BASIC ELECTRICAL ENGNEERING

Department of Electrical and Electronics Engineering

GOKARAJU RANGARAJU
INSTITUTE OF ENGINEERING AND TECHNOLOGY
(Autonomous)

Bachupally, Hyderabad – 500090

GOKARAJU RANGARAJU INSTITUTE OF ENGINEERING AND TECHNOLOGY
BASIC ELECTRICAL ENGINEERING

Course Code: GR20A1008 **L/T/P/C: 2/1/0/3**
I Year I semester
Course Objectives:
1. Introduce the fundamentals of Electrical Engineering.
2. Understand magnetic circuits, DC circuits and AC single phase & three phase circuits
3. Provide foundation in theory and applications of Transformers and DC machines
4. Understand the basic principles of AC Electrical machinery and their applications.
5. Impart the knowledge of Electrical Installations.

Course Outcomes:
At the end of this course, students will able to
1. Understand and analyze basic electric circuits with suitable theorems.
2. Solve 1-phase and 3-phase balanced sinusoidal systems.
3. Interpret the working principle of Electrical machines.
4. Appraise the applications of Induction motors and synchronous generators used in Industries.
5. Identify the components of Low Voltage Electrical Installations.

UNIT I
D.C. CIRCUITS
Electrical circuit elements (R, L and C), voltage and current sources, KVL&KCL, analysis of simple circuits with dc excitation. Thevenin's and Norton's theorems, Super position and Reciprocity theorems. Time-domain analysis of first-order RL and RC circuits.
UNIT II
A.C. CIRCUITS
Representation of sinusoidal waveforms, average and rms values, phasor representation, real power, reactive power, apparent power, power factor. Analysis of single-phase AC circuits consisting of R, L, C, RL, RC, RLC combinations (series and parallel), resonance in series RLC circuit. Locus Diagram. Three-phase balanced circuits, voltage and current relations in star and delta connections.

UNIT III
DC MACHINES AND TRANSFORMERS
DC Motor and Generator: Construction, Principle of operation and Applications. Ideal and practical transformer, equivalent circuit, losses in transformers and efficiency, regulation. Autotransformer and three-phase transformer connections.

UNIT IV
AC MACHINES
Generation of rotating magnetic fields, Construction and working of a three-phase induction motor, Significance of torque-slip characteristic, Loss components and efficiency. Single-phase induction motor, Construction, working, torque-speed characteristics. Construction and working of synchronous generators.

UNIT V
ELECTRICAL INSTALLATIONS
Power system overview. Components of LT Switchgear: Switch Fuse Unit (SFU), MCB, ELCB, MCCB, Types of Wires and Cables, Earthing. Types of Batteries, Important Characteristics for Batteries. Elementary calculations for energy consumption, power factor improvement and battery backup.

Text Books:

1. Basic Electrical Engineering - D.P. Kothari and I.J. Nagrath, 3rd edition 2010, Tata McGraw Hill.
2. D.C. Kulshreshtha, “Basic Electrical Engineering”, McGraw Hill, 2009.
3. L.S. Bobrow, Fundamentals of Electrical Engineering”, Oxford University Press, 2011
4. Electrical and Electronics Technology, E. Hughes, 10th Edition, Pearson, 2010
5. Electrical Engineering Fundamentals, Vincent Deltoro, Second Edition, Prentice Hall India, 1989

Reference Books:

1. C. K. Alexander and M. N. O. Sadiku, “Electric Circuits”, McGraw Hill Education, 2004.
2. K. V. V. Murthy and M. S. Kamath, “Basic Circuit Analysis”, Jaico Publishers, 1999.
3. Circuit Theory (Analysis and Synthesis) by A.Chakrabarti-DhanpatRai& Co.
4. P. S. Bimbhra, “Electrical Machinery”, Khanna Publishers, 2011.

Unit I

DC CIRCUITS

Voltage:

According to the structure of an atom, we know that there are two types of charges: positive and negative. A force of attraction exists between these positive and negative charges. A certain amount of energy (work) is required to overcome the force and move the charges through a specific distance. All opposite charges possess a certain amount of potential energy because of the separation between them. The difference in potential energy of the charges is called the *potential difference*.

Potential difference in electrical terminology is known as voltage, and is denoted either by V or v. It is expressed in terms of energy (W) per unit charge (Q), i.e.,

$$V = \frac{W}{Q} \quad \text{or} \quad v = \frac{dw}{dq}$$

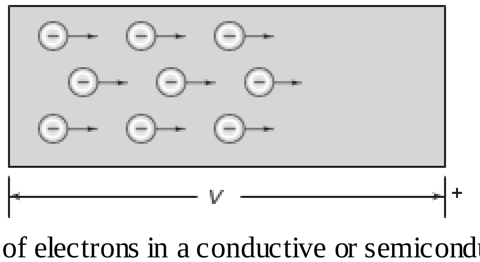
dw is the small change in energy, and
dq is the small change in charge.

where energy (W) is expressed in joules (J), charge (Q) in coulombs (C), and voltage (V) in volts (V). One volt is the potential difference between two points when one joule of energy is used to pass one coulomb of charge from one point to the other.

Current:

There are free electrons available in all semiconductive and conductive materials. These free electrons move at random in all directions within the structure in the absence of external pressure or voltage. If a certain amount of voltage is applied across the material, all the free electrons move in one direction depending on the polarity of the applied voltage, as shown in Fig 1.

This movement of electrons from one end of the material to the other end constitutes an electric current, denoted by either I or i. The conventional direction of current flow is opposite to the flow of -ve charges,i.e. the electrons.



Current is defined as the rate of flow of electrons in a conductive or semiconductive material. It is measured by the number of electrons that flow past a point in unit time. Expressed mathematically,

$$I = \frac{Q}{t}$$

where I is the current, Q is the charge of electrons, and t is the time, or

$$i = \frac{dq}{dt}$$

where dq is the small change in charge, and dt is the small change in time.

In practice, the unit *ampere* is used to measure current, denoted by A. One ampere is equal to one coulomb per second. One is the charge carried by 6.25×10^{18} electrons.

Power and energy:

Energy is the capacity for doing work, i.e. energy is nothing but stored work. Energy may exist in many forms such as mechanical, chemical, electrical and so on. Power is the rate of change of energy, and is denoted by either P or p. If a certain amount of energy is used over a certain length of time, then

$$P = \frac{Energy}{time} = \frac{W}{t} \text{ or } p = \frac{dw}{dt}$$

where dw is the change in energy and dt is the change in time.
Wecan also write

$$p = \frac{dw}{dt} = \frac{dw}{dq} \times \frac{dq}{dt}$$

$$= v \times i = vi \text{ W}$$

Energy is measured in joules (J), time in seconds (s), and power in watts (W).

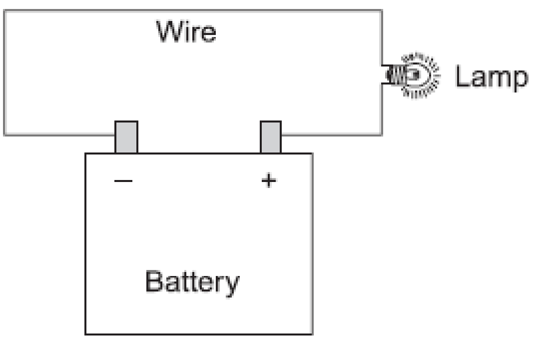
By definition, one watt is the amount of power generated when one joule of energy is consumed in one second. Thus, the number of joules consumed in one second is always equal to the number of watts.

The Circuit:

Simply put, an electric circuit consists of three parts: (1) energy source, such as battery or generator, (2) the load or sink, such as lamp or motor, and (3) connecting wires as shown in Fig 2. This arrangement represents a simple circuit. A battery is connected to a lamp with two wires. The purpose of the circuit is to transfer energy from source (battery) to the load (lamp). And this is accomplished by the passage of electrons through wires around the circuit.

The current flows through the filament of the lamp, causing it to emit visible light. The current flows through the battery by chemical action. A closed circuit is defined as a circuit in which the current has a complete path to flow. When the current path is broken so that current cannot flow, the circuit is called an open circuit.

More specifically, interconnection of two or more simple circuit elements (viz. voltage sources, resistors, inductors and capacitors) is called an electric network. If a network contains at least one closed path, it is called an electric circuit. By definition, a simple circuit element is the mathematical model of two terminal electrical devices, and it can be completely characterized by its voltage and current. Evidently then, a physical circuit must provide means for the transfer of energy.



Broadly, network elements may be classified into four groups, viz.,

1. Active or passive
2. Unilateral or bilateral
3. Linear or nonlinear
4. Lumped or distributed

Active or passive

Energy sources (voltage or current sources) are active elements, capable of delivering power to some external device. Passive elements are those which are capable only of receiving power. Some passive elements like inductors and capacitors are capable of storing a finite amount of energy, and return it later to an external element. More specifically, an active element is capable of delivering an average power greater than zero to some external device over an infinite time interval. For example, ideal sources are active elements. A passive element is defined as one that cannot supply average power that is greater than zero over an infinite time interval. Resistors, capacitors, and inductors fall into this category.

Unilateral or bilateral

In the bilateral element, the voltage-current relation is the same for current flowing in either direction. In contrast, a unilateral element has different relations between voltage and current for the two possible directions of current. Examples of bilateral elements are elements made of high conductivity materials in general. Vacuum diodes, silicon diodes, and metal rectifiers are examples of unilateral elements.

Linear or nonlinear

An element is said to be linear, if its voltage-current characteristic is at all times a straight line through the origin. For example, the current passing through a resistor is proportional to the voltage applied through it, and the relation is expressed as V α I or V =IR. A linear element or network is one which satisfies the principle of superposition, i.e., the principle of homogeneity and additivity. An element which does not satisfy the above principle is called a nonlinear element.

Lumped or distributed

Lumped elements are those elements which are very small in size and in which simultaneous actions take place for any given cause at the same instant of time. Typical lumped elements are capacitors, resistors, inductors and transformers. Generally, the elements are considered as lumped when their size is very small compared to the wave length of the applied signal. Distributed elements, on the other hand, are those which are not electrically separable for analytical purposes. For example, a transmission line which has distributed resistance, inductance and capacitance along its length may extend for hundreds of miles.

Resistance parameter:

When a current flow in a material, the free electrons move through the material and collide with other atoms. These collisions cause the electrons to lose some of their energy. This loss of energy per unit charge is the drop in potential across the material. The amount of energy lost by the electrons is related to the physical property of the material. These collisions restrict the movement of electrons. The property of a material to restrict the flow of electrons is called resistance, denoted by R. The symbol for the resistor is shown in Fig 3. The unit of resistance is ohm (Ω). Ohm is defined as the resistance offered by the material when a current of one ampere flows between two terminals with one volt applied across it.

According to Ohm's law, the current is directly proportional to the voltage and inversely proportional to the total resistance of the circuit, i.e.,

Figure 3

$$I=\frac{V}{R} \quad \text{or} \quad i=\frac{v}{R}$$

We can write the above equation in terms of charge as follows.

$$V=R\frac{dq}{dt}, \text{ or } i=\frac{v}{R}=Gv$$

where G is the conductance of a conductor. The units of resistance and conductance are ohm (Ω) and mho(℧) respectively. When current flows through any resistive material, heat is generated by the collision of electrons with other atomic particles. The power absorbed by the resistor is converted to heat. The power absorbed by the resistor is given by

$$P = vi = (iR)i = i^2 R$$

where i is the current in the resistor in amps, and v is the voltage across the resistor in volts. Energy lost in a resistance in time t is given by

$$W=\int_0^t p\,dt = \frac{v^2}{R}t = i^2Rt$$

where v is the volts

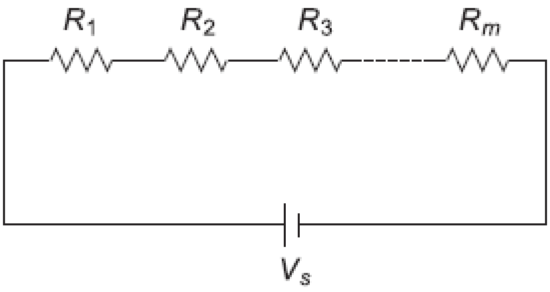
R is in ohms,

t is in seconds, and

W is in joules.

Series and parallel Combinations of Resistors:

Series: When the circuit is connected in series, the total resistance of the circuit increses as the number of resistors connected in series increases. If we consider m resistors connected in series as shown in Fig 4. the voltage drop across each element is



Therefore,

$$V_{R1} = IR_1$$
$$V_{R2} = IR_2$$
$$V_{R3} = IR_3$$
$$\vdots$$
$$V_{R_m} = IR_m$$

Applying KVL to the above circuit $V_s = V_{R1} + V_{R2} + \ldots\ldots\ldots V_{Rm}$

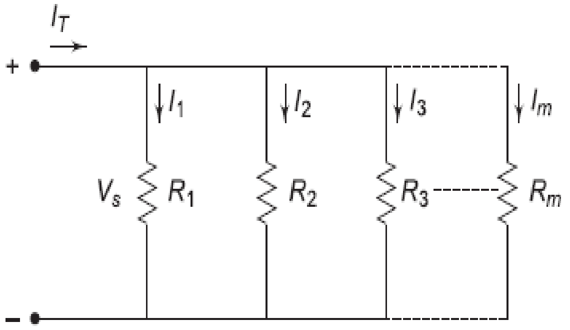
$I R_s = I R_1 + I R_2 + \ldots \ldots\ldots\ldots I R_m$

$R_s = R_1 + R_2 + \ldots\ldots\ldots\ldots IR_m$

Parallel: When the circuit is connected in parallel, the total resistance of the circuit decreases as the number of resistors connected in parallel increases. If we consider m parallel branches in a circuit as shown in Fig 5. the current equation is

IT = I1 + I2 +.... + Im

The same voltage is applied across each resistor. By applying Ohm's law, the current in each branch is given by



$$I_1 = \frac{V_s}{R_1}, I_2 = \frac{V_s}{R_2}, \ldots I_m = \frac{V_s}{R_m}$$

According to Kirchhoff's current law,

$$I_T = I_1 + I_2 + I_3 + \ldots + I_m$$

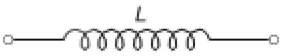
$$\frac{V_s}{R_T} = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} + \ldots + \frac{V_s}{R_m}$$

From the above equation, we have

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_m}$$

Inductance parameter:

A wire of certain length, when twisted into a coil becomes a basic inductor .If current is made to pass through an inductor, an electromagnetic field is formed. A change in the magnitude of the current changes the electromagnetic field. Increase in current expands the fields, and decrease in current reduces it. Therefore, a change in current produces change in the electromagnetic field,which induces a voltage across the coil according to Faraday's law of electromagnetic induction.



The unit of inductance is henry, denoted by H. By definition, the inductance is one henry when current through the coil, changing at the rate of one ampere per second, induces one volt across the coil. The symbol for inductance is shown in Fig 6.

The current-voltage relation is given by

$$V=L\frac{di}{dt}$$

where v is the voltage across inductor in volts, and i is the current through inductor in amps. We can rewrite the above equations as

$$di=\frac{1}{L}vdt$$

Integrating both sides, we get

$$\int_0^t di=\frac{1}{L}\int_0^t vdt$$

$$i(t)-i(0)=\frac{1}{L}\int_0^t vdt$$

$$i()=\frac{1}{L}\int_0^t vdt+i(0)$$

From the above equation, we note that the current in an inductor is dependent upon the integral of the voltage across its terminals and the initial current in the coil, $i(0)$.

The power absorbed by the inductor is

$$P=vi=Li\frac{di}{dt} \quad \text{watts}$$

The energy stored by the inductor is

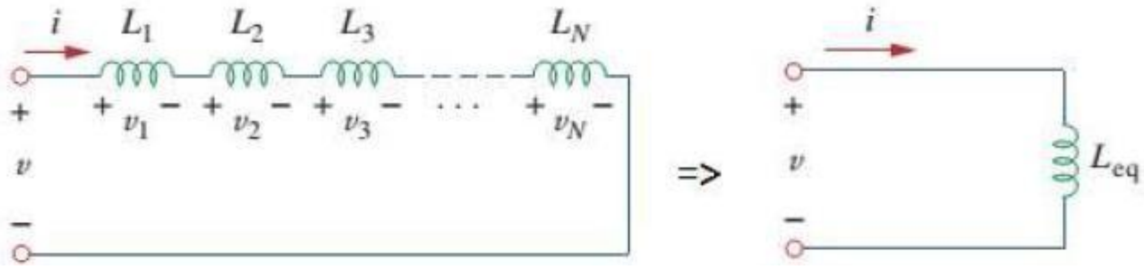
$$W=\int_0^t pdt$$

$$=\int_0^t Li\frac{di}{dt}dt=\frac{1}{2}Li^2$$

From the above discussion, we can conclude the following:

1. The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to dc.
2. A small change in current within zero time through an inductor gives an infinite voltage across the inductor, which is physically impossible. In a fixed inductor, the current cannot change abruptly.
3. The inductor can store finite amount of energy, even if the voltage across the inductor is zero, and
4. A pure inductor never dissipates energy, only stores it. That is why it is also called a non-dissipative passive element. However, physical inductors dissipate power due to internal resistance.

Series and Parallel Combinations of Inductors:



Series: Consider N inductors are connected in series, and voltage drop across each inductor is v1, v2, … . . vn.

According to the KVL v = v1 + v2 … . . +vn

According to the KVL

$$v = v_1 + v_2 \ldots\ldots + v_n$$

But $v = L\frac{di}{dt}$

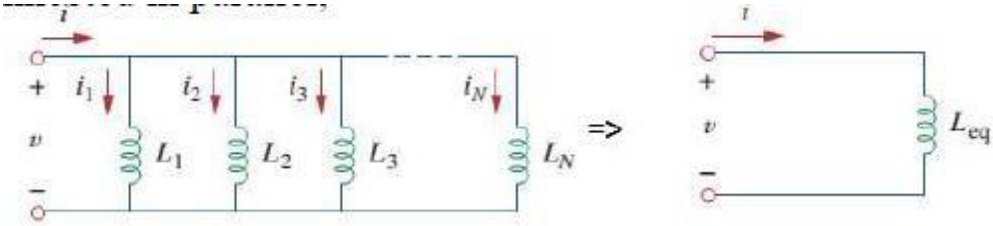
$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} \ldots\ldots + L_n\frac{di}{dt}$$

$$v = (L_1 + L_2 \ldots\ldots + L_n)\frac{di}{dt}$$

$$v = L_{eq}\frac{di}{dt}$$

So $L_{eq} = L_1 + L_2 \ldots\ldots + L_n$

Parallel: Consider N inductor are connected in parallel



According to KCL

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

But $i_k = \frac{1}{L_k}\int_{t_0}^{t} v\,dt + i_k(t_0)$; hence,

$$\begin{aligned} i &= \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0) \\ &\quad + \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0) \\ &= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0) \\ &\quad + \cdots + i_N(t_0) \\ &= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0) \end{aligned}$$

where

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

Capacitance parameter:

Any two conducting surfaces separated by an insulating medium exhibit the property of a capacitor. The conducting surfaces are called electrodes, and the insulating medium is called dielectric. A capacitor stores energy in the form of an electric field that is established by the opposite charges on the two electrodes. The electric field is represented by lines of force between the positive and negative charges, and is concentrated within the dielectric. The amount of charge per unit voltage that is capacitor can store is its capacitance, denoted by C. The unit of capacitance is Farad denoted by F. By definition, one Farad is the amount of capacitance when one coulomb

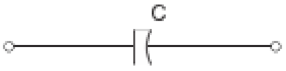


Figure 7

A capacitor is said to have greater capacitance if it can store more charge per unit voltage and the capacitance is given by

$$C = \frac{Q}{V} \qquad \text{or } c = \frac{q}{v}$$

(lowercase letters stress instantaneous values)

$$i = C\frac{dv}{dt}$$

where *v* is the voltage across capacitor and *i* is the current through it.

$$dv = \frac{1}{C} idt$$

Integrating both sides, we have

$$\int_0^t dv = \frac{1}{C}\int_0^t idt$$

$$v(t) - v(0) = \frac{1}{C}\int_0^t idt$$

$$v(t) = \frac{1}{C}\int_0^t idt + v(0)$$

where *v* (0) indicates the initial voltage across the capacitor.
From the above equation, the voltage in a capacitor is dependent upon the integral of the current through it, and the initial voltage across it.

The power absorbed by the capacitor is given by

$$P = vi = Cv\frac{dv}{dt} \text{ watts}$$

The energy stored by the inductor is

$$W = \int_0^t pdt$$

$$= \int_0^t Cv\frac{dv}{dt}dt = \frac{1}{2}Cv^2$$

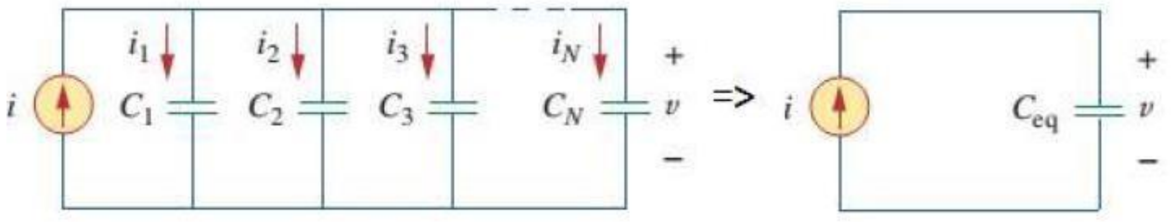
From the above discussion, we can conclude the following:

1. The current in a capacitor is zero if the voltage across it is constant; that means, the capacitor acts as an open circuit to dc
2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible. In a fixed capacitance the voltage cannot change abruptly.
3. The capacitor can store a finite amount of energy, even if the current through it is zero, and

4. A pure capacitor never dissipates energy, but only stores it; that is why it is called *non-dissipative passive element.* However, physical capacitors dissipate power due to internal resistance.

Series and Parallel Connection of Capacitors:

Parallel: Let N Capacitors are connect in Parallel



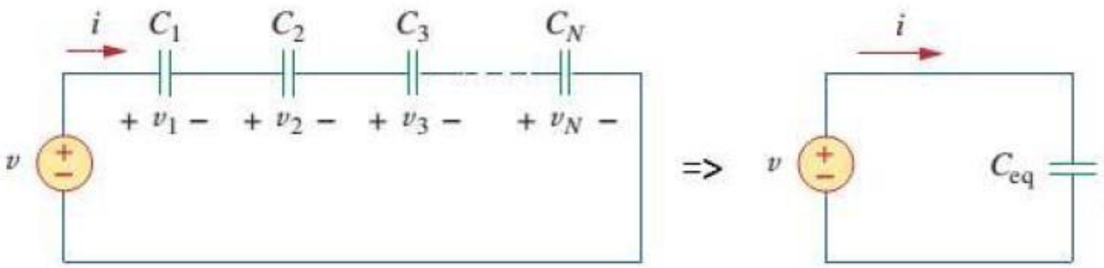
Apply KCL

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$
$$= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

where

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$$

Series: Consider N capacitors are connected in series



Apply KVL above circuit (left)

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

But $v_k = \frac{1}{C_k}\int_{t_0}^{t} i(\tau)d\tau + v_k(t_0)$. Therefore,

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)d\tau + v_2(t_0)$$

$$+ \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

where

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}}$$

Energy sources:

According to their terminal voltage–current characteristics, electrical energy sources are categorised into ideal voltage sources and ideal current sources. Further they can be divided into independent and dependent sources.

An ideal voltage source is a two-terminal element in which the voltage vs is completely independent of the current is through its terminals. The representation of ideal constant voltage source is shown in Fig. 8(a)

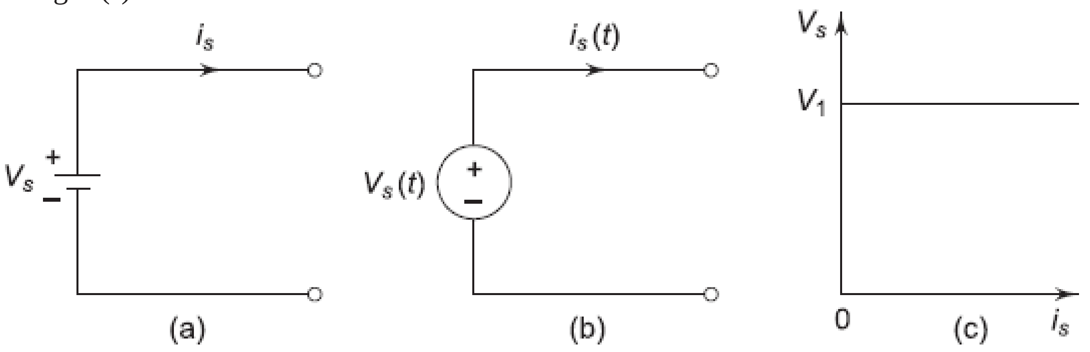


Figure 8

If we observe the v–i characteristics for an ideal voltage source as shown in Fig. 8(c) at any time, the value of the terminal voltage vs is constant with respect to the value of current is. Whenever vs=0, the voltage source is the same as that of a short circuit. Voltage sources need not have constant magnitude; in many cases the specified voltage may be time-dependent like a sinusoidal waveform. This may be represented as shown in Fig. 8(b). In many practical voltage sources, the internal resistance is represented in series with the source as shown in Fig. 9(a). In this, the voltage across the terminals falls as the current through it increases, as shown in Fig. 9(b)

The terminal voltage v_t depends on the source current as shown in Fig. 9(b),

where $v_t = v_S - i_S R$.

An ideal constant current source is a two-terminal element in which the current is completely independent of the voltage vs across its terminals. Like voltage sources we can have current sources of constant magnitude is or sources whose current varies with time is(t). The representation of an ideal current source is shown in Fig.9 (a)

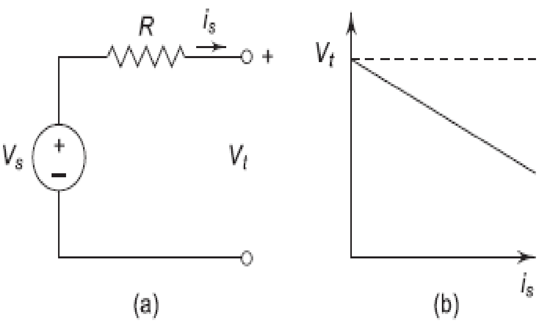


Figure 9

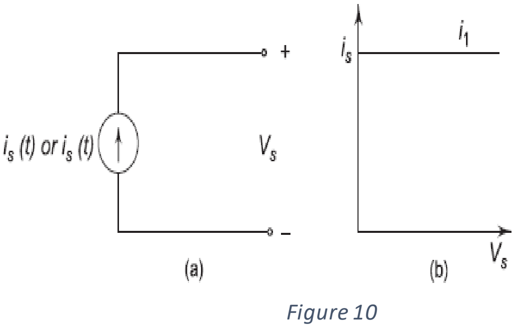


Figure 10

If we observe the v – i characteristics for an ideal current source as shown in Fig. 9(b), at

any time the value of the current is is constant with respect to the voltage across it. In many practical current sources, the resistance is in parallel with a source as shown in Fig. 10(a). In this the magnitude of the current falls as the voltage across its terminals increases. Its terminal v – i characteristic is shown in Fig. 10 (b). The terminal current is given by it = is any time the value of the current is is constant with respect to the voltage across it. In many practical current sources, the resistance is in parallel with a source as shown in Fig. 11(a). In this the magnitude of the current falls as the voltage across its terminals increases. Its terminal v – i characteristic is shown in Fig. 11(b). The terminal currentgiven by it = is – (vs/R), where R is the internal resistance of the ideal current source.

The two types of ideal sources we have discussed are independent sources for which voltage and current are independent and are not affected by other parts of the circuit. In the case of dependent sources, the source voltage or current is not fixed, but is dependent on the voltage or current existing at some other location in the circuit.

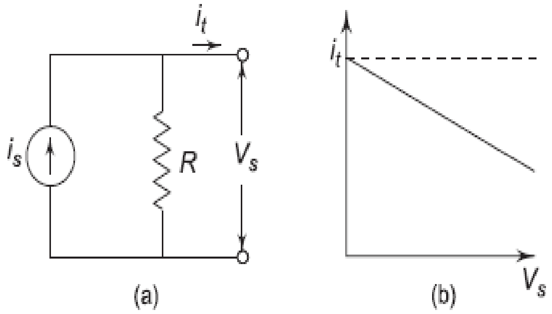


Figure 11

Dependent or controlled sources are of the following types:

1.voltage controlled voltage source (VCVS)

2.current controlled voltage source (CCVS)

3.voltage controlled current source (VCCS)

4.current controlled current source (CCCS)

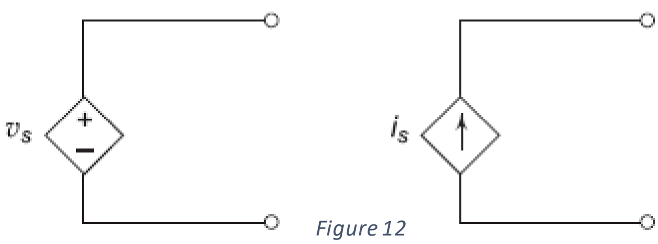


Figure 12

These are represented in a circuit diagram by the symbol shown in Fig12. These types of sources mainly occur in the analysis of equivalent circuits of transistors.

Kirchhoff's voltage law:

Kirchhoff's voltage law states that the algebraic sum of all branch voltages around any closed path in a circuit is always zero at all instants of time. When the current passes through a resistor, there is a loss of energy and, therefore, a voltage drop. In any element, the current always flows from higher potential to lower potential. Consider the circuit in Fig. It is customary to take the direction of current I as indicated in the figure, i.e. it leaves the positive terminal of the voltage source and enters into the negative terminal.

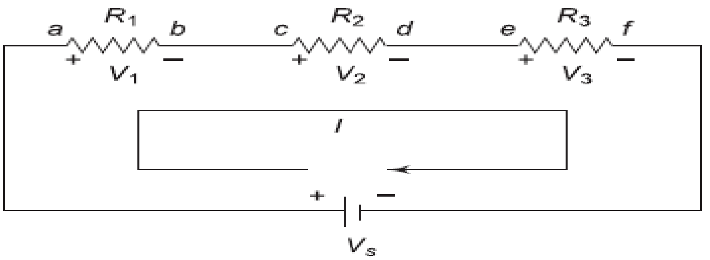


Figure 13

As the current passes through the circuit, the sum of the voltage drop around the loop is equal to the total voltage in that loop. Here, the polarities are attributed to the resistors As the current passes through the circuit, the sum of the voltage drop around the loop is equal to the total voltage in that loop. Here, the polarities are attributed to the resistors

$$\therefore \quad V_s = V_1 + V_2 + V_3$$

Consider the problem of finding out the current supplied by the source V in the circuit shown in Fig.14

Our first step is to assume the reference current direction and to indicate the polarities for different elements. (See Fig 14).

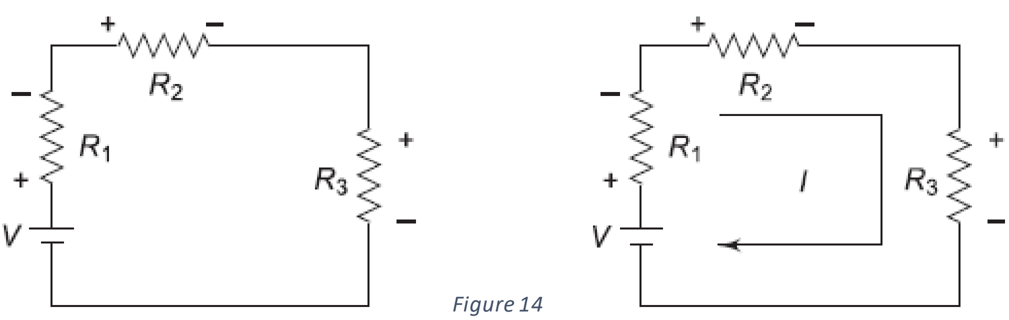


Figure 14

By using Ohm's law, we find the voltage across each resistor as follows.

$$V_{R1} = IR_1, V_{R2} = IR_2, V_{R3} = IR_3$$

where VR1, VR2 and VR3 are the voltages across R1, R2 and R3, respectively. Finally, by applying Kirchhoff's law, we can form the equations

$$V = V_{R1} + V_{R2} + V_{R3}$$

$$V = IR_1 + IR_2 + IR_3$$

From the above equation, the current delivered by the source is given by

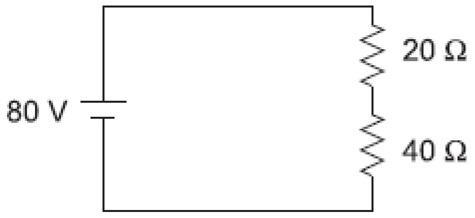
$$I = \frac{V}{R1+R2+R3}$$

Voltage Division:

The series circuit acts as a voltage divider. Since the same current flows through each resistor, the voltage drops are proportional to the values of resistors. Using this principle, different voltages can be obtained from a single source, called a voltage divider. For example, the voltage across a 40 Ω resistor is twice that of 20Ω in a series circuit shown in Fig 15.

In general, if the circuit consists of a number of series resistors, the total current is given by the total voltage divided by equivalent resistance. This is shown in Fig 15.

The current in the circuit is given by I = Vs/(R1 +R2+ … + Rm). The voltage across any resistor is nothing but the current passing through it, multiplied by that particular resistor.



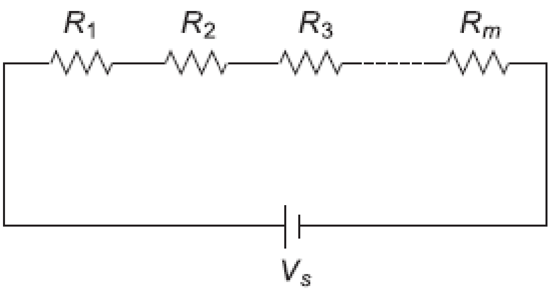


Figure 15

$$\text{Therefore,} \quad V_{R1} = IR_1$$
$$V_{R2} = IR_2$$
$$V_{R3} = IR_3$$
$$\vdots$$
$$V_{R_m} = IR_m$$
$$\text{or} \quad V_{R_m} = \frac{V_s(R_m)}{R_1 + R_2 + \ldots + R_m}$$

From the above equation, we can say that the voltage drop across any resistor, or a combination of resistors in a series circuit is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage, i.e.,

$$V_m = \frac{R_m}{R_T} V_s$$

where Vm is the voltage across mth resistor, Rm is the resistance across which the voltage is to be determined and RT is the total series resistance.

Power in series Circuit:

The total power supplied by the source in any series resistive circuit is equal to the sum of the powers in each resistor in series,i.e

$$P_s = P_1 + P_2 + P_3 + \ldots + P_m$$

where m is the number of resistors in series, PS is the total power supplied by source, and Pm is the power in the last resistor in series. The total power in the series circuit is the total voltage applied to a circuit, multiplied by the total current. Expressed mathematically,

$$P_s = V_s I = I^2 R_T = \frac{V_s^2}{R_T}$$

where Vs is the total voltage applied, RT is the total resistance, and I is the total current.

Kirchoffs Current Law:

Kirchhoff's current law states that the sum of the currents entering into any node is equal to the sum of the currents leaving that node. The node may be an interconnection of two or more branches. In any parallel circuit, the node is a junction point of two or more branches. The total current entering into a node is equal to the current leaving that node. For example, consider the circuit shown in Fig 16. which contains two nodes

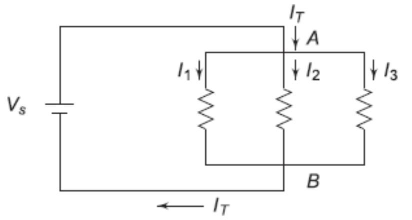


Figure 16

A and B. The total current IT entering node A is divided into I1, I2 and I3. These currents flow out of the node A. According to Kirchhoff's current law, the current into node A is equal to the total current out of the node A: that is, IT = I1 + I2 + I3. If we consider the node B, all three currents I1, I2, I3 are entering B, and the total current IT is leaving the node B, Kirchhoff's current law formula at this node is therefore the same as at the node A.

$$I_T = I_1 + I_2 + I_3$$

In general, the sum of the currents entering any point or node or junction equal to sum of the currents leaving from that point or node or junction as shown in Fig.

I1 + I2 + I4 + I7 = I3 + I5 + I6

If all of the terms on the right side are brought over to the left side, their signs change to negative and a zero is left on the right side, i.e.

I1 + I2 + I4 +I7 – I3 – I5 – I6 = 0

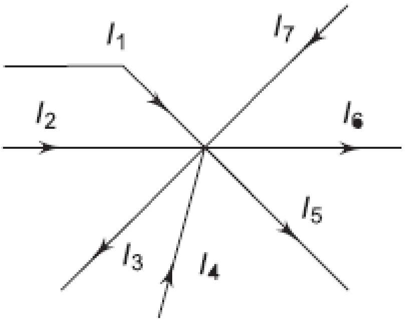


Figure 17

This means that the algebraic sum of all the currents meeting at a junction is equal to zero.

Current Division:

In a parallel circuit, the current divides in all branches. Thus, a parallel circuit acts as a current divider. The total current entering into the parallel branches is divided into the branches currents according to the resistance values. The branch having higher resistance allows lesser current, and the branch with lower resistance allows more current. Let us find the current division in the parallel circuit shown in Fig 18.

The voltage applied across each resistor is Vs. The current passing through each resistor is given by

$$I_1 = \frac{V_s}{R_1}, I_2 = \frac{V_s}{R_2}$$

If RT is the total resistance, which is given by R1R2/(R1 +R2), Total current

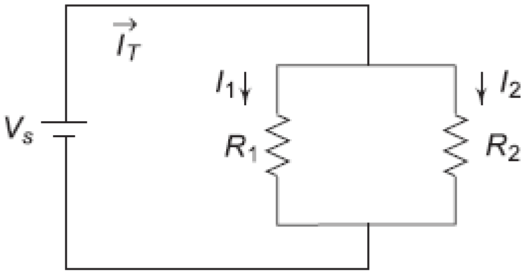


Figure 18

$$I_T = \frac{V_s}{R_T} = \frac{V_s}{R_1 R_2}(R_1 + R_2)$$

or $$I_T = \frac{I_1 R_1}{R_1 R_2}(R_1 + R_2) \text{ since } V_s = I_1 R_1$$

$$I_1 = I_T \cdot \frac{R_2}{R_1 + R_2}$$

Similarly, $$I_2 = I_T \cdot \frac{R_1}{R_1 + R_2}$$

From the above equations, we can conclude that the current in any branch is equal to the ratio of the opposite branch resistance to the total resistance value, multiplied by the total current in the circuit. In general, if the circuit consists of m branches, the current in any branch can be determined by

$$I_i = \frac{R_T}{R_i + R_T} I_T$$

where Ii represents the current in the ith branch,
Ri is the resistance in the ith branch,
RT is the total parallel resistance to the ith branch, and
IT is the total current entering the circuit.

Powers in Parallel Circuit:

The total power supplied by the source in any parallel resistive circuit is equal to the sum of the powers in each resistor in parallel, i.e.,

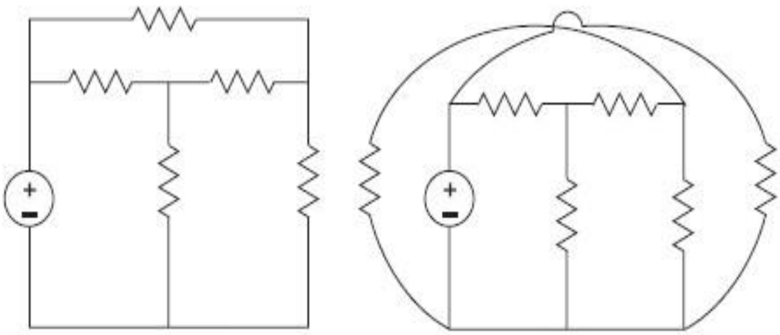
Ps = P1 + P2 + P3 + ... + Pm

where m is the number of resistors in parallel, Ps is the total power, and Pm is the power in the last resistor.

Mesh Analysis:

Mesh and nodal analysis are two basic important techniques used in finding solutions for a network. The suitability of either mesh or nodal analysis to a particular problem depends mainly on the number of voltage sources or current sources. If a network has a large number of voltage sources, it is useful to use mesh analysis; as this analysis requires that all the sources in a circuit be voltage sources. Therefore, if there are any current sources in a circuit they are to be converted into equivalent voltage sources, if, on the other hand, the network has more current sources, nodal analysis is more useful. Mesh analysis is applicable only for planar networks. For non-planar circuits, mesh analysis is not applicable. A circuit is said to be planar if it can be drawn on a plane surface without crossovers. A non-planar circuit cannot be drawn on a plane surface without a crossover.

Figure (a) is a planar circuit. Figure (b) is a non-planar circuit and Fig. (c) is a planar circuit which looks like a non-planar circuit. It has already been discussed that a loop is a closed path. A mesh is defined as a loop which does not contain any other loops within it. To apply mesh analysis, our first step is to check whether the circuit is planar or not and the second is to select mesh currents. Finally, writing Kirchhoff's voltage law equations in terms of unknowns and solving them leads to the final solution.



(a) (b) (c)

Observation of Fig 19 indicates that there are two loops abefa, and bcdeb in the network. Let us assume loop currents I1 and I2 with directions as indicated in the figure 19. Considering the loop abefa alone, we observe that current I1 is passing through R1, and (I1 – I2) is passing through R2. By applying Kirchhoff's voltage law, we can write

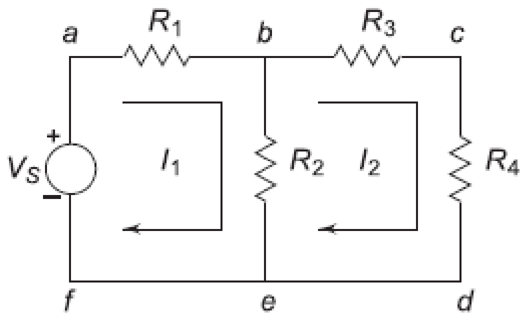


Figure 19

$$Vs = I1R1 + R2(I1 – I2)$$

Similarly, if we consider the second mesh bcdeb, the current I2 is passing through R3 and R4, and (I2 – I1) is passing through R2. By applying Kirchhoff's voltage law around the second mesh, we have

$$R2\ (I2 – I1) + R3\ I2 + R4\ I2 = 0$$

By rearranging the above equations, the corresponding mesh current equations are

$$I1(R1 + R2) – I2\ R2 = Vs$$

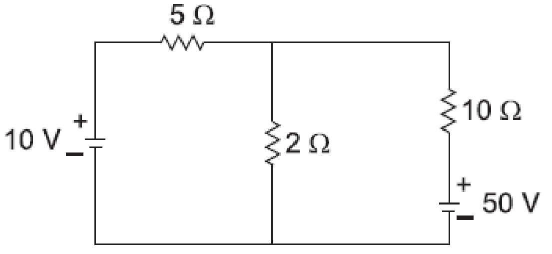
$$– I1\ R2 + (R2 + R3 + R4)I2 = 0$$

By solving the above equations, we can find the currents I1 and I2. If we observe above Fig the circuit consists of five branches and four nodes, including the reference node. The number of mesh currents is equal to the number of mesh equations.

And the number of equations 5 branches – (nodes – 1). In above Fig, the required number of mesh currents would be 5 – (4 – 1) = 2.

In general, if we have B branches and N nodes including the reference node then the number of linearly independent mesh equations M = B – (N – 1).

Example: Write the mesh current equations in the circuit shown in Fig, and determine the currents.



Solution: Assume two mesh currents in the direction as indicated in below figure.

The mesh current equations are

$5I1 + 2(I1 – I2) = 10$

$10I2 + 2(I2 – I1) + 50 = 0$

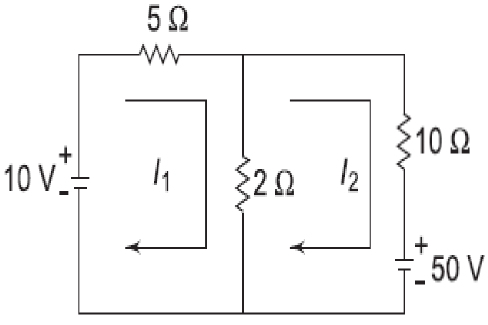
We can rearrange the above equations as

$7I1 – 2I2 = 10$

$– 2I1 + 12I2 = – 50$

By solving the above equations, we have

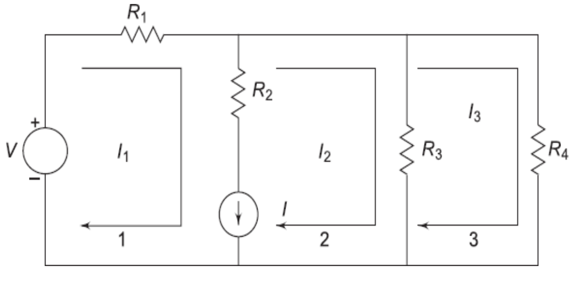
I1 =0.25 A, and I2 = – 4.125 A



Here, the current in the second mesh, I2, is negative; that is the actual current I2 flows opposite to the assumed direction of current in the circuit of above Fig.

<u>Super mesh Analysis:</u>

Suppose any of the branches in the network has a current source; then it is slightly difficult to apply mesh analysis straightforward because first we should assume an unknown voltage across the current source, writing mesh equations as before, and then relate the source current to the assigned mesh currents. This is generally a difficult approach. One way to overcome this difficulty is by applying the supermesh technique. Here we have to choose the kind of supermesh. A supermesh is constituted by two adjacent loops that have a common current source. As an example, consider the network shown in below figure.



Here, the current source I is in the common boundary for the two meshes 1 and 2. This current source creates a supermesh, which is nothing but a combination of meshes 1 and 2.

$$R1I1 + R3(I2 – I3) = V \text{ or } R1I1 + R3I2 – R4I3 = V$$

Considering the mesh 3, we have

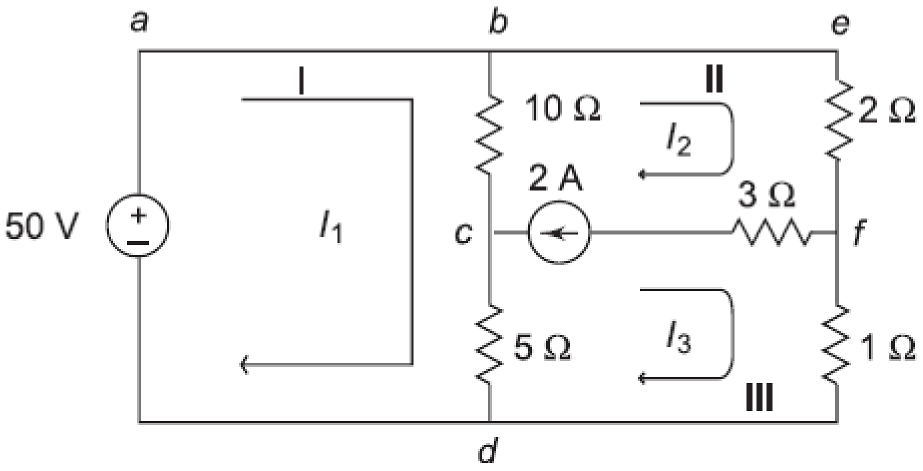
$$R3(I3 – I2) + R4I3 = 0$$

Finally, the current I from the current source is equal to the difference between the two mesh currents, i.e.

$$I1 – I2 = I$$

We have, thus, formed three mesh equations which we can solve for the three unknown currents in the network.

Example:Determine the current in the 5 V resistor in the network given in Fig.



Solution: From the first mesh, i.e. abcda, we have

$$50 = 10(I1 – I2) + 5(I1 – I3)$$

$$\text{or} \quad 15I1 – 10I2 – 5I3 = 50 \ldots\ldots(1)$$

From the second and third meshes, we can form a supermesh

$$10(I2 – I1) + 2I2 + I3 + 5(I3 – I1) = 0$$

$$\text{or} – 15I1 + 12I2 + 6I3 = 0(2)$$

The current source is equal to the difference between II and III mesh currents,

$$\text{i.e.} \quad I2 – I3 = 2 \text{ A}\ldots\ldots(3)$$

Solving (1), (2), and (3), we have

I1 = 19.99 A, I2 = 17.33 A, and I3 = 15.33 A

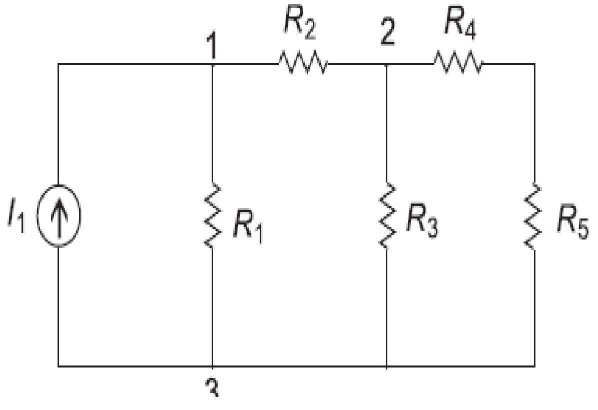
The current in the 5 Ω resistor = I1 – I3

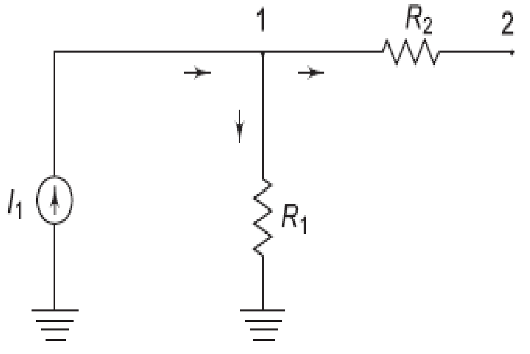
= 19.99 – 15.33 = 4.66 A

the current in the 5 Ω resistor is 4.66 A.

Nodal Analysis:

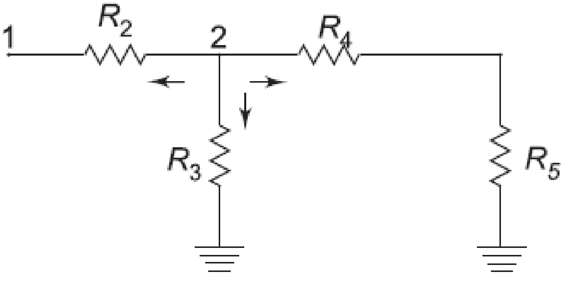
In previous section we discussed simple circuits containing only two nodes, including the reference node. In general, in an N-node circuit, one of the nodes is choosen as reference or datum node, then it is possible to write N – 1 nodal equations by assuming N – 1 node voltages. For example, a 10-node circuit requires nine unknown voltages and nine equations. Each node in a circuit can be assigned a number or a letter. The node voltage is the voltage of a given node with respect to one particular node, called the reference node, which we assume at zero potential. In the circuit shown in below figure the node 3 is assumed as the reference node. The voltage at the node 1 is the voltage at that node with respect to the node 3. Similarly, the voltage at the node 2 is the voltage at that node with respect to the node 3. Applying Kirchhoff's current law at the node 1; the current entering is equal to the current leaving (see Fig.)





$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

Here, V1 and V2 are the voltages at nodes 1 and 2, respectively. Similarly, at the node 2, the current entering is equal to the current leaving as shown in Fig.



$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5} = 0$$

Rearranging the above equations, we have

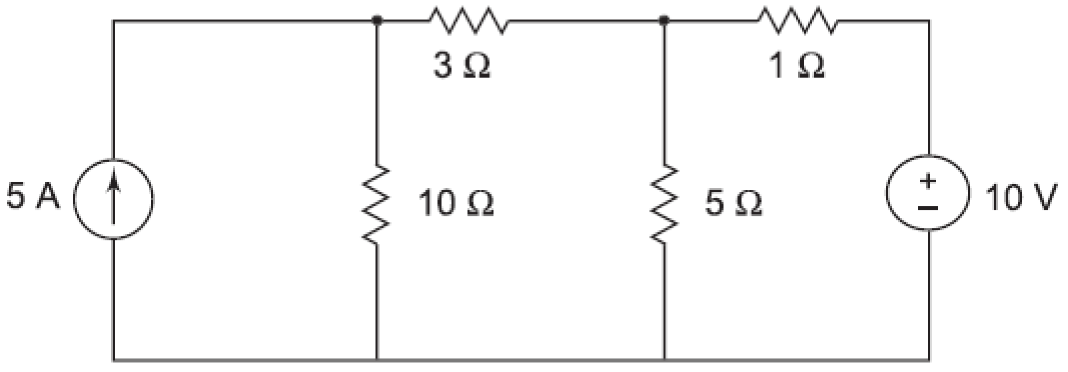
$$V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - V_2\left[\frac{1}{R_2}\right] = I_1$$

$$-V_1\left[\frac{1}{R_2}\right] + V_2\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right] = 0$$

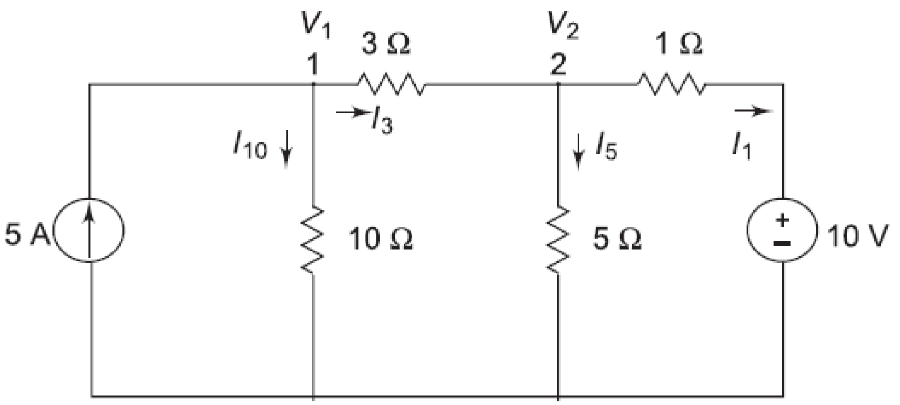
From the above equations, we can find the voltages at each node.

Example:

Write the node voltage equations and determine the currents in each branch for the network shown in Fig.



Solution: The first step is to assign voltages at each node as shown in Fig.



Applying Kirchhoff's current law at the node 1, we have

$$5=\frac{V_1}{10}+\frac{V_1-V_2}{3}$$

$$\text{or}\quad V_1\left[\frac{1}{10}+\frac{1}{3}\right]-V_2\left[\frac{1}{3}\right]=5$$

Applying Kirchhoff's current law at the node 2, we have

$$\frac{V_2-V_1}{3}+\frac{V_2}{5}+\frac{V_2-10}{1}=0$$

$$\text{or}\quad -V_1\left[\frac{1}{3}\right]+V_2\left[\frac{1}{3}+\frac{1}{5}+1\right]=10$$

From Eqs we can solve for V1 and V2 to get

$$V_1 = 19.85\text{ V},\ V_2 = 10.9\text{ V}$$

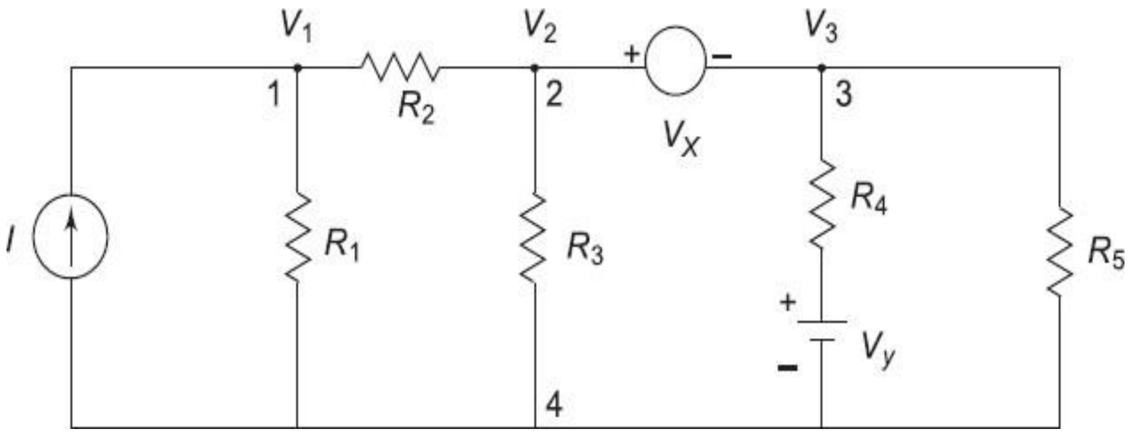
$$I_{10}=\frac{V_1}{10}=1.985\text{ A},\ I_3=\frac{V_1-V_2}{3}=\frac{19.85-10.9}{3}=2.98\text{ A}$$

$$I_5=\frac{V_2}{5}=\frac{10.9}{5}=2.18\text{ A},\ I_1=\frac{V_2-10}{1}=0.9\text{ A}$$

Supernodal Analysis:

Suppose any of the branches in the network has a voltage source; then it is slightly difficult to apply nodal analysis. One way to overcome this difficulty is to apply the supernode technique. In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff's current law as usual. This is explained with the help of Fig.

It is clear from Fig, that the node 4 is the reference node. Applying Kirchhoff's current law at the node 1, we get



$$I = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

Due to the presence of voltage source Vx in between nodes 2 and 3, it is slightly difficult to find out the current. The supernode technique can be conveniently applied in this case.

Accordingly, we can write the combined equation for nodes 2 and 3 as under.

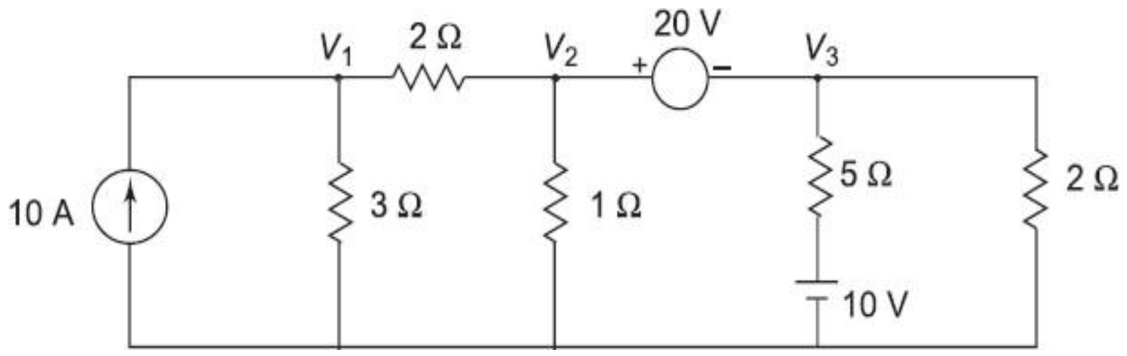
$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_3 - V_y}{R_4} + \frac{V_3}{R_5} = 0$$

The other equation is

$V2 - V3 = Vx$

From the above three equations, we can find the three unknown voltages.

Example: Determine the current in the 5 Ω resistor for the circuit shown in Fig.



Solution: At node 1

The voltage between nodes 2 and 3 is given by

V2 – V3 = 20

The current in the 5 Ω resistor

$$I_5 = \frac{V_3 - 10}{5}$$

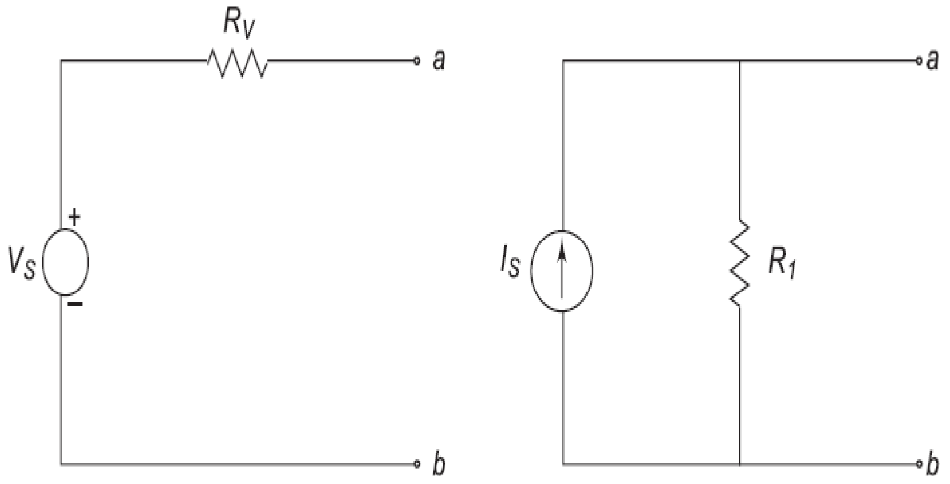
Solving above Eqs we obtain

V3 = –8.42 V

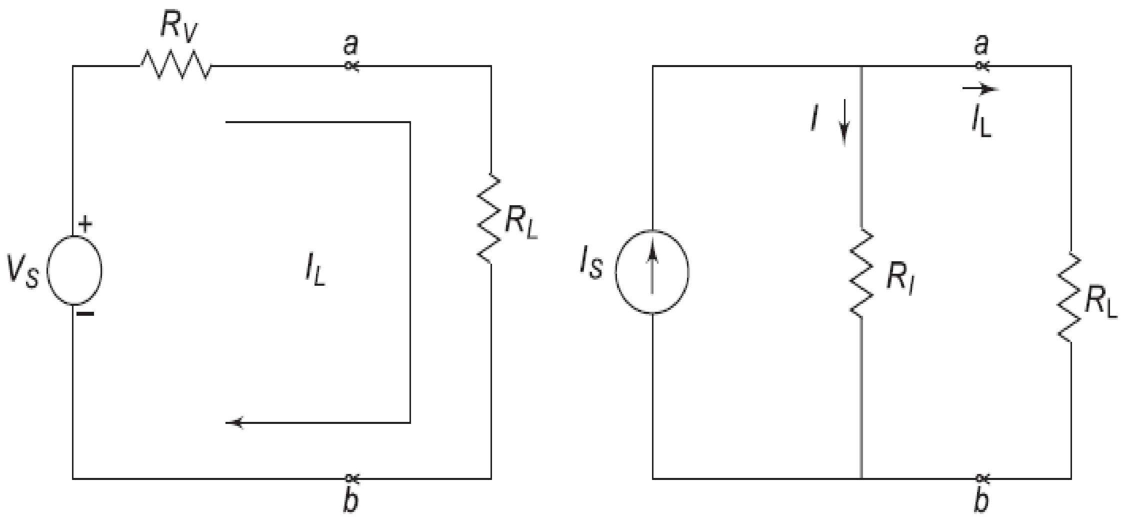
$$\therefore \text{Current } I_5 = \frac{-8.42 - 10}{5} = -3.68 \text{ A (current towards node 3) i.e. the current flows towards the node 3.}$$

Source Transformation Technique:

In solving networks to find solutions, one may have to deal with energy sources. It has already been discussed in privious section that basically, energy sources are either voltage sources or current sources. Sometimes it is necessary to convert a voltage source to a current source and vice-versa. Any practical voltage source consists of an ideal voltage source in series with an internal resistance. Similarly, a practical current source consists of an ideal current source in parallel with an internal resistance as shown in Fig. Rv and Ri represent the internal resistances of the voltage source Vs, and current source Is, respectively.



Any source, be it a current source or a voltage source, drives current through its load resistance, and the magnitude of the current depends on the value of the load resistance. Figure below represents a practical voltage source and a practical current source connected to the same load resistance RL.



From Fig. , the load voltage can be calculated by using Kirchhoff's voltage law as

$$V_{ab} = V_s - I_L R_v$$

The open-circuit voltage $V_{OC} = V_s$

The short-circuit current $I_{SC} = \dfrac{V_s}{R_v}$

$$I_L = I_s - I = I_S - \frac{V_{ab}}{R_I}$$

The open-circuit voltage $V_{OC} = I_S R_I$

The short-circuit current $I_{SC} = I_S$

The above two sources are said to be equal, if they produce equal amounts of current and voltage when they are connected to identical load resistances. Therefore, by equating the open-circuit voltages and short- circuit currents of the above two sources, we obtain

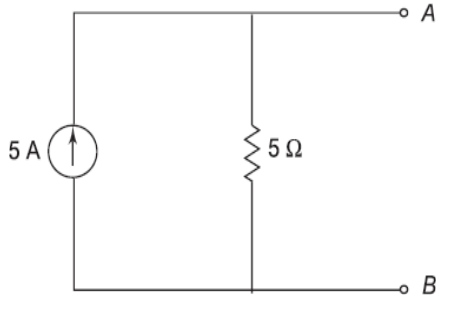
$$V_{OC} = I_s R_I = V_S \quad , \quad I_{SC} = I_S = \frac{V_s}{R_v}$$

It follows that $R_1 = R_V = R_s \;\therefore\; V_s = I_S R_S$

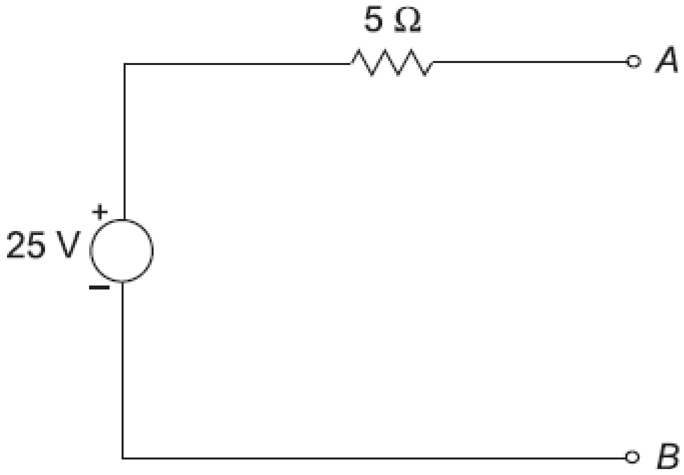
where RS is the internal resistance of the voltage or current source. Therefore, any practical voltage source, having an ideal voltage VS and internal series resistance RS can be replaced by a current source IS =VS /RS in parallel with an internal resistance RS. The reverse transformation is also possible. Thus, a practical current source in parallel with an internal resistance RS can be replaced by a voltage source VS = Is Rs in series with an internal resistance RS.

Example:

Determine the equivalent voltage source for the current source shown in Fig.



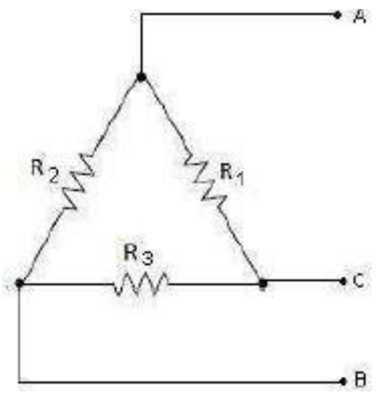
Solution :The voltage across terminals A and B is equal to 25 V. Since the internal resistance for the current source is 5 Ω, the internal resistance of the voltage source is also 5 Ω. The equivalent voltage source is shown in Fig.

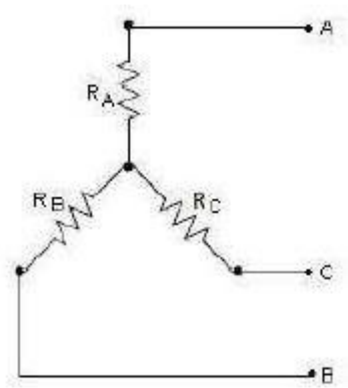


STAR – DELTA TRANSFORMATION:

Star connection and delta connection are the two different methods of connecting three basic elements which cannot be further simplified into series or parallel.

The two ways of representation can have equivalent circuits in either form.





Assume some voltage source across the terminals AB

$$R_{eq} = R_a + R_b$$

$$R_{eq} = R_1(R_2 + R_3)/(R_1 + R_2 + R_3)$$

Therefore $\quad + \quad = R_1(R_2 + R_3)/(R_1 + R_2 + R_3)$ ....(1)

Similarly $\quad R_b + \quad = R_3(R_1 + R_2)/(R_1 + R_2 + R_3)$ ....(2)

$$R_c + R_a = R_2(R_3 + R_1)/(R_1 + R_2 + R_3) \ldots\ldots\ldots\ldots\ldots (3)$$

Subtracting (2) from (1) and adding to (3) ,

$R_a \quad = \quad R_1R_2/(R_1 + R_2 + R_3) \ldots\ldots\ldots\ldots\ldots (4)$

$R_b \quad = \quad R_1R_3/(R_1 + R_2 + R_3) \ldots\ldots\ldots\ldots\ldots (5)$

$R_c \quad = \quad R_2R_3/(R_1 + R_2 + R_3) \ldots\ldots\ldots\ldots\ldots (6)$

A delta connection of $R_1R_2R_3$ can be replaced by an equvivalent star connection with the values from equations (4),(5),(6).

Multiply (4)(5) ; (5)(6) ; (4)(6) and then adding the three we get,

$$R_aR_b \;+\; \;+\; {}_a \qquad = \quad R_1R_2R_3/(R_1 + R_2 + R_3)$$

Dividing LHS by R_a gives R_3 , by R_b gives R_2 , by $R_c$gives R_1.

$R_1 \quad = \quad (R_aR_b + R_bR_c + R_cR_a)$ /

$R_2 \quad = \quad (R_aR_b + R_bR_c + R_cR_a)$ /

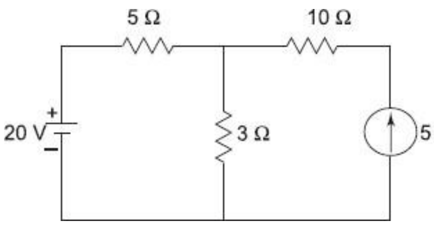
$R_3 \quad = \quad (R_aR_b + R_bR_c + R_cR_a)$ /

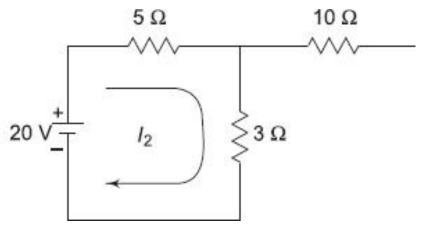
Superposition theorem

The superposition theorem states that in any linear network containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non-operative; that is, while considering the effect of individual sources, other ideal voltage sources and ideal current sources in the network are replaced by short circuit and open circuit across their terminals. This theorem is valid only for linear systems. This theorem can be better understood with a numerical example.

Consider the circuit which contains two sources as shown in below figure

Now let us find the current passing through the 3 Ω resistor in the circuit. According to the superposition theorem, the current I2 due to the 20 V voltage source with 5 A source open circuited = 20/(5 + 3) = 2.5 A(see Fig.)





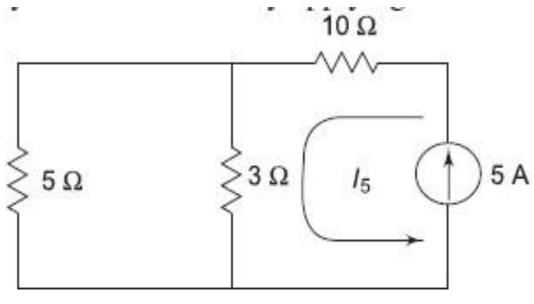
The current I_5 due to the 5 A source with the 20 V source short circuited is

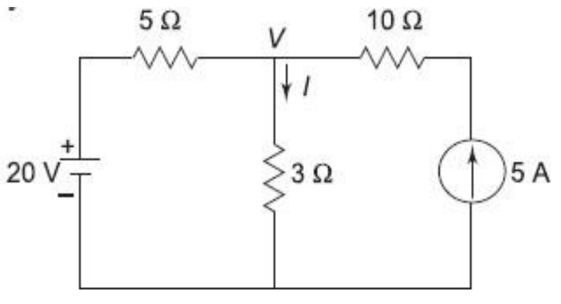
$$I_5 = 5\times\frac{5}{(3+5)} = 3.125\,\text{A}$$

The current passing through 3Ω Resistor

(2.5+3.125)=5.625A

Let us verify the above result by applying nodal analysis.





The current passing in the 3 Ω resistor due to both sources should be5.625 A. Applying nodal analysis to above figure, we have

$$\frac{V-20}{5}+\frac{V}{3}=5$$

$$V\left[\frac{1}{5}+\frac{1}{3}\right]=5+4$$

$$V=9\times\frac{15}{8}=16.875\ \text{V}$$

The current passing through the 3 Ω resistor is equal to $V/3$,

$$\text{i.e.}\quad I=\frac{16.875}{3}=5.625\,\text{A}$$

So the superposition theorem is verified. Let us now examine the power responses.

Power dissipated in the 3Ω resistor due to the voltage source acting alone

$$P_{20} = (I_2)^2R = (2.5)^2\,3 = 18.75\ \text{W}$$

Power dissipated in the 3Ω resistor due to the current source acting alone

$$P_5 = (I_5)^2R = (3.125)^2\,3 = 29.29\ \text{W}$$

Power dissipated in the 3 Ω resistor when both the sources are acting simultaneously is given by

$$P = (5.625)^2 \times 3 = 94.92\ \text{W}$$

From the above results, the superposition of $P20$ and $P5$ gives

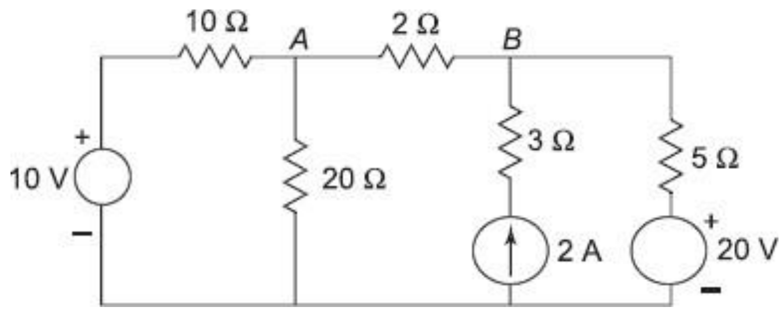
$$P_{20} + P_5 = 48.04\ \text{W}$$

Which is not equal to P=94.92W

We can, therefore, state that the superposition theorem is not valid for power responses. It is applicable only for computing voltage and current responses.

Example:

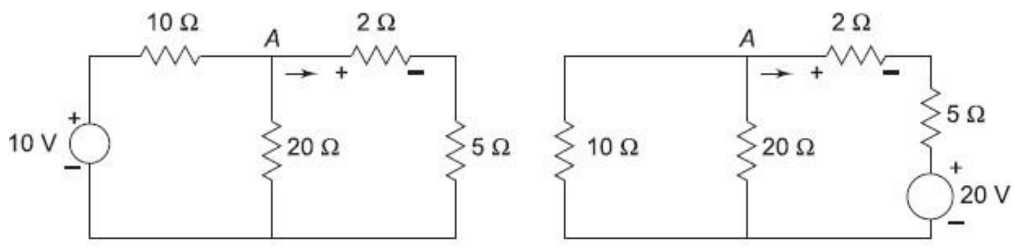
Find the voltage across the 2 Ω resistor in Fig. by using the superposition theorem.



Solution :Let us find the voltage across the 2 Ω resistor due to individual sources. The algebraic sum of these voltages gives the total voltage across the 2 Ω resistor.

Our first step is to find the voltage across the 2 Ω resistor due to the 10 V source, while other sources are set equal to zero.

The circuit is redrawn as shown in Fig. (a).



(a) (b)

Assuming a voltage V at the node 'A' as shown in Fig. (a), the current equation is

$$\frac{V-10}{10}+\frac{V}{20}+\frac{V}{7}=0$$

$$V\,[0.1 + 0.05 + 0.143] = 1$$

$$\text{or} \quad V = 3.41\text{ V}$$

The voltage across the 2 Ω resistor due to the 10 V source is

$$V_2=\frac{V}{7}\times 2=0.97\text{ V}$$

Our second step is to find out the voltage across the 2 Ω resistor due to the 20 V source, while the other

sources are set equal to zero. The circuit is redrawn as shown in Fig. (b)
Assuming voltage V at the node A as shown in Fig. (b), the current equation is

$$\frac{V-20}{7}+\frac{V}{20}+\frac{V}{10}=0$$

$$V[0.143 + 0.05 + 0.1] = 2.86$$

$$\text{or} \quad V=\frac{2.86}{0.293}=9.76 \text{ V}$$

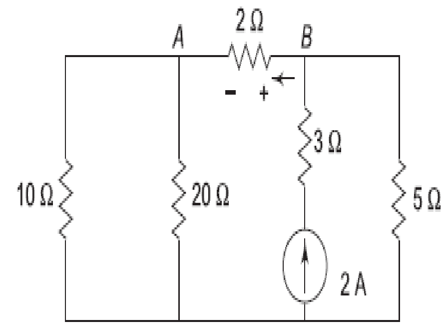


Figure C

The voltage across the 2Ω resistor due to the 20 V source is

$$V_2=\left(\frac{V-20}{7}\right)\times 2=-2.92 \text{ V}$$

The last step is to find the voltage across the 2 Ω resistor due to the 2 A current source, while the other sources are set equal to zero. The circuit is redrawn as shown in Fig. (c).

$$\text{The current in the 2 } \Omega \text{ resistor} = 2\times\frac{5}{5+8.67}$$

$$=\frac{10}{13.67}=0.73 \text{ A}$$

$$\text{The voltage across the 2 } \Omega \text{ resistor} = 0.73 \times 2 = 1.46 \text{ V}$$

The algebraic sum of these voltages gives the total voltage across the 2 Ω resistor in the network

V = 0.97 – 2.92 – 1.46 5 = -3.41 V

The negative sign of the voltage indicates that the voltage at ‘A’ is negative.

Thevenin’s theorem:

In many practical applications, it is always not necessary to analyse the complete circuit; it requires that the voltage, current, or power in only one resistance of a circuit be found. The use of this theorem provides a simple, equivalent circuit which can be substituted for the original network. Thevenin’s theorem states that any two terminal linear network having a number of voltage current sources and resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a resistance, where the value of the voltage source is equal to the open-circuit voltage across the two terminals of the network, and resistance is equal to the equivalent resistance measured between the terminals with all the energy sources are replaced by their internal resistances. According to Thevenin’s theorem, an equivalent circuit can be found to replace the circuit in below Fig.

In the circuit, if the 24 Ω load resistance is connected to Thevenin's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experienced in the original circuit. To verify this, let us find the current passing through the 24 Ω resistance due to the original circuit.

$$I_{24} = I_T \times \frac{12}{12+24}$$

$$\text{where} \quad I_T = \frac{10}{2+(12\|24)} = \frac{10}{10} = 1 \text{ A}$$

$$\therefore \quad I_{24} = 1 \times \frac{12}{12+24} = 0.33 \text{ A}$$

The voltage across the 24 Ω resistor = 0.33 × 24 = 7.92 V.

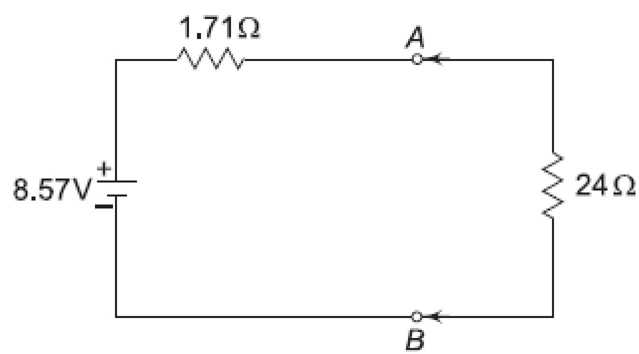
The Thevenin voltage is equal to the open-circuit voltage across the terminals 'AB', i.e. the voltage across the 12 Ω resistor. When the load resistance is disconnected from the circuit, the Thevenin voltage

$$V_{Th} = 10 \times \frac{12}{14} = 8.57 \text{ V}$$

The resistance into the open-circuit terminals is equal to the Thevenin resistance

$$R_{Th} = \frac{12 \times 2}{14} = 1.71 \ \Omega$$

Thevenin's equivalent circuit is shown in Fig.



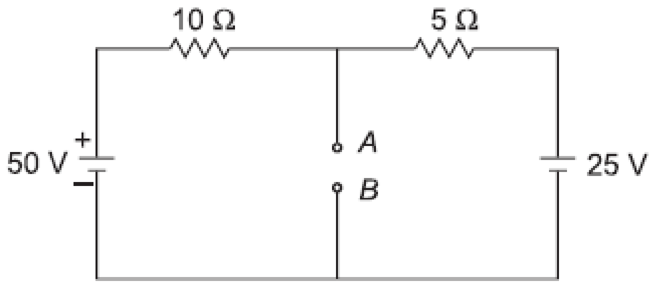
Now let us find the current passing through the 24 Ω resistance and voltage across it due to Thevenin's equivalent circuit.

$$I_{24} = \frac{8.57}{24+1.71} = 0.33 \text{ A}$$

The voltage across the 24 Ω resistance is equal to 7.92 V. Thus, it is proved that RL (= 24 Ω) has the same values of current and voltage in both the original circuit and Thevenin's equivalent circuit.

Example:

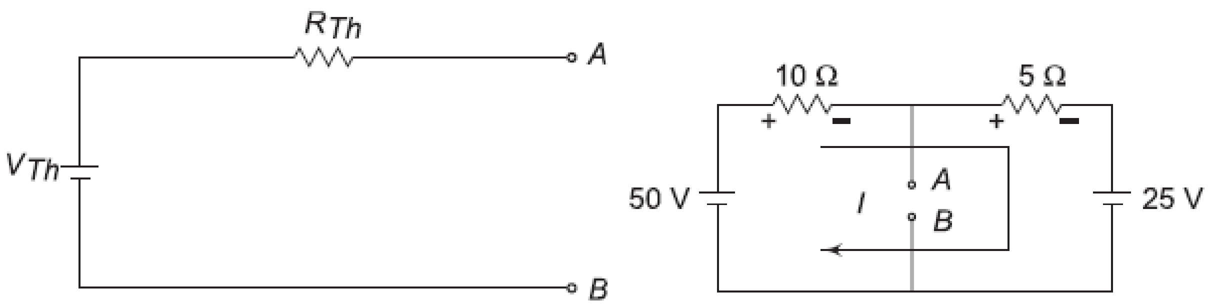
Determine the Thevenin's equivalent circuit across 'AB' for the given circuit shown in Fig.



Solution: The complete circuit can be replaced by a voltage source in series with a resistance as shown in Fig.

Where VTh is the voltage across terminals AB, and RTh is the resistance seen into the terminals AB.

To solve for VTh, we have to find the voltage drops around the closed path as shown in Fig.



(a) (b)

We have $50 - 25 = 10I + 5I$

or $15I = 25$

$$\therefore \quad I = \frac{25}{15} = 1.67\,\text{A}$$

Voltage across 10 Ω = 16.7 V

Voltage drop across 5 Ω = 8.35 V

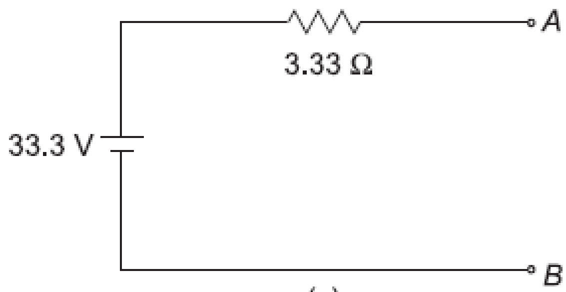
or $V_{Th} = V_{AB} = 50 - V_{10}$

$= 50 - 16.7 = 33.3\ \text{V}$

To find RTh, the two voltage sources are removed and replaced with short circuit. The resistance at terminals AB then is the parallel combination of the 10 Ω resistor and 5 Ω resistor; or

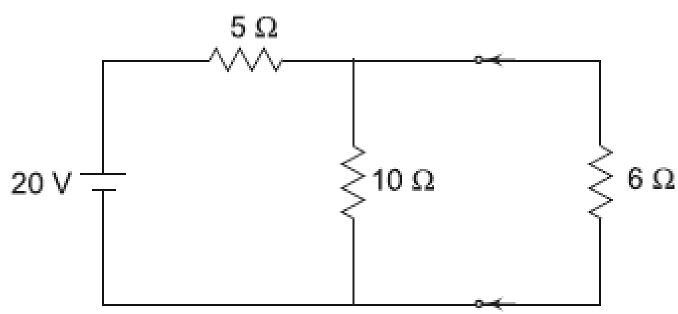
$$R_{Th} = \frac{10 \times 5}{15} = 3.33\ \Omega$$

Thevenin's equivalent circuit is shown in Fig.(c).



(c)

Norton's Theorem: Another method of analysing the circuit is given by Norton's theorem, which states that any two terminal linear network with current sources, voltage sources and resistances can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance. The value of the current source is the short-circuit current between the two terminals of the network and the resistance is the equivalent resistance measured between the terminals of the network with all the energy sources are replaced by their internal resistance.



According to Norton's theorem, an equivalent circuit can be found to replace the circuit in Fig.

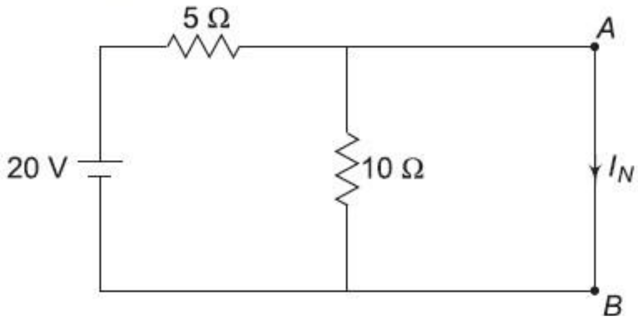
In the circuit, if the load resistance of 6 Ω is connected to Norton's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experiences in the original circuit. To verify this, let us find the current passing through the 6 Ω resistor due to the original circuit.

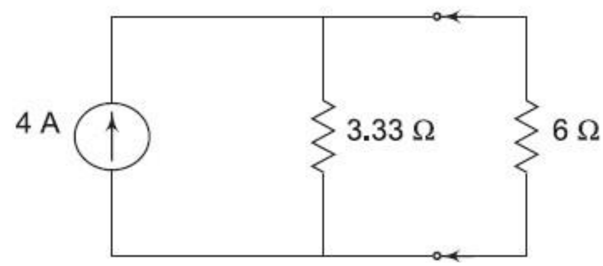
$$I_6 = I_T \times \frac{10}{10+6}$$

$$\text{where} \quad I_T = \frac{20}{5+(10\|6)} = 2.285\,\text{A}$$

$$\therefore \quad I_6 = 2.285 \times \frac{10}{16} = 1.43\,\text{A}$$

$$R_N = \frac{5 \times 10}{15} = 3.33\ \Omega$$





i.e. the voltage across the 6 Ω resistor is 8.58 V. Now let us find Norton's equivalent circuit. The magnitude of the current in the Norton's equivalent circuit is equal to the current passing through short-circuited terminals as shown in below figure.

$$\text{Here,} \quad I_N = \frac{20}{5} = 4\ \text{A}$$

Norton's resistance is equal to the parallel combination of both the 5 Ω and 10 Ω resistors.

The Norton's equivalent source is shown in Fig above figure.

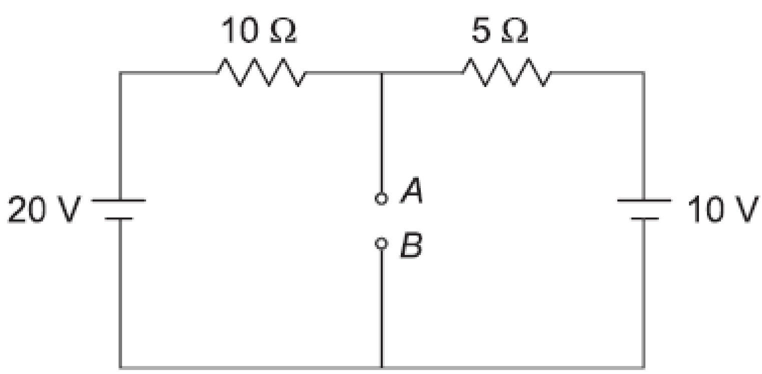
Now let us find the current passing through the 6 Ω resistor and the voltage across it due to Norton's equivalent circuit.

$$I_6 = 4 \times \frac{3.33}{6 + 3.33} = 1.43\ \text{A}$$

The voltage across the 6 Ω resistor = 1.43 × 6 = 8.58 V

Thus, it is proved that RL (= 6 Ω) has the same values of current and voltage in both the original circuit and Norton's equivalent circuit.

Example: Determine Norton's equivalent circuit at terminals AB for the circuit shown in Fig.

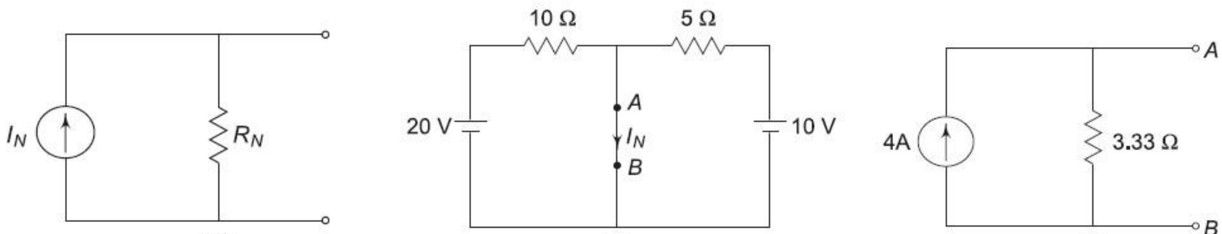


Solution The complete circuit can be replaced by a current source in parallel with a single resistor as shown in below figure(a), where IN is the current passing through the short circuited output terminals AB and RN is the resistance as seen into the output terminals.

To solve for IN , we have to find the current passing through the terminals AB as shown in below figure(b). From Fig, the current passing through the terminals AB is 4 A. The resistance at terminals AB is The parallel combination of the 10 Ω resistor and the 5 Ω resistor,

or $$R_N = \frac{10 \times 5}{10+5} = 3.33\ \Omega$$

Norton's equivalent circuit is shown in Fig. 1.75(c).



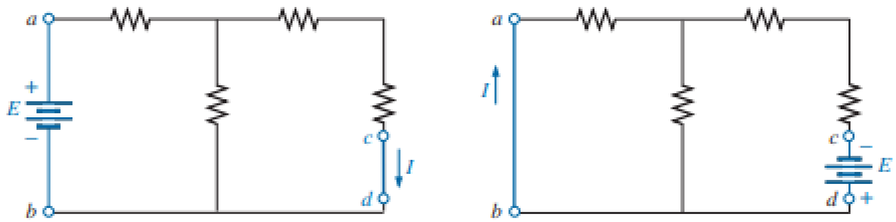
(a) (b) (c)

Reciprocity Theorem:

The reciprocity theorem is applicable only to single-source networks. It is, therefore, not a theorem used in the analysis of multisource networks described thus far. The theorem states the following:

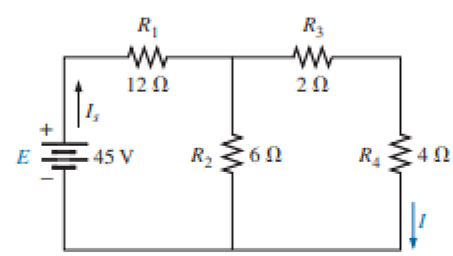
The current I in any branch of a network due to a single voltage source E anywhere else in the network will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.

In other words, the location of the voltage source and the resulting current may be interchanged without a change in current. The theorem requires that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position.



(a) (b)

In the representative network in Fig. (a), the current I due to the voltage source E was determined. If the position of each is interchanged as shown in Fig. (b), the current I will be the same value as indicated. To demonstrate the validity of this statement and the theorem, consider the network in Fig., in which values for the elements of Fig. (a) have been assigned.



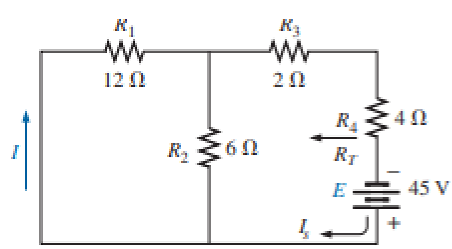
The total resistance for Fig .a

$$R_T = R_1 + R_2 \| (R_3 + R_4) = 12\ \Omega + 6\ \Omega \| (2\ \Omega + 4\ \Omega)$$
$$= 12\ \Omega + 6\ \Omega \| 6\ \Omega = 12\ \Omega + 3\ \Omega = 15\ \Omega$$

and $$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{15\ \Omega} = 3\text{ A}$$

with $$I = \frac{3\text{ A}}{2} = \mathbf{1.5\ A}$$

For Fig.b



$$R_T = R_4 + R_3 + R_1 \| R_2$$
$$= 4\ \Omega + 2\ \Omega + 12\ \Omega \| 6\ \Omega = 10\ \Omega$$

and $$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{10\ \Omega} = 4.5\text{ A}$$

so that $$I = \frac{(6\ \Omega)(4.5\text{ A})}{12\ \Omega + 6\ \Omega} = \frac{4.5\text{ A}}{3} = \mathbf{1.5\ A}$$

Steady state and Trasient Response:

A circuit having constant sources is said to be in steady state if the currents and voltages do not change with time. Thus, circuits with currents and voltages having constant amplitude and constant frequency, sinusoidal functions are also considered to be in a steady state. That means that the amplitude or frequency of a sinusoid never changes in a steady state circuit.

In a network containing energy storage elements, with change in excitation, the currents and voltages change from one state to another state. The behaviour of the voltage or current when it is changed from one state to another is called the transient state. The time taken for the circuit to change from one steady state to another steady state is called the transient time. The application of KVL and KCL to circuits containing energy storage elements results in differential, rather than algebraic, equations. When we consider a circuit containing storage elements which are independent of the sources, the response depends upon the nature of the circuit and is called the natural response. Storage elements deliver their energy to the resistances. Hence, the response changes with time, gets saturated after some time, and is referred to as the transient response. When we consider sources acting on a circuit, the response depends on the nature of the source or sources. This response is called forced response. In other words, the complete response of a circuit consists of two parts: the forced response and the transient response. When we consider a differential equation, the complete solution consists of two parts: the complementary function and the particular solution. The complementary function dies out after a short interval, and is referred to as the transient response or source free response. The particular solution is the steady state response, or the forced response. The first step in finding the complete solution of a circuit is to form a differential equation for the circuit. By obtaining the differential equation, several methods can be used to find out the complete solution.

DC response of an RL circuit:

Consider a circuit consisting of a resistance and inductance as shown in Fig. 1.76. The inductor in the circuit is initially uncharged and is in series with the resistor. When the switch S is closed, we can find the complete solution for the current. Application of Kirchhoff's voltage law to the circuit results in the following differential equation.

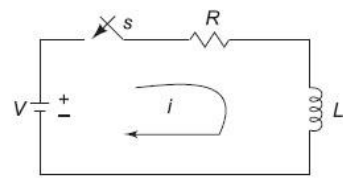


Fig. 1.76

$$V = Ri + L\frac{di}{dt} \tag{1.66}$$

$$\text{or} \quad \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \tag{1.67}$$

In the above equation, the current i is the solution to be found and V is the applied constant voltage. The voltage V is applied to the circuit only when the switch S is closed. The above equation is a linear differential equation of first order. Comparing it with a non-homogeneous differential equation

$$\frac{dx}{dt} + Px = K \tag{1.68}$$

whose solution is

$$x = e^{-pt}\int Ke^{+Pt}dt + ce^{-Pt} \tag{1.69}$$

where c is an arbitrary constant. In a similar way, we can write the current equation as

$$i = ce^{-(R/L)t} + e^{-(R/L)t}\int \frac{V}{L}e^{(R/L)t}dt$$

$$\therefore \quad i = ce^{-(R/L)t} + \frac{V}{R} \tag{1.70}$$

To determine the value of c in Eq. (1.70), we use the initial conditions. In the circuit shown in Fig. 1.76, the switch S is closed at $t = 0$. At $t = 0^-$, i.e. just before closing the switch S, the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at $t = 0^+$ just after the switch is closed, the current remains zero.

Thus, at $t = 0, i = 0$

Substituting the above condition in Eq. (1.71), we have

$$0 = c + \frac{V}{R}$$

Hence, $$c = -\frac{V}{R}$$

Substituting the value of c in Eq. (1.71), we get

$$i = \frac{V}{R} - \frac{V}{R}\exp\left(-\frac{R}{L}t\right)$$

$$i = \frac{V}{R}\left(1 - \exp\left(-\frac{R}{L}t\right)\right) \tag{1.71}$$

Equation (1.71) consists of two parts, the steady state part V/R, and the transient part $(V/R)e^{-(R/L)t}$. When the switch S is closed, the response reaches a steady-state value after a time interval as shown in Fig. 1.77.

Here, the transition period is defined as the time taken for the current to reach its final or steady state value from its initial value. In the transient part of the solution, the quantity L/R is important in describing the curve since L/R is the time required for the current to reach from its initial value of zero to the final value V/R. The time constant of a function $\frac{V}{R}e^{-\left(\frac{R}{L}\right)t}$ is the time at which the exponent of e is unity, where e is the base of the natural logarithms. The term L/R is called the *time constant* and is denoted by τ

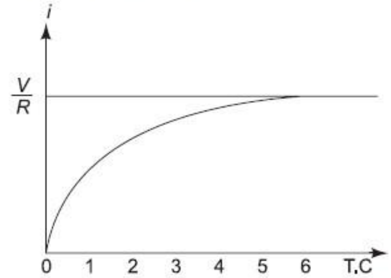


Fig. 1.77

$$\therefore \quad \tau = \frac{L}{R} \text{ sec}$$

$\therefore$ the transient part of the solution is

$$i = -\frac{V}{R}\exp\left(-\frac{R}{L}t\right) = \frac{V}{R}e^{-t/\tau}$$

At one TC, i.e. at one time constant, the transient term reaches 36.8 percent of its initial value.

$$i(\tau) = -\frac{V}{R}e^{-t/\tau} = -\frac{V}{R}e^{-1} = -0.368\frac{V}{R}$$

Similarly,

$$i(2\tau) = -\frac{V}{R}e^{-2} = -0.135\frac{V}{R}$$

$$i(3\tau) = -\frac{V}{R}e^{-3} = -0.0498\frac{V}{R}$$

$$i(5\tau) = -\frac{V}{R}e^{-5} = -0.0067\frac{V}{R}$$

After 5 TC, the transient part reaches more than 99 percent of its final value.
In Fig. 1.76, we can find out the voltages and powers across each element by using the current.
Voltage across the resistor is

$$v_R = Ri = R\times\frac{V}{R}\left[1-\exp\left(-\frac{R}{L}t\right)\right]$$

$$\therefore \quad v_R = V\left[1-\exp\left(-\frac{R}{L}t\right)\right]$$

Similarly, the voltage across the inductance is

$$v_L = L\frac{di}{dt}$$

$$= L\frac{V}{R}\times\frac{R}{L}\exp\left(-\frac{R}{L}t\right) = V\exp\left(-\frac{R}{L}t\right)$$

The responses are shown in Fig. 1.78.

Power in the resistor is

$$p_R = v_R i = V\left\{1-\exp\left(-\frac{R}{L}t\right)\right\}\left\{1-\exp\left(-\frac{R}{L}t\right)\right\}\frac{V}{R}$$

$$=\frac{V^2}{R}\left\{1-2\exp\left(-\frac{R}{L}t\right)+\exp\left(-\frac{2R}{L}t\right)\right\}$$

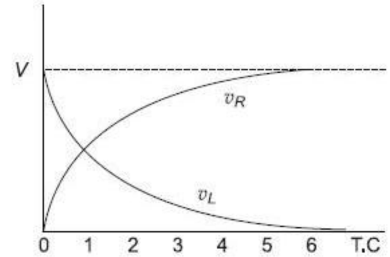


Fig. 1.78

Power in the inductor is

$$p_L = v_L i = V\exp\left(-\frac{R}{L}t\right)\times\frac{V}{R}\left\{1-\exp\left(-\frac{R}{L}t\right)\right\}$$

$$=\frac{V^2}{R}\left\{\exp\left(-\frac{R}{L}t\right)-\exp\left(-\frac{2R}{L}t\right)\right\}$$

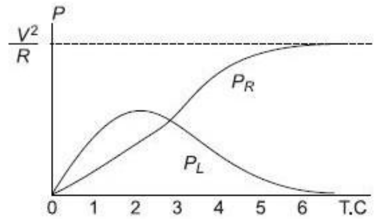


Fig. 1.79

The responses are shown in Fig. 1.79.

Example:

A series RL circuit with $R = 30\ \Omega$ and $L = 15$ H has a constant voltage $V = 60$ V applied at $t = 0$ as shown in Fig. 1.80. Determine the current i, the voltage across resistor and the voltage across the inductor.

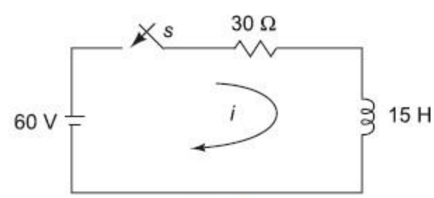


Fig. 1.80

Solution By applying Kirchhoff's voltage law, we get

$$15\frac{di}{dt}+30i=60$$

$$\therefore\quad \frac{di}{dt}+2i=4$$

The general solution for a linear differential equation is

$$i = ce^{-Pt}+e^{-Pt}\int Ke^{Pt}\,dt$$

where $P = 2$, $K = 4$

$$\therefore\quad i = ce^{-2t}+e^{-2t}\int 4e^{2t}\,dt$$

$$\therefore\quad i = ce^{-2t}+2$$

At $t = 0$, the switch S is closed.

Since the inductor never allows sudden changes in currents, at $t = 0^+$, the current in the circuit is zero.

Therefore, at $t = 0^+$, $i = 0$

$$\therefore\quad 0 = c + 2$$

$$\therefore\quad c = -2$$

Substituting the value of c in the current equation, we have

$$i = 2(1 - e^{-2t})\ \text{A}$$

Voltage across the resistor $v_R = iR$

$$= 2(1 - e^{-2t}) \times 30 = 60\,(1 - e^{-2t})\text{ V}$$

Voltage across the inductor $v_L = L\dfrac{di}{dt}$

$$= 15 \times \frac{d}{dt} 2(1 - e^{-2t}) = 30 \times 2e^{-2t} = 60e^{-2t}\text{ V}$$

DC response of an RC circuit:

Consider a circuit consisting of resistance and capacitance as shown in Fig. 1.81. The capacitor in the circuit is initially uncharged, and is in series with a resistor. When the switch S is closed at $t = 0$, we can determine the complete solution for the current. Application of the Kirchhoff's voltage law to the circuit results in the following differential equation.

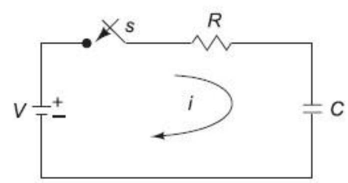


Fig. 1.81

$$V = Ri + \frac{1}{C}\int i\,dt \tag{1.72}$$

By differentiating the above equation, we get

$$0 = R\frac{di}{dt} + \frac{i}{C} \tag{1.73}$$

or
$$\frac{di}{dt} + \frac{1}{RC}i = 0 \tag{1.74}$$

Equation (1.74) is a linear differential equation with only the complementary function. The particular solution for the above equation is zero. The solution for this type of differential equation is

$$i = ce^{-t/RC} \tag{1.75}$$

Here, to find the value of c, we use the initial conditions.

In the circuit shown in Fig. 1.81, the switch S is closed at $t = 0$. Since the capacitor never allows sudden changes in voltage, it will act as a short circuit at $t = 0^+$. So, the current in the circuit at $t = 0^+$ is V/R

$\therefore$ At $t = 0$, the current $i = \dfrac{V}{R}$

Substituting this current in Eq. (1.75), we get

$$\frac{V}{R} = c$$

$\therefore$ the current equation becomes

$$i = \frac{V}{R}e^{-t/RC} \tag{1.76}$$

When the switch S is closed, the response decays with time as shown in Fig. 1.82.

In the solution, the quantity RC is the time constant, and is denoted by τ,

where $\tau = RC$ seconds

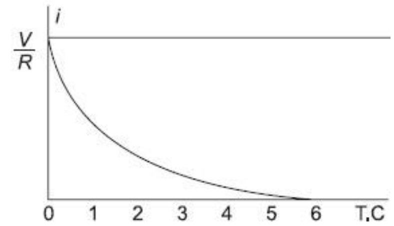


Fig. 1.82

After 5 TC, the curve reaches 99 percent of its final value. In Fig. 1.81, we can find out the voltage across each element by using the current equation.

Voltage across the resistor is

$$v_R = Ri = R \times \frac{V}{R} e^{-(1/RC)t}; v_R = Ve^{-t/RC}$$

Similarly, voltage across the capacitor is

$$\begin{aligned} v_C &= \frac{1}{C}\int i\, dt \\ &= \frac{1}{C}\int \frac{V}{R} e^{-t/RC} dt \\ &= -\left(\frac{V}{RC} \times RC\, e^{-t/RC}\right) + c = -Ve^{-t/RC} + c \end{aligned}$$

At $t = 0$, voltage across capacitor is zero

$$\therefore \quad c = V$$

$$\therefore \quad v_C = V(1 - e^{-t/RC})$$

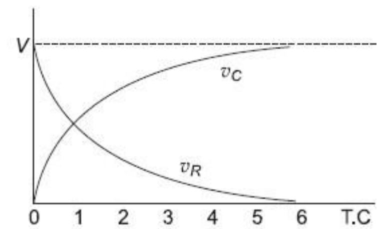


Fig. 1.83

The responses are shown in Fig. 1.83.

Power in the resistor

$$p_R = v_R i = Ve^{-t/RC} \times \frac{V}{R} e^{-t/RC} = \frac{V^2}{R} e^{-2t/RC}$$

Power in the capacitor

$$\begin{aligned} p_C &= v_C i = V(1 - e^{-t/RC}) \frac{V}{R} e^{-t/RC} \\ &= \frac{V^2}{R}(e^{-t/RC} - e^{-2t/RC}) \end{aligned}$$

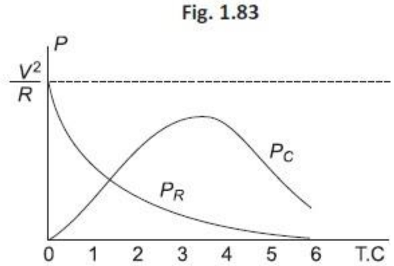


Fig. 1.84

The responses are shown in Fig. 1.84.

Example:

A series RC circuit consists of a resistor of 10 Ω and a capacitor of 0.1 F as shown in Fig. 1.85. A constant voltage of 20 V is applied to the circuit at t = 0. Obtain the current equation. Determine the voltages across the resistor and the capacitor.

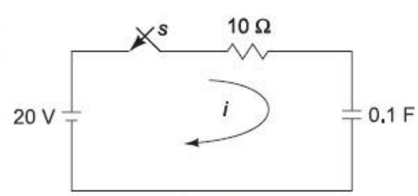


Fig. 1.85

Solution By applying Kirchhoff's law, we get

$$10i + \frac{1}{0.1}\int i\, dt = 20$$

Differentiating with respect to t, we get

$$10\frac{di}{dt}+\frac{i}{0.1}=0$$

$$\therefore \quad \frac{di}{dt}+i=0$$

The solution for the above equation is $i = ce^{-t}$

At $t = 0$, the switch S is closed. Since the capacitor does not allow sudden changes in the voltage, the current in the circuit is $i = V/R = 20/10 = 2$ A.

At $t = 0$, $i = 2$ A.

$\therefore$ the current equation $i = 2e^{-t}$

Voltage across the resistor is $v_R = i \times R = 2e^{-t} \times 10 = 20e^{-t}$ V

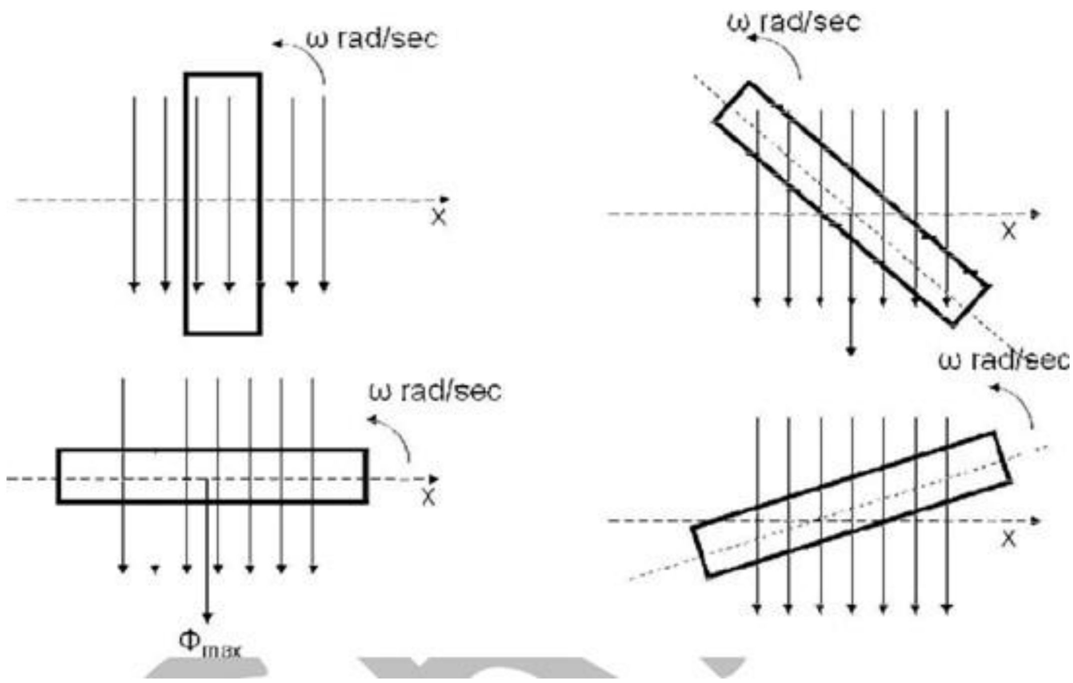
Voltage across the capacitor is $v_C = V\left(1-e^{-\frac{t}{RC}}\right)$

$= 20(1 - e^{-t})$ V

UNIT-2

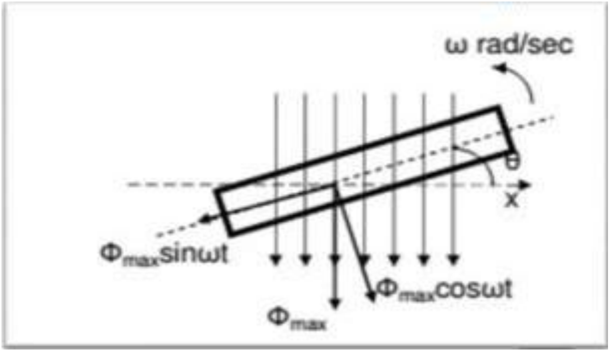
Generation of sinusoidal AC voltage

Consider a rectangular coil of N turns placed in a uniform magnetic field as shown in the figure. The coil is rotating in the anticlockwise direction at an uniform angular velocity of ω rad/sec.



When the coil is in the vertical position, the flux linking the coil is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero. When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil according to **Faradays Law**. When the coil reaches the horizontal position, the flux linking the coil is maximum, and hence the emf induced is also maximum. When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces. Next when the coil comes to the vertical position, the emf induced becomes zero. After that the same cycle repeats and the emf is induced in the opposite direction. When the coil completes one complete revolution, one cycle of AC voltage is generated.

The generation of sinusoidal AC voltage can also be explained using mathematical equations. Consider a rectangular coil of N turns placed in a uniform magnetic field in the position shown in the figure. The maximum flux linking the coil is in the downward direction as shown in the figure. This flux can be divided into two components, one component acting along the plane of the coil $\Phi_{max}\sin\omega t$ and another component acting perpendicular to the plane of the coil $\Phi_{max}\cos\omega t$.



The component of flux acting along the plane of the coil does not induce any flux in the coil. Only the component acting perpendicular to the plane of the coil ie $\Phi_{max}\cos\omega t$ induces an emf in the coil.

$$\emptyset = \emptyset_{max}\cos(\omega t)$$

$$e = -N\frac{d\emptyset}{dt}$$

$$e = -N\frac{d(\emptyset_{max}\cos(\omega t))}{dt}$$

$$e = N\emptyset_{max}\omega * \sin(\omega t)$$

$$e = E_{max}\sin(\omega t)$$

Hence the emf induced in the coil is a sinusoidal emf. This will induce a sinusoidal current in the circuit given by

$$i = i_m sin\,(\omega t)$$

Where

i *is instantaneous value,*
i_m *maximum value*
ω *is angular velocity*

Angular Frequency (ω)

Angular frequency is defined as the number of radians covered in one second(ie the angle covered by the rotating coil). The unit of angular frequency is rad/sec.

$$\omega = \frac{2\pi f}{T}$$

Problem-An alternating current i is given by $i = 141.4\, sin\,(314t)$

Find i) The maximum value
ii) Frequency
iii) Time Period
iv) The instantaneous value when $t = 3ms$ $i = 141.4\, sin(314t)$.

Solution:

$i = i_m sin\,(\omega t)$________________(1)

Compare given equation with eq-1,

$$Maximum\ value\ i_m = 141.4Volts$$
$$\omega = 314\ raad/sec$$
$$f = \omega/2\pi = 50Hz$$
$$T = 1/f = 0.02sec.$$
$$t = 3msec\ ,$$
$$i\ =\ 141.4\ sin(\ 314 * 3 * 10^{-3}) = 114.35A$$

Explain about phasor, and Lead and lagging ?

Phasor Representation:

An alternating quantity can be represented using

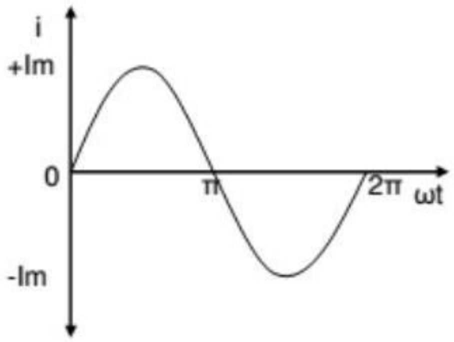
i) Waveform

ii) Equations

iii) Phasor

A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor.** A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity

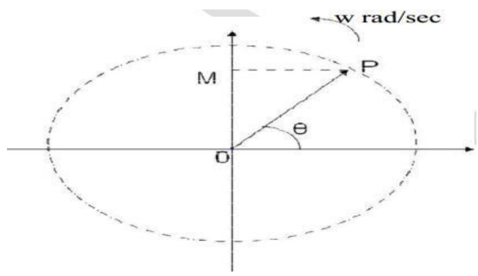
The waveform and equation representation of an alternating current is as shown. This sinusoidal quantity can also be represented using phasors.



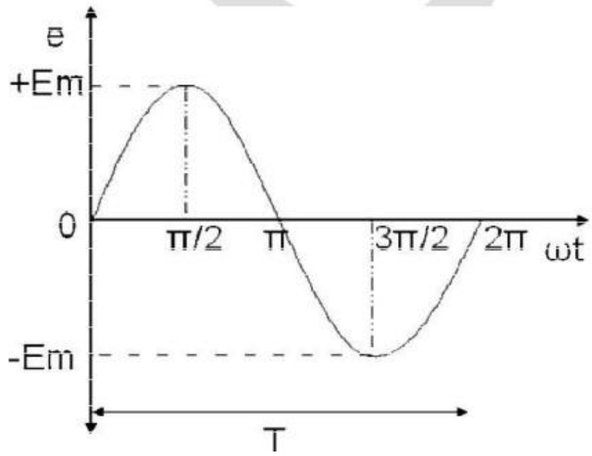
$$i = I_m \sin \omega t$$

In phasor form the above wave is written as $\bar{I} = I_m \angle 0^0$

Draw a line OP of length equal to I_m. This line OP rotates in the anticlockwise direction with a uniform angular velocity ω rad/sec and follows the circular trajectory shown in figure. At any instant, the projection of OP on the y-axis is given by OM=OPsinθ = $I_m$sinωt. Hence the line OP is the phasor representation of the sinusoidal current.

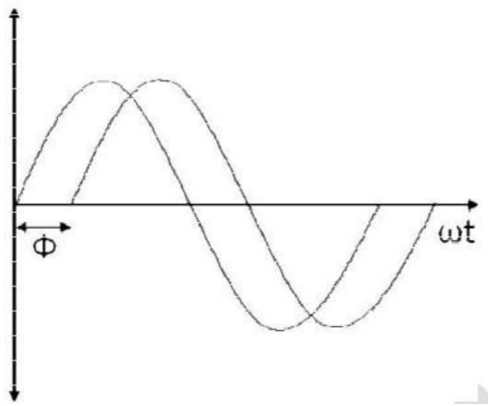


Phase



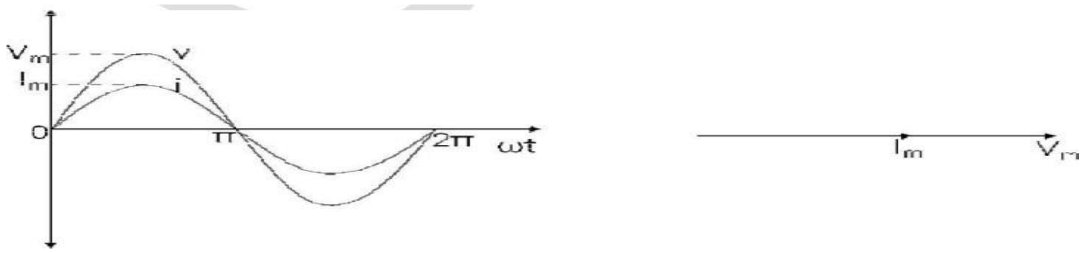
Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference
Phase of $+E_m$ is $\pi/2$rad or T/4 sec
Phase of $-E_m$ is $\pi/2$rad or 3T/4 sec.

Phase Difference



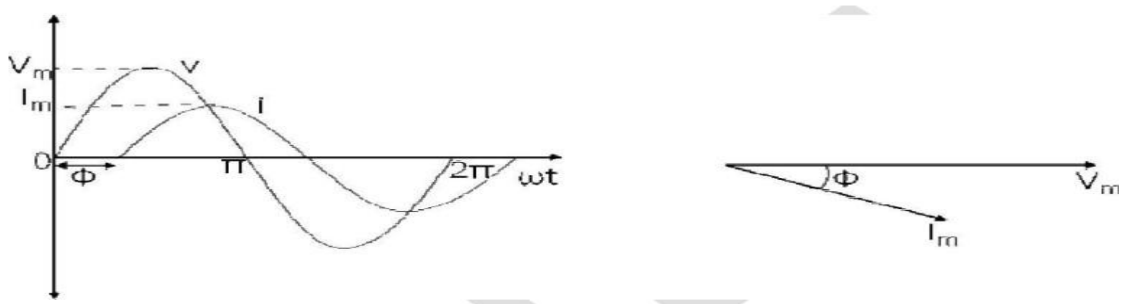
When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

In Phase
Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.



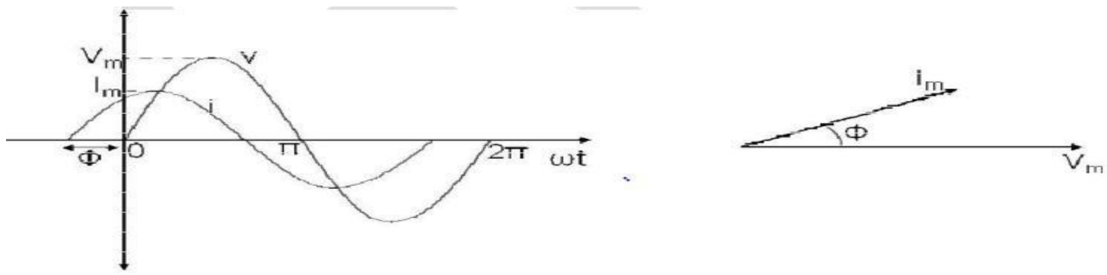
$$v = v_m \sin(\omega t)$$
$$i = i_m \sin(\omega t)$$

Lagging
In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation is as shown.



$$v = v_m \sin(\omega t) => \bar{V} = V_m\angle 0^0$$
$$i = i_m \sin(\omega t - \theta) => \bar{I} = I_m\angle - \theta$$

Leading
In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown.

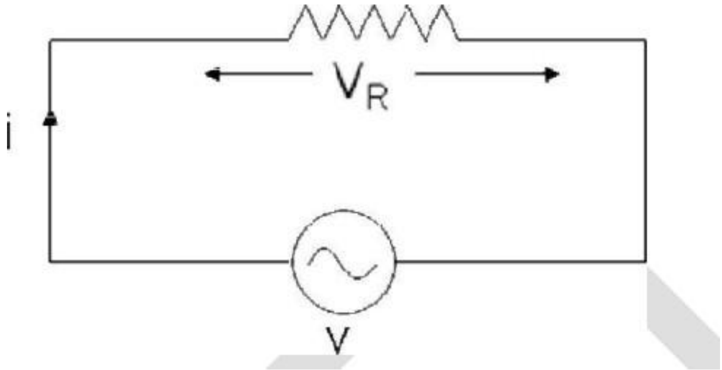


$$v = v_m \sin(\omega t) => \bar{V} = V_m\angle 0^0$$
$$i = i_m \sin(\omega t + \theta) => \bar{I} = I_m\angle \theta^0$$

Exercise Problem: Find sum and draw phasor diagram.
$\boldsymbol{i_1 = 4cos(\omega t + 30^0)A \ and \ i_2 = 5sin(\omega t - 20^0)A,}$

Q: Explain Phasor Relationship with Circuit Elements:

AC circuit with a pure resistance



Consider an AC circuit with a pure resistance R as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin(wt)$$

The current flowing in the circuit is i. The voltage across the resistor is given as V_R which is the same as v.
Using ohms law, we can write the following relations

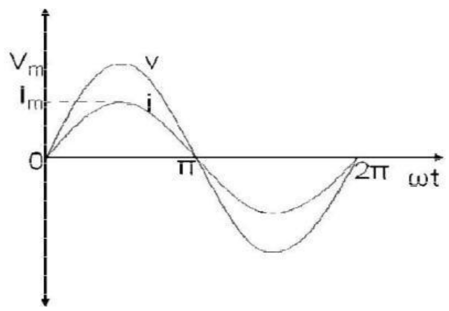
$$i = \frac{v}{R} = \frac{v_m \sin(\omega t)}{R}$$

$$i = i_m \sin(\omega t)$$

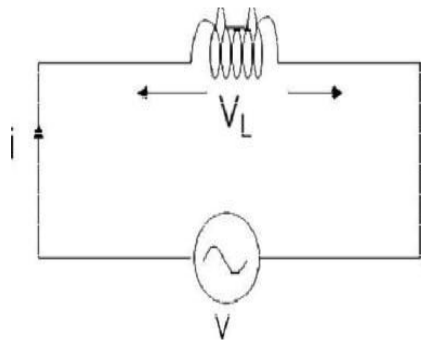
Where

$$i_m = \frac{v_m}{R}$$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase. Hence the voltage and current waveforms and phasors can be drawn as below.



AC circuit with a pure inductance



Consider an AC circuit with a pure inductance L as shown in the figure. The alternating voltage v is given by

$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the inductor is given as V_L which is the same as v.
We can find the current through the inductor as follows.

$$i = L\frac{di}{dt}$$

$$v_m \sin(\omega t) = L\frac{di}{dt}$$

$$di = \frac{v_m}{L} * \sin(\omega t)$$

$$i = \frac{v_m}{L} * \int \sin(\omega t)\, dt$$

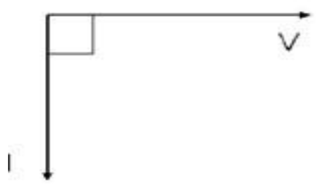
$$i = \frac{v_m}{\omega L} * -\cos(\omega t)$$

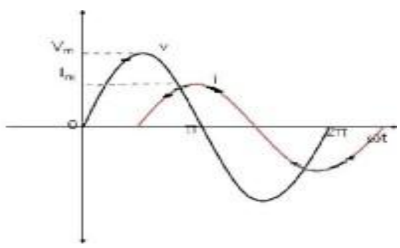
$$i = \frac{v_m}{\omega L} * sin(\omega t - \frac{\pi}{2})$$

$$i = i_m * sin(\omega t - \frac{\pi}{2})$$

Where $i_m = \frac{v_m}{\omega L}$

From equation (1) and (2) we observe that in a pure inductive circuit, the current lags behind the voltage by 90^0. Hence the voltage and current waveforms and phasors can be drawn as below.





Inductive reactance

The inductive reactance X_L is given as

$$X_L = 2\pi f L$$

$$i_m = \frac{v_m}{X_L}$$

$$in\ phasor\ form\ \bar{V} = j\omega L\bar{I}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms

Problem:

The voltage $v = \mathbf{12\cos(60}\boldsymbol{t}\mathbf{+45^0)}$ is applied to a 0.1H inductor. Find the steady-state current through the inductor.

Solution: from equation $\bar{V} = j\omega L\bar{I}$

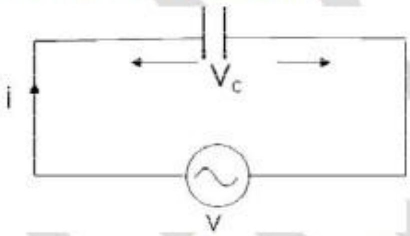
$$\bar{\bar{V}} = 12\angle 45^0\ volts$$

$$\omega = 60\frac{rad}{s}$$

$$\bar{I} = \frac{12\angle 45^0}{j * 60 * 0.1} = \frac{12\angle 45^0}{60 * 0.1\angle 90^0} = 2A$$

$$i = 2\cos(60t - 45^0)\ A$$

AC circuit with a pure capacitance



Consider an AC circuit with a pure capacitance C as shown in the figure. The alternating voltage v is given by

$$v = v_m \sin(\omega t) => \bar{V} = V_m\angle 0^0 \_\_\_\_\_\_\_\_\_(1)$$

The current flowing in the circuit is i. The voltage across the capacitor is given as V_C which is the same as v.

We can find the current through the capacitor as follows

$$q = Cv$$

$$q = Cv_m \sin(\omega t)$$

$$\frac{dq}{dt} = \omega C v_m \cos(\omega t)$$

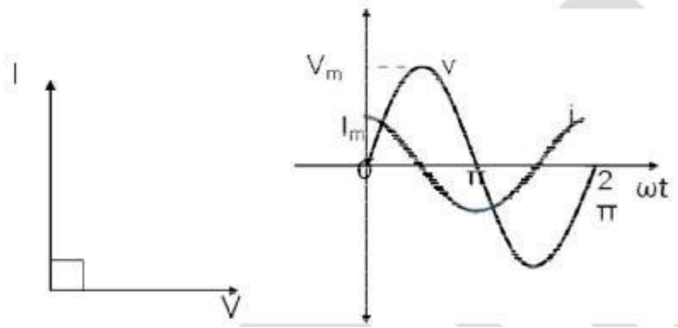
$$i = \omega C v_m \cos(\omega t)$$

$$i = \omega C v_m \sin(\omega t + \frac{\pi}{2}) \_\_\_\_\_\_\_\_\_(2)$$

$$i = i_m \sin(\omega t + \frac{\pi}{2})$$

$$i_m = \omega C v_m => X_C = \frac{v_m}{i_m} = \frac{1}{\omega C}$$

From equation (1) and (2) we observe that in a pure capacitive circuit, the current leads the voltage by 90^0. Hence the voltage and current waveforms and phasors can be drawn as below.



Capacitive reactance

The capacitive reactance X_C is given as

$$X_C = \frac{1}{2\pi f C}$$
$$i_m = \frac{v_m}{X_C}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms

$$\bar{V} = V_m\angle 0^0 = V + j0$$
$$\bar{I} = I_m\angle 90^0 = 0 + j\,I$$

$$\frac{\bar{V}}{\bar{I}} = \frac{V_m\angle 0^0}{I_m\angle 90^0} = \frac{\mathbf{V_m}}{\mathbf{I_m}}\angle -\mathbf{90^0} = \mathbf{X_C}\angle -\mathbf{90^0}$$

Impedance:

relationship between Current and Voltage to different circuit elements are,

1. To Resistor: $R = \frac{\bar{V}}{\bar{I}}$
2. To Capacitor: $X_C = \frac{\bar{V}}{\bar{I}}$
3. To inductor: $X_L = \frac{\bar{V}}{\bar{I}}$

This shows that a pure resistance within an AC circuit produces a relationship between its voltage and current phasors in exactly the same way as it would relate the same resistors voltage and current relationship within a DC circuit.

Reciprocal of impedance is called as *Admittance* and units are mhos.

Impedance Combinations:

Series:

Let us assume are N impedances are connected in series than equivalent impedance is obtained by

Apply KVL in loop

$$V = (I\,Z_1 + IZ_2 + , \ldots \ldots . IZ_n)$$

$$\frac{V}{I} = Z_1 + Z_2 + \cdots Z_n$$

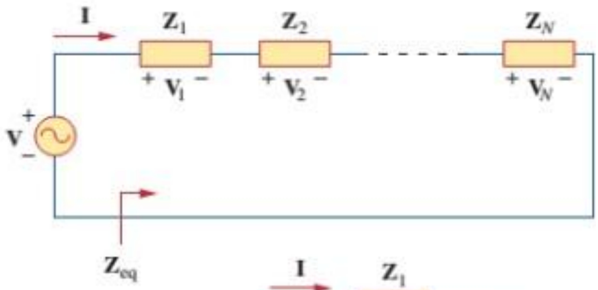
$$Z_{eq} = Z_1 + Z_2 + \cdots Z_n$$

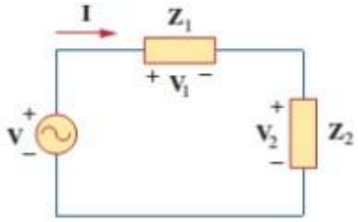
Suppose N=2:

$$Z_{eq} = Z_1 + Z_2$$

$$V_1 = \frac{Z_1 V}{Z_1 + Z_2}$$

$$V_2 = \frac{Z_2 V}{Z_1 + Z_2}$$





Parallel:

Let us assume are N impedances are connected in parallel ,than equivalent is given by

Apply current (I) between two nodes and assume voltage across nodes=V

According to the KCL $I = I_1 + I_2 \ldots I_n$

$$\frac{V}{Z_{eq}} = \frac{V}{Z_1} + \frac{V}{Z_2} + \cdots + \frac{V}{Z_n}$$

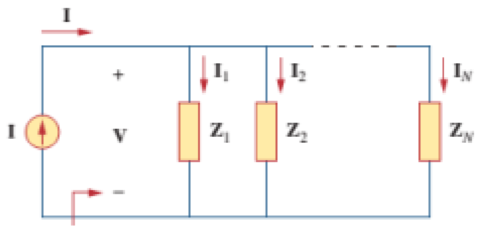
$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$$

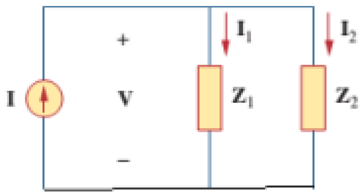
Suppose N=2; than

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

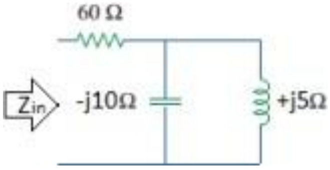
$$I_1 = \frac{Z_2}{Z_1 + Z_2} I$$

$$I_2 = \frac{Z_1}{Z_1 + Z_2} I$$

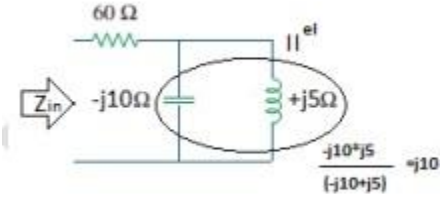




Problem:
Find equivalent impedance of below circuit.

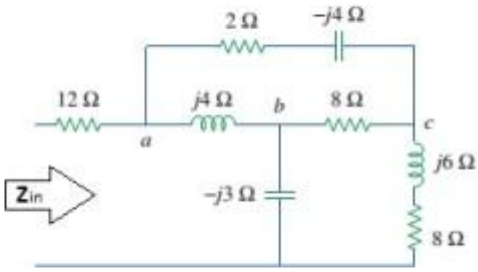


Solution: -j10 and +j5 are in ||el after simplifying



Therefore Z_{in} = (60 series in with j10) = 60+ j10.

Problem:
Find impedance Z_{in} of below circuit.



Solution:

The delta network connected to nodes *a*, *b*, and *c* can be converted to the *Y* network of Fig. 9.29. We obtain the *Y* impedances as follows using Eq. (9.68):

$$\mathbf{Z}_{an} = \frac{j4(2 - j4)}{j4 + 2 - j4 + 8} = \frac{4(4 + j2)}{10} = (1.6 + j0.8)\ \Omega$$

$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad \mathbf{Z}_{cn} = \frac{8(2 - j4)}{10} = (1.6 - j3.2)\ \Omega$$

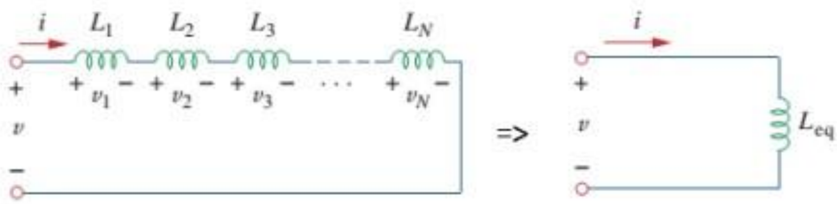
The total impedance at the source terminals is

$$\begin{aligned}\mathbf{Z} &= 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \parallel (\mathbf{Z}_{cn} + j6 + 8)\\ &= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8)\\ &= 13.6 + j0.8 + \frac{j0.2(9.6 + j2.8)}{9.6 + j3}\\ &= 13.6 + j1 = 13.64\underline{/4.204^\circ}\ \Omega\end{aligned}$$

Series and Parallel combination of Inductors:

Series: Consider N inductor are connected in series, and voltage drop across each inductor is

$v_1, v_2, \ldots\ldots v_n$.



According to the KVL

$v = v_1 + v_2 \ldots\ldots + v_n$

But $v = L\frac{di}{dt}$

$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} \ldots\ldots + L_n\frac{di}{dt}$

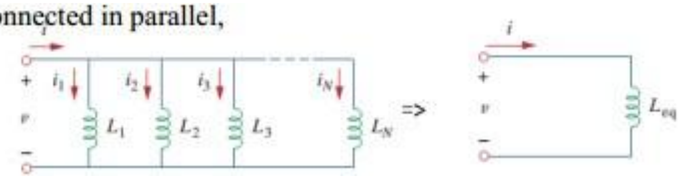
$v = (L_1 + L_2 \ldots\ldots + L_n)\frac{di}{dt}$

$v = L_{eq}\frac{di}{dt}$

So $L_{eq} = L_1 + L_2 \ldots\ldots + L_n$

The equivalent inductance of series connected inductors is the sum of the individual inductors.

Parallel: Consider N inductor are connected in parallel,



According to KCL

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

But $i_k = \frac{1}{L_k}\int_{t_0}^{t} v\,dt + i_k(t_0)$; hence,

$$i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0) + \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0)$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)$$

$$= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0)$$

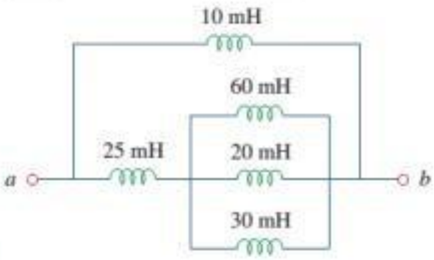
where

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

The equivalent impedance of parallel inductors is reciprocal of the sum of the reciprocals of the individual inductances.

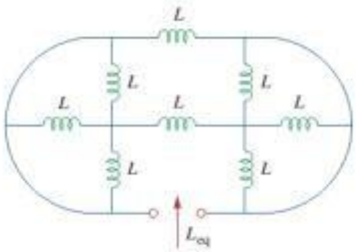
Problem:

Find L_{eq} between 'ab' terminals in below figure.



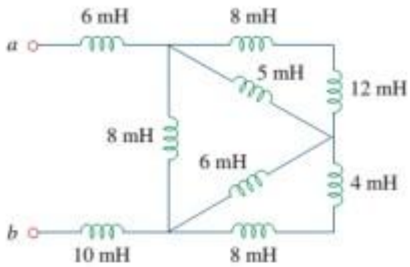
Problem:

Find L_{eq} In below figure.



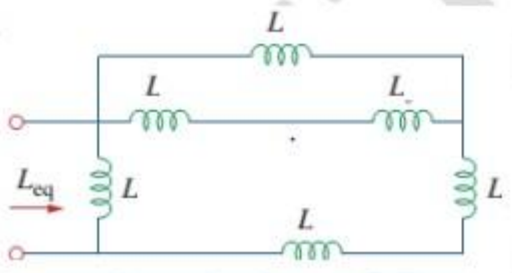
Problem:

Find L_{eq} between 'ab' terminals in below figure?

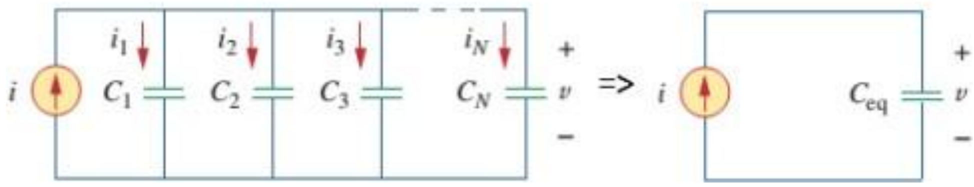


Problem:

Find L_{eq} in below figure.



Series and parallel connection of capacitors:
Parallel Connection of Capacitors:
Consider N capacitors are connected in parallel,



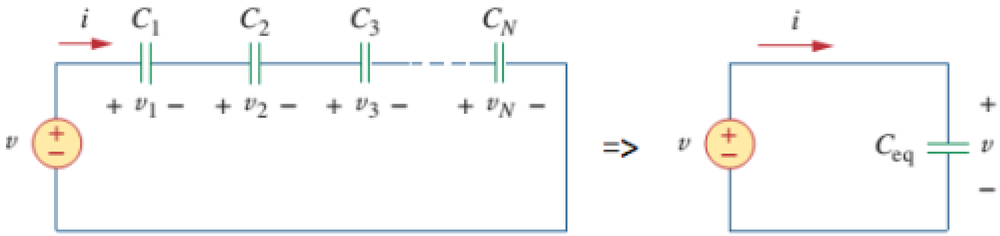
Apply KCL
$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$
$$= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

where

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$$

The equivalent capacitance of N parallel connected capacitors is the sum of all individual capacitance.

Series connection of capacitors: Consider N capacitors are connected in series.



Apply KVL above circuit (left)

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

But $v_k = \frac{1}{C_k}\int_{t_0}^{t} i(\tau)d\tau + v_k(t_0)$. Therefore,

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)d\tau + v_2(t_0)$$
$$+ \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)d\tau + v_N(t_0)$$
$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + v_2(t_0)$$
$$+ \cdots + v_N(t_0)$$
$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

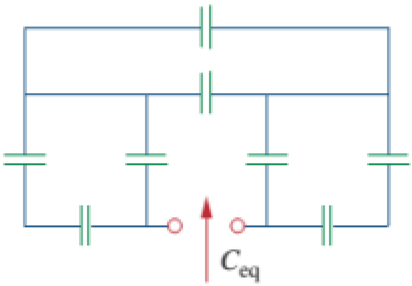
where

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}}$$

The equivalent capacitance of series connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitors.

Problem:

Find C_{eq} in below figure, If the value of all capacitor 4mf



RMS and Average values, Form factor, Steady State Analysis of Series, Parallel and Series Parallel combinations of R, L,C with Sinusoidal excitation, Instantaneous power, Average power, Real power, Reactive power and Apparent power, concept of Power factor, Frequency.

Q: Define Average value, RMS value and Form Factor and Calculate for sinusoidal wave.

Average Value

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value.

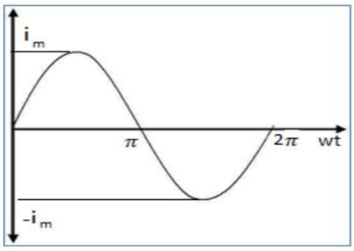
$$Average\ value = \frac{Area\ under\ one\ cycle}{Base}$$

$$v_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v\ d(\omega t)$$

For Symmetrical waveforms, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area. Hence for symmetrical waveforms, the average value is calculated for half cycle.

$$Average\ value = \frac{Area\ under\ one\ cycle}{Base}$$

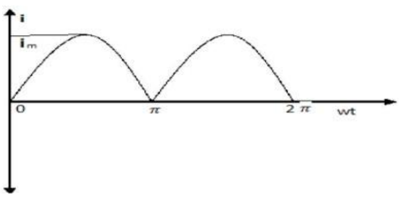
$$v_{avg} = \frac{1}{\pi}\int_0^{\pi} v\ d(\omega t)$$



Average value of a sinusoidal current:

$$i = i_m \sin(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\ d(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i_m \sin(\omega t) d(\omega t)$$
$$i_{avg} = \frac{2i_m}{\pi} = 0.637 i_m$$

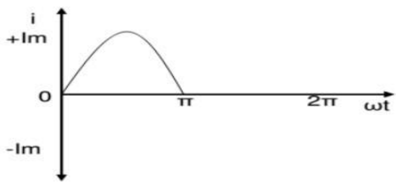
Average value of a full wave rectifier output



$$i = i_m \sin(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\ d(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i_m \sin(\omega t) d(\omega t)$$
$$i_{avg} = \frac{2i_m}{\pi} = 0.637 i_m$$

Average value of a half wave rectifier output

$$i = i_m \sin(\omega t)$$
$$i_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i\ d(\omega t)$$
$$i_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i_m \sin(\omega t) d(\omega t)$$
$$i_{avg} = \frac{2i_m}{2\pi} = 0.318 i_m$$



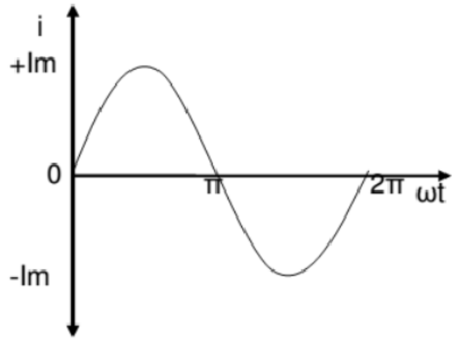
RMS or Effective Value

The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

$$RMS = \sqrt{\frac{Area\ under\ squared\ curve}{Base}}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2\, d(\omega t)}$$

RMS value of a sinusoidal current:



$$i = i_m \sin(\omega t)$$

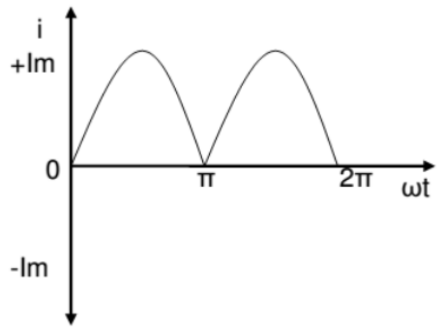
$$i_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_m^2 sin^2(\omega t)\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{i_m^2}{\pi}\int_0^{\pi} \frac{(1-\cos(2\omega t))}{2}\, d(\omega t)}$$

$$i_{rms} = \frac{i_m}{\sqrt{2}} = 0.707\, i_m$$

RMS value of a full wave rectifier output:



$$i = i_m \sin(\omega t)$$

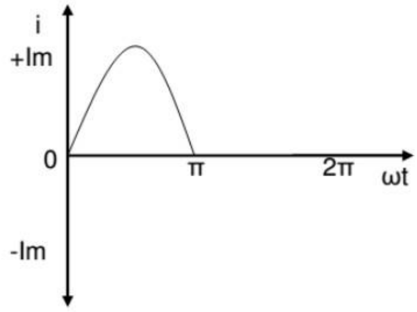
$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_m^2 sin^2(\omega t)\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{i_m^2}{\pi}\int_0^{\pi} \frac{(1-\cos(2\omega t))}{2}\, d(\omega t)}$$

$$i_{rms} = \frac{i_m}{\sqrt{2}} = 0.707 i_m$$

RMS value of a half wave rectifier output



$$i = i_m \sin(\omega t) \qquad 0 \le \omega t \le 180^0$$

$$i = 0 \qquad 180^0 \le \omega t \le 360^0$$

$$i_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i_m^2 sin^2(\omega t)\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{i_m^2}{2\pi}\int_0^{\pi} \frac{(1-\cos(2\omega t)}{2}\, d(\omega t)}$$

$$i_{rms} = \frac{i_m}{2} = 0.5 i_m$$

Form Factor:

It is the ratio of RMS value to the average value of an alternating quantity is known as Form Factor

$$FF = \frac{RMS\ value}{Avg\ Value}$$

Peak Factor or Crest Factor:

It is the ratio of maximum value to the RMS value of an alternating quantity is known as the peak factor.

$$PF = \frac{Maximum\ value}{RMS\ Value}$$

For a sinusoidal waveform and For full wave rectifier output:

$$i_{avg} = \frac{2i_m}{\pi} = 0.637i_m$$

$$i_{rms} = \frac{i_m}{\sqrt{2}} = 0.707i_m$$

$$FF = \frac{i_{rms}}{i_{avg}} = \frac{0.637\,i_m}{0.707\,i_m} = 1.11$$

$$PF = \frac{i_{rms}}{i_{avg}} = \frac{i_m}{0.707\,i_m} = 1.414$$

For a Half Wave Rectifier Output:

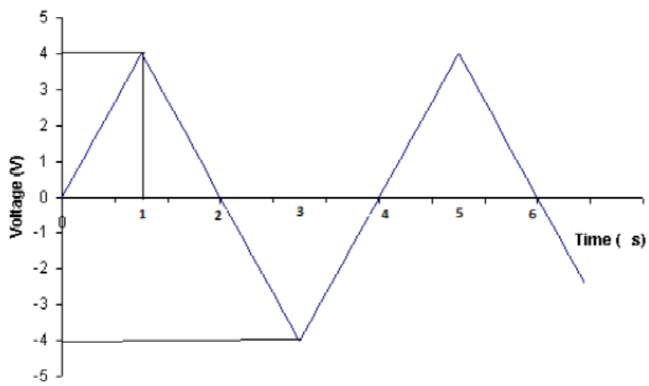
$$i_{rms} = \frac{i_m}{2} = 0.5i_m$$

$$i_{avg} = \frac{2i_m}{2\pi} = 0.318i_m$$

$$FF = \frac{i_{rms}}{i_{avg}} = \frac{0.318i_m}{0.5i_m} = 1.57$$

$$PF = \frac{i_{rms}}{i_{avg}} = \frac{i_m}{0.5i_m} = 2$$

Problem: Find Form Factor of figure show below.



Solution:

From figure:

$$V(t) = 4 * t \qquad 0 \le t \le 1$$
$$= 4 - 4(t-1) \quad 1 \le t \le 2$$

Average value:

$$1/t \int_0^{t/2} v(t)dt = 1/4\left(\int_0^1 4t\,dt + \int_1^2 4 - 4(t-1)\,dt\right)$$

$$¼\left(4 * \frac{t^2}{2}\Big]_0^1 + 4t]_0^1 - 4 * \frac{t^2}{2}\Big]_1^2 + 4t]_1^2\right)$$

$$\left(\frac{1}{4}\right) * (4 * [1/2] + 4 - 4[4/2 - 1/2] + 4) = 0.25$$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt\ put\ T = t/2}$$

$$V_{RMS} = \sqrt{\frac{1}{2}\int_0^2 v(t)^2\,dt}$$

$$V_{RMS} = \sqrt{\frac{1}{2}\left(\int_0^1 16*(t)^2\,dt + \int_1^2 16*(1-1(t-1))^2\,dt\right)}$$

$$V_{RMS} = \sqrt{\frac{16}{2}\left(\int_0^1 t^2\,dt + \int_1^2 (-(t-2))^2\,dt\right)}$$

$$V_{RMS} = \sqrt{8*\left(\int_0^1 t^2\,dt + \int_1^2 (t^2-4t+4)\,dt\right)}$$

$$V_{RMS} = \sqrt{8*\left(\int_0^1 t^2\,dt + \int_1^2 (t-2)^2 dt\right)}$$

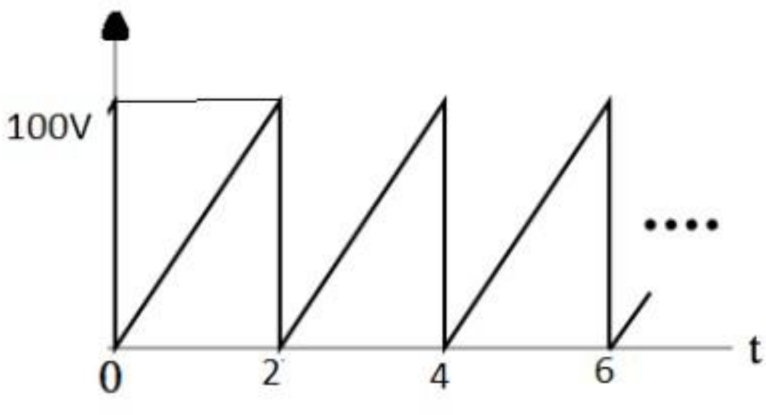
$$V_{RMS} = \sqrt{8*\left\{\left(\frac{t^3}{3}\right)_0^1 + (4t)_1^2 - \left(\frac{4t^2}{2}\right)_1^2 + \left(\frac{t^3}{3}\right)_1^2\right\}}$$

$$V_{RMS} = \sqrt{8*\left\{\frac{1}{3}-\frac{0}{3}+4(2-1)-\left(4*\frac{2^2}{2}-4*\frac{1^2}{2}\right)+\frac{2^3}{3}-\frac{1^3}{3}\right\}}$$

$$V_{RMS} = \sqrt{14*\frac{8}{3}}$$

Problem:

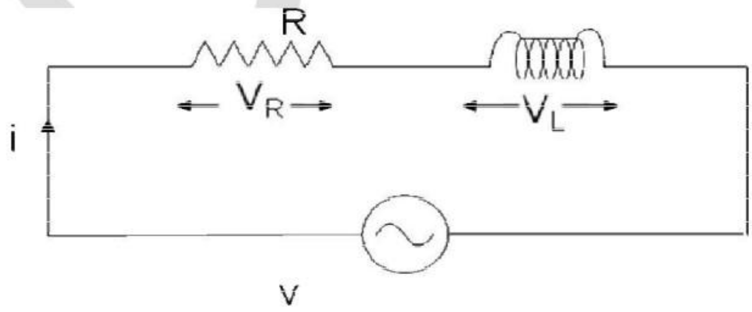
Calculate Average value, Rms value and Form factor of the sawtooth wave show in the figure.



.

Steady State Analysis of Series, Parallel and Series Parallel combinations of R, L,C with Sinusoidal excitation:

R-L Series Circuit:



Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure. The alternating voltage v is given by

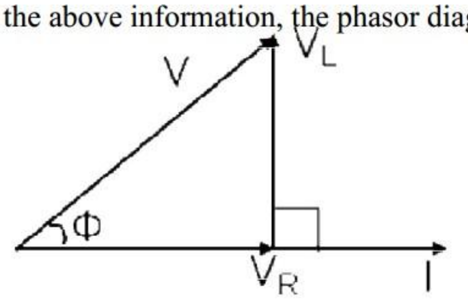
$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the inductor is V_L

V_R=IR is in phase with I

V_L=IX_L leads current by 90 degrees

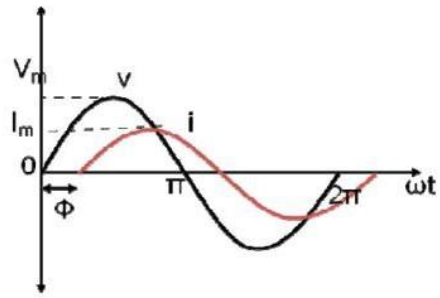
With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_L leads the current by 90^0. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ.

The waveform and equations for an RL series circuit can be drawn as below.

$$V = V_m \sin(\omega t)$$
$$I = I_m \sin(\omega t - \emptyset)$$



From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}$$
$$V_R = IR$$
$$V_L = IX_L$$
$$V = \sqrt{(IR)^2 + (IX_L)^2}$$
$$V = I\sqrt{R^2 + X_L{}^2}$$
$$V = IZ$$
$$where\ Z = \sqrt{R^2 + X_L{}^2}$$

The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms(Ω)

Phase angle:

$$\emptyset = tan^{-1}(\frac{V_L}{V_R})$$

$$\emptyset = tan^{-1}(\frac{IX_L}{IR})$$
$$\emptyset = tan^{-1}(\frac{X_L}{R})$$
$$\emptyset = tan^{-1}(\frac{\omega L}{R})$$

Power Factor:

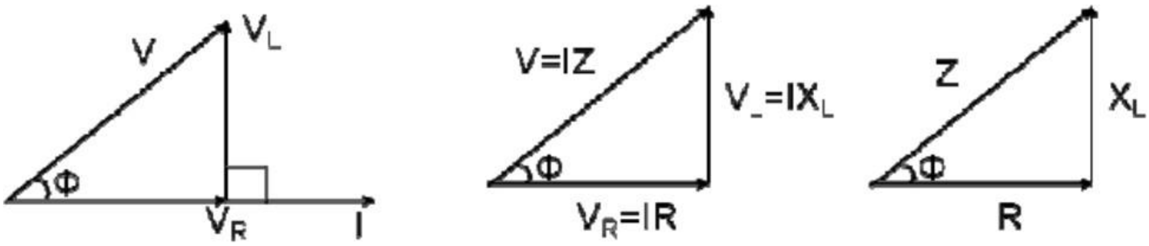
The power factor in an AC circuit is defined as the cosine of the angle between voltage and current ie., cosΦ.

Problem:

A series RL circuit has a resistor 36Ω and impedance of circuit is 10 Ω, then find power factor

Impedance Triangle:

We can derive a triangle called the impedance triangle from the phasor diagram of an RL series circuit as shown



The impedance triangle is right angled triangle with R and X_L as two sides and impedance as the hypotenuse. The angle between the base and hypotenuse is Φ. The impedance triangle enables us to calculate the following things.

1. Impedance $Z = \sqrt{R^2 + X_L{}^2}$
2. Power Factor $cos\Phi = R/Z$
3. Phase angle $\emptyset = tan^{-1}(\frac{X_L}{R})$
4. Whether current is leading or lagging.

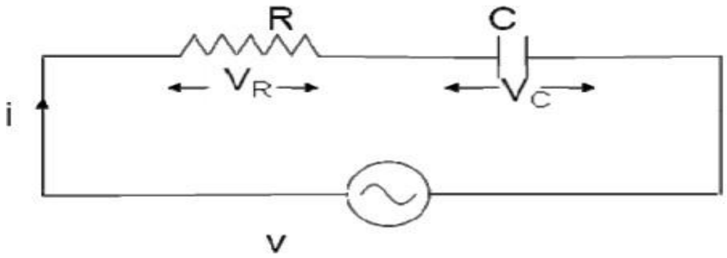
Problem:

A 200 V, 50 Hz, inductive circuit takes a current of 10A, lagging 30 degree. Find (i) the resistance (ii) reactance (iii) inductance of the coil.

Solution:

$$Z = \frac{V}{I} = \frac{200}{10} = 20\,\Omega$$
$$i)R = Zcos(\emptyset) = 20 * cos(30^0) = 17.32\Omega$$
$$\text{ii) } X_L = Z\sin(\emptyset) = 20 * \sin(30^0) = 10\Omega$$
$$iii)L = \frac{X_L}{2\pi f} = \frac{10}{2 * 3.14 * 50} = 0.0318H$$

Explain the behavior of AC through RC Series Circuit:



Consider an AC circuit with a resistance R and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

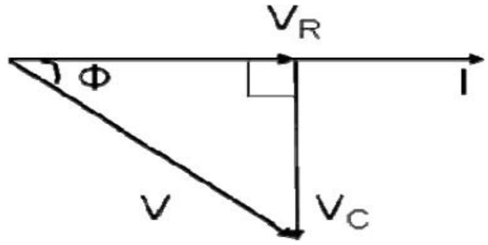
$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is *i*. The voltage across the resistor is V_R and that across the capacitor is

V_R=IR is in phase with I

V_c =IXclags current by 90 degrees

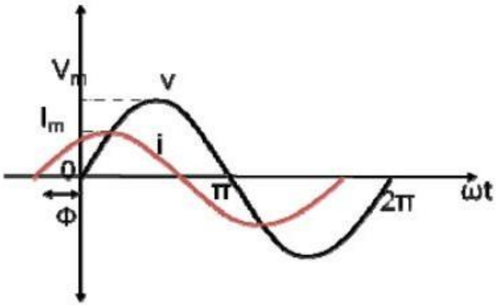
With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_C lags behind the current by 90^0. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

The waveform and equations for an RC series circuit can be drawn as below.

$$V = V_m \sin(\omega t)$$
$$I = I_m \sin(\omega t + \emptyset)$$



From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_C^2}$$
$$V_R = IR$$
$$V_C = IX_C$$
$$V = \sqrt{(IR)^2 + (IX_C)^2}$$
$$V = I\sqrt{R^2 + X_C{}^2}$$
$$V = IZ$$
$$where\ impedance\ Z = \sqrt{R^2 + X_C{}^2}$$

The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms(Ω).

Phase angle:

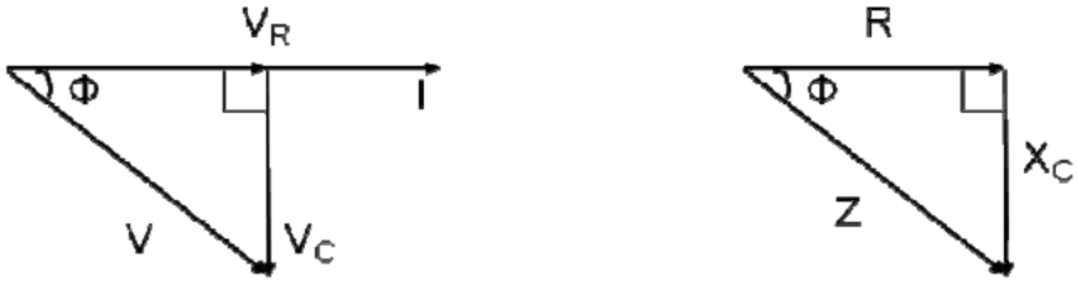
$$\emptyset = tan^{-1}(\frac{V_C}{V_R})$$

$$\emptyset = tan^{-1}(\frac{IX_C}{IR})$$

$$\emptyset = tan^{-1}(\frac{X_C}{R})$$

$$\emptyset = tan^{-1}\left(\frac{\frac{1}{\omega C}}{R}\right) = tan^{-1}(\frac{1}{R\omega C})$$

Impedance triangle:



Phasor algebra for RC series circuit.

$$\bar{V} = V + j0 = V\angle 0^o$$

$$\bar{Z} = R - j\,X_C = Z\angle - \emptyset$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle \emptyset$$

Problem:

A Capacitor of capacitance 79.5μF is connected in series with a non inductive resistance of 30 across a 100V, 50Hz supply. Find (i) impedance (ii) current (iii) phase angle

Solution:

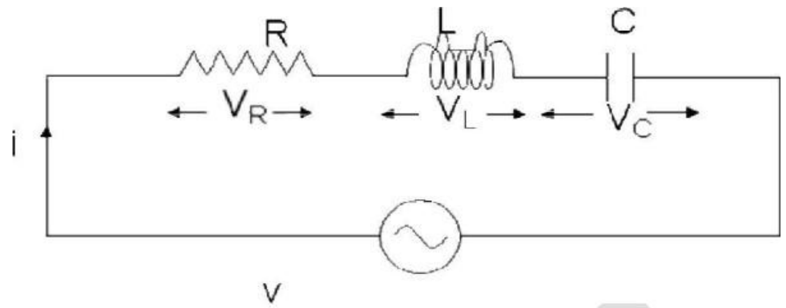
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2*3.141*50*79.5*10^{-6}} = 40\Omega$$

$$i)\ Z = \sqrt{R^2 + {X_L}^2} = \sqrt{30^2 + 40^2} = 50\Omega$$

$$ii)\ I = V/Z = 100/50 = 2A$$

$$iii)\ Phase\ angle = Tan^{-1}\left(\frac{X_C}{R}\right) = Tan^{-1}\left(\frac{40}{30}\right) = 53^o$$

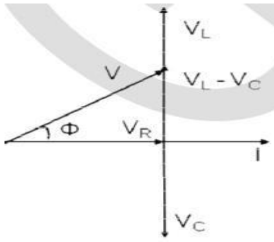
Behavior AC with R-L-C Series circuit:

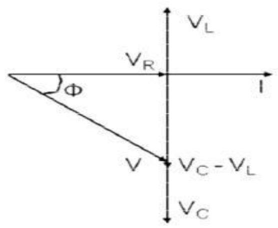


Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

$$V = V_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the resistor is V_R, the voltage across the inductor is V_L and that across the capacitor is V_C.
$V_R = IR$ is in phase with I
$V_L = IX_L$ leads the current by 90 degrees
$V_C = IX_C$ lags behind the current by 90 degrees
With the above information, the phasor diagram can be drawn as shown. The current I is taken as the reference phasor. The voltage V_R is in phase with I, the voltage V_L leads the current by 90^o and the voltage V_C lags behind the current by 90^o. There are two cases that can occur $V_L > V_C$ and $V_L < V_C$ depending on the values of X_L and X_C. And hence there are two possible phasor diagrams. The phasor $V_L - V_C$ or $V_C - V_L$ is drawn and then the resultant voltage V is drawn.





From the phasor diagram we observe that when $V_L > V_C$, the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ. When $V_L < V_C$, the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived
as follows.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$
$$V = I\sqrt{(R)^2 + (X_L - X_C)^2}$$
$$V = IZ$$
$$Where\ impedance\ is\ Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Phase\ angle\ \emptyset = Tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = Tan^{-1}\left(\frac{IX_L - IX_C}{IR}\right) = Tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

From the expression for phase angle, we can derive the following three cases

Case (i): When $X_L > X_C$

The phase angle Φ is positive and the circuit is inductive. The circuit behaves like a series RL circuit.

Case (ii): When $X_L < X_C$

The phase angle Φ is negative and the circuit is capacitive. The circuit behaves like a series RC circuit.

Case (iii): When $X_L = X_C$

The phase angle Φ = 0 and the circuit is purely resistive. The circuit behaves like a pure resistive circuit.

The voltage and the current can be represented by the following equations. The angle Φ is positive or negative depending on the circuit elements

$$V = V_m \sin(\omega t)$$

$$I = I_m \sin(\omega t \pm \emptyset)$$

Phasor algebra for RLC series circuit.

$$V = V + j0 = V\angle 0^o$$

$$\bar{Z} = R + j\,(X_L - X_C) = Z\angle\emptyset$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle - \emptyset$$

Problem:

A 230 V, 50 Hz ac supply is applied to a coil of 0.06 H inductance and 2.5 resistance connected in series with a 6.8 μF capacitor. Calculate (i) Impedance (ii) Current (iii) Phase angle between current and voltage (iv) power factor

Solution:

$$X_L = 2\pi f L = 1 * 3.14 * 50 * 0.06 = 18.84\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2*3.141*50*6.8*10^{-6}} = 468\ \Omega$$

$$i) \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2.5^2 + (18.84 - 468)^2} = 449.2\ \Omega$$

$$I = \frac{V}{Z} = \frac{230}{4492} = 0.512A$$

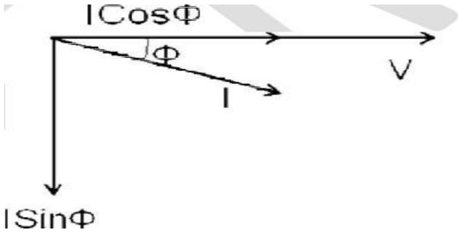
$$\emptyset = Tan^{-1}\left(\frac{X_L - X_C}{R}\right) = Tan^{-1}\left(\frac{18.84 - 468}{30}\right) = -89.7^o$$

$$Power\ factor = \cos(\emptyset) = \cos(-89.7) = 0.0056\ Lead$$

Power:

In an AC circuit, the various powers can be classified as
1. Real or Active power or Average power.
2. Reactive power
3. Apparent power

Real or active power in an AC circuit is the power that does useful work in the circuit. Reactive power flows in an AC circuit but does not do any useful work. Apparent power is the total power in an AC circuit.



Instantaneous Power:

The instantaneous power is product of instantaneous values of current and voltages and it can be derived as follows.

$$P = vi$$

$$p = V_m \sin(\omega t + \theta_v) * I_m \sin(\omega t + \theta_i)$$

From trigonometric expression:

$$\cos(A - B) - \cos(A + B) = 2\sin(A)\sin(B)$$

$$p = \frac{V_m I_m}{2}(\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i))$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) - \frac{V_m I_m}{2}\cos(2\omega t)$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average Power:

From instantaneous power we can find average power over one cycle as following.

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{V_m I_m}{2}\cos(\theta_v - \theta_i) - \frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i)\right)d(\omega t)$$

$$P = \frac{1}{2\pi}\left(\frac{V_m I_m}{2}\cos(\theta_v - \theta_i) * (2\pi - 0)\right) - \frac{1}{2\pi}\int_0^{2\pi} - \frac{V_m I_m}{2}\cos(2\omega t)\, d(\omega t)$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} = V_{RMS} * I_{RMS}\cos(\theta_v - \theta_i)$$

As seen above the average power is the product of the RMS voltage and the RMS current.

Problem:

Calculate the instantaneous power and average power absorbed by the passive linear network. If $v = 330\cos(10t + 20^0)\ volts$ and $i = 33\sin(10t + 60^0)\ A$.

Problem:

Calculate the average power absorbed by an impedance Z=30-j70 ohms when applied a voltage $\overline{V} = 120\angle 0^0$ is applied across it.

Problem:

$v(t) = 120\cos(377t + 45°)$ V and $i(t) = 10\cos(377t - 10°)$ A

find the instantaneous power and the average power absorbed by the passive linear network

Solution:

The instantaneous power is given by

$$p = vi = 1200\cos(377t + 45°)\cos(377t - 10°)$$

Applying the trigonometric identity

$$\cos A\cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

gives

$$p = 600[\cos(754t + 35°) + \cos 55°]$$

or

$$p(t) = 344.2 + 600\cos(754t + 35°) \text{ W}$$

The average power is

$$P = \frac{1}{2}V_m I_m\cos(\theta_v - \theta_i) = \frac{1}{2}120(10)\cos[45° - (-10°)]$$

$$= 600\cos 55° = 344.2 \text{ W}$$

Real Power:

The power due to the active component of current is called as the active power or real power. It is denoted by P.

$$P = V * I\cos(\emptyset) = I^2 R\cos(\emptyset)$$

Real power is the power that does useful power. It is the power that is consumed by the resistance. The unit for real power in Watt(W).

Reactive Power:

The power due to the reactive component of current is called as the reactive power. It is denoted by Q.

$$Q = V * I\sin(\emptyset) = I^2 X_L \sin(\emptyset)$$

Reactive power does not do any useful work. It is the circulating power in the L and C components. The unit for reactive power is Volt Amperes Reactive (VAR).

Apparent Power:

The apparent power is the total power in the circuit. It is denoted by S.

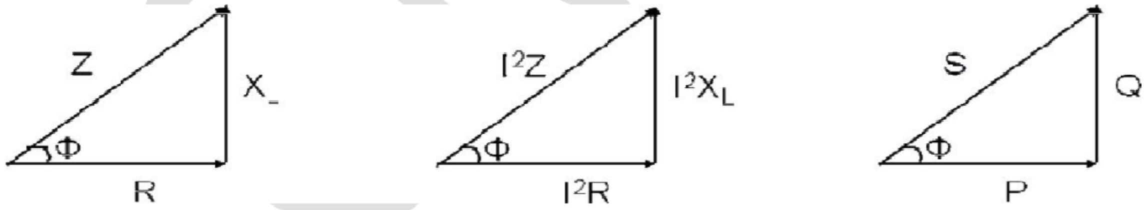
$$S = VI = I^2 Z$$

$$S = \sqrt{P^2 + Q^2}$$

The unit for apparent power is Volt Amperes (VA).

Power Triangle:

From the impedance triangle, another triangle called the power triangle can be derived as shown.



The power triangle is right angled triangle with P and Q as two sides and S as the hypotenuse. The angle between the base and hypotenuse is Φ. The power triangle enables us to calculate the following things.

Apparent Power $S = \sqrt{P^2 + Q^2}$

$$Power\ factor = \cos(\emptyset) = \frac{P}{Q} = \frac{Real\ Power}{Reactive\ Power}$$

The power Factor in an AC circuit can be calculated by any one of the following Methods

$= Cosine\ of\ angle\ between\ V\ and\ I$

$= \frac{Resistance}{Impedance} = \frac{R}{Z}$

$= \frac{Real\ power}{Apparent\ power}$

Problem:

A coil having a resistance of 7 and an inductance of 31.8mH is connected to 230V, 50Hz supply. Calculate (i) the circuit current (ii) phase angle (iii) power factor (iv) power consumed v) Reactive power vi) Apparent power.

Solution:

$$X_L = 2\pi f L = 2 * 3.14 * 50 * 31.8 * 10^{-3} = 10\Omega$$

$$Z = \sqrt{R^2 + {X_L}^2} = \sqrt{7^2 + 10^2} = 12.2\ \Omega$$

$$i)\ I = \frac{V}{Z} = \frac{230}{12.2} = 18.85A$$

$$ii)\ \emptyset = Tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}(\frac{10}{7}) = -53^0\ lag$$

$$iii) PF = \cos(\emptyset) = \cos(-53) = 0.537\ lag$$

$$iv) Power\ Consumed\ P = VI\ cos(\emptyset) = 230 * 18.85 * 0.537 = 2.484kW$$

$$v) Reactive\ Power\ Q = VI\sin(\emptyset) = 230 * 18.85 * 0.795 = 3.462k\ VAR$$

$$vi) Apperant\ Power = \sqrt{P^2 + Q^2} = \sqrt{2.48^2 + 3.46^2} = 4.25kVA$$

Problem:

A current of (120-j50)A flows through a circuit when the applied voltage is (8+j12)V. Determine (i) impedance (ii) power factor (iii) power consumed and reactive power.

Solution:

$$\bar{V} = 8 + j\ 12$$

$$\bar{I} = 120 - j50$$

$$i)\ \bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{8+j\ 12}{120-j50} = 0.02 + j0.11 = 0.11\angle 79.7^0$$

$$Z = 0.11\Omega$$

$$\emptyset = 79.7^0$$

$$ii)\ pf = \cos(\emptyset) = \cos(79.7^0) = 0.179\ lag$$

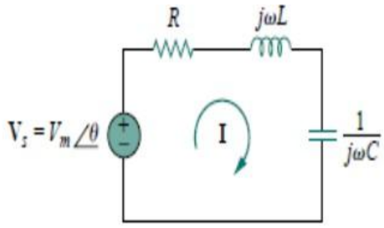
$$iii) S = VI^* = (8 + j12)(120 + j50) = 360 + j1840$$

$$But\ S = P + jQ$$

$$P = 360W$$

$$S = 1860VAR$$

SERIES RESONANCE:
The most prominent feature of the frequency response of a circuit may be the sharp peak (or *resonant peak*) exhibited in its amplitude characteristic.The concept of resonance applies in several areas of science and engineering.Resonance occurs in any system that has a complex conjugate pair of poles; it is the cause of oscillations of stored energy from one form to another.
It is the phenomenon that allows frequency discrimination in communications networks. Resonance occurs in any circuit that has at least one inductor and one capacitor.
Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective. They are used in many applications such as selecting the desired stations in radio and TV receivers.
Consider the series *RLC* circuit shown in the frequency domain. The input impedance is



$$Z = H(\omega) = \frac{Vs}{I} = R + j\omega L + \frac{1}{j\omega C} \quad ....(1)$$

$$Z = R + j(\omega L - \frac{1}{j\omega C}) \quad ..(2)$$

Resonance results when the imaginary part of the transfer function is zero, or

$$Im(Z) = \omega L - \frac{1}{\omega C} = 0 \quad ..(3)$$

The value of ω that satisfies this condition is called the ***resonant frequency*** ω_0. Thus, the resonance condition is

$$\omega_0 L = \frac{1}{\omega 0 C} \quad ..(4)$$

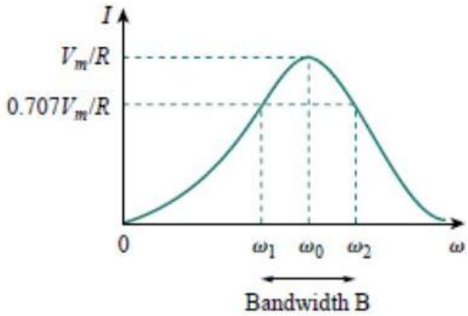
$$..(5) \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}}$$

$$\text{Or Since } \omega_0 = 2\pi f_0, \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \quad ..(6)$$

Note that at resonance: The impedance is purely resistive, thus, **Z** = *R*. In other words, the *LC* series combination acts like a short circuit, and the entire voltage is across *R*. The voltage **V***s* and the current **I** are in phase, so that the power factor is unityThe magnitude of the transfer function **H***(ω)* = **Z***(ω)* is minimum. The inductor voltage and capacitor voltage can be much more than the source voltage. The frequency response of the circuit's current magnitude

$$I = |I| = \frac{Vm}{\sqrt{R^2 + \left((\omega L - \frac{1}{\omega C})^2\right)}} \quad (7)$$

is shown in Fig. the plot only shows the symmetry illustrated in this graph when the frequency axis is a logarithm.



The average power dissipated by the *RLC* circuit is $P(\omega) = \frac{1}{2} I^2 R$ The highest power dissipated occurs at resonance, when $I = Vm/R$, so That $P(\omega_0) = \frac{1}{2}\frac{v_{m^2}}{R}$ ..(8)

At certain frequencies $\omega = \omega 1, \omega 2$, the dissipated power is half the maximum value; that is,

$$P(\omega_1) = P(\omega_2) = \frac{(Vm/\sqrt{2})^2}{2R} = \frac{V_{m^2}}{4R} \quad ..(9)$$

Hence, $\omega 1$ and $\omega 2$ are called the **half-power frequencies**. The half-power frequencies are obtained by setting Z equal to $\sqrt{2}$R and writing $\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

Solving for ω, we obtain

$$\omega_1 = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}} \quad ..(10)$$
$$\omega_2 = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

We can relate the half-power frequencies with the resonant frequency. From Eqs. (5) and (10),

$$\omega_0 = \sqrt{\omega 1 \omega 2}$$

It is seen that the resonant frequency is the geometric mean of the halfpower frequencies. Notice that $\omega 1$ and $\omega 2$ are in general not symmetrical around the resonant frequency $\omega 0$, because the frequency response is not generally symmetrical. However, as will be explained shortly,
symmetry of the half-power frequencies around the resonant frequency is often a reasonable approximation. Although the height of the curve in Fig.,it is determined by *R*, the width of the curve depends on other factors. The width of the response curve depends on the *bandwidth B*, which is defined as the difference between the two half-power frequencies,

$$B = \omega_2 - \omega_1 \quad (11)$$

This definition of bandwidth is just one of several that are commonly used. Strictly speaking, *B*in Eq. (14.35) is a half-power bandwidth, because it is the width of the frequency band between the half-power frequencies.

The “sharpness” of the resonance in a resonant circuit is measured quantitatively by the **quality factor Q**. At resonance, the reactive energy in the circuit oscillates between the inductor and the capacitor. The quality factor relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation:

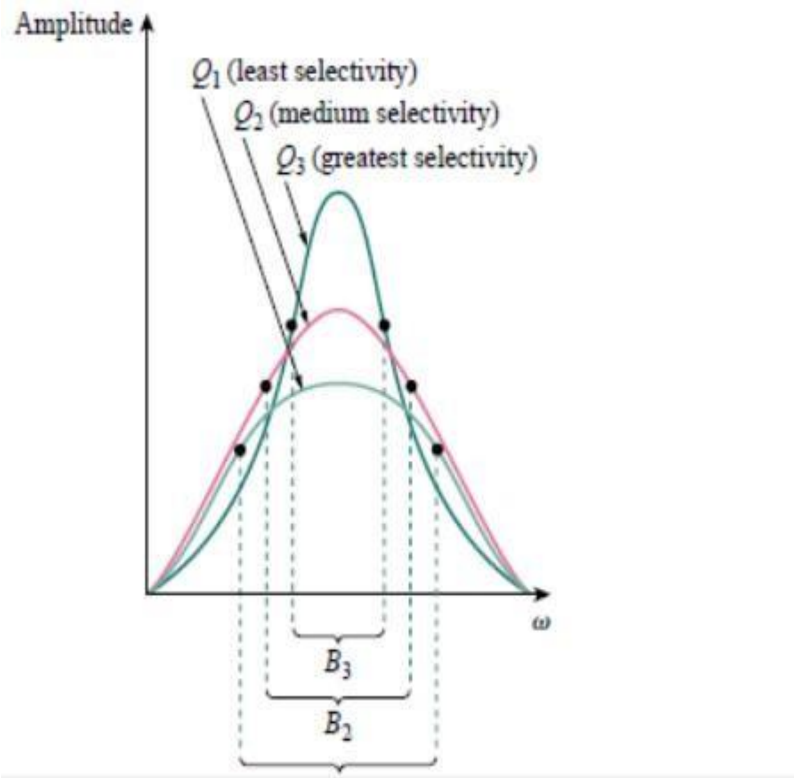
$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}} \quad ..(12)$$

It is also regarded as a measure of the energy storage property of a circuit in relation to its energy dissipation property. In the series *RLC* circuit, the peak energy stored is 1/2L , while the energy dissipated in one period is 1/2(R)(1/f). Hence,

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(\frac{1}{f})} = \frac{2\pi f L}{R} \quad ..(13)$$

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \quad ..(14)}$$

Notice that the quality factor is dimensionless. The relationship between the bandwidth *B* and the quality factor *Q* is obtained by substituting Eq.10 into Eq. (11) and utilizing Eq. (14).



$$\boxed{B = \frac{R}{L} = \frac{\omega_0}{Q}}$$

Or B = CR Thus, The qualityfactor of a resonant circuit is the ratio of its resonant frequency to its bandwidth. As illustrated in Fig, the higher the value of *Q*, the more selective the circuit is but the smaller the bandwidth.

The ***selectivity*** of an*RLC* circuit is the ability of the circuit to respond to a certain frequency and discriminate against all other frequencies. If the band of frequencies to be selected or rejected is narrow, the quality factor of the resonant circuit must be high. If the band of frequencies is wide, the quality factor must be low. A resonant circuit is designed to operate at or near its resonant frequency. It is said to be a *high-Q circuit* when its quality factor is equal to or greater than 10. For high-*Q* circuits ($Q \geq 10$), the halfpower frequencies are, for all practical purposes, symmetrical around the resonant frequency and can be approximated as

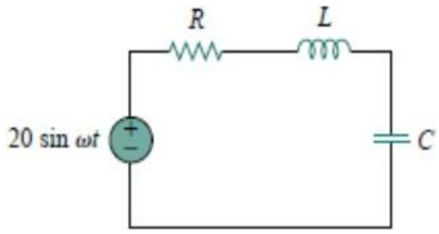
$$\boxed{\omega_1 \cong \omega_0 - \frac{B}{2} \;;\; \omega_1 \cong \omega_0 + \frac{B}{2}}$$

High-*Q* circuits are used often in communications networks. We see that a resonant circuit is characterized by five related parameters: the two half-power frequencies $\omega 1$ and $\omega 2$, the resonant frequency $\omega 0$, the bandwidth *B*, and the quality factor *Q*.

Characteristic	Series Circuit
Resonant frequency , ω_0	$\frac{1}{\sqrt{LC}}$
Quality Factor,Q	$\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 CR}$
Bandwidth B	$\frac{\omega 0}{Q}$
Half Power frequencies ω_1 , ω_2	$\omega_0 \sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega 0}{2Q}$
For $Q \geq 10$, ω_1 , ω_2	$\omega_0 \pm \frac{B}{2}$

Problem:

In the circuit in Fig. 14.24, $R = 2$ &, $L = 1$ mH, and $C = 0.4\,\mu$F.(a) Find the resonant frequency and the half-power frequencies. (b) Calculate the quality factor and bandwidth. (c) Determine the amplitude of the current at $\omega 0$, $\omega 1$, and $\omega 2$.



Solution:

(a) The resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} * 0.4 * 10^{-6}}} = 50 \text{ krad/s}$$

The lower half-power frequency is

$$\omega_1 = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

$-1 + \sqrt{1 + 2500}$ krad/s = 49 krad/s

Similarly, the upper half-power frequency is

$\omega_2 = 1 + \sqrt{1 + 2500}$ krad/s = 51 krad/s

(b) The bandwidth is

$$B = \omega 2 - \omega 1 = 2 \text{ krad/s or } B = \frac{R}{L} = \frac{2}{10^{-3}} = 2\text{krad/s}$$

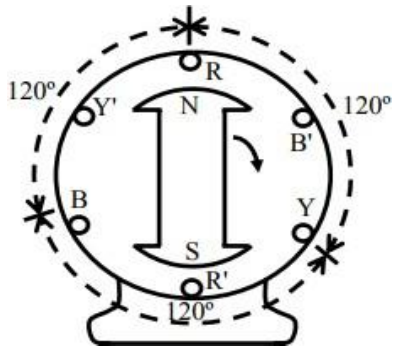
The quality factor is

$$Q = \frac{\omega_0}{B} = \frac{50}{2} = 25$$

THREE PHASE BALANCED SYSTEM

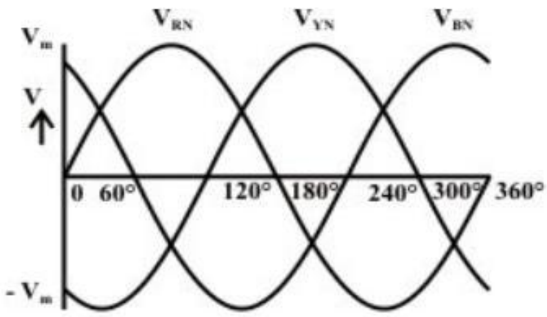
Generation of Three-phase Balanced Voltages:

The generation of single-phase voltage, using a multi-turn coil placed inside a magnet, was described. It may be noted that, the scheme shown was a schematic one, whereas in a machine, the windings are distributed in number of slots. Same would be the case with a normal three-phase generator. Three windings, with equal no. of turns in each one, are used, so as to obtain equal voltage in magnitude in all three phases. Also to obtain a balanced three-phase voltage, the windings are to be placed at an electrical angle of with each other, such that the voltages in each phase are also at an angle of with each other. The waveforms in each of the three windings (R, Y & B), are also shown in Fig below. The windings are in the stator, with the poles shown in the rotor, which is rotating at a synchronous speed of (r/min, or rpm).



Schematic diagram of three windings of stator for the generation of three phased balanced voltage (2-pole rotor).

Three-phase Voltages for Star Connection :

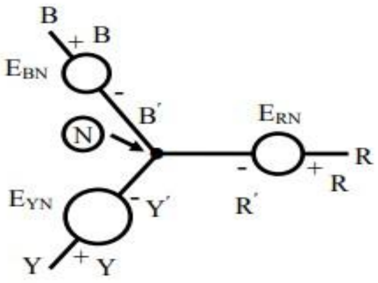


The connection diagram of a star (Y)-connected three-phase system is shown in Fig. 18.2a, along with phasor representation of the voltages (in fig). These are in continuation of the figures 18.1a-b. Three windings for three phases are R (+) & R′(−),Y (+) & Y′(−), and B (+) & Y′(−). Taking the winding of one phase, say phase R as an example, then R with sign (+) is taken as start, and R′ with sign (−) is taken as finish. Same is the case with two other phases. For making star (Y)-connection, R′, Y′ & B′ are connected together, and the point is taken as neutral, N. Three phase voltages are:

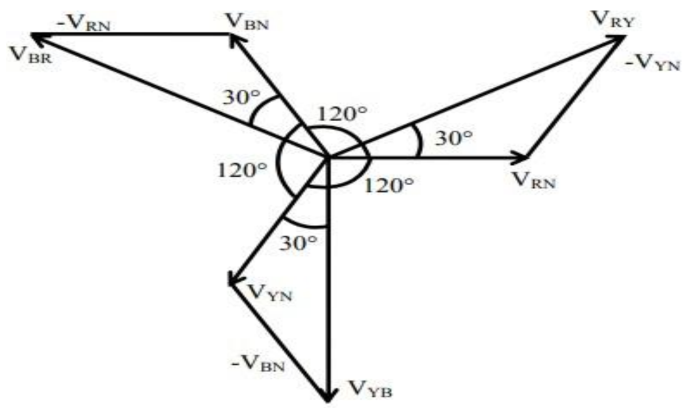
$$e_{RN} = E_m \sin\theta; \qquad e_{YN} = E_m \sin(\theta - 120°);$$
$$e_{BN} = E_m \sin(\theta - 240°) = E_m \sin(\theta + 120°)$$

It may be noted that, if the voltage in phase R (e_{RN}) is taken as reference as stated earlier, then the voltage in phase Y(e_{YN}) lags e_{RN} by 120 degrees , and the voltage in phase B(e_{BN}) lags e_{YN} by 120 degrees , or leads e_{RN} by 120 degrees . The phasors are given as:

$E_{RN} \angle 0° = E(1.0 + j0.0)$: $\quad E_{YN} \angle -120° = E(-0.5 - j0.866)$;
$E_{BN} \angle +120° = E(-0.5 + j0.866)$.



(a)



(b)

(a) Three-phase balanced voltages, with the source star-connected (the phase sequence, R-Y-B)
(b) Phasor diagram of the line and phase voltages

The phase voltages are all equal in magnitude, but only differ in phase. This is also shown in Fig.b. The relationship between E and is Em E = Em/√2 . The phase sequence is R-Y-B. It can be observed from Fig.1 that the voltage in phase Y attains the maximum value, after $\theta = \omega \cdot t = 120°$ from the time or angle, after the voltage in phase R attains the maximum value, and then the voltage in phase B attains the maximum value. The angle of lag or lead from the reference phase, R is stated earlier.

Relation between the Phase and Line Voltages for Star Connection:

Three line voltages (Fig. 18.4) are obtained by the following procedure. The line voltage, E_{RY} is

$$E_{RY} = E_{RN} - E_{YN} = E\angle 0° - E\angle -120° = E\,[(1+j0)-(-0.5-j0.866)]$$

$$= E\,(1.5 + j0.866) = \sqrt{3}\,E\angle 30°$$

The magnitude of the line voltage, E_{RY} is $\sqrt{3}$ times the magnitude of the phase voltage E_{RN}, and E_{RY} leads E_{RN} by $30°$. Same is the case with other two line voltages as shown in brief (the steps can easily be derived by the procedure given earlier).

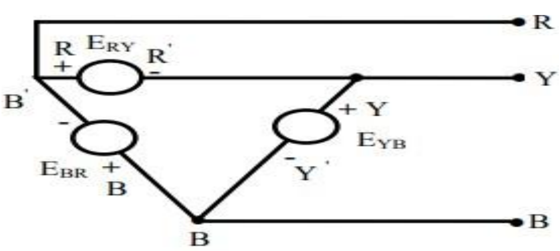
$$E_{YB} = E_{YN} - E_{BN} = E\angle -120° - E\angle +120° = \sqrt{3}E\angle -90°$$

$$E_{BR} = E_{BN} - E_{RN} = E\angle +120° - E\angle 0° = \sqrt{3}E\angle +150°$$

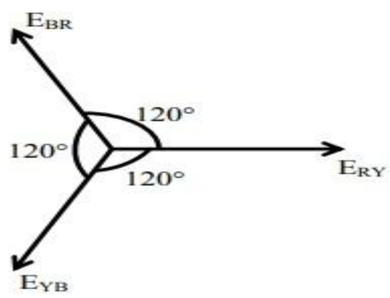
So, the three line voltages are balanced, with their magnitudes being equal, and the phase angle being displaced from each other in sequence by $120°$. Also, the line voltage, say E_{RY}, <u>leads</u> the corresponding phase voltage, E_{RN} by $30°$

Relation between the Phase and Line Voltages for Delta Connection

The connection diagram of a delta (Δ)-connected three-phase system is shown in Fig. 18.4a, along with phasor representation of the voltages. For making delta (Δ)-connection, the start of one winding is connected to the finish of the next one in sequence, for instance, starting from phase R, R' is to connected to Y, and then Y' to B, and so on. The line and phase voltages are the same in this case, and are given as



(a)

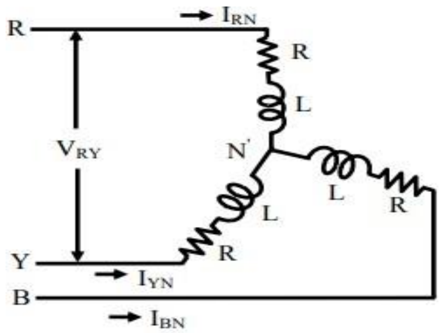


(b)

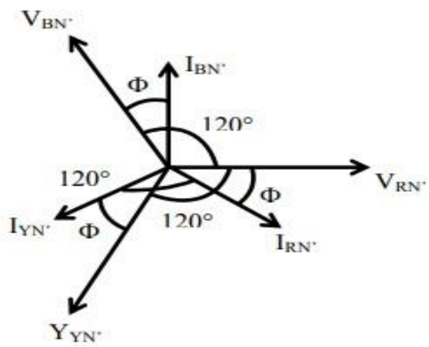
(a) Three-phase balanced voltages, with the source delta-connected (the phase sequence, R-Y-B)

(b) Phasor diagram of the line and phase voltages

Currents for Circuit with Balanced Load (Star-connected):



(a)



(b)

(a) Circuit diagram for a three-phase balanced star-connected load
(b) Phasor diagram of the phase voltages, and the line & phase currents

A three-phase star (Y)-connected balanced load is fed from a balanced three-phase supply, which may be a three-wire one. A balanced load means that, the magnitude of the impedance per phase, is same, i.e., $|Z_p| = |Z_{RN'}| = |Z_{YN'}| = |Z_{BN'}|$, and their angle is also same, as $\phi_p = \phi_{RN'} = \phi_{YN'} = \phi_{BN'}$. In other words, if the impedance per phase is given as, $Z_p \angle \phi_p = R_p + j\,X_p$, then $R_p = R_{RN'} = R_{YN'} = R_{BN'}$, and also $X_p = X_{RN'} = X_{YN'} = X_{BN'}$. The magnitude and phase angle of the impedance per phase are: $Z_p = \sqrt{R_p^2 + X_p^2}$, and $\phi_p = \tan^{-1}(X_p / R_p)$. For balanced load, the magnitudes of the phase voltages, $|V_p| = |V_{RN'}| = |V_{YN'}| = |V_{BN'}|$ are same, as those of the source voltages per phase $|V_{RN}| = |V_{YN}| = |V_{BN}|$, if it is connected in star, as given earlier. So, this means that, the point N', star point on the load side is same as the star point, N of the load side. The phase currents (Fig. b) are obtained as,

$$I_{RN'} \angle -\phi_p = \frac{V_{RN} \angle 0^\circ}{Z_{RN'} \angle \phi_p} = \frac{V_{RN}}{Z_{RN'}} \angle -\phi_p$$

$$I_{YN'} \angle -(120^\circ + \phi_p) = \frac{V_{YN} \angle -120^\circ}{Z_{YN'} \angle \phi_p} = \frac{V_{YN}}{Z_{YN'}} \angle -(120^\circ + \phi_p)$$

$$I_{BN'} \angle (120^\circ - \phi_p) = \frac{V_{BN} \angle +120^\circ}{Z_{BN'} \angle \phi_p} = \frac{V_{BN}}{Z_{BN'}} \angle (120^\circ - \phi_p)$$

In this case, the phase voltage, V_{RN} is taken as reference. This shows that the phase currents are equal in magnitude, i.e., $(|I_p| = |I_{RN'}| = |I_{YN'}| = |I_{BN'}|)$, as the magnitudes of the voltage and load impedance, per phase, are same, with their phase angles displaced from each other in sequence by 120°. The magnitude of the phase currents, is expressed as $|I_p| = |(V_p / Z_p)|$. These phase currents are also line currents $(|I_L| = |I_R| = |I_Y| = |I_B|)$, in this case.

Unit-3

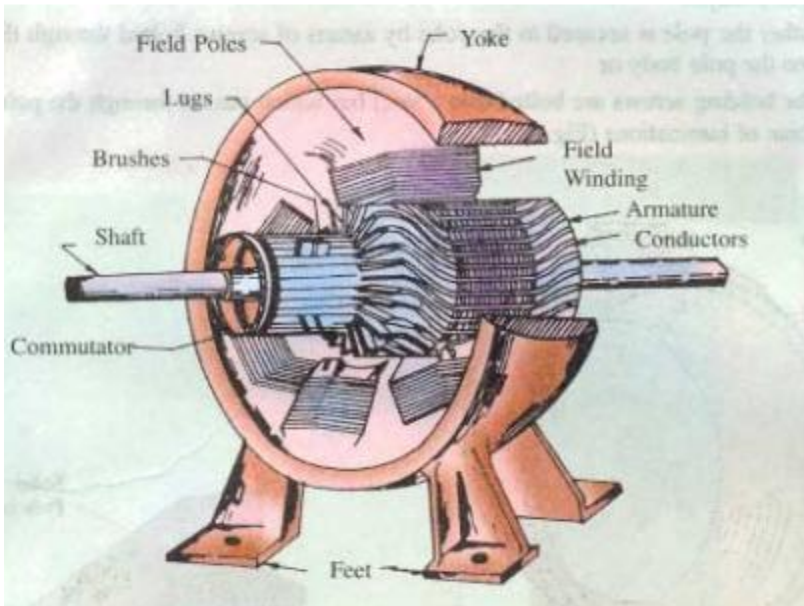
DC Motor:

An electrical motor is a machine which converts electrical energy in to mechanical energy.

Its action is based on the principle that when a current carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by FLEMINGS LEFT hand rule and whose magnitude is given by F=BIL Newton.

The same DC machine can be operated both like a generator as well as motor. So the construction of the machine will be same for both DC motor and DC generator.

Construction of Dc Machine:



The different parts of the machine and their working are explained as follows.

Yoke:

The outer frame of the machine is called yoke and it serves two purposes.

1) Mechanical support for the poles and a protecting cover for the whole machine.

2) It carries magnetic flux produced by pole. This is made of cast iron/cast steel or rolled steel.

Pole Cores and Pole Shoes:

These are built of the thin laminations of annealed steel which are riveted together under hydraulic pressure. The thickness of laminations vary from 1mm to 0.25mm. The laminated poles are secured to the yoke by means of screws bolted through the yoke into the pole body.

The pole shoe serve two purposes.

1) The spread out the flux in the air gap and also being of large cross section, reduce the reluctance of the magnetic path.

2) They support exciting (or) field coils.

Pole Coils:

The field or pole coils are former wound and placed on the pole core. The current passing through these coils electromagnetics the poles which produces the necessary flux that is cut by the revolving armature.

Armature Core:

The houses the armature coils and causes them to rotate and hence cut the magnetic flux.

It is a cylindrical or drum shaped and built up of circular sheet steel discs or laminations approximately

0.5mm thickness.

Usually the laminations are performed for air ducts which permits axials flow of air through the armature for cooling purpose. A complete circular lamination is made up of four or six or even eight segmental laminations. The two keyways are notched in each segment and are dovetailed or wedge shaped to make the laminations self locking in position. The purpose of laminations is to reduce the eddy current losses.

Armature winding:

These are usually former wound. These are first wound in the form of flat rectangular coils and are them pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots which are lined with tough insulating material, the slot insulation is folded over above the armature conductors placed in the slot and is secured in place by special hard wooden or fiber wedge.

Commutator:

The function of commutator is to facilitate collection of current(or supply) from armature conductors. It rectifies AC current in armature conductors into DC current in external load circuit if generator or rectifies DC current from supply to ac current in armature if motor. Commutator segments are insulated from each other by mica. Number of segments are equal to number of armature coils.

Brushes and Bearings:

Their function is to collect current from commutator. These are made of carbon or graphite.

Working :

When the field magnets are excited and its armature conductors are supplied with current from the supply mains, they experience a force tending to rotate the armature. Armature conductors under N pole are assumed to carry current downwards and those under S-pole carry current upwards. By applying FLEMINGS LEFT hand rule, the direction of the force on each conductor can be found. It will be seen that each conductor experiences a force F which tends to rotate the armature in anti-clockwise direction. These forces collectively produce a driving torque which sets the armature rotating.

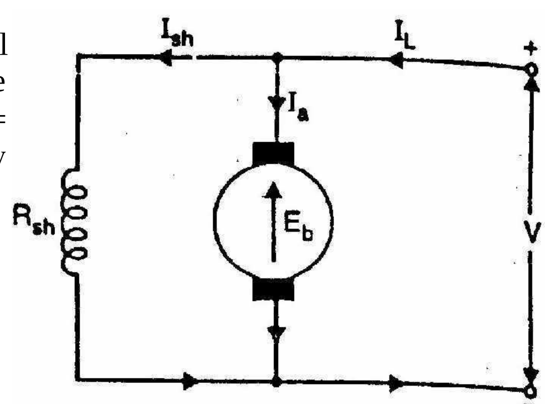
BACK OR COUNTER EMF

When the armature of a DC motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence EMF is induced in them as in a generator. The induced EMF acts in opposite direction to the applied voltage V (Lenz's law) and in known as back or counter EMF E_b. The back EMF E_b (= PΦZN/60 A) is always less than the applied voltage V, although this difference is small when the motor is running under normal conditions.

Net voltage across armature circuit = V - E_b

If R_a is the armature circuit resistance, then, $I_a = \dfrac{V - E_b}{R_a}$

Since V and R_a are usually fixed, the value of E_b will determine the current drawn by the motor. If the speed of the motor is high, then back EMF E_b (= PΦZN/60 A) is large and hence the motor will draw less armature current and vice-versa.



Voltage Equation of DC Motor

Let in a DC motor (See

Fig. 4.3), V = applied voltage
E_b = back EMF
R_a = armature resistance I_a = armature current
Since back EMF E_b acts in opposition to the applied voltage V, the net voltage across the armature circuit is V- E_b.
The armature current I_a is given by;

$$I_a = \frac{V - E_b}{R_a}$$

Or $V = E_b + I_aR_a$

This is known as voltage equation of the DC motor

Applications of DC Motors

The main applications of the three types of direct current motors are given below.

Series Motors

The series DC motors are used where high starting torque is required and variations in speed are possible. For example – the series motors are used in the traction system, cranes, air compressors, Vaccum Cleaner, Sewing machine, etc.

Shunt Motors

The shunt motors are used where constant speed is required and starting conditions are not severe. The various applications of DC shunt motor are in Lathe Machines, Centrifugal Pumps, Fans, Blowers, Conveyors, Lifts, Weaving Machine, Spinning machines, etc.

Compound Motors

The compound motors are used where higher starting torque and fairly constant speed is required. The examples of usage of compound motors are in Presses, Shears, Conveyors, Elevators, Rolling Mills, Heavy Planners, etc.

The small DC machines whose ratings are in fractional kilowatt are mainly used as control device such in techno generators for speed sensing and in servo motors for positioning and tracking.

Dc Generator

Working Principle

The **working principle of the DC generator** is based on Faraday's laws of **electromagnetic induction**. When a conductor is located in an unstable magnetic field, an electromotive force gets induced within the conductor. The induced e.m.f magnitude can be measured from the equation of **the electromotive force of a generator**.

If the conductor is present with a closed lane, the current which is induced will flow in the lane. In this generator, field coils will generate an electromagnetic field as well as the armature conductors are turned into the field. Therefore, an electromagnetically induced electromotive force (e.m.f) will be generated within the armature conductors. The path of induced current will be provided by **Fleming's right-hand rule**

EMF equation of a generator

Let P = number of poles

Ø = flux/pole in webers

Z = total number of armature conductors.

= number of slots x number of conductors/slot

N = armature rotation in revolutions (speed for armature) per minute (rpm)

A = No. of parallel paths into which the 'z' no. of conductors are divided.

E = emf induced in any parallel path

E_g = emf generated in any one parallel path in the armature.

Average emf generated/conductor = dØ/dt volt

Flux current/conductor in one revolution

dt = d x p

In one revolution, the conductor will cut total flux produced by all poles = d x p

No. of revolutions/second = N/60

Therefore, Time for one revolution, dt = 60/N second

According to Faraday's laws of Electromagnetic Induction, emf generated/conductor = dØ/dt=□ x p x N / 60 volts

This is emf induced in one conductor.

For a simplex wave-wound generator

No. of parallel paths = 2

No. of conductors in (series)in one path = Z/2

EMF generated/path = ØPN/60 x Z/2 = ØZPN/120 volt

For a simple lap-wound generator

Number of parallel paths = P

Number of conductors in one path = Z/P

EMF generated/path = ØPN/60 (Z/P) =

ØZN/60 A = 2 for simplex – wave winding

A =P for simplex lap-winding

Applications of DC Generators

The applications of the various types of DC Generators are as follows:-

Separately Excited DC Generators

- Separately excited DC Generators are used in laboratories for testing as they have a wide range of voltage output.

- Used as a supply source of DC motors.

Shunt wound Generators

- DC shunt-wound generators are used for lighting purposes.
- Used to charge the battery.
- Providing excitation to the alternators.

Series Wound Generators

- DC series wound generators are used in DC locomotives for regenerative braking for providing field excitation current.
- Used as a booster in distribution networks.
- Over compounded cumulative generators are used in lighting and heavy power supply.
- Flat compounded generators are used in offices, hotels, homes, schools, etc.
- Differentially compounded generators are mainly used for arc welding purpose

Transformers

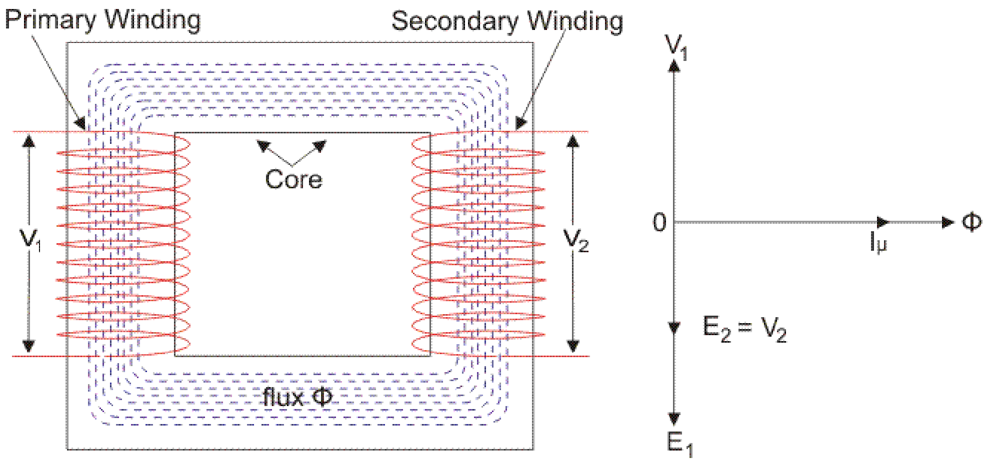
A **transformer** is a static electrical device that transfers electrical energy between two or more circuits through electromagnetic induction. A varying current in one coil of the transformer produces a varying magnetic field, which in turn induces a varying electromotive force (emf) or "voltage" in a second coil. Power can be transferred between the two coils, without a metallic connection between the two circuits. Transformers are used to increase or decrease the alternating voltages in electric power applications.

Ideal and Practical Transformer:

An *ideal transformer* is an imaginary transformer which does not have any loss in it, means no core losses, copper losses and any other losses in transformer. Efficiency of this transformer is considered as 100%. There is zero leakage reactance of transformer. As we said, whenever we place a low reluctance core inside the windings, maximum amount of flux passes through this core, but still there is some flux which does not pass through the core but passes through the insulation used in the transformer.

In an **ideal transformer**, this leakage flux is also considered nil. That means, 100% flux passes through the core and links with both the primary and secondary windings of transformer. Although every winding is desired to be purely inductive but it has some resistance in it which causes voltage drop and I^2R loss in it.

In such **ideal transformer model**, the windings are also considered ideal, that means resistance of the winding is zero. Now if an alternating source voltage V_1 is applied in the primary winding of that ideal transformer, there will be a counter self emf E_1 induced in the primary winding which is purely 180^o in phase opposition with supply voltage V_1. For developing counter emf E_1 across the primary winding, it draws current from the source to produce required magnetizing flux. As the primary winding is purely inductive, that current 90^o lags from the supply voltage. This current is called magnetizing current of transformer I_μ.



Secondary Induced Voltage Ideal Transformer

This alternating current I_μ produces an alternating magnetizing flux Φ which is proportional to that current and hence in phase with it. As this flux is also linked with secondary winding through the core of transformer, there will be another emf E_2 induced in the secondary winding, this is mutually induced emf. As the secondary is placed on the same core where the primary winding is placed, the emf induced in the secondary winding of transformer, E_2 is in the phase with primary emf E_1 and in phase opposition with source voltage V_1.

A practical transformer differs from the ideal transformer in many respects.

The practical transformer has

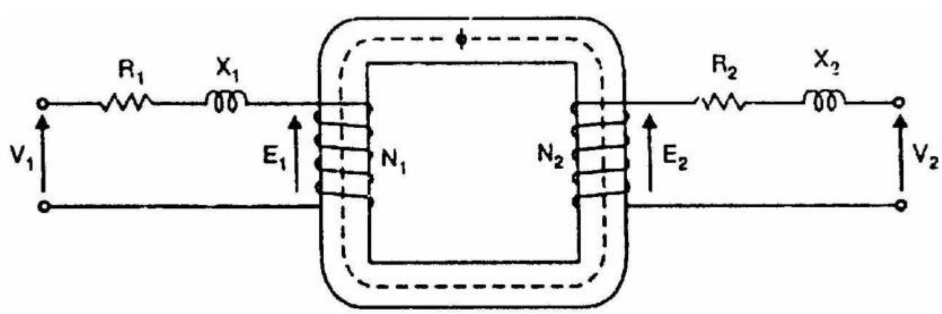
1. iron losses
2. winding resistances and
3. magnetic leakage, giving rise to leakage reactances.

1. Iron Losses

Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it. These two losses together are known as iron losses or core losses. The iron losses depend upon the supply frequency, maximum flux density in the core, volume of the core etc. It may be noted that magnitude of iron losses is quite small in a practical transformer.

2. Winding resistances

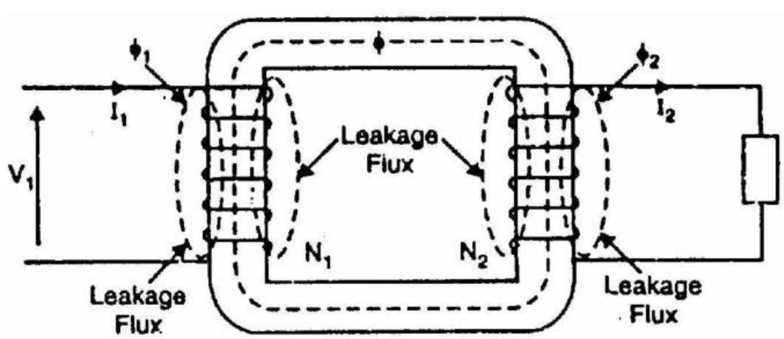
Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance R_1 and secondary resistance R_2 act in series with the respective windings as shown in figure. When current flows through the windings, there will be power loss as well as a loss in voltage due to IR drop. This will affect the power factor and E_1 will be less than V_1 while V_2 will be less than E_2.



3. Leakage reactances

Both primary and secondary currents produce flux. The flux ϕ which links both the windings is the useful flux and is called mutual flux. However, primary current would produce some flux ϕ which would not link the secondary winding. Similarly, secondary current would produce some flux ϕ that would not link the primary winding. The flux such as ϕ_1 or ϕ_2 which links only one winding is called leakage flux. The leakage flux paths are mainly through the air. The effect of these leakage fluxes would be the same as though inductive reactance were connected in series with each winding of transformer that had no leakage flux as shown in figure.

In other words, the effect of primary leakage flux ϕ_1 is to introduce an inductive reactance X_1 in series with the primary winding as shown. Similarly, the secondary leakage flux ϕ_2 introduces an inductive reactance X_2 in series with the secondary winding. There will be no power loss due to leakage reactance. However, the presence of leakage reactance in the windings changes the power factor as well as there is voltage loss due to IX drop.



Problems:

1. An ideal transformer has an input voltage of 480 V. The output current and voltage are 10 A and 120 V. Determine the value of input current.

$$I_1 = \frac{V_2 I_2}{V_1} = \frac{(120)(10)}{480} = 2.5 \text{ A}$$

2. A transformer has 600 primary turns connected to a 1.5 kV supply. Determine the number of secondary turns for a 240 V output voltage, assuming no losses.

For a transformer, $\frac{N_1}{N_2} = \frac{V_1}{V_2}$

Secondary turns, $N_2 = N_1\left(\frac{V_2}{V_1}\right) = (600)\left(\frac{240}{1500}\right) = \mathbf{96\ turns}$

3. An ideal transformer with a turns ratio 2:9 is fed from a 220 V supply. Determine its output voltage.

$\frac{N_1}{N_2} = \frac{2}{9}$ and $V_1 = 220\,\text{V}$

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which,

Output voltage, $V_2 = V_1\left(\frac{N_2}{N_1}\right) = (220)\left(\frac{9}{2}\right)$ = **990 V**

4. A transformer has 800 primary turns and 2000 secondary turns. If the primary voltage is 160 V, determine the secondary voltage assuming an ideal transformer.

$$\frac{N_1}{N_2} = \frac{800}{2000} \quad \text{and} \quad V_1 = 160\,\text{V}$$

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **output voltage**, $V_2 = V_1\left(\frac{N_2}{N_1}\right) = (160)\left(\frac{2000}{800}\right)$ = **400 V**

5. An ideal transformer with a turns ratio 3:8 has an output voltage of 640 V. Determine its input voltage.

$$\frac{N_1}{N_2} = \frac{3}{8} \quad \text{and} \quad V_2 = 640\,\text{V}$$

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **input voltage**, $V_1 = V_2\left(\frac{N_1}{N_2}\right) = (640)\left(\frac{3}{8}\right)$ = **240 V**

6. An ideal transformer has a turns ratio of 12:1 and is supplied at 192 V. Calculate the secondary voltage.

$$\frac{N_1}{N_2} = \frac{12}{1} \quad \text{and} \quad V_1 = 192\,\text{V}$$

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **output voltage**, $V_2 = V_1\left(\frac{N_2}{N_1}\right) = (192)\left(\frac{1}{12}\right)$ = **16 V**

7. A transformer primary winding connected across a 415 V supply has 750 turns. Determine how many turns must be wound on the secondary side if an output of 1.66 kV is required

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **secondary turns**, $N_2 = N_1\left(\frac{V_2}{V_1}\right) = (750)\left(\frac{1660}{415}\right)$ = **3000 turns**

8. An ideal transformer has a turns ratio of 15:1 and is supplied at 180 V when the primary current is 4 A. Calculate the secondary voltage and current

$$\frac{N_1}{N_2} = \frac{12}{1}, \quad V_1 = 220\,\text{V} \quad \text{and} \quad I_1 = 4\,\text{A}$$

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, **output voltage**, $V_2 = V_1\left(\frac{N_2}{N_1}\right) = (180)\left(\frac{1}{15}\right)$ = **12 V**

$\frac{N_1}{N_2} = \frac{I_2}{I_1}$ from which, **secondary current**, $I_2 = I_1\left(\frac{N_1}{N_2}\right) = (4)\left(\frac{15}{1}\right)$ = **60 A**

9. A 10 kVA, single-phase transformer has a turns ratio of 12:1 and is supplied from a 2.4 kV supply. Neglecting losses, determine (a) the full load secondary current, (b) the minimum value of load resistance which can be connected across the secondary winding without the kVA rating being exceeded, and (c) the primary current.

$$10000 = V_1 I_1 = V_2 I_2, \quad \frac{N_1}{N_2} = \frac{12}{1} \quad \text{and} \quad V_1 = 2400\,\text{V}$$

(a) $\frac{N_1}{N_2} = \frac{V_1}{V_2}$ from which, output voltage, $V_2 = V_1\left(\frac{N_2}{N_1}\right) = (2400)\left(\frac{1}{12}\right)$ = 200 V

10000 VA = $V_2 I_2$ = 200 I_2 from which, **secondary current**, $I_2 = \frac{10000}{200}$ = **50 A**

(b) **Load resistance**, $R_L = \frac{V_2}{I_2} = \frac{200}{50}$ = **4 Ω**

(c) $\frac{N_1}{N_2} = \frac{I_2}{I_1}$ from which, **primary current**, $I_1 = I_2\left(\frac{N_2}{N_1}\right) = (50)\left(\frac{1}{12}\right)$ = **4.17 A**

10. A transformer has 500 turns of the primary winding and 10 turns of the secondary winding.

a) Determine the secondary voltage if the secondary circuit is open and the primary voltage is 120 V.

b) Determine the current in the primary and secondary winding, given that the secondary winding is connected to a resistance load 15 Ω?

Given:

Number of turns of the primary coil: $N_1 = 500$
Number of turns of the secondary coil: $N_2 = 10$
a) Primary voltage: $U_1 = 120$ V
b) Secondary winding connected to a resistance load: $R = 15\ \Omega$

To determine:

a) Secondary voltage: U_2 = ? (V)
b) Current in the primary winding: I_1 = ? (A)
Current in the secondary winding: I_2 = ? (A)

To solve this, the ratio of the voltage on the secondary and on the primary coil is the same as the ratio of the number of turns of both coils:

$$k=U_2 / U_1=N_2 / N_1$$

So, unknown secondary voltage:

$$U_2= (N_2 / N_1)*U_1$$
$$U_2= (10/500)*120V=2.4V$$

The electric power P_1 is determined as follows

$$P_1=U_1I_1$$

The electric power P_2 can be expressed as:

$$P_2=U_2I_2= (U_2)^2 /R$$

We substitute into previous relationship and evaluate the primary current:

$$U_1I_1= (U_2)^2 / R$$
$$I_1= (U_2)^2 /U_1R$$

Primary current:

$$I_1=((2.4)^2/120)*15A=3.2mA$$

Secondary current:

$$I2=2.4 / 15 = 0.16A$$

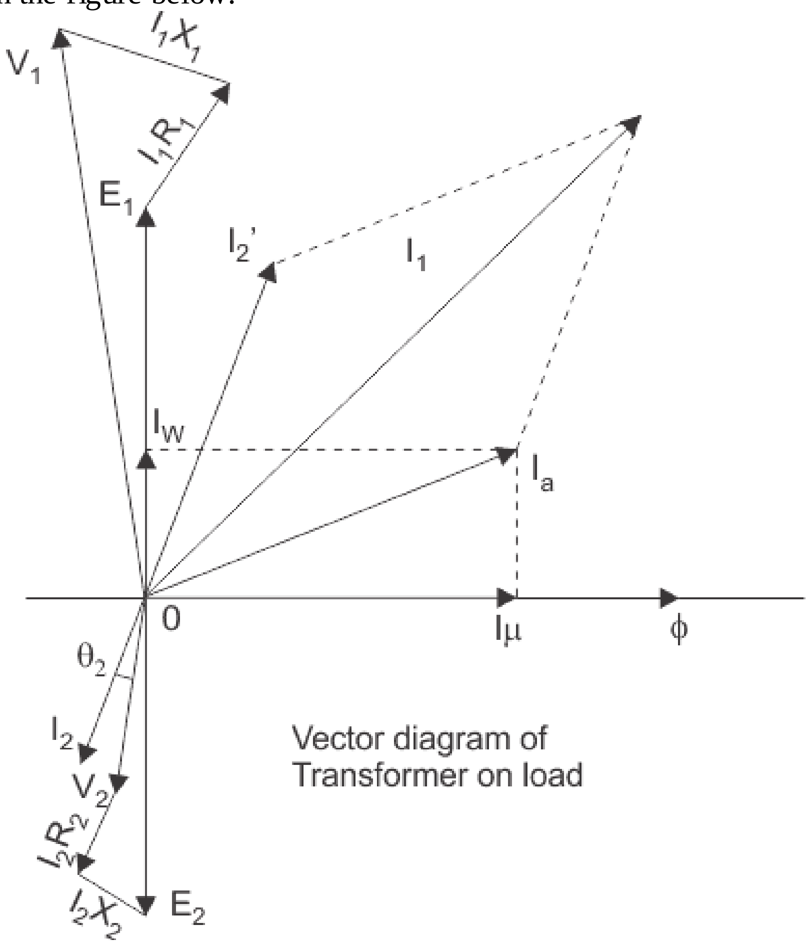
Equivalent Circuit of Transformer:

Equivalent impedance of transformer is essential to be calculated because the electrical power transformer is an electrical power system equipment for estimating different parameters of electrical power system which may be required to calculate total internal impedance of an electrical power transformer, viewing from primary side or secondary side as per requirement. This calculation requires equivalent circuit of transformer referred to primary or equivalent circuit of transformer referred to secondary sides respectively.

Percentage impedance is also very essential parameter of transformer. Special attention is to be given to this parameter during installing a transformer in an existing electrical power system. Percentage impedance of different power transformers should be properly matched during parallel operation of power transformers. The percentage impedance can be derived from equivalent impedance of transformer so, it can be said that equivalent circuit of transformer is also required during calculation of % impedance.

Equivalent Circuit of Transformer Referred to Primary

For drawing equivalent circuit of transformer referred to primary, first we have to establish general equivalent circuit of transformer then, we will modify it for referring from primary side. For doing this, first we need to recall the complete vector diagram of a transformer which is shown in the figure below.



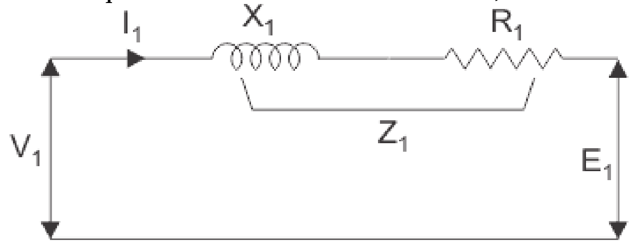
Let us consider the transformation ratio be

$$K = \frac{N_1}{N_2} = \frac{E_1}{E_2}$$

In the figure above, the applied voltage to the primary is V_1 and voltage across the primary winding is E_1. Total current supplied to primary is I_1. So the voltage V_1 applied to the primary is partly dropped by I_1Z_1 or $I_1R_1 + j.I_1X_1$ before it appears across primary winding.
The voltage appeared across winding is countered by primary induced emf E_1. So voltage equation of this portion of the transformer can be written as,

$$V_1 - (I_1R_1 + jI_1X_1) = E_1$$

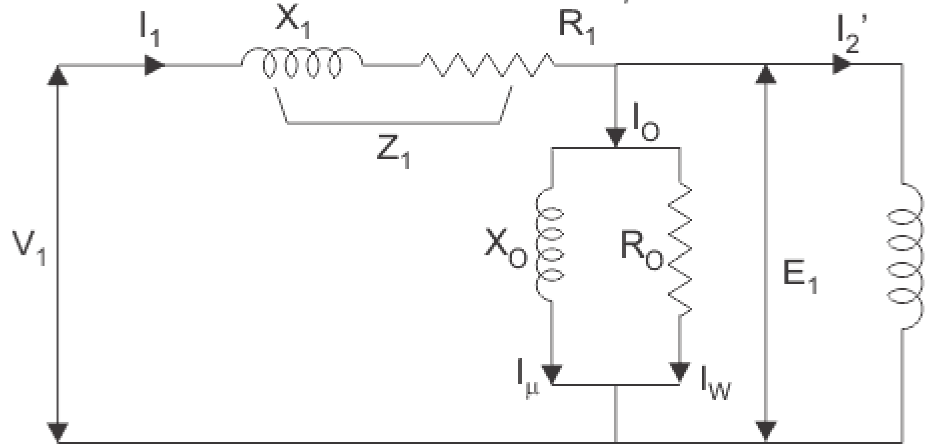
The equivalent circuit for that equation can be drawn as below,



Equivalent Circuit

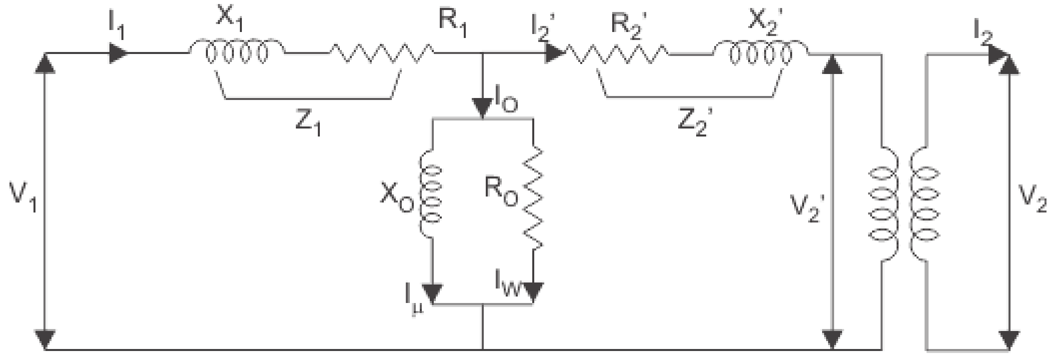
From the vector diagram above, it is found that the total primary current I_1 has two components, one is no - load component I_o and the other is load component I_2'. As this primary current has two components or branches, so there must be a parallel path with primary winding of transformer. This parallel path of current is known as excitation branch of equivalent circuit of transformer. The resistive and reactive branches of the excitation circuit can be represented as

$$R_0 = \frac{E_1}{I_w} \; and \; X_0 = \frac{E_1}{I_\mu}$$



Equivalent Circuit of Primary Side of Transformer

The load component I_2' flows through the primary winding of transformer and induced voltage across the winding is E_1 as shown in the figure right. This induced voltage E_1 transforms to secondary and it is E_2 and load component of primary current I_2' is transformed to secondary as secondary current I_2. Current of secondary is I_2. So the voltage E_2 across secondary winding is partly dropped by I_2Z_2 or $I_2R_2 + j.I_2X_2$ before it appears across load. The load voltage is V_2. The complete equivalent circuit of transformer is shown below.



Equivalent Circuit of Transformer referred to Primary

Now if we see the voltage drop in secondary from primary side, then it would be ′K′ times greater and would be written as $K.Z_2.I_2$. Again I_2'.

$$\Rightarrow I_2 = I_2' \frac{N_1}{N_2}$$

$N_1 = I_2.N_2 \Rightarrow I_2 = KI_2'$

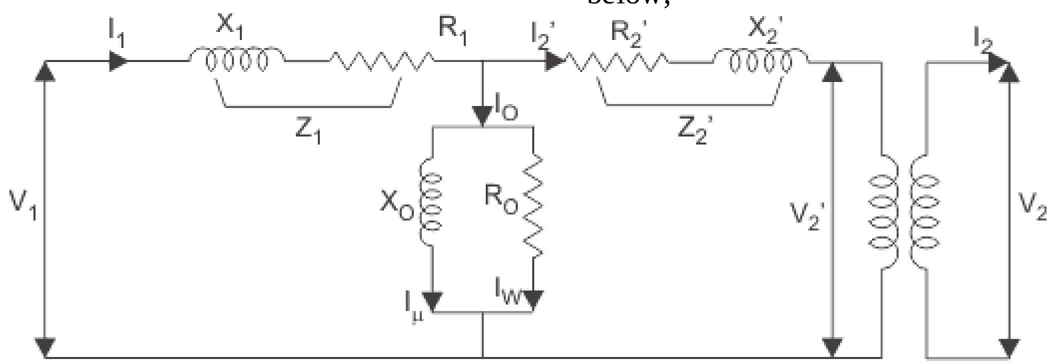
Therefore, $KZ_2I_2 = KZ_2KI_2' = K^2Z_2I_2'$

From above equation, secondary impedance of transformer referred to primary is,

$$Z_2' = K^2Z_2$$

$$Hence,\ R_2' = K^2R_2\ and\ X_2 = K^2X_2$$

So, the complete equivalent circuit of transformer referred to primary is shown in the figure below,



Equivalent Circuit of Transformer referred to Primary

Equivalent Circuit of Transformer Referred to Secondary

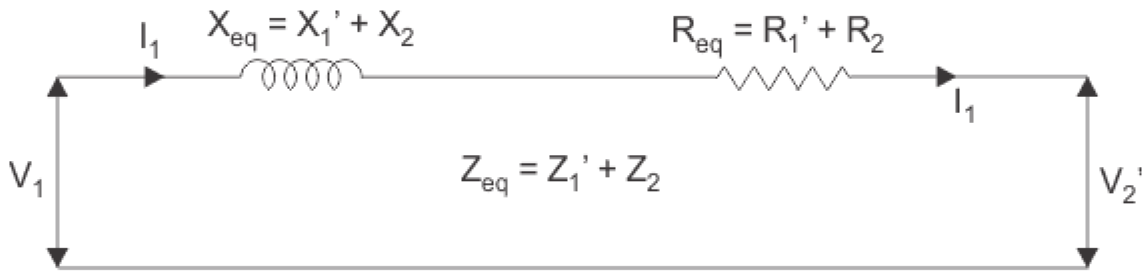
In a similar way, the approximate equivalent circuit of transformer referred to secondary can be drawn. Where equivalent impedance of transformer referred to secondary, can be derived as

$$Z_1 = \frac{Z_1}{K^2}$$

$$Therefore,\ R_1' = \frac{R_1}{K^2}$$

$$X_1' = \frac{X_1}{K^2}$$

$$Here,\ V_1' = \frac{V_1}{K}$$



Approximate Equivalent Circuit of Transformer referred to Secondary

Problems:

A 15-kVA, 2400:240-V, 60 Hz transformer has the following equivalent circuit parameters:

$R_1 = 2.5\Omega$; $R_2 = 0.025\Omega$; $X_1 = 7.0\Omega$; $X_2 = 0.070\Omega$ $R_c = 32\Omega$ $Xm = 11.5\Omega$

If the transformer is supplying a 10-kW, 0.8 PF lagging load at rated voltage, calculate: (a) Input voltage (b) Input current, and (c) Input power factor

Solution:

Reflect R_2 and X_2 to the primary side:.

$$R_2' = \left(\frac{N_1}{N_2}\right)^2 R_2 = \left(\frac{2400}{240}\right)^2 (0.025) = 2.5\ \Omega$$

$$X_2' = \left(\frac{N_1}{N_2}\right)^2 X_2 = \left(\frac{2400}{240}\right)^2 (0.070) = 7.0\ \Omega$$

(a,b) Assume $\bar{V}_2$ on reference; then,

$$a\bar{V}_2 = (10)(240\angle 0°) = 2400\angle 0°\ \text{V}$$

The load current is found by

$$I_2 = \frac{P_2}{V_2(PF)} = \frac{10{,}000}{240(0.8)} = 52.08\ \text{A}$$

$$\bar{I}_2 = I_2\angle -\cos^{-1}(0.8) = 52.08\angle -36.87°\ \text{A}$$

The reflected load current is

$$\bar{I}_2' = \frac{1}{10}\bar{I}_2 = 5.208\angle -36.87° \text{ A}$$

The excitation branch voltage is found by use of KVL.

$$\bar{E} = \bar{I}_2'\left(R_2' + jX_2'\right) + a\bar{V}_2 = \left(5.208\angle -36.87°\right)\left(7.433\angle 70.35°\right) + 2400\angle 0°$$

$$\bar{E} = 2432.381\angle 0.50° \text{ V}$$

The excitation branch current follows as

$$\bar{I}_o = \bar{I}_c + \bar{I}_m = \frac{\bar{E}}{R_c} + \frac{\bar{E}}{jX_m} = \frac{2432.381\angle 0.50°}{32{,}000} + \frac{2432.381\angle 0.50°}{11{,}500\angle 90°}$$

$$\bar{I}_o = 0.224\angle -69.70° \text{ A}$$

The input current can now be determined by KCL.

$$\bar{I}_1 = \bar{I}_2' + \bar{I}_o = 5.208\angle -36.87° + 0.224\angle -69.70°$$

$$\bar{I}_1 = 5.398\angle -38.16° \text{ A}$$

Application of KVL yields the input voltage.

$$\bar{V}_1 = \bar{I}_1\left(R_1 + jX_1\right) + \bar{E} = 5.398\angle -38.16°\left(7.433\angle 70.35°\right) + 2432.381\angle 0.50°$$

$$\bar{V}_1 = 2466.61\angle 0.99° \text{ V}$$

$$V_1 = 2466.61 \text{ V}$$

(*c*) With $\bar{V}_1$ and $\bar{I}_1$ known, the input power factor is given by

$$PF_{in} = \cos\left(\angle\bar{V}_1 - \angle\bar{I}_1\right) = \cos\left(0.99° + 38.16°\right) = 0.775 \text{ lagging}$$

Types of Losses in a Transformer:

There are various types of losses in the transformer such as iron losses, copper losses, hysteresis losses, eddy current losses, stray loss, and dielectric losses. The hysteresis losses occur because of the variation of the magnetization in the core of the transformer and the copper loss occur because of the transformer winding resistance.

Iron Losses

Iron losses are caused by the alternating flux in the core of the transformer as this loss occurs in the core it is also known as Core loss. Iron loss is further divided into hysteresis and eddy current loss.

Hysteresis Loss

The core of the transformer is subjected to an alternating magnetising force, and for each cycle of emf, a hysteresis loop is traced out. Power is dissipated in the form of heat known as hysteresis loss and given by the equation shown below

$$P_h = K\eta B_{max}^{1.6} f\, V \quad \text{watts}$$

Where

$K\eta$ is a proportionality constant which depends upon the volume and quality of the material of the core used in the transformer.

- f is the supply frequency
- Bmax is the maximum or peak value of the flux density

The iron or core losses can be minimised by using silicon steel material for the construction of the core of the transformer.

Eddy Current Loss

When the flux links with a closed circuit, an emf is induced in the circuit and the current flows, the value of the current depends upon the amount of emf around the circuit and the resistance of the circuit. Since the core is made of conducting material, these EMFs circulates currents within the body of the material. These circulating currents are called Eddy Currents. They will occur when the conductor experiences a changing magnetic field. As these currents are not responsible for doing any useful work, and it produces a loss (I^2R loss) in the magnetic material known as an Eddy Current Loss. The eddy current loss is minimised by making the core with thin laminations.

The equation of the eddy current loss is given as

$$P_e = K_e B_m^2 t^2 f^2\, V \qquad \text{watts}$$

Where,

- K_e – co-efficient of eddy current. Its value depends upon the nature of magnetic material like volume and resistivity of core material, thickness of laminations
- B_m – maximum value of flux density in wb/m^2
- T – thickness of lamination in meters
- F – frequency of reversal of magnetic field in Hz
- V – volume of magnetic material in m^3

Copper Loss Or Ohmic Loss:

These losses occur due to ohmic resistance of the transformer windings. If $I_1$and I_2 are the primary and the secondary current. R_1 and R_2 are the resistance of primary and secondary winding then the copper losses occurring in the primary and secondary winding will be $I_1^2R_1$ and $I_2^2R_2$ respectively.

Therefore, the total copper losses will be

$$P_c = I_1^2 R_1 + I_2^2 R_2$$

These losses varied according to the load and known hence it is also known as variable losses. Copper losses vary as the square of the load current.

Stray Loss:

The occurrence of these stray losses is due to the presence of leakage field. The percentage of these losses are very small as compared to the iron and copper losses so they can be neglected.

Dielectric Loss

Dielectric loss occurs in the insulating material of the transformer that is in the oil of the transformer, or in the solid insulations. When the oil gets deteriorated or the solid insulation get damaged, or its quality decreases, and because of this, the efficiency of transformer is effected

What is Voltage Regulation ?

The voltage regulation is the percentage of voltage difference between no load and full load voltages of a transformer with respect to its full load voltage.

Explanation of Voltage Regulation of Transformer

Say an electrical power transformer is open circuited, means load is not connected with secondary terminals. In this situation, the secondary terminal voltage of the transformer will be its secondary induced emf E_2. Whenever full load is connected to the secondary terminals of the transformer, rated current I_2 flows through the secondary circuit and voltage drop comes into picture. At this situation, primary winding will also draw equivalent full load current from source. The voltage drop in the secondary is I_2Z_2 where Z_2 is the secondary impedance of transformer.

Now if at this loading condition, any one measures the voltage between secondary terminals, he or she will get voltage V_2 across load terminals which is obviously less than no load secondary voltage E_2 and this is because of I_2Z_2 voltage drop in the transformer.

Expression of Voltage Regulation of Transformer, represented in percentage, is

$$Voltage\ regulation(\%) = \frac{E_2 - V_2}{V_2} \times 100\%$$

1. A 6 kVA, 100 V/500 V, single-phase transformer has a secondary terminal voltage of 487.5 V when loaded. Determine the regulation of the transformer.

$$\textbf{Regulation} = \frac{\text{no load secondary voltage} - \text{terminal voltage on load}}{\text{no load secondary voltage}} \times 100\%$$

$$= \frac{500 - 487.5}{500} \times 100\% = \frac{12.5}{500} \times 100\% = \mathbf{2.5\%}$$

2. A transformer has an open circuit voltage of 110 volts. A tap-changing device operates when the regulation falls below 3%. Calculate the load voltage at which the tap-changer operates.

$$\textbf{Regulation} = \frac{\text{no load secondary voltage} - \text{terminal voltage on load}}{\text{no load secondary voltage}} \times 100\%$$

Hence, $$3 = \frac{110 - V_2}{110} \times 100\%$$

from which, $$\frac{3(110)}{100} = 110 - V_2$$

and $$V_2 = 110 - \frac{3(110)}{100}$$ = **106.7 V = voltage at which the tap-changer operates.**

Efficiency Of Transformer

Just like any other electrical machine, efficiency of a transformer can be defined as the output power divided by the input power. That is efficiency = output / input .
Transformers are the most highly efficient electrical devices. Most of the transformers have full load efficiency between 95% to 98.5% . As a transformer being highly efficient, output and input are having nearly same value, and hence it is impractical to measure the efficiency of transformer by using output / input. A better method to find efficiency of a transformer is using, efficiency = (input - losses) / input = 1 - (losses / input).

Condition For Maximum Efficiency

Let, Copper loss = I12R1
Iron loss = Wi

$$\text{efficiency} = 1 - \frac{\text{losses}}{\text{input}} = 1 - \frac{I_1{}^2R_1 + W_i}{V_1I_1\cos\Phi_1}$$

$$\eta = 1 - \frac{I_1R_1}{V_1\cos\Phi_1} - \frac{W_i}{V_1I_1\cos\Phi_1}$$

differentiating above equation with respect to I_1

$$\frac{d\eta}{dI_1} = 0 - \frac{R_1}{V_1\cos\Phi_1} + \frac{W_i}{V_1I_1{}^2\cos\Phi_1}$$

$$\eta \text{ will be maximum at } \frac{d\eta}{dI_1} = 0$$

Hence efficiency η will be maximum at

$$\frac{R_1}{V_1\cos\Phi_1} = \frac{W_i}{V_1I_1{}^2\cos\Phi_1}$$

$$\frac{I_1{}^2R_1}{V_1I_1{}^2\cos\Phi_1} = \frac{W_i}{V_1I_1{}^2\cos\Phi_1}$$

$$I_1{}^2R_1 = W_i$$

Hence, efficiency of a transformer will be maximum when copper loss and iron losses are equal. That is Copper loss = Iron loss.

Problems:

1. A single-phase transformer has a voltage ratio of 6:1 and the h.v. winding is supplied at 540 V. The secondary winding provides a full load current of 30 A at a power factor of 0.8 lagging. Neglecting losses, find (a) the rating of the transformer, (b) the power supplied to the load, (c) the primary current.

$$\frac{V_1}{V_2} = \frac{6}{1} \quad \text{and} \quad V_1 = 540\,\text{V} \quad \text{hence,} \quad V_2 = \frac{540}{6} = 90\text{ V} \quad \text{and} \quad I_2 = 30\text{ A}$$

(a) **Rating of transformer** = $V_2 I_2 = 90 \times 30$ = **2700 VA** or **2.7 kVA**

(b) **Power supplied to load** = V I cos ϕ = (2700)(0.8) since power factor = cos ϕ = 0.8
= **2.16 kW**

(c) $\frac{V_1}{V_2} = \frac{I_2}{I_1}$ from which, **primary current,** $I_1 = I_2\left(\frac{V_2}{V_1}\right) = (30)\left(\frac{1}{6}\right)$ = **5 A**

2. A single-phase transformer is rated at 40 kVA. The transformer has full-load copper losses of 800 W and iron losses of 500 W. Determine the transformer efficiency at full load and 0.8

power factor.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{\text{input power} - \text{losses}}{\text{input power}} = 1 - \frac{\text{losses}}{\text{input power}}$$

Full-load output power = V I cos ϕ = (40)(0.8) = 32 kW
Total losses = 800 + 500 = 1.3 kW
Input power = output power + losses = 32 + 1.3 = 33.3 kW

Hence, **efficiency**, $\eta = 1 - \frac{1.3}{33.3}$ = 0.961 or **96.10%**

3. Determine the efficiency of the transformer in problem 2 at half full-load and 0.8 power factor.

Half full load power output = $\frac{1}{2}(40)(0.8)$ = 16 kW

Copper loss (or I^2R loss) is proportional to current squared

Hence, copper loss at half full load = $\left(\frac{1}{2}\right)^2(800)$ = 200 W

Iron loss == 500 W (constant)
Total loss = 200 + 500 = 700 W or 0.7 kW
Input power at half full load = output power at half full load + losses = 16 + 0.7 = 16.7 kW

Hence, efficiency, $\eta = 1 - \frac{\text{losses}}{\text{input power}} = 1 - \frac{0.7}{16.7}$ = 0.9581 or **95.81%**

4. A 100 kVA, 2000 V/400 V, 50 Hz, single-phase transformer has an iron loss of 600 W and a full- load copper loss of 1600 W. Calculate its efficiency for a load of 60 kW at 0.8 power factor.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{\text{input power} - \text{losses}}{\text{input power}} = 1 - \frac{\text{losses}}{\text{input power}}$$

Full-load output power = V I cos ϕ = (100)(0.8) = 80 kW
Load power = 60 kW

Hence the transformer is at $\frac{60}{80} = \frac{3}{4}$ full load

Hence, copper loss at 3/4 load = $\left(\frac{3}{4}\right)^2(1600)$ = 900 W

Total losses = 900 + 600 = 1.5 kW
Input power = output power + losses = 60 + 1.5 = 61.5 kW

Hence, **efficiency**, $\eta = 1 - \frac{1.5}{61.5}$ = 0.9756 or **97.56%**

All Day Efficiency Of Transformer:

As we have seen above, ordinary or commercial efficiency of a transformer can be given as

$$\text{ordinary efficiency} = \frac{\text{output (in watts)}}{\text{input (in watts)}}$$

But in some types of transformers, their performance can not be judged by this efficiency. For example, distribution transformers have their primaries energized all the time. But, their secondaries supply little load all no-load most of the time during day. That is, when secondaries of transformer are not supplying any load (or supplying only little load), then only core losses of transformer are considerable and copper losses are absent (or very little).

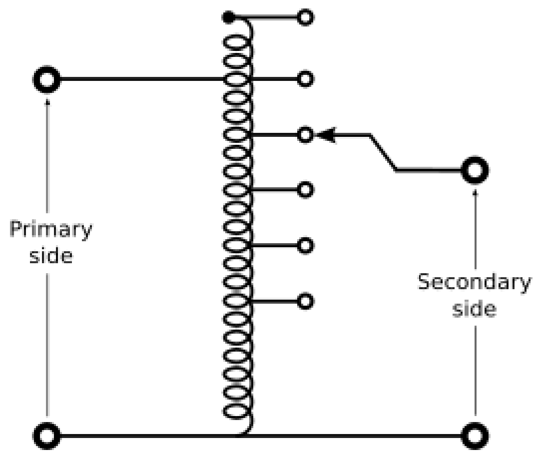
Copper losses are considerable only when transformers are loaded. Thus, for such transformers copper losses are relatively less important. The performance of such transformers is compared on the basis of energy consumed in one day.

$$\text{All day efficiency } = \frac{\text{output (in kWh)}}{\text{input (in kWh)}} \quad \text{(for 24 hours)}$$

All day efficiency of a transformer is always less than ordinary efficiency of it.

AUTO – TRANSFORMER:

An **Auto-transformer** (sometimes called ***auto-step down transformer***)[1] is an electrical transformer with only one winding. The "auto" prefix refers to the single coil acting alone and **not** to any kind of automatic mechanism. In an autotransformer, portions of the same winding act as both the primary and secondary sides of the transformer. In contrast, an ordinary transformer has separate primary and secondary windings which are not electrically connected.



The winding has at least three taps where electrical connections are made. Since part of the winding does "double duty", autotransformers have the advantages of often being smaller, lighter, and cheaper than typical dual-winding transformers, but the disadvantage of not providing electrical isolation between primary and secondary circuits. Other advantages of autotransformers include lower leakage reactance, lower losses, lower excitation current, and increased VA rating for a given size and mass.

The primary voltage is applied across two of the terminals, and the secondary voltage taken from two terminals, almost always having one terminal in common with the primary voltage. The primary and secondary circuits therefore have a number of windings turns in common. Since the volts-per-turn is the same in both windings, each develops a voltage in proportion to its number of turns.

In an autotransformer part of the current flows directly from the input to the output, and only part is transferred inductively, allowing a smaller, lighter, cheaper core to be used as well as requiring only a single winding.

Now we will discuss the savings of copper in auto transformer compared to conventional two winding transformer. We know that weight of copper of any winding depends upon its length and cross-sectional area. Again length of conductor in winding is proportional to its number of turns and cross-sectional area varies with rated current. So weight of copper in winding is directly proportional to product of number of turns and rated current of the winding.

Therefore, weight of copper in the section AC proportional to, $(N_1 - N_2)I_1$ and similarly, weight of copper in the section BC proportional to,

$$N_2(I_2 - I_1)$$

Hence, total weight of copper in the winding of auto transformer proportional to,

$(N_1 - N_2)I_1 + N_2(I_2 - I_1)$

$\Rightarrow N_1I_1 - N_2I_1 + N_2I_2 - N_2I_1$

$\Rightarrow N_1I_1 + N_2I_2 - 2N_2I_1$

$\Rightarrow 2N_1I_1 - 2N_2I_1 (Since,\ N_1I_1 = N_2I_2)$

$\Rightarrow 2(N_1I_1 - N_2I_1)$

In similar way it can be proved, the weight of copper in two winding transformer is proportional $N_1I_1 - N_2I_2$

to, $\Rightarrow 2N_1I_1 \qquad (Since,\ in\ a\ transformer\ N_1I_1 = N_2I_2)$

$N_1I_1 + N_2I_2$

$\Rightarrow 2N_1I_1$ (Since, in a transformer $N_1I_1 = N_2I_2$)

Let's assume, W_a and W_{tw} are weight of copper in auto transformer and two winding transformer respectively,

$$Hence,\ \frac{W_a}{W_{tw}} = \frac{2(N_1I_1 - N_2I_1)}{2(N_1I_1)}$$

$$= \frac{N_1I_1 - N_2I_1}{N_1I_1} = 1 - \frac{N_2I_1}{N_1I_1}$$

$$= 1 - \frac{N_2}{N_1} = 1 - k$$

$$\therefore W_a = W_{tw}(1 - k)$$

$$\Rightarrow W_a = W_{tw} - kW_{tw}$$

$\therefore$ Saving of copper in auto transformer compared to two winding transformer,

$$\Rightarrow W_{tw} - W_a = kW_{tw}$$

Advantages of using Auto Transformers

1. For transformation ratio = 2, the size of the **auto transformer** would be approximately 50% of the corresponding size of two winding transformer. For transformation ratio say 20 however the size would be 95 %. The saving in cost of the material is of course not in the same proportion. The saving of cost is appreciable when the ratio of transformer is low, that is lower than 2. Thus auto transformer is smaller in size and cheaper.
2. An auto transformer has higher efficiency than two winding transformer. This is because of less ohmic loss and core loss due to reduction of transformer material.
3. Auto transformer has better voltage regulation as voltage drop in resistance and reactance of the single winding is less.

Disadvantages of Using Auto Transformer

1. Because of electrical conductivity of the primary and secondary windings the lower voltage circuit is liable to be impressed upon by higher voltage. To avoid breakdown in the lower voltage circuit, it becomes necessary to design the low voltage circuit to withstand higher voltage.
2. The leakage flux between the primary and secondary windings is small and hence the impedance is low. This results into severer short circuit currents under fault conditions.
3. The connections on primary and secondary sides have necessarily needs to be same, except when using interconnected starring connections. This introduces complications due to changing primary and secondary phase angle particularly in the case of delta/delta connection.
4. Because of common neutral in a star/star connected auto transformer it is not possible to earth neutral of one side only. Both their sides should have their neutrality either earth or isolated.
5. It is more difficult to maintain the electromagnetic balance of the winding when voltage adjustment tappings are provided. It should be known that the provision of tapping on an auto transformer increases considerably the frame size of the transformer. If the range of tapping is very large, the advantages gained in initial cost is lost to a great event.

Applications of Auto Transformers

1. Compensating voltage drops by boosting supply voltage in distribution systems.
2. Auto transformers with a number of tapping are used for starting induction and synchronous motors.
3. **Auto transformer** is used as variac in laboratory or where continuous variable over broad ranges are required.

Problems:

1. A single-phase auto transformer has a voltage ratio of 480 V:300V and supplies a load of 30 kVA at 300 V. Assuming an ideal transformer, calculate the current in each section of the winding.

Rating = 30 kVA = $V_1 I_1 = V_2 I_2$

Hence, **primary current**, $I_1 = \frac{30 \times 10^3}{480}$ = **62.5 A**

and **secondary current**, $I_2 = \frac{30 \times 10^3}{300}$ = **100 A**

Hence, **current in common part of winding =** $I_2 - I_1$ = 100 – 62.5 = **37.5 A**

2. Calculate the saving in the volume of copper used in an auto transformer compared with a double-wound transformer for (a) a 300 V:240 V transformer, and (b) a 400 V:100 V transformer.

(a) For a 300 V:240 V transformer, $x = \frac{V_2}{V_1} = \frac{240}{300} = 0.80$

From equation (20.12), volume of copper in auto transformer

= (1 – 0.80)(volume of copper in a double-wound transformer)

= (0.20)(volume of copper in a double-wound transformer)

Hence, saving is **80%**

(b) For a 400 V:1000 V transformer, $x = \frac{V_2}{V_1} = \frac{100}{400} = 0.25$

From equation (20.12), volume of copper in auto transformer

= (1 – 0.25)(volume of copper in a double-wound transformer)

= (0.75)(volume of copper in a double-wound transformer)

Hence, saving is **25%**

Three Phase Transformer Connection:

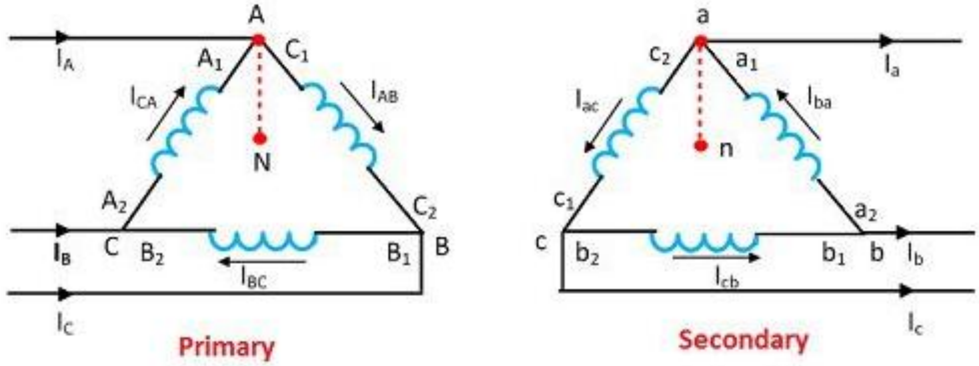
The three phase transformer consists three transformers either separate or combined with one core. The primary and secondary of the transformer can be independently connected either in star or delta. There are four possible connections for a 3-phase transformer bank.

1. Δ – Δ (Delta – Delta) Connection
2. Y – Y (Star – Star) Connection
3. Δ – Y (Delta – Star) Connection
4. Y – Δ (Star – Delta) Connection

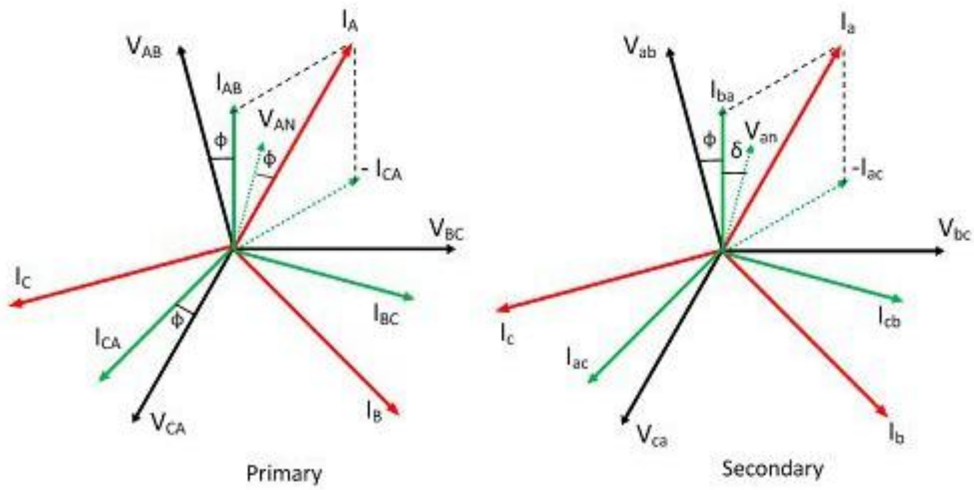
The choice of connection of three phase transformer depends on the various factors likes the availability of a neutral connection for grounding protection or load connections, insulation to ground and voltage stress, availability of a path for the flow of third harmonics, etc. The various types of connections are explained below in details.

1. Delta-Delta (Δ-Δ) Connection:

The delta-delta connection of three identical single phase transformer is shown in the figure below. The secondary winding a_1a_2 is corresponding to the primary winding A_1A_2, and they have the same polarity. The polarity of the terminal **a** connecting $\mathbf{a_1}$ and $\mathbf{c_2}$ is same as that connecting $\mathbf{A_1}$ and $\mathbf{C_2}$. The figure below shows the phasor diagram for lagging power factor **cosφ**.



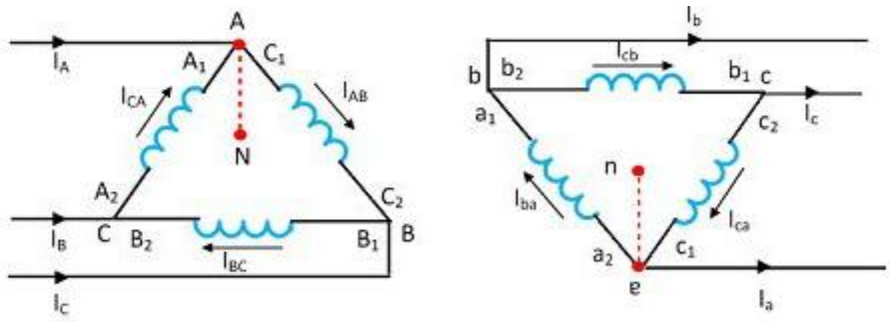
Delta-Delta Connection of Transformer



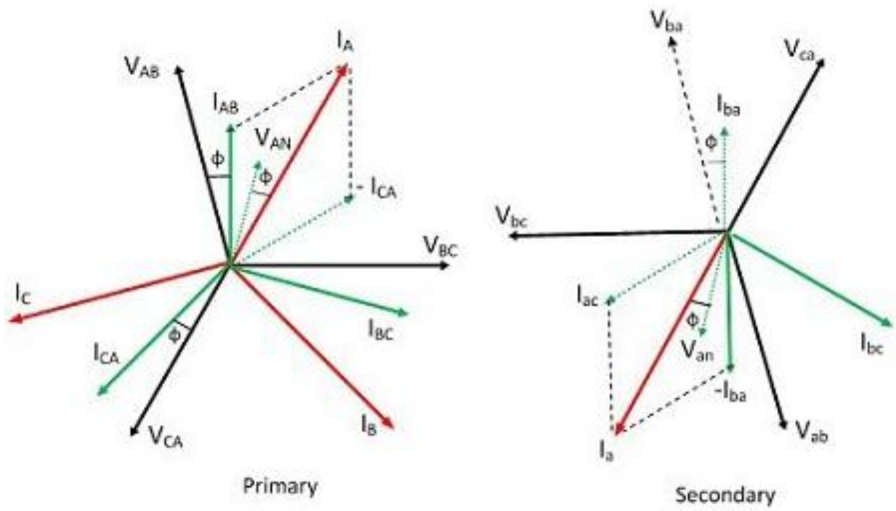
Phasor Diagram of Delta-Delta Connection of Transformer

The magnetising current and voltage drops in impedances have been neglected. Under the balanced condition, the line current is √3 times the phase winding current. In this configuration, the corresponding line and phase voltage are identical in magnitude on both primary and secondary sides. The secondary line-to-line voltage is in phase with the primary line-to-line voltage with a voltage ratio equal to the turns ratio.

If the connection of the phase windings is reversed on either side, the phase difference of 180° is obtained between the primary and the secondary system. Such a connection is known as an 180° connection. The delta-delta connection with 180° phase shift is shown in the figure below. The phasor diagram of a three phase transformer shown that the secondary voltage is in phase opposition with the primary voltage.



180° Phase Shift of Delta-Delta Connection of Transformer



180 Phase Shift of Delt Delta Connection of Transformer

The delta-delta transformer has no phase shift associated with it and problems with unbalanced loads or harmonics.

Advantages of delta–delta connection of transformer:

The following are the advantages of the delta-delta configuration of transformers.

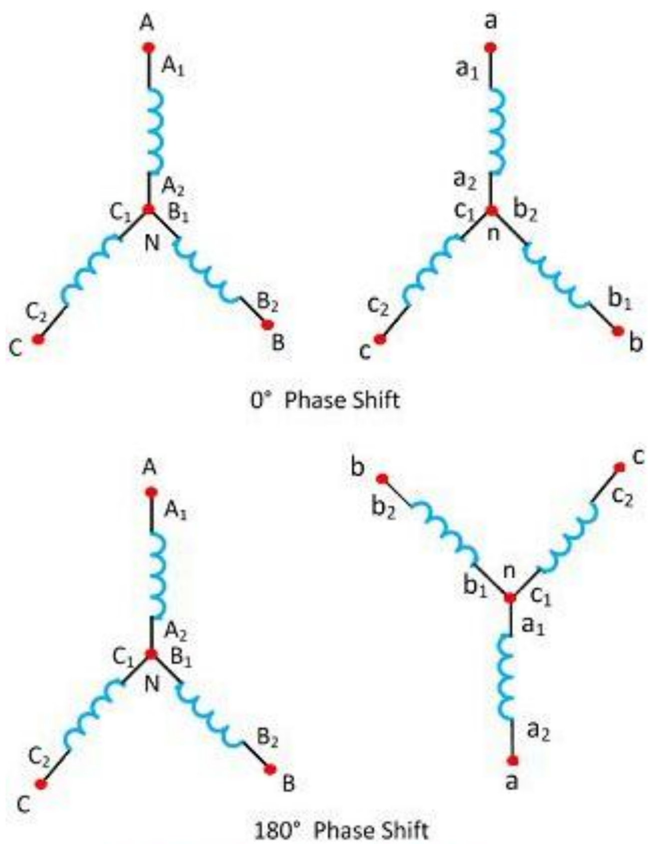
1. The delta-delta transformer is satisfactory for a balanced and unbalanced load.
2. If one transformer fails, the remaining two transformers will continue to supply the three-phase power. This is called an open delta connection.
3. If third harmonics present, then it circulates in a closed path and therefore does not appear in the output voltage wave.

The only **disadvantage** of the delta-delta connection is that there is no neutral. This connection is useful when neither primary nor secondary requires a neutral and the voltage are low and moderate.

2. Star-Star (Y-Y) Connection of Transformer

The star-star connection of three identical single phase transformer on each of the primary and secondary of the transformer is shown in the figure below. The phasor diagram is similar as in delta-delta connection.

Connection	Phase Voltage	Line Volatge	Phase Current	Line Current
Star	$V_P = V_L \div \sqrt{3}$	$V_L = \sqrt{3} \times V_P$	$I_P = I_L$	$I_L = I_P$
Delta	$V_P = V_L$	$V_L = V_P$	$I_P = I_L \div \sqrt{3}$	$I_L = \sqrt{3} \times I_P$



Star-Star Connection of Transformer

The phase current is equal to the line current, and they are in phase. The line voltage is three times the phase voltage. There is a phase separation of 30º between the line and phase voltage. The 180º phase shift between the primary and secondary of the transformer is shown in the figure above.

Problems Associated With Star-Star Connection

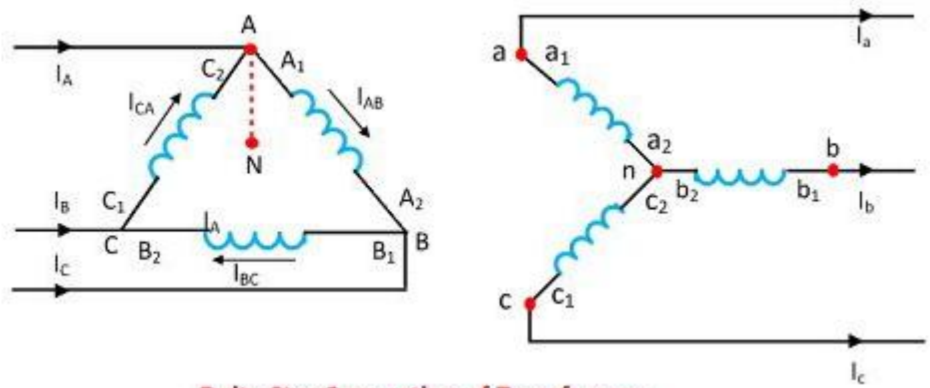
The star-star connection has two very serious problems. They are

1. The Y-Y connection is not satisfactory for the unbalance load in the absence of a neutral connection. If the neutral is not provided, then the phase voltages become severely unbalance when the load is unbalanced.
2. The Y-Y connection contains a third harmonics, and in balanced conditions, these harmonics are equal in magnitude and phase with the magnetising current. Their sum at the neutral of star connection is not zero, and hence it will distort the flux wave which will produce a voltage having a harmonics in each of the transformers.

The unbalanced and third harmonics problems of Y-Y connection can be solved by using the solid ground of neutral and by providing tertiary windings.

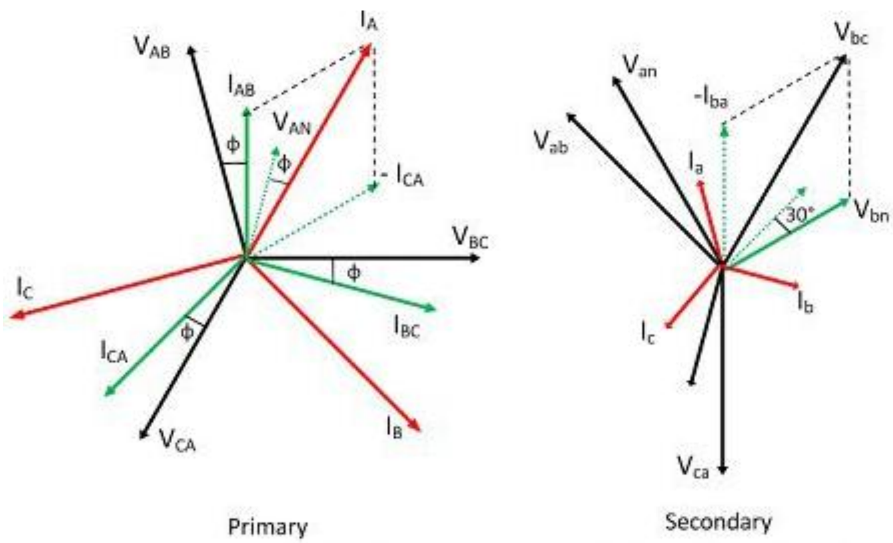
3. Delta-Star (Δ-Y) Connection

The Δ-Y connection of the three winding transformer is shown in the figure below. The primary line voltage is equal to the secondary phase voltage. The relation between the secondary voltages is $V_{LS} = \sqrt{3}\, V_{PS}$.



Delta Star Connection of Transformer

The phasor diagram of the Δ-Y connection of the three phase transformer is shown in the figure below. It is seen from the phasor diagram that the secondary phase voltage V_{an} leads the primary phase voltage V_{AN} by 30°. Similarly, V_{bn} leads V_{BN} by 30° and V_{cn} leads V_{CN} by 30°.This connection is also called +30° connection.



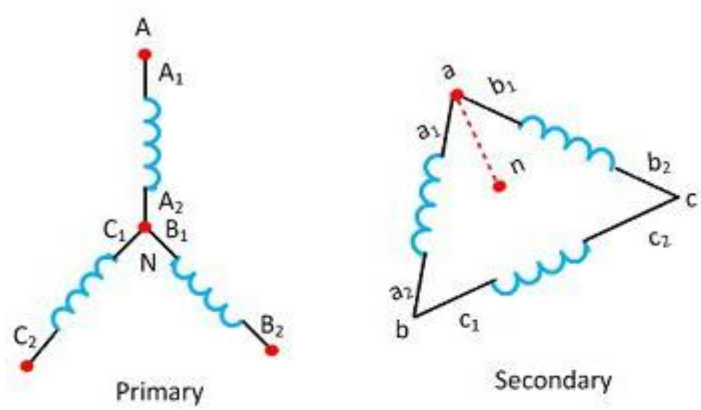
Phasor Diagram of Delta-Star Connection of Transformer

By reversing the connection on either side, the secondary system voltage can be made to lag the primary system by 30°. Thus, the connection is called -30° connection.

4. Star-Delta (Y-Δ) Connection

The star-delta connection of three phase transformer is shown in the figure above. The primary line voltage is √3 times the primary phase voltage. The secondary line voltage is equal to the secondary phase voltage. The voltage ratio of each phase is

$$\frac{V_{pP}}{V_{pS}} = a$$



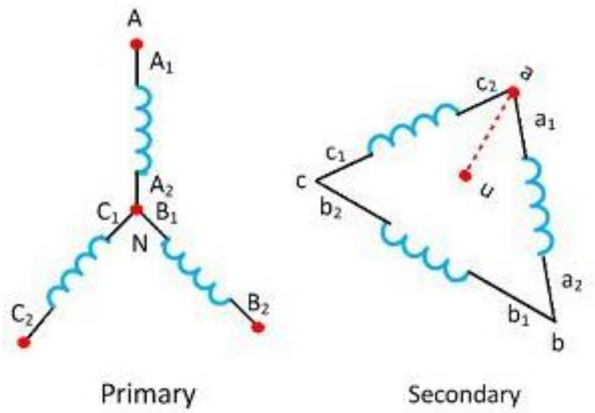
Star-Delta Connection of Transformer

Therefore line-to-line voltage ratio of Y-Δ connection is

$$\frac{V_{lP}}{V_{LS}} = \frac{\sqrt{3}V_{pP}}{V_{pS}} = \sqrt{3}a$$

The phasor diagram of the configuration is shown in the figure above. There is a phase shift of 30 lead exists between respective phase voltage. Similarly, 30° leads exist between respective phase voltage. Thus the connection is called +30° connection.

The phase shows the star-delta connection of transformer for a phase shift of 30° lag. This connection is called – 30° connection. This connection has no problem with the unbalanced load and thirds harmonics. The delta connection provided balanced phase on the Y side and provided a balanced path for the circulation of third harmonics without the use of the neutral wire.



Star-Delta Connetion of a transformer

Primary-Secondary Configuration	Line Voltage Primary or Secondary	Line Current Primary or Secondary
Delta – Delta	$V_L \Rightarrow nV_L$	$I_L \Rightarrow \frac{I_L}{n}$
Delta – Star	$V_L \Rightarrow \sqrt{3}.nV_L$	$I_L \Rightarrow \frac{I_L}{\sqrt{3}.n}$
Star – Delta	$V_L \Rightarrow \frac{nV_L}{\sqrt{3}}$	$I_L \Rightarrow \sqrt{3}.\frac{I_L}{n}$

Star – Star	$V_L \Rightarrow nV_L$	$I_L \Rightarrow \dfrac{I_L}{n}$

Where: n equals the transformers "turns ratio" (T.R.) of the number of secondary windings N_S, divided by the number of primary windings N_P. (N_S/N_P) and V_L is the line-to-line voltage with V_P being the phase-to-neutral voltage

Problem: A three-phase transformer has 600 primary turns and 150 secondary turns. If the supply voltage is 1.5 kV determine the secondary line voltage on no-load when the windings are connected (a) delta-star, (b) star-delta.

(a) For a delta connection, $V_L = V_P$

hence, primary phase voltage, $V_{P_1} = 1.5\text{ kV} = 1500\text{ V}$

Secondary phase voltage, $V_{P_2} = V_{P_1}\left(\dfrac{N_2}{N_1}\right) = (1500)\left(\dfrac{150}{600}\right) = 375\text{ V}$

For a star connection, $V_L = \sqrt{3}\,V_P$

hence, **secondary line voltage** $= \sqrt{3}(375) =$ **649.5 V**

(b) For a star connection, $V_L = \sqrt{3}\,V_P$ or $V_P = \dfrac{V_L}{\sqrt{3}}$

Primary phase voltage, $V_{P_1} = \dfrac{V_{L_1}}{\sqrt{3}} = \dfrac{1500}{\sqrt{3}} = 866.0\text{ V}$

For a delta connection, $V_L = V_P$

$\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2}$ from which, secondary phase voltage, $V_{P_2} = V_{P_1}\left(\dfrac{N_2}{N_1}\right) = (866.0)\left(\dfrac{150}{600}\right)$

$=$ **216.5 V = secondary line voltage**

UNIT – 4

THREE PHASE INDUCTION MOTORS

INTRODUCTION

The three-phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control. We usually prefer d.c. motors when large speed variations are required. Nevertheless, the 3-phase induction motors are simple, rugged, low-priced, easy to maintain and can be manufactured with characteristics to suit most industrial requirements. In this chapter, we shall focus our attention on the general principles of 3-phase induction motors.

THREE-PHASE INDUCTION MOTOR

Stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding). Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name. The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a "transformer-type" a.c. machine in which electrical energy is converted into mechanical energy.

Advantages

(i) It has simple and rugged construction.
(ii) It is relatively cheap.
(iii) It requires little maintenance.
(iv) It has high efficiency and reasonably good power factor.
(v) It has self starting torque.

Disadvantages

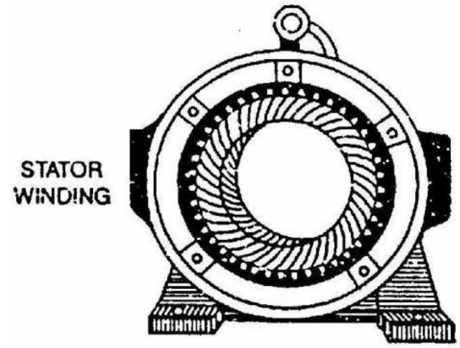
(i) It is essentially a constant speed motor and its speed cannot be changed easily.
(ii) Its starting torque is inferior to d.c. shunt motor.

Construction

A 3-phase induction motor has two main parts (i) stator and (ii) rotor. The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor.

Stator

It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations [See Fig. (8.1)]. The insulated connected to form a balanced 3-phase star or delta connected circuit. The 3-phase stator winding is wound for a definite number of poles as per requirement of speed. Greater the number of poles, lesser is the speed of the motor and vice-versa. When 3-phase supply is given to the stator winding, a rotating magnetic field (See Sec. 8.3) of constant magnitude is produced. This rotating field induces currents in the rotor by electromagnetic induction.



Rotor

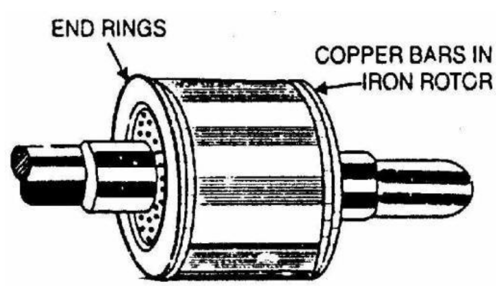
The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types:

(i) Squirrel cage type (ii) Wound type

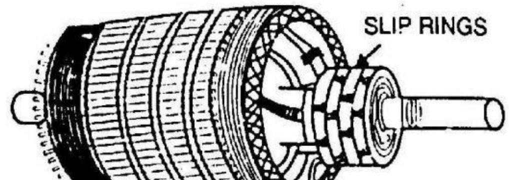
(i) **Squirrel cage rotor**.

It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminum bar is placed in each slot. All these bars are joined at each end by metal rings called end rings [See Fig. (8.2)]. This forms a permanently short-circuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator.

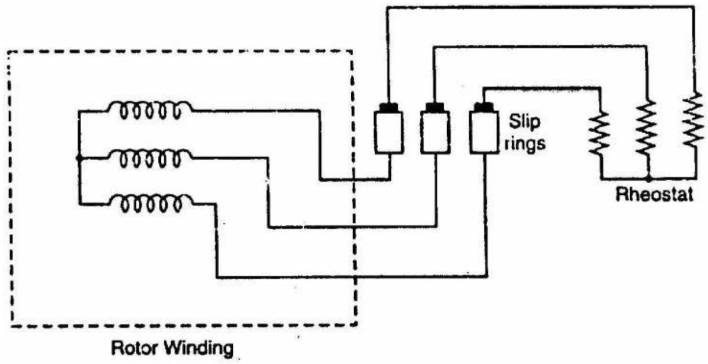
Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of 3-phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque.



(ii) **Wound rotor**. It consists of a laminated cylindrical core and carries a 3- phase winding, similar to the one on the stator [See Fig. (8.3)]. The rotor winding is uniformly distributed in the slots and is usually star-connected. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The three brushes are connected to a 3-phase star-connected rheostat as shown in Fig. (8.4). At starting, the



external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed.



The external resistances are used during starting period only. When the motor attains normal speed, the three brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.

Rotating Magnetic Field Due to 3-Phase Currents

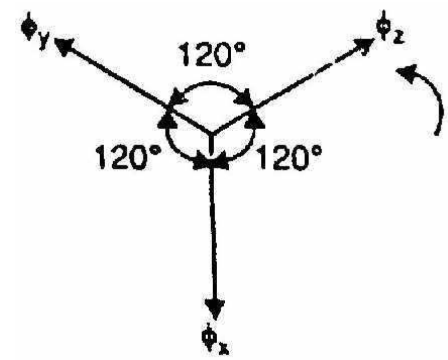
When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do no remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating Held. It can be shown that magnitude of this rotating field is constant and is equal to 1.5 ϕm where ϕm is the maximum flux due to any phase.

To see how rotating field is produced, consider a 2-pole, 3-phase winding as shown in Fig. The three phases X, Y and Z are energized from a 3-phase source and currents in these phases are indicated as I_x, I_y and I_z [See Fig.]. Referring to Fig, the fluxes produced by these currents are given by:

$$\phi_x = \phi_m \sin \omega t$$
$$\phi_y = \phi_m \sin (\omega t - 120°)$$
$$\phi_z = \phi_m \sin (\omega t - 240°)$$

Here ϕm is the maximum flux due to any phase. Fig. shows the phasor diagram of the three fluxes. We shall now prove that this 3-phase supply produces a rotating field of constant magnitude equal to 1.5 ϕ_m.

Speed of rotating magnetic field



The speed at which the rotating magnetic field revolves is called the synchronous speed (Ns). Referring to Fig. (8.6 (ii)), the time instant 4 represents the completion of one-quarter cycle of alternating current Ix from the time instant 1. During this one quarter cycle, the field has rotated through 90°. At a time instant represented by 13 or one complete cycle of current Ix from the origin, the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, fur P poles, the rotating field makes one revolution in P/2 cycles of current.

$$\therefore \quad \text{Cycles of current} = \frac{P}{2} \times \text{revolutions of field}$$

$$\text{or} \quad \text{Cycles of current per second} = \frac{P}{2} \times \text{revolutions of field per second}$$

Since revolutions per second is equal to the revolutions per minute (Ns) divided by 60 and the number of cycles per second is the frequency f,

$$\therefore \quad f = \frac{P}{2} \times \frac{N_s}{60} = \frac{N_s P}{120}$$

$$\text{or} \quad N_s = \frac{120\ f}{P}$$

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

PRINCIPLE OF OPERATION

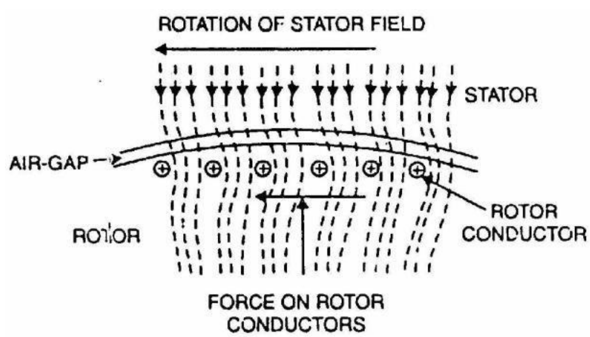
Consider a portion of 3-phase induction motor as shown in Fig. (8.13). The operation of the motor can be explained as under:

(i) When 3-phase stator winding is energized from a 3-phase supply, a rotating magnetic field is set up which rotates round the stator at synchronous speed Ns (= 120 f/P).

(ii) The rotating field passes through the air gap and cuts the rotor conductors, which as yet, are stationary. Due to the relative speed between the rotating flux and the stationary rotor, e.m.f.s are induced in the rotor conductors. Since the rotor circuit is short-circuited, currents start flowing in the rotor conductors.

(iii) The current-carrying rotor conductors are placed in the magnetic field produced by the stator. Consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which tends to move the rotor in the same direction as the rotating field.

(iv) The fact that rotor is urged to follow the stator field (i.e., rotor moves in the direction of stator field) can be explained by Lenz's law. According to this law, the direction of rotor currents will be such that they tend to oppose the cause producing them. Now, the cause producing the rotor currents is the relative speed between the rotating field and the stationary rotor conductors. Hence to reduce this relative speed, the rotor starts running in the same direction as that of stator field and tries to catch it.



Slip

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the suitor field speed (Ns). This difference in speed depends upon load on the motor.

The difference between the synchronous speed Ns of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.,

$$\%\text{ age slip, } s = \frac{N_s - N}{N_s} \times 100$$

(i) The quantity Ns – N is sometimes called slip speed.
(ii) When the rotor is stationary (i.e., N = 0), slip, s = 1 or 100 %.
(iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

Effect of Slip on the Rotor Circuit

When the rotor is stationary, s = 1. Under these conditions, the per phase rotor e.m.f. E_2 has a frequency equal to that of supply frequency f. At any slip s, the relative speed between stator field and the rotor is decreased. Consequently, the rotor e.m.f. and frequency are reduced proportionally to sEs and sf respectively. At the same time, per phase rotor reactance X2, being frequency dependent, is reduced to sX_2.

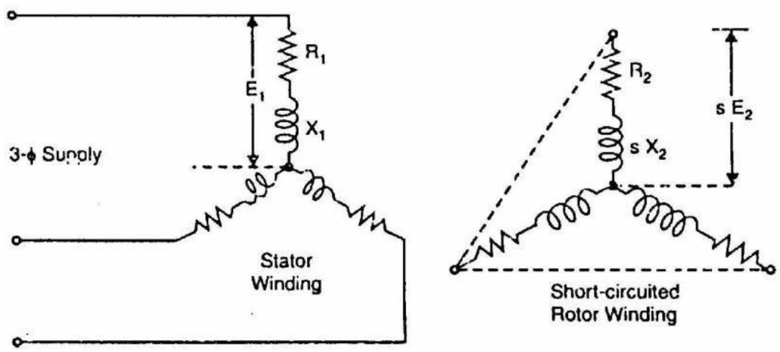
Thus at any slip s,

Rotor e.m.f./phase = sE_2
Rotor reactance/phase = sX_2
Rotor frequency = sf

where E_2,X_2 and f are the corresponding values at standstill.

Rotor Current

Fig. (8.14) shows the circuit of a 3-phase induction motor at any slip s. The rotor is assumed to be of wound type and star connected. Note that rotor e.m.f./phase and rotor reactance/phase are s E2 and sX2 respectively. The rotor resistance/phase is R2 and is independent of frequency and, therefore, does not depend upon slip. Likewise, stator winding values R1 and X1 do not depend upon slip.

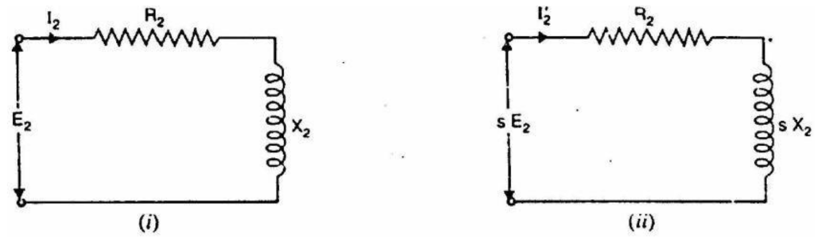


Since the motor represents a balanced 3-phase load, we need consider one phase only; the conditions in the other two phases being similar.

At standstill. Fig. (8.15 (i)) shows one phase of the rotor circuit at standstill.

$$\text{Rotor current/phase, } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

$$\text{Rotor p.f., } \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$



When running at slip s. Fig. (8.15 (ii)) shows one phase of the rotor circuit when the motor is running at slip s.

$$\text{Rotor current, } I'_2 = \frac{sE_2}{Z'_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{Rotor p.f., } \cos\phi'_2 = \frac{R_2}{Z'_2} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Rotor Torque

The torque T developed by the rotor is directly proportional to:
(i) Rotor current
(ii) Rotor e.m.f.
(iii) Power factor of the rotor circuit

$$T \propto E_2 I_2 \cos\phi_2$$

$$T = K E_2 I_2 \cos\phi_2$$

where I_2 = rotor current at standstill
E_2 = rotor e.m.f. at standstill
$\cos\phi_2$ = rotor p.f. at standstill

Starting Torque (T_s)

Let E_2 = rotor e.m.f. per phase at standstill
X_2 = rotor reactance per phase at standstill
R_2 = rotor resistance per phase

$$\text{Rotor impedance/phase, } Z_2 = \sqrt{R_2^2 + X_2^2} \qquad \text{...at standstill}$$

$$\text{Rotor current/phase, } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \qquad \text{...at standstill}$$

$$\text{Rotor p.f., } \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \qquad \text{...at standstill}$$

Starting torque, $T_s = K\, E_2\, I_2 \cos\phi_2$

$$= K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$= \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

Generally, the stator supply voltage V is constant so that flux per pole ϕ set up by the stator is also fixed. This in turn means that e.m.f. E2 induced in the rotor will be constant.

$$\therefore \qquad T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} = \frac{K_1 R_2}{Z_2^2}$$

where K_1 is another constant.

It is clear that the magnitude of starting torque would depend upon the relative values of R2 and X2 i.e., rotor resistance/phase and standstill rotor reactance/phase.

Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

$$\text{Now} \qquad T_s = \frac{K_1 R_2}{R_2^2 + X_2^2}$$

Differentiating eq. (i) w.r.t. R_2 and equating the result to zero, we get,

$$\frac{dT_s}{dR_2} = K_1 \left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{\left(R_2^2 + X_2^2\right)^2} \right] = 0$$

$$\text{or} \qquad R_2^2 + X_2^2 = 2R_2^2$$

$$\text{or} \qquad R_2 = X_2$$

Hence starting torque will be maximum when:

Rotor resistance/phase = Standstill rotor reactance/phase

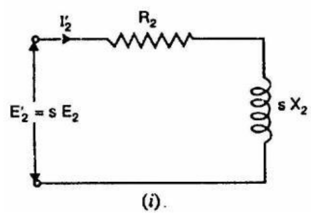
Torque Under Running Conditions

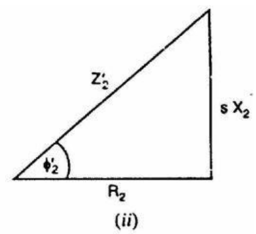
Let the rotor at standstill have per phase induced e.m.f. E_2, reactance X_2 and resistance R_2. Then under running conditions at slip s,
Rotor e.m.f./phase, $E_2` = sE_2$
Rotor reactance/phase, $X_2` = sX_2$

$$\text{Rotor impedance/phase, } Z'_2 = \sqrt{R_2^2 + (sX_2)^2}$$

$$\text{Rotor current/phase, } I'_2 = \frac{E'_2}{Z'_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{Rotor p.f., } \cos\phi'_m = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$





Running Torque,

$$T_r \propto E_2` I_2` \cos\phi_2`$$

$$\propto \phi\, I_2` \cos\phi_2` \qquad (\Theta\ E_2` \propto \phi)$$

$$\propto \phi \times \frac{s\,E_2}{\sqrt{R_2^2 + (s\,X_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (s\,X_2)^2}}$$

$$\propto \frac{\phi\, s\, E_2\, R_2}{R_2^2 + (s\,X_2)^2}$$

$$= \frac{K\,\phi\, s\, E_2\, R_2}{R_2^2 + (s\,X_2)^2}$$

$$= \frac{K_1\, s\, E_2^2\, R_2}{R_2^2 + (s\,X_2)^2} \qquad (\because E_2 \propto \phi)$$

If the stator supply voltage V is constant, then stator flux and hence E_2 will be constant.

$$\therefore \qquad T_r = \frac{K_2 \, s \, R_2}{R_2^2 + (s \, X_2)^2}$$

where K_2 is another constant.

It may be seen that running torque is:
(i) directly proportional to slip i.e., if slip increases (i.e., motor speed decreases), the torque will increase and vice-versa.
(ii) directly proportional to square of supply voltage ($\Theta E_2 \propto V$) .

Maximum Torque under Running Conditions

$$T_r = \frac{K_2 \, s \, R_2}{R_2^2 + s^2 \, X_2^2}$$

In order to find the value of rotor resistance that gives maximum torque under running conditions, differentiate exp. (i) w.r.t. s and equate the result to zero i.e.,

$$\frac{dT_r}{ds} = \frac{K_2 \left[R_2 \left(R_2^2 + s^2 \, X_2^2 \right) - 2 s X_2^2 (s \, R_2) \right]}{\left(R_2^2 + s^2 \, X_2^2 \right)^2} = 0$$

$$\text{or} \qquad \left(R_2^2 + s^2 \, X_2^2 \right) - 2 s X_2^2 = 0$$

$$\text{or} \qquad R_2^2 = s^2 \, X_2^2$$

$$\text{or} \qquad R_2 = s \, X_2$$

Thus for maximum torque (T_m) under running conditions:

Rotor resistance/phase = Fractional slip × Standstill rotor reactance/phase

For maximum torque, $R_2 = s \, X_2$. Putting $R_2 = s \, X_2$ in the above expression, the maximum torque Tm is given by;

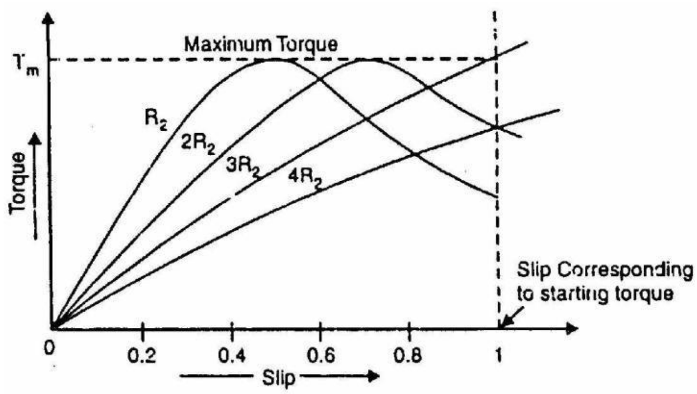
$$T_m \propto \frac{1}{2 \, X_2}$$

Torque-Slip Characteristics

The motor torque under running conditions is given by;

$$T = \frac{K_2 \, s \, R_2}{R_2^2 + s^2 \, X_2^2}$$

If a curve is drawn between the torque and slip for a particular value of rotor resistance R_2, the graph thus obtained is called torque-slip characteristic. Fig. shows a family of torque-slip characteristics for a slip-range from s = 0 to s = 1 for various values of rotor resistance.



The following points may be noted carefully:
(i) At s = 0, T = 0 so that torque-slip curve starts from the origin.
(ii) At normal speed, slip is small so that s X2 is negligible as compared to R2.

$$T \propto s / R_2$$

$$\propto s \qquad \text{... as } R_2 \text{ is constant}$$

Hence torque slip curve is a straight line from zero slip to a slip that corresponds to full-load.
(iii) As slip increases beyond full-load slip, the torque increases and becomes maximum at s = R2/X2. This maximum torque in an induction motor is called pull-out torque or break-down torque. Its value is at least twice the full-load value when the motor is operated at rated voltage and frequency.
(iv) when slip increases beyond that corresponding to maximum torque, the term $s^2 X_2^2$ increases very rapidly so that R_2^2 may be neglected as compared to $s_2 R_2^2$.

$$\therefore \quad T \propto s / s^2 X_2^2$$

$$\propto 1/s \qquad \text{... as } X_2 \text{ is constant}$$

Thus the torque is now inversely proportional to slip. Hence torque-slip curve is a rectangular hyperbola.
(v) The maximum torque remains the same and is independent of the value of rotor resistance. Therefore, the addition of resistance to the rotor circuit does not change the value of maximum torque but it only changes the value of slip at which maximum torque occurs.

POWER STAGES IN AN INDUCTION MOTOR

The input electric power fed to the stator of the motor is converted into mechanical power at the shaft of the motor. The various losses during the energy conversion are:

1. Fixed losses

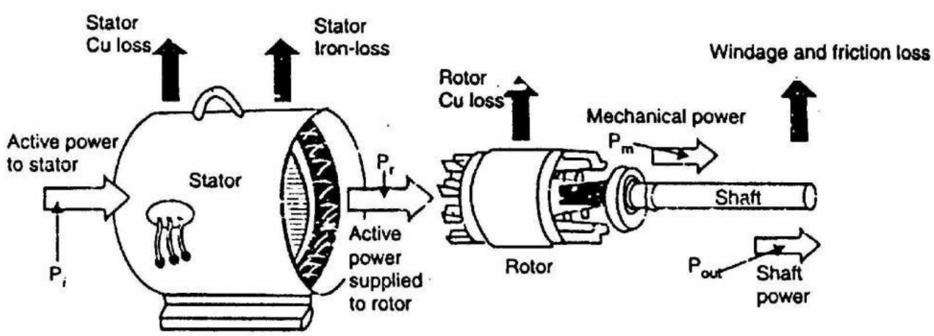
(i) Stator iron loss
(ii) Friction and windage loss

The rotor iron loss is negligible because the frequency of rotor currents under normal running condition is small.

2. Variable losses

(i) Stator copper loss
(ii) Rotor copper loss

Fig. shows how electric power fed to the stator of an induction motor suffers losses and finally converted into mechanical power.



The following points may be noted from the above diagram:

(i) Stator input, Pi = Stator output + Stator losses
= Stator output + Stator Iron loss + Stator Cu loss

(ii) Rotor input, Pr = Stator output

It is because stator output is entirely transferred to the rotor through airgap by electromagnetic induction.

(iii) Mechanical power available, Pm = Pr – Rotor Cu loss

This mechanical power available is the gross rotor output and will produce a gross torque Tg.

(iv) Mechanical power at shaft, Pout = Pm – Friction and windage loss

Mechanical power available at the shaft produces a shaft torque Tsh.

Clearly, Pm – Pout = Friction and windage loss

Induction Motor Torque

The mechanical power P available from any electric motor can be expressed as:

$$P = \frac{2\pi N T}{60} \quad \text{watts}$$

where N = speed of the motor in r.p.m.
T = torque developed in N-m

$$\therefore \quad T = \frac{60}{2\pi}\frac{P}{N} = 9.55\frac{P}{N} \text{ N - m}$$

Efficiency of Three Phase Induction Motor

Efficiency is defined as the ratio of the output to that of input,

$$Efficiency,\ \eta = \frac{output}{input}$$

Rotor efficiency of the three phase induction motor ,

$$= \frac{rotor\ output}{rotor\ input}$$

= Gross mechanical power developed / rotor input

$$= \frac{P_m}{P_2}$$

Three phase induction motor efficiency,

$$= \frac{power\ developed\ at\ shaft}{electrical\ input\ to\ the\ motor}$$

Three phase induction motor efficiency

$$\eta = \frac{P_{out}}{P_{in}}$$

Problems:

1. A 3 φ 4 pole 50 hz induction motor runs at 1460 r.p.m. find its %age slip.

Solution

N_s = 120f/p = 120*50/4 = 1500r.p.m.

Running speed of motor = n= 1460r.p.m.

Slip S=(N_s–N)/ N_s*100 =(1500-1460) x 100 / 1500 = 2.667%

2. A 12 pole 3 φ alternator driver at speed of 500 r.p.m. supplies power to an 8 pole 3 φ induction motor. If the slip of motor is 0.03p.u, calculate the speed.

Solution

Frequency of supply from alternator, f=PN/120 =12*500/120 = 50hz

where P= no of poles on alternater

N=alternator speed is r.p.m.

Synchronous speed of 3 φ induction motor N=120f/Pm =120*50/8 = 750 r.p.m.

Speed of 3 φ induction motor N=Ns (1-s)=750(1-0.03) = 727.5 r.p.m.

3. A motor generates set used for providing variable frequency ac supply consists of a 3-φ synchronous and 24 pole 3 φ synchronous generator. The motor generate set is fed from 25hz, 3 φ ac supply. A 6 pole 3 φ induction motor is electrically connected to the terminals of the synchronous generator and runs at a slip of 5%. Find
   i) the frequency of generated voltage of synchronous generator
   ii) the speed at which induction motor is running

Solution

Speed of motor generator setN_s=(120*f_1(supply freq))/(no of pole on syn motor)

=120*25/10 = 300 r.p.m.

(1) frequency of generated voltage

f_z=speed of motor gen set voltage *no of poles on syn gen/120 = 300*24/120 = 60hz

(2) Speed of induction motor , N_m=N_s(1-s)=120 f_z /P_m(1-s) = 120*60/6(1-0.05) = 1140r.p.m.

4. A 3-φ 4 pole induction motor is supplied from 3φ 50Hz ac supply. Find
   (1) synchronous speed
   (2) rotor speed when slip is 4%
   (3) the rotor frequency when runs at 600r.p.m.

Solution

1) N_s =120f/p=120*50/4 = 1500 r.p.m.

2) speed when slip is 4% or .04

N=N_s (1-s)=1500(1-0.04) = 1440 r.p.m.

3) slip when motor runs at 600 r.p.m.

S'=(N_s –N)/N_s=(1500-600)/1500 = 0.6

Rotor frequency f' = S'f = 0.6*50 = 30Hz.

5. A 4 pole 50 Hz 3 φ induction motor running at full load, develops a torque of 160N-m, when rotor makes 120 complete cycles per minute, find what power output

   Solution

Supply frequency f = 50Hz

Rotor e.m.f. frequency = f = 120/60 = 2Hz

Slip S = f'/f = 2/50 = 0.04

N_s = 120f/p = 120 *50/4 = 1500 r.p.m.

Shaft power output = T_{sh} * 2πN/160 = 160 * 2 π * 1440/60 = 24127W

6. The power input to a 500V 50Hz, 6 pole, 3 φ squirrel case inductor motor running at 975 r.p.m. is 40kw. The stator losses are 1 kw and friction and windage losses are 2kw. Find:
   1) Slip
   2) Rotor Cu loss

3) Brake hp

Solution:

i) $N_s = 120f/P = 120*50/6 = 1000$ r.p.m.

$S = (N_s - N)/N_s = (1000 - 975)/1000 = 0.025$

Power input to station $P_1 = 40$Kw

Stator output power = P_1 – stator losses = 40 -1 = 39kw

Power input to rotor P_2 = Stator output power = 39 KW

ii) Rotor Cu loss = $sP_2 = 0.025 * 39 = 0.975$KW

$P_{mech} = P_2 - P_{cu} = 39 - 0.975 = 38.025$

iii) Motor output = P_{mech} – friction and windage loss = 38.025 – 2 = 36.025KW

SINGLE-PHASE MOTORS

INTRODUCTION

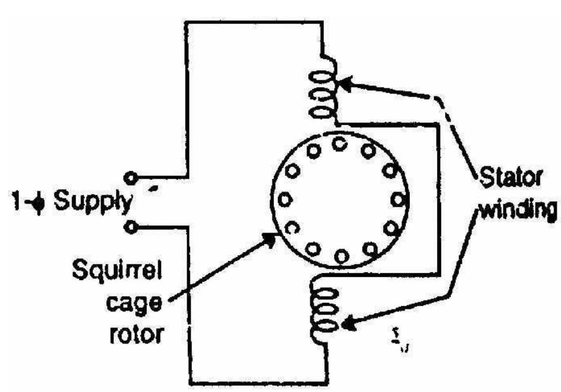
As the name suggests, these motors are used on single-phase supply. Single-phase motors are the most familiar of all electric motors because they are extensively used in home appliances, shops, offices etc. It is true that single-phase motors are less efficient substitute for 3-phase motors but 3-phase power is normally not available except in large commercial and industrial establishments. Since electric power was originally generated and distributed for lighting only, millions of homes were given single-phase supply. This led to the development of single-phase motors. Even where 3-phase mains are present, the single-phase supply may be obtained by using one of the three lines and the neutral. In this chapter, we shall focus our attention on the construction, working and characteristics of commonly used single-phase motors.

SINGLE-PHASE INDUCTION MOTORS

A single phase induction motor is very similar to a 3-phase squirrel cage induction motor. It has
(i) a squirrel-cage rotor identical to a 3-phase motor and
(ii) a single-phase winding on the stator.

Unlike a 3-phase induction motor, a single-phase induction motor is not self starting but requires some starting means. The single-phase stator winding produces a magnetic field that pulsates in strength in a sinusoidal manner. The field polarity reverses after each half cycle but the field does not rotate. Consequently, the alternating flux cannot produce rotation in a stationary squirrel-cage rotor. However, if the rotor of a single-phase motor is rotated in one direction by some mechanical means, it will continue to run in the direction of rotation. As a matter of fact, the rotor quickly accelerates until it reaches a speed slightly below the synchronous speed. Once the motor is running at this speed, it will continue to rotate even though single-phase current is flowing through the stator winding. This method of starting is generally not convenient for large motors. Nor can it be employed fur a motor located at some inaccessible spot.

Fig. shows single-phase induction motor having a squirrel cage rotor and a single-phase distributed stator winding. Such a motor inherently docs not develop any starting torque and, therefore, will not start to rotate if the stator winding is connected to single-phase a.c. supply. However, if the rotor is started by auxiliary means, the motor will quickly attain me final speed. This strange behaviour of single-phase induction motor can be explained on the basis of double-field revolving theory.



DOUBLE-FIELD REVOLVING THEORY

The double-field revolving theory is proposed to explain this dilemma of no torque at start and yet torque once rotated. This theory is based on the fact that an alternating sinusoidal flux ($\phi = \phi_m \cos\omega t$) can be represented by two revolving fluxes, each equal to one-half of the maximum value of alternating flux (i.e., $\phi_m/2$) and each rotating at synchronous speed ($N_s = 120$ f/P, $\omega = 2\pi f$) in opposite directions. The above statement will now be proved. The instantaneous value of flux due to the stator current of a single-phase induction motor is given by;

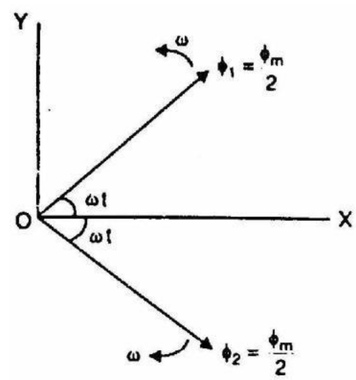
$$\phi = \phi_m \cos\omega t$$

Consider two rotating magnetic fluxes ϕ_1 and ϕ_2 each of magnitude $\phi_m/2$ and rotating in opposite directions with angular velocity ω.
Let the two fluxes start rotating from OX axis at t = 0. After time t seconds, the angle through which the flux vectors have rotated is at. Resolving the flux vectors along-X-axis and Y-axis, we have,

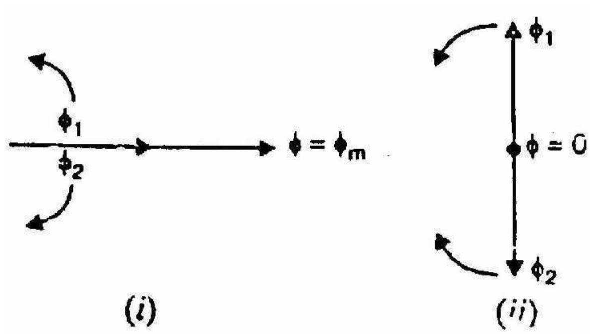
$$\text{Total X-component} = \frac{\phi_m}{2}\cos\omega t + \frac{\phi_m}{2}\cos\omega t = \phi_m \cos\omega t$$

$$\text{Total Y-component} = \frac{\phi_m}{2}\sin\omega t - \frac{\phi_m}{2}\sin\omega t = 0$$

$$\text{Resultant flux, } \phi = \sqrt{(\phi_m \cos\omega t)^2 + 0^2} = \phi_m \cos\omega t$$

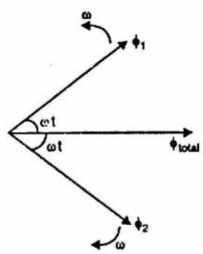


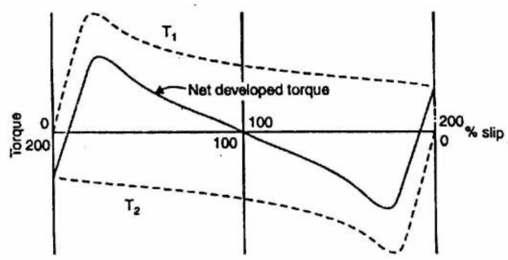
Thus the resultant flux vector is $\phi = \phi_m \cos\omega t$ along X-axis. Therefore, an alternating field can be replaced by two relating fields of half its amplitude rotating in opposite directions at synchronous speed. Note that the resultant vector of two revolving flux vectors is a stationary vector that oscillates in length with time along X-axis. When the rotating flux vectors are in phase the resultant vector is $\phi = \phi_m$; when out of phase by 180° the resultant vector $\phi = 0$. Let us explain the operation of single-phase induction motor by double-field revolving theory.



Rotor at standstill

Consider the case that the rotor is stationary and the stator winding is connected to a single-phase supply. The alternating flux produced by the stator winding can be presented as the sum of two rotating fluxes ϕ_1 and ϕ_2, each equal to one half of the maximum value of alternating flux and each rotating at synchronous speed (N_s = 120 f/P) in opposite directions as shown in Fig. Let the flux ϕ_1 rotate in anti clockwise direction and flux ϕ_2 in clockwise direction. The flux ϕ_1 will result in the production of torque T_1 in the anti clockwise direction and flux ϕ_2 will result in the production of torque T_2 In the clockwise direction. At standstill, these two torques are equal and opposite and the net torque developed is zero. Therefore, single-phase induction motor is not self-starting. This fact is illustrated in Fig. (ii). Note that each rotating field tends to drive the rotor in the direction in which the field rotates. Thus the point of zero slip for one field corresponds to 200% slip for the other as explained later. The value of 100% slip (standstill condition) is the same for both the fields.





Rotor running

Now assume that the rotor is started by spinning the rotor or by using auxiliary circuit, in say clockwise direction. The flux rotating in the clockwise direction is the forward rotating flux (ϕ_f) and that in the other direction is the backward rotating flux (ϕ_b). The slip w.r.t. the forward flux will be,

$$s_f = \frac{N_s - N}{N_s} = s$$

Where N_s = synchronous speed
N = speed of rotor in the direction of forward flux

The rotor rotates opposite to the rotation of the backward flux. Therefore, the slip w.r.t. the backward flux will be

$$s_b = \frac{N_s - (-N)}{N_s} = \frac{N_s + N}{N_s} = \frac{2N_s - N_s + N}{N_s}$$

$$= \frac{2N_s}{N_s} - \frac{(N_s - N)}{N_s} = 2 - s$$

$$\therefore \quad s_b = 2 - s$$

Thus fur forward rotating flux, slip is s (less than unity) and for backward rotating flux, the slip is 2 – s (greater than unity). Since for usual rotor resistance/reactance ratios, the torques at slips of less than unity arc greater than those at slips of more than unity, the resultant torque will be in the direction of the rotation of the forward flux. Thus if the motor is once started, it will develop net torque in the direction in which it has been started and will function as a motor.

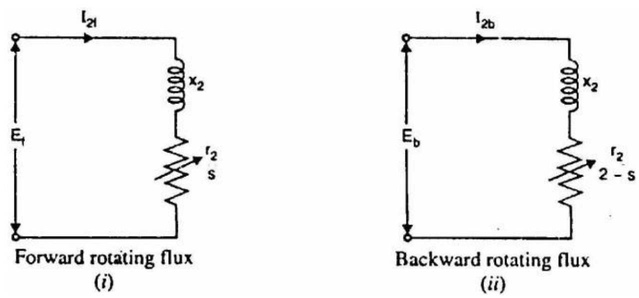
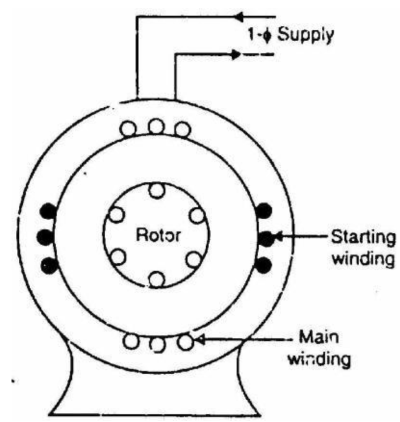


Fig. shows the rotor circuits for the forward and backward rotating fluxes. Note that $r_2 = R_2/2$, where R_2 is the standstill rotor resistance i.e., r_2 is equal to half the standstill rotor resistance. Similarly, $x_2 = X_2/2$ where X_2 is the standstill rotor reactance. At standstill, s = 1 so that impedances of the two circuits are equal. Therefore, rotor currents are equal i.e., $I_2f = I_2b$. However, when the rotor rotates, the impedances of the two rotor circuits are unequal and the rotor current I_2b is higher (and also at a lower power factor) than the rotor current I_2f. Their m.m.f.s, which oppose the stator m.m.f.s, will result in a reduction of the backward rotating flux. Consequently, as speed increases, the forward flux increases, increasing the driving torque while the backward flux decreases, reducing the opposing torque. The motor-quickly accelerates to the final speed.

MAKING SINGLE-PHASE INDUCTION MOTOR SELF-STARTING



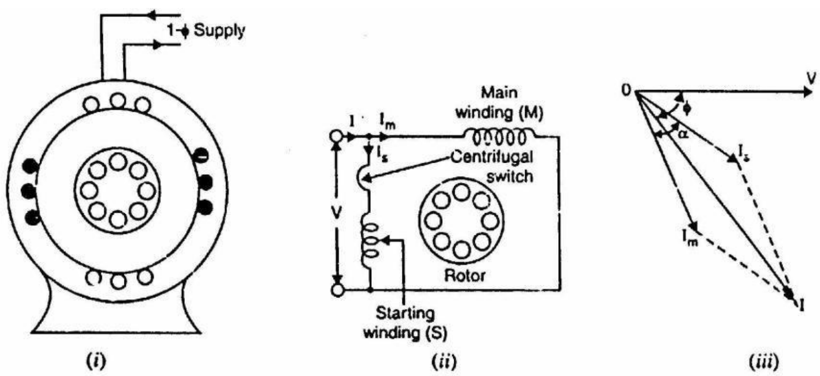
The single-phase induction motor is not self starting and it is undesirable to resort to mechanical spinning of the shaft or pulling a belt to start it. To make a single-phase induction motor self-starting, we should somehow produce a revolving stator magnetic field. This may be achieved by converting a single-phase supply into two-phase supply through the use of an additional winding.

When the motor attains sufficient speed, the starting means (i.e., additional winding) may be removed depending upon the type of the motor. As a matter of fact, single-phase induction motors are classified and named according to the method employed to make them self-starting.

Split-phase motors - started by two phase motor action through the use of an auxiliary or starting winding
Capacitor motors - started by two-phase motor action through the use of an auxiliary winding and a capacitor.
Shaded-pole motors - started by the motion of the magnetic field produced by means of a shading coil around a portion of the pole structure.

Split-Phase Induction Motor

The stator of a split-phase induction motor is provided with an auxiliary or starting winding S in addition to the main or running winding M. The starting winding is located 90° electrical from the main winding [See Fig.] and operates only during the brief period when the motor starts up. The two windings are so resigned that the starting winding S has a high resistance and relatively small reactance while the main winding M has relatively low resistance and large reactance as shown in the schematic connections in Fig.(ii). Consequently, the currents flowing in the two windings have reasonable phase difference c (25° to 30°) as shown in the phasor diagram in Fig. (iii).



Operation

(i) When the two stator windings are energized from a single-phase supply, the main winding carries current I_m while the starting winding carries current I_s.
(ii) Since main winding is made highly inductive while the starting winding highly resistive, the currents I_m and I_s have a reasonable phase angle a (25° to 30°) between them as shown in Fig. (iii). Consequently, a weak revolving field approximating to that of a 2-phase machine is produced which starts the motor. The starting torque is given by;

$$T_s = k\, I_m\, I_s \sin\alpha$$

where k is a constant whose magnitude depends upon the design of the motor.
(iii) When the motor reaches about 75% of synchronous speed, the centrifugal switch opens the circuit of the starting winding. The motor then operates as a single-phase induction motor and continues to accelerate till it reaches the normal speed. The normal speed of the motor is below the synchronous speed and depends upon the load on the motor.

Characteristics

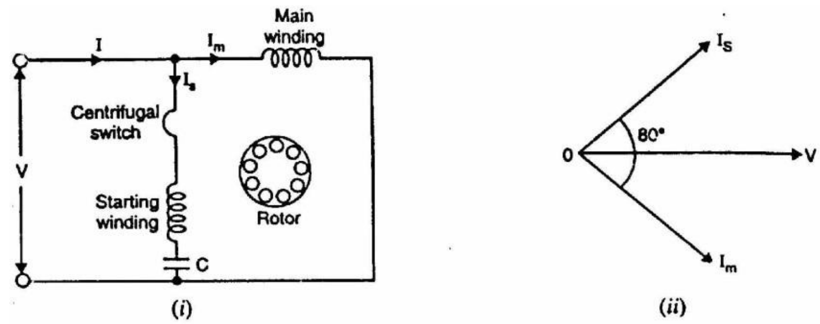
(i) The sinning torque is 15 to 2 times the full-loud torque mid (lie starting current is 6 to 8 times the full-load current.
(ii) Due to their low cost, split-phase induction motors are most popular single-phase motors in the market.
(iii) Since the starting winding is made of fine wire, the current density is high and the winding heats up quickly. If the starting period exceeds 5 seconds, the winding may burn out unless the motor is protected by built-in-thermal relay. This motor is, therefore, suitable where starting periods are not frequent.
(iv) An important characteristic of these motors is that they are essentially constant-speed motors. The speed variation is 2-5% from no-load to full load.
(v) These motors are suitable where a moderate starting torque is required and where starting periods are infrequent e.g., to drive: (a) fans (b) washing machines (c) oil burners (d) small machine tools etc. The power rating of such motors generally lies between 60 W and 250 W.

Capacitor-Start Motor

The capacitor-start motor is identical to a split-phase motor except that the starting winding has as many turns as the main winding. Moreover, a capacitor C is connected in series with the starting winding as shown in Fig. (i). The value of capacitor is so chosen that Is leads I_m by about 80° (i.e., $\alpha \sim 80°$) which is considerably greater than 25° found in split-phase motor [See Fig. (ii)]. Consequently, starting torque ($T_s = k\ I_m\ I_s \sin\alpha$) is much more than that of a split-phase motor Again, the starting winding is opened by the centrifugal switch when the motor attains about 75% of synchronous speed. The motor then operates as a single-phase induction motor and continues to accelerate till it reaches the normal speed.

Characteristics

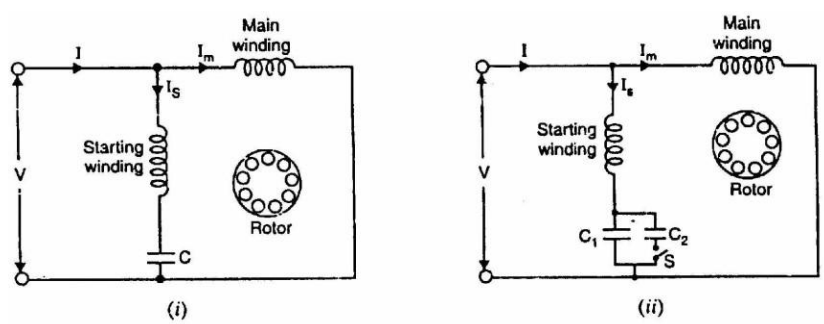
(i) Although starting characteristics of a capacitor-start motor are better than those of a split-phase motor, both machines possess the same running characteristics because the main windings are identical.
(ii) The phase angle between the two currents is about 80° compared to about 25° in a split-phase motor. Consequently, for the same starting torque, the current in the starting winding is only about half that in a split-phase motor. Therefore, the starting winding of a capacitor start motor heats up less quickly and is well suited to applications involving either frequent or prolonged starting periods.



(iii) Capacitor-start motors are used where high starting torque is required and where the starting period may be long e.g., to drive:
(a) compressors (b) large fans (c) pumps (d) high inertia loads
The power rating of such motors lies between 120 W and 7-5 kW.

Capacitor-Start Capacitor-Run Motor

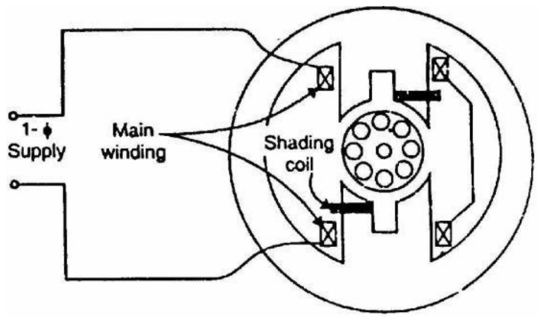
This motor is identical to a capacitor-start motor except that starting winding is not opened after starting so that both the windings remain connected to the supply when running as well as at starting. Two designs are generally used.
(i) In one design, a single capacitor C is used for both starting and running as shown in Fig. (i). This design eliminates the need of a centrifugal switch and at the same time improves the power factor and efficiency of the motor.



(ii) In the other design, two capacitors C_1 and C_2 are used in the starting winding as shown in Fig. (ii). The smaller capacitor C_1 required for optimum running conditions is permanently connected in series with the starting winding. The much larger capacitor C_2 is connected in parallel with C_1 for optimum starting and remains in the circuit during starting. The starting capacitor C_1 is disconnected when the motor approaches about 75% of synchronous speed. The motor then runs as a single-phase induction motor.
Characteristics
(i) The starting winding and the capacitor can be designed for perfect 2-phase operation at any load. The motor then produces a constant torque and not a pulsating torque as in other single-phase motors.
(ii) Because of constant torque, the motor is vibration free and can be used in:
(a) Hospitals (b) studios and (c) other places where silence is important.

Shaded-Pole Motor

The shaded-pole motor is very popular for ratings below 0.05 H.P. (~ 40 W) because of its extremely simple construction. It has salient poles on the stator excited by single-phase supply and a squirrel-cage rotor as shown in Fig. A portion of each pole is surrounded by a short-circuited turn of copper strip called shading coil.

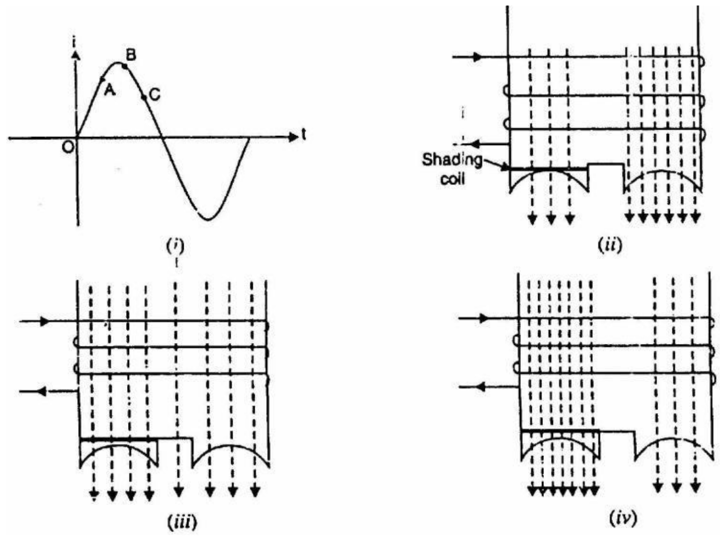


Operation

The operation of the motor can be understood by referring to Fig. which shows one pole of the motor with a shading coil.

(i) During the portion OA of the alternating-current cycle [See Fig.], the flux begins to increase and an e.m.f. is induced in the shading coil. The resulting current in the shading coil will be in such a direction (Lenz's law) so as to oppose the change in flux. Thus the flux in the shaded portion of the pole is weakened while that in the unshaded portion is strengthened as shown in Fig. (ii).

(ii) During the portion AB of the alternating-current cycle, the flux has reached almost maximum value and is not changing. Consequently, the flux distribution across the pole is uniform [See Fig. (iii)] since no current is flowing in the shading coil. As the flux decreases (portion BC of the alternating current cycle), current is induced in the shading coil so as to oppose the decrease in current. Thus the flux in the shaded portion of the pole is strengthened while that in the unshaded portion is weakened as shown in Fig. (iv).



(iii) The effect of the shading coil is to cause the field flux to shift across the pole face from the un-shaded to the shaded portion. This shifting flux is like a rotating weak field moving in the direction from un-shaded portion to the shaded portion of the pole.
(iv) The rotor is of the squirrel-cage type and is under the influence of this moving field. Consequently, a small starting torque is developed. As soon as this torque starts to revolve the rotor, additional torque is produced by single-phase induction-motor action. The motor accelerates to a speed slightly below the synchronous speed and runs as a single-phase induction motor.

Characteristics
(i) The salient features of this motor are extremely simple construction and absence of centrifugal switch.
(ii) Since starting torque, efficiency and power factor are very low, these motors are only suitable for low power applications e.g., to drive:

(a) Small fans (6) toys (c) hair driers (d) desk fans etc.

The power rating of such motors is upto about 30 W.

Torque Speed Characteristic of an Induction Motor

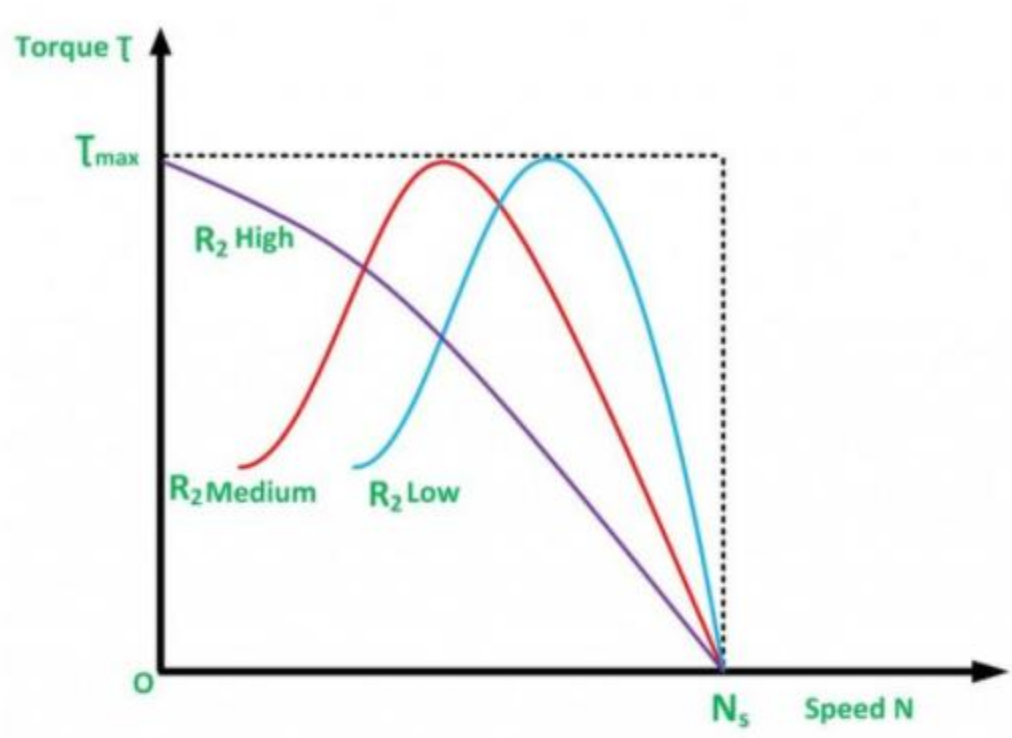
Torque Speed Characteristic is the curve plotted between the torque and the speed of the induction motor. As we have already discussed the torque of the induction motor in the topic Torque Equation of an Induction motor. The equation of the torque is given as shown below.

$$T = \frac{k\, s\, R_2 E_{20}^2}{R_2^2 + (sX_{20})^2} \quad \dots\dots\dots\dots (1)$$

At the maximum torque, the speed of the rotor is expressed by the equation shown below.

$$N_M = N_S\,(1 - s_M) \quad \dots\dots\dots (2)$$

The curve below shows the **Torque Speed Characteristic.**



The maximum torque is independent of the rotor resistance. But the exact location of the maximum torque T_{max} is dependent on it. Greater, the value of the R_2, the greater is the value of the slip at which maximum torque occurs. As the rotor resistance increases, the pullout speed of the motor decreases. In this condition, the maximum torque remains constant.

ALTERNATORS

CONSTRUCTION & PRINCIPLE OF SYNCHRONOUS MACHINES

INTRODUCTION

In overall world, almost all generating stations produce electricity by using alternators. A Synchronous generator is a machine for converting mechanical power from a prime mover to ac electric power at a specific voltage and frequency. A synchronous machine rotates at a constant speed called synchronous speed. A Synchronous motor is a Synchronous Machine that converts electric power into mechanical power. Synchronous generator is usually of 3-phase type, because of the several advantages of 3-phase generation, transmission and distribution. A Synchronous generator, when supply isolated load, acts as a voltage source whose frequency is determined by its prime mover speed by using the equation, $N_S = 120f/P$. Synchronous generators are usually run in parallel, connected through long distance transmission lines.

SYNCHRONOUS GENERATOR (or) ALTERNATOR

The machine generating a.c. e.m.f is called alternator or synchronous generator. Alternator consists essentially of two parts namely the armature and field magnet system.

Small AC generators are commonly made with stationary field magnet system and revolving armature (the armature rotates in the field system). In large A.C generators (modern A.C generators) have stator as armature and rotor as field, in practice.

Difference between DC Generator and AC Generator

In the case of DC generator, the nature of the induced emf in the armature conductors is alternating type. With the help of commutator and brush arrangement, the alternating emf is converted into a unidirectional emf and made available to the external circuit. If the commutator is dropped from a DC generator and induced emf is tapped outside from an armature directly, the nature of such emf will be alternating. Such a machine without commutator, providing an alternating emf to the external circuit is called an alternator.

The induced emf is basically due the effect of the relative motions present between an armature and the field. Such a relative motion is achieved by rotating armature with the help of prime mover, in case of a DC generator. As armature is connected to commutator in a DC generator, armature must be a rotating part while field is a stationary part.

The alternators work on the principle of electromagnetic induction when there is a relative motion between the conductors and the flux, e.m.f gets induced in the conductors. The only difference in practical alternator and a d.c. generator is that in an alternator the armature conductors are stationary and field is rotating.

When the field rotates with respect to the stationary conductors, alternating emf is induced in the armature. For a fixed number of poles, the Synchronous generator is to be rotated at a constant speed to keep the generator emf constant. Such a speed of the Synchronous generator is called Synchronous Speed. Synchronous speed is given by

$$N_S = \frac{120\,f}{P}$$

Where f = frequency of generated e.m.f

P = Number of poles

No of Poles P	Synchronous speed for a supply Frequency of 50hz	Synchronous speed for a supply Frequency of 60hz
2	3000	3600
4	1500	1800
6	1000	1200
8	750	900
10	600	720
12	500	600

Advantages of Stationary Armature and Rotating Field System

1. The output current can be taken directly from fixed terminals on the stator without using slip rings, brushes etc
2. Armature winding is more complex than the field winding. Therefore, it is easy to place armature winding on the stationary armature.
3. The problem of sparking at the slip rings can be minimised by keeping field rotating which is low voltage circuit compared to armature.
4. In modern alternators, high voltage is generated, therefore heavy insulation is provided and it is easy to insulate the high voltage winding when it is placed on stationary structure.
5. It is easier to build and properly balance high – speed rotors when they carry the field structure.
6. Rotating field is comparatively light and can run with high speeds. So higher output can be obtained from an alternator of given size.
7. Better cooling system can be provided when the armature is kept stationary.
8. Only two slip rings are required for the supply of direct current to the rotor and since the exciting current is supplied at a low voltage of 125V or 250 V, there is no difficulty in insulating them.

Due to all these reasons the most of the medium and large alternators are constructed with revolving field.

Operating Principle of an Alternator

The A.C generator or alternator consists of two parts i.e an armature winding (stator) and magnetic field (rotor), similar to DC generator. It operates on the same fundamental principle of electromagnetic induction as DC generators i.e when the flux linking a conductor changes, an emf is induced in the conductor.

When the rotor rotates, the stator conductors are cut by the magnetic flux, hence they have induced emf produced in them. The magnetic poles are alternatively N and S so, they induce an emf and hence current in the armature conductors, which first flow in one direction and then in the other direction. Hence, an alternating emf is produced in the stator conductor, whose frequency depends on the number of N and S poles moving past in a conductor in one second and whose direction is given by Fleming’s right hand rule.

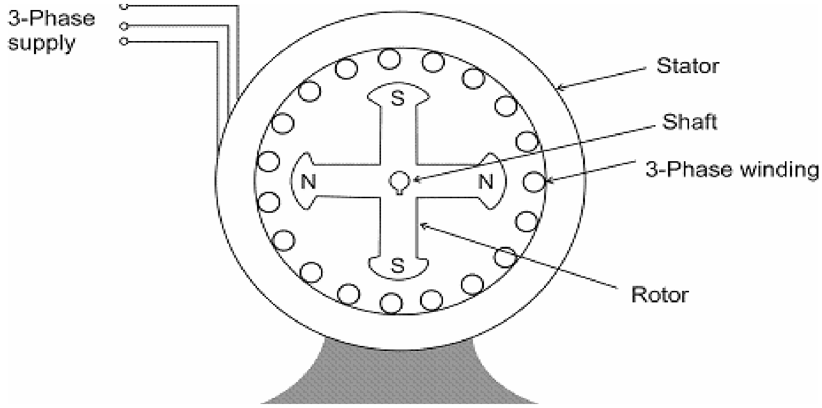


Fig. Three-phase Alternator

Construction of Alternator

The two main parts of a synchronous machine are stator and rotor. The stator is the stationary part of the machine, and it carries the armature winding, in which the voltage is generated. The output of the machine is taken from the stator. The rotor is the rotating part of the machine; it produces the main field flux.

Construction of Stator

The stator consists of a cast-iron frame, which supports the armature core, having slots on its inner periphery for housing the armature conductors. The various parts of the stator are the stator frame, stator core, stator windings and cooling arrangement. The stator core is build up of thin laminations (about 0.5mm thickness), which are insulated from each other in order to reduce the eddy current losses. Frame does not carry any flux and serves as the support to the core.

The stator frame with core and stator winding is the heaviest component of the entire alternator. The frame must be rigid i.e of god mechanical strength in order to withstand the forces and torques arising during operation. The stator core is provided with air passages and ventilating air ducts for cooling purpose. A 3-phase winding is put in the slot cut on the inner periphery of the stator as shown in the fig.6.2. These slots may be open, semi closed and closed according to speed and size of machine. Open slots are most commonly used because the coil can be freely wound and insulated properly. These slots provide the facility of removal and replacement of defective coils. The semi-closed slots are used to provide better performance over open slots. The totally closed slots are rarely used.

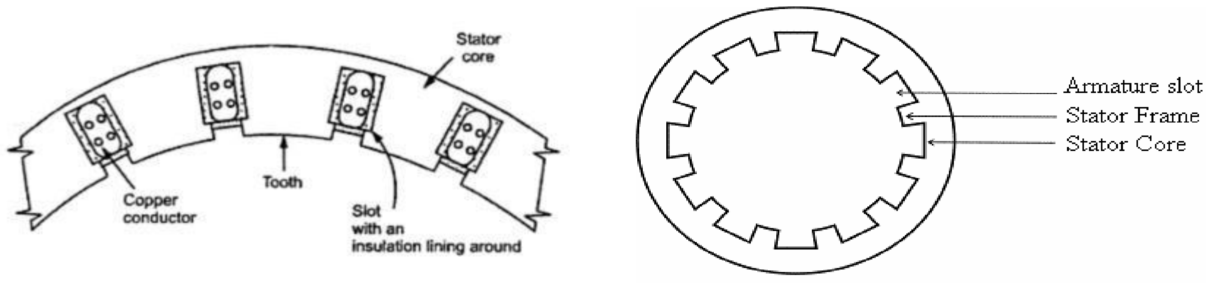


Fig. Stator of an alternator and cross section of an alternator

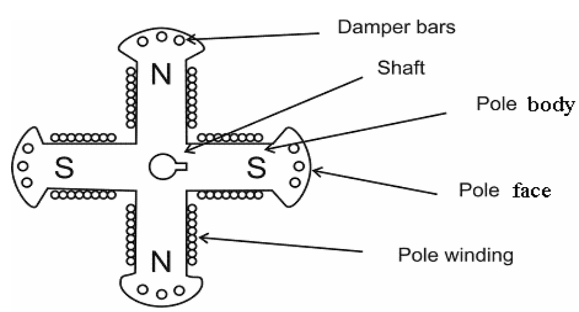
Construction of Rotor

Alternators are classified according to their pole construction as:
(a) Salient pole-type or Projected pole-type rotor
(b) Non-Salient pole-type or Smooth cylindrical pole-type rotor

a) Salient Pole Type Rotor

In this rotor poles are projected out from the surface of the rotor. This type of rotor is used for slow speed machines. The Pole face is so shaped that the radial gap length increases from the pole centre to pole tips. This makes the flux distribution over the armature uniform to generate sinusoidal waveform of e.m.f. The rotor is subjected to changing magnetic field, it is made up of thin steel laminations to reduce eddy current losses. The concentrated field winding is provided on the pole shoe. The pole faces are usually provided with slots for damper winding. The damper bars are short circuited at both the ends by copper rings. These dampers are used in preventing hunting in alternators.

The salient pole field structure has the following special feature



i) They have large diameter and short axial length.

ii) The pole shoe covers about 2/3rd of pole pitch.

iii) Poles are laminated to reduce eddy current losses.

iv) These employed with hydraulic turbines or I.C. engines.

ii) Non-salient pole type rotor

A non-salient pole rotor machine is also called a cylindrical rotor machine. The rotor consists of a smooth solid forged steel cylinder, having a number of slots milled out at intervals along the outer periphery for accommodating field-coils. The cylindrical construction of the rotor gives better balance and quieter-operation and also less windage losses. According to design, two-third of the rotor periphery is slotted, to accommodate the field winding and one-third of it is left for the formation of poles. The field winding used is of distributed type. As the air gap is uniform, the flux distribution is almost sinusoidal.

Constructionally, these rotors have smaller diameter and larger axial length. Their operating speed ranges from 1500 rpm to 3000 rpm. The prime movers used to drive such type of rotors are generally steam turbines, electric motors etc.

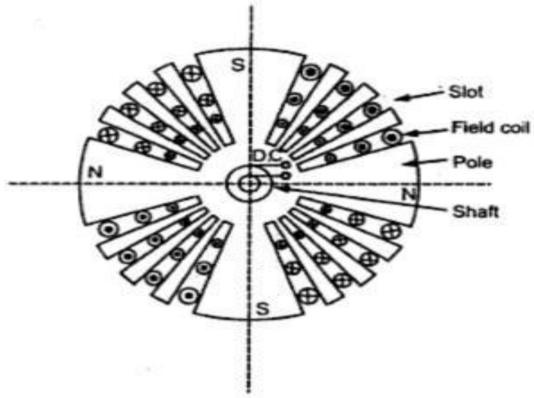


Fig. Smooth cylindrical rotor

Comparison between Salient pole and Non-salient pole type Rotor

SI.No	Salient Pole Rotor	Non-Salient Pole Rotor
1	Poles are projecting out from the surface.	Un-slotted portion of cylinder acts as pole and hence poles are non projecting.
2	Air-gap is non-uniform.	Air-gap is uniform due to smooth cylindrical periphery.
3	It has larger diameter and smaller axial length.	It has smaller diameter and larger axial length.
4	Preferred for low and medium speed Alternators.	Preferred for high speed alternators
5	The field winding is concentrated winding.	The field winding is distributed winding.
6	As the air gap is non-uniform the flux distribution is non-sinusoidal.	As the air gap is uniform the flux distribution is sinusoidal.
7	Mechanically weak.	The cylindrical construction of the rotor gives better balance and quieter-operation and also less windage losses.
8	Prime movers used are water turbines, IC engines.	Prime movers used are steam turbines, electric motors.

Armature Windings

The winding through which a d.c current is passed to produce the main flux is called the field winding. The winding in which voltage is induced is called the armature winding. The armature windings of dc machine are usually closed circuit windings but alternators windings may be either closed giving delta connections or open giving star connections. Some basic terms related to the armature winding are defined as follows.

a) **Conductor:** The length of a wire lying in the magnetic field and in which emf is induced is called a conductor.
b) **Turn:** A conductor in slot, when connected to a conductor in another slot forms a turn. So a turn consists of two conductors along with their end connections as shown in fig. 6.5.
c) **Coil:** A coil is formed by connecting several turns in series as shown in fig.6.5.
d) **Winding:** A winding is formed by connecting several coils in series, as shown in fig.6.5.
e) **Coil side:** when coil consists of many turns, part of the coil in each slot is called coil side of a coil.

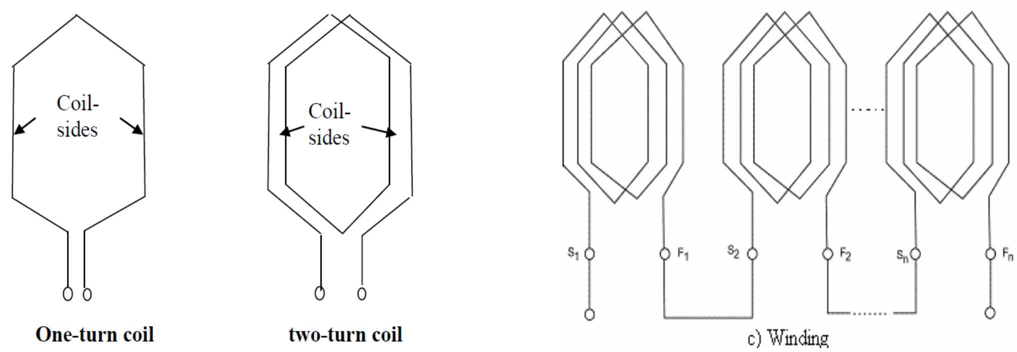
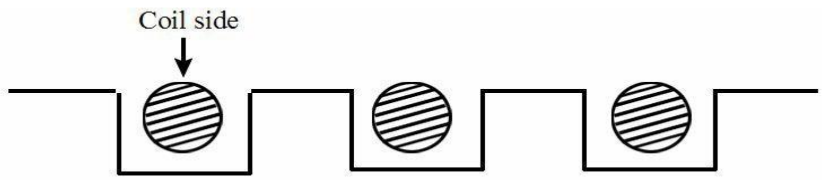
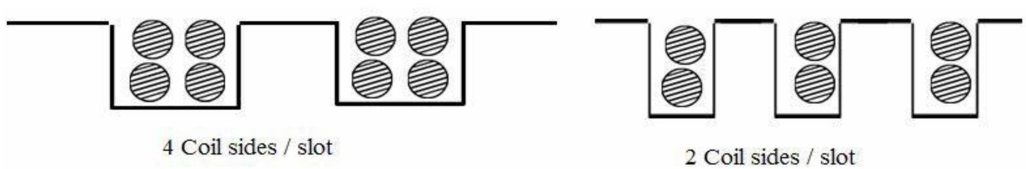


Fig.

f) **Single layer winding:** The winding in which single coil side occupies the total slot area, then it is called single layer winding. Single layer winding is used only in small AC machines.



g) **Double layer winding:** The winding in which even number of coil sides placed in two layers, then it is called double layer winding. Double layer windings are most commonly used.



h) **Concentrated winding:** In this case all the winding turns are wound together to form one multi turn coil. All the turns have same magnetic axis.

Ex: Field winding of a DC machine, transformer winding

i) **Distributed winding:** In this case all the winding turns are arranged in several full pitch or fractional pitch coils.(or) In this winding, conductors of a given phase are distributed in various slots around the periphery of air gap.
Ex: Armature winding of a DC machine

Problem:

1. A 60-cycle alternator has 2 poles. What is the speed of the alternator?
A 60-cycle alternator has a speed of 120 rpm. How many poles has it?

Solution:

(a) $f = [PS / (120)]$ cles/sec (1)

Where f = frequency, P = number of poles, S = rpm

$$\therefore S = [(120f) / P]$$
$$= [(120 \times 60) / 2]$$
$$= 3600 \text{ rpm}$$

(b) This may be solved without using Eqn. (1) directly.

The 2-pole 60-cycle alternator rotates at 3,600 rpm. Therefore the 60-cycle, 120 rpm Alternator must have

$$[(3{,}600) / (120)]\ 2 = 60 \text{ poles}$$

UNIT 5

ELECTRICAL INSTALLATIONS

5.1| Low Voltage Switchgear or LV Switchgear

Generally electrical switchgear rated upto 1 KV is termed as low voltage switchgear. The term LV Switchgear includes low voltage circuit breakers, switches, off load electrical isolators, HRC fuses, earth leakage circuit breaker, miniature circuit breakers (MCB) and molded case circuit breakers (MCCB) etc i.e. all the accessories required to protect the LV system.

The most common use of LV switchgear is in LV distribution board. This system has the following parts

Incomer

The incomer feeds incoming electrical power to the incomer bus. The switchgear used in the incomer should have a main switching device.

The switchgear devices attached with incomer should be capable of withstanding abnormal current for a short specific duration in order to allow downstream devices to operate. But it also be cable of interrupting maximum value of the fault current generated in the system. It must have interlocking arrangement with downstream devices. Generally air circuit breakers are preferably used as interrupting device. Low voltage air circuit breaker is preferable for this purpose because of the following features.

1. Simplicity

2. Efficient performance

3. High normal current rating up to 600 A

4. High fault withstanding capacity upto 63 kA

Although air circuit breakers have long tripping time, big size, high cost but still they are most suitable for low voltage switchgear for the above mentioned features.

Sub - Incomer

Next downstream part of the LV Distribution board is sub - incomer. These sub - incomers draw power from main incomer bus and feed this power to feeder bus. The devices installed as parts of a sub - incomer should have the following features

1. Ability to achieve economy without sacrificing protection and safety

2. Need for relatively less number of inter - locking since it cover limited are of network. ACBs and switch fuse units are generally used as sub - incomers along with molted case circuit breakers(MCCB)

Feeders

Different feeders are connected to the feeder bus to feeds different loads like, motor loads, lighting loads, industrial machinery loads, air conditioner loads, transformer cooling system loads etc. All feeders are primarily protected by switch fuse unit and in addition to that, depending upon the types of load connected to the feeders, the different switchgear devices are chosen for different feeders. Let's discuss in details

1. Motor Feeder

2. Motor feeder should be protected against over load, short circuit, over current up to locked rotor condition and single phasing.

1. Industrial Machinery Load Feeder

2. Feeder connected industrial machinery load like oven, electroplating bath etc are commonly protected by MCCBl and switch fuse units

3. Lighting Load Feeder

This is protected similar to industrial machinery load but additional earth leakage current protection is provided in this case to reduce any damage to life and property that could be caused by harmful leakages of current and fire.

In LV switchgear system, electrical appliances are protected against short circuit and over load conditions by electrical fuses or electrical circuit breaker. However, the human operator is not adequately protected against the faults occurs inside the appliances. The problem can be overcome by using earth leakage circuit breaker. This operates on low leakage current. The earth leakage circuit breaker can detect leakage current as low as 100 mA and is capable of disconnecting the appliance in less than 100 msec.

Fuses and MCBs

A fuse is a piece of conducting wire having low melting point. It is rated for a certain current. It is included at many stages of installation to protect various circuits and entire wiring system. The fuse is always put in the live wire. Whenever the current in a subcircuit exceeds the rated current, the fuse melts and breaks the circuit. This is the cheapest type of protection that can be provided to an electrical installation. To avoid any damage to the installation and risk of fire, one should never use a fuse of rating higher than the circuit is meant for.

The fuse wire for smaller current ratings (say, up to 10 A) are made of lead-tin alloy (36 % lead and 64 % tin). For higher current ratings, fuse wires are made of copper, zinc, lead, tin, aluminium, etc. In case a fuse melts, it can be replaced without much expense.

In modern practices, we use a MCB (mini-circuit breaker) in place of a fuse. A fuse cut-out assembly made of porcelain is comparatively cheaper (costing about Rs. 25/-) than an MCB (costing about Rs. 120/-), but occupies more space, and is quite cumbersome to install and operate. Furthermore, due to frequent sparking it may cause fire hazard. On the other hand, an MCB is a neat and clean device, which simply trips off itself whenever the current in the circuit exceeds its rating. On correcting the fault, it can simply be switched on again.

An Earth-leakage circuit breaker (ELCB) is a safety device used in electrical installations with high Earth impedance to prevent shock. It detects small stray voltages on the metal enclosures of electrical equipment, and interrupts the circuit if a dangerous voltage is detected. Once widely used, more recent installations instead use residual current circuit breakers which instead detect leakage current directly.
There are two types of Earth-leakage circuit breaker:

- voltage operated (referred as ELCB)
- current operated (referred to as RCCB)

Voltage-operated (ELCB)

Voltage ELCBs have been in widespread use since then, and many are still in operation but are no longer installed in new construction. A voltage-operated ELCB detects a rise in potential between the protected interconnected metalwork (equipment frames, conduits, enclosures) and a distant isolated Earth reference electrode. They operate at a detected potential of around 50 volts to open a main breaker and isolate the supply from the protected premises.

A voltage-operated ELCB has a second terminal for connecting to the remote reference Earth connection.
Thee Earth circuit is modified when an ELCB is used; the connection to the Earth rod is passed through the ELCB by connecting to its two Earth terminals. One terminal goes to the installation Earth CPC (circuit protective conductor, aka Earth wire), and the other to the Earth rod (or sometimes other type of Earth connection).

Disadvantages
Compared with a current-sensing system, voltage sensing systems have several disadvantages which include:

- A wire break in the fault to load section, or in the earth to ground section, will disable operation of the ELCB.
- Requirement of an additional third wire from the load to the ELCB.
- Separate devices cannot be grounded individually.

- Any additional connection to Earth on the protected system can disable the detector.

Molded Case Circuit Breaker (MCCB): Molded case circuit breakers are a type of electrical protection device that is commonly used when load currents exceed the capabilities of miniature circuit breakers. They are also used in applications of any current rating that require adjustable trip settings, which are not available in plug-in circuit breakers and MCBs.

Definition and Function

A molded case circuit breaker, abbreviated MCCB, is a type of electrical protection device that can be used for a wide range of voltages, and frequencies of both 50 Hz and 60 Hz. The main distinctions between molded-case and miniature circuit breaker are that the MCCB can have current ratings of up to 2,500 amperes, and its trip settings are normally adjustable. An additional difference is that MCCBs tend to be much larger than MCBs. As with most types of circuit breakers, an MCCB has three main functions:

1. Protection against overload – currents above the rated value that last longer than what is normal for the application.

2. Protection against electrical faults – During a fault such as a short circuit or line fault, there are extremely high currents that must be interrupted immediately.

3. Switching a circuit on and off – This is a less common function of circuit breakers, but they can be used for that purpose if there isn't an adequate manual switch.

The wide range of current ratings available from molded-case circuit breakers allows them to be used in a wide variety of applications. MCCBs are available with current ratings that range from low values such as 15 amperes, to industrial ratings such as 2,500 amperes. This allows them to be used in both low-power and high-power application.

Molded Case Circuit Breaker Operating Mechanism:

At its core, the protection mechanism employed by MCCBs is based on the same physical principles used by all types of thermal-magnetic circuit breakers.

Overload protection is accomplished by means of a thermal mechanism. MCCBs have a bimetallic contact what expands and contracts in response to changes in temperature. Under normal operating conditions, the contact allows electric current through the MCCB. However, as soon as the current exceeds the adjusted trip value, the contact will start to heat and expand until the circuit is interrupted. The thermal protection against overload is designed with a time delay to allow short duration overcurrent, which is a normal part of operation for many devices. However, any overcurrent conditions that last more than what is normally expected represent an overload, and the MCCB is tripped to protect the equipment and personnel.

On the other hand, fault protection is accomplished with electromagnetic induction, and the response is instant. Fault currents should be interrupted immediately, no matter if their duration is short or long. Whenever a fault occurs, the extremely high current induces a magnetic field in a solenoid coil located inside the breaker – this magnetic induction trips a contact and current is interrupted. As a complement to the magnetic protection mechanism, MCCBs have internal arc dissipation measures to facilitate interruption.

As with all types of circuit breakers, the MCCB includes a disconnection switch which is used to trip the breaker manually. It is used whenever the electric supply must be disconnected to carry out field work such as maintenance or equipment upgrades.

5.2 | WIRES AND CABLES FOR INTERNAL WIRING

For domestic wiring, the most extensively used conductor material is *copper* or *aluminium**. To prevent any leakage of current from the conductor and also to provide mechanical strength, it is surrounded by an insulation and sheath. Normally, the cables are classified according to the insulation used over the conductor. The selection of suitable cable for an installation depends upon the following considerations :

(i) The nature of conditions under which the cable is to be used (for example, underground, hanging in air, in damp conditions, etc.).
(ii) The operating voltage.
(iii) The current capacity of the installation.

The operating conditions decide the type of insulation and other protection needed around the conductor of the cable. The operating voltage determines the thickness of the insulation. The current capacity of the installation determines the cross-sectional area or size of the cable-conductor. Following types of cables are available in market :

(1) Elastomer Insulated Cables

These cables have insulation coating of following different rubber-like materials :

(i) ***Vulcanised Indian Rubber (VIR)*** The insulation covering has 36 % natural rubber and balance materials such as sulphur, carbon, wax, etc. The insulation is applied either as a single coating by extrusion or in two or more layers. It is then vulcanised.

(ii) ***Butyl Rubber Coatings*** Such coatings are more resistant to oxidation and ozone than the natural rubber. These can be used for maximum conductor temperature of 85 °C.

(iii) ***Ethylene-Propylene Rubber Coatings*** These coatings possess good electrical insulation properties, and are resistant to heat, humidity, chemicals and ozone. Hence, these cables can be directly buried underground.

These are suitable for hazardous industries, oil refineries, power plants, etc.

(iv) ***Silicon Rubber Coatings*** These coatings can safely withstand conductor temperature of 150 °C for long durations and of 200 °C for short durations.

(v) ***Polychloroprene Coatings*** These coatings are superior to natural rubber in resistance to outdoor weather.

Furthermore, these coatings possess self-extinguishing property, if ignited externally.
These cables can safely withstand conductor temperature of 60 °C.

(2) Polyvinyl Chloride (PVC) Insulated Cables

The PVC coverings are chemically inert towards oxygen, many alkalis and oils. These are non-inflammable, insoluble in common liquids, and possess good mechanical strength, high dielectric strength and long life. However, these coatings have a tendency of softening when subjected to even moderate temperature; hence these are used only if the conductor temperature does not exceed 65 °C. Because of many advantages, the PVC cables have replaced VIR cables for low voltage installation.

(3) Lead Sheathed Cables

These are mainly employed for underground laying of service mains. In such cables, paper insulation is provided by helically wrapping paper tapes of desired thickness around the conductor. The insulated cover is vacuum dried and then impregnated under pressure with mineral oil or some other suitable liquid compound. Finally, the insulation is covered with a lead (or aluminium) sheath to protect it from moisture, soil, etc

(4) Flexible cables(or Chords)

Flexible wires are made up of a large number (say, 17 or 21) of very fine wires, called *strands*, twisted together to make a composite wire or cable. This is insulated by PVC, rubber, or plastic.Normally, two such wires having different colour insulation-coatings are twisted together to make a flexible chord. Such chords are used for the socket outlet to portable apparatus (such as table lamp, radio, battery charger, etc.) or from ceiling roses to a lamp holder or a ceiling fan. The chords used for hanging the pendant light-fittings from a ceiling rose must have sufficient mechanical strength to support the weight of the light-fittings.

The flexible chords used for connecting appliances such as microwave oven, OTG, refrigerator, washing machine, etc. must incorporate an earth continuity conductor so as to avoid any hazard of electric shock. The advantage of using flexible chords is the easiness of handling due to their flexibility. However, care must be taken to protect the insulation cover over the conductors against mechanical damage.

5.3 Specification of Wires

The conductor material, insulation, size and the number of cores, specifies the electrical wires. These are important parameters as they determine the current and voltage handling capability of the wires. The conductors are usually of either copper or aluminium.

Various insulating materials like PVC, TRC, and VIR are used. The wires may be of single strand or multistrand. Wires with combination of different diameters and the number of cores or strands are available

For example: The VIR conductors are specified as 1/20/,3/22,. ... 7/20
The numerator indicates the number of strands while the denominator corresponds to the diameter of the wire in SWG (Standard Wire Gauge). SWG 20 corresponds to a wire of diameter 0.914 mm, while SWG 22 corresponds to a wire of diameter 0.737 mm.

Wiring may be the simplest wiring like controlling the appliances from single point. Or two-way and three-way control is also possible with reasonable complexity.

5.4 EARTHING

Earthing is to connect any electrical equipment to earth with a very low resistance wire, making it to attain earth's potential. The wire is usually connected to a copper plate placed at a depth of 2.5 to 3 meters from the ground level. The potential of the earth is considered to be at zero for all practical purposes as the generator (supply) neutral is always earthed. The body of any electrical equipment is connected to the earth by means of a wire of negligible resistance to safely discharge electric energy, which may be due to failure of the insulation, or line coming in contact with the casing, etc. Earthing brings the potential of the body of the equipment to ZERO i.e. to the earth's potential, thus protecting the operating personnel against electrical shock. The body of the electrical equipment is not connected to the supply neutral due to long transmission lines and intermediate substations, the same neutral wire of the generator will not be available at the load end. Even if the same neutral wire is running, it will have a self-resistance, which is higher than the human body resistance. Hence, the body of the electrical equipment is connected to earth only.

Necessity of Earthing:

1. The protect the operating personnel from danger of shock in case they come in contact with the charged frame due to defective insulation.
2. To maintain the line voltage constant under unbalanced load condition.
3. To protect the equipments.
4. T protect large buildings and all machines fed from overhead lines against lightning.

5.5 Methods of Earthing

The important methods of earthing are the plate earthing and the pipe earthing. The earth resistance for copper
wire is 1 ohm and that of *GI* wire less than 3 ohms. The earth resistance should be kept as low as possible so that the neutral of any electrical system, which is earthed, is maintained almost at the earth potential. The typical value of the earth resistance at powerhouse is 0.5 ohm and that at substation is 1 ohm.

Plate Earthing In this method, a copper plate of 60 cm × 60 cm × 3.18 cm or a *GI* plate of the size 60 cm × 60 cm × 6. 35 cm is used for earthing. The plate is placed vertically down inside the ground at a depth of 3 m and is embedded in alternate layers of coal and salt for a thickness of 15 cm. In addition, water is poured for keep-ing the earth electrode resistance value well below a maximum of 5 ohms. The earth wire is securely bolted to the earth plate. A cement masonry chamber is built with a cast iron cover for easy regular maintenance.

Pipe Earthing Earth electrode made of a *GI* (galvanized) iron pipe with 38 mm diameter and length of 2 m (depending on the current) with 12 mm holes on the surface is placed upright at a depth of 4.75 m in a perma-nently wet ground. To keep the value of the earth resistance at the desired level, the area (15 cm) surrounding

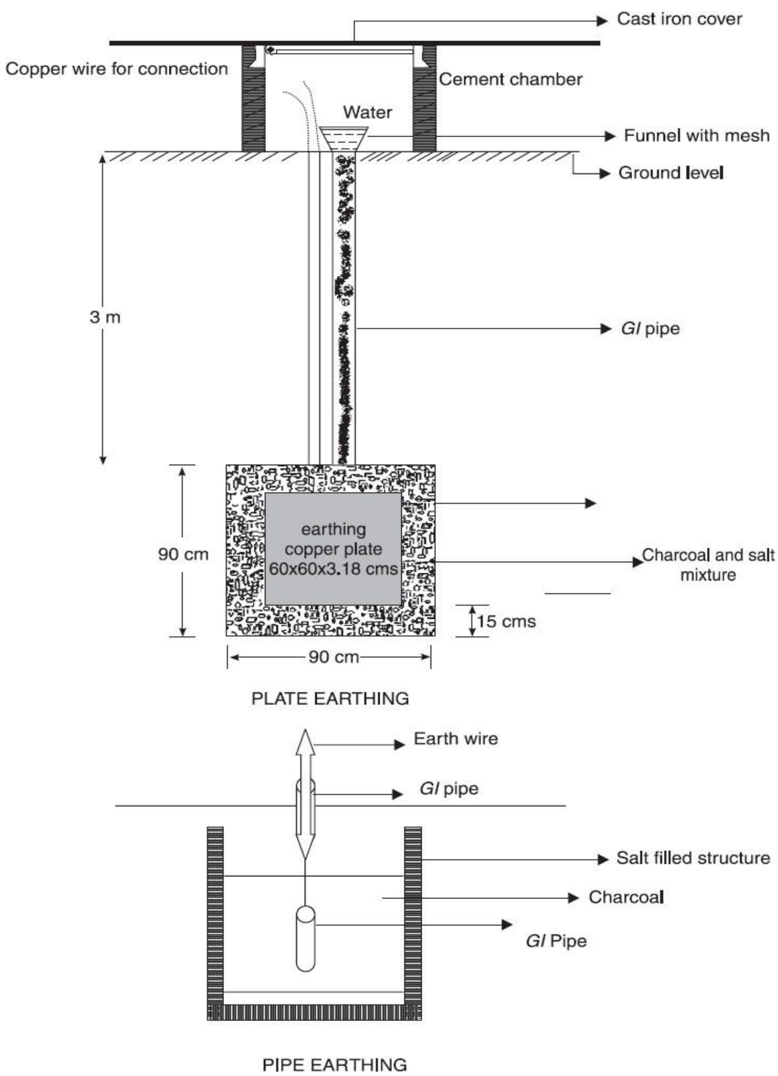


Fig. 5.1 *Pipe earthing*

the *GI* pipe is filled with a mixture of salt and coal. The efficiency of the earthing system is improved by pouring water through the funnel periodically. The *GI* earth wires of sufficient cross-sectional area are run through a 12.7 mm diameter pipe (at 60 cms below) from the 19 mm diameter pipe and secured tightly at the top as shown in the figure.

When compared to the plate earth system, the pipe earth system can carry larger leakage currents as a much larger surface area is in contact with the soil for a given electrode size. The system also enables easy maintenance as the earth wire connection is housed at the ground level.

5.6 EARTHING OF ELECTRICAL INSTALLATION

An electrical equipment or appliance is said to be ***earthed***, if its outer frame and its other parts not carrying any current are connected to the earth so as to attain as nearly zero potential as possible. In practice, all equipments and machinery, as well as electric poles, towers, neutral wires, etc., are earthed. The purpose of earthing is to ensure that all parts of the system other than live parts (i.e., the parts which are supposed to carry currents) are maintained at the earth potential at all times.

Satisfactory earthing is the most important part of an electrical installation, as it provides safety and be-cause the operation of all the protective devices depends on it. However, unfortunately, it becomes the most neglected part of the installation as it involves extra expense without showing any immediate benefits.

Objective of Earthing

The main objective of earthing is to provide ***safety*** of operation. In case, the insulation of windings placed inside the machine ever becomes weak, a part of the operating current gets diverted to the surface. When a person touches such a machine, the surface current finds a path through his body to earth. If this ‘leakage current’ is high, the person gets an electric shock which may even cause death. By earthing the machine, the shock hazard is avoided as the leakage current gets an outlet to earth.

Another objective of earthing, though not widely used nowadays, is to ***save conducting material***. The earth itself provides the return path for the current. In earlier days, we were using this technique of saving conductor material in case of telegraph. We still use this technique of providing return path for the current through *earth* (either the chassis or a common conductor) in case of automobiles and electronic equipments, not so much for saving the conductor but for avoiding complications in laying the return wire.

Earthing also helps in protecting high-rise buildings from atmospheric lightning. A forked metal rod or thick wire, called the *lightning conductor*, sticks out from the top of the building, chimney, tower, etc. Its other end is buried deep into the ground. Whenever lightning occurs, the electricity passes directly from the top of lightning conductor to the earth, thereby protecting the building from any damage.

Methods of Earthing

Earthing should be done in a way so that on a short circuit, the earth-loop impedance is low enough to pass 3 times the current if fuses are used, and 1.5 times the current if MCBs are used. The metal work of the installation should be solidly earthed without using any switch or fuse in the circuit.

In an earthing system, the metallic body (such as plate or pipe) which is embedded in the earth is called *earth electrode*. For effective earthing, the resistance offered by the earth electrode along with the soil in which the electrode is embedded should be quite low. *Galvanised iron* (GI) or *copper* is used to make an earth electrode. Copper, although costly, is a better choice as it is least affected by moisture and is not easily rusted.

Furthermore, to increase the conductivity of the soil around the earth electrode, alternate layers of common salt and charcoal are filled. There are following two earthing methods commonly used.

(1) ***Plate Earthing*** As shown in Fig. 5.2*a*, a plate of following dimensions is used as earth electrode

(i) *Copper Plate* : 0.3 m x 0.3 m x 6.35 mm

(ii) *GI Plate* : 0.3 m x 0.3 m x3.2 mm

The plate is buried to a depth not less than 2 m and at least 0.6 m away from the foundation of any building.

The layers of common salt and charcoal are 30-mm and 80-mm thick, respectively.

(2) ***Pipe Earthing*** As shown in Fig. 5.2*b*, a GI pipe with a few holes at its lower end is buried to a depth not less than 2 m and at least 0.6 m away from the foundation of any building. Normally, the size of pipe is either 2 m long and 38 mm diameter or 1.37 m and 51 mm diameter. However, for dry and rocky soil, we use longer pipes. Alternate layers of common salt and charcoal have thickness of 30 mm and 80 mm, respectively.

In both the methods shown in Fig. 5.2, to maintain good conductivity of the soil, an arrangement is made for pouring water into the earth pit surrounding the earth electrode. This is especially needed during sum-mer. As the pipe has much larger contact area with soil, it can handle larger leakage currents than the plate earthing of same electrode size. The earth wire (made of copper) is tightly fastened to the earth electrode by means of nut and bolt.

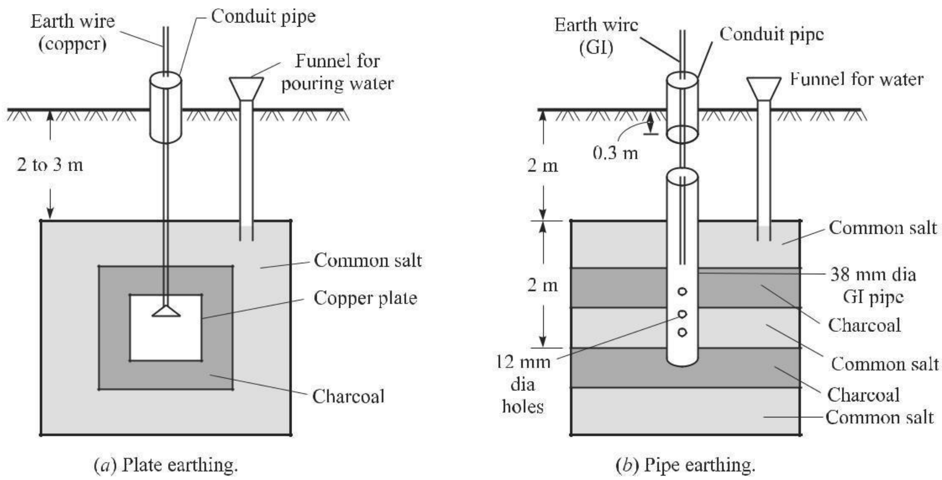


Fig. 5.2 *Earthing methods.*

5.7 Battery Technology

Introduction

With all the new applications, which have developed over the past few decades, the market for consumer batteries has grown correspondingly. Nowadays, the average family may have many batteries at any one time in and around the home. Many of these batteries are of advanced design and construction, giving greatly improved performance as a result of developments in materials science and technology. Although most small consumer batteries are still of the primary variety, there is growing trend to adopt secondary (rechargeable) batteries as being more economical.

A cell has only two electrodes (or half cells) that generate electrical energy and the EMF thus, generated depend on the magnitude of the electrode potentials of the two electrodes. Depending on the specific purpose, higher voltage can be achieved by coupling a number of cells in series. The arrangement of two or more cells coupled in series is called a battery. *The working principle of a battery is the transformation of free energy change of redox reactions of the electrode active materials of cells into electrical energy.*

Batteries are the indispensable source of portable energy. Nowadays, wide varieties of batteries are avail-able in the market; and these find extensive applications in modern technology. The batteries are constructed as per the desired requirements and also to suit specific applications.

Depending on the type of battery, they are used in electronic gadgets, pacemakers, calculators, power sup-plies, telecommunication equipment, and so on. High power batteries are also being tested and used in small cars, to minimize the air pollution problem. Nowadays, batteries of long shelf life, power, recharging capacity, tolerance to extreme conditions and reliability are desirable, and have wide applications too.

Basic concepts of Batteries:

A galvanic cell is a device that generates electrical energy at the expense of decrease of free energy of electrode reactions of a cell. *A **battery** is a device that consists of one or more of cells connected in series or parallel or both and converts the chemical energy by means of an electrochemical oxidation–reduction reaction depending on their desired output voltage and capacity* (Fig. 5.3).

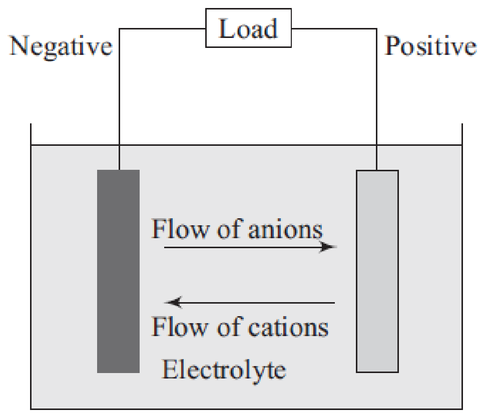


Fig. 5.3 *An electrochemical cell.*

The cell consists of three major aspects:

1. The anode (or the –ve electrode) is oxidised during the electrochemical reaction and liberates electrons to the external circuit.

$$M \rightarrow M^{n+} + ne$$

The materials having the following properties are preferable as an anodic material. Efficiency as a reducing agent, high coulombic output (Ah/g), good conductivity, stability, ease of fabrication and low cost.

2. The cathode (or positive electrode) is reduced during the electrochemical reaction, which accepts electrons from the external circuit.

$$M^{n+} + ne \rightarrow M$$

The cathode must be an efficient oxidizing agent, be stable when in contact with the electrolyte, and have useful working voltage.

3. The electrolyte provides the medium for transfer of ions inside the cell between the anode and the cathode. The electrolyte must have good ionic conductivity. The battery itself can be built in many shapes and configurations—cylindrical, button and flat.

Battery Characteristics

A battery may be specifically designed, constructed and used based on its characteristic properties. The few important characteristics of batteries are mentioned below.

1. Free energy change

Whenever electrode reactions occur in the presence of active materials in a battery, there occurs a decrease in free energy of the redox system

$$-\Delta G = nEF$$

where
F = Faraday (96500 C or 26.8 Ah)
n = number of electrons involved in electrode reactions.
E = potential, Volts.

2. EMF of a battery

The net voltage (or EMF) of battery depends on the total number of cells, which constitute a battery. The overall reaction of a cell of a battery is given as

$M_1 + M_2^{n+}$ m$M_1^{n+} + M_2$

Thus, the EMF of a cell in a battery, E_B is

$$E_B = E^{o}_{cell} - \frac{0.0591}{n}\log_{10}\frac{[M_1^{n+}]}{[M_2^{n+}]}$$

where $E^{o}_{cell} = E^{o}_{cathode} - E^{o}_{anode}$ and $\frac{[M_1^{n+}]}{[M_2^{n+}]}$ is the ratio of the ionic concentrations of M_1^{n+} and M_2^{n+}

It is evident, therefore, that the EMF of the cell of a battery is dependent on.

(i) p.d between the cathode and anode,
(ii) the ratio of the ionic concentration of M_1^{n+} and M_2^{n+}, and
(iii) the temperature.

Summarizing, it can be remarked that

- EMF of the battery is higher, if the electrode potential difference between the two electrodes is more.

- EMF of the cell decreases with the increasing molar concentration of $[M_1^{n+}]$ in the numerator of the expression.If the temperature of the battery increases, the EMF of the cell gets reduced marginally.

3. Amps/Ampere-hour

Also known as *Amperes*. This is the rate at which electrons flow in a wire. The units are coulombs per second, or since an electron has a charge of 1.602 $x10^{-19}$ coulombs, an amp is 6.24 x 10^{+18} electrons per second.
One ampere-hour is equal to a current of one ampere flowing for one hour. A unit quantity of electricity used as a measure of the amount of electrical charge that may be obtained from a storage battery before it requires recharging.

4. Capacity

The capacity of a battery is expressed as the total quantity of electricity involved in the electrochemical reaction and is defined in terms of coulombs or amperes-hours (Ah) or the total number of ampere hour or watt-hours that can be withdrawn from a fully charged cell or battery under specified conditions of discharge is termed as the capacity of a battery. The theoretical capacity of a battery is the quantity of electricity involved in the electro-chemical reaction. It is denoted Q and is given by

$Q = xnF$

where x = the number of moles of reaction,
n = the number of electrons transferred per mole of reaction and
F = Faraday's constant. The capacity is usually given in terms of mass, not the number of moles:

$$Q = \frac{nF}{M_r}$$

where M_r = molecular mass.

This gives the capacity in units of ampere-hours per gram (Ah/g). The ampere-hour capacity of a battery on discharge is determined by a number of factors, among which the following are the most important: final limiting voltage; quantity of electrolyte; discharge rate; density of electrolyte; design of separators; temperature, age and life history of the battery; and number, design and dimensions of electrodes.

The total capacity, Ah or Wh, that will be obtained from a cell or battery at defined discharge rates and other specified discharge or operating conditions is known as its ‘available capacity’.

5. Power

The *power* generated by a battery can be calculated as

$W = V.I$, where V = cell voltage and I = cell current (or rate)

since $V = IR$; $w = I^2R$ and $w = V^2/R$.

The *energy* generated by a battery is power x time or

$$E = V.I.\ t = qV$$

where q = charge = rate x time

6. Power density

The *power density* is usually discussed in terms of the cell mass:

Power density = Energy (E)/time (t)/mass (kg)
=Energy (qV)/time (t)/mass (kg) of cell
=Power/mass (units are W/kg)

The ratio of the power delivered by a cell or a battery to its weight, w/kg, is also known as the power density of a battery. During the discharge of a battery, the power density decreases.

This is related to the *energy density* at a given discharge rate and indicates how rapidly the cell can be discharged and how much power generated. A cell with high energy density may exhibit a significant voltage and capacity drop at higher discharge rates and power density, therefore, has a low power density.

7. Energy density

The energy density or capacity—determined by the voltage of the cell and the amount of charge that can be stored, $E = qV$, this parameter is usually evaluated on a weight or volume basis:

Theoretical. weight. capacity = qV/mass (units are W h/kg)
Theoretical. volume. capacity = qV/volume (units are Watt h/L)

The energy density of a cell or a battery is also described as the ratio of the energy output of a cell or battery to its weight, Wh/kg.

8. Efficiency
The ratio of the output of a battery on the discharge to the input required to restore it to the initial state of charge under specified conditions

i) 'Voltage efficiency' is described as the ratio of average voltage during discharge to average voltage during recharge under specified conditions.

ii) 'Watt-hour efficiency' is known as the ratio of watt-hours delivered on discharge of a battery to the watt-hour needed to restore it to its original state under specified conditions of charge and discharge.

iii) Ampere-hour efficiency—The ratio of the output of a secondary cell or a battery, measured in ampere-hours, to the input required to restore the initial state of charge, under specified conditions (also coulombic efficiency).

9. Cycle life
For rechargeable batteries, the duration of satisfactory performance, measured in years or in the *number of charge/discharge cycles*. In practice, end of life is usually considered to be reached when the cell or battery delivers approximately 80 percent of rated ampere-hour capacity.

Shelf Life—The duration of storage under specified conditions at the end of which a cell or a battery still retains the ability to give a specified performance.

For a dry cell, the period of time (measured from date of manufacture), at a storage temperature of 21°C (69°F), after which the cell retains a specified percentage (usually 90 percent) of its original energy content.

10. Tolerance to service conditions
The battery duty may require that it provides power continuously, intermittently or at an irregular rate.

It may also be expected to perform its duty or be stored under a range of conditions (e.g.−40 to +55°C to be successful in the extremes of winter and summer).

The battery may also have to be tolerant to various types of misuse, including occasional shorting, vibration and shock.

Commercial Cell or Battery

An assembly of electrochemical cells is called a commercial battery, provided it satisfies the following requisites.

- Long shelf life
- Recharging capacity
- Easily portable
- Compact and lightweight and

Classification of Batteries

Electrochemical cells or batteries are identified as primary (non-rechargeable) or secondary (rechargeable), depending on their capability of being electrically recharged. The batteries are classified as (i) primary, ii) secondary, and (iii) reserve batteries.

1. Primary batteries

The working principle of a primary battery is the conversion of the free energy change of the active materials during electrode processes into the electrical energy. *A battery which is not intended to be recharged and discarded when the battery has delivered all its electrical energy is known as a primary battery.*

The net cell reaction of a primary battery is irreversible and as long as the active materials are present in a battery, the cell generates electrical energy. In other words, primary batteries cannot be recharged.

Example: Zn–MnO_2 dry cell.

2. Secondary or rechargeable batteries

A secondary battery is known as a galvanic battery, which after discharge, may be restored to the fully charged state by the passage of an electric current through the cell in the opposite direction to that of the discharge.

In other words, the net cell reactions of battery can be reversed. They are storage devices for electrical energy and are known as 'storage batteries'.

Examples: Lead–acid battery and Ni–Cd battery.

The secondary batteries have advantages over the other primary batteries that the net cell reactions can be reversed during the charging process and the current can be drawn during the discharge process. The second-ary batteries have better cycle life and capacity, so that it can be used over and over again. The secondary batteries are classified into two types:

- Acid storage battery—lead–acid battery.
- Alkaline storage battery—Ni–Cd battery.

3. Reserve batteries

In these reserve types of batteries, vital component is separated from the rest of the battery prior to activa-tion. Under this condition, chemical deterioration or self-discharge is essentially eliminated, and the battery is capable of long-term storage. Usually, an electrolyte is the component that is isolated.

These batteries are used, for example, to deliver high power for relatively short periods of time, in missiles, torpedoes and other weapon systems.

Example: LiV_2O_5 cell.

Important Applications of Batteries

Batteries have a wide variety of applications and their market is very potential

1. Car batteries

2. High specific power is required for the large mechanical load, low operating temps can arise, safety and environmental concerns an important issue. The Pb–acid cell is currently used exclusively.

3. Electrically powered vehicles use batteries to replace the combustion engine. The batteries will need sufficient energy and power densities, and environmental safety, and cost issues become especially important for batteries produced on a large scale.

4. Secondary cells for portable high-power apps such as portable computers (a rapidly growing market— currently these use NiCd, NiMH or lithium ion cells), power tools, flashlights, etc.

5. Primary cells for portable low-power applications such as watches, metres, cameras, calculators. In these devices, low power densities are often acceptable, but low self-discharge rates and high energy densities are desirable to provide long service life. Lithium batteries are most often used.

6. Military applications: Examples include power for missile or torpedo guidance, drive or activation (these can obviously be primary cells), and communication devices. Fuel cells are currently used extensively.

7. Pacemakers

8. Cost is not a primary issue, but concerns focus on a long service life, safety and discharge profile. A lithium–iodine cell is typically used.

9. Hearing aids: Usually use mercury, silver oxide or zinc–air batteries because the high volumetric energy densities allow for a smaller battery.

5.8 Elementary Calculations for Energy Consumption:

what is energy measured in? The primary way electricity consumption is measured is with the unit “watt-hour”.

Lighting and common household appliances such as air conditioning units, computers and toasters are all products requiring electricity to function.

A watt (w) is a measure of this electric power and each of these household products should be marked with a watt rating to reflect their usage. Most common household appliances owned by consumers include a compliance badge, which is used to indicate the amount of electrical power that the particular product requires to function correctly.

To put this into perspective, a light bulb could have a 40-watt rating, an average toaster could have a 600-watt rating and an air conditioner could have a 4000-watt rating. Multiplying the watt – or the unit of energy required – by the duration of its usage will offer the amount of total electricity consumed.

The standard measure of electricity consumption is the amount of watts expended over the period of one hour, which is also known as a watt-hour. This means if a 40 watt light bulb is turned on for one hour, it will use 40 watt-hours of electricity.

When people receive an electricity bill, it will record the number of kilowatt-hours (kWh) consumed during that period by the household. A kilowatt-hour is 1000 watt-hours, which means using a 4000 watt air-conditioner for one hour will consume 4 kWh of electricity. This total consumption is what is used to calculate the pricing of an energy bill, which is delivered to customers

The power is the rate at which energy is transferred from one place to another or from one form to another

The formula that links energy and power is: **Energy = Power x Time.**
The unit of energy is the joule, the unit of power is the watt, and the unit of time is the second. If we know the power in watts of an appliance and how many seconds it is used we can calculate the number of joules of electrical energy which have been converted to sortie other form.

E.g. If a 40 watt lamp is turned on for one hour, how many joules of electrical energy have been converted by the lamp?

Energy (w) = Power x Time Energy = 40 x 3600 = 14,400 joules

Note: if an appliance has a rating of one watt it means it converts one joule of electrical energy to some other form every second. Because the joule is such a small unit, quantities of energy are often given in kilojoules. I.e, thousands of joules.

Therefore the above answer could be written as 14.4 kJ.

The Kilowatt Hour (kWh)

Because the joule is so small, electrical energy supplied to consumers is bought by the UNIT. The UNIT is the kilowatt hour (kWh). One kilowatt hour is the amount of energy that would be converted by a one thousand watt appliance when used for one hour

Example

A consumer uses a 6 kW immersion heater, a 4 kW electric stove and three 100 watt lamps for 10 hours. How many units (kWh) of electrical energy have been converted.

Total power in kilowatts = 6 + 4 + 300/1000. = 10.3kW.

Energy in kilowatt hours = Power in watts x time in hours
=10.3 x 10
= 103 kilowatt hours

Electrical supply authorities use the kWh as the unit for measuring electrical energy to householders.

Revision Exercise

1 How much heat energy is converted by a 1kw heater in half a minute?
2 An electric toaster is rated at 500 watts. Determine the amount of heat energy it converts to heat in one minute.
In the calculations of energy so far the values of the power have been given.
However, if enough information is given the volume of the power can be calculated first and then the value put into the energy formula.

Worked Example - 1

Calculate the heat produced by an electric iron, which has a resistance of 30 ohms and takes a current of 3 amperes when it is switched on for 15 seconds.

Power = $I^2R = 3^2 \times 30$
= 270 watts
Energy = Power x Time
= 270 x 15
= 4050 joules

Worked Example - 2

A d-c generator has an e.m.f of 200 volts and provides a current of 10 amps. How much energy does it provide each minute?
Energy = Power x Time
Power = V x I
= 200 x 10
= 2000 watts

Energy = 2000 x 60
= 120,000 Joules or 120 kJ.

5.9 Power factor Improvement:

Need for power factor Improvement:
Improving power factor means reducing the phase difference between voltage and current. Since majority of loads are of inductive nature, they require some amount of reactive **power** for them to function. This reactive **power** is provided by the capacitor or bank of capacitors installed parallel to the load.

Methods for Power Factor Improvement
The following devices and equipment are used for Power Factor Improvement.

1. Static Capacitor
2. Synchronous Condenser
3. Phase Advancer

1. Static Capacitor
We know that most of the industries and power system loads are inductive that take lagging current which decrease the system power factor . For Power factor improvement purpose, Static capacitors are connected in parallel with those devices which work on low power factor. These static capacitors provides leading current which neutralize (totally or approximately) the lagging inductive component of load current (i.e. leading component neutralize or eliminate the lagging component of load current) thus power factor of the load circuit is improved.
These capacitors are installed in Vicinity of large inductive load e.g Induction motors and transformers etc, and improve the load circuit power factor to improve the system or devises efficiency.

Suppose,here is a single phase inductive load which is taking lagging current (I) and the load power factor is Cosθ as shown in fig-1.

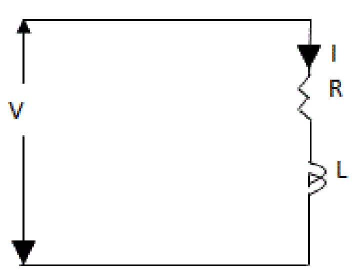


fig – 1

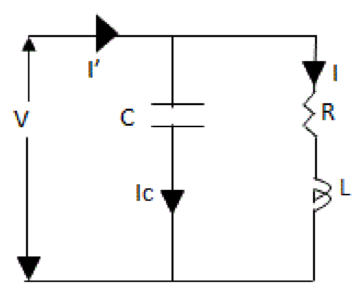


fig-2

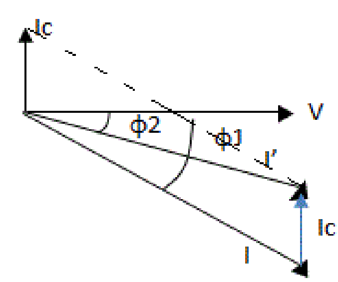


fig-3

In fig-2, a Capacitor (C) has been connected in parallel with load. Now a current (Ic) is flowing through Capacitor which lead 90° from the supply voltage (Note that Capacitor provides leading Current i.e., In a pure capacitive circuit, Current leading 90° from the supply Voltage, in other words, Voltage are 90° lagging from Current). The load current is (I). The Vectors combination of (I) and (Ic) is (I') which is lagging from voltage at θ2 as shown in fig 3.

It can be seen from fig 3 that angle of θ2 < θ1 i.e. angle of θ2 is less than from angle of θ2. Therefore Cosθ2 is less than from Cosθ1 (Cosθ2> Cosθ1). Hence the load power factor is improved by capacitor.

Also note that after the power factor improvement, the circuit current would be less than from the low power factor circuit current. Also, before and after the power factor improvement, the active component of current would be same in that circuit because capacitor eliminates only the re-active component of current. Also, the Active power (in Watts) would be same after and before power factor improvement.

Advantages:

- Capacitor bank offers several advantages over other methods of power factor improvement.
- Losses are low in static capacitors
- There is no moving part, therefore need low maintenance
- It can work in normal conditions (i.e. ordinary atmospheric conditions)
- Do not require a foundation for installation
- They are lightweight so it is can be easy to installed

Disadvantages:

- The age of static capacitor bank is less (8 – 10 years)
- With changing load, we have to ON or OFF the capacitor bank, which causes switching surges on the system
- If the rated voltage increases, then it causes damage it
- Once the capacitors spoiled, then repairing is costly

2. Synchronous Condenser

When a Synchronous motor operates at No-Load and over-exited then it's called a synchronous Condenser. Whenever a Synchronous motor is over-exited then it provides leading current and works like a capacitor.

When a synchronous condenser is connected across supply voltage (in parallel) then it draws leading current and partially eliminates the re-active component and this way, power factor is improved. Generally, synchronous condenser is used to improve the power factor in large industries.

Advantages:

- Long life (almost 25 years)
- High Reliability
- Step-less adjustment of power factor.
- No generation of harmonics of maintenance
- The faults can be removed easily
- It's not affected by harmonics.
- Require Low maintenance (only periodic bearing greasing is necessary)

Disadvantages:

- It is expensive (maintenance cost is also high) and therefore mostly used by large power users.
- An auxiliary device has to be used for this operation because synchronous motor has no self starting torque
- It produces noise

3. Phase Advancer

Phase advancer is a simple AC exciter which is connected on the main shaft of the motor and operates with the motor's rotor circuit for power factor improvement. Phase advancer is used to improve the power factor of induction motor in industries.

As the stator windings of induction motor takes lagging current 90° out of phase with Voltage, therefore the power factor of induction motor is low. If the exciting ampere-turns are excited by external AC source, then there would be no effect of exciting current on stator windings. Therefore the power factor of induction motor will be improved. This process is done by Phase advancer.

Advantages:

- Lagging kVAR (Reactive component of Power or reactive power) drawn by the motor is sufficiently reduced because the exciting ampere turns are supplied at slip frequency (fs).
- The phase advancer can be easily used where the use of synchronous motors is Unacceptable

Disadvantage:

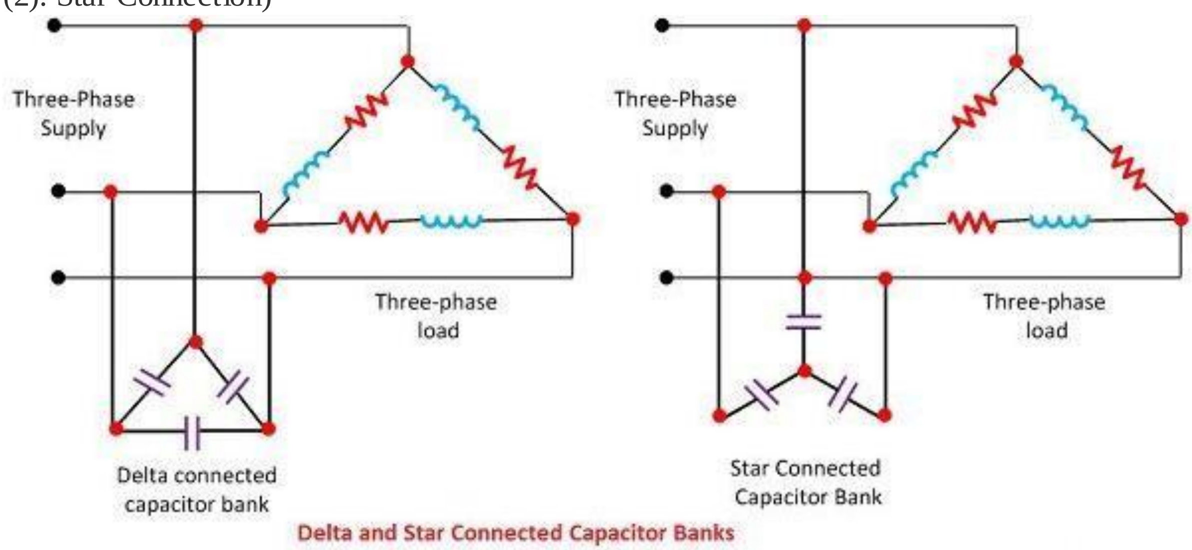
- Using Phase advancer is not economical for motors below 200 H.P. (about 150kW)

Power Factor Improvement in single phase and three phase star & delta connections

Power factor improvement in three phase system by connecting a capacitor bank in

(1). Delta connection

(2). Star Connection)



Power Factor Improvement in single phase and three phase star delta connection

5.10 Battery Back up:

Also known as an uninterruptible power supply or UPS, a battery backup is a means of providing a continual supply of power to electronic equipment, even when the main source of power fails for some reason. The battery backup is only one of several solutions that are used to keep systems operating even when a utility or other power sources is rendered inoperative for a period of time. Business often include this type of backup along with other solutions as a means of keeping computers and other necessary systems running even in the event of a blackout or some a natural disaster that cuts off the main power supply for anything from a few minutes to several days.

Like all types of emergency power, a battery backup makes it possible to switch to an alternative source of power as soon as the main power source fails. Today, it is not unusual for a UPS to be connected to banks of computer workstations, servers, and even telecommunication equipment. When the flow of electricity is interrupted, the backup battery automatically activates, routes power through a surge protector of some type, and allows the connected devices to continue functioning. Often, this occurs with little to no downtime at all.

Most battery backup devices have a limited amount of battery power held in reserve. Earlier designs normally allowed the batteries to supply power for anywhere from five to fifteen minutes before exhausting the supply. This makes them ideal for maintaining operations while a second source of alternative power is put into place and activated. For example, one or more UPS devices may serve as the first phase of the emergency power strategy, and supply power for the ten minutes it takes authorized personnel to connect and activate power generators that are capable of supplying power for several hours.

While the idea of battery backup was once associated more with business settings, the use of this type of device in the home has become more common.

UPS devices for home use are usually somewhat compact in size, but just as powerful as many of the larger commercial models of years past. The home battery backup may be used to power essential appliances, keep healthcare equipment up and running, or simply power a desktop computer during a power outage.

Over the years, the cost for a reliable battery backup device has decreased significantly, making this type of support equipment affordable for businesses of all sizes, as well as for use around the home

Battery Backup Systems:

Often, this occurs with little to no downtime at all. Most battery backup devices have a limited amount of battery power held in reserve.

There are a number of different UPS systems that provide a battery backup with surge protector. Standby UPS systems provide basic battery backup protection in the event of a power failure, as well as surge protection.

Computer Battery Backup:

Often, this occurs with little to no downtime at all. Most battery backup devices have a limited amount of battery power held in reserve.

The second use is as a battery backup so that computers or other equipment can be safely shut down in the event of a total loss of electricity.

Uninterruptable Power Supply:(UPS)

An uninterruptible power supply (UPS) is a battery-driven power supply that helps protect electronic equipment from a sudden loss of power.

Potential damage from changes in voltage can be minimized by a using an uninterruptable power supply. One of the most common situations in which power dips occur is when an individual adds a high-current electric item to an already loaded circuit.

Major Roles of UPS

When there is any failure in main power source, the UPS will supply the power for a short time. This is the prime role of UPS. In addition to that, it can also able to correct some general power problems related to utility services in varying degrees. The problems that can be corrected are voltage spike (Sustained over voltage), Noise, Quick reduction in input voltage, Harmonic distortion and the instability of frequency in mains.

Types of UPS

Generally, the UPS system is categorised into On-line UPS, Off- line UPS and Line interactive UPS. Other designs include Standby on-line hybrid, Standby-Ferro, Delta conversion On-Line.

Off-line UPS

This UPS is also called as Standby UPS system which can give only the most basic features. Here, the primary source is the filtered AC mains (shown in solid path in figure

1). When the power breakage occurs, the transfer switch will select the backup source (shown in dashed path in figure 1.

Thus we can clearly see that the stand by system will start working only when there is any failure in mains. In this system, the AC voltage is first rectified and stored in the storage battery connected to the rectifier.

When power breakage occurs, this DC voltage is converted to AC voltage by means of inverter and given to the load connected to it. This is the least expensive UPS system and it provides surge protection in addition to back up. The transfer time can be about 25 milliseconds which can be related to the time taken by the UPS system to detect the utility voltage that is lost. The block diagram is shown below.

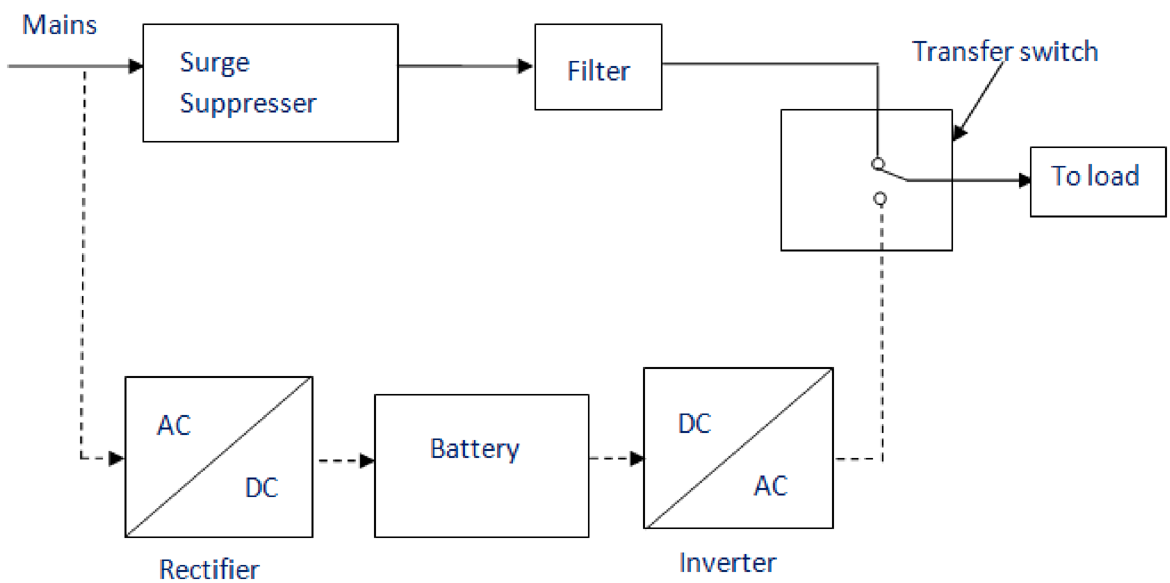


Figure 1

On-line UPS:

In this **type of UPS**, double conversion method is used. Here, first the AC input is converted into DC by rectifying process for storing it in the rechargeable battery. This DC is converted into AC by the process of inversion and given to the load or equipment which it is connected (figure 2). This type of UPS is used where electrical isolation is mandatory. This system is a bit more costly due to the design of constantly running converters and cooling systems. Here, the rectifier which is powered with the normal AC current is

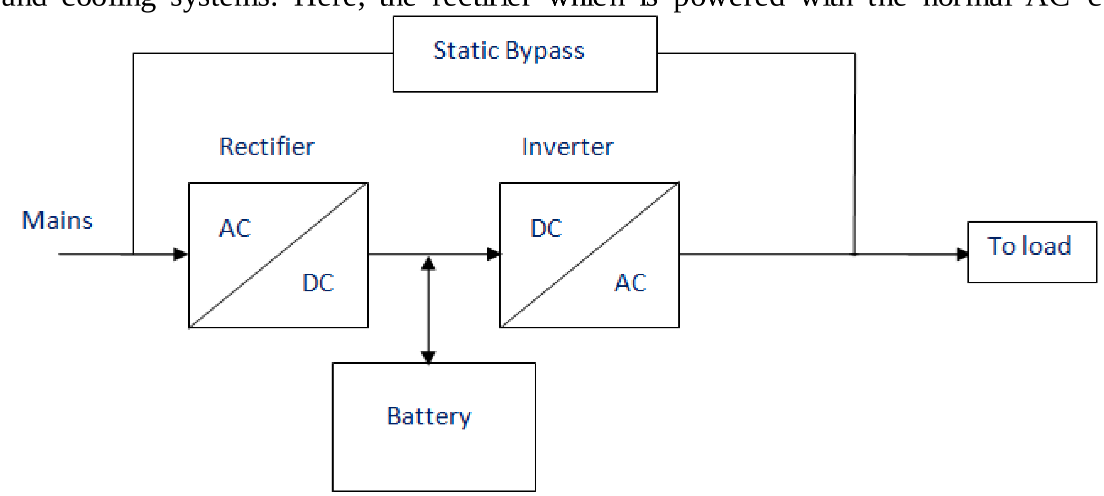


Figure 2

directly driving the inverter. Hence it is also known as Double conversion UPS. The block diagram is shown below.

When there is any power failure, the rectifier have no role in the circuit and the steady power stored in the batteries which is connected to the inverter is given to the load by means of transfer switch. Once the power is restored, the rectifier begins to charge the batteries. To prevent the batteries from overheating due to the high power rectifier, the charging current is limited. During a main power breakdown, this UPS system operates with zero transfer time. The reason is that the backup source acts as a primary source and not the main AC input. But the presence of inrush current and large load step current can result in a transfer time of about 4-6 milliseconds in this system.

Line Interactive UPS :

For small business and departmental servers and webs, line interactive UPS is used. This is more or less same as that of off-line UPS. The difference is the addition of tap changing transformer. Voltage regulation is done by this tap-changing transformer by changing the tap depending on input voltage. Additional filtering is provided in this UPS result in lowertransient loss. The block diagram is shown below.

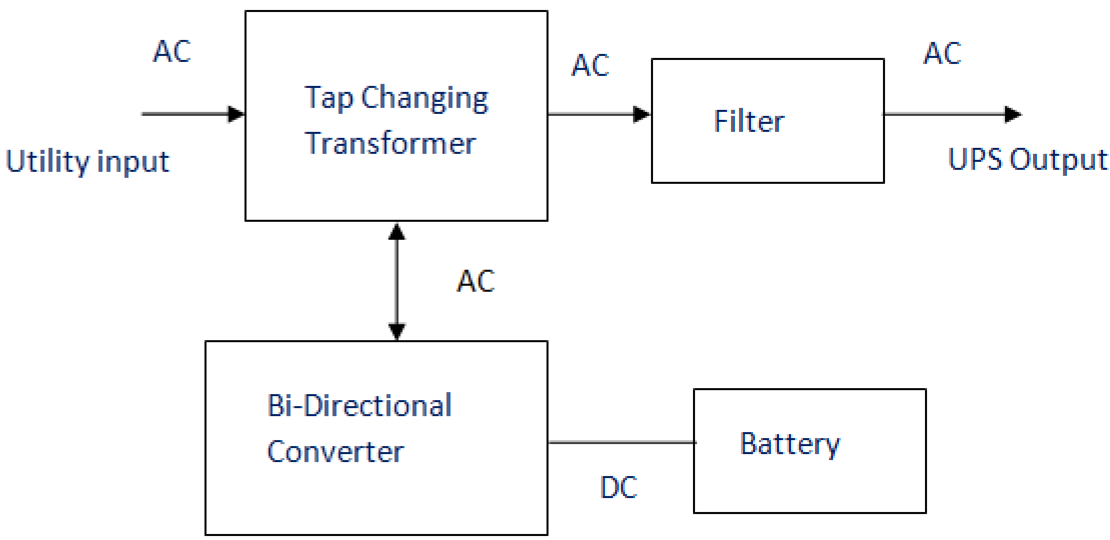


Figure 3