EE 207 [Spring 24-25] ***HW-7 Solutions***

1. Consider a long-channel silicon PMOSFET with a body doping of $10^{17} cm^{-3}$. The insulator is complex: it comprises 1nm of SiO_2 grown on the Si, and 4nm of a high-k dielectric XO_2 deposited on top of the SiO_2. Atop that, we have a metal (cobalt) gate, with a workfunction $\varphi_m = 5.0eV$. Assume no charge inside the oxide layers.

Si: electron affinity $\chi_{Si} = 4.05eV$, bandgap $E_g = 1.1eV$, permittivity $\in_{Si} = 11.8$, intrinsic carrier concentration $n_i = 1.5\times10^{10} cm^{-3}$, assume equal electron and hole effective mass

SiO_2: permittivity $\in_{SiO_2} = 3.9$, band-offsets with Si – $\Delta E_C = 3eV,\ \Delta E_V = 5eV$

XO_2: permittivity $\in_{XO_2} = 7.8$, everything else same as SiO_2, no band-offsets with SiO_2

Free space permittivity $\in_0 = 8.85\times10^{-12} Fm^{-1}$

[You may use Boltzmann statistics for all calculations. Assume all interfaces are ideal.]

Calculate the flat-band voltage V_{FB} and threshold voltage V_T. Sketch the equilibrium band-diagram.

We start by calculating the semiconductor workfunction.

In the n-type bulk, we have:

$$\varphi_F \cong E_F - E_i = \frac{k_B T}{q}\ln\left(\frac{N_d}{n_i}\right)$$
$$\Rightarrow \varphi_F = 0.026eV\cdot\ln\left(\frac{10^{17}}{1.5\times10^{10}}\right)$$
$$\Rightarrow \varphi_F = 0.41eV ...[1]$$

The workfunction in this case can now be obtained as:

$$\psi_s = \chi_s + \left(E_c - E_i\right) - \varphi_F$$

Since the electron and hole effective masses are identical, the intrinsic Fermi level will lie exactly at the middle of the gap, and so we may write:

$$\psi_s = \chi_s + \left(E_c - E_i\right) - \varphi_F = \chi_s + E_g/2 - \varphi_F$$
$$\Rightarrow \psi_s = \left(4.05 + 1.1/2 - 0.41\right)eV$$
$$\Rightarrow \psi_s = 4.19eV ...[2]$$

In this ideal case, the flat-band voltage is given by the difference in metal and semiconductor Fermi levels:

$$V_{FB} = \psi_{ms} = \psi_m - \psi_s$$
$$\Rightarrow V_{FB} = (5.0 - 4.19)eV$$
$$\Rightarrow V_{FB} = 0.81eV \ldots[3]$$

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The threshold voltage is given by: $V_T = V_{FB} - \dfrac{Q_d}{C_{ox}} - 2\varphi_F \ldots[4]$

where the depletion charge is given by:

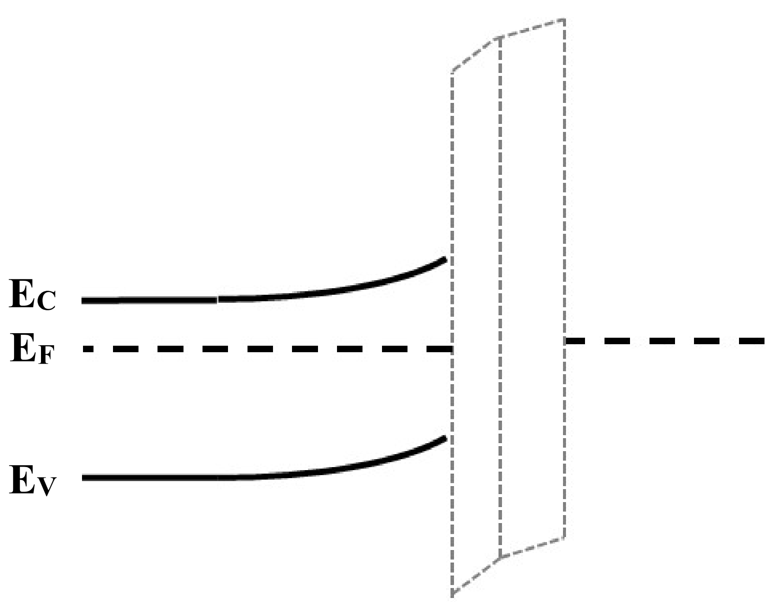
$$Q_d = qN_d W_{\max} = qN_d \sqrt{\frac{2\varepsilon_s \cdot 2\varphi_F}{qN_d}}$$
$$\Rightarrow Q_d = 2\sqrt{\varepsilon_s qN_d \varphi_F} = 1.7 \times 10^{-3} Cm^{-2} \ldots[5]$$

For calculating the oxide capacitance, we note that the high-k oxide XO_2 has double the permittivity of SiO_2, so we can consider it to be equivalent to SiO_2 of half the thickness. Thus, the 1nm-SiO_2/4nm-XO_2 stack is equivalent to 3nm SiO_2.

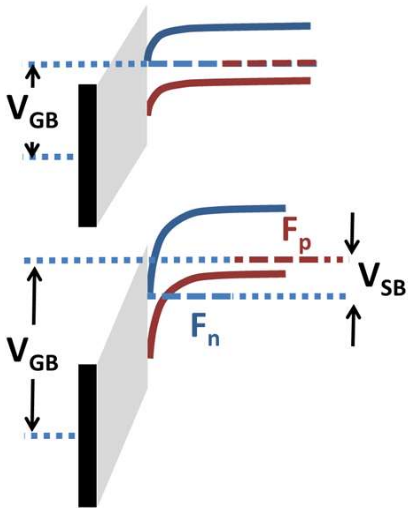
$$\therefore C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 11.5 \times 10^{-3} Fm^{-2} \ldots[6]$$

Using [3], [5], [6] in [4]:

$$V_T = \left(0.81 - \frac{1.7}{11.5} - 2 \times 0.41\right)V = -0.16V$$



2. In order to control leakage in my CMOS based circuit, I use a body-biasing scheme, wherein a voltage is applied to the substrate (body) with respect to the grounded source, so as to increase the threshold voltage. This effect is illustrated with the band diagrams below.



First, explain the effect of body bias on theshold voltage intuitively through these band diagrams. Second, calculate the threshold voltage V_T, with and without the body bias.

The following parameters may be useful:

Parameter	Values
V_{FB}(V)	0
$\|2\varphi_F\|$ (mV)	200
$V_{ox}(\psi_S=\|2\varphi_F\|)$ (mV)	100
$\|V_{SB}\|$ (V)	1.6

From the figure, it is clear that for a positive V_{SB}, the quasi-Fermi level for electrons (tied to the source) will be pushed down w.r.t. the body, i.e. w.r.t. the quasi-Fermi level for holes, which is tied to the body. Since the threshold for this NMOSFET is based upon creating an n-type (electron) inversion layer, that will now require higher voltage. This is used to reduce source-to-drain leakage in the OFF-state.

In the normal case, i.e. without body bias:

$$V_T \cong V_{GB}\left(\psi_s = |2\varphi_F|\right) = V_{GS}\left(\psi_s = |2\varphi_F|\right) = |2\varphi_F| + V_{ox}\left(\psi_s = |2\varphi_F|\right)$$

$$\Rightarrow V_T = 200mV + 100mV = 300mV$$

With an applied body bias, I can write, w.r.t. the body:

$$V_{GB} = \psi_s + V_{ox}\left(\psi_s\right) = \psi_s - \frac{Q_d\left(\psi_s\right)}{C_{ox}} = \psi_s + \frac{\sqrt{2\varepsilon_s q N_a \psi_s}}{C_{ox}}$$

In this case, *at threshold*:

$$\because \psi_s = |2\varphi_F| + V_{SB}$$

$$\Rightarrow V_{GB} = \psi_s + V_{ox}\left(\psi_s\right) = \left(|2\varphi_F| + V_{SB}\right) + \frac{\sqrt{2\varepsilon_s q N_a\left(|2\varphi_F| + V_{SB}\right)}}{C_{ox}}$$

$$V_{GS} = V_{GB} - V_{SB} = |2\varphi_F| + \frac{\sqrt{2\varepsilon_s q N_a \left(|2\varphi_F| + V_{SB}\right)}}{C_{ox}}$$

Thus, with body-bias: $V_T = |2\varphi_F| + \dfrac{\sqrt{2\varepsilon_s q N_a \left(|2\varphi_F| + V_{SB}\right)}}{C_{ox}}$

$$V_T = |2\varphi_F| + \frac{\sqrt{2\varepsilon_s q N_a |2\varphi_F|}}{C_{ox}} \cdot \left(\frac{\sqrt{|2\varphi_F| + V_{SB}}}{\sqrt{|2\varphi_F|}} \right)$$

$$V_T = 200mV + 100mV \cdot \left(\frac{\sqrt{200 + 1600}}{\sqrt{200}} \right) = 500mV$$

Thus the body-bias raises the threshold voltage, and thereby reduces the leakage. (This decides the sign of the body-bias.)