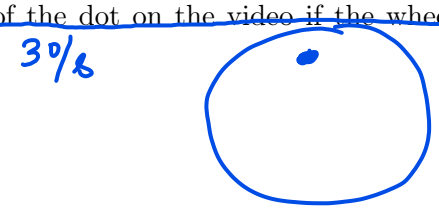
Questions on Signals and Systems

1. f_1 and f_2 represent two positive numbers that indicate frequencies in Hertz. Identify all frequencies present in the signal $\cos(2\pi f_1 t)\cos(2\pi f_2 t)$. Ans: $(f_1 \pm f_2)$.

2. An audio signal $x_1(t)$ has only frequencies up to 15 kHz. Another audio signal $x_2(t)$ has frequencies up to 20 kHz. What is the minimum rate at which samples of these signals individual signals can be used to reconstruct them losslessly? What about $x_1(t) + x_2(t)$? Ans: The indvidual maximum frequencies can be thought of similar to the rotating wheels example done in class. There, we saw that, taking frames at MORE than twice the maximum frequency is sufficient to represent the signal accurately. Therefore, $x_1(t)$ needs to be sampled at MORE than 30 kHz, and $x_2(t)$ at more than 40 kHz. In the case of $x_1(t)+x_2(t)$, this is like the two wheels on the same screen, so the faster wheel dominates. So, the sampling rate required is still more than 40 kHz.

3. A camera captures frames at 30 fps. A wheel that has one dot on its rim is rotating at various speeds, and is captured by the camera. Find the speed and direction of the dot on the video if the wheel is physically moving at

   a. 10 rps clockwise

   b. 15 rps clockwise

   c. 25 rps clockwise

   d. 30 rps clockwise

   Ans: This was solved in class. In any case, using sampling theorem, rotation speeds less than half of the frame rate will be captured, so that directly gives us that 10 rps clockwise would be represented accurately in the video. 15 rps would result in two grey dots at opposite ends. 30 rps is also simple: the dot would appear static. Finally, for 25 rps, you can infer that the speed would be 5 rps anti-clockwise.

4. A wireless provider has leased frequencies between 900 MHz and 905 MHz in a particular area. If 25 kHz are required to support a single audio call, how many simultaneous calls can the provide support? Ans: 5 MHz / 25 kHz = 200.

5. In a communication system, a rectangular pulse whose length is 1 ms is used to send the bit 1, and a zero signal for 1 ms is used to send 0.

   a. What is the data rate achieved by this system?

   b. If the duration of the bit 1 signal is kept the same, but the bit 0 signal is reduced to 0.5 ms, and the bits 0 and 1 are equally frequent, find the new data rate.

   c. Could there be any issues in making the duration of the pulses vary small?

   Ans: For (a), the data rate will be 1000 bps. In the (b) case, since the average bit duration reduces to 0.75 ms, the new data rate is 1333.33 bps. It is, however, not always possible to increase data rates this way due to limitations in the medium and susceptibility to electronics and noise related issues.