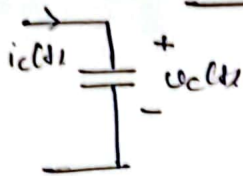


Assignment 4 (Solutions)

1)



$$i_c(t) = C \frac{du_c}{dt}$$

$$\Rightarrow u_c(t) dt$$

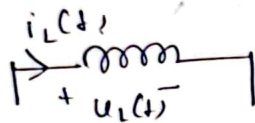
$$\Rightarrow i_c(t) dt = C du_c$$

$$\Rightarrow \int_{u_c(t)}^{u_c(t+T)} du_c = \int_t^{t+T} i_c(\tau) d\tau$$

$$\Rightarrow u_c(t+T) - u_c(t) = \left(\frac{1}{T} \int_t^{t+T} i_c(\tau) d\tau \right) \cdot T$$

$$\Rightarrow \boxed{\langle I_c(t) \rangle = 0} \quad \boxed{\text{QED}}$$

2)

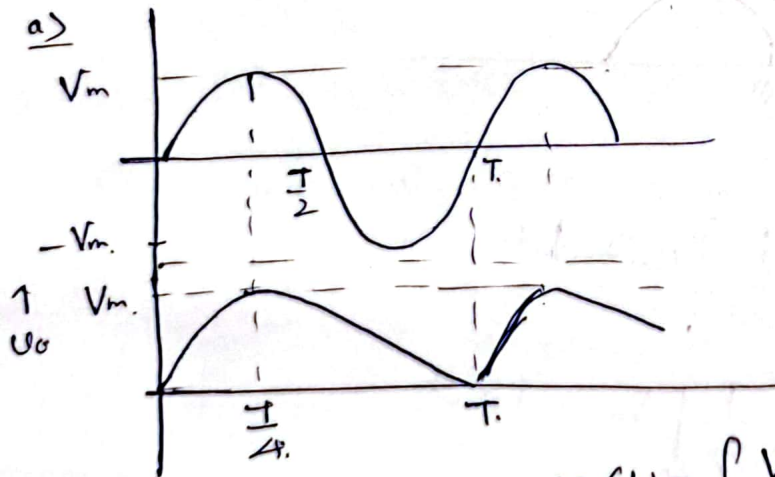


$$u_L(t) = L \frac{di_L}{dt}$$

$$\Rightarrow \int_{i_L(t)}^{i_L(t+T)} di_L(t) = \int_t^{t+T} u_L(\tau) d\tau$$

$$\Rightarrow \boxed{\langle u_L(t) \rangle = 0} \quad \boxed{\text{QED}}$$

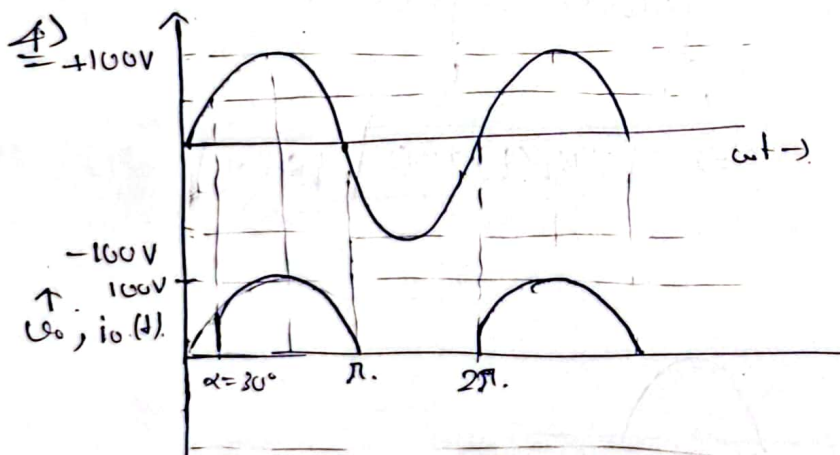
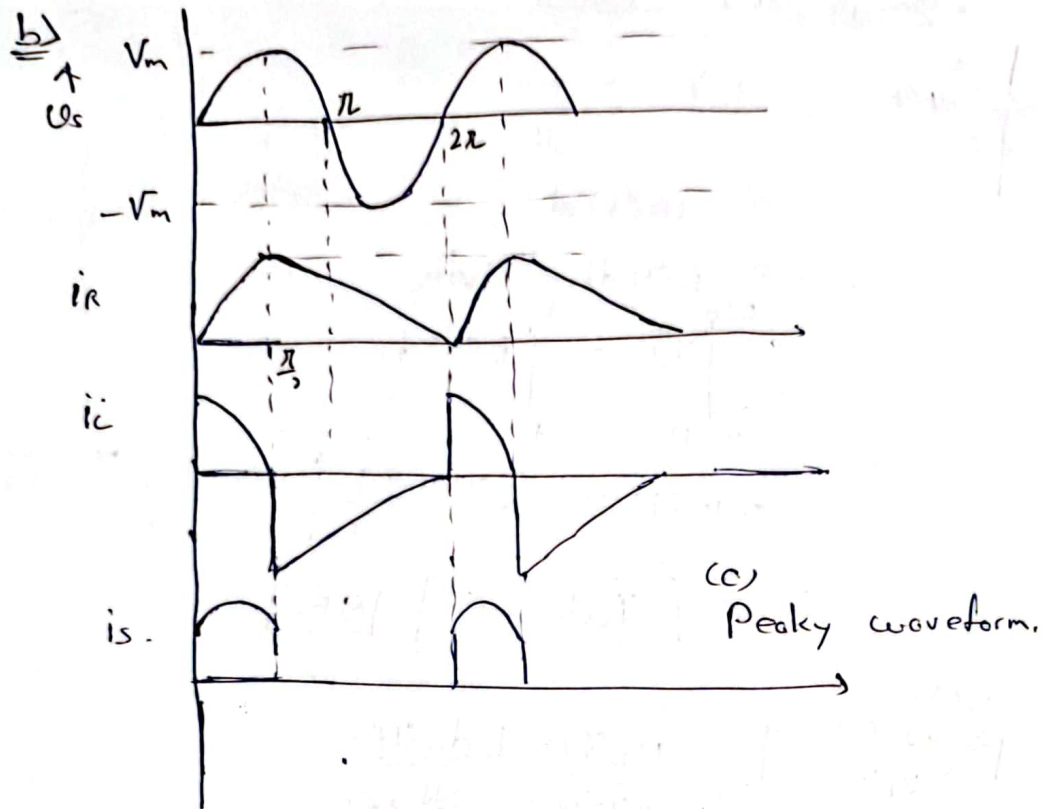
3) a)



$$u_0(t) = \begin{cases} V_m \sin \omega t, & 0 < t \leq \frac{T}{4} \\ 0 - (0 - V_m) e^{-(t - T/4)/\tau}, & \frac{T}{4} \leq t < T \end{cases}$$

$$= \begin{cases} V_m \sin \omega t, & 0 < t \leq \frac{T}{4} \\ V_m e^{-(t - T/4)/\tau}, & \frac{T}{4} \leq t < T \end{cases}$$

$$\text{where } \boxed{\tau = RC}$$



$$\begin{aligned}
 V_o &= \frac{1}{2\pi} \int_{\alpha}^{\pi} v_s(t) d\omega t \\
 &= \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t d\omega t \\
 &= \frac{V_m}{2\pi} [\cos \alpha + 1]
 \end{aligned}$$

$$V_o = \frac{100}{2\pi} \left[1 + \frac{\sqrt{3}}{2} \right] = 29.69 \text{ V.}$$

$$V_{o_{rms}}^2 = \frac{V_m^2}{2\pi} \left[\int_{\alpha}^{\pi} (1 - \cos 2\omega t) d\omega t \right] = \frac{V_m^2}{2\pi} \left[(\pi - \alpha) + \frac{1}{2} \sin 2\alpha \right]$$

$$V_{rms}^2 = \frac{V_m^2}{2\pi} \left[(\pi - \alpha) + \frac{1}{2} \sin 2\alpha \right]$$

$$V_{rms} = 69.68 \text{ V.}$$

$$\underline{c)} \quad P = \frac{V_{rms}^2}{R} = \frac{69.68^2}{R} = \frac{4855.83}{R} \text{ W.}$$

$$= 485.58 \text{ W.}$$

$$\underline{d)} \quad V_o = 23.87 = \frac{V_m}{2\pi} (1 + \cos \alpha) = 23.87$$

$$\Rightarrow \boxed{\alpha = 60^\circ}$$

$$\omega t = \alpha$$

$$\Rightarrow t = \frac{\alpha}{\omega} = \frac{\pi}{3 \times 2\pi f} = \frac{1}{6 \times 50} = 3.33 \text{ ms.}$$

5)

$$u_o(t) = u_L(t) + E + iR$$

$$\Rightarrow \langle u_o(t) \rangle = \langle u_L(t) \rangle + \langle E \rangle + \langle i \rangle R.$$

$$\Rightarrow \langle i(t) \rangle = \frac{50 - 10}{10} = \frac{40}{10} = 4 \text{ A.}$$

$$\underline{6)} \quad 1) \quad V_o = DV_{dc} = 0.5 \times 100 = 50 \text{ V.}$$

$$2) \quad I_L = I_o = \frac{50}{10} = 5 \text{ A.}$$

$$3) \quad P_{in} = P_{out}$$

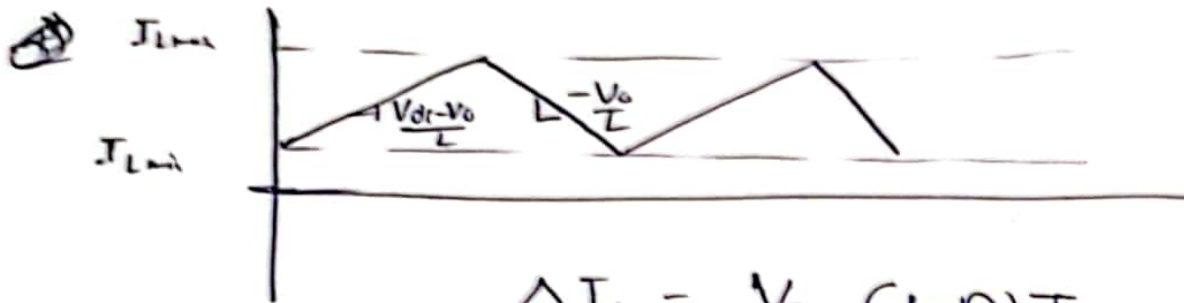
$$\Rightarrow 100 \times I_{sw} = 50 \times 5.$$

$$\Rightarrow I_{sw} = \frac{250}{100} = 2.5 \text{ A.}$$

$$i_{sw} + i_D + i_L = i_L$$

$$\Rightarrow \langle i_{sw} \rangle + \langle i_D \rangle = \langle i_L \rangle$$

$$\Rightarrow I_D = I_L - I_{sw} = 5 - 2.5 = 2.5 \text{ A.}$$



$$\Delta I_L = \frac{V_o \cdot (1-D) T_s}{L}$$

$$= \frac{50}{60 \times 10^{-3}} \times \frac{1}{2} \times \frac{1}{2 \times 10^{-4}}$$

$$= \frac{5}{24} = 208.33 \text{ mA} = 0.208 \text{ A.}$$

$$I_{Lmin} = I_L - \frac{\Delta I_L}{2}$$

$$= 5 - 0.104$$

$$= 4.896 \text{ A.}$$

$$I_{Lmax} = I_L + \frac{\Delta I_L}{2} = 5 + 0.104 = 5.104 \text{ A.}$$