

Introduction to Electrical Engineering

Course Code: EE 103

Department: Electrical Engineering

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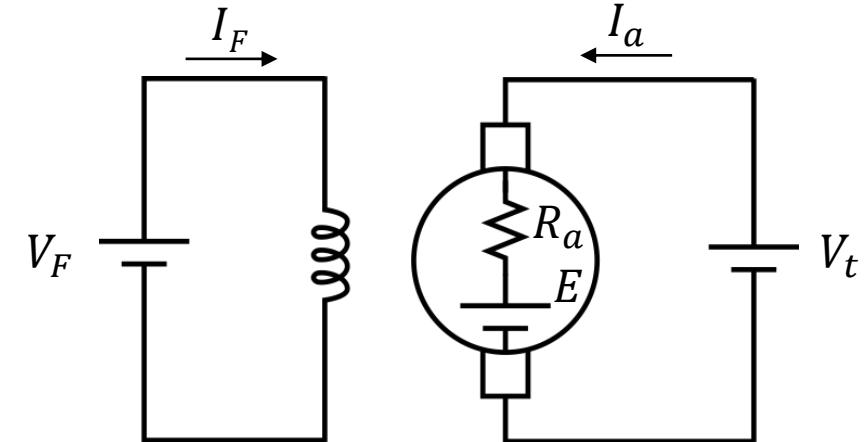
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Review:

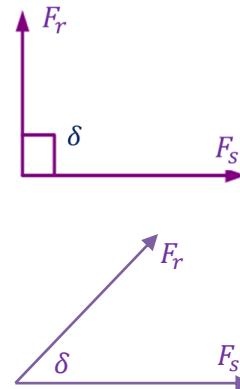
$$EI_a = V_t I_a - I_a^2 R_a$$

$$T = \frac{EI_a}{\omega} = \frac{(K\phi\omega)I_a}{\omega} = K\phi I_a$$



Advantages of DC machines:

- The armature and field MMF can be controlled independently
- Torque control is simple
- 'Torque per ampere' is maximum
- In DC machine → Stator field is 'time-invariant'
→ Stator winding is concentrated and connected to a DC supply



Limitations of DC machines:

- Commutator and brushes wear over time
→ require regular maintenance
- Short-circuiting commutator segments give rise to spark → can not be used in explosive environments

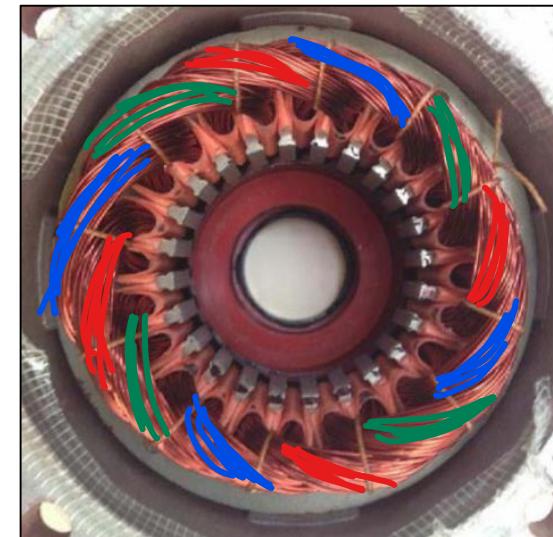
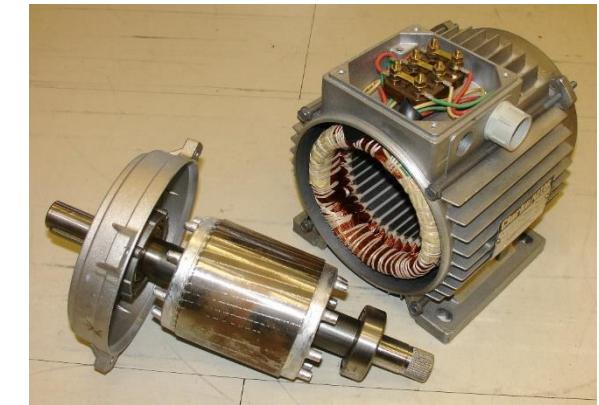


Wikipedia



Asynchronous machine Stator → AC, Rotor → AC

- Induction motor is also known as asynchronous machine
- Of all the motors used, more than 80% are Induction motor*
- “Almost 70% of the machinery in industrial applications uses three-phase induction motors”\$
- In induction machine → Stator winding is distributed



*Induction Motor Market Size, Share & Industry Analysis, <https://www.fortunebusinessinsights.com/industry-reports/induction-motor-market-101639>

\$ Induction Motor Market - Growth, Trends, and Forecast (2020 - 2025), <https://www.mordorintelligence.com/industry-reports/induction-motor-market>

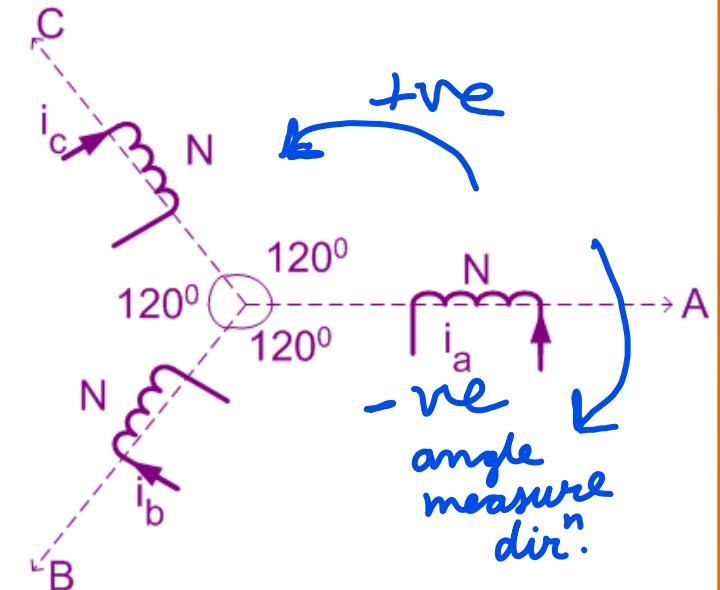


Induction machine

- Consider 3 coils of 'N' turns, displaced in space by 120°
- Let i_a, i_b & i_c are given as

$$\begin{cases} i_a = I_m \sin(\omega_s t) \\ i_b = I_m \sin\left(\omega_s t - \frac{2\pi}{3}\right), \\ i_c = I_m \sin\left(\omega_s t - \frac{4\pi}{3}\right), \end{cases} \quad \text{where } \omega_s = 2\pi f$$

- Current in each coil produces a pulsating magnetic field
- Amplitude & direction depend on the instantaneous value of 'I' flowing through it
- Each phase winding produces a similar magnetic field displaced by 120° in space from each other



$$B = \frac{\mu_0 I R^2}{2(R^2 + r^2)^{3/2}}$$



Induction machine

Magnitude and position of the resultant field can be determined as follows

- Resolve the field produced by individual coil along x & y axes
- Determine $\sum x$ & $\sum y$ components
- Resultant,

$$R = \sqrt{(\sum x)^2 + (\sum y)^2}$$
$$\theta = \tan^{-1} \frac{\sum y}{\sum x}, \text{ w.r.t axis of coil 'A'}$$

$$i_a + i_b + i_c = 0$$

$$\begin{aligned}\sum x &= Ni_a + Ni_b \cos(-120) + Ni_c \cos(-240) \\ &= Ni_a - \frac{1}{2}Ni_b - \frac{1}{2}Ni_c \\ \therefore i_a + i_b + i_c &= 0, \Rightarrow \sum x = \frac{3}{2}Ni_a\end{aligned}$$

$$\begin{aligned}\sum y &= 0 + Ni_b \sin(-120) + Ni_c \sin(-240) \\ &= -\frac{\sqrt{3}}{2}Ni_b + \frac{\sqrt{3}}{2}Ni_c \\ \Rightarrow \sum y &= \frac{\sqrt{3}}{2}N(i_c - i_b)\end{aligned}$$



i_a, i_b & i_c are sinusoidal varying quantities

$$i_a = I_m \sin(\omega_s t)$$

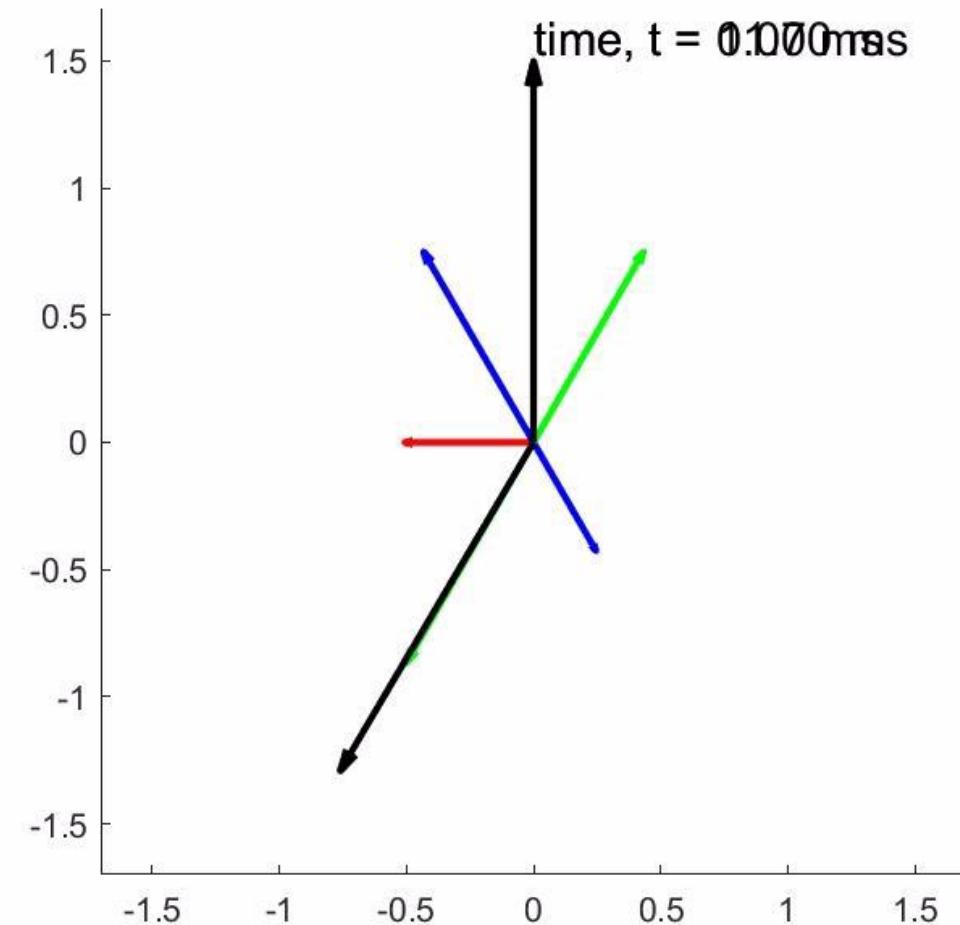
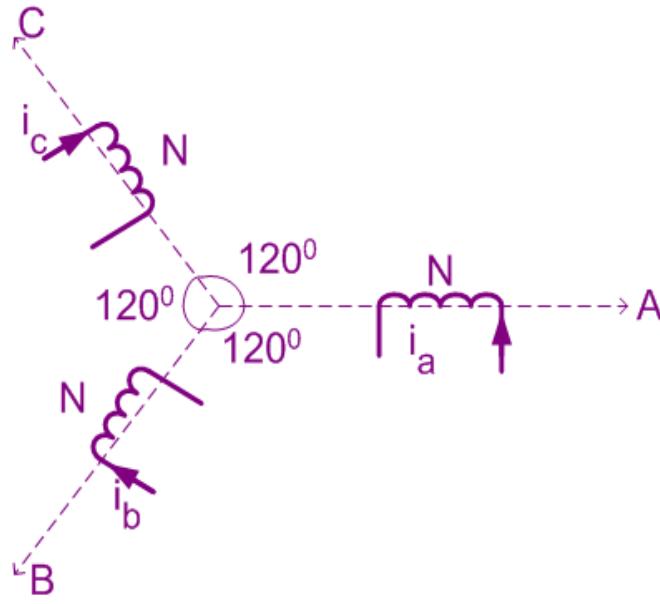
$$i_b = I_m \sin\left(\omega_s t - \frac{2\pi}{3}\right),$$

$$i_c = I_m \sin\left(\omega_s t - \frac{4\pi}{3}\right),$$

$\omega_s t$	i_a	i_b	i_c	Σx	Σy	R	θ
0°	0	$-\frac{\sqrt{3}}{2} I_m$	$\frac{\sqrt{3}}{2} I_m$	0	$\frac{3}{2} NI_m$	$\frac{3}{2} NI_m$	$\frac{\pi}{2}$
30°	$\frac{I_m}{2}$	$-I_m$	$\frac{I_m}{2}$	$\frac{3}{4} NI_m$	$\frac{3\sqrt{3}}{4} NI_m$	$\frac{3}{2} NI_m$	$\frac{\pi}{3}$
90°	I_m	$-\frac{I_m}{2}$	$-\frac{I_m}{2}$	$\frac{3}{2} NI_m$	0	$\frac{3}{2} NI_m$	0
180°	0	$\frac{\sqrt{3}}{2} I_m$	$-\frac{\sqrt{3}}{2} I_m$	0	$\frac{3}{2} NI_m$	$\frac{3}{2} NI_m$	$-\frac{\pi}{2}$



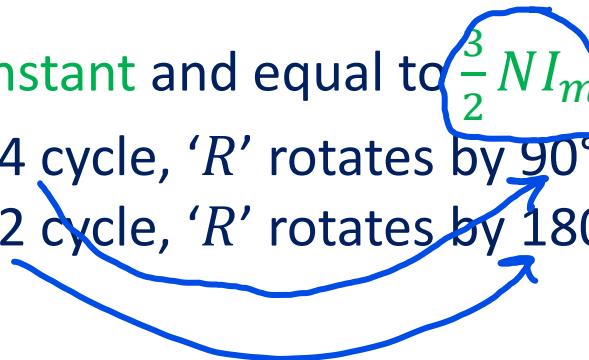
Three phase currents and the resultant MMF



Three phase currents and the resultant MMF

Observations:

- Magnitude of 'R' is **constant** and equal to $\frac{3}{2} NI_m$ where I_m is the peak value
- Input 'I' completes 1/4 cycle, 'R' rotates by 90°
- Input 'I' completes 1/2 cycle, 'R' rotates by 180°



Conclusion:

- The result of displacing 3 windings by 120° in space and displacing the winding 'I' by 120° in time phase is a single revolving field of constant magnitude
- For a given winding arrangement, **speed of rotation** is determined by **supply frequency** of the input alone
- This speed of rotation of field is called 'synchronous speed' and it is denoted by N_s

$$N_s = \frac{120f}{P}$$

→ N_s is in RPM, f is the supply frequency in Hz, and P is the number of poles which 3
(How did we arrive at this expression?? Hint: Compare induced voltage angular frequency with rotational speed of the coil)

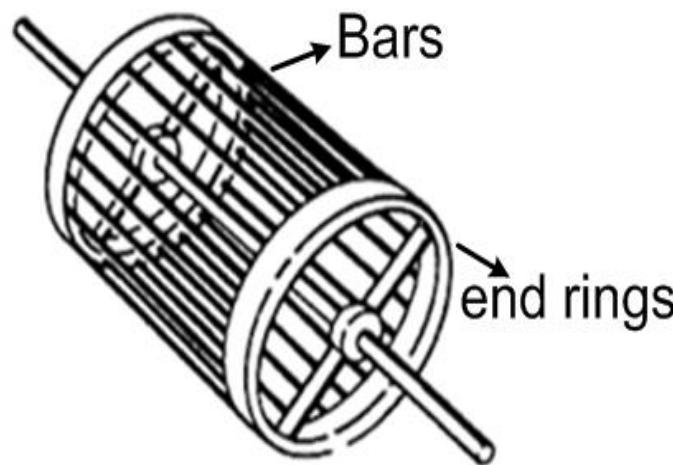


Rotor: two types of construction

1. **squirrel cage** → aluminum/ copper bars are embedded in the rotor slots and permanently shorted at both ends by Al/Cu end rings

⇒ electrically closed circuit

⇒ no additional ' R ' can be connected



Wikipedia

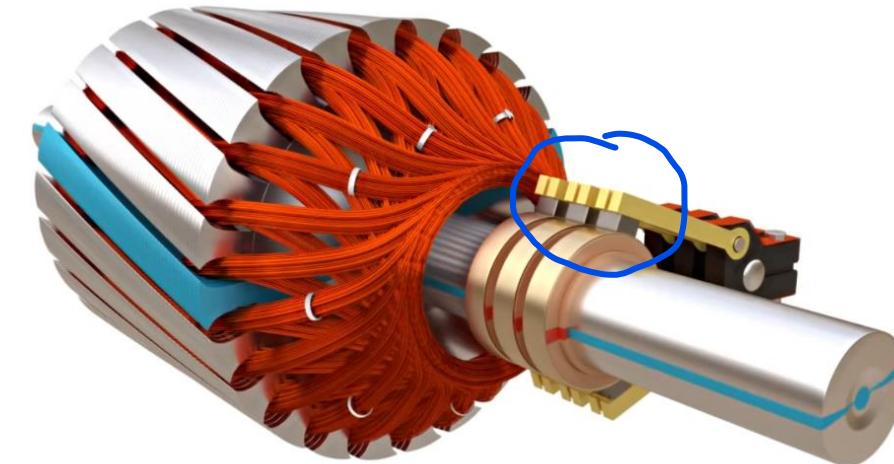
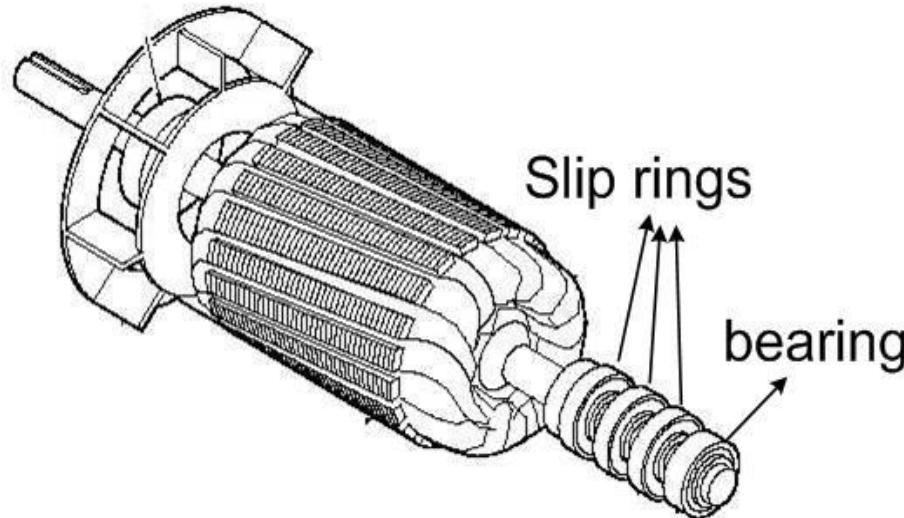


Rotor: two types of construction

2. **slip ring rotor** three phase winding is placed in rotor slots

⇒ three terminals of the windings are connected to three rings fixed to rotor shaft

⇒ external '*R*'/'*Z*'/'*V*' source can be connected to these rings via. brushes



Youtube: Slip ring Induction Motor, How it works ?



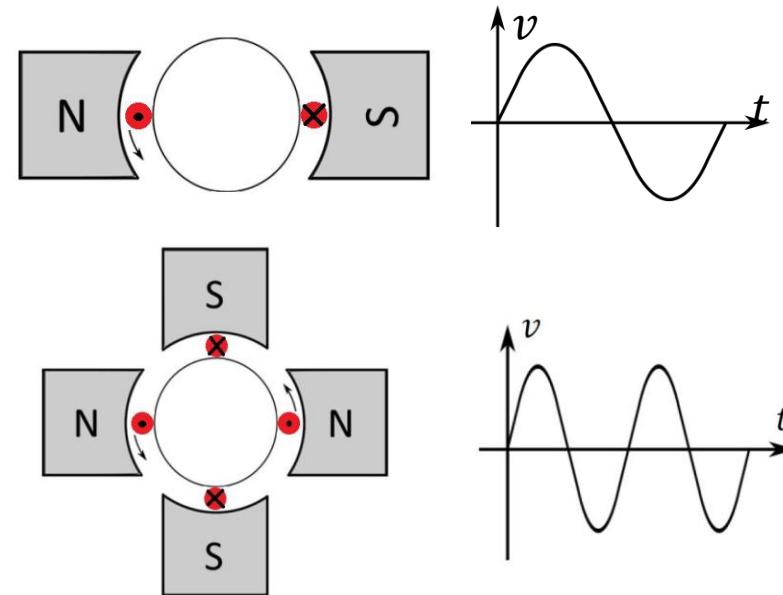
- Number of cycles induced voltage completes in one rotation depends on number of poles
- Thus, if the conductors are rotated at 3000 rpm, the frequency of the induced emf for

2-pole \rightarrow 50 Hz

4-pole \rightarrow 100 Hz

$$N_s = \frac{120f}{P}$$

Suppose, we want the induced voltage of 50 Hz in 4-pole machine \rightarrow Decrease the rotational speed to 1500 rpm



In case of induction machine, 3 coils of 'N' turns, displaced in space by 120° , and excited by i_a, i_b & i_c as follows

$$i_a = I_m \sin(\omega_s t)$$

$$i_b = I_m \sin\left(\omega_s t - \frac{2\pi}{3}\right),$$

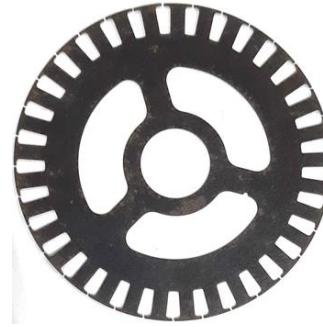
$$i_c = I_m \sin\left(\omega_s t - \frac{4\pi}{3}\right), \quad \text{where } \omega_s = 2\pi f$$

- For this arrangement (2-poles), if $f = 50$ Hz, the synchronous speed is 3000 rpm.
- If winding arrangement is modified to have 4-pole, then the synchronous speed of 3000 will result in $f = 100$ Hz.
- But, if $f = 50$ Hz for 4-pole machine, then synchronous speed will be 1500 rpm (half of 3000 rpm)





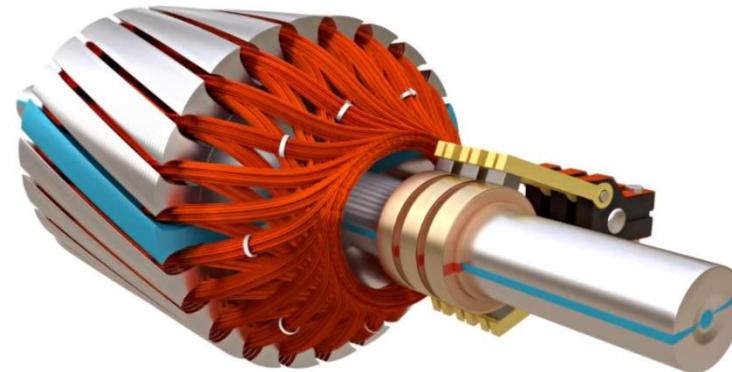
Stator for Concentrated winding



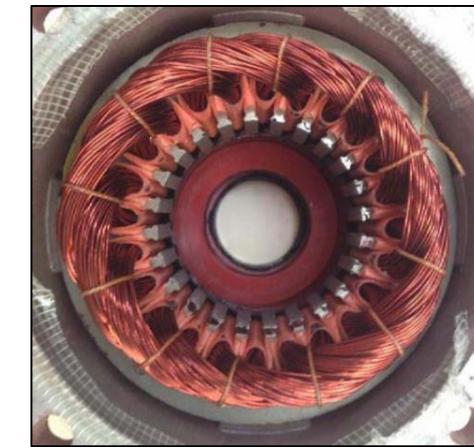
Stator for distributed winding



Squirrel Cage Rotor



Slip Ring Rotor



Stator with Distributed winding

- Windings in the stator of an induction motor **are distributed** → To produce **sinusoidal MMF distribution in space**



Induction motor: Principle of operation

- In both the rotor types, when the rotor is at rest, synchronously rotating stator field induces voltage of stator supply frequency in the rotor
∴ rotor is at rest, relative speed between stator field & rotor is

$$N_s - 0 = N_s \rightarrow \text{maximum}$$

∴ relative speed between field and conductor (rotor) is maximum,
→ induced emf, and therefore, rotor ' I ' is maximum.

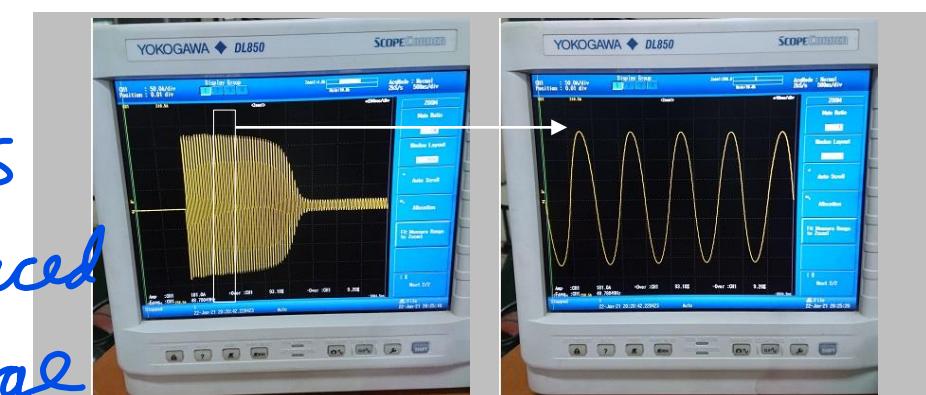
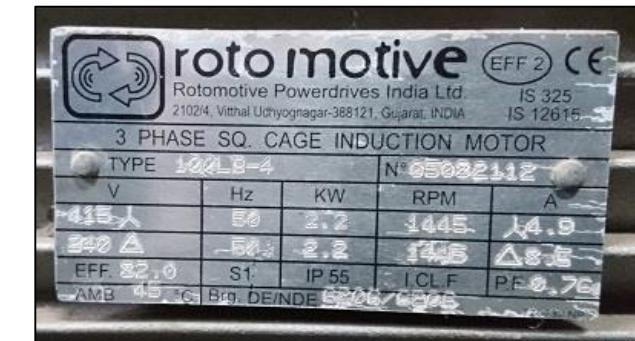
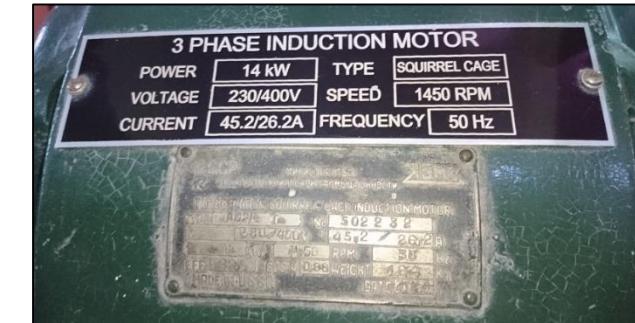
- Now, for any current in rotor there is an equivalent ' I ' in stator
(discussed in 2nd course)

∴ if an IM is started at rated ' V ' & ' f ', a high ' I ' will be drawn from the source

→ This high ' I ' could be \approx (6 - 7) times the rated current if it's of motor

? given or induced induced voltage

Name plates of IM



Induction motor: Principle of operation

- Current carrying conductor placed in magnetic field experiences a force
 \therefore conductor (rotor) starts rotating
- As the speed \uparrow , relative speed between stator field & rotor \downarrow

As a result:

- Induced 'V' in the rotor and hence the current \downarrow
- Frequency of induced 'V' & 'I' in rotor \downarrow
- rotor eventually reach a steady state speed N_r where, $N_r < N_s$

- N_r can not be $= N_s$ because at $N_r = N_s$, relative speed between rotor (conductor) and stator field is zero
→ no emf, and therefore, no 'I' and no force or torque

- $N_s - N_r \rightarrow$ slip speed

- Slip:
$$S = \frac{N_s - N_r}{N_s}$$

$$F_S, F_\delta, N_S, N_\delta,$$

Rotor speed,
stator speed?

$$N_s = \frac{120f}{P}$$

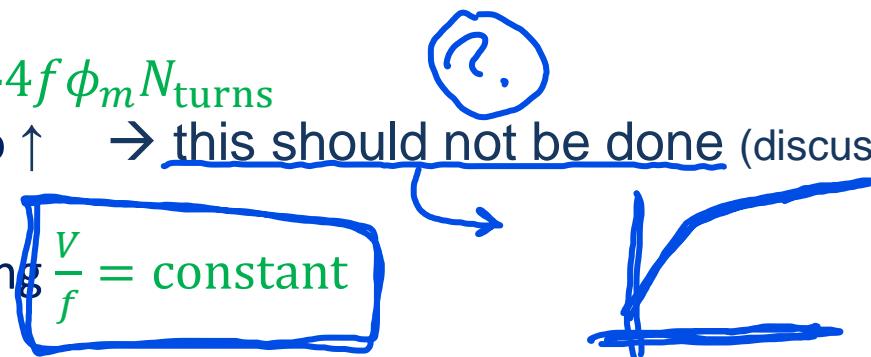


Induction motor: Speed Control

- Consider a 4-pole, 50 Hz, Induction motor $\rightarrow N_s = \frac{120 \times 50}{4} = 1500$ RPM
 - $N_r \approx 1485$ RPM when no external torque is applied
 - Apply external torque (T_L) to the shaft
 - Rotor will slow down $\rightarrow (N_s - N_r) \uparrow \therefore 'V'$ induced in the rotor increased $\therefore 'I' \uparrow$
 - Torque developed (T_e) by the rotor \uparrow
 - N_r will fall until $T_e > T_L$
 - At steady state, $T_e = T_L$ (neglecting friction)
 - Corresponding speed is less than 1485 RPM $\rightarrow \therefore$ speed of IM varies with application of external load
- This variation is 2 - 3% of N_s , which is small!! What if a wide variation is required? (Fan, electric vehicle)

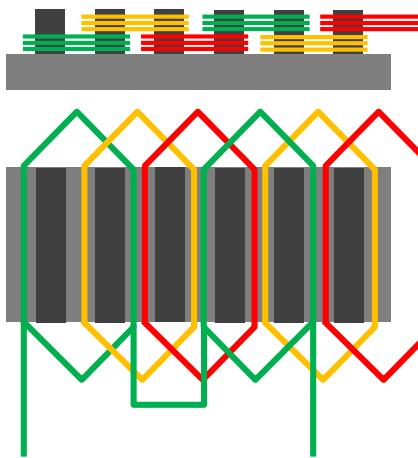
- We have to change the N_s (in FANS, ' N_s ' is not changed!)
- N_s can be changed by changing the supply frequency..
- To reduce the speed, ' f ' \downarrow Recall that: $V = 4.44f\phi_m N_{\text{turns}}$
- If the ' V ' is kept constant and ' f ' is reduced, ϕ_m tries to $\uparrow \rightarrow$ this should not be done (discussed in 3rd course)
- Instead, ϕ_m is kept constant. This can be done by keeping $\frac{V}{f} = \text{constant}$
 \Rightarrow both, ' V ' and ' f ' are changed

$$T = k \phi I_a$$

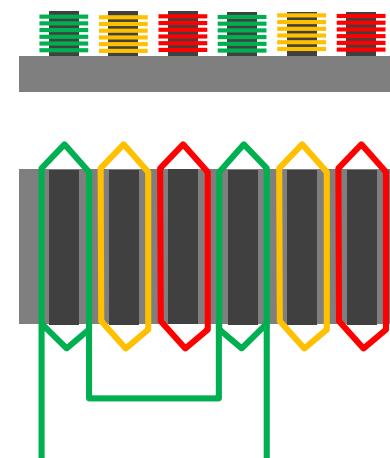


Distributed and Concentrated Windings

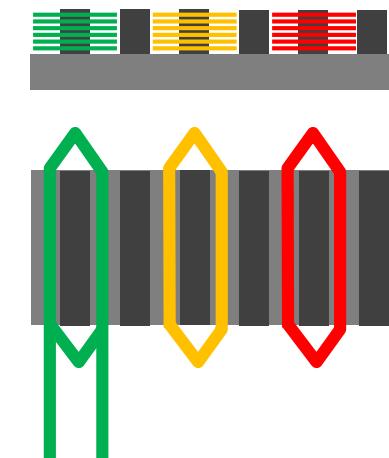
2-layer distributed winding



2-layer concentrated winding



1-layer concentrated winding

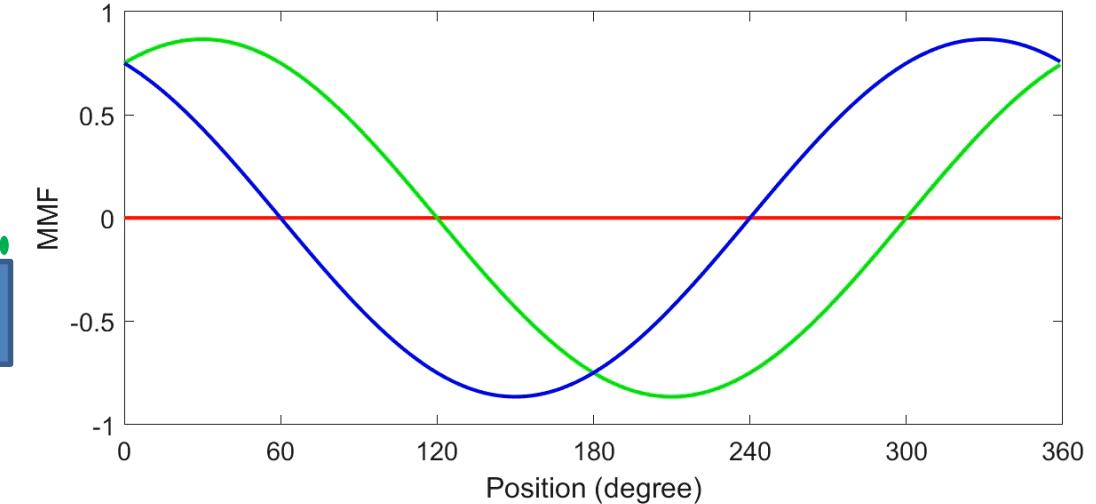


Distributed winding:

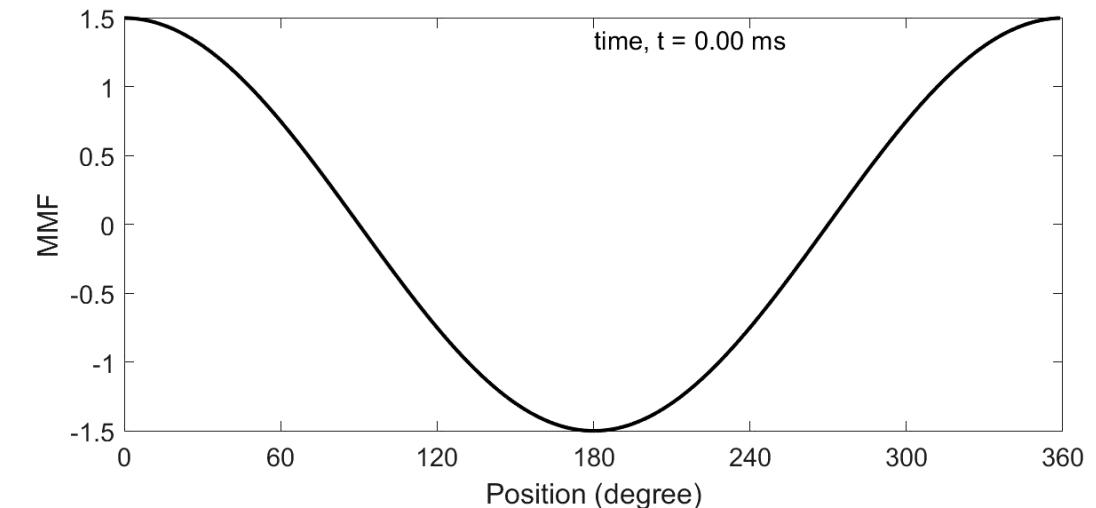
- Larger end-windings → Larger copper losses
- Larger frame size required
- Lower harmonic content in MMF



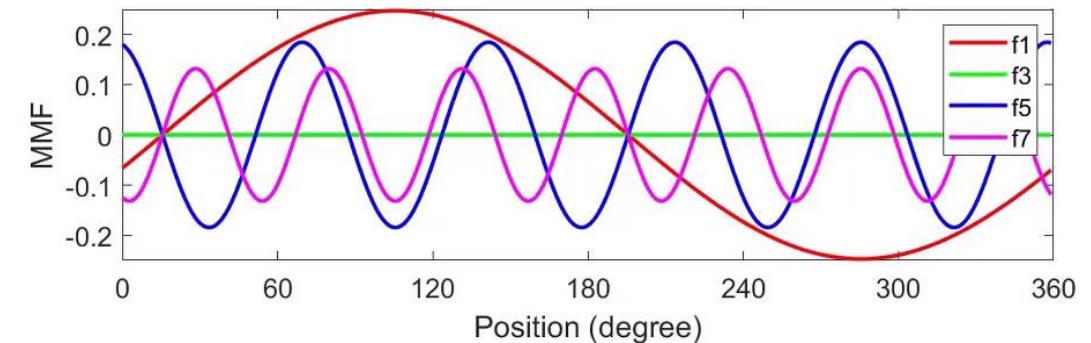
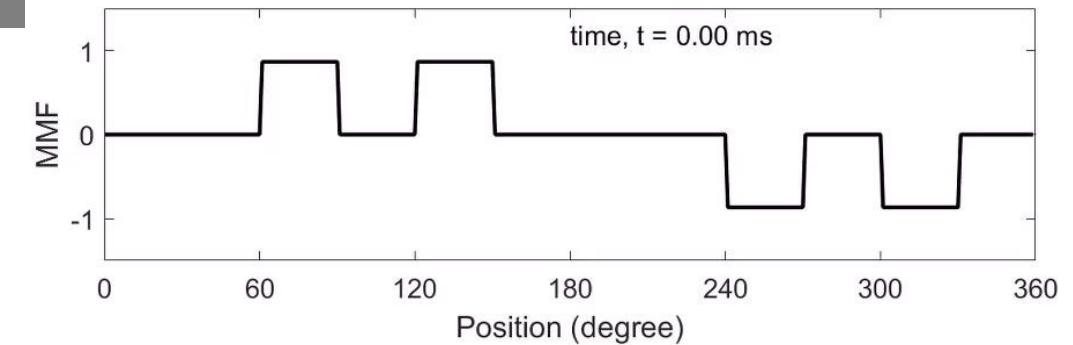
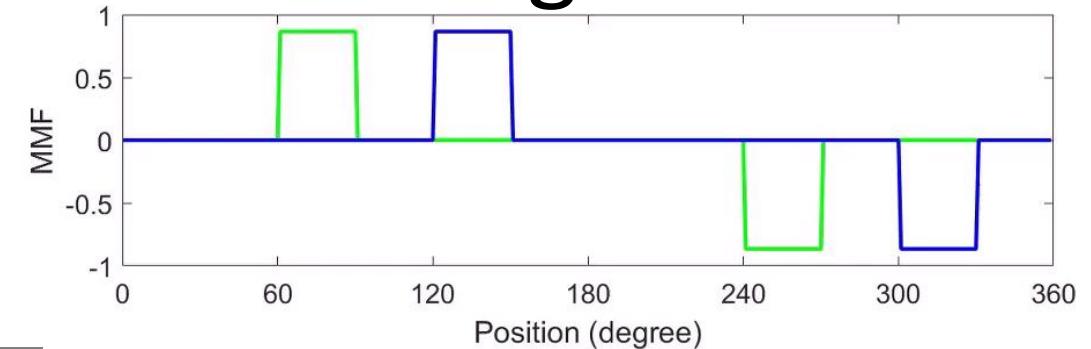
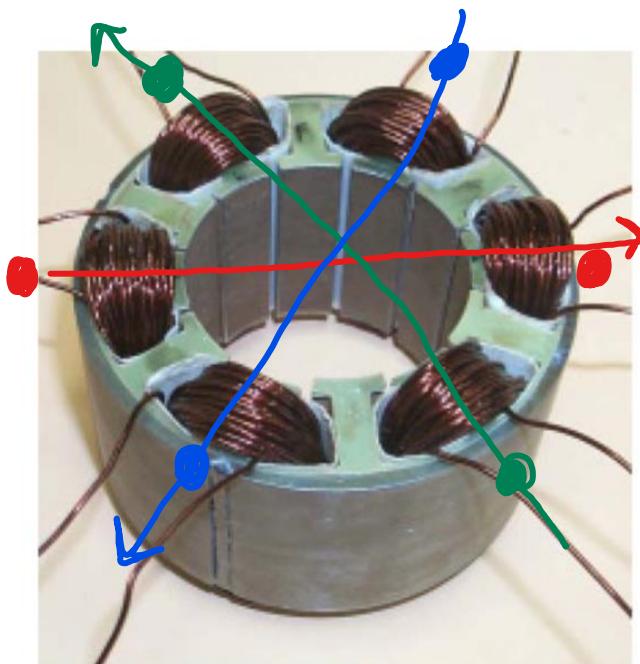
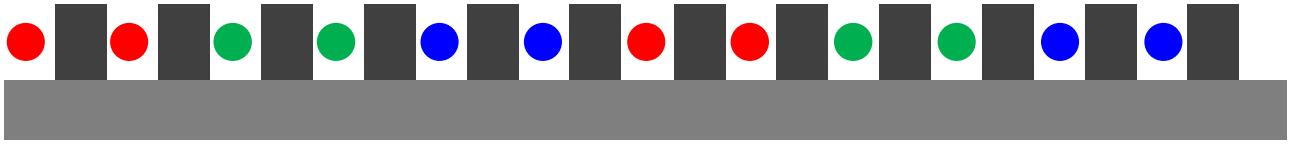
Stator MMF due to distributed windings



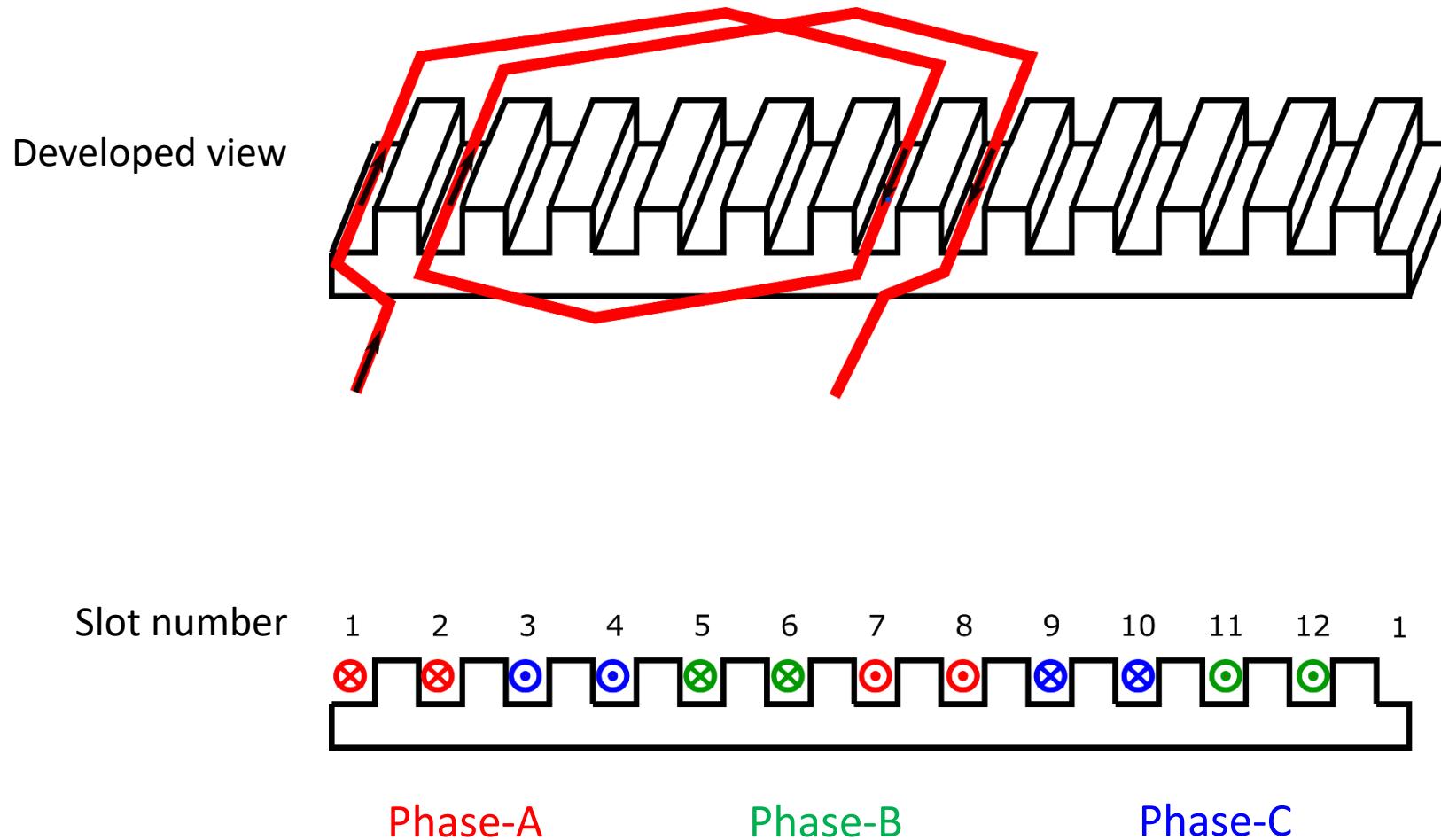
<https://www.icrfq.net/motor-winding/>



Stator MMF due to concentrated windings



Distributed winding with spp = 2 (12-slot, 2-pole)



Synchronous Motor

Generally,

- Stator of Induction Motor = Stator of Synchronous Machine
- Rotor → Winding excited by DC or Permanent Magnet (PMSM/ BLDC)

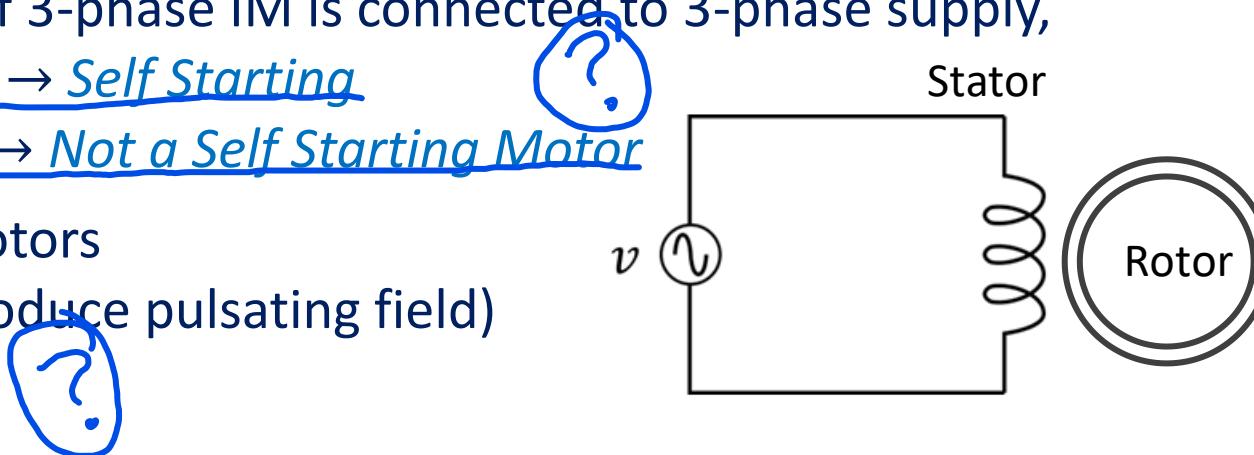
When Stator Winding of 3-phase IM is connected to 3-phase supply,

- Rotor starts rotating → Self Starting
- Synchronous Motor → Not a Self Starting Motor

So, are single phase motors

(1-phase supply will produce pulsating field)

- Average Torque = 0



When Stator Winding of 3-phase, 2-pole IM (=SM) is connected to 3-phase, 50Hz supply,

- F_s starts rotating at 3000rpm ($\frac{120f}{P}$)



Synchronous Motor

Since the field produced by rotor is DC

- Speed of F_r = Speed of Rotor (N_r)
- At $t = 0, N_r = 0$ (Rotor is stationary)
 $= 0^+, N_r = 0$ (Speed cannot change instantaneously, Rotor has its own inertia)
- Speed of $F_s \neq$ Speed of F_r , $T_{avg} = 0$?

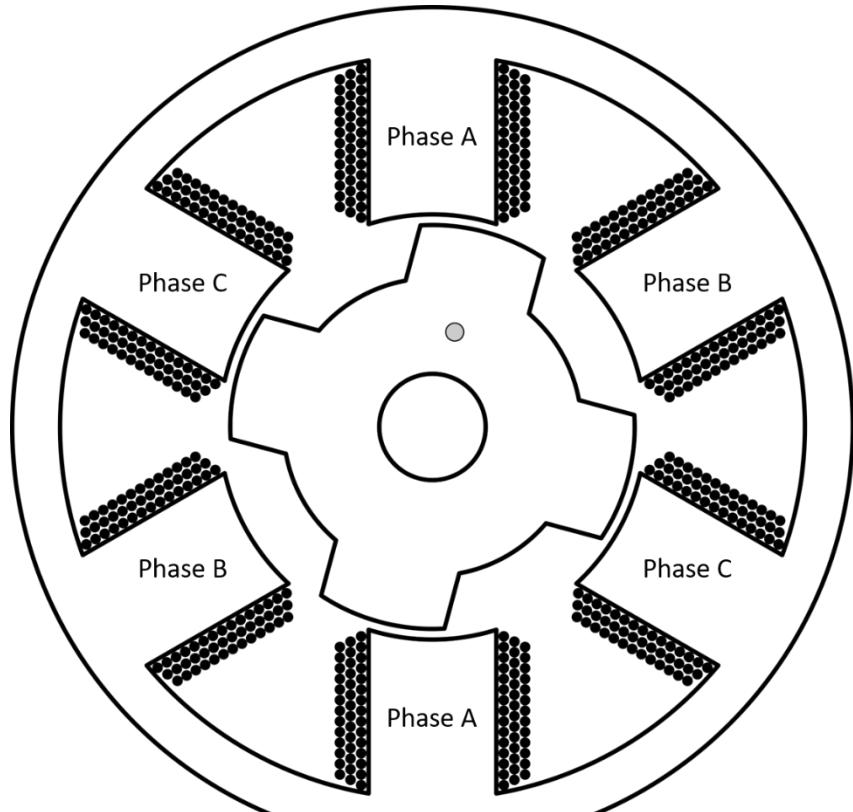
T_{avg} is finite, when Speed of F_s = Speed of F_r , ($N_s = N_r$)

→ Hence the name Synchronous Motor

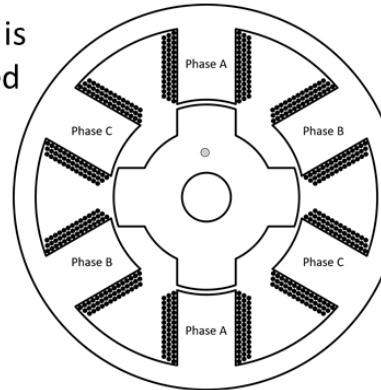


Torque production without a current carrying conductor in a magnetic field..

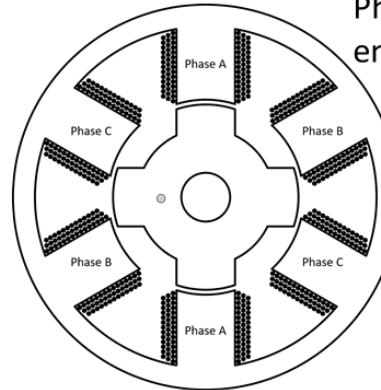
- Consider the following 'assembly'
- Rotor is made of ferromagnetic material
- No. of stator teeth not equal to rotor teeth



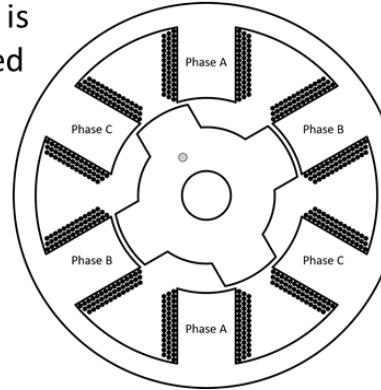
Phase A is energized



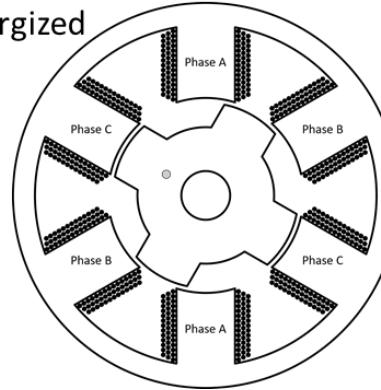
Phase A is energized



Phase B is energized



Phase C is energized



- Continuous rotation is achieved by repeating the sequence of phase excitation
- What would happen if the rotor is simply 'cylindrical'? Why??

