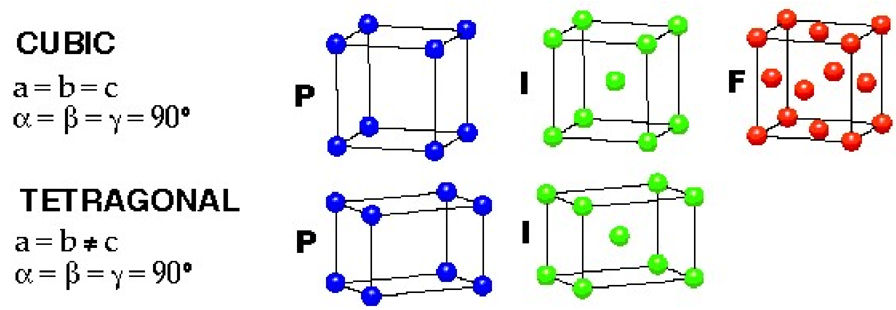
Spring 2024-25 EE207 HW-1

1. A tetragonal lattice is produced by reducing the symmetry of a cubic lattice as shown below ($c \neq a$). That is, by taking a cubic lattice and stretching it along one direction, namely the c - axis. Here I denotes body-centered, and F denotes face-centered lattices.



(a) Obviously, the body-centered tetragonal (BCT) may be produced by stretching the BCC lattice along the c-axis. Perhaps not so obviously, it may also be produced by stretching the FCC lattice along the same direction. Show (with illustrations if you like) that the BCT lattice may be viewed as a stretched FCC. Show that for $c/a = \sqrt{2}$ it is identical to FCC. (Hint: look at neighboring planes perpendicular to the c-axis.)

(b) Sketch schematically the packing fraction for the BCT lattice as a function of c/a (for $c \geq a$), assuming close-packing.

2. A set of primitive vectors of the 3-D simple hexagonal lattice (viz. the figure below if you ignore layer B) are $\vec{a}_1 = \frac{a\sqrt{3}}{2}\hat{x} + \frac{a}{2}\hat{y}$, $\vec{a}_2 = -\frac{a\sqrt{3}}{2}\hat{x} + \frac{a}{2}\hat{y}$ and $\vec{a}_3 = c\hat{z}$. This means that $n_1\vec{a}_1 + n_2\vec{a}_2 + n_3\vec{a}_3$ with integral n_1, n_2, n_3 gives you all the points on this lattice.

Consider now the hexagonal close-packed (hcp) crystal structure illustrated below (including layer B now). It consists of two interpenetrating simple hexagonal lattices labeled A and B. For ideal 'close-packing', imagine hard spheres at every lattice point that just touch the neighboring spheres. What is the c/a ratio in this case? What happens if c/a is much greater than this value? Can you provide an example of this situation in nature?