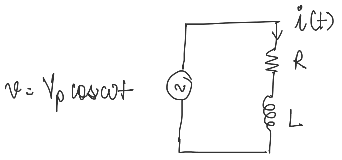
Lecture 2

Tuesday, 9 January 2024 3:20 PM

$V_p = \sqrt{2}\, V_{rms}$

* Phasors :-

What is a phasor?



Circuit is in sinusoidal steady state. ⇒ Source was connected to the ckt a long time ago.

$i(t) = ??$

To solve the equation using Laplace Transform, first write the corresponding differential equation and then take Laplace Transform of the equation and solve for the variables.

We can use differential equations or laplace to solve the above ckt.

Another way to analyze the ckt is through phasors.

If the elements of a circuit have common input (current or voltage) then assume the input's expression and use it in the equations.

This circuit is linear $\quad i(t) = I_p \cos(\omega t + \theta)$

($i(t)$ will be a sinusoid of the same frequency as $v(t)$ with some phase shift θ)

$$v = iR + L\frac{di}{dt} \qquad (\text{KVL in above ckt})$$

$$V_p \cos\omega t = I_p R \cos(\omega t + \theta) - L I_p \omega \sin(\omega t + \theta) \quad \text{— (A)}$$

$$(\omega t + \tfrac{\pi}{2})$$

$$V_p \sin\omega t = I_p R \sin(\omega t + \theta) + L I_p \omega \cos(\omega t + \theta) \quad \text{— (B)}$$

{ Placing a voltage source phase shifted by $\pi/2$ rads. The current is also going to be phase shifted by $\pi/2$ rads. }

$A + jB$ on both sides

$$V_p[\cos\omega t + j\sin\omega t] = I_p R[\cos(\omega t + \theta) + j\sin(\omega t + \theta)] + L I_p \omega[\cos(\omega t + \theta + \pi/2) + j\sin(\omega t + \theta + \pi/2)]$$

$$V_p e^{j\omega t} = I_p R e^{j(\omega t + \theta)} + L I_p \omega e^{j(\omega t + \theta + \pi/2)}$$

$$\frac{V_p}{\sqrt{2}} \cancel{e^{j\omega t}} = \frac{I_p}{\sqrt{2}} R e^{j\theta} \cancel{e^{j\omega t}} + L \frac{I_p}{\sqrt{2}} \omega \cancel{e^{j\omega t}} e^{j(\theta + \pi/2)}$$

$$V_{rms} = I_{rms} R e^{j\theta} + j I_{rms} L \omega e^{j\theta}$$ } Rectangular form

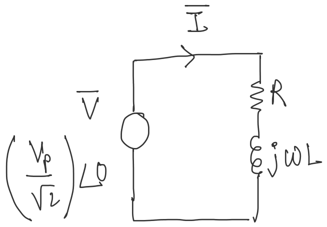
$\bar{V} = \left(\frac{V_p}{\sqrt{2}}\right)\angle 0 \quad ; \quad \bar{I} = \left(\frac{I_p}{\sqrt{2}}\right)\angle\theta$ } That's how we will define phasors.

$$\bar{V} = R\bar{I} + \underbrace{j\omega L}_{X_L}\bar{I}$$

Things to check before applying phasor analysis } ⇒
1. Linear circuit
2. Sinusoidal steady state
3. Sinusoid of only one frequency

[Is it possible to have a linear circuit with two frequencies?]

{ yes ! If we have 2 seperate sources of different frequencies
↳ To solve such circuits use superposition principle. }



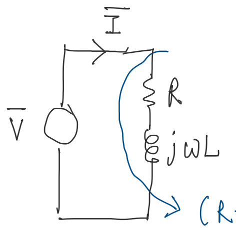
$\bar{I} = ??$
$= I_{rms}\angle\theta$
$i(t) = I_{rms}\sqrt{2}\cos(\omega t + \theta)$

$\bar{V} \rightarrow V_{rms}\angle\theta$
$\bar{V} \rightarrow V_{rms}\cos\theta + j\,V_{rms}\sin\theta$
$\bar{V} \rightarrow V_{rms}e^{j\theta}$

{ They all are equivalent to each other }

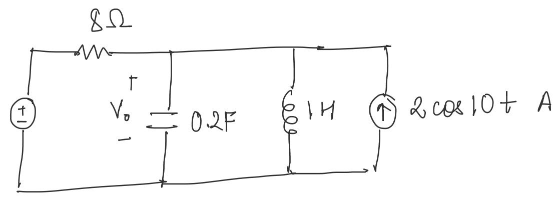
	R	L	C
Time domain ↓	↓	↓	↓
Frequency domain	R	$j\omega L$	$\frac{1}{j\omega C}$
		↓	↓
		X_L	X_C

Impedance →



$(R + j\omega L) = Z$ } Impedance

eg.



$V_0 = ??$

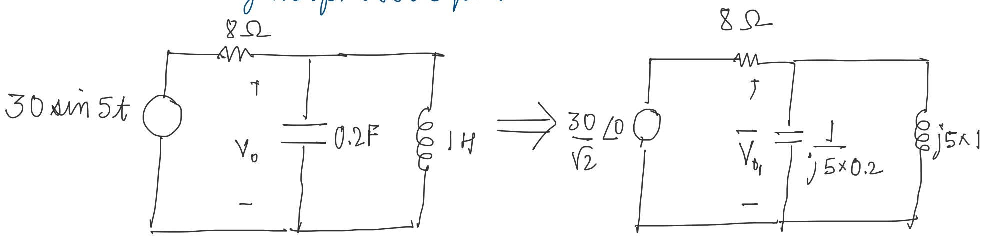
Answer :- $4.631\sin(5t - 1.416) + 1.051\cos(10t - 1.505)$ V

Solution:—

Using superposition principle.

Case 1 :- Considering only voltage source. Current source is open circuited.

Converting to phasor equivalent ckt



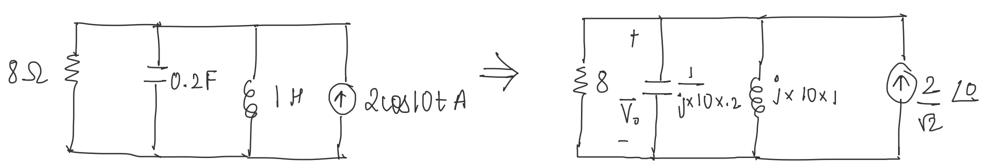
$$\overline{V_{0_1}} = \frac{\frac{30\angle 0}{\sqrt{2}}}{8 + \frac{\frac{1}{j}\times 5j}{\frac{1}{j}+5j}} \times \left(\frac{\frac{1}{j}\times 5j}{\frac{1}{j}+5j}\right)$$

$$= \frac{\frac{30}{\sqrt{2}}}{8 - \frac{5}{4}j} \times \left(-\frac{5}{4}j\right)$$

$$\overline{V_{0_1}} = 3.275\angle -1.416$$

$$v_{0_1}(t) = \sqrt{2}\times 3.275 \sin(5t - 1.416)$$
$$= 4.631 \sin(5t - 1.416)\ V$$

Case 2 :- Considering only current source. Voltage source is short circuited.



$$\overline{V_{0_2}} = \frac{2}{\sqrt{2}} \times \frac{1}{\left(\frac{1}{8} + 2j + \frac{1}{10j}\right)}$$

$$\overline{V_{0_2}} = 0.743\angle -1.505$$

$$v_{0_2}(t) = 1.051 \cos(10t - 1.505)\ V$$

$$V_0(t) = v_{0_1}(t) + v_{0_2}(t)$$
$$v_0(t) = 4.631 \sin(5t - 1.416) + 1.051 \cos(10t - 1.505)\ V$$

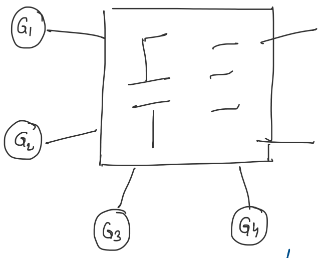
——— x ———

Solve assignment 1 : till Question 8

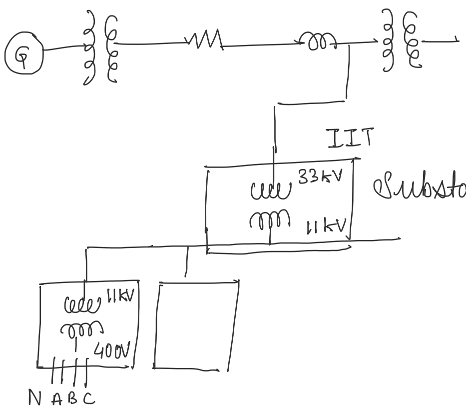
Phasor $\rightarrow V_{rms}\angle\theta$

Brief Idea about power system :-

Generator $\rightarrow$ an electrical machine which is generating voltage.
There is no single generator in a power system
There are multiple sources in a power grid.



All the generators are operating at same frequency.
Generators are able to generate voltage at same frequency using Governers.

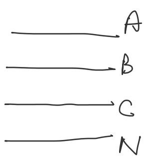


Single line Diagram (SLD)

Transformer → step up the generated voltage → Transmission lines

Different ways of representing it (We use R, L ckt) for simplicity

3 phase 4 wire system



$v_{AN} = V \cos \omega t$ — phase A

$v_{BN} = V \cos(\omega t - 120^\circ)$ — phase B

$v_{CN} = V \cos(\omega t - 240^\circ)$ — phase C

Step up transformers step up the voltage generated by the generators to a higher voltage to reduce losses while transmitting power over long distances using transmission lines. Substations near load step down the voltage to 11kV. The voltage is again stepped down to line voltage of 400V to be fed to the buildings.

In the last few year distributed generations (DGs) have picked up.
There is a possibility of power flowing in both directions.