KVL
(for Planar graphs: for this course)
Then : $F - E + V = 2$:
Interpret 2 as: one unnecessary KCL and one unnecessary KVL
(unnecessary: redundant)
$(F - 1)$ = independent KVL
$(V - 1)$ = independent KCL
and E number of device-laws
Together make the across-voltage and through-currents a 'determined system'

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