EE 207 [Autumn 24-25] ***HW-4 Solutions***

1. We have a semiconductor with bandgap 1.4eV, an intrinsic carrier concentration of 1.8×10^6 cm^{-3} at room temperature, n-doping of $10^{14} cm^{-3}$, and electron (hole) mobility of 8000 (400) $cm^2V^{-1}s^{-1}$ Assume identical electron and hole effective mass. Suppose it is irradiated with light such that the steady-state electron-hole pair generation rate close to the top surface is $10^{20}s^{-1}cm^{-3}$.

(a) What is the steady-state concentration of electrons and holes close to the surface? Draw a quantitatively labeled band-diagram for that region in this scenario.

(b) Now, the excess minority carriers at the surface will diffuse into the bulk of the semiconductor, recombining as they go. The minority carrier lifetime is 100ns (1ns $=10^{-9}$s). What is their diffusion length λ? Derive an equation describing the steady-state minority carrier concentration as a function of depth (you do not need to solve it).

(a) $\Delta p = G \cdot \tau_p$

$$p = p_0 + \Delta p \approx \Delta p = 10^{20} cm^{-3}s^{-1} \cdot 10^{-7} s = 10^{13} cm^{-3}$$

$$\Delta n = \Delta p$$

$$n = n_0 + \Delta n = n_0 + \Delta p = 1.1 \times 10^{14} cm^{-3}$$

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$\Rightarrow 10^{14} cm^{-3} = 1.8 \times 10^{06} cm^{-3} \cdot e^{(E_F - E_i)/kT}$$

$$\Rightarrow E_F - E_i = 0.026eV \cdot \ln\left(10^{14}/1.8 \times 10^6\right) = 0.464eV$$

$$n = n_i e^{(F_n - E_i)/kT}$$

What are they? Fn vs Ef

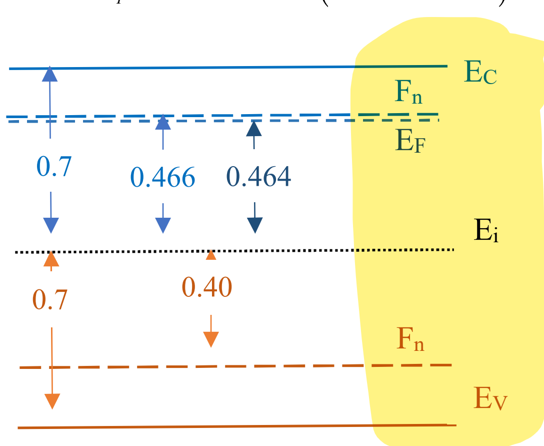
$$\Rightarrow 1.1 \times 10^{14} cm^{-3} = 1.8 \times 10^{06} cm^{-3} \cdot e^{(F_n - E_i)/kT}$$

$$\Rightarrow F_n - E_i = 0.026eV \cdot \ln\left(1.1 \times 10^{14}/1.8 \times 10^6\right) = 0.466eV$$

$$p = n_i e^{(E_i - F_p)/kT}$$

$$\Rightarrow 10^{13} cm^{-3} = 1.8 \times 10^{06} cm^{-3} \cdot e^{(E_i - F_p)/kT}$$

$$\Rightarrow E_i - F_p = 0.026eV \cdot \ln\left(10^{13}/1.8 \times 10^6\right) = 0.404eV$$



(b) $\lambda_p = \sqrt{D_p \cdot \tau_p}$

Einstein relation: $D_p = \mu_p kT/q$

$D_p = 400cm^2V^{-1}s^{-1} \cdot 0.026V = 10.4cm^2s^{-1}$

$\lambda_p = \sqrt{10.4cm^2s^{-1} \cdot 10^{-7}s} = 1.02 \times 10^{-3}cm = 10.2\mu m$

Continuity: $\dfrac{\partial p}{\partial t} = -\dfrac{1}{q}\dfrac{\partial J_p}{\partial x} + (G_p - R_p)$

Steady-state: $0 = -\dfrac{1}{q}\dfrac{\partial J_p}{\partial x} + (G_p - R_p)$

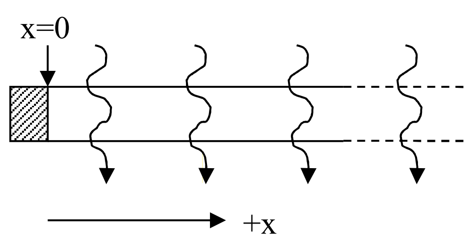
Inside the semiconductor: $G_p = 0; R_p = \Delta p/\tau_p$

Diffusion current: $J_p = -qD_p\, \partial p/\partial x$

$$\therefore D_p \frac{\partial^2 p}{\partial x^2} - \frac{\Delta p}{\tau_p} = 0$$

$$\Rightarrow \frac{\partial^2}{\partial x^2}\Delta p = \frac{\Delta p}{D_p \tau_p} = \frac{\Delta p}{\lambda_p^2}$$

2. Consider the thin, semi-infinite, n-type semiconductor sample subjected to radiation shown below, resulting in a uniform steady-state generation rate of holes G_p deep in the bulk of the sample. Assume the recombination rate of excess holes is given by $R_p = \delta p/\tau_p$, where τ_p is the hole lifetime; the hole diffusivity is D_p and diffusion length is $\lambda_p = \sqrt{D_p \tau_p}$.



(a) The contact at $x = 0$ acts as a sink to force the excess hole concentration to zero there. Derive an expression for the variation of the excess hole concentration with position x.

(b) In reality, the contact will not be the perfect sink that it is assumed to be in part (a). Suggest a way to quantify its effectiveness as a sink. (Please try to come up with your idea/s before looking up how this is conventionally done.)

(a) Suppose the excess hole concentration as a function of position is given by $\delta p(x)$. The steady-state hole continuity equation for this case is:

$$-\frac{1}{q}\frac{\partial J_p}{\partial x}+G_p-R_p=0\text{...[1]}$$

The (diffusion) current is given by: $J_p=-qD_p\frac{\partial p}{\partial x}=-qD_p\frac{\partial(\delta p)}{\partial x}$...[2]

Using [2] in [1], we get: $D_p\frac{\partial^2 p}{\partial x^2}+G_p-R_p=0$...[3]

$$\Rightarrow D_p\frac{\partial^2(\delta p)}{\partial x^2}+G_p-\frac{\delta p}{\tau_p}=0$$

$$\Rightarrow D_p\tau_p\frac{\partial^2(\delta p)}{\partial x^2}+G_p\tau_p-\delta p=0$$

$$\Rightarrow \frac{\partial^2(\delta p)}{\partial x^2}=\frac{\delta p-G_p\tau_p}{D_p\tau_p}=\frac{\delta\tilde{p}}{\lambda_p^2}$$

$$\therefore \frac{\partial^2(\delta\tilde{p})}{\partial x^2}=\frac{\delta\tilde{p}}{\lambda_p^2}\text{...[4]}$$

where $\delta\tilde{p}=\delta p-G_p\tau_p$. This admits solutions of the well-known form:

$$\boxed{\delta\tilde{p}(x)=Ae^{x/\lambda_p}+Be^{-x/\lambda_p}\text{...[5]}}$$

Since the excess hole concentration cannot increase without bound far away from the contact, we must have $A=0$. Therefore:

$$\delta\tilde{p}(x)=Be^{-x/\lambda_p}$$

$$\Rightarrow \delta p(x)=G_p\tau_p+Be^{-x/\lambda_p}\text{...[6]}$$

Also, since the hole concentration is forced to zero at the contact, we have:

$$\delta p(0)=0\Rightarrow G_p\tau_p+B=0$$

$$\Rightarrow B=-G_p\tau_p\text{...[7]}$$

Using [7] in [6]:

$$\therefore \delta p(x)=G_p\tau_p\left(1-e^{-x/\lambda_p}\right)$$

The rise of the excess hole concentration rises from zero at the contact to its bulk value is mathematically identical to that of a charging capacitor in an RC circuit.

(b) Where the contact is not a perfect sink, it is characterized by the (in this case, hole) surface recombination velocity:

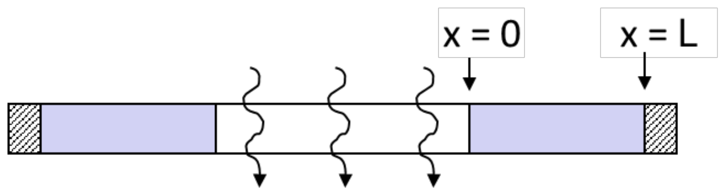
$$S_p = \frac{\text{Hole current flowing into contact}}{\text{Excess hole charge at } x = 0^+}$$

$S_p \to 0$ corresponds to the case where surface recombination at the contact is practically non-existent. If that were the case here, the excess carrier concentration would have the bulk value everywhere, as if the contact does not exist.

$S_p \to \infty$ implies a perfect sink with the excess hole concentration vanishing at the contact. This is what we have here.

This shows that for extreme surface recombination the excess hole concentration decreases to zero at the contact as might be expected; in fact, its rise from zero at the contact to the bulk value is mathematically identical to that of a charging capacitor in an RC circuit.

3. Consider a semiconductor bar illuminated in the middle – illustrated below and discussed in class. The difference here is that the length of the unilluminated segment is not infinite, it is L.



Suppose the generation rate in the illuminated segment is G, and the minority carrier lifetime everywhere is τ. Assume that L is much less than the minority carrier recombination length λ, so that recombination of minority carriers can be neglected as they diffuse from x=0 to x=L. Derive an expression for the excess minority carrier concentration as a function of position in this region. Assume that the contact at x=L forces the excess carrier concentration to zero there.

For $x \in [0, L]$, the steady-state continuity equation is given by:

$$\frac{\partial n}{\partial t} = 0 = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G_n - R_n) = \frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$

Now, since the current is due to diffusion: $J_n = eD_n \frac{\partial n}{\partial x} \Rightarrow D_n \frac{\partial^2 n}{\partial x^2} = \frac{\delta n}{\tau_n} \Rightarrow \frac{\partial^2 (\delta n)}{\partial x^2} = \frac{\delta n}{\lambda_n^2}$

Neglecting recombination, we have: $\frac{\partial^2 (\delta n)}{\partial x^2} = 0$...[1]

The boundary conditions are as follows. At $x = 0$, the excess electron concentration would have the following value under steady-state generation: $\delta n(0) = G\tau$. Also, the perfect contact at $x = L$ forces $\delta n(L) = 0$.

Integrating [1] successively, we get $\frac{\partial(\delta n)}{\partial x} = A$, and $\delta n(x) = Ax + B$...[2], where A and B are constants. Using the boundary conditions in [2], we find:

$$B = \delta n(0) = G\tau$$

and, $AL + B = 0$

$$\Rightarrow A = -B/L = -\delta n(0)/L = -G\tau/L$$

$$\therefore \delta n(x) = \delta n(0)[1 - x/L] = G\tau[1 - x/L]$$

4. We have discussed in class that majority carriers typically respond much faster than minority carriers. Therefore, we focused on the slower minority carrier response which dominates the dynamics, while just assuming charge neutrality due to the fast majority carriers. Here we examine the majority carrier response. Imagine a slab of metal (so we do not have to worry about minority carriers, or recombination) contacted at the bottom, whose top surface is subjected to a pulse-like disturbance that produces a non-equilibrium (excess) carrier concentration. This gives rise to a transient electric field and a transient current (pure drift, subject to Ohm's Law) flowing from the top surface towards the bottom contact.

(a) What are the relevant equations to analyze this scenario?

(b) Put them together to derive an expression for the time-dependence of the excess majority charge density at the top surface, and thereby, the majority carrier response time.

(a) The relevant equations are the continuity equation (with no G-R), the drift current (there is no diffusion), and the Gauss Law (or Poisson Equation) for electrostatics.

(b) The continuity equation in this case gives us:

$$\frac{\partial n}{dt} = \frac{1}{q}\frac{\partial J_n}{\partial x} \text{ where } J_n = nq\mu E = \sigma E$$

Suppose the equilibrium concentration is n_0. Then, we have:

$$\Rightarrow \frac{\partial(n_0 + \delta n)}{dt} = \frac{1}{q}\frac{\partial(\sigma E)}{\partial x}$$

$$\Rightarrow \frac{\partial(\delta n)}{dt} = \frac{\sigma}{q}\frac{\partial E}{\partial x} \text{...[1]}$$

Now, the Gauss Law for electrostatics gives us:

$$\frac{\partial E}{\partial x} = \frac{-q(\delta n)}{\varepsilon} \text{...[2]}$$

From [1] and [2], we get:

$$\frac{\partial(\delta n)}{dt} \approx -\frac{\sigma}{\varepsilon}(\delta n) = -\frac{(\delta n)}{\tau}$$

The majority carrier response time being $\tau = \varepsilon/\sigma$. Clearly the higher the conductivity (in the case of a semiconductor, the higher the doping), the faster is the majority carrier response.