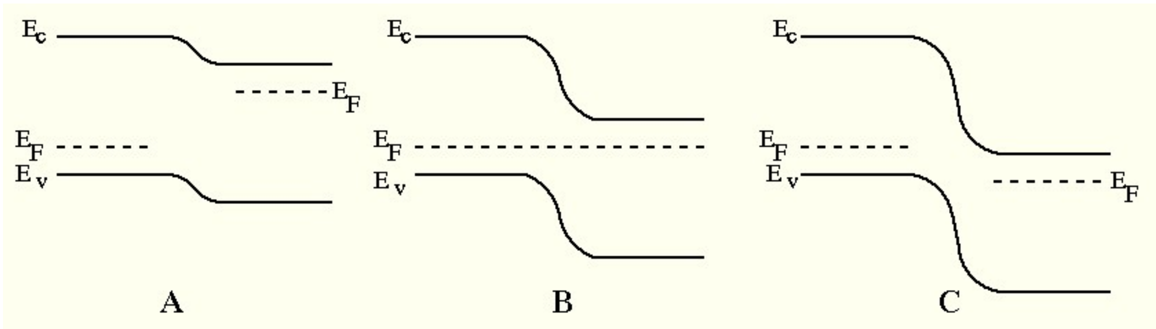
EE 207 [Spring 24-25] ***HW-5***

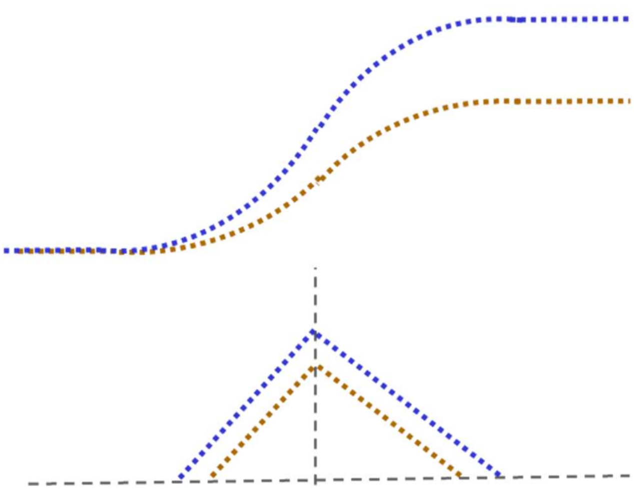
1. Consider a p-n junction with comparable n and p doping.

(a) Draw the band diagram schematically when the junction is in reverse bias. Show the quasi-fermi levels clearly. Draw the profiles of the electric field and potential in comparison with their equilibrium profiles (i.e. on the same figure).

This comparative band diagram is from some textbook. It uses the symbol E_F for quasi Fermi level as well.



The following are the (unlabeled) comparative potential and field profiles. Note, in the electric field profile, that the slope in equilibrium and reverse bias is the same on both sides – because it is proportional to the doping concentration. The slope of the potential is of course different because the fields are greater in reverse bias. Ditto for the total drop, which is just the area under the field curve.



(b) Assuming this to be silicon, with uniform n and p side doping of 10^{15} and $5\text{x}10^{15}$ cm^{-3} respectively, calculate the depletion widths and potential drops on either side of the junction in equilibrium, and, for a reverse bias of 1V.

First, we calculate the contact potential (build-in voltage) as:

$$V_0 \cong \frac{k_B T}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.63eV$$

Then, the total depletion width may be calculated as:

$$W \cong \sqrt{\frac{2\varepsilon_0 \epsilon_{Si}(V_0 + V_R)}{e}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

In equilibrium, $W(V_R = 0) = 1\ \mu m$

For a reverse bias of 1V, $W(V_R = 1V) = 1.6\ \mu m$

The depletion region widths in the n and p regions are related by:

$$N_a x_p = N_d x_n \Rightarrow 5x_p = x_n$$

$$\therefore V_R = 0 \Rightarrow x_p = W/6 = 0.165\ \mu m; x_n = 5x_p = 0.824\ \mu m$$

And, $V_R = 1\ V \Rightarrow x_p = 0.265\ \mu m; x_n = 1.326\ \mu m$

Similarly the voltage drops on the two sides are related by the following expression, which follows from the area under the electric field profile:

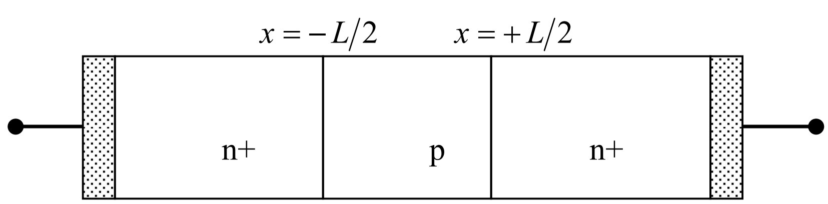
$$V_p/V_n = E_m x_p / E_m x_n = x_p/x_n$$

$$\therefore V_R = 0 \Rightarrow V_p = V_0/6 = 0.105\ V; V_n = 5V_p = 0.525\ V$$

And, $V_R = 1\ V \Rightarrow V_p = (V_0 + V_R)/6 = 0.272\ V; V_n = 5V_p = 1.358\ V$

2. Consider an n+/p/n+ structure shown below. It is made of a semiconductor material with relative permittivity κ_s and bandgap E_g. The thickness of the p-region is L. Assume that the doping in the n+ region (N_d) is non-degenerate but much larger than the doping in the p-region (N_a), both being uniform. All questions pertain to the equilibrium situation.

(a) For warm-up, show that for a single long n+/p junction, the potential drop is almost wholly in the p region. Thus, we assume that the depletion width W is in the p region.



(a) For a long n+/p junction, charge neutrality implies: $N_d x_n = N_a x_p$

Since $N_d \gg N_a$ this implies that: $x_p \gg x_n \Rightarrow W \cong x_n + x_p \approx x_p$, i.e. the depletion region is almost wholly on the p-side.

To see the potential drop, we start with the peak electric field (at the interface): $\in_m = \frac{qN_d x_n}{\varepsilon_s} = \frac{qN_a x_p}{\varepsilon_s}$

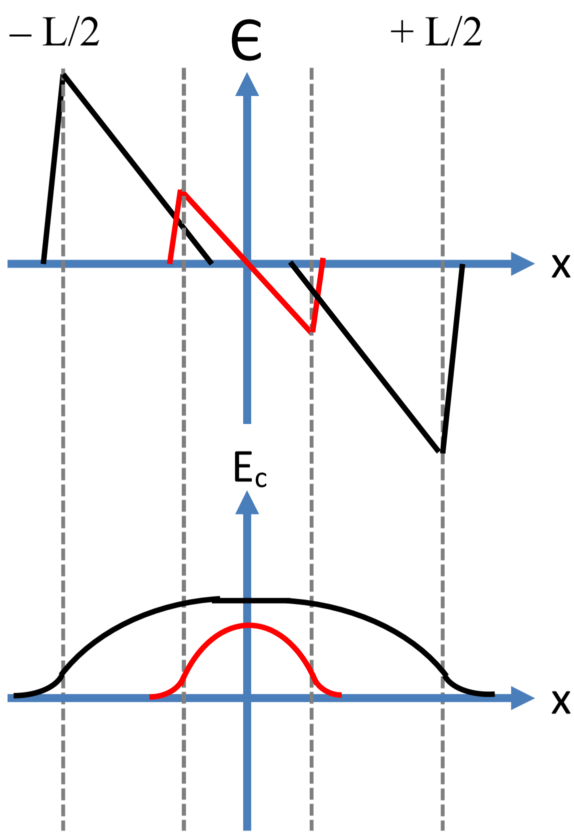
This implies that the built-in potential drop on the n and p sides will be given by:

$\varphi_{n,p} = \frac{1}{2} \in_m \cdot x_{n,p}$

Therefore, $x_p \gg x_n \Rightarrow \varphi_p \gg \varphi_n$.

(b) Now, draw the electric field profiles and comparative band-diagrams for the n+/p/n+ structure when (i) $L > 2W$ and (ii) $L < 2W$.

For part (i), we can consider this as two uncouples junctions, n+/p and p/n+. Therefore the electric field and band diagrams will be as shown in black below:



For part (ii), the calculations are somewhat different; the results are shown in red above. Here the total space charge in the p-region is N_a throughout, giving a total space-charge (per unit area) of $N_a L$. The electric field as a function of position is:

$$\in(x)=\int_{-L/2}^{x}\frac{\rho(x)}{\varepsilon_s}dx=-\frac{qN_a x}{\varepsilon_s}$$

The potential may be obtained by integration:

$$\varphi(x)=\frac{qN_a x^2}{2\varepsilon_s}+\varphi(0)$$

Since most of the voltage drop is in the p-region (see part a), we assume that the potential goes to zero at the end-points of the p-region. That is:

$$\varphi(\pm L/2)=0=\frac{qN_a L^2}{8\varepsilon_s}+\varphi(0)$$

$$\varphi(x)=\frac{qN_a\left(x^2-(L/2)^2\right)}{2\varepsilon_s}$$

The band-bending will have the opposite sign. Only the conduction band is shown.

(c) As you can tell, electrons see a potential barrier between the n+ regions – plot the barrier height schematically as a function of L covering both regimes (i) and (ii).

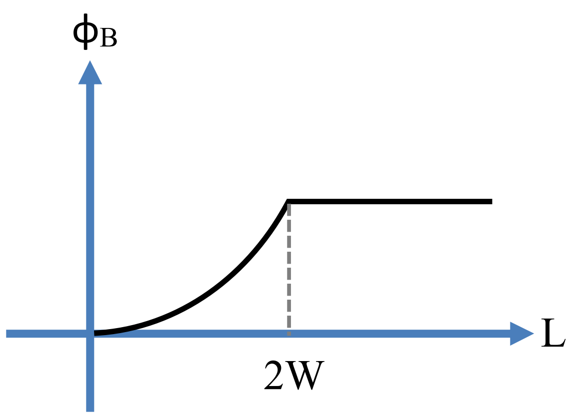
(b) From the above, the barrier is obviously:

$$\varphi_B=\frac{qN_a(L/2)^2}{2\varepsilon_s}\text{ for }L/2<W$$

From standard p-n junction theory (barrier for a n+/p junction):

$$\varphi_B=\frac{qN_a W^2}{2\varepsilon_s}=\frac{kT}{q}\ln\frac{N_a N_d}{n_i^2}\text{ for }L/2>W$$

Thus the barrier height as a function of device (p-region) length is as follows:



3. Consider a long semiconductor homo-junction that is n^{++} on one side and p^{++} on the other, with fully-ionized doping concentration N_d and N_a respectively. The doping on both sides is high enough that the Fermi-level is inside the corresponding band. The intrinsic carrier concentration is $n_i = \sqrt{N_C N_V}\exp\left(-E_g/2k_BT\right)$, where the symbols have their usual meaning. Please feel free to make *reasonable* approximations.

(a) First, some background. Show that the equilibrium electron concentration in a bulk homogeneous n^{++} semiconductor may be written as $n_0 \simeq \alpha_n (E_F - E_C)^{\frac{3}{2}}$ where α_n is a material parameter. [The hole concentration in a bulk homogeneous p^{++} semiconductor may, similarly, be expressed as $p_0 \simeq \alpha_p (E_V - E_F)^{\frac{3}{2}}$.]

The (3D) density of states for electrons is given by:

$$g_{3D} \simeq \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^*}{\hbar^2}\right)^{\frac{3}{2}} (E - E_C)^{\frac{1}{2}}$$

The electron density is then obtained as follows:

$$n_0 = \int_{E_C}^{\infty} dE \cdot g_{3D}(E) f_{FD}(E)$$

For a degenerately doped system, the electron population due to the thermal tail above the Fermi energy is negligible compared to that due to the filled states below it.

Thus: $n_0 \simeq \int_{E_C}^{E_F} dE \cdot g_{3D}(E)$

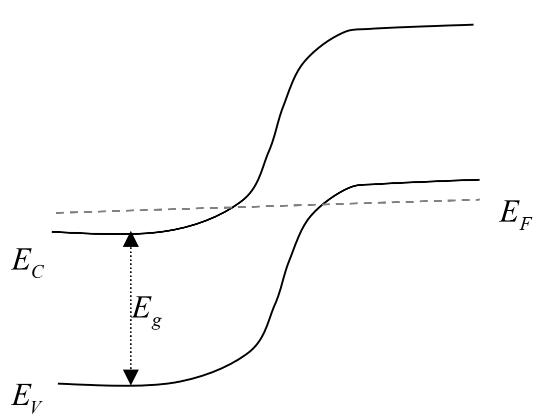
$$\Rightarrow n_0 \simeq \int_{E_C}^{E_F} dE \cdot \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^*}{\hbar^2}\right)^{\frac{3}{2}} (E - E_C)^{\frac{1}{2}}$$

$$\Rightarrow n_0 \simeq \frac{1}{3\pi^2}\left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}} (E_F - E_C)^{\frac{3}{2}}$$

Note that this power law dependence can be inferred from the DOS and the step-like statistics, even without going through the full calculation.

(b) Draw, schematically, the equilibrium band-diagram for this device.

The equilibrium band diagram is as follows:



(c) Derive an expression for the equilibrium built-in potential.

$$n_0 = N_d \simeq \frac{1}{3\pi^2}\left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}} (E_{Fn} - E_C)^{\frac{3}{2}} = \alpha_n (E_{Fn} - E_C)^{\frac{3}{2}}$$
$$p_0 = N_a \simeq \frac{1}{3\pi^2}\left(\frac{2m_p^*}{\hbar^2}\right)^{\frac{3}{2}} \left(E_V - E_{Fp}\right)^{\frac{3}{2}} = \alpha_p \left(E_V - E_{Fp}\right)^{\frac{3}{2}}$$

Therefore:

$$E_{Fn} = E_C + \left(n_0/\alpha_n\right)^{2/3} = E_C + \left(N_d/\alpha_n\right)^{2/3}$$
$$E_{Fp} = E_V - \left(p_0/\alpha_p\right)^{2/3} = E_V - \left(N_a/\alpha_p\right)^{2/3}$$

When a junction is formed, the Fermi level must be flat everywhere in equilibrium, implying that the built-in potential is given by the difference in initial Fermi levels:

$$\varphi_{bi} = E_{Fn} - E_{Fp} = E_g + \left(N_d/\alpha_n\right)^{2/3} + \left(N_a/\alpha_p\right)^{2/3}$$

(d) Write down an expression for the forward-bias current in this device, along with the justification thereof.

$E_{Fn} - E_{Vn} \gg k_B T$
$E_{Cp} - E_{Fp} \gg k_B T$
Therefore the minority carrier concentrations on the two sides are given by:

$$n_{P0} = N_C e^{-\left(E_C - E_{Fp}\right)/k_B T}$$
$$p_{N0} = N_V e^{-\left(E_{Fn} - E_V\right)/k_B T}$$

Here the pre-factors on the RHS are the effective density of states in the conduction band and valence band respectively, given by:

$$N_C = 2\left(\frac{2\pi m_n^* k_B T}{\hbar^2}\right)^{3/2}$$
$$N_V = 2\left(\frac{2\pi m_p^* k_B T}{\hbar^2}\right)^{3/2}$$

{Note that we <u>cannot use the usual formulas</u> $n_{P0} = n_i^2/N_a$, $\mathrm{p}_{N0} = n_i^2/N_d$ or $n_{P0} = n_i e^{\left(E_{Fp} - E_i\right)/k_B T}$, $\mathrm{p}_{N0} = n_i e^{\left(E_i - E_{Fn}\right)/k_B T}$ since the Boltzmann statistics does not apply to the majority carriers.}

From the above, we may write:

$$E_{Fn} - E_C = \left(N_d/\alpha_n\right)^{2/3} \Rightarrow E_{Fn} - E_V = E_g + \left(N_d/\alpha_n\right)^{2/3}$$
$$E_V - E_{Fp} = \left(N_a/\alpha_p\right)^{2/3} \Rightarrow E_C - E_{Fp} = E_g + \left(N_a/\alpha_p\right)^{2/3}$$

Thus, we finally get:

$$n_{P0} = N_C e^{-\left(E_g + \left(N_a/\alpha_p\right)^{2/3}\right)/k_B T}$$
$$p_{N0} = N_V e^{-\left(E_g + \left(N_d/\alpha_n\right)^{2/3}\right)/k_B T}$$

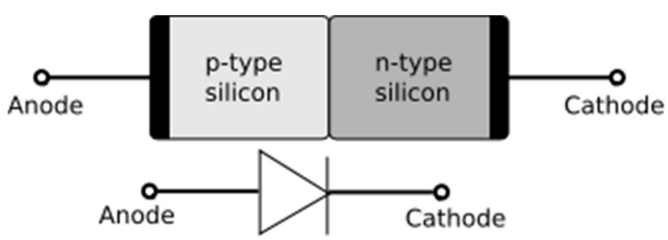
Now, in terms of these, the minority carrier concentrations at the depletion layer boundaries under forward bias may still be expressed by the <u>usual formulas</u> that are based on the Boltzmann statistics:

$$n\left(x_P\right) = n_{P0} e^{qV/k_B T}$$
$$p\left(x_N\right) = p_{N0} e^{qV/k_B T}$$

And, therefore, the diode current may be written as:

$$I = qA\left(\frac{n_{P0} D_n}{L_n} + \frac{p_{N0} D_p}{L_p}\right)\left(e^{qV/k_B T} - 1\right)$$

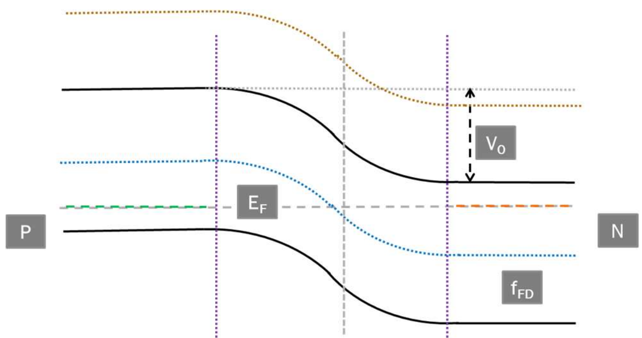
4. Let us try to derive the p-n diode I-V relation using a different method. Consider the electron current density from left to right $J_{n,L\to R}\left(V\right)$ and that from right to left $J_{n,R\to L}\left(V\right)$ in the diode illustrated below (drawing a band-diagram might help).



(a) Taking the difference between them, I get: $J_n\left(V\right) = J_{n0}\left[\mathrm{e}^{qV/k_B T} - 1\right]$. How? Explain with detailed reasoning. Why is it that drift/diffusion considerations seem to have no role now?

The electron flux from left to right, parameterized by $J_{n,L\to R}\left(V\right)$ constitutes minority carriers 'rolling down a potential hill'. Therefore it has largely no dependence on the height of this hill, which is modulated by the potential. Therefore:

$$J_{n,L\to R}(V) \simeq J_{n,L\to R}(0)$$



The electron flux from right to left parameterized by $J_{n,R\to L}(V)$, on the other hand, depends exponentially on the voltage because the density of electrons above the barrier is modulated exponentially (because of the Boltzmann distribution) by a shift in the quasi-Fermi level under bias.

At equilibrium, we must have: $J_{n,L\to R}(0) = J_{n,R\to L}(0)$

$$J_n = J_{n,R\to L}(V) - J_{n,L\to R}(V)$$
$$\Rightarrow J_n = J_{n,R\to L}(0)\mathrm{e}^{qV/k_BT} - J_{n,L\to R}(0)$$

$$J_n = J_{n,L\to R}(0)\mathrm{e}^{qV/k_BT} - J_{n,L\to R}(0) = J_{n0}\left[\mathrm{e}^{qV/k_BT} - 1\right]$$

Drift and diffusion have not come up explicitly, but the electron flux from right to left, that is from the heavily n-doped to the p-doped region is actually due to diffusion. And the electric field due to the potential barrier in equilibrium can be considered to set up the countering drift flux.

(b) Derive J_{n0} in terms of device parameters (doping, lifetime, mobility etc.) with reasoning.

We will estimate $J_{n0} = J_{n,L\to R}$ using the charge control method discussed in class. This current is related to the generation of electrons within a minority carrier diffusion length of the junction. This generated charge (per unit area) is given by:

$$Q_n = q\frac{n_i^2}{N_a}\lambda_n$$

These are the charges that can make their way to the junction within the minority carrier lifetime τ_n and go downhill to give rise to the current. Thus:

$$J_{n0} = q\frac{n_i^2}{N_a}\frac{\lambda_n}{\tau_n} = q\frac{n_i^2}{N_a}\frac{D_n}{\lambda_n} \text{ since } \lambda_n^2 = D_n\tau_n .$$

Thus, we recover the normal diode current equation.