Tutorial Sheet EE113 Control Systems Module

Consider $G(s)$ transfer function as $G(s) := \dfrac{Y(s)}{U(s)}$ (throughout, $u \equiv$ input, $y \equiv$ output).

for

$$u(t) \to \boxed{G} \to y(t)$$

LTI system.

Though $G(s)$ is not Laplace transform of a signal, but a transfer fn, when $U(s)=1$ (i.e. $u(t)=\delta$), $\mathcal{L}^{-1}(G(s)) =: h(t)$ the impulse response of system G.

Q-1 For system $\dot{y} + 3y = 2u$, $\boxed{h(t)=0 \text{ for } t<0}$

a) find impulse response & transfer fn $G(s)$.

b) Now when $u(t) = be^{at}$ (for $t \geq 0$, & $u=0$ for $t<0$) (take $b=5$) then find $y(t)$ by: $\mathcal{L}^{-1}(G(s)\cdot U(s))$ (using partial fraction expansion)

c) Convolve $u(t) = 5e^{at}$ & $h(t)$ to check if you get $y(t)$ as in Q1 b). ($h(t)$ is "signature" in this sense).

d) In output $y(t)$, consider coefficient of exponent e^{at} and find ratio of coefficients in output/input (i.e. b).

Check if this ratio is $G(s)\Big|_{s=a}$.

Thus for "eigenfunction" e^{at}, scaling is just $G(s)\Big|_{s=a}$ to get scaled e^{at} as output.

Q-2: Unless otherwise stated explicitly, assume all functions(t) are zero for $t<0$.

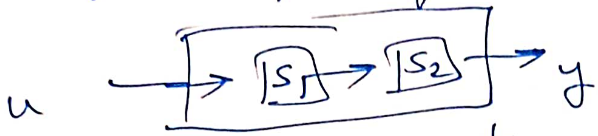
(a) Show that $\displaystyle\int_0^t u(\tau)\,h(t-\tau)\,d\tau = \int_0^t u(t-\tau)\,h(\tau)\,d\tau = \int_{-\infty}^{\infty} u(\tau)\,h(t-\tau)\,d\tau$

(b) Convolve e^{5t} and constant function C_0 (Note; both are zero for $t<0$)

(c) for Q2b, find their Laplace transforms, multiply them, & then take inverse Laplace transform of product).

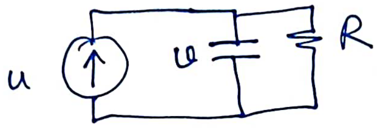
Q-3: Suppose LTI system S_1 & LTI system S_2 are in series and system S_1 has impulse response e^{5t} & S_2 : Co
(both responses $= 0$ for $t < 0$).

Find impulse response of this "series" connection



Series connect

Q-4: Consider



$C = 1F$, $R = 1000\Omega$
input = current = $u(t)$
& output = voltage across capacitor $= y$.

(a) Find transfer fn from u to y: call it $G(s)$

(b) what should resistance R be for the pole to at -5?
at -1?

(c) Plot $|G(j\omega)|$ versus ω (for $\omega \in (-\infty, \infty)$).

(d) Is this a low pass filter or high pass filter.

(e) cut-off frequency?

Q-5 (a) Consider $\ddot{\theta} - \theta = F$ as equation of a pendulum, θ is deviation from desired position. Is this regular or inverted pendulum?

(b) Find feedback law $F = k_p\theta$ such that closed loop
- poles are in open LHP.
- poles are in closed LHP.

(c) Find feedback law $F = k_p\theta + k_d\dot{\theta}$ such that
- poles are at "time-constant of decay" $= 0.3$ seconds.
- further with oscillation frequency 2 rad/s.

Q-6: For $\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = u$. Find range of k s.t. closed loop is stable. $(u = ky)$.