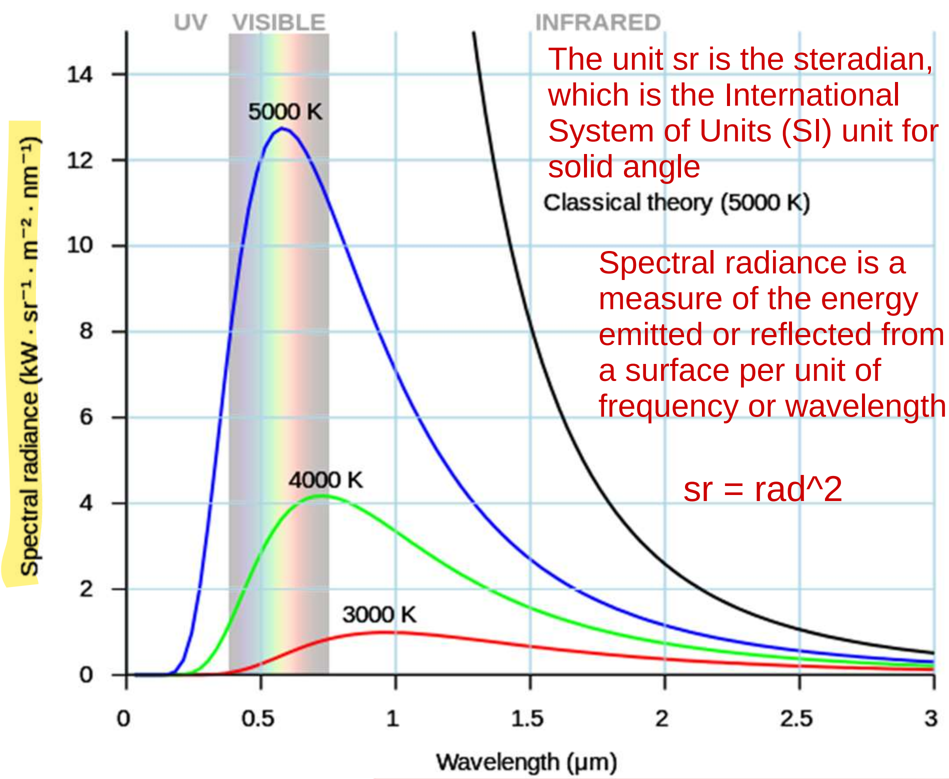
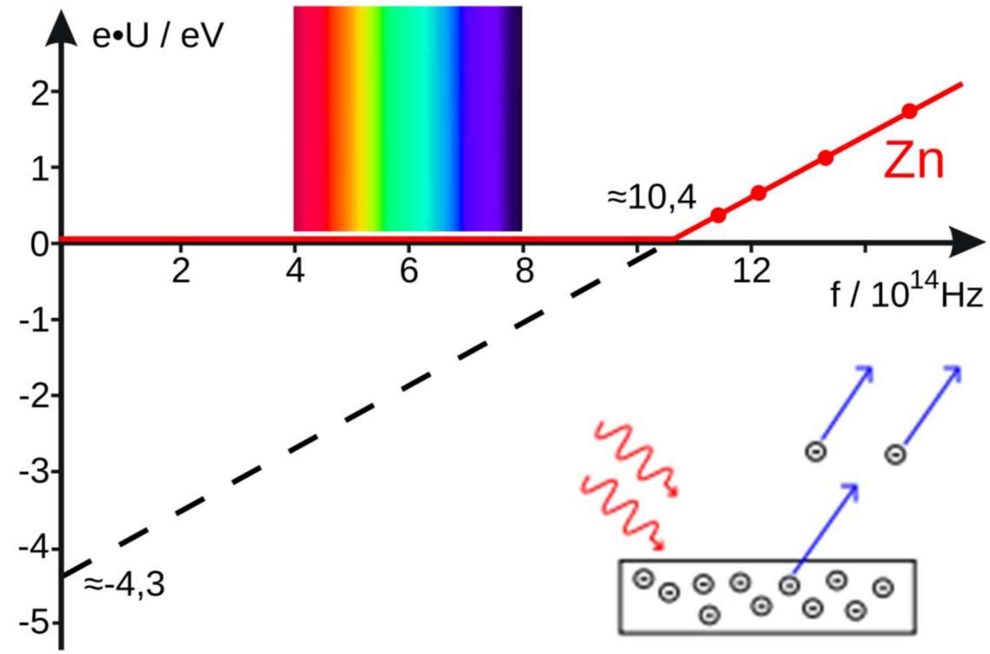
"Old Quantum Theory": historical perspective





Black-body radiation (Planck, 1900)

Photoelectric Effect (Einstein, 1905)

Quantum Stat. Mech.

Specific heats of solids (Einstein, 1907)

Special Relativity (Einstein 1905): $E^2 = (m_0c^2)^2 + (pc)^2$

Light emitted and absorbed by 'oscillators' in quanta: $E_n - E_{n-1} = h\nu$

Light *is* a particle: $E = h\nu$
→ $p = h/\lambda$

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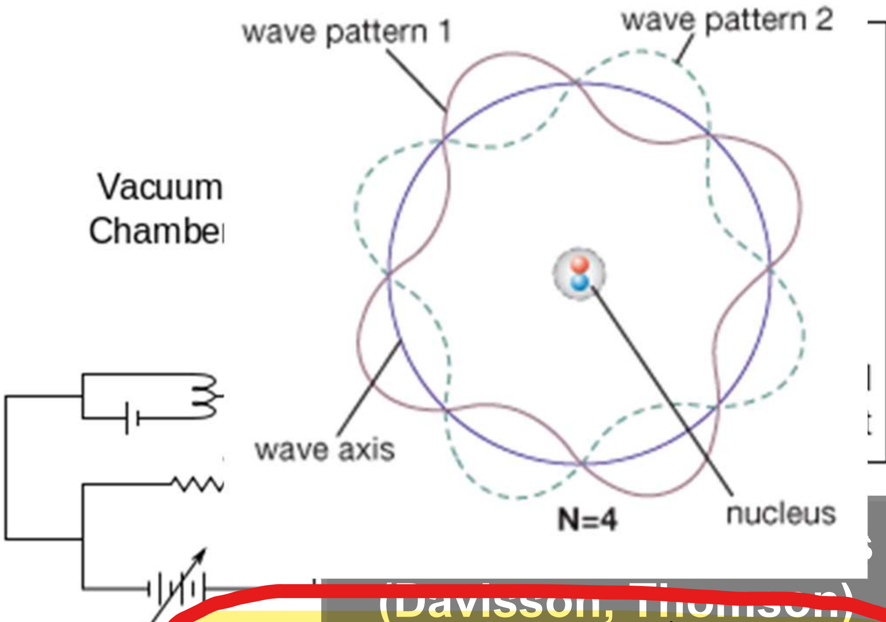
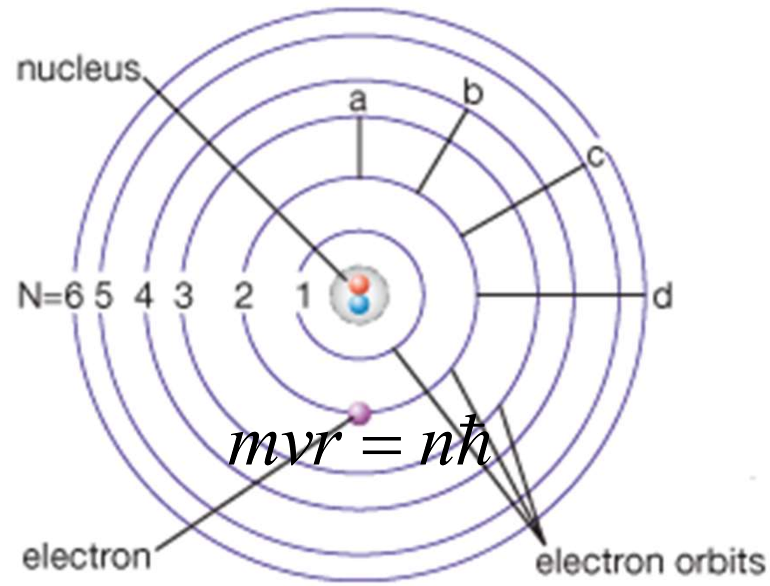


(Davisson, Thomson)

$$n\lambda = 2\pi r \Rightarrow nh/p = 2\pi r$$

Matter waves (de Broglie, 1924): λ = h/p

Electrons are waves!



$$mvr = n\hbar$$

Hydrogen emission spectra (Lyman, Balmer, Paschen, Brackett, Pfund)

Hydrogen atom (Bohr, 1913)

$$E_n = -\frac{m_0 q^4}{2\left(4\pi\varepsilon_0 n\hbar\right)^2} = \frac{-13.6eV}{n^2}$$

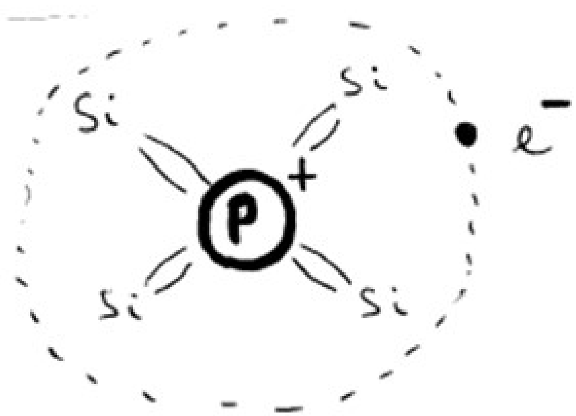
$$r_n = \frac{4\pi\varepsilon_0 \left(n\hbar\right)^2}{m_0 q^2} = 0.53\dot{\text{A}} \cdot n^2$$

Question 0

Bohr Model for hydrogen atom

$$r_n = \frac{4\pi\varepsilon_0 (n\hbar)^2}{m_0 q^2} = 0.53\dot{\text{A}} \cdot n^2 \qquad E_n = -\frac{m_0 q^4}{2(4\pi\varepsilon_0 n\hbar)^2} = \frac{-13.6eV}{n^2}$$

"HYDROGENIC" MODEL FOR DOPANTS



$$\boxed{\begin{array}{l} m_0 \to m^* \\ \varepsilon_0 \to \varepsilon_0 \varepsilon_r \end{array}}$$

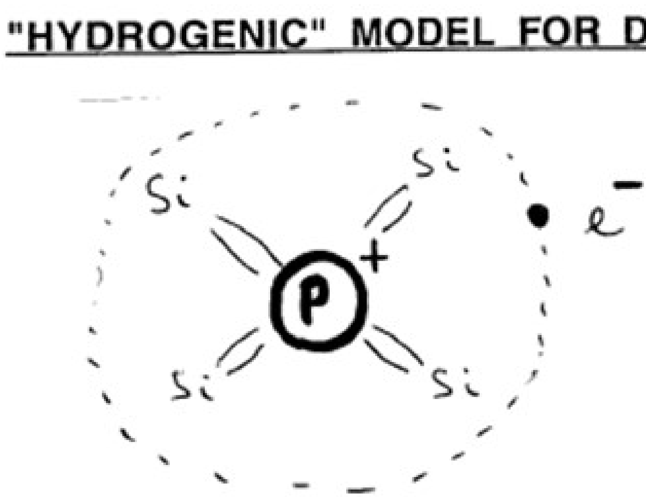
Dopants give rise to carriers because:

a. That is their job, and they understand their job
b. It is more difficult to ionize inside a semiconductor
c. It is easier to ionize inside a semiconductor than in vacuum ✓
d. It is always thermodynamically favorable to give up the carrier

Question 0

Typically, in a semiconductor: $m^* \sim 0.1 m_0, \varepsilon_r \sim 10$
$\Rightarrow r_d \sim 5nm, E_d \sim 10meV$

Will dopant ionization happen at all temperatures?



$$m_0 \to m^*$$

$$\varepsilon_0 \to \varepsilon_0 \varepsilon_r$$

$$r_d = 0.53 A \frac{\varepsilon_r}{\left(m^*/m_0\right)}$$

$$E_d = -13.6 eV \frac{\left(m^*/m_0\right)}{\varepsilon_r^2}$$

Dopants give rise to carriers because:

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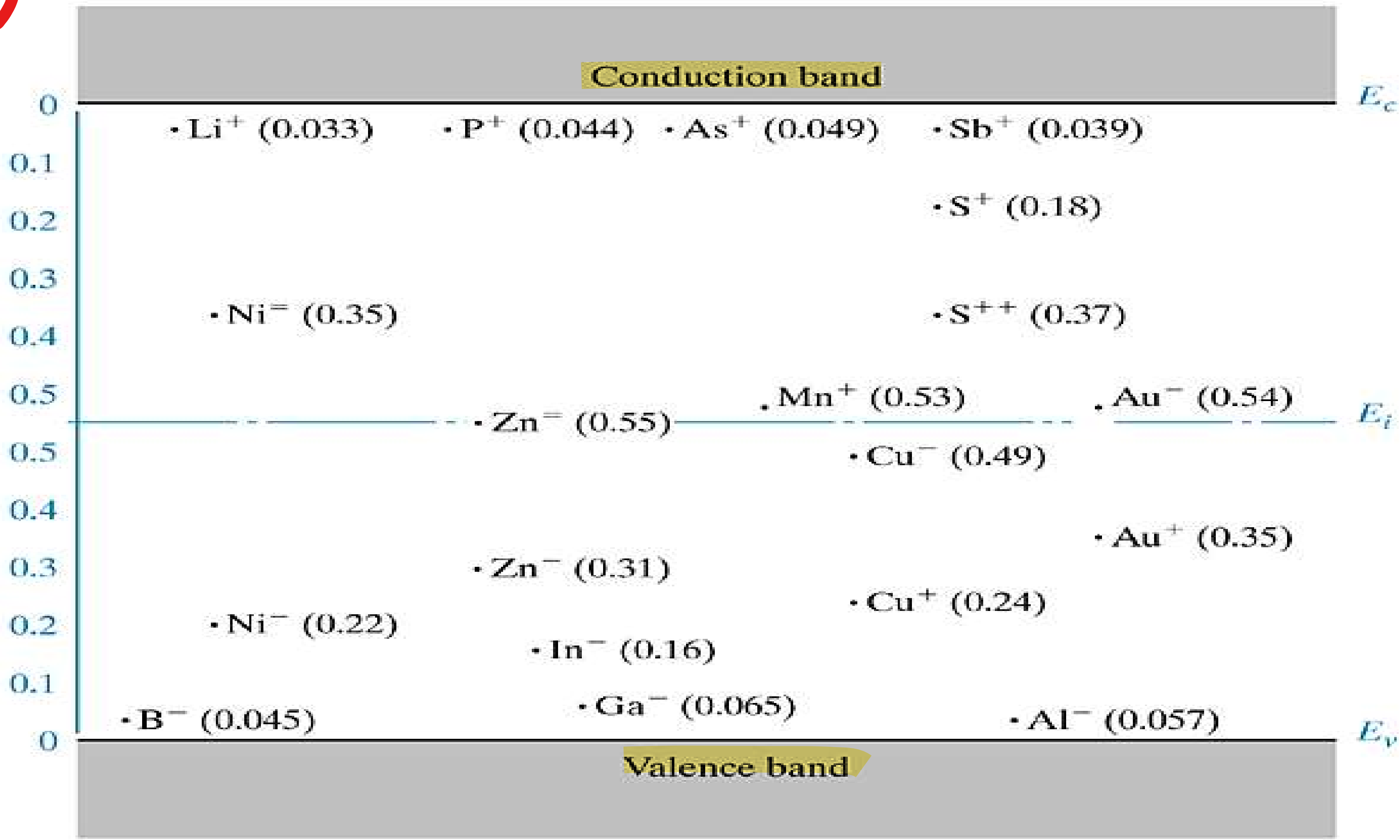
Question 0



Real impurities: shallow vs. deep, donor/acceptor vs. trap

Schrödinger Wave Equation

Plane wave amplitude

$$\Psi \sim \exp[i(k\,x - \omega\,t)]$$

Momentum

$$p = \frac{h}{\lambda} = \frac{2\pi\hbar}{2\pi/k} = \hbar k$$

Energy

$$E = h\nu = \hbar\omega$$

$$-i\hbar\frac{\partial\Psi}{\partial x} = (\hbar k)\Psi$$

$$i\hbar\frac{\partial\Psi}{\partial t} = (\hbar\omega)\Psi$$

$$H = \frac{p^2}{2m} + V$$

$$p \rightarrow -i\hbar\frac{\partial}{\partial x}$$

$$E \rightarrow i\hbar\frac{\partial}{\partial t} \cong H$$

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

$$H\psi = E\psi$$

Time-dependent Schrodinger Equation

Time-independent Schrodinger Equation

Solutions to the Schrödinger Equation

$$H_n \psi_n = E_n \psi_n \qquad \Psi = \psi_n(x) e^{-iE_n t/\hbar} \to \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar}$$

Probability of finding particle in the neighborhood of x:

$$d^3x \; |\psi_n|^2 = d^3x \; \psi_n^* \psi_n$$

Orthonormal basis

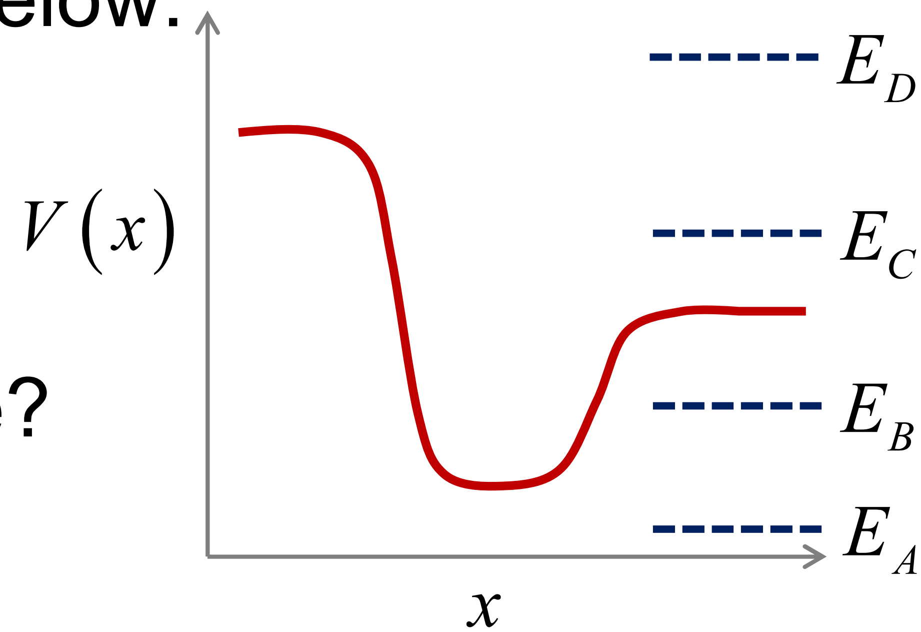
$$\int_x d^3x \; \psi_m^* \psi_n = \delta_{mn} = \begin{matrix} 0 \Leftarrow m \neq n \\ 1 \Leftarrow m = n \end{matrix}$$

Expectation value of operator A (e.g. p, $p^2/2m + V$) in state ψ_n

$$\int_x d^3x \; \psi_n^* \hat{A} \psi_n$$

Question 1

Consider the potential $V(x)$ and the states A, B, C and D whose energies are shown below:



Which of the following is false?

a. State A cannot exist.

b. State B is a bound state.

c. State C is a bound state.

d. State D is a scattering state.

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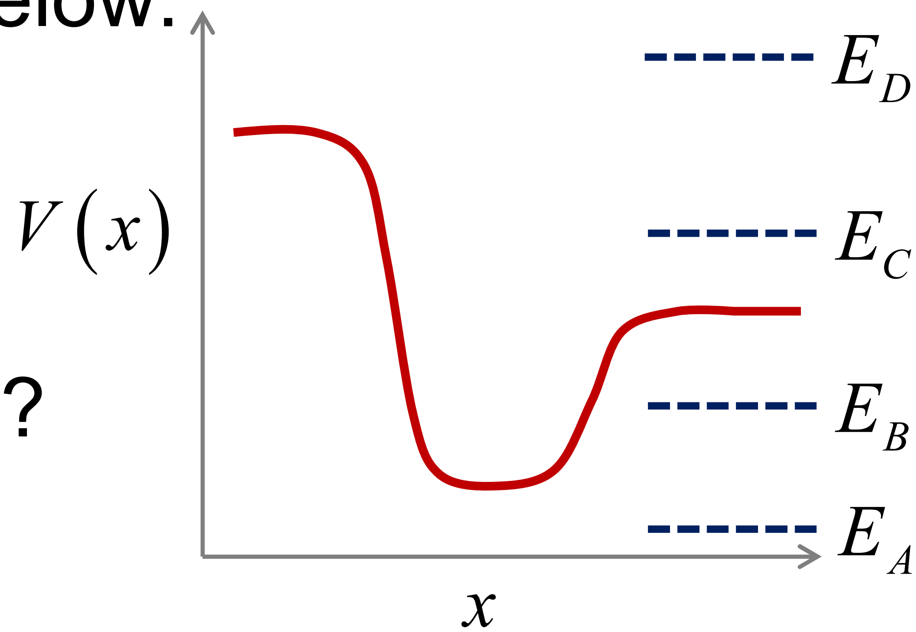
Consider the potential $V(x)$ and the states A, B, C and D whose energies are shown below:



Which of the following is false?

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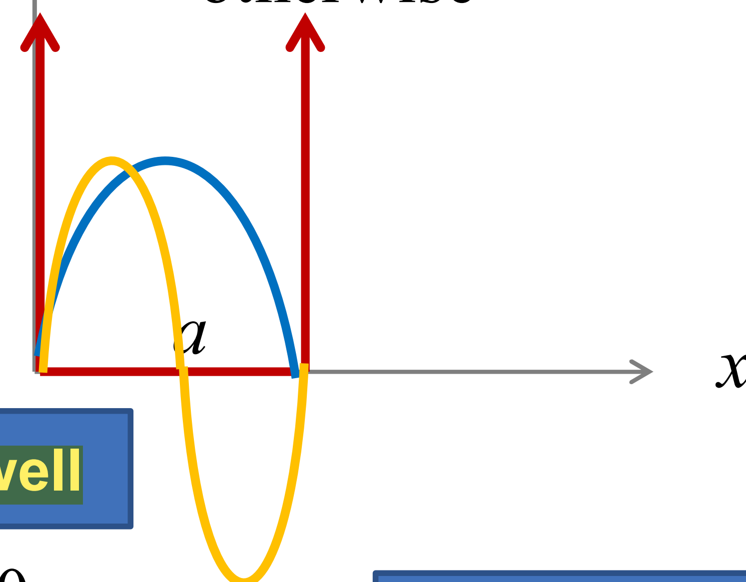
The infinite rectangular well

$$H\psi = E\psi$$

$$\Rightarrow \left(\frac{p^2}{2m} + V\right)\psi = E\psi$$

$$\Rightarrow \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

$$V(x) = \begin{matrix} 0 \Leftarrow 0 \le x \le a \\ \infty \text{ otherwise} \end{matrix}$$



Inside well

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \Rightarrow \frac{d^2\psi}{dx^2} = -k^2\psi$$

$$\Rightarrow \psi = A\sin(kx) + B\cos(kx),\ k = \sqrt{2mE/\hbar^2}$$

Outside well

$$\psi(x) = 0$$

Boundary condition: ψ is continuous

$$\psi(0) = 0 \qquad \psi(a) = 0$$

$$\Rightarrow B = 0 \qquad \Rightarrow ka = n\pi$$

$$\boxed{\begin{aligned} \psi_n &= A\sin\left(\frac{n\pi x}{a}\right) \\ E_n &= \frac{\hbar^2 n^2 \pi^2}{2ma^2} \end{aligned}}$$

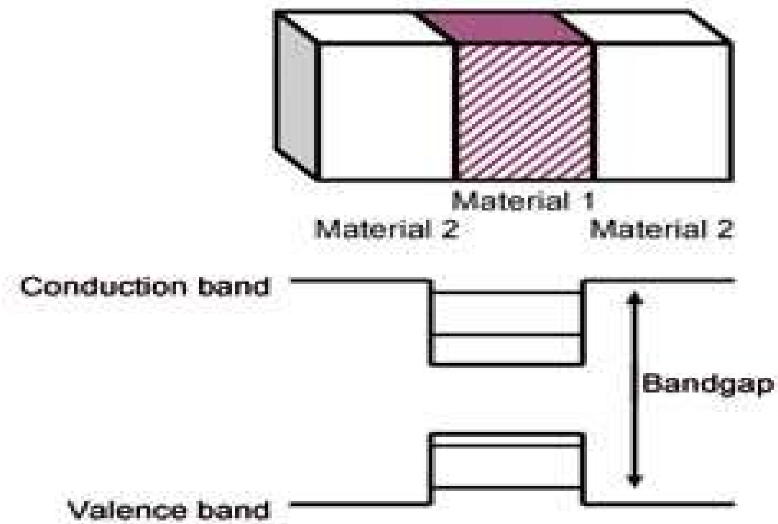
Normalization

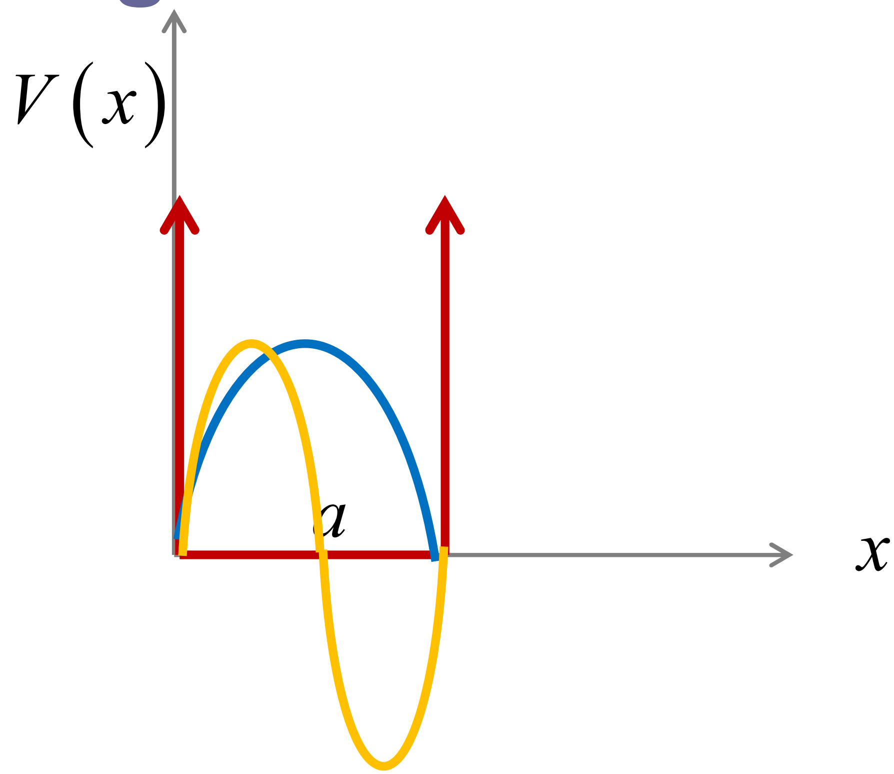
$$\int_{-\infty}^{\infty} dx\left|\psi_n\right|^2 = 1$$

$$\int_{-\infty}^{\infty} dx\left|A\right|^2 \sin^2\left(\frac{n\pi x}{a}\right) = 1$$

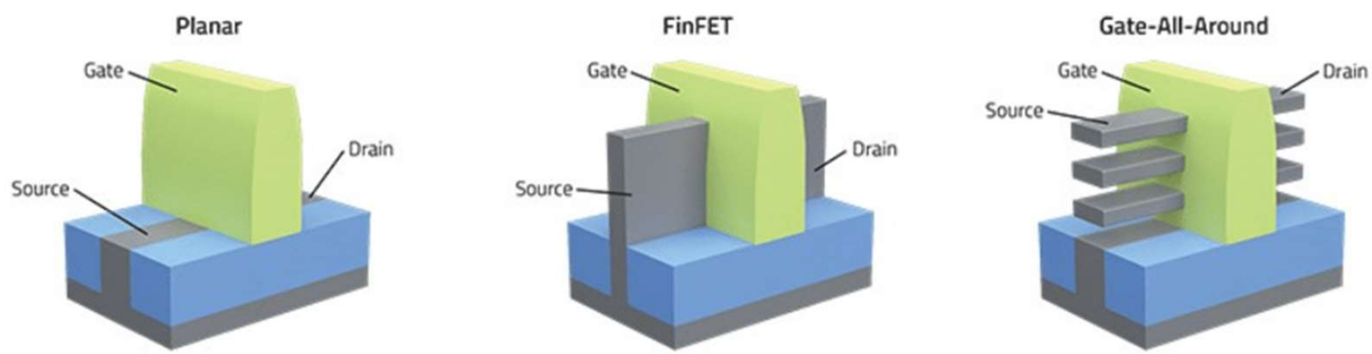
$$\Rightarrow A = \sqrt{2/a}$$

The infinite rectangular well





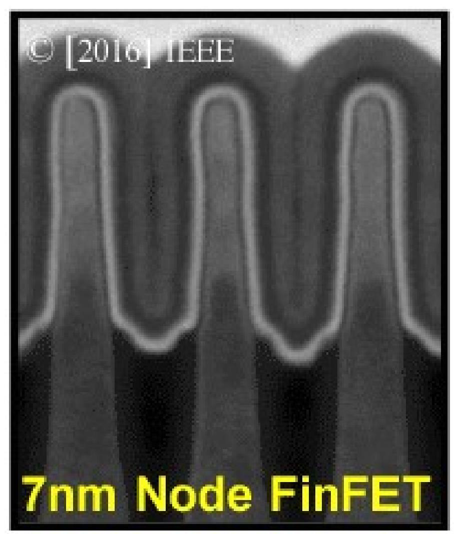
Quantization or Quantum Confinement: MOSFET, QW Laser…

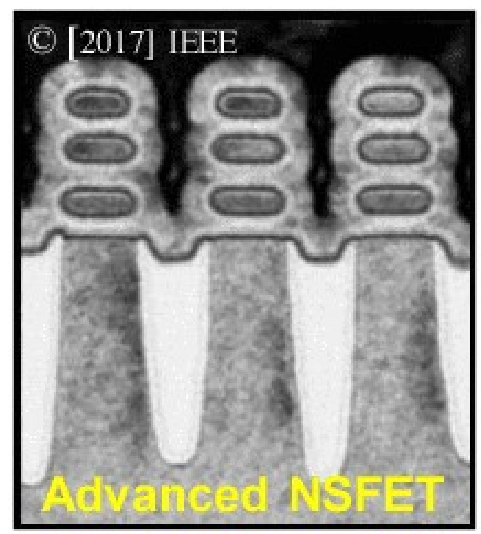


Question 2

Consider electrons at the bottom of the conduction band in a semiconductor fin (see figure) at room temperature. Assume $m^* = m_0$ for this semiconductor. The fin thickness for which one needs to consider quantum confinement of electrons in the fin is less than about:

a. 10 Å
b. 100 Å
c. 1000 Å
d. Never





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d. Never

$$E - E_C = \frac{p^2}{2m^*} \sim k_B T$$

$$\Rightarrow p = \sqrt{2m^* k_B T}$$

$$p = \hbar k = \frac{h}{\lambda}$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{2m^* k_B T}} = 7.6nm$$

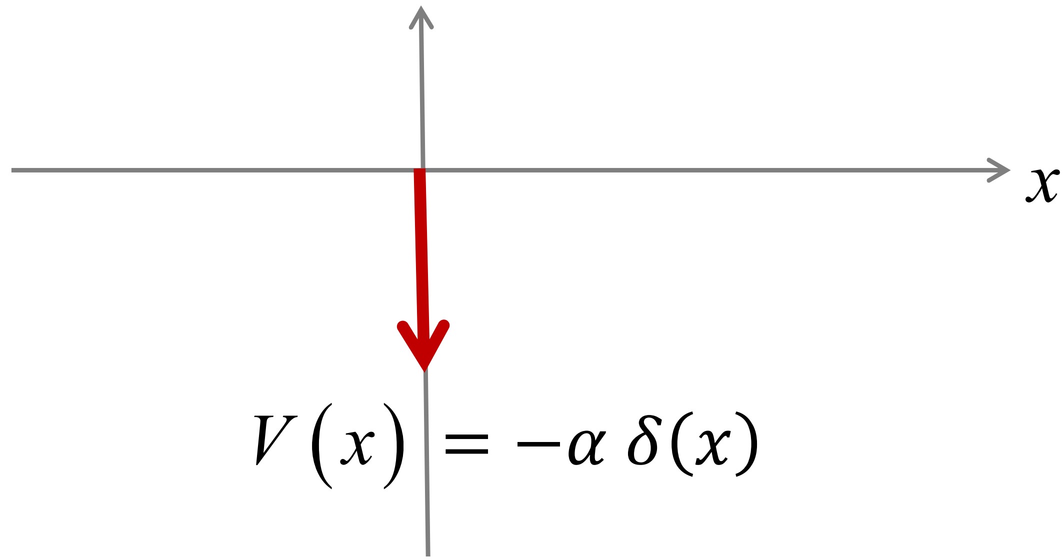
The delta-function barrier

Delta-function

$$\delta(x) = 0 \Leftarrow x \neq 0$$

$$\int_{-\infty}^{+\infty} \delta(x)\, dx = 1$$

$$\Rightarrow \int_{-\infty}^{+\infty} f(x+a)\,\delta(x)\, dx = f(a)$$



$$V(x) = -\alpha\, \delta(x)$$

x < 0

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \Rightarrow \frac{d^2\psi}{dx^2} = \kappa^2\psi \qquad \Rightarrow \psi = C_- e^{+\kappa x}$$

$$\kappa = \left(\frac{2m(-E)}{\hbar^2}\right)$$

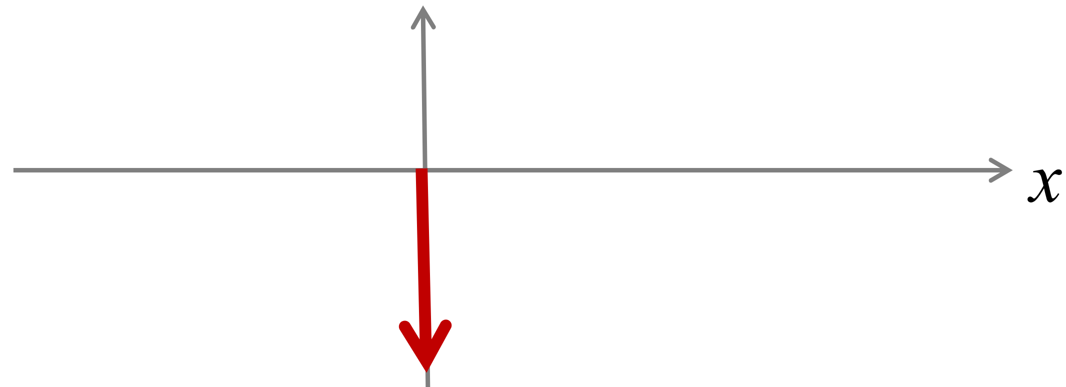
x > 0

$$\psi = C_+ e^{-\kappa x}$$

Boundary condition:
ψ is continuous

$$C_- = C_+ = \psi(0)$$

The delta-function barrier



$$V(x) = -\alpha\,\delta(x)$$

x < 0

$$\psi = \psi(0)e^{+\kappa x}$$

x > 0

$$\psi = \psi(0)e^{-\kappa x}$$

Boundary condition:
ψ' is continuous
unless V is ∞

$$\int_{0^-}^{0^+} dx\left[\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} - \alpha\cdot\delta(x)\psi\right] = \int_{0^-}^{0^+} dx\cdot E\psi$$

$$\Rightarrow \frac{-\hbar^2}{2m}\left[\frac{d\psi}{dx}\right]_{0^-}^{0^+} - \alpha\cdot\psi(0) = 0 \Rightarrow \Delta\psi'(0) = -\frac{2m\alpha}{\hbar^2}\psi(0)$$

$$\Delta\psi'(0) = -\kappa\psi(0) - \kappa\psi(0) = -2\kappa\psi(0) = -\frac{2m\alpha}{\hbar^2}\psi(0) \Rightarrow \kappa = \frac{m\alpha}{\hbar^2}$$

$$\Rightarrow -\frac{2mE}{\hbar^2} = \left(\frac{m\alpha}{\hbar^2}\right)^2 \Rightarrow E = -\frac{m\alpha^2}{2\hbar^2}$$

Finis

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