

**Example 1:** A single phase Transformer is rated 110/440V, 4 KVA having Leakage reactance at LT side is 0.2 Ω, Determine the Leakage reactance in p.u.

**Solution:** Given:  $V_B = 110V$ ,  $S_B = 4 \text{ KVA}$ ,  $Z_{Act} = 0.2\Omega$

We know that,

$$Z_B = \frac{(\text{BaseKV})^2}{\text{BaseMVA}}$$

$$Z_B \text{ at LT side} = \frac{(110 \times 10^{-3})^2 \times 1000}{4} = 3.025 \Omega$$

$$Z_{p.u.} = \frac{Z_{Act}}{Z_{Base}} = \frac{0.2}{3.025} = 0.0661 \text{ p.u.}$$

For Secondary side:

$$Z'_1 = Z_1 \left( \frac{N_2}{N_1} \right)^2 \text{ where, } N_1 \& N_2 \text{ are the no. of turns on Primary and Secondary side.}$$

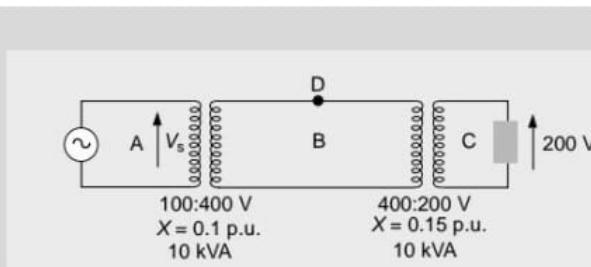
$$Z'_1 = 0.2 \left( \frac{440}{110} \right)^2 = 3.2$$

and  $Z_B = \frac{(440 \times 10^{-3})^2 \times 1000}{4} = 48.4\Omega$

$$Z_{p.u.} = \frac{3.2}{48.4} = 0.0661 \text{ p.u.}$$

### Example 2.4

In the network of Figure 2.19, two single-phase transformers supply a 10 kVA resistance load at 200 V. Show that the p.u. load is the same for each part of the circuit and calculate the voltage at point D.



**Figure 2.19** Network with two transformers-p.u. approach

*Solution*

The load resistance is  $(200^2 / 10 \times 10^3)$ , that is,  $4\Omega$ .

In each of the circuits A, B, and C a different voltage exists, so that each circuit will have its own base voltage, that is, 100 V in A, 400 V in B, and 200 V in C.

Although it is not essential for rated voltages to be used as bases, it is essential that the voltage bases used be related by the turns ratios of the transformers. If this is not so the simple p.u. framework breaks down. The same volt-ampere base is used for all the circuits as  $V_1 I_1 = V_2 I_2$  on each side of a transformer and is taken in this case as 10 kVA. The per unit impedances of the transformers are already on their individual equipment bases of 10 kVA and so remain unchanged.

The base impedance in C

$$Z_{base} = \frac{V_{base}^2}{\text{base VA}} = \frac{200^2}{10000} = 4\Omega$$

The load resistance (p.u.) in C

$$= \frac{4}{4} = 1 \text{ p.u.}$$

In B the base impedance is

$$Z_{base} = \frac{V_{base}^2}{\text{base VA}} = \frac{400^2}{10000} = 16\Omega$$

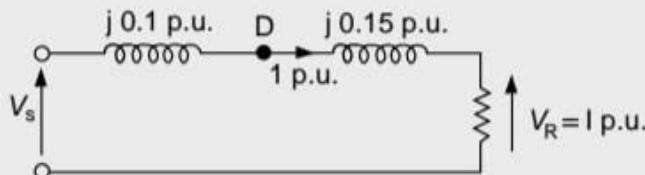
and the load resistance (in ohms) referred to B is

$$= 4 \times N^2 = 4 \times 2^2 = 16\Omega$$

Hence the p.u. load referred to B

$$\frac{16}{16} = 1 \text{ p.u.}$$

Similarly, the p.u. load resistance referred to A is also 1 p.u. Hence, if the voltage bases are related by the turns ratios the load p.u. value is the same for all circuits.



**Figure 2.20** Equivalent circuit with p.u. values, of network in Figure 2.19

An equivalent circuit may be used as shown in Figure 2.20. Let the volt-ampere base be 10 kVA; the voltage across the load ( $V_R$ ) is 1 p.u. (as the base voltage in C is 200 V).

84/514

The base current at voltage level C of this single phase circuit

$$I_{base} = \frac{\text{base VA}}{V_{base}} = \frac{10000}{200} = 50 \text{ A}$$

The corresponding base currents in the other circuits are 25 A in B, and 100 A in A.

The actual load current is  $200\text{V}/4\Omega = 50 \text{ A} = 1 \text{ p.u.}$

Hence the supply voltage  $V_s$

$$\mathbf{V}_s = 1(j0.1 + j0.15) + 1 \text{ p.u.}$$

$$V_s = \sqrt{1^2 + 0.25^2} = 1.03 \text{ p.u.}$$

$$= 1.03 \times 100 = 103 \text{ V}$$

The voltage at point D in Figure 2.17

$$\mathbf{V}_D = 1(j0.15) + 1 \text{ p.u.}$$

$$V_D = \sqrt{1^2 + 0.15^2} = 1.012 \text{ p.u.}$$

$$= 1.012 \times 400 = 404.8 \text{ V}$$

It is a useful exercise to repeat this example using ohms, volts and amperes.

A summary table of the transformation of the circuit of Figure 2.19 into per unit is shown.

Section of network	$S_{base}$ (common for network)	$V_{base}$ (chosen as transformer turns ratio)	$I_{base}$ (calculated from $S_{base}/V_{base}$ )	$Z_{base}$ (calculated from $V_{base}^2/S_{base}$ )
A	10 kVA	100 V	100 A	1 Ω
B	10 kVA	400 V	25 A	16 Ω
C	10 kVA	200 V	50 A	4 Ω

8-3

1500 KVA, 6.6 KV, 3-Φ Y gen.

$$\textcircled{1} \quad R = 0.4 \Omega \quad X = 6 \Omega$$

$$\text{PF} = 0.8 \text{ lag}$$

\textcircled{2} Find  $V_t$  for same EF [as above case \textcircled{1}] at 0.8 PF lead.

$$\rightarrow \textcircled{1} \quad I_{\text{full-load}} = \frac{S}{\sqrt{3} V_t(\text{lag})} = \frac{1500}{\sqrt{3} \times 6.6} = 131.216 \text{ A}$$

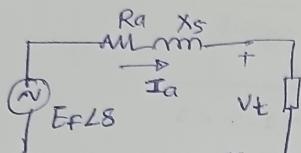
$$I_a = 131.216 \angle -36.86^\circ$$

$$\bar{E}_f = \bar{V}_t + I_a Z_s$$

$$Z = 0.4 + j6$$

$$Z = 6.013 \angle 86.18^\circ$$

$$|E_f|_{LS} = |V_t|_{LS} + (I_a \angle 0^\circ) (R_a + jX_s)$$



$$= \frac{6.6 \times 10^3}{\sqrt{3}} + (131.216 \angle -36.86^\circ)(6.013 \angle 86.18^\circ)$$

$$= 3810.51 + 789 \angle 49.325^\circ$$

$$= 4324.754 + j598.392$$

$$\therefore |E_f|_{ph} = 4365.955 \text{ V}$$

\textcircled{2} now for  $|E_f| = 4365.955 \text{ V}$ , find  $V_t$

(OR)

$$\bar{E}_f = \bar{V}_t + (R_a + jX_s)(I_a \angle \phi)$$

$$E_f LS = V_t \angle 0^\circ + (R_a + jX_s)(I_a \cos \phi + jI_a \sin \phi)$$

~~Take  $V_t$  as reference~~Take  $I_a$  as reference

(for lag PF) :-  $\bar{E}_f = V_t \angle \phi + (R_a + jX_s) I_a \angle 0^\circ$

$$E_f = V_t \cos \phi + jV_t \sin \phi + I_a R_a + jI_a X_s$$

$$\Rightarrow |E_f| = \sqrt{(V_t \cos \phi + I_a R_a)^2 + (V_t \sin \phi + I_a X_s)^2}$$



$\rightarrow$  For Simplification, use this formula directly

$\hookrightarrow$  No need to find  $\delta$  in this eqn.

Q-4

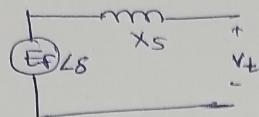
$$X_S = 0.8 \text{ pu} \quad \text{PF} = 1$$

$$V_t = 1.1 \text{ pu} \quad R_a = 0$$

$$P = 1 \text{ pu}$$

angle leading source reactance

angle of voltage behind sync. gen. Reactance



voltage behind  $X_S$  is  $E_F$   
find angle of  $E_F \Rightarrow \delta$ ?

$$P = \frac{E_F V_t}{X_S} \sin \delta$$

$$1 = \frac{E_F (1.1)}{0.8} \sin \delta \quad (1)$$

∴ first find  $E_F$ .

$$\rightarrow E_F = V_t + j I_a X_S$$

$$|E_F| = \sqrt{(1.1)^2 + (0.909)^2}$$

$$|E_F| = 1.3186 \text{ pu}$$

$$I_a = \frac{P}{\sqrt{3} V \cos \phi}$$

$$I_a = \frac{1}{\sqrt{3} \times 1.1 \times 1}$$

$$I_a = 0.909$$

NOTE: In pu system,  
no term of  $\sqrt{3}$ .

∴ from eqn (1)

$$\sin \delta = \frac{1 \times 0.8}{1.3186 \times 1.1}$$

$$\boxed{\delta = 33.47^\circ}$$

Q-5

$$E_F = 1.3 \text{ pu} \quad \delta = ?$$

$$X_S = 1.1 \text{ pu}$$

$$P = 0.6 \text{ pu}$$

$$\underline{I_B}: V_t = 1 \text{ pu}$$

$$P = 0.6 = \frac{1.3 \times 1}{1.1} \sin \delta$$

$$\delta = 30.51^\circ$$

$$\delta = \frac{1.3 \times 1}{1.1} \cos(30.51^\circ) - \frac{(1)^2}{1.1}$$

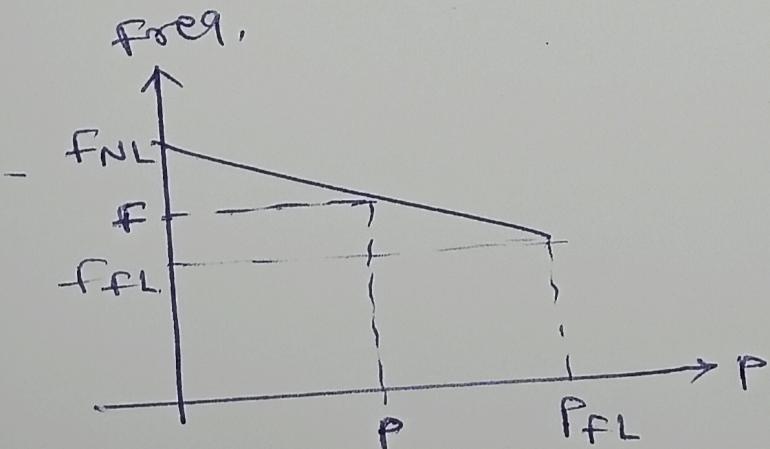
$$\delta = 0.109 \text{ pu}$$

Q-7

200 MVA sync. gen.

assume UPF

$$\text{droop} = \frac{f_{NL} - f_{FL}}{P_{FL}} = 0.04$$



$$\frac{f_{FL} - f}{P_{FL} - P} = \frac{f_{NL} - f_{FL}}{P_{FL}}$$

$$\frac{1.5}{\Delta P} = \frac{0.04 \times f_{FL}}{200}$$

$$\boxed{\Delta P = 150 \text{ mw}}$$

Q-6

100 MVA, 13.8 KV, full load, PF = 0.9 lag

$$V_t = 13.8 \text{ KV}$$

$$V_t = 13.8 \times 0.95 \text{ KV}$$

$$X_s = 1 \Omega$$

$$I_a = \frac{100 \times 10^6}{\sqrt{3} \times 13.8 \times 10^3} = 4.183 \text{ KA}$$

for  $V_t = 1 \text{ pu}$ :  $E_{f1} = \frac{13800}{\sqrt{3}} + j(4.183 \times 10^3 \angle -25.84^\circ) (1)$

$$E_{f1} \approx 10489.55 \text{ V}$$

for  $V_t = 0.95 \text{ pu}$   $E_{f2} = \frac{13800}{\sqrt{3}} \times 0.95 + j(4.183 \times 10^3 \angle -25.84^\circ) (1)$

$$E_{f2} \approx 10118.74$$

$$\% \text{ increase} = \frac{E_{f1} - E_{f2}}{E_{f2}} \times 100\% \approx 3.6\%$$