EE 207 [Autumn 24-25] ***HW – 3***

1. The effective mass tensor is defined as $\overleftrightarrow{M}_{ij}^{-1} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon}{\partial k_j \partial k_i}$.

(a) Calculate it for electrons in a 2D material with the following dispersion relation. $E = E_C + \frac{\hbar^2 k_x^2}{2m_{11}} + \frac{\hbar^2 k_y^2}{2m_{22}} + \frac{\hbar^2 k_x k_y}{m_{12}}$

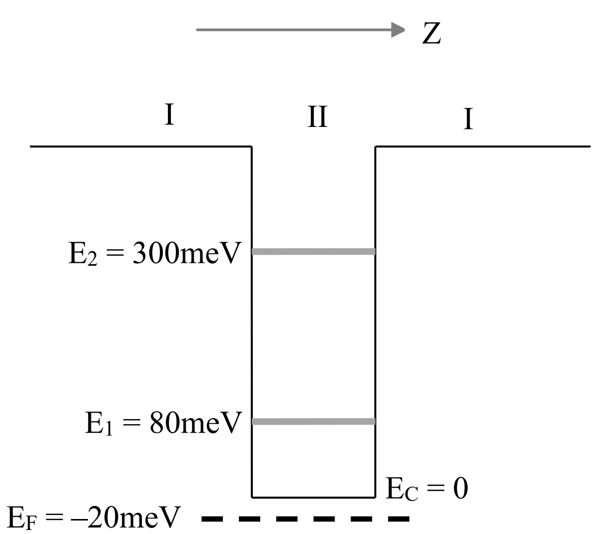
(b) Calculate the electron acceleration for an external electric field parallel to the X-axis. Explain in physical terms, any peculiarity you observe in the result.

2. The dispersion relation for band electrons in a 2D material is given by: $E = \frac{\hbar^2 k_x^2}{2m_1} + \frac{\hbar^2 k_y^2}{2m_2}$.

(a) What is the equal energy surface? What will be the radius of a circle of equal area?

(b) Show that the density of states (DOS) can still be written in the form $g = m^*/\pi\hbar^2$ which was derived for a spherical dispersion relation. m^* is called the 'density of states effective mass'. It is some average of m_1 and m_2 that must give you the correct DOS. Write down m^* in terms of m_1 and m_2 (hint: consider the second part of 3a).

3. Consider the quantum well for electrons formed in the heterostructure shown below. It has two bound states as shown below. Assuming a constant, isotropic effective mass everywhere, the 2D density of states is given by $g = m^*/\pi\hbar^2$. Take $g = 2\times10^{14}\,cm^{-2}eV^{-1}$. Then calculate the 2D carrier density at 300K. State clearly and justify any assumptions you make.



4. Consider particles in 3D, with dispersion relation $E = \hbar c|k|$.

(a) Calculate the density of states.

(b) Assume these are bosons. Write down the energy density as a function of wavelength $\lambda = 2\pi/k$. Comment on the result.

5. Consider the free electron Fermi gas in three dimensions (3D). The 3D density-of-states (DOS) for this case is given by $g_{3D}(E) = \frac{1}{2\pi^2}\left(\frac{2m_0}{\hbar^2}\right)^{3/2} E^{1/2}$. Assume $T = 0K$. Suppose $k_F = \sqrt{2m_0 E_F / \hbar^2}$ is the Fermi wavevector and $2a_0$ is the average distance between electrons. Show that $k_F a_0 \sim 1$. Provide a physical interpretation for this result.