Equilibrium electronic density – DOS * FD

DOS in a band

$$g(E) = \frac{\sqrt{2}}{\pi^2}\left(\frac{m^*}{\hbar}\right)^{3/2}\sqrt{E} \rightarrow g_c(E) = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^*}{\hbar}\right)^{3/2}\sqrt{E - E_C}$$

Occupation probability

$$f_{FD}(E) = \frac{1}{e^{\beta(E-E_F)} + 1}$$

Electron density

$$n = \int_{E_C}^{E_{\max}} g_c(E) f_{FD}(E)\, dE \qquad p = \int_{E_V}^{E_{\min}} g_v(E)\left[1 - f_{FD}(E)\right] dE$$

Hole density

$$n = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^*}{\hbar}\right)^{3/2} \int_{E_C}^{E_{\max}} \frac{(E - E_C)^{1/2}\, dE}{e^{(E-E_F)/k_B T} + 1}$$

$$\boxed{E_{\max} \rightarrow \infty}$$

$$n = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^* k_B T}{\hbar}\right)^{3/2} \int_0^{\infty} \frac{\eta^{1/2}\, d\eta}{e^{(\eta-\eta_F)} + 1}$$

$$\boxed{\begin{aligned} \eta &= (E - E_C)/k_B T \\ \eta_F &= (E_F - E_C)/k_B T \end{aligned}}$$

Equilibrium electronic density – DOS * FD

$$n = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^* k_B T}{\hbar}\right)^{3/2} \int_0^\infty \frac{\eta^{1/2} d\eta}{e^{(\eta-\eta_F)}+1} = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^* k_B T}{\hbar}\right)^{3/2} F_{1/2}(\eta_F)$$

$F_{1/2}$: Complete Fermi integral

Boltzmann approximation:
Fermi-Dirac → Maxwell-Boltzmann

$(\eta - \eta_F) \gg 1$
$\Rightarrow (E - E_F) \gg k_B T$
$\Rightarrow (E_C - E_F) \gg k_B T$

$$n = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^* k_B T}{\hbar}\right)^{3/2} e^{\eta_F} \int_0^\infty \eta^{1/2} e^{-\eta} d\eta$$

$$n = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^* k_B T}{\hbar}\right)^{3/2} e^{\eta_F} \frac{1}{2}\Gamma\left(\frac{1}{2}\right)$$

$$n = \frac{\sqrt{2}}{\pi^2}\left(\frac{m_n^* k_B T}{\hbar}\right)^{3/2} e^{\eta_F} \frac{\sqrt{\pi}}{2}$$

$$n = \frac{1}{\sqrt{2}}\left(\frac{m_n^* k_B T}{\pi\hbar}\right)^{3/2} e^{-(E_C-E_F)/k_B T}$$

Effective DOS

$$N_C \cong \frac{1}{\sqrt{2}}\left(\frac{m_n^* k_B T}{\pi\hbar}\right)^{3/2}$$

Equilibrium carrier concentrations

	Extrinsic	Intrinsic	
Electrons	$n = N_C e^{-(E_C - E_F)/k_B T}$	$n_i = N_C e^{-(E_C - E_i)/k_B T}$	Why?
Holes	$p = N_V e^{-(E_F - E_V)/k_B T}$	$n_i = N_V e^{-(E_i - E_V)/k_B T}$	

$$np = N_C N_V e^{-(E_C - E_F)/k_B T} e^{-(E_F - E_V)/k_B T} = N_C N_V e^{-(E_C - E_V)/k_B T} = N_C N_V e^{-E_G/k_B T} = n_i^2$$

$$n = n_i e^{(E_F - E_i)/k_B T}$$

$$p = n_i e^{(E_F - E_i)/k_B T}$$

$$\boxed{np = n_i^2}$$

$$n_i = \sqrt{N_C N_V} \cdot e^{-E_G/2k_B T}$$

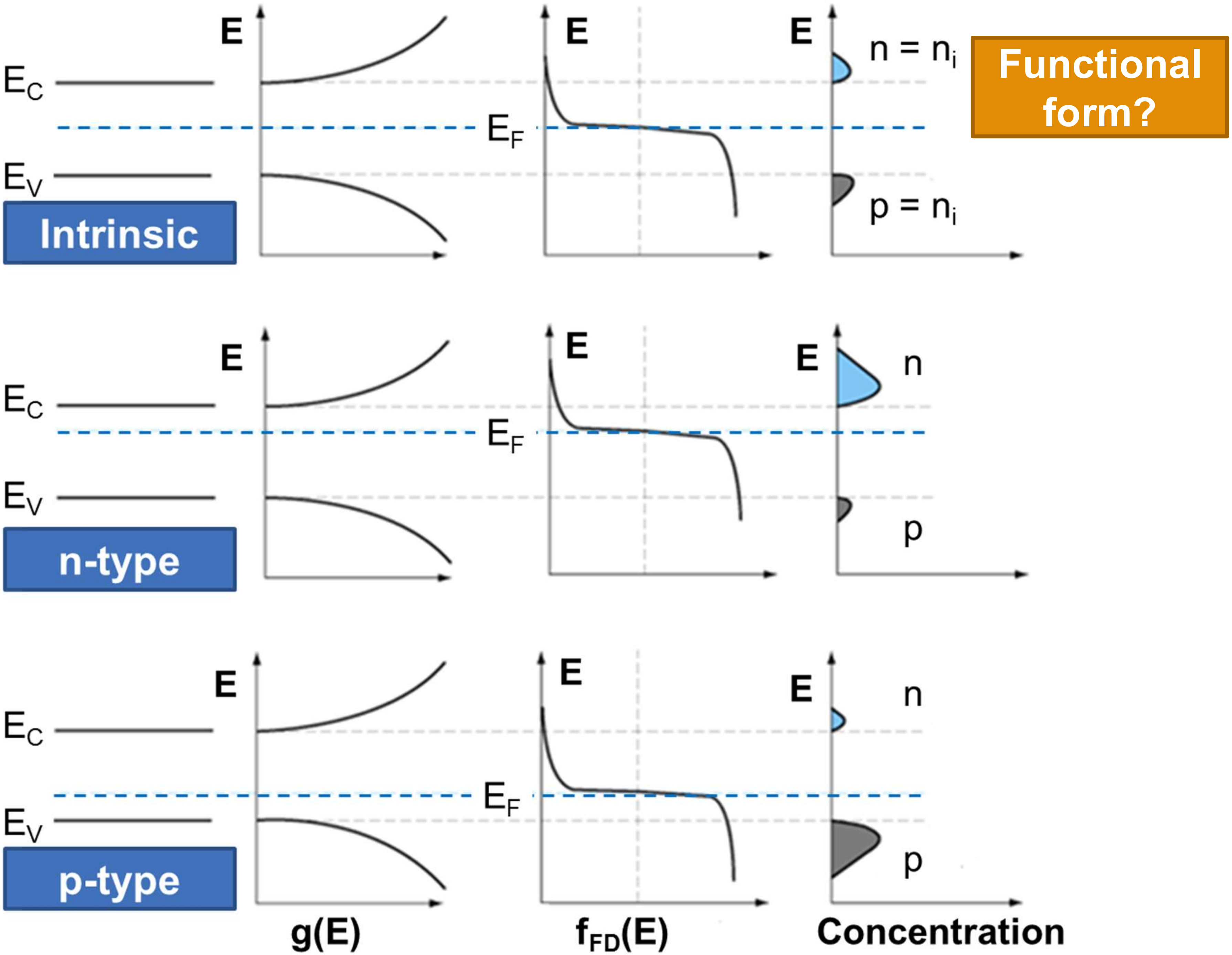
Exponential bandgap dependence

$$\frac{N_C e^{-(E_C - E_i)/k_B T}}{N_V e^{-(E_i - E_V)/k_B T}} = 1 \Rightarrow \ln N_C - \frac{E_C - E_i}{k_B T} - \ln N_V + \frac{E_i - E_V}{k_B T} = 0$$

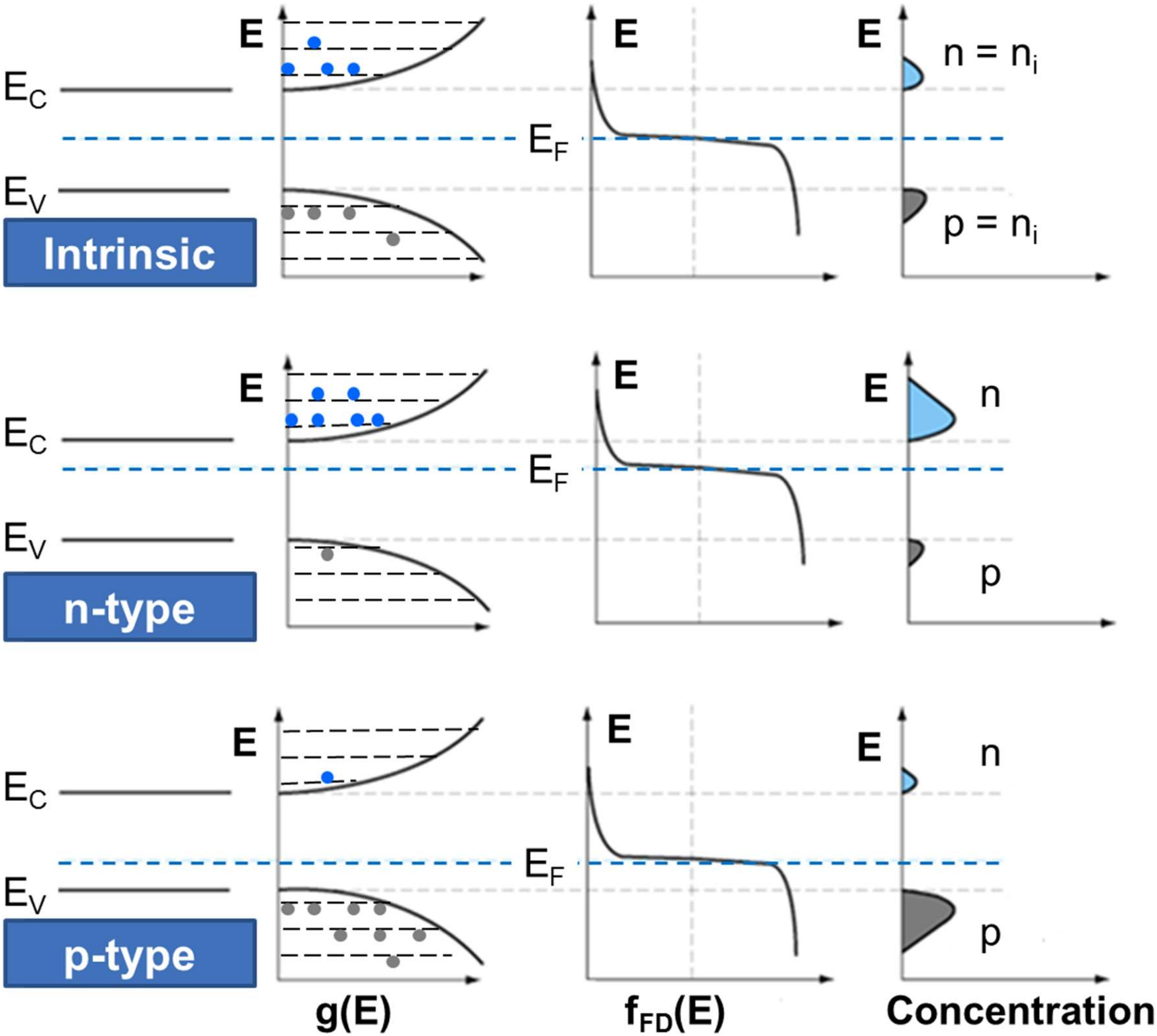
Intrinsic Fermi level

$$E_i = \frac{E_C + E_V}{2} - \frac{k_B T}{2} \cdot \ln\left(\frac{N_C}{N_V}\right) = \frac{E_C + E_V}{2} - \frac{3k_B T}{4} \cdot \ln\left(\frac{m_n^*}{m_p^*}\right)$$

Equilibrium carrier concentrations



Equilibrium carrier concentrations



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