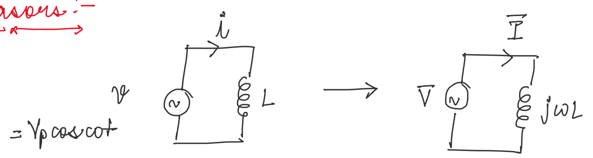
Lecture 3

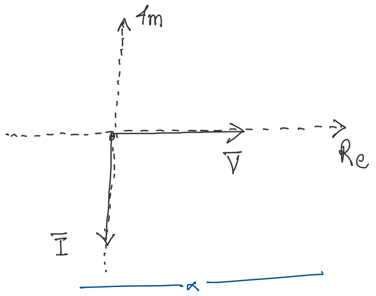
Friday, 12 January 2024 3:31 PM

Phasors :-



$$\bar{I} = \frac{\bar{V}}{j\omega L} = \frac{1}{\omega L}\bar{V}\angle -90^\circ$$

Current in the inductor lags voltage applied across it by 90°



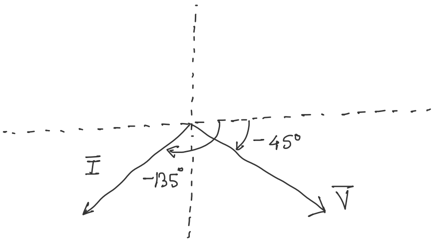
$\bar{V} = \frac{V_p}{\sqrt{2}}\angle 0^\circ$

$\bar{I} = \frac{1}{\omega L}\frac{V_p}{\sqrt{2}}\angle -90^\circ$

If $v(t) = V_p\cos(\omega t - 45^\circ)$

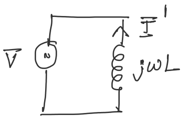
$\bar{V} = \frac{V_p}{\sqrt{2}}\angle -45^\circ$

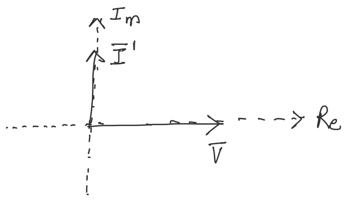
$\bar{I} = \frac{V_p}{\omega L\sqrt{2}}\angle -135^\circ$



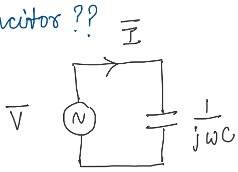
If we want to plot $\bar{I}' = -\bar{I}$

$$\bar{I}' = \frac{-\bar{V}}{j\omega L} = \frac{-j}{j\times j}\frac{\bar{V}}{\omega L} = j\frac{\bar{V}}{\omega L} = \frac{\bar{V}}{\omega L}\angle 90^\circ$$



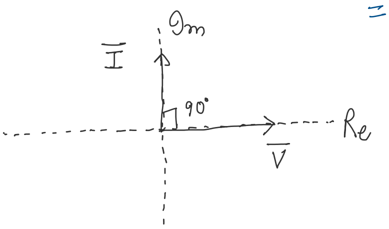


For capacitor ??



$$\bar{I} = \frac{\bar{V}}{\frac{1}{j\omega C}} = \bar{V}j\omega C$$

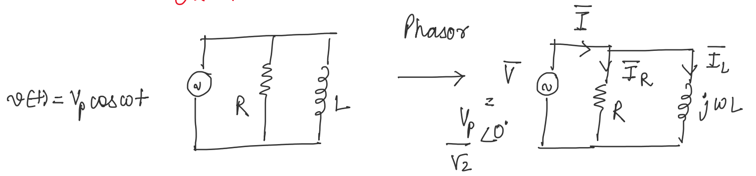
$$= \omega C\bar{V}\angle 90^\circ$$



Power in single phase circuit :-

$X_L = L\omega$

$X_c = \frac{1}{C\omega}$



What is the real "power" absorbed by the load

Real "power" = ?? (Average power)

Inst. power in inductor $= V_p \cos\omega t \, \frac{V_p}{\omega L} \cos(\omega t - 90°)$

Avg. power in inductor $= \frac{1}{2\pi}\int_0^{2\pi} V_p \cos\omega t \, \frac{V_p}{\omega L} \cos(\omega t - 90°) \, d(\omega t)$

$= 0$

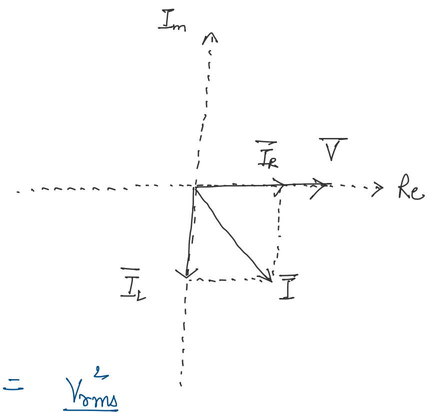
Avg Real power "L" $= 0$

Inst. Real power "R" $= V_p \cos\omega t \, \frac{V_p}{R} \cos(\omega t)$

Avg. power "R" $= \frac{1}{2\pi}\int_0^{2\pi} \frac{V_p^2}{R} \cos^2(\omega t) \, d(\omega t) = \frac{V_p^2}{2R}$

$= \frac{V_{rms}^2}{R}$

Total avg power absorbed by load $= \frac{V_{rms}^2}{R}$



Real Power (Active Power) $= \frac{V_{rms}^2}{R}$

$= |\bar{I}_R||\bar{V}_R| = |\bar{V}||\bar{I}| \cos\theta$

Real power or Active power

$\theta \rightarrow$ angle b/w $\bar{V}$ & $\bar{I}$

Suppose in the class room we don't have any resistance as load.

There is no real power consumption.

But the I_{rms} flowing into the room is non-zero.

The flow of current leads to losses in the transmission line due to the resistance within them.

How to quantify the power consumed ??

We introduce

Reactive power $= |\bar{V}||\bar{I}| \sin\theta$

Reactive power = voltage × Reactive current component.

Reactive current $= |\bar{I}| \sin\theta$

Reactive power is also known as imaginary power.

Reactive power or Imaginary power

Inductors and capacitors draw different sort of current.

Nomenclature :- inductor absorbs reactive power
capacitor supplies reactive power.

reactive power supplied by the capacitor - reactive power absorbed by the inductor = Net reactive power

Active power = P
Reactive power = Q

At substations we place capacitor banks to cater to the reactive power needs of the inductor and also to reduce the resistive losses in the transmission lines.

Complex power $S = P + jQ$

complex power also known as apparent power

Complex power or Apparent power

S, Z are just complex values and not phasors.

$$S = P + jQ = V_{rms} I_{rms} (\cos\theta + j\sin\theta)$$
$$= V_{rms} I_{rms} e^{j\theta}$$
$$= |\bar{V}| \bar{I}^*$$
$$= \bar{V} \bar{I}^*$$

$\angle\theta$

Convention

$\bar{V} = V_{rms} \angle 0$

$\bar{I} = I_{rms} \angle -\theta$

$\Rightarrow V_{rms} I_{rms} \angle \theta$

$\bar{I}^*$

$\Rightarrow \bar{V} \bar{I}^*$

To meet the convention that inductor consumes reactive power and capacitor supplies reactive power ⇒ we take conjugate of I.

Units :-

Unit of real power $P \rightarrow$ W, kW

unit of reactive power $Q \rightarrow$ VAr, kVAr

Unit of Apparent power $S \rightarrow$ VA, kVA

$$|S| = \sqrt{P^2 + Q^2}$$

Take care of units here.

Which unit will you use W or VAr
We use VA

Power factor :-

$$\text{Power factor} = \frac{|P|}{|S|} = \frac{|\bar{V}||\bar{I}|\cos\theta}{|\bar{V}||\bar{I}|} = \cos\theta$$

$\cos\theta = \frac{R}{Z}$

Power factor → Lagging (inductor)
→ Leading (capacitor)

A small water pump $P = 1\,kW$
$PF = 0.9$ lagging

$Q = ?$
$S = ?$

Fan $P = 30\,W$ or $40\,W$
$pf = 0.8$ lagging, 0.85 lagging

2012 Blackouts :-

In the figure below, we see the grid frequency in the northern (green) & central (red) part of India. Due to overloading, the protection system of grid disconnected the grids of two regions. In northern region, the demand was more than the supply, therefore the frequency of voltage generated dropped. In the central region, the supply was more than the demand, hence the frequency increased. The grid frequency deviated more than the permissible limit, resulting in the system getting unstable which led to the blackout.