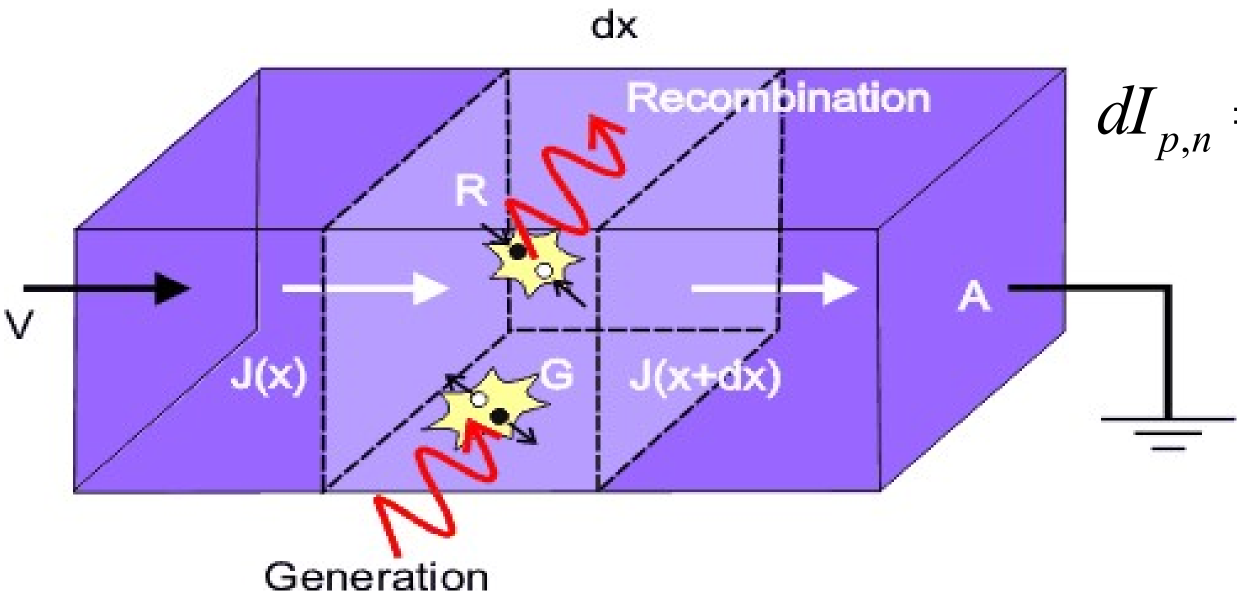
Continuity equation for carriers



$$dI_{p,n} = A \cdot dJ_{p,n} = \pm e\left(G_{p,n} - R_{p,n}\right) \cdot Adx$$

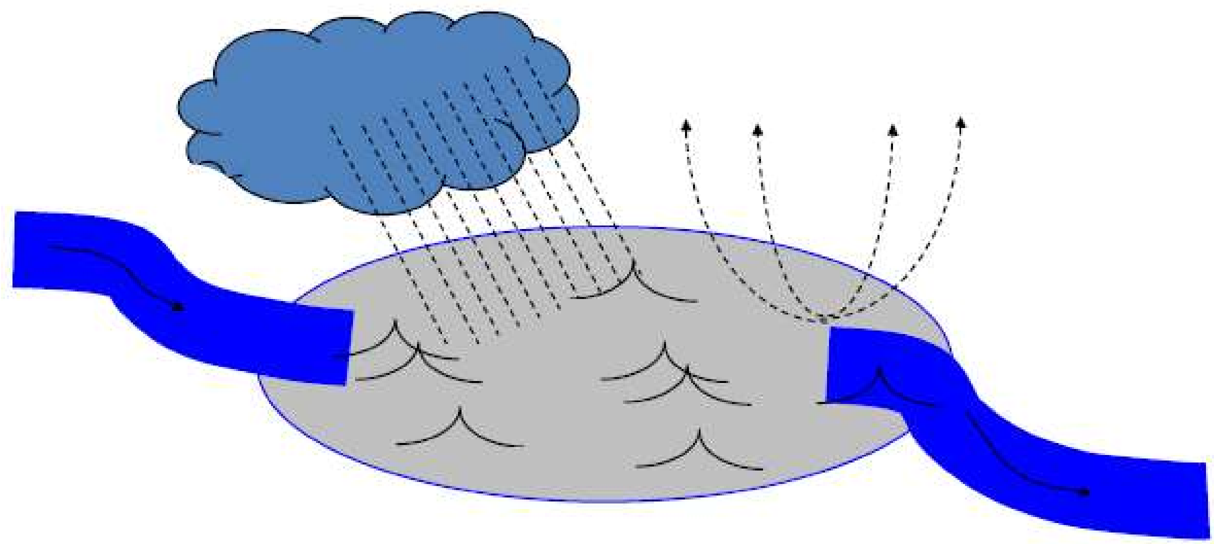
$$\Rightarrow \frac{\partial J_{p,n}}{\partial x} = \pm e\left(G_{p,n} - R_{p,n}\right)$$

Steady-state

Time-dependence

$$\frac{\partial \rho}{\partial t} = -\frac{\partial J_p}{\partial x} + e\left(G_p - R_p\right)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{e}\frac{\partial J_p}{\partial x} + \left(G_p - R_p\right)$$



$$\boxed{\frac{\partial p}{\partial t} = -\frac{1}{e}\vec{\nabla} \cdot \vec{J}_p + \left(G_p - R_p\right)}$$

$$\boxed{\frac{\partial n}{\partial t} = +\frac{1}{e}\vec{\nabla} \cdot \vec{J}_n + \left(G_n - R_n\right)}$$

Continuity from Ampere's Law?

Shockley Equations

Continuity

Drift-Diffusion

Electrons

$$\frac{\partial n}{\partial t} = +\frac{1}{e}\vec{\nabla}\cdot\vec{J}_n + \left(G_n - R_n\right)$$

$$J_n = ne\mu_n\mathcal{E} + eD_n\frac{dn}{dx}$$

Holes

$$\frac{\partial p}{\partial t} = -\frac{1}{e}\vec{\nabla}\cdot\vec{J}_p + \left(G_p - R_p\right)$$

$$J_p = pe\mu_p\mathcal{E} - eD_p\frac{dp}{dx}$$

Fluid-like flow ← Boltzmann Transport Equation

Electrostatics: Gauss/Poisson

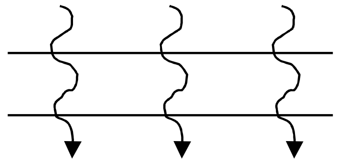
$$\vec{D} = \in\vec{E}$$

$$\vec{E} = -\vec{\nabla}V$$

$$\vec{\nabla}\cdot\vec{D} = e\left(p - n + N_d^+ - N_a^-\right)$$

Important! Recap!

Case study: uniform steady-state illumination



$$G_n = G_p = G$$

What is the density of excess minority carriers (electrons)?

Continuity

$$\frac{\partial n}{\partial t} = \frac{1}{e}\frac{\partial J_n}{\partial x} + \left(G_n - R_n\right)$$

Steady-state

$$0 = \frac{1}{e}\frac{\partial J_n}{\partial x} + \left(G_n - R_n\right)$$

Uniform

$$J_n = 0 \Rightarrow G_n = R_n$$

Low-level injection

$$R_n = \frac{\delta n}{\tau_n}$$

$$\boxed{\delta n = G\tau_n}$$

Case study: uniform transient illumination

$$G_n(t) = G_p(t) = G \cdot \Theta(t) \qquad \delta n(t=0) = 0$$

How does the density of minority electrons vary with time?

Continuity $$\frac{\partial n}{\partial t} = \frac{1}{e}\frac{\partial J_n}{\partial x} + (G_n - R_n) \qquad R_n = \delta n / \tau_n$$

$$\frac{\partial n}{\partial t} = \left(G - \frac{\delta n}{\tau_n}\right) \Rightarrow \frac{\partial(\delta n)}{\partial t} = \left(G - \frac{\delta n}{\tau_n}\right) \qquad J_n = 0$$ **Uniform**

$$\Rightarrow \frac{\partial(\delta n - G\tau_n)}{\partial t} = -\frac{\delta n - G\tau_n}{\tau_n}$$

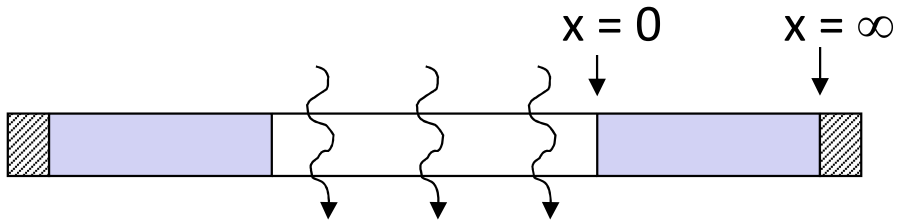
$$\Rightarrow (\delta n - G\tau_n) = (\delta n - G\tau_n)\Big|_{t=0} e^{-t/\tau_n} = -G\tau_n e^{-t/\tau_n} \qquad \delta n(0) = 0$$

$$\boxed{\delta n(t) = G\tau_n\left(1 - e^{-t/\tau_n}\right)}$$

Case study: diffusion with recombination

Steady-state

$$\delta n(x=0) = G\tau_n$$



$$\delta n(x=\infty) = 0$$

How does the density of minority electrons vary along x?

$$x \in [0,\infty) \qquad \frac{\partial n}{\partial t} = 0 = \frac{1}{e}\frac{\partial J_n}{\partial x} + (G_n - R_n) = \frac{1}{e}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n}$$

Steady-state **No generation**

$$J_n = eD_n\frac{\partial n}{\partial x} \Rightarrow D_n\frac{\partial^2 n}{\partial x^2} = \frac{\delta n}{\tau_n} \Rightarrow \frac{\partial^2(\delta n)}{\partial x^2} = \frac{\delta n}{\lambda_n^2} \qquad \lambda_n^2 = D_n\tau_n$$

$$\delta n(x) = Ae^{+x/\lambda_n} + Be^{-x/\lambda_n} \qquad A = 0; B = \delta n(0)$$

Boundary conditions

$$\boxed{\delta n(x) = \delta n(0)e^{-x/\lambda_n} = G\tau_n e^{-x/\lambda_n}}$$

Without recombination?

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Artwork Sources:

1. Prof. Sanjay Banerjee
2. Prof. M.A. Alam
3. www.pveducation.org
4. britneyspears.ac