EE111

Lecture Notes

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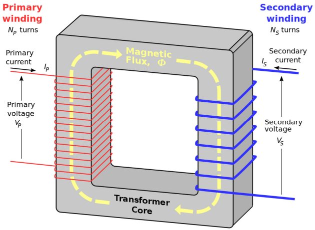
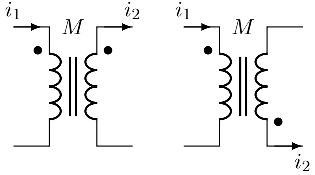
Contents

Mutual Inductance

- When two circuits are magnetically connected
- Voltage induced in the second circuit is linked to time varying current in the first circuit by parameter called mutual inductance
- Coupling is generally indicated with dot markings. The dot mark basically indicates the polarity of the induced voltage and depends on how the coils are physically wound.
- Mutually Coupled coils have self and mutual inductances

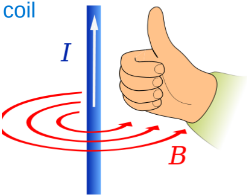




Mutual Inductance

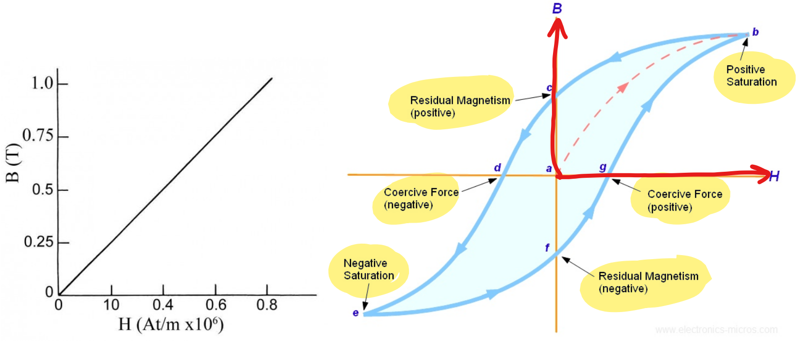
- Consider N turn coil
- Flux lines are designated as ϕ which depends on current (i)
- Direction of flux depends on the direction of current and is obtained by Flemings Right Hand Rule

Flux linking the coils = the total flux through a coil



- Flux Linking Coil is designated as $\lambda = N\phi(\text{webers})$
- $\phi = \mathbf{P}Ni$. $\mathbf{P}$ is permeance of material which is magnetic property of the material. For non-magnetic material ϕvs.i is linear and for ferromagnetic materialsϕvs.i is nonlinear
- $\lambda = \mathbf{P}N^2 i \rightarrow v = \frac{d\lambda}{dt} = \mathbf{P}N^2 \frac{di}{dt}$
- $L = \mathbf{P}N^2 \rightarrow$inductance
- We can show that $\boxed{M = k\sqrt{L_1 L_2}}$

$L = \frac{N^2}{R}$



Linear (left) and Non linear (right) B-H or $\phi - i$ curves

Mutual Inductance

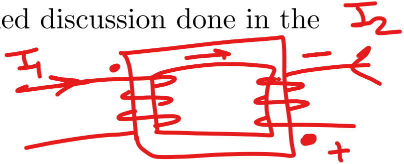
$\phi = LI$

$$\phi_1 = \phi_{11} + \phi_{12}$$
$$\mathbf{P}_1 N_1^2 i = \mathbf{P}_{11} N_1^2 i + \mathbf{P}_{12} N_2^2 i$$
$$v = N_1^2 \left(\mathbf{P}_{11} + \mathbf{P}_{12}\right) \frac{di}{dt}$$
$$\frac{d\lambda_2}{dt} = N_1 N_2 \mathbf{P}_{12} \frac{di}{dt}$$
$$L = N_1^2 \left(\mathbf{P}_{11} + \mathbf{P}_{12}\right)$$
$$M = N_1 N_2 \mathbf{P}_{12}$$

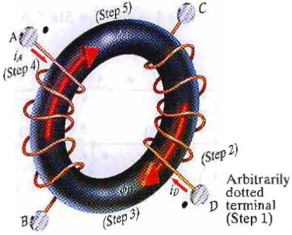
$$\mathbf{P}_{21} = \mathbf{P}_{12}$$
$$L_1 L_2 = N_1^2 N_2^2 \mathbf{P}_1 \mathbf{P}_2$$
$$L_1 L_2 = \left(N_1 N_2 \mathbf{P}_{12}\right)^2 \left(1 + \frac{\mathbf{P}_{11}}{\mathbf{P}_{12}}\right)\left(1 + \frac{\mathbf{P}_{22}}{\mathbf{P}_{12}}\right)$$
$$L_1 L_2 = \frac{M^2}{k^2}$$
$$\boxed{M = k\sqrt{L_1 L_2} \qquad 0 \le k \le 1} \tag{1}$$

Simplified Dot Marking Procedure

Use this process instead of the detailed discussion done in the class.



- Arbitrarily select a terminal of one of the coils and mark it with a dot.
- Define current entering into the terminal and find direction of the flux using right hand rule.
- Arbitrarily pick one terminal of the second coil and define current entering in the coil.
- Find the direction of the flux produced by current in the second coil.
- Compare the directions of the two fluxes. If the fluxes have the same reference direction, place a dot on the terminal of the second coil where the test current enters. If the fluxes have different reference direction place a dot on the terminal of the second coil where the test current leaves.

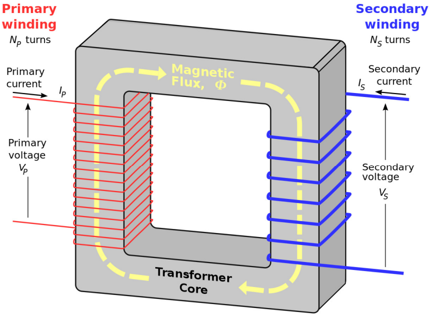


Construction of Transformers

• The main components of a transformer are the windings and a core. It basically consists of mutually coupled coils and the coupling occurs through the core.

• There are two general types of transformer construction – Shell type and Core type.

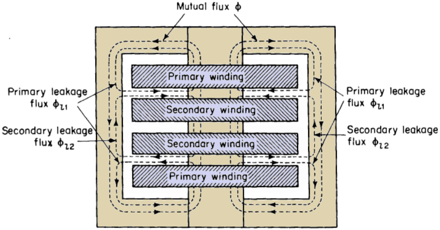
• They differ in the manner in which the windings are wound around the core.



Construction of Transformers

Shell type Transformers

- The core limbs surround the windings.
- The windings are divided and wound over the central limb in an interleaved manner.
- The flux in the central limb divides equally and returns through the outer limbs.
- They are used in low voltage low power applications.



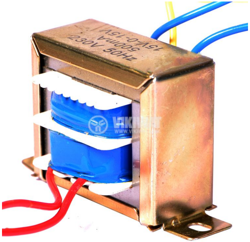
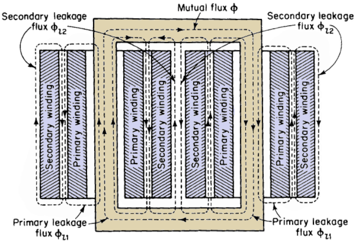


Figure: Shell type Transformer

Construction of Transformers

Core type Transformers

- The windings suround the core. They are are divided into half and wound in a concentric manner.
- The LV coil is placed adjacent to the steel core, and the HV winding is placed outside it.
- Lesser insulation and iron requirements.
- They are used for high voltage high power applications.



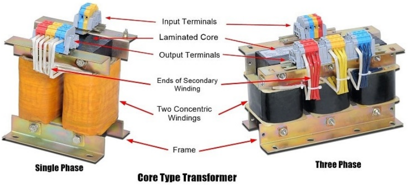


Figure: Core type Transformer

Construction of Transformers

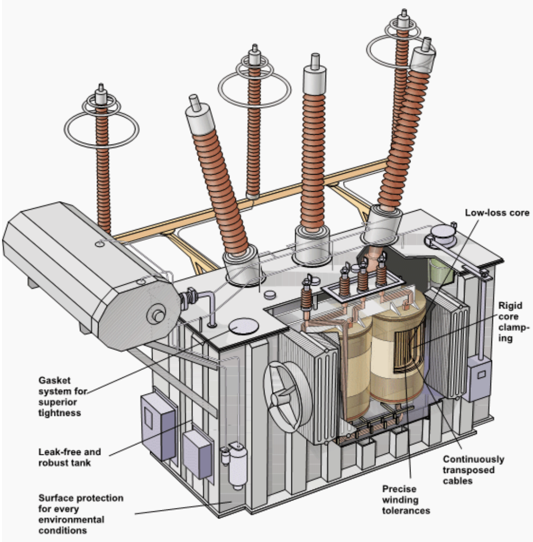
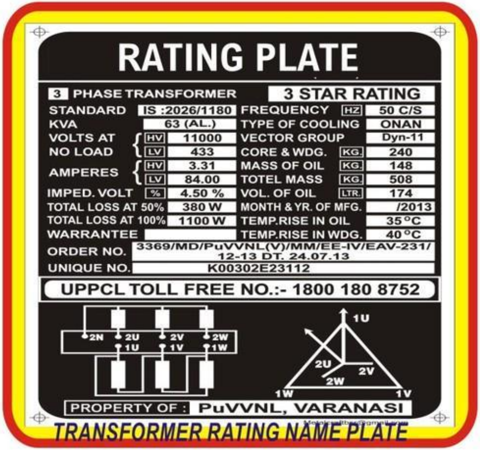


Figure: Internal view of a transformer



Transformer windings (l) and name plate of a transformer (r)

Construction of Transformers

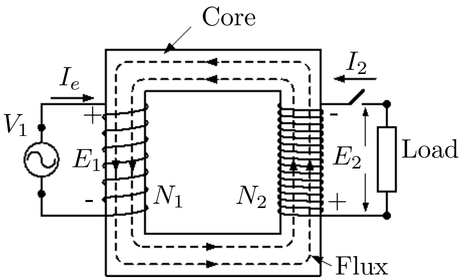
Size Comparison : A worker next to a high voltage power transformer

Principle of Transformer Action

The schematic of the transformer is shown in Figure.

For analysis, the transformer is considered as ideal, meaning –

- Winding resistances are negligible
- All the magnetic flux is confined to the core
- There are no losses in the machine.



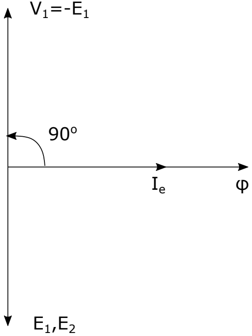


Figure: Ideal Transformer (left) and Phasor Diagram (right)

Primary Winding

• Initially, we assume that the secondary side is open.

• Sinusoidal V_1 is applied and I_e flows in the primary side.

• Since V_1 and I_e are sinusoidal, flux ϕ is also sinusoidal.

•$\phi = \phi_{max} sin(\omega t)$ is set up in the primary side coil.

• Due to alternating ϕ, emf induced in the primary coil will have a polarity such as to oppose the cause, I_e as per Lenz's law.

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \omega \phi_{max} cos(\omega t) = E_{1max} sin(\omega t - \frac{\pi}{2})$$

, where $E_{1max} = 2\pi \mathrm{f} N_1 \phi_{max}$.

• The primary winding acts as load for the source, V_1, and so the polarity of E_1 will be as shown in Figure.

• RMS value of primary winding emf $E_1 = \sqrt{2}\pi f N_1 \phi_{max}$.

Secondary Winding

• The alternating flux also induces a voltage on secondary side.

$$e_2 = -N_2\frac{d\phi}{dt} = -N_2\omega\phi_{max}cos(\omega t) = E_{2max}sin(\omega t - \frac{\pi}{2})$$

where $E_{2max} = 2\pi \mathrm{f} N_2\phi_{max}$.

• Now, if the secondary side is connected to a load, current flows in the secondary winding.

• Current direction must be such that it opposes the core flux.

• Thus, the current will flow from bottom to top through load.

• The secondary winding acts as a source for the load. Hence, the polarity of E_2 will be as shown in Figure.

• RMS value of secondary winding emf $E_2 = \sqrt{2}\pi f N_2\phi_{max}$.

• The winding may be wound in such a way that '+' terminal goes to the top.

Principle of Transformer Action

Effect of Secondary Side Current

- When load current flows in the secondary side, a mmf will be established in the secondary winding.
- It tends to oppose (reduce) main flux (by Lenz's Law).
- If flux reduces, E_1 will reduce. However, if primary side resistance is neglected, $|E_1| = |V_1|$ (source voltage), and V_1 will not change.
- Hence any change in E_1 is countered by increasing the primary current (say $I_1^{'}$), such that

$$\text{Primary MMF } N_1 I_1^{'} = \text{Secondary MMF } N_2 I_2$$

Turns Ratio and Leakage Flux

$$E_1 = \sqrt{2}\pi f N_1 \phi_{max} \qquad E_2 = \sqrt{2}\pi f N_2 \phi_{max}$$
$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \sqrt{2}\pi f \phi_{max} \therefore \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

Turns Ratio

The quantity $\frac{N_1}{N_2}$ is termed as turns ratio.
Hence, the primary and secondary voltages are related by the turns ratio.

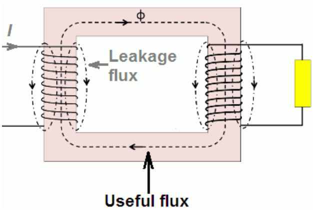
Leakage Flux

In iron core transformers, some flux leaks into the non-magnetic material surrounding the core, such as air, insulation, etc.
This flux is wasted and it decreases the amount of useful flux.
It is desirable to reduce this leakage flux by appropiate construction and winding placement.

Leakage and Magnetizing Inductance

Leakage Inductance

- Leakage flux in effect represents a reduction in actual voltage producing the main flux.
- This is modelled as an inductive voltage drop. It is inductive since the voltage drop across it produces the leakage flux.
- Thus, this inductance is termed leakage inductance.



Magnetizing Inductance

- It is used to model the main flux producing current.
- E_1 lags the flux producing current I_e by $90°$. Also, $E_1 = -V_1$.
- Hence, V_1 leads I_e by $90°$.
- Thus, I_e, in effect, flows through an equivalent reactance X_m, such that $I_e = \frac{V_1}{X_m}$.
- This reactance X_m is termed as the magnetizing inductance.

Equivalent Circuit

The equivalent circuit of any machine or device is useful to understand its performance and analyse its behaviour.

• The equivalent circuit of a practical transformer deviates from that of an ideal one in many ways.

• Starting from the circuit of an ideal transformer, actual equivalent circuit can be arrived at in a stepwise manner.

Developing transformer equivalent circuit

• Winding resistances, which were previously neglected, are now considered for both primary and secondary sides.

• They lead to voltage drops in both the sides.

• Voltage drops across the leakage reactances in both primary and secondary sides are also included.

Equivalent Circuit

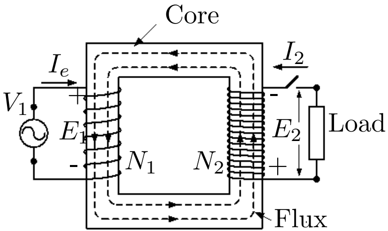


Figure: Idealised transformer circuit

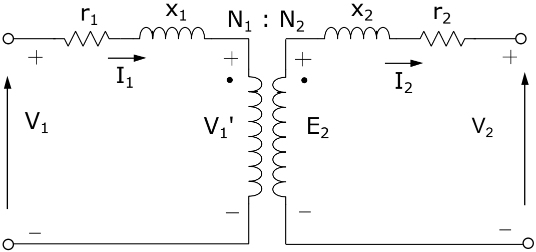


Figure: Equivalent circuit with Exciting Current neglected

- The primary current can be divided into two parts.
  - Load component $I_1{}'$ which counteracts secondary mmf.
  - No load or excitation current I_e.
  - Excitation current again consists of
    —Core loss component I_c, representing core losses.
    —Magnetizing component I_m.
- Modelling these primary current components gives the exact equivalent circuit.

Modelling Core loss component I_c

- Core loss occurs in the transformer core and causes heating. It depends on applied voltage.
- It can be modelled as a resistive loss.
- Hence, R_c in parallel with V_1 represents core loss given by

$$P_c = \frac{{V_1}^2}{R_c}.$$

Modelling magnetizing component I_m

- This is the component of I_e (and not I_e itself as previously shown for convenience) that produces the main flux.
- I_m flows through magnetizing reactance X_m placed in parallel to applied voltage.

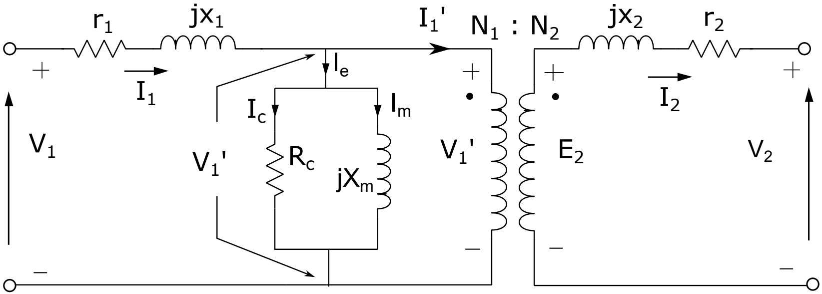


Figure: Exact Equivalent circuit

Equivalent Circuit

Transferring quantities across sides

- Quantities (such as resistance, current, etc.) can be transferred across sides.
- Example – secondary drop $I_2 r_2$ can be transferred by

$$\text{Secondary resistive drop transferred to primary} = (I_2 r_2)\frac{N_1}{N_2}$$
$$= (I_1 \frac{N_1}{N_2} r_2)\frac{N_1}{N_2} = I_1 {r_2}', \text{ where } {r_2}' = r_2 \frac{{N_1}^2}{{N_2}^2}$$

- Thus secondary resistive drop is transferred to primary by placing a resistance ${r_2}'$.
- Here, ${r_2}'$ is called secondary resistance referred to primary.
- Therefore, effective primary resistance $r_{e1} = r_1 + {r_2}'$.
- Similarly, primary to secondary transformation can be done.
- The leakage reactance can also be transferred in the same way.

Equivalent Circuit

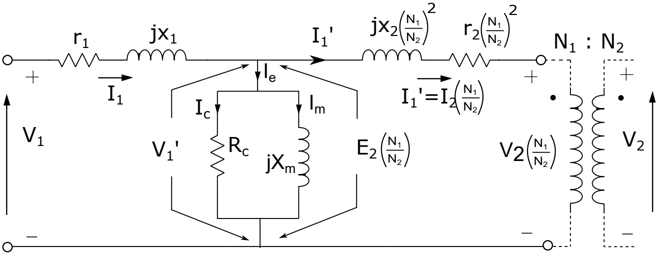


Figure: Equivalent circuit referred to Primary

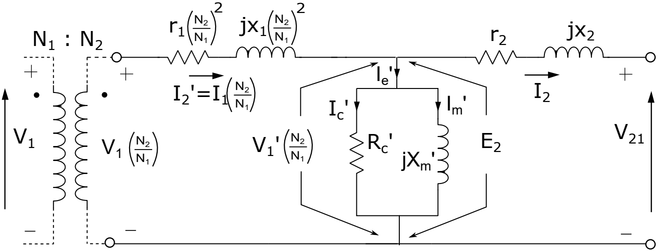


Figure: Equivalent circuit referred to Secondary

Phasor Diagram

The phasor diagram of the equivalent circuit referred to primary can be drawn in stepwise manner.
The magnetizing current I_m is taken as reference.
There are 2 general cases : output current I_2 $(= I_1^{'})$ either **lags** or **leads** output voltage $V_2^{'}$ depending upon load conditions.

I_2 lags $V_2^{'}$

- Secondary induced emf E_2 leads I_m by 90° ($\because$ I_m lags $V_1^{'}$, and $E_2 = V_1^{'}$).
- Voltage drops $I_2 r_2^{'}$ and $jI_2 x_2^{'}$ added to $V_2^{'}$ gives E_2.
- $I_c = \frac{V_1^{'}}{R_c}$ will be in phase with $V_1^{'}$.
- Phasor sum of I_m and I_c gives I_e.
- Phasor sum of I_e and I_2 gives I_1.
- Finally, adding the drops $I_1 r_1$ and $jI_1 x_1$ to $V_1^{'}$ gives V_1.

I_2 leads V_2'

Similar process can be followed when I_2 leads V_2'.

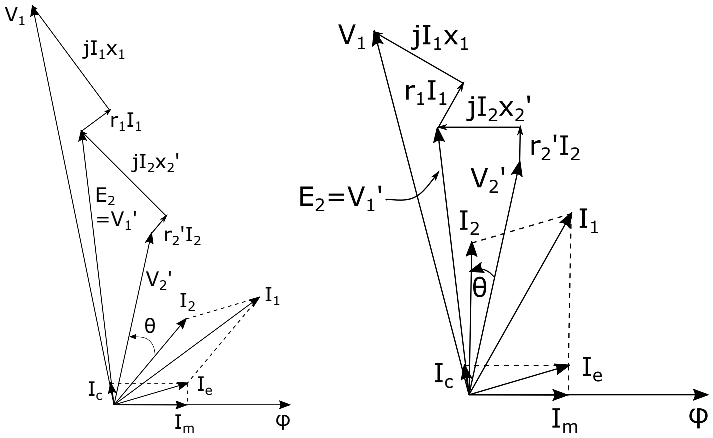


Figure: Phasor Diagrams (left) lagging I_2 (right) leading I_2

Transformer Tests

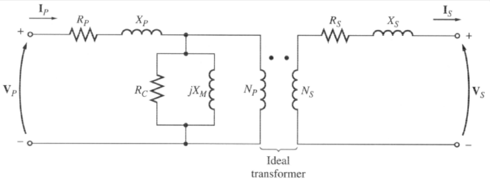
Main parameters of the equivalent circuit

- R_{01} as referred to primary (or secondary R_{02})
- The equivalent leakage reactance X_{01} as referred to primary (or secondary X_{02})
- Magnetising susceptance B_0 (or reactance X_0)
- core loss conductance G_0 (or resistance R_o)

To be determined by 2 tests

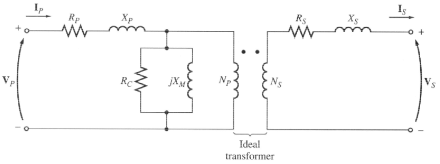
The above constants can be easily determined by two tests

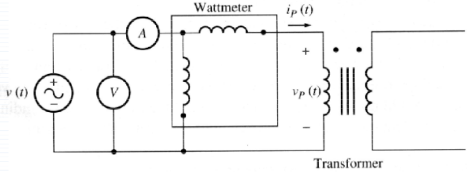
- Open circuit tests (O.C. test/No Load test)
- Short Circuit tests (S.C. test/Impedance test)

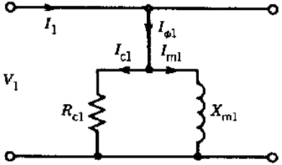


Circuit Parameters : Open-Circuit Test

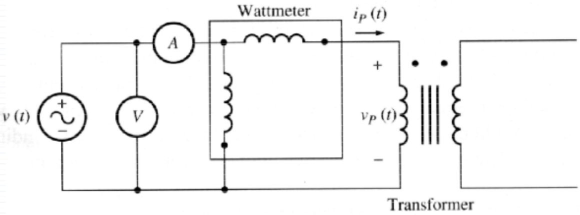
- Transformer's secondary winding is open-circuited.
- Primary winding is connected to a full-rated line voltage. All the input current must be flowing through the excitation branch of the transformer.
- The series elements R_p and X_p are too small in comparison to R_C and X_M to cause a significant voltage drop, so essentially all the input voltage is dropped across the excitation branch.
- Input voltage, input current, and input power to the transformer are measured.

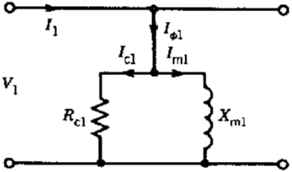






Circuit Parameters : Open-Circuit Test





The magnitude of excitation admittance

$$|Y_E| = \frac{I_{OC}}{V_{OC}}$$

The open-circuit power factor (we can calculate power facter angle accordingly)

$$PF = cos(\theta) = \frac{P_{OC}}{V_{OC} I_{OC}}$$

The power factor is always lagging for a transformer, so the current will lag the voltage by the angle q. Therefore, the admittance Y_E is:

$$Y_E = \frac{1}{R_C} = j\frac{1}{X_M} = \frac{I_{OC}}{V_{OC}} \angle - cos^{-1}(PF)$$

Therefore, it is possible to determine values of R_C and X_M in the open-circuit test.

Short-Circuit Test

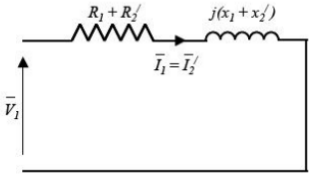
Fairly low input voltage is applied to the primary side of the transformer. This voltage is adjusted until the current in the secondary winding equals to its rated value.
The input voltage, current, and power are again measured.

Since the input voltage is low, the current flowing through the excitation branch is negligible; therefore, all the voltage drop in the transformer is due to the series elements in the circuit. The magnitude of the series impedance referred to the primary side of the transformer is:

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

The power factor of the current is given by:

$$PF = cos(\theta) = \frac{P_{SC}}{V_{SC} I_{SC}}$$



Circuit Parameters : Short-Circuit Test

Therefore :

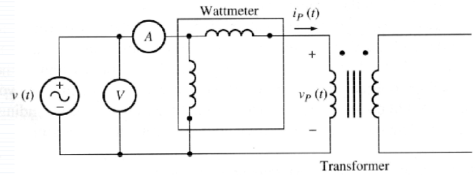
$$Z_{SE} = \frac{V_{SC}\angle 0^o}{I_{SC}\angle -\theta^o} = \frac{V_{SC}}{I_{SC}}\angle \theta^o$$

Since the serial impedance Z_{SE} is equal to:

$$Z_{SE} = R_{eq} + jX_{eq}$$

$$Z_{SE} = (R_P + a^2 R_S) + j(X_P + a^2 X_S)$$

It is possible to determine the total series impedance referred to the primary side of the transformer. However, there is no easy way to split the series impedance into primary and secondary components.
The same tests can be performed on the secondary side of the transformer. The results will yield the equivalent circuit impedances referred to the secondary side of the transformer.

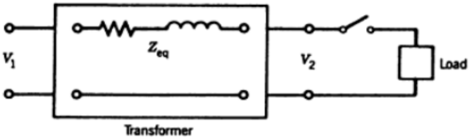


$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

$$PF = cos(\theta) = \frac{P_{SC}}{V_{SC}I_{SC}}$$

Transformer Voltage Regulation

At no-load, $V_{2|NL} = \dfrac{V_1}{a}$
After closing the switch,
$V_{2|L} = V_{2|NL} \pm \Delta V_2$



Definition

Because a real transformer has series impedance within it, the output voltage of a transformer varies with the load even if the input voltage remains constant. To compare transformers in this respect, the quantity called a full-load voltage regulation (VR) is defined.

The voltage regulation of a transformer is the change in the magnitude of the secondary terminal voltage from no-load to full-load.

$$VR = \frac{V_{s,nl} - V_{s,\beta}}{V_{s,\beta}}.100\% = \frac{V_p/a - V_{s,\beta}}{V_{s,\beta}}.100\%$$

where, $V_{s,nl}$ and $V_{s,fl}$ are the secondary no load and full load voltages. Note that VR of an ideal transformer is zero.

Inrush Current

Transformer Inrush Current

The transformer inrush current or inrush of magnetizing current is the maximum instantaneous current drawn by the primary of the transformer when their secondary is open circuit. During the inrush current, the maximum value attained by the flux is over twice the normal flux.

The steady-state magnetizing current for a transformer is very low, but the momentary current when it is first energized can be quite high.

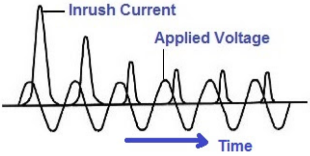
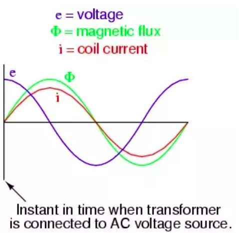
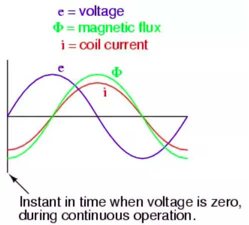
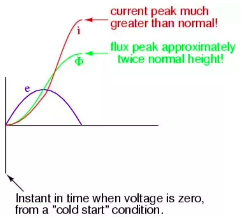


Figure: Transformer Inrush Current

Inrush Current







When connected when voltage is at its positive peak, magnetic flux of rapidly increasing value must be generated. The result is that winding current increases rapidly, but actually no more rapidly than under normal conditions.

When connected at voltage equal to zero, both flux and winding current are at its negative peak. As voltage builds to its peak, flux and current build to maximum rate of change and on upwards till voltage descends to zero.

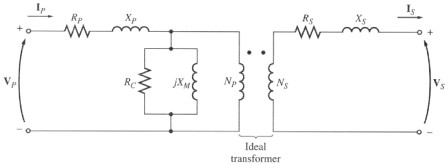
When a transformer is taken off-line, a certain amount of residual flux remains in the core. When voltage is reapplied, the flux introduced builds upon that already existing in the core. In order to maintain the level, the transformer can draw current well in excess of the transformers rated full-load current (Inrush Current).

Transformer Efficiency

The efficiency of a transformer is defined as,

$$\eta = \frac{P_{out}}{P_{in}}.100\% = \frac{P_{out}}{P_{out} + P_{loss}}.100\%$$

Note : The same equation describes the efficiency of motors and generators.



Considering the transformer equivalent circuit, we notice 3 types of losses:

- Copper losses (I^2R) - are accounted for by the series resistance.
- Hysteresis losses - are accounted for by the resister R_c.
- Eddy current losses - are accounted for by the resistor R_c.

Since the output power is, $P_{OUT} = V_s I_s cos(\theta_s)$

The transformer efficiency is, $\eta = \dfrac{V_s I_s cos(\theta)}{P_{Cu} + P_{core} + V_s I_s cos(\theta)}.100\%$