Spring 2024-25 **EE 207**

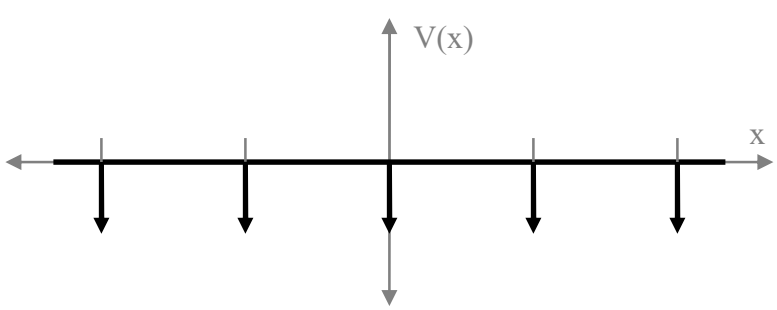
HW – 2

1. Consider a delta-function potential well $V = -\alpha \cdot \delta(x)$, where $\alpha \in \mathbb{R}$ and $\alpha > 0$.

   (a) Integrate the Schrodinger equation to prove that the discontinuity in the first-derivative of the wavefunction at $x = 0$ is given, in this case, by $\Delta\left(\frac{d\psi}{dx}\right) = \frac{-2m\alpha}{\hbar^2}\psi(0)$ (symbols have their usual meanings).

   (b) Consider continuum states, i.e. states with $E > 0$. Show that the transmission and reflection probabilities are given by $T = \frac{1}{1+\left(m\alpha^2/2\hbar^2 E\right)}$ and $R = \frac{1}{1+\left(2\hbar^2 E/m\alpha^2\right)}$, respectively.

   (c) What are the allowed bound state energies?

   (d) What would be the tunneling probability as a function of E for a delta-function barrier $V = +\alpha \cdot \delta(x)$?

2. Let us model the periodic electric potential in a (one-dimensional) crystalline solid by the "Dirac-comb" potential: $V(x) = -\alpha \sum_{j=0}^{N-1} \delta(x - ja)$ illustrated below, where N is the number of atoms. Use the Bloch theorem and the known solution to the single Dirac potential well (from problem #1 above) to find out the allowed energies. You should also find some forbidden 'gaps' in the spectrum, and thus obtain bands. How many allowed states do you have in each band? Consider the filling of these allowed states by electrons contributed by the atoms (note: remember the Pauli Exclusion Principle for electrons). Comment on the situation when each of the N atoms contributes 1 free electron, versus when they contribute 2 each.