Assignment 3a - solutions

Thursday, 22 February 2024 8:52 AM

EE114 - Power Engineering 1

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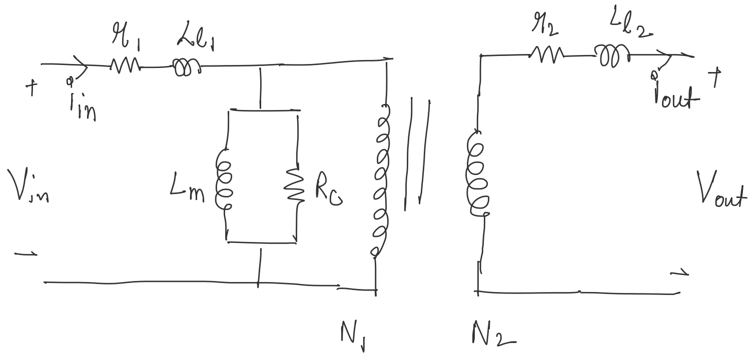
** Answers are rounded upto 3-decimal places

Ques 1 :-

$N_1 = 300$

$N_2 = 100$

$\mathcal{R} = 2.5 \times 10^4$ AT/Wb

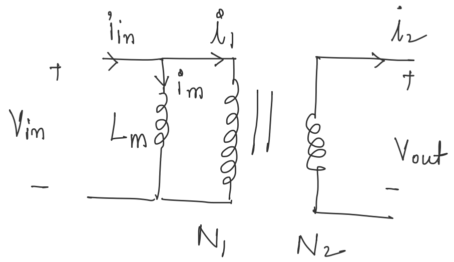


No leakage flux $\rightarrow$ $L_{l_1} = L_{l_2} = 0$

No iron loss $\rightarrow$ $R_c = 0$

No wire resistance $\rightarrow$ $r_1 = r_2 = 0$

So the equivalent ckt is



a) $V_{in} \rightarrow 240V, 50\,Hz$ supply

Secondary of the transformer is open circuited

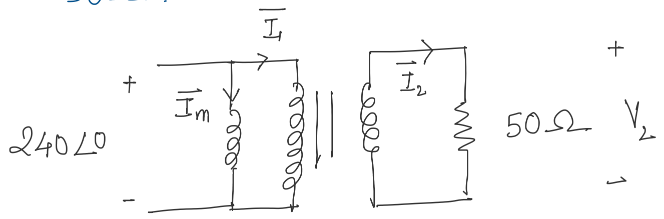
So $i_2 = 0 \Rightarrow$ Reflected i_1 on primary side is also 0. Only current that flows in the equivalent ckt of the transformer is in the magnetizing inductance branch.

$$\boxed{L_m = \frac{N_1^2}{\mathcal{R}}} = \frac{300^2}{2.5\times10^4} = 3.6\ H$$

$$\vec{I}_m = \frac{\vec{V}_{in}}{j\omega L_m} = \frac{240}{j2\pi 50\times3.6}$$

$$\vec{I}_m = 0.212\angle-90^\circ\ A$$

b) Now secondary is connected to resistance of 50Ω.



$$\boxed{\frac{\vec{V}_1}{\vec{V}_2} = \frac{N_1}{N_2}}$$

$$\vec{V}_2 = \frac{100}{300}\times240\angle0$$

$$\vec{V}_2 = 80\angle0$$

$$\vec{I}_2 = \vec{V}_2/R = 80\angle0/50 = 1.6\angle0\ A$$

c) Method 1 :-

$$N_1\vec{I}_1 = N_2\vec{I}_2$$

$$\vec{I}_1 = \frac{100}{300}\times\vec{I}_2 = \frac{100}{300}\times1.6\angle0$$

$$\vec{I}_1 = 0.533\angle0^\circ$$

$$\boxed{\vec{I}_{in} = \vec{I}_1 + \vec{I}_m}$$

$$= 0.533\angle0 + 0.212\angle90^\circ$$

$$\vec{I}_{in} = 0.574\angle-21.690^\circ$$

Method 2 :-

Rout referred to the primary side of the transformer

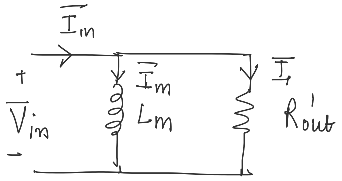
$$R_{out}' = R_{out}\times\left(\frac{N_1}{N_2}\right)^2$$

$$= 50\times\left(\frac{300}{100}\right)^2$$

$$R_{out}' = 450\,\Omega$$

Equivalent ckt :

Equivalent ckt :-



$$\bar{I}_{in} = \bar{I}_m + \bar{I}_1$$

$$\bar{I}_{in} = \frac{\bar{V}_{in}}{j\omega L_m} + \frac{\bar{V}_{in}}{R_{out}'}$$

$$\bar{I}_{in} = 0.212\angle{-90^\circ} + 0.533\angle 0^\circ$$

$$\bar{I}_{in} = 0.574\angle{-21.690^\circ}$$

Ques 2 :-

$$f = 50\ Hz$$
$$\mu_i = 4500\ \mu_0$$
$$A_c = 0.05\ m^2$$
$$l = 2\ m$$
$$N_1 = 600$$
$$N_2 = 200$$

a)
$$\boxed{V_1 = 4.44 f \phi_{peak} N_1 = 4.44 f B_{peak} A_c N_1}$$

Derivation :-

$$V = V_m \cos\omega t$$
$$e = N\frac{d\phi}{dt}$$
$$\phi = \frac{1}{N}\int e\,dt$$
$$\phi = \frac{V_m}{N\omega}\sin\omega t$$
$$\phi_{peak} = \frac{V_{peak}}{N\omega}$$
$$V_{peak} = N\omega\phi_{peak}$$
$$V_{rms_1} = \frac{N\omega}{\sqrt{2}}\phi_{peak}.$$
$$= \frac{N\times 2\pi f}{\sqrt{2}}\phi_{peak}$$
$$V_{rms_1} = 4.44\ Nf\ \phi_{peak}$$
$$V_{rms_1} = 4.44\times 600\times 50\times 1\times\sqrt{2}\times 0.05$$
$$= 9418.662\ V$$
$$V_{rms_2} = \frac{N_2}{N_1}\times V_1 = 3139.553\ V$$

b)
$$\boxed{\mathcal{L}_m = \frac{N_1^2}{\mathcal{R}}}$$

$$\mathcal{R} = \frac{l}{\mu A_c} = \frac{2}{4500 \times 4\pi \times 10^{-7}} \times \frac{1}{0.05}$$

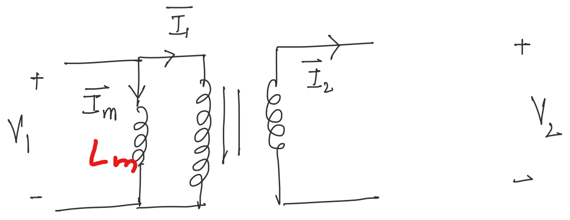
$$\mathcal{R} = 7073.553$$

$$L_m = \frac{600^2}{7073.553} = 50.894\ H$$

$$\bar{I}_m = \frac{\bar{V}_{in}}{j\omega L}$$

$$= \frac{9418.662}{j \times 2\pi 50 \times 50.894}$$

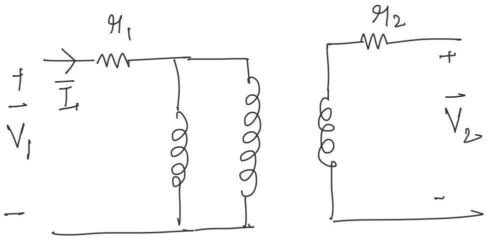
$$\bar{I}_m = 0.589 \angle -90^\circ$$



c) No iron losses $R_c = 0$

No leakage inductance $L_{l_1} = L_{l_2} = 0$

Equivalent ckt



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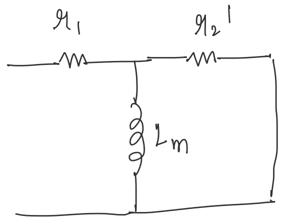

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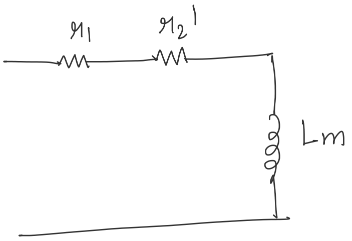

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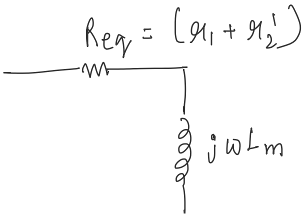


Approximate equivalent circuit

$$X_m = j\omega L_m = j \times 2\pi 50 \times 50.894\ \Omega$$

$$X_m = 15.99\ k\Omega$$

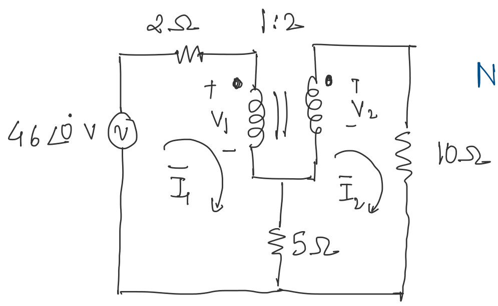
Approximately, losses in transformer at full load

$$\boxed{P_{loss} = I^2 R_{eq}}$$

$$2000 = 80^2 \times R_{eq}$$

$$R_{eq} = 0.3125\,\Omega$$

Ques 3 :-



$$N_1 \bar{I}_1 = N_2 \bar{I}_2$$

$$\bar{I}_2 = \frac{1}{2}\bar{I}_1$$

$$\frac{\bar{V}_1}{\bar{V}_2} = \frac{1}{2}$$

$$\bar{V}_2 = 2\bar{V}_1$$

Choose the current direction according to the dot convention.
Here, if we reverse the direction of current I2 then the relation between I1 and I2 would be I1 = - I2.

First Loop

$$-46\angle 0^\circ + 2\bar{I}_1 + \bar{V}_1 + 5(\bar{I}_1 - \bar{I}_2) = 0$$

$$\bar{V}_1 + \frac{9}{2}\bar{I}_1 = 46\angle 0^\circ \quad \text{—— (i)}$$

$$-\bar{V}_2 + 10\bar{I}_2 + 5(\bar{I}_2 - \bar{I}_1) = 0$$

$$-2\bar{V}_1 + \frac{5}{2}\bar{I}_1 = 0$$

$$\bar{V}_1 = \frac{5}{4}\bar{I}_1 \quad \text{—— (ii)}$$

From (i) & (ii)

$$\frac{5}{4}\bar{I}_1 + \frac{9}{2}\bar{I}_1 = 46\angle 0^\circ$$

$$\bar{I}_1 = 8\angle 0\ A$$

$$\bar{I}_2 = 4\angle 0\ A$$

$$P_{loss} \text{ in } 10\,\Omega \text{ resistor} = |\bar{I}_2|^2 R = 4^2 \times 10 = 160\,W$$

Ques 4.

$$P_{core_1} = 3000\,W \ @\ 50\ Hz$$

$$P_{core_2} = 5000\,W \ @\ 75\ Hz$$

$$P_{core} = P_{hys} + P_{eddy} = a f B_m^{1.6} + b f^2 B_m^2$$

... frequencies

Since B_m is kept constant at both frequencies

$$P_{core} = Af + Bf^2$$

At 50 Hz $\quad 50A + 2500B = 3000$

$$75A + 5625B = 5000$$

$$A = 46.667$$
$$B = 0.267$$

@ 50 Hz $\quad P_{hys} = 46.667 \times 50 = 2333.35\ W$

$$P_{eddy} = 0.267 \times 50^2 = 667.5\ W$$

@ 75 Hz $\quad P_{hyst} = 3500.025\,W$

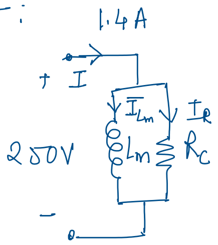
$$P_{eddy} = 1501.875\ W$$

@ 60 Hz $\quad P_{hyst} = 2800.02\,W$

$$P_{eddy} = 961.2\ W$$

Ques 5. OC test:

LV side :-



$$P_{oc} = 105\,W$$

$$\frac{V^2}{R_c} = 105\,W$$

$$R_c = \frac{250^2}{105}$$

$$R_c = 595.238\,\Omega$$

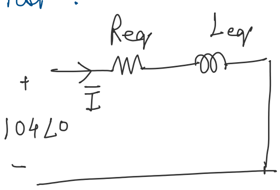
$$I_R = \frac{250}{595.238} = 0.42\,A$$

$$I^2 = I_R^2 + I_{Lm}^2$$

$$I_{Lm} = \sqrt{I^2 - I_R^2} = \sqrt{1.4^2 - 0.42^2} = 1.336\,A$$

$$1.336 = \frac{250}{X_m} \Rightarrow X_m = 187.126\,\Omega$$

SC test :- $\qquad$ HV side



$P_{cu} = 320\,W \Rightarrow 8^2 \times R_{eq} = 320$

$$R_{eq} = 5\,\Omega$$

Referred to LV: $R_{eq} = 0.05\,\Omega$

$$I = \frac{104}{(R_{eq}^2 + X_{eq}^2)^{1/2}}$$

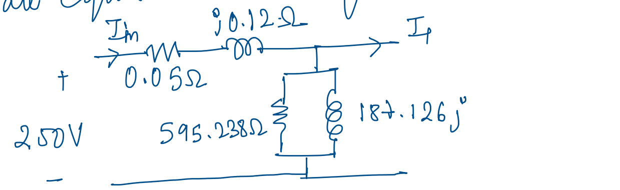
$$8 = \frac{104}{\sqrt{5^2 + X_{eq}^2}}$$

$$64 = \frac{104^2}{5^2 + X_{eq}^2}$$

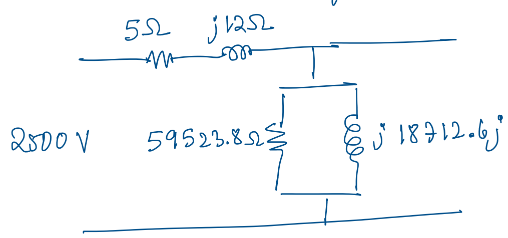
$$X_{eq} = \sqrt{\frac{104^2}{64} - 5^2} = 12\,\Omega$$

Referred to LV $R_{eq} = 0.12\,\Omega$

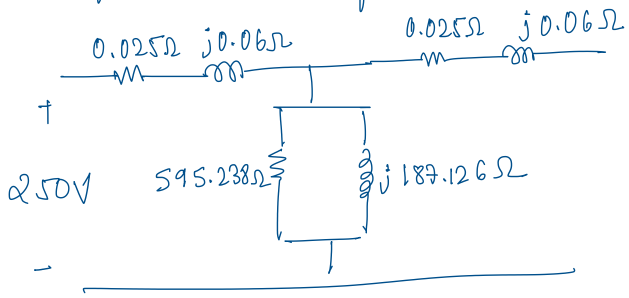
Approximate equivalent ckt referred to LV side



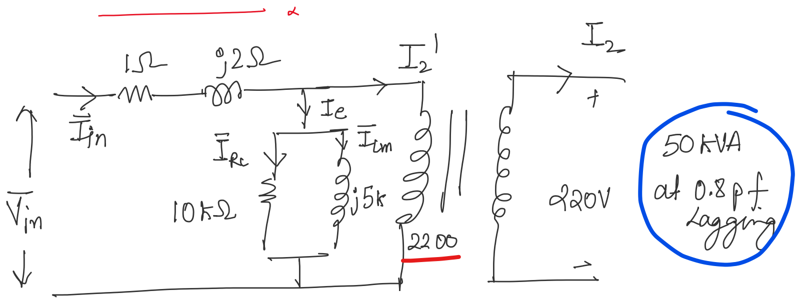
Approximate equivalent ckt referred to HV side



Exact equivalent ckt referred to LV side



Ques6



$$\boxed{S_{out} = |\bar{V}_{out}| \times |\bar{I}_2|}$$

$$50\times10^3 = 220 \times I_2$$

$$\boxed{\cos^{-1}(0.8) = 36.87^\circ}$$

$$|\bar{I}_2| = 227.273$$

$$\bar{I}_2 = 227.273 \angle{-36.87^\circ}\ A$$

$$\bar{I}_2' = \frac{N_2}{N_1} \times \bar{I}_2$$

$$= \frac{220}{2200} \times 227.273 \angle{-36.87^\circ}$$

$$I_2' = 22.727 \angle -36.87$$

a) $\text{Iron loss} = V^2/R_C$

$$= \frac{2200^2}{10\times 10^3} = 484\ W$$

$$I_{R_C} = \frac{2200}{10\times 10^3} = 0.22$$

$$I_{Lm} = \frac{2200}{j\times 5\times 10^3} = -j0.44$$

$$\boxed{\bar{I}_e = I_{R_C} + I_{Lm}}$$

$$\bar{I}_e = 0.22 - j0.44 \} \text{ c}$$

b) $\bar{I}_1 = \bar{I}_e + \bar{I}_2' = 23.168 \angle -37.414$

$$P_{cu} = 23.168^2 \times 1 = 536.756\ W$$

d) $\bar{V}_i = 2200 + 23.168 \angle -37.414 [1+2j]$

$$\bar{V}_i = 2246.669 \angle 0.58^\circ$$

$$\boxed{S_{in} = |\bar{V}_i||I_i|}$$

$$= 2246.669 \times 23.168$$

$$S_{in} = 52050.823\ W$$

$$\boxed{P_{in} = Re\{\bar{V}_i\ \bar{I}_1^*\}}$$

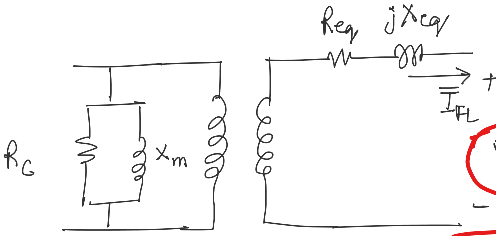
$$= Re\{2246.669 \times 23.168 \angle 0.58 + 37.414^\circ\}$$

$$P_{in} = 41019.967\ W$$

$$\text{input } Pf = \frac{P_{in}}{S_{in}} = \frac{41019.967}{52050.823}$$

$$= 0.788$$

Ques 7.



$$\boxed{\bar{V}_{rated} = 2300\angle 0^\circ}$$

$$\boxed{|\bar{I}_{FL}| = 10.87\ A}$$

$$\bar{V}_{FL} = \bar{V}_{rated}$$

$$\bar{V}_{NL} = \bar{V}_{rated} + x\bar{I}_{FL}(R_{eq} + jX_{eq})$$

$$= \bar{V}_{rated} + x\bar{I}_{FL}(R_{eq} + jX_{eq})$$

$$= \bar{V}_{rated} + x|\bar{I}_{FL}| R_{eq}(\cos\theta + j\sin\theta)$$

$$+ x|\bar{I}_{FL}| X_{eq}(\cos\theta + j\sin\theta)$$

$$= V_{rated} + x|I_{FL}|(R_{eq}\cos\theta - X_{eq}\sin\theta) + j|I_{FL}|x(R_{eq}\sin\theta + X_{eq}\cos\theta)$$

$$|\bar{V}_{NL}| = \sqrt{\left[|\bar{V}_{rated}| + x|\bar{I}_{fl}|(R_{eq}\cos\theta - X_{eq}\sin\theta)\right]^2 + \left[|\bar{I}_{fl}|x(\cancel{R_{eq}\sin\theta + X_{eq}\cos\theta})\right]^2}$$

$\rightarrow 0$ Very small w.r.t first term as it has $|\bar{V}_{rated}|$ added to it

$$|\bar{V}_{NL}| = |\bar{V}_{rated}| + x|\bar{I}_{fl}|(R_{eq}\cos\theta - X_{eq}\sin\theta)$$

If lagging $|V_{NL}| = |V_{rated}| + x|\bar{I}_{fl}|(R_{eq}\cos\theta + X_{eq}\sin\theta)$

If leading $|V_{NL}| = |V_{rated}| + x|\bar{I}_{fl}|(R_{eq}\cos\theta - X_{eq}\sin\theta)$

$$\text{Voltage Regulation} = \frac{|\bar{V}_{NL}| - |\bar{V}_{FL}|}{|\bar{V}_{FL}|}$$

$$= \frac{x|\bar{I}_{fl}|(R_{eq}\cos\theta \pm X_{eq}\sin\theta)}{V_{rated}}$$

Let $R = Z_{eq}\cos\phi$

$X_{pu} = Z_{eq}\sin\phi$

$$= \frac{x|\bar{I}_{fl}|Z_{eq}[\cos\theta\cos\phi \pm \sin\theta\sin\phi]}{V_{rated}}$$

$$\text{Voltage regulation} = \frac{x|I_{fl}|Z_{eq}}{V_{rated}}[\cos\theta\cos\phi \pm \sin\theta\sin\phi]$$

In case of lagging load

$$\text{Voltage regulation} = \frac{x|\bar{I}_{fl}|Z_{eq}}{V_{rated}}[\cos(\theta-\phi)]$$

For maximum regulation $x = 1$

& $\theta = \phi$

In case of leading load

$$\text{Voltage regulation} = \frac{x|\bar{I}_{fl}|Z_{eq}}{V_{rated}}[\cos(\theta+\phi)]$$

Voltage regulation will always be better (less) for leading load as compared to lagging load

For our question :-

$$Z_{base} = \frac{2300^2}{25\times 10^3} = 211.6$$

$$\theta = \phi = \tan^{-1}\left(\frac{X_{eq}}{R_{eq}}\right) = \tan^{-1}\left(\frac{5}{4}\right) = 51.34^\circ$$

pf of load = 0.625 lagging

regulation = Z_{pu} = $\frac{\sqrt{16+25}}{211.6}$ = 0.03026

= 3.026 %

Ques 8 :-

$$\boxed{P_{core} = \frac{V_{rated}^2}{R_c}}$$

$$R_c = \frac{V_{rated}^2}{P_{core}} = \frac{240^2}{100} = 576\,\Omega$$

$$\boxed{P_{cu} = \left(\frac{I_{rated}}{2}\right)^2 R_{eq}}$$

$$\boxed{I_{rated} = \frac{10\times10^3}{240}} = 41.667$$

$$60 = \frac{41.667^2}{4} \times R_{eq}$$

$$R_{eq} = 0.138\,\Omega$$

$$\boxed{\eta = \frac{P_{out}}{P_{out} + P_{loss}}}$$

$$\boxed{P_{out} = V_{out} I_{out} \cos\theta} = 240 \times 41.667 \times 0.8$$

$$P_{out} = 8000.064$$

$$P_{loss} = P_{core} + P_{cu} = 100 + 239.587$$

$$= 339.587$$

$$\eta = 95.93\%$$

Ques 9.

$$P_{core} = 200\,W$$

$$P_{cu} = I^2 R_{eq}$$

$$= (x I_{rated})^2 R_{eq}$$

$$0.8^2 \times P_{cu}^{rated} = 500$$

$$P_{cu}^{rated} = 781.25\,W$$

$$\boxed{\eta = \frac{P_{out}}{P_{out} + P_{core} + x^2 P_{cu}^{rated}}}$$

(i) $$\eta = \frac{5\times10^3\times0.8}{5\times10^3\times0.8 + 200 + 0.25^2\times781.25} = 94.144\%$$

(ii) $$\eta = \frac{10\times10^3\times0.9}{10\times10^3\times0.9 + 200 + 0.5^2\times781.25} = 95.792\%$$

(iii) $$\eta = \frac{20\times10^3}{20\times10^3 + 200 + 781.5\times1^2} = 95.323\%$$

(iv) $$\eta = \frac{10\times10^3\times0.8}{} = 95.291\%$$

$0.8 \times 10 \times 10^3 + 200 + 781.5 \times 0.5$

Fraction of rated load at which efficiency is maximum (at upf)

$$\eta = \frac{x P_{rated}}{x P_{rated} + P_{core} + x^2 P_{cu}}$$

$$\frac{d\eta}{dx} = \frac{(x P_{rated} + P_{core} + x^2 P_{cu}) P_{rated} - x P_{rated}(P_{rated} + 2x P_{cu})}{(x P_{rated} + P_{core} + x^2 P_{cu})^2}$$

$$\cancel{x P_{rated}} + P_{core} + x^2 P_{cu} - \cancel{x P_{rated}} - 2x^2 P_{cu} = 0$$

$$x = \sqrt{\frac{P_{core}}{P_{cu}}} = \sqrt{\frac{200}{781.25}} = 50.596\%$$

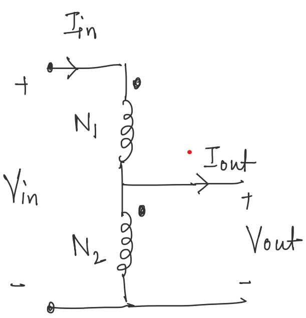
At 0.8 pf lagging & $x \times P_{rated}$ loading

$$\eta = \frac{x \times S_{rated} \cos\theta}{x \times S_{rated} \cos\theta + P_{core} + x^2 P_{cu}}$$

$$\eta_{max} \text{ at } x = \sqrt{\frac{P_{core}}{P_{cu}}} = 50.596\%$$

$$\eta_{max} = 95.291\%$$

Ques 10 :-



$P_{out} = 5$ kW at upf

$V_{out} = 115$ V

$V_{in} = 230$ V

a) Voltage transformation ratio (V_{HV}/V_{LV})

$$= \frac{V_{in}}{V_{out}}$$

$$= \frac{230}{115} = 2$$

b) $P_{out} = V_{out} I_{out} \cos\theta_2$

$5 \times 10^3 = 115 \times I_{out}$

$I_{out} = 43.478$ A

c) Assuming ideal transformer

c) Assuming ideal transformer

$$V_{in} I_{in} = V_{out} I_{out}$$

$$I_{in} = \frac{115}{230} \times 43.478$$

$$I_{in} = 21.739 \text{ A}$$

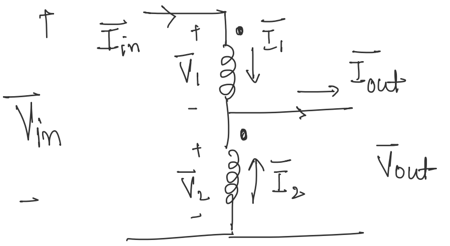
d) Given Number of turns in primary side $N_1 + N_2 = 400$

$$\frac{V_{in}}{V_{out}} = \frac{N_1 + N_2}{N_2}$$

$$\frac{230}{115} = \frac{400}{N_2}$$

$N_2 = 200 \rightarrow$ No of turns in secondary

e)



$$\bar{I}_{out} = \bar{I}_1 + \bar{I}_2$$

$$\bar{I}_2 = \bar{I}_{out} - \bar{I}_1$$

$$\bar{I}_{out} = 43.478 \text{ A}$$

$$\bar{I}_{in} = \bar{I}_1 = 21.739 \text{ A}$$

$$\Rightarrow \bar{I}_2 = 21.739 \text{ A}$$

Power transformed in the windings and supplied to load

$$= \bar{V}_1 \cdot \bar{I}_1 \text{ or } \bar{V}_2 \bar{I}_2$$

$$= 115 \times 21.739$$

$$= \boxed{2.5 \text{ kW} = \bar{V}_2 \cdot \bar{I}_2}$$

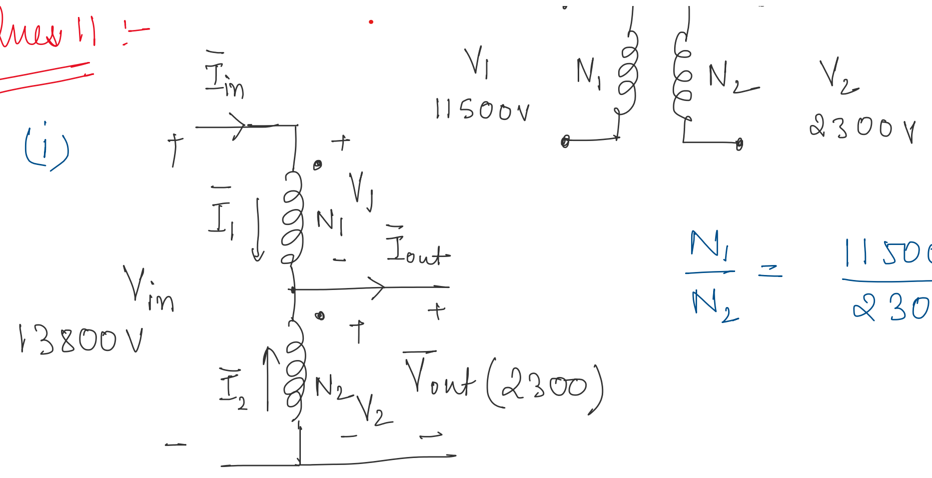
Rest of the output power is conducted directly from the supply to the load

$$= 5 \text{ kW} - 2.5 \text{ kW}$$

$$= 2.5 \text{ kW}$$

Ques 11 :-

(i)



$$\frac{N_1}{N_2} = \frac{11500}{2300}$$

$$\frac{\bar{I}_1}{\bar{I}_2} = \frac{N_2}{N_1}$$

For 2 winding configuration

$$\boxed{V_2 I_2 = V_{out} I_2 = S_{1\phi}}$$

For autotransformer

$$V_{out} \times I_{out} = S_{Auto}$$

$$S_{Auto} = V_{out} \times (I_1 + I_2)$$

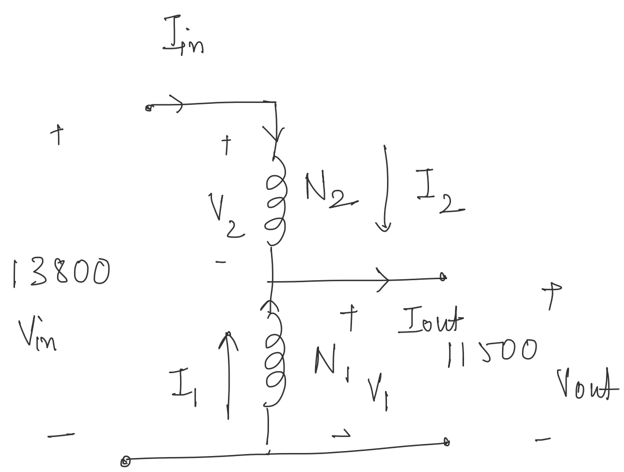
$$= V_{out} \times \left(\frac{N_2}{N_1} + 1\right) I_2$$

$$S_{Auto} = S_{1\phi}\left(1 + \frac{N_2}{N_1}\right)$$

$$= 100KVA\left(1 + \frac{2300}{11500}\right)$$

$$= 120kVA$$

(ii)



$$S_{1\phi} = V_1 I_1$$

$$S_{Auto} = V_1 (I_1 + I_2)$$

$$= V_1 \left(I_1 + \frac{N_1}{N_2} I_1 \right)$$

$$= V_1 I_1 \left(1 + \frac{N_1}{N_2} \right)$$

$$= S_{1\phi} \left(1 + \frac{N_1}{N_2} \right)$$

$$= 100 \left(1 + \frac{11500}{2300} \right)$$

$$= 600 \text{ kVA}$$