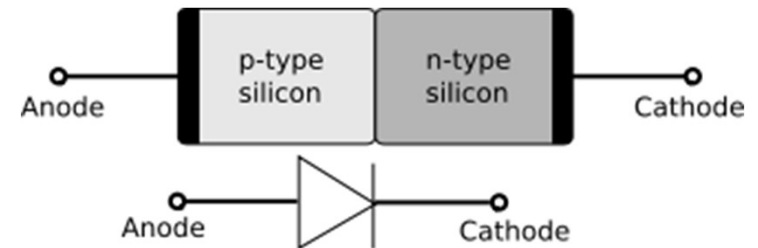


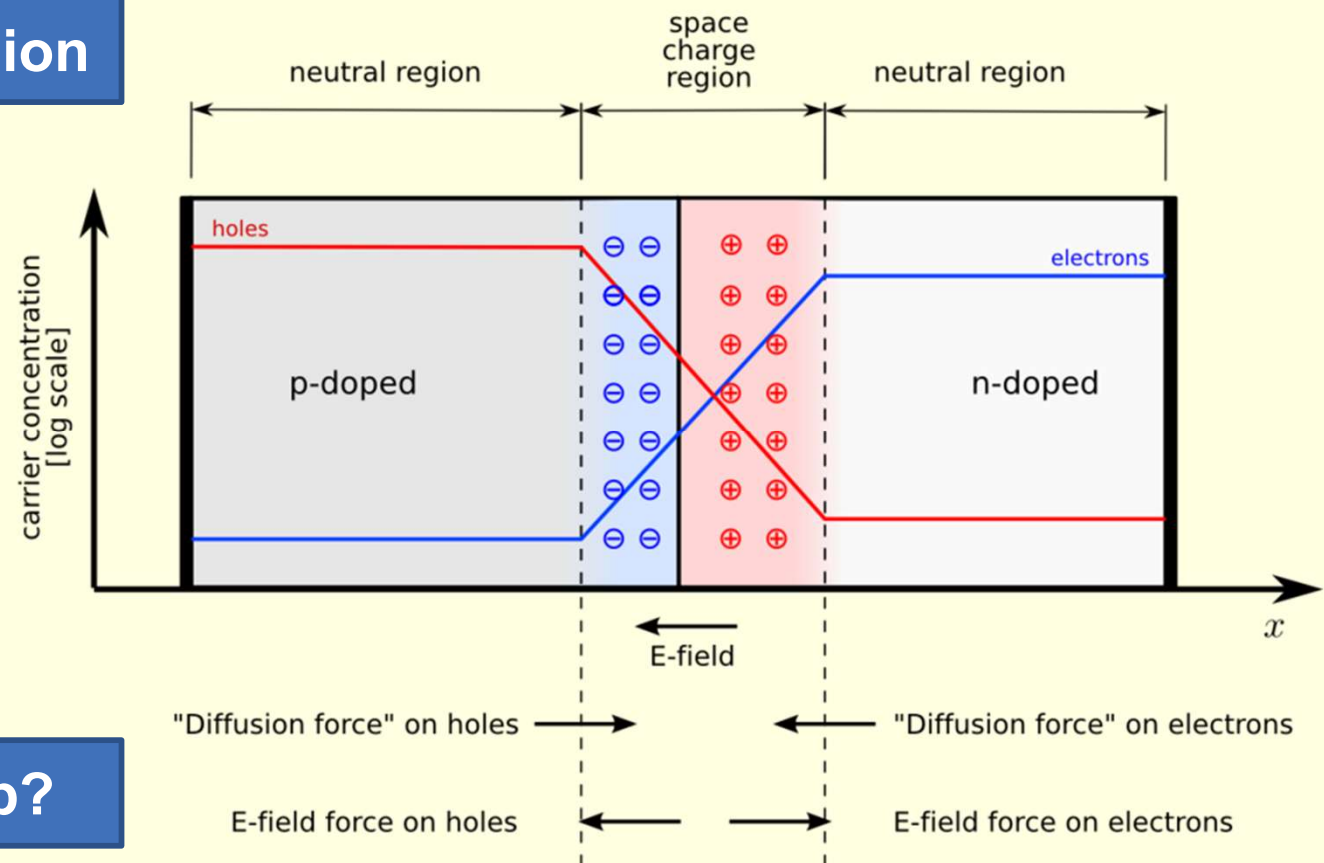
Idealized p-n junction diode



**Ideal approximation:
1-D, abrupt**

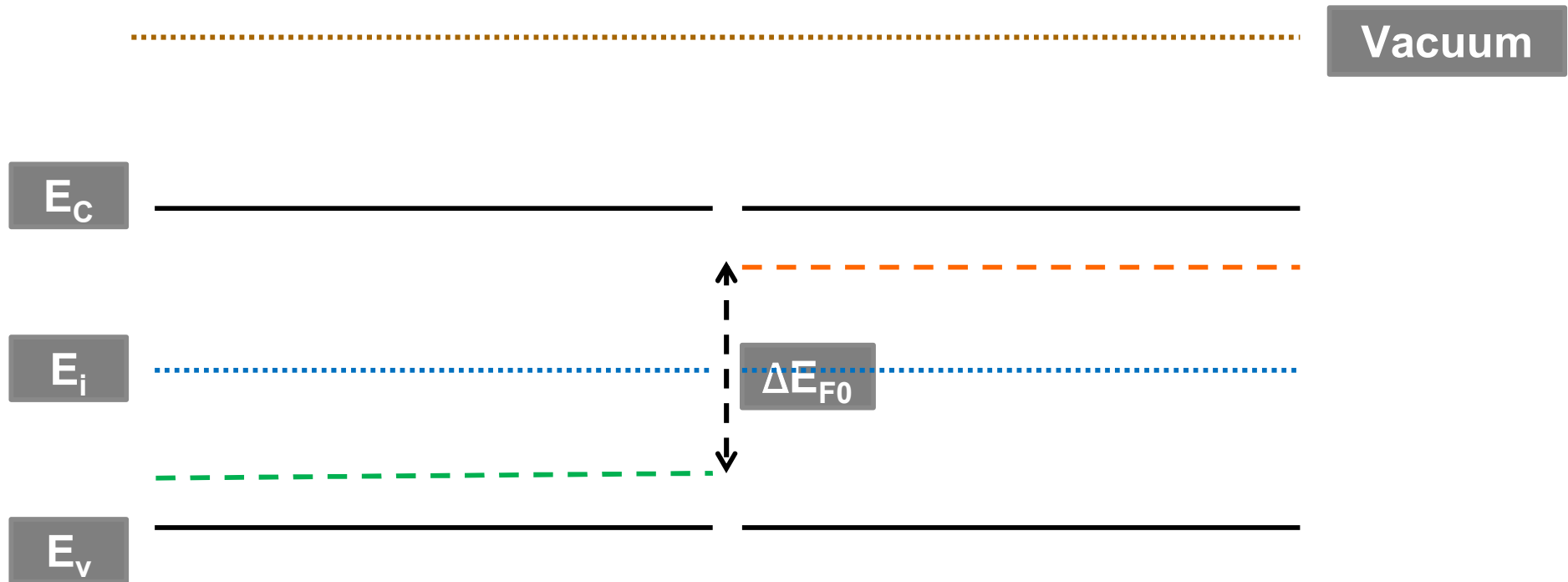


Inhomogeneity → diffusion



Why/where does it stop?

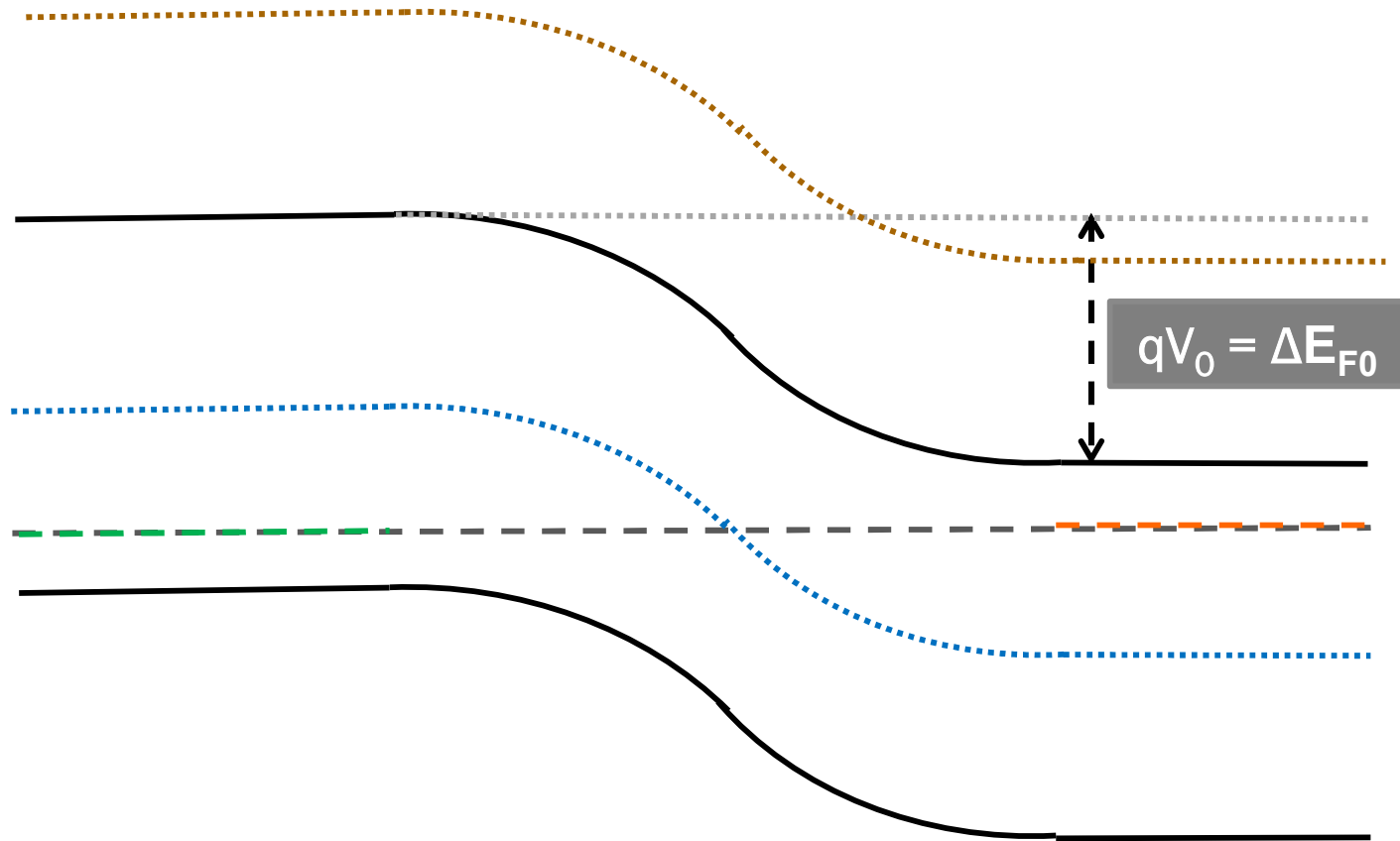
Equilibrium barrier height...



$$E_{Fp} = E_i - \ln\left(\frac{p_{p0}}{n_i}\right) = E_i - k_B T \ln\left(\frac{N_A}{n_i}\right) \quad E_{Fn} = E_i + \ln\left(\frac{n_{n0}}{n_i}\right) = E_i + k_B T \ln\left(\frac{N_D}{n_i}\right)$$

$$\Delta E_{F0} = E_{Fn} - E_{Fp} = k_B T \ln\left(\frac{N_D}{n_i}\right) + k_B T \ln\left(\frac{N_A}{n_i}\right)$$

...from band diagram



$$V_0 = \frac{k_B T}{q} \ln \left(\frac{N_D}{n_i} \right) + \frac{k_B T}{q} \ln \left(\frac{N_A}{n_i} \right)$$

**Contact potential /
built-in voltage**

$$V_0 = \frac{k_B T}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

Hetero-junction?

Equilibrium with potentials/band-bending

Equilibrium, homogeneous

$$n_0 = n_i \exp\left(\frac{E_F - E_{i0}}{k_B T}\right) \qquad p_0 = n_i \exp\left(\frac{E_{i0} - E_F}{k_B T}\right)$$

Equilibrium, inhomogeneous

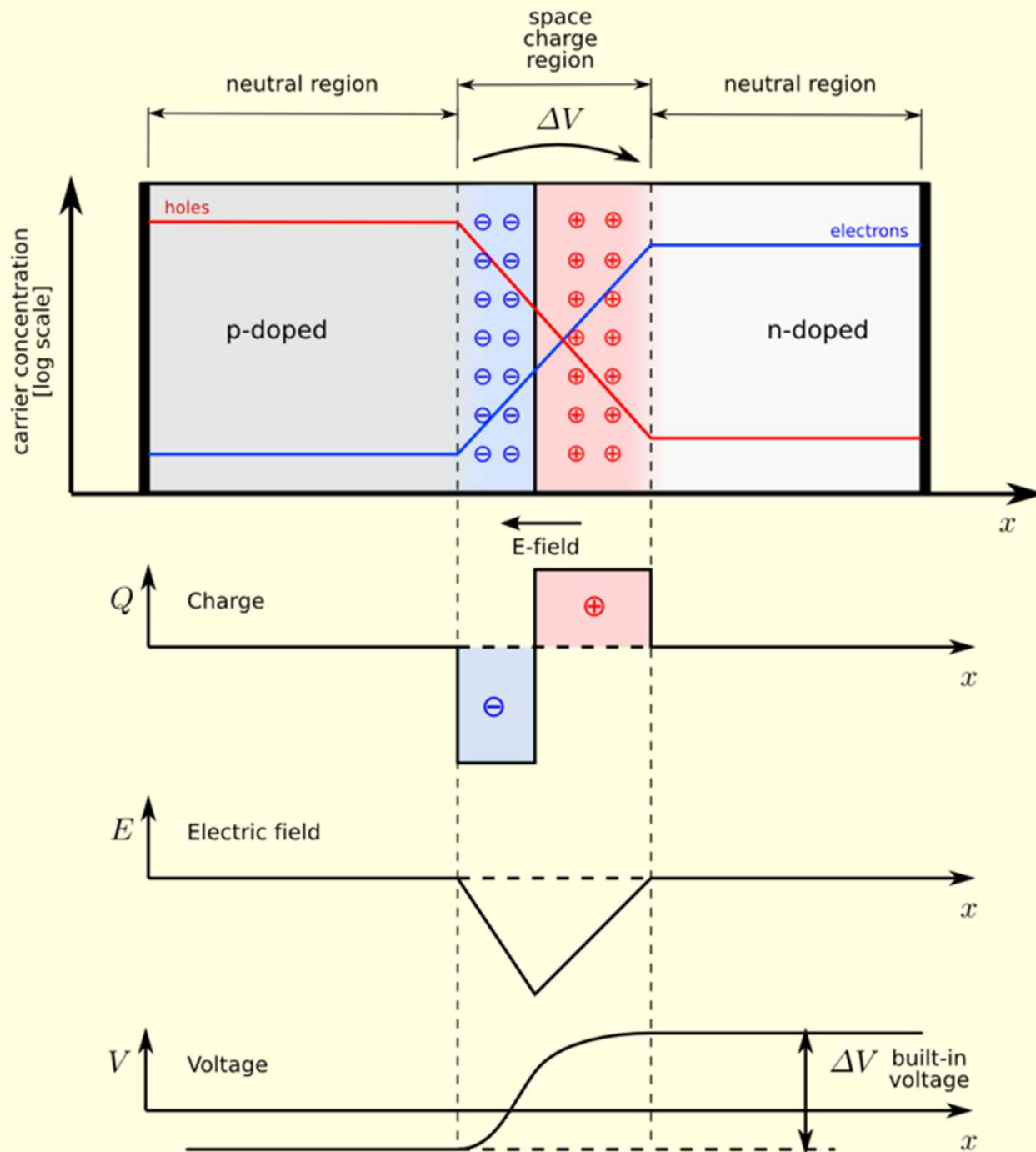
$$n = n_i \exp\left(\frac{E_F - E_{i0} + eV}{k_B T}\right) = n_0 \exp\left(\frac{+eV}{k_B T}\right) \qquad E_i = E_{i0} - eV$$

$$p = n_i \exp\left(\frac{E_{i0} - E_F - eV}{k_B T}\right) = p_0 \exp\left(\frac{-eV}{k_B T}\right)$$

Drift = Diffusion?

Contact potential?

Barrier width – electrostatics



Charge density

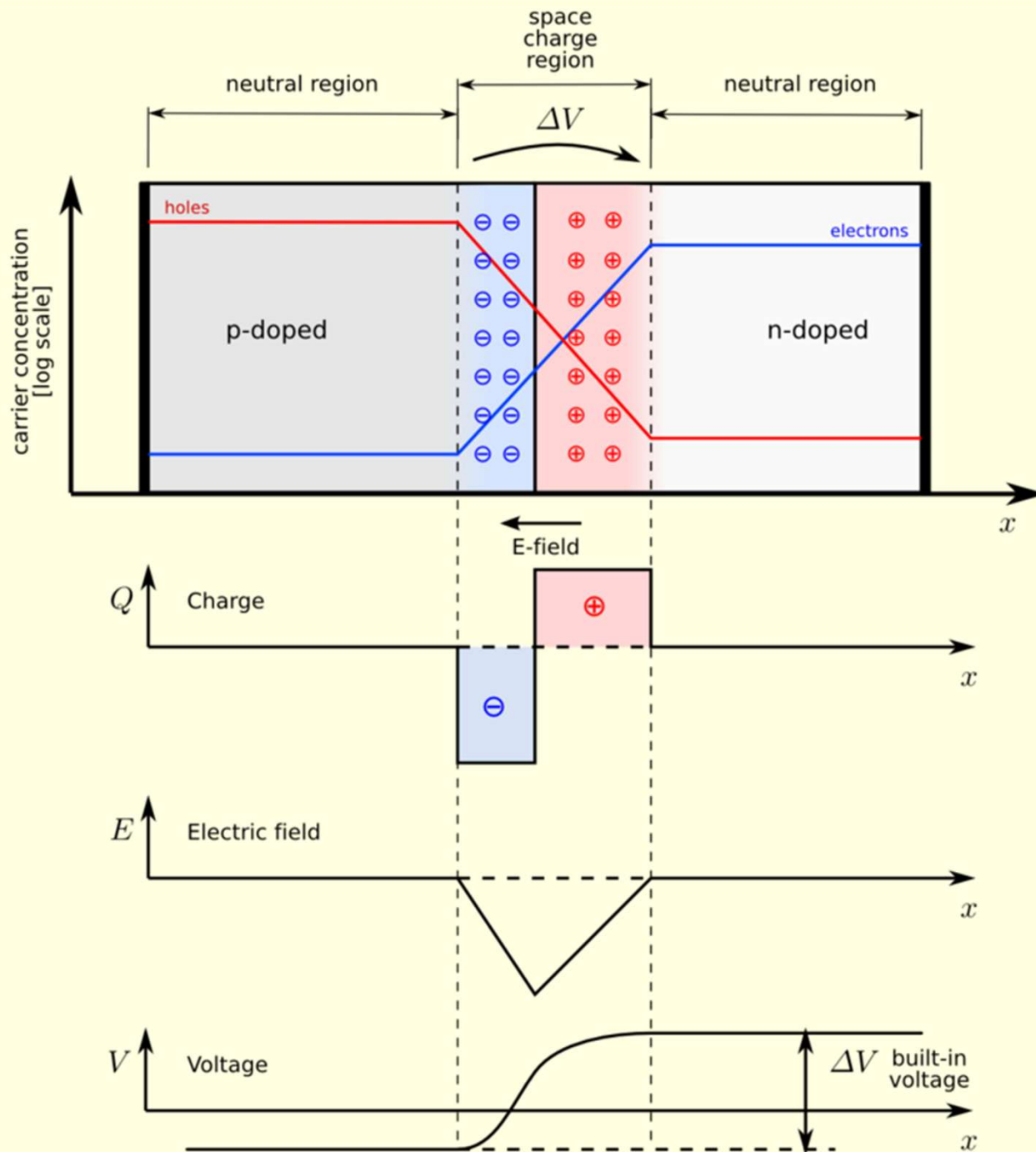
$$\begin{aligned} -eN_A, -x_p \leq x \leq 0 \\ \rho(x) = +eN_D, +x_n \geq x \geq 0 \\ 0, x \leq -x_p \cup x \geq x_n \end{aligned}$$

Depletion approximation

$$N_A x_p = N_D x_n$$

Charge neutrality

Barrier width – electrostatics



Electric field

$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon}$$

Gauss' Law

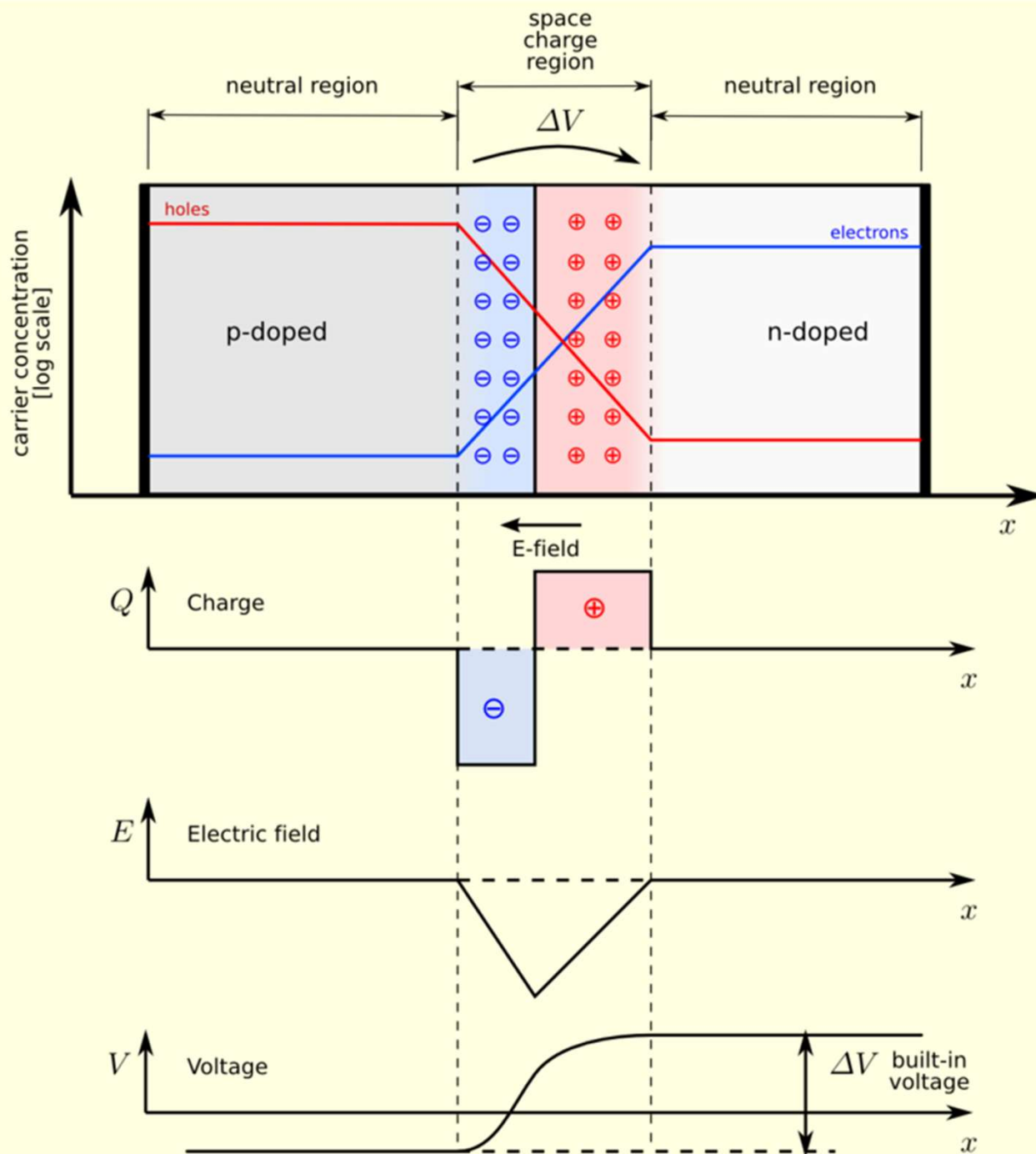
Boundary condition?

$$\rho(x) = \begin{cases} -eN_A, & -x_p \leq x \leq 0 \\ +eN_D, & +x_n \leq x \leq 0 \end{cases}$$

$$E = \begin{cases} -eN_A(x + x_p)/\epsilon \\ eN_D(x - x_n)/\epsilon \end{cases}$$

$$E_m = \frac{-eN_A x_p}{\epsilon} = \frac{-eN_D x_n}{\epsilon}$$

Barrier width – electrostatics



Electrostatic potential

$$E = -\frac{dV}{dx}$$

$$E_m = \frac{-eN_A x_p}{\epsilon} = \frac{-eN_D x_n}{\epsilon}$$

$$x_p = \frac{-\epsilon E_m}{eN_A}; \quad x_n = \frac{-\epsilon E_m}{eN_D}$$

$$V_0 = \frac{(-E_m) \cdot W}{2} = \frac{(-E_m) \cdot (x_p + x_n)}{2}$$

$$W = \sqrt{\frac{2\epsilon V_0}{e} \left(\frac{1}{N_A} + \frac{1}{N_D} \right)}$$

Depletion layer width

Depletion layer width

Equilibrium

$$W = \sqrt{\frac{2\epsilon V_0}{e} \left(\frac{1}{N_A} + \frac{1}{N_D} \right)}$$

One-sided junction?

Voltage drop?

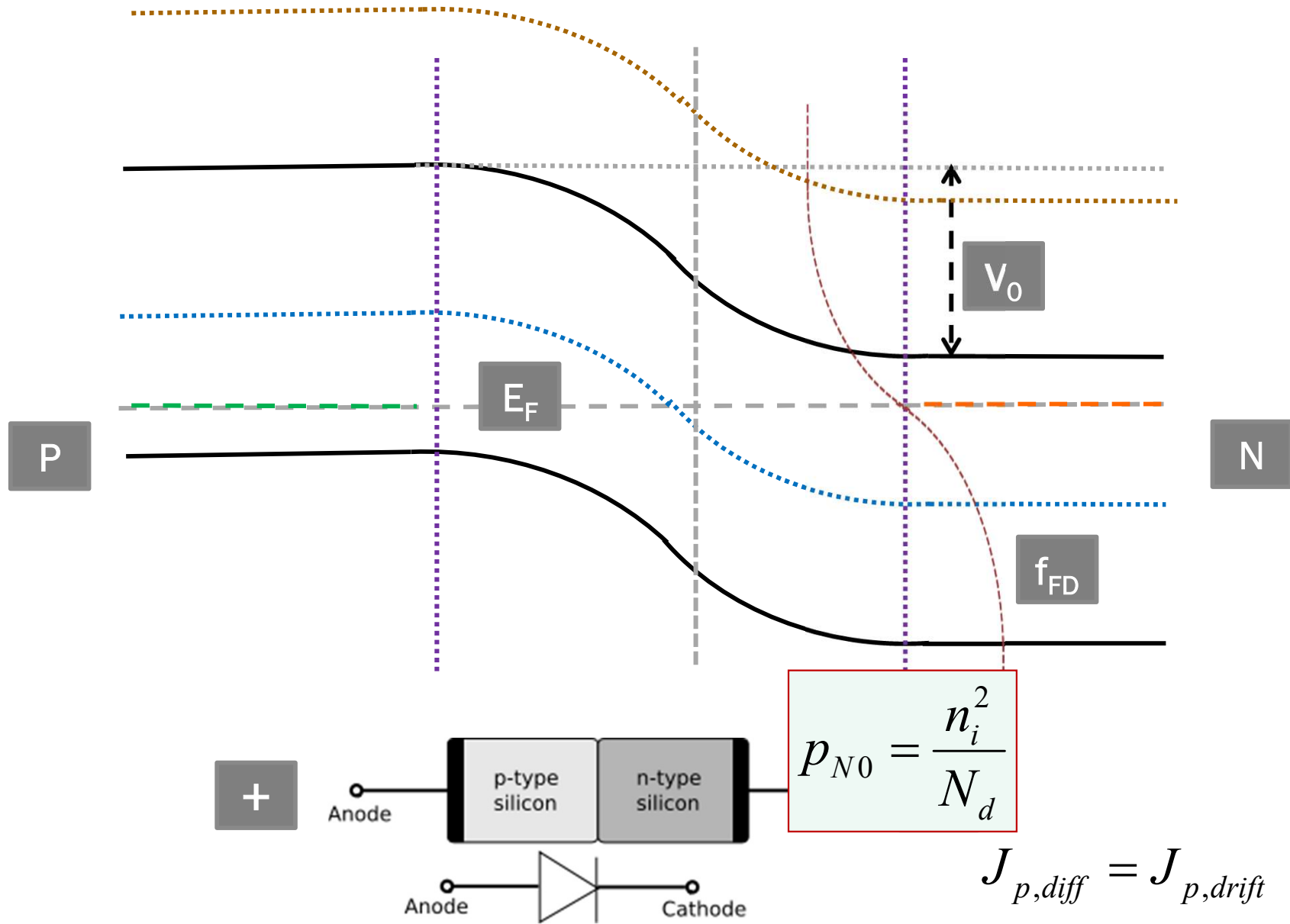
With bias

$$W = \sqrt{\frac{2\epsilon (V_0 - V)}{e} \left(\frac{1}{N_A} + \frac{1}{N_D} \right)}$$

Forward/Reverse?

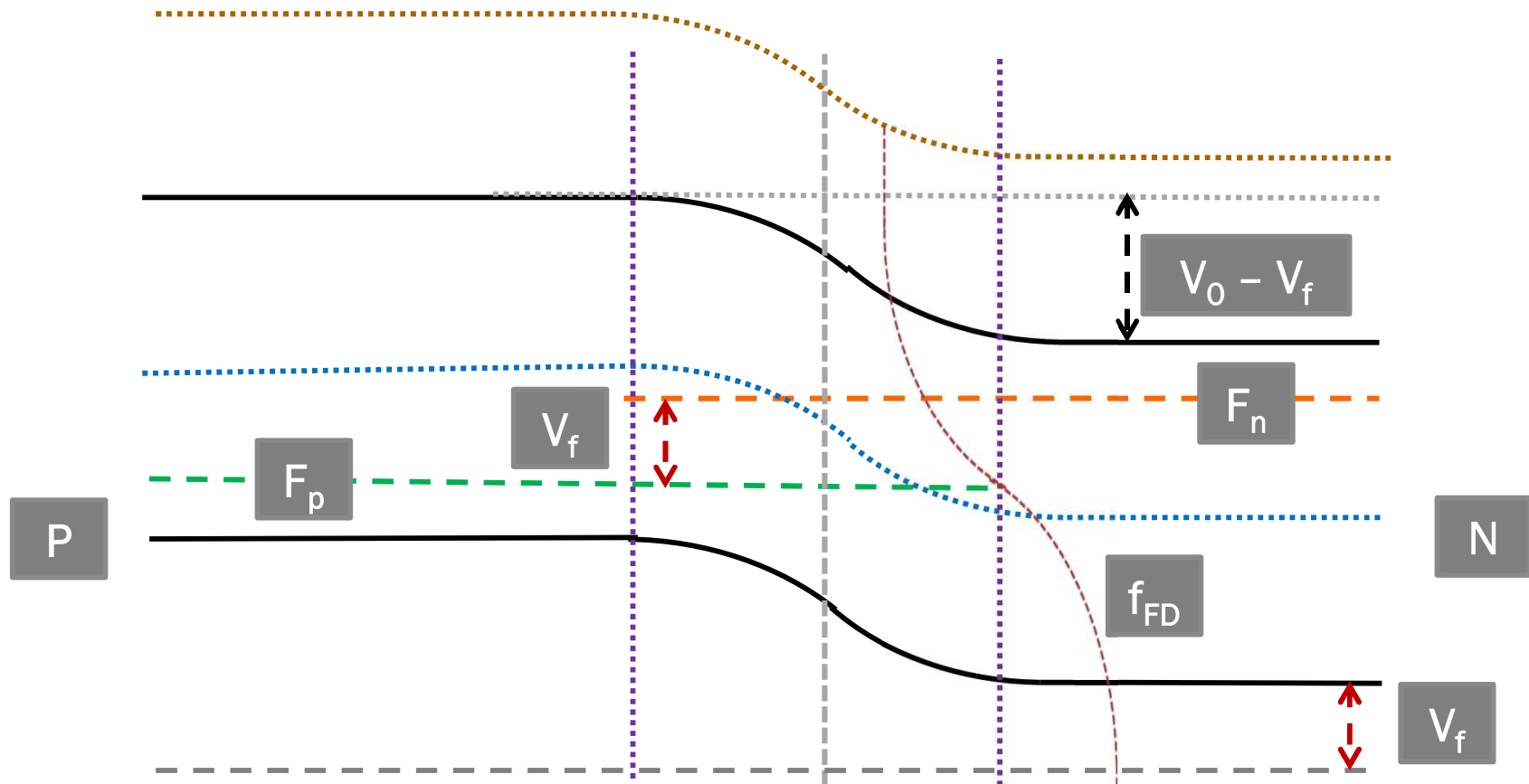
Fermi level?

P-N junction in equilibrium



What happens under forward bias?

P-N junction in forward-bias



$$\frac{p(x_N)}{p_{N0}} = \exp(qV_f/k_B T)$$

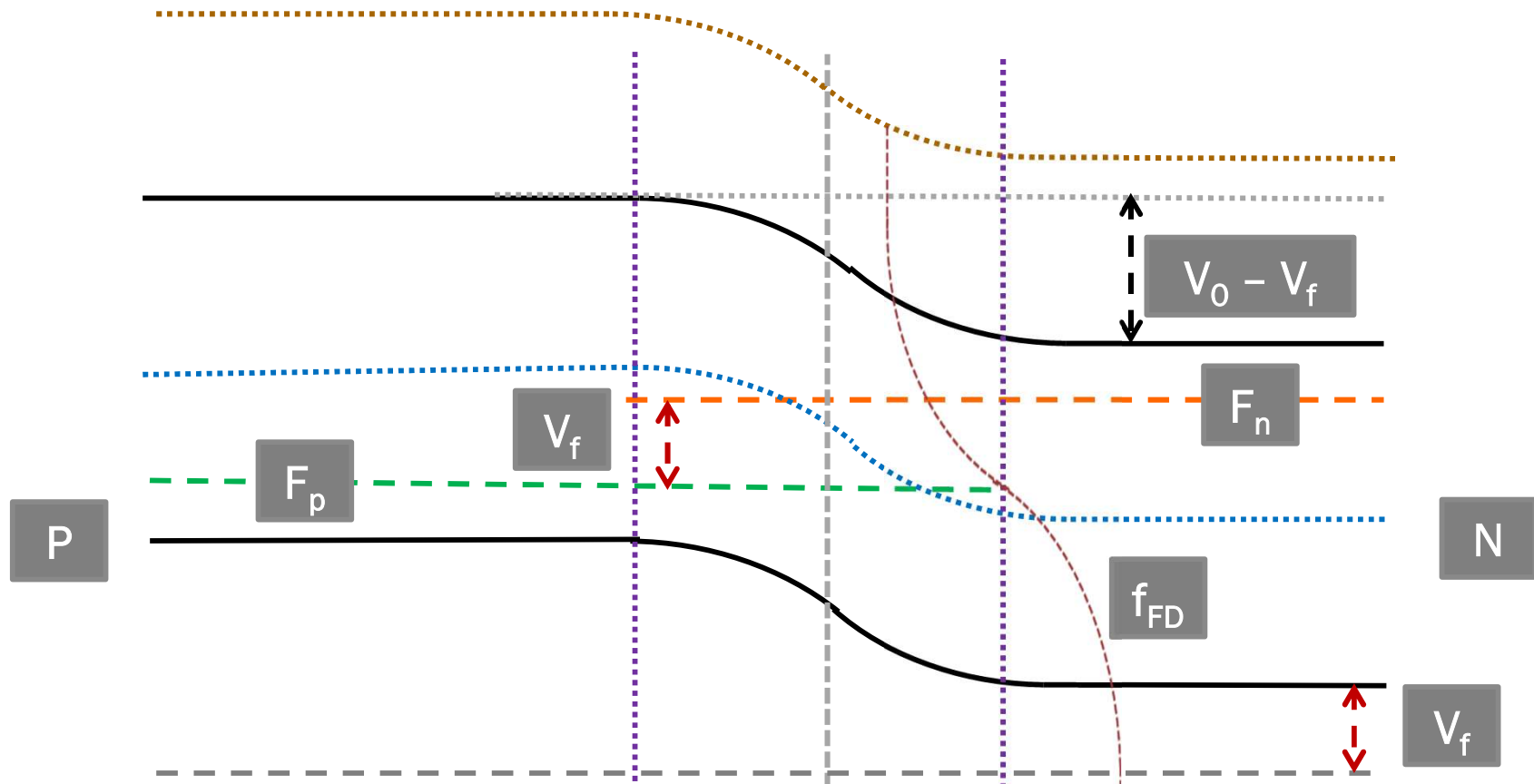
$$J_{p,diff} \gg J_{p,drift}$$

$$p(x_N) = N_v \exp\left[\left(E_v(x_N) - F_p\right)/k_B T\right]$$

$$p(x_N) = N_v \exp\left[\left(E_{v0} + qV_f - F_p\right)/k_B T\right]$$

$$p(x_N) = N_v \exp\left[\left(E_{v0} - E_F\right)/k_B T\right] \cdot \exp\left(+qV_f/k_B T\right)$$

P-N junction in forward-bias



$$\frac{n(x_P)}{n_{P0}} = \exp(qV_f/k_B T) = \frac{p(x_N)}{p_{N0}}$$

$$\Delta p(x_N) = p_{N0} \left[\exp(qV_f/k_B T) - 1 \right]$$

Minority carrier injection ← Boltzmann statistics

Forward-bias current (steady-state)

$$\frac{n_i^2}{N_a} \left(e^{qV_f/k_B T} - 1 \right) = n_{p0} \left(e^{qV_f/k_B T} - 1 \right) = \Delta n(x_P)$$

P

$$W_P \gg \lambda_n = \sqrt{D_n \tau_n}$$

$$J_{n,diff}(x_P) = \frac{qD_n}{\lambda_n} \frac{n_i^2}{N_a} \left(e^{qV_f/k_B T} - 1 \right)$$

$$\begin{aligned} \Delta p(x_N) &= p_{N0} \left(e^{qV_f/k_B T} - 1 \right) \\ &= \frac{n_i^2}{N_d} \left(e^{qV_f/k_B T} - 1 \right) \end{aligned}$$

N

$$W_N \gg \lambda_p = \sqrt{D_p \tau_p}$$

$$\Delta p(x) = \Delta p(x_N) e^{-(x-x_N)/\lambda_p}$$

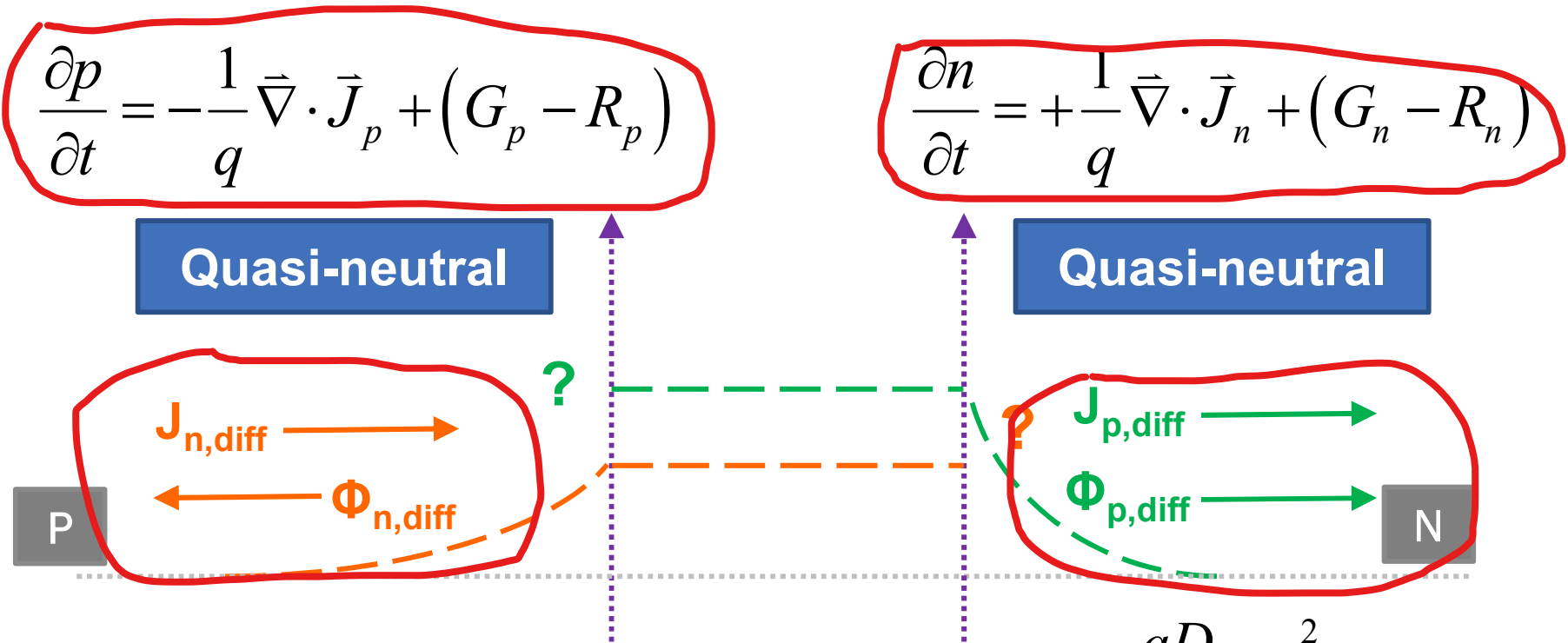
$$J_{p,diff}(x_N) = -qD_p \left. \frac{dp}{dx} \right|_{x=x_N} = \frac{qD_p}{\lambda_p} \Delta p(x_N)$$

Continuity → J is constant (?)

How to sum up its components?

$$J_{p,diff}(x_N) = \frac{qD_p}{\lambda_p} \frac{n_i^2}{N_d} \left(e^{qV_f/k_B T} - 1 \right)$$

Forward-bias current (steady-state)



Minority carriers:

$$J_{diff} \ll J_{drift}$$

Total current

$$J = J_{p,diff}(x_N) + J_{n,diff}(x_P)$$

$$J_{p,diff}(x_N) = \frac{qD_p}{\lambda_p} \frac{n_i^2}{N_d} (e^{qV/k_B T} - 1)$$

$$J_{p,diff}(x) = \frac{qD_p}{\lambda_p} \frac{n_i^2}{N_d} (e^{qV/k_B T} - 1) e^{-(x-x_N)/\lambda_p}$$

$$J = \left(\frac{qD_p}{\lambda_p} \frac{n_i^2}{N_d} + \frac{qD_n}{\lambda_n} \frac{n_i^2}{N_a} \right) (e^{qV/k_B T} - 1) = J_0 (e^{qV/k_B T} - 1)$$

Narrow-base?

Reverse-bias current (steady-state)

Reverse-bias band-diagram?



$$J = J_0 \left(e^{qV/k_B T} - 1 \right) \xrightarrow{V < 0} -J_0$$

Why is the reverse current bias-independent?

Small-signal response

Charge stored in (nonlinear) element as function of voltage

Taylor Series

$$Q(V) = Q(V_0 + v) = Q(V_0) + v \cdot \left. \frac{dQ}{dV} \right|_{V=V_0} + \dots$$

DC bias + AC *small-signal*

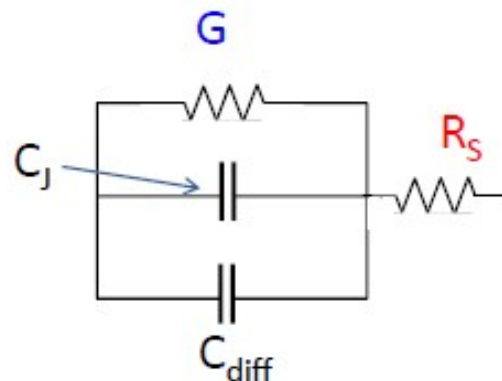
$$q = Q(V_0 + v) - Q(V_0) = v \cdot \left. \frac{dQ}{dV} \right|_{V=V_0} = C(V_0) \cdot v$$

Small-signal response: linearization; DC property

Small-signal
capacitance

What about current as a function of voltage?

Diode small-signal model



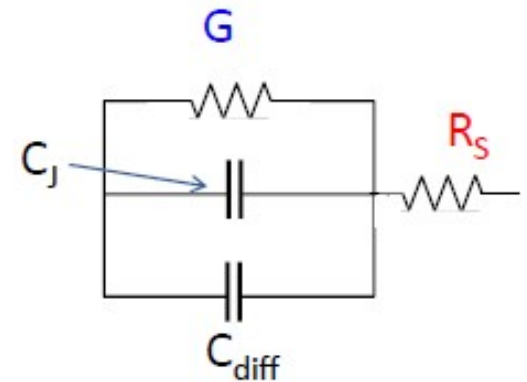
Forward-bias conductance

Diode current

$$I = I_0 \left[e^{\beta q (V - R_S I) / m} - 1 \right]$$

m: ideality factor
(G-R, high-injection)

$$V = \frac{m}{\beta q} \ln \left(\frac{I + I_0}{I_0} \right) + R_S I$$



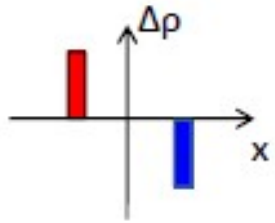
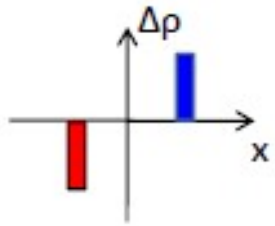
Small-signal resistance

$$\frac{dV}{dI} = \frac{m}{\beta q (I + I_0)} + R_S = \frac{1}{G} + R_S$$

Where is the bias-dependence?

Junction capacitance

Reverse-bias depletion width



$$\delta V = \frac{q}{\varepsilon} \frac{N_a N_d}{N_a + N_d} W \delta W \quad \Leftarrow (V_0 + V) = \frac{q}{2\varepsilon} \frac{N_a N_d}{N_a + N_d} W^2$$

$$\frac{N_d \delta W}{N_a + N_d} = \frac{N_d (\delta x_n + \delta x_p)}{N_a + N_d} = \frac{N_a \delta x_p + N_d \delta x_p}{N_a + N_d} = \delta x_p$$

$$\delta V = \frac{W}{\varepsilon} q N_a \delta x_p = \frac{W}{\varepsilon} \delta Q$$

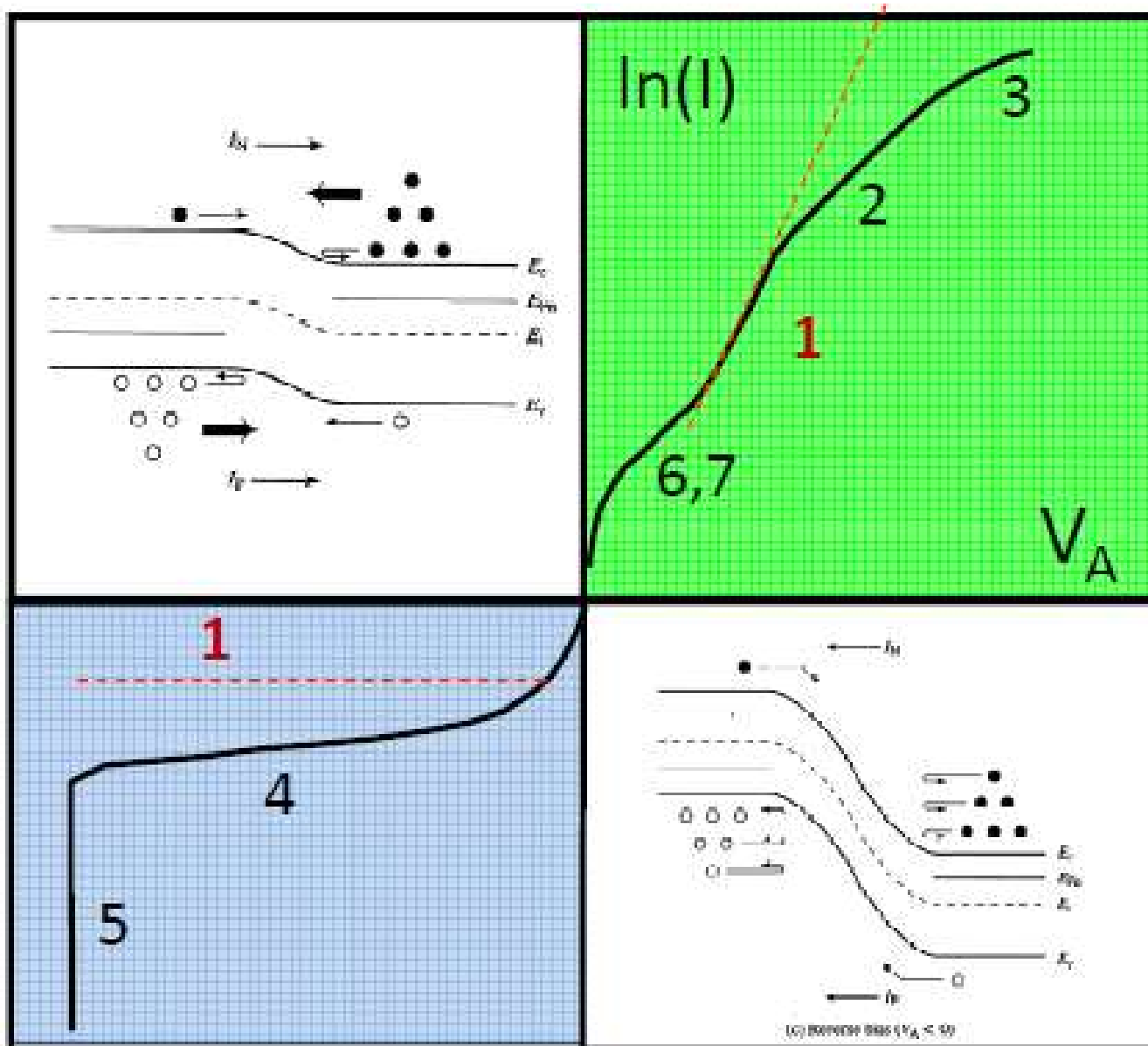
$$C_j = \frac{\varepsilon}{W}$$

Bias-dependent/non-linear

Majority or minority response?

Parallel-plate! Why/how?

P-N diode: regimes of I-V



1. Ideal

2. High-injection

3. Ohmic

4. G-R in depletion

5. Breakdown

6. G-R in depletion

7. Inter-band tunneling

Finis

Artwork Sources:

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