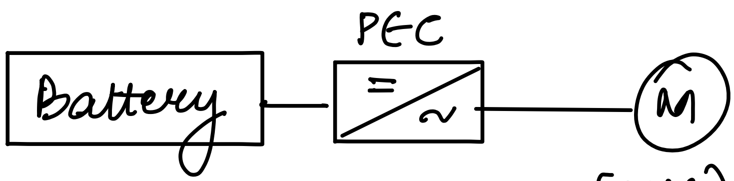
Power Electronics

PEC is usually static (no moving parts)

Eg Electric Vehicles.

should be able to handle both sides power flow



(IM) (3φ)

(38V – 55V)

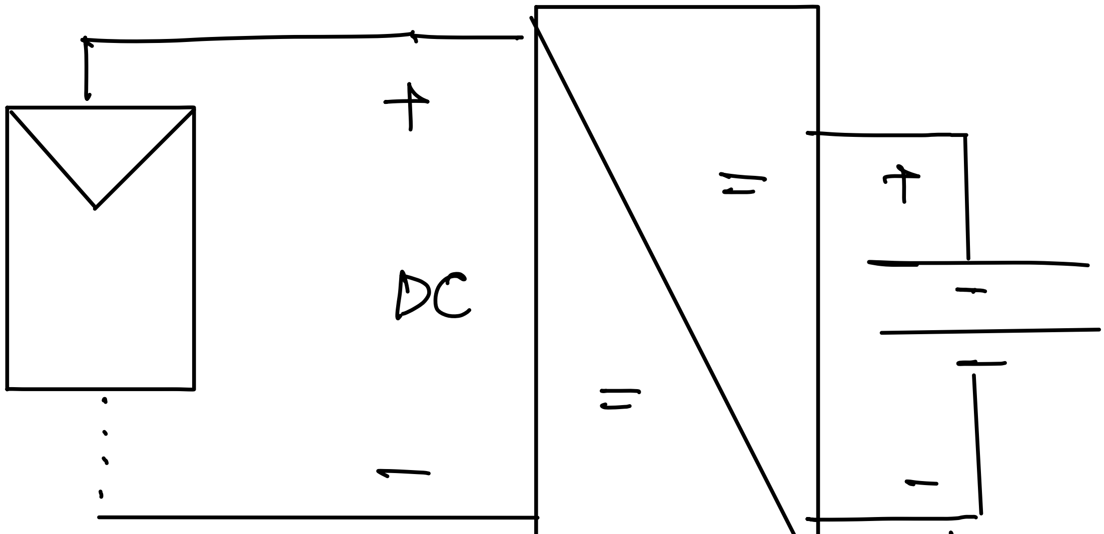
30 V_{rms}, 50Hz

⟷ Power flow

PEC
- DC–DC
- DC–AC
- AC–DC

} a.k.a. inverters

Eg Solar Photovoltaic



battery.

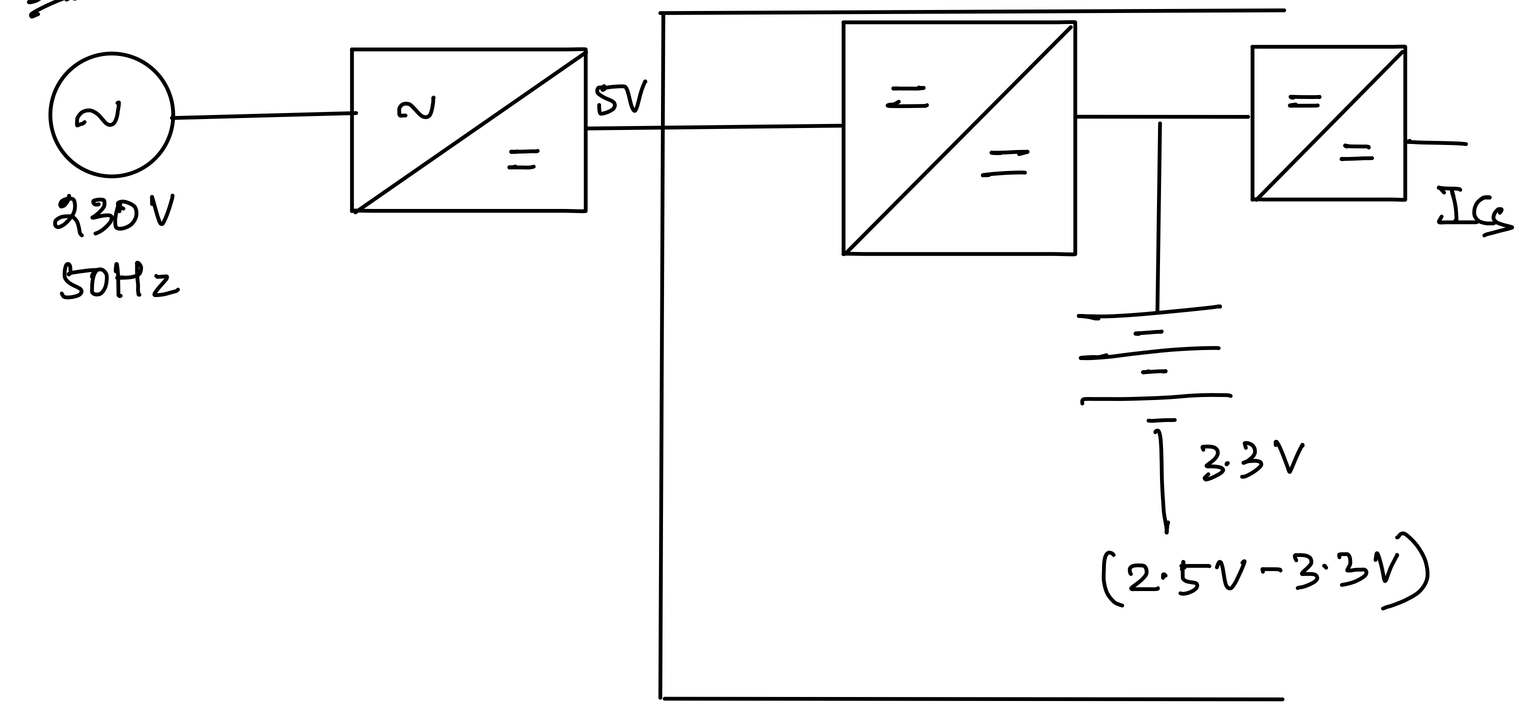
400 V nominal

200 V – 400 V range

100 V nominal

110 V – 70 V range

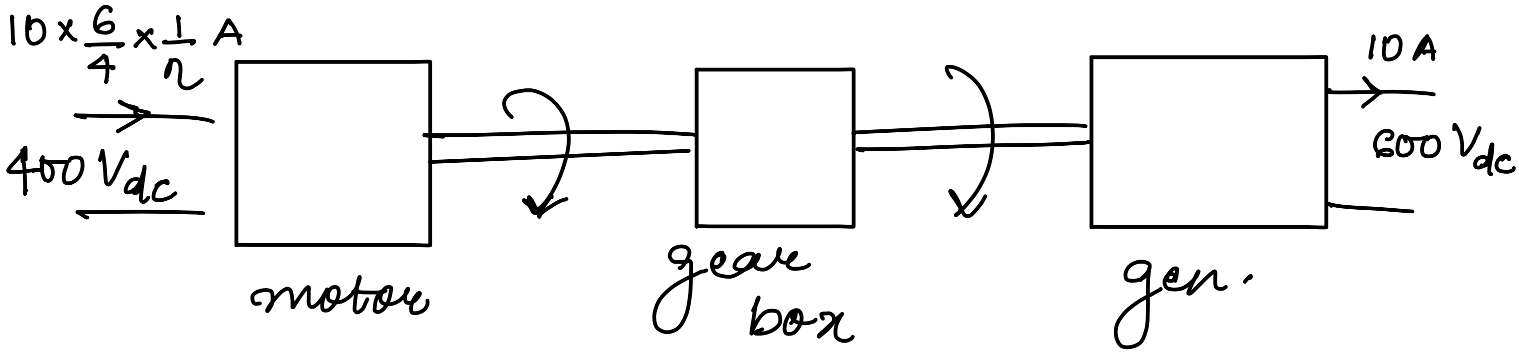
eg Mobile Phones



DC- DC Conversion

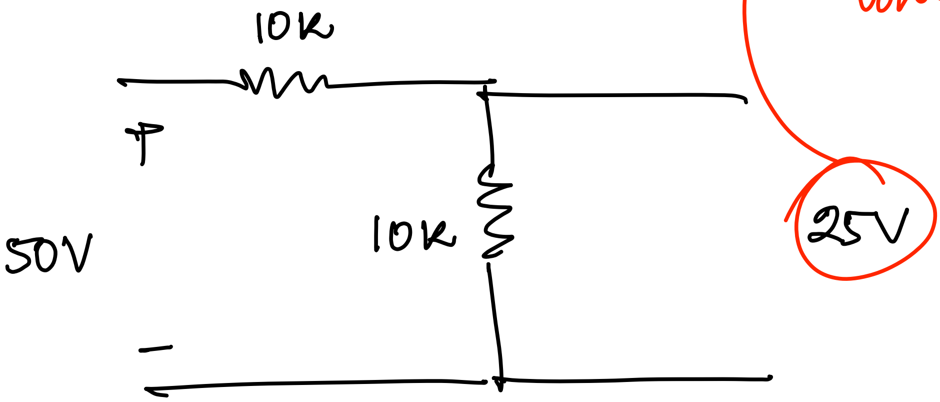
1) Using Motor-Generator Set
late 19th century and early 20th century.

$400\ V_{dc} \rightarrow 600\ V_{dc}$



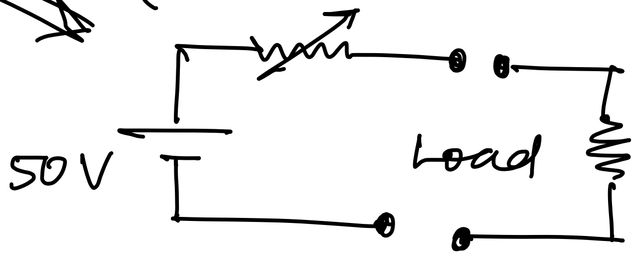
2) Voltage Divider Circuit

50 V_{dc} ⟶ 25 V_{dc}



This 25V is only at no load. Agar R_L aagya toh the Vout will change

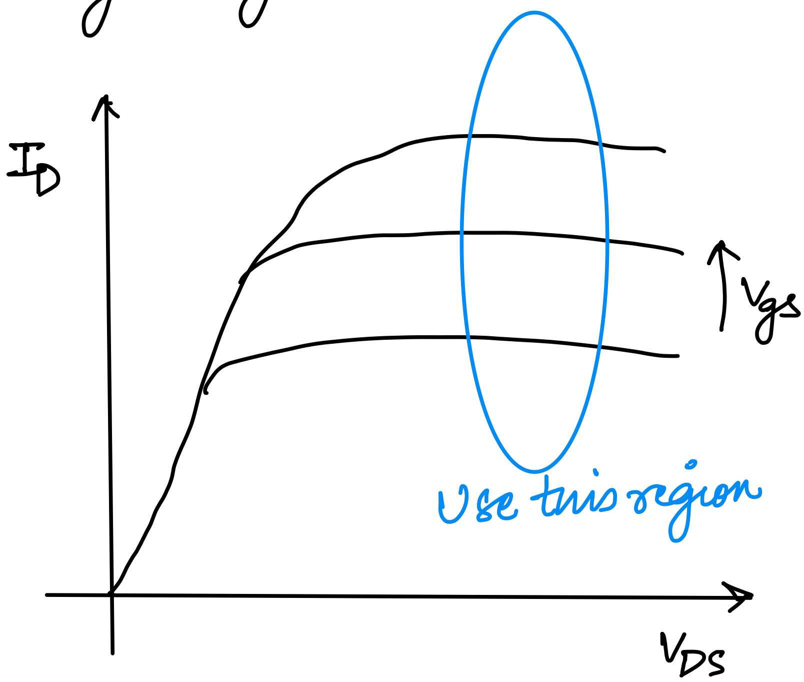
Partly solves η with variable R.



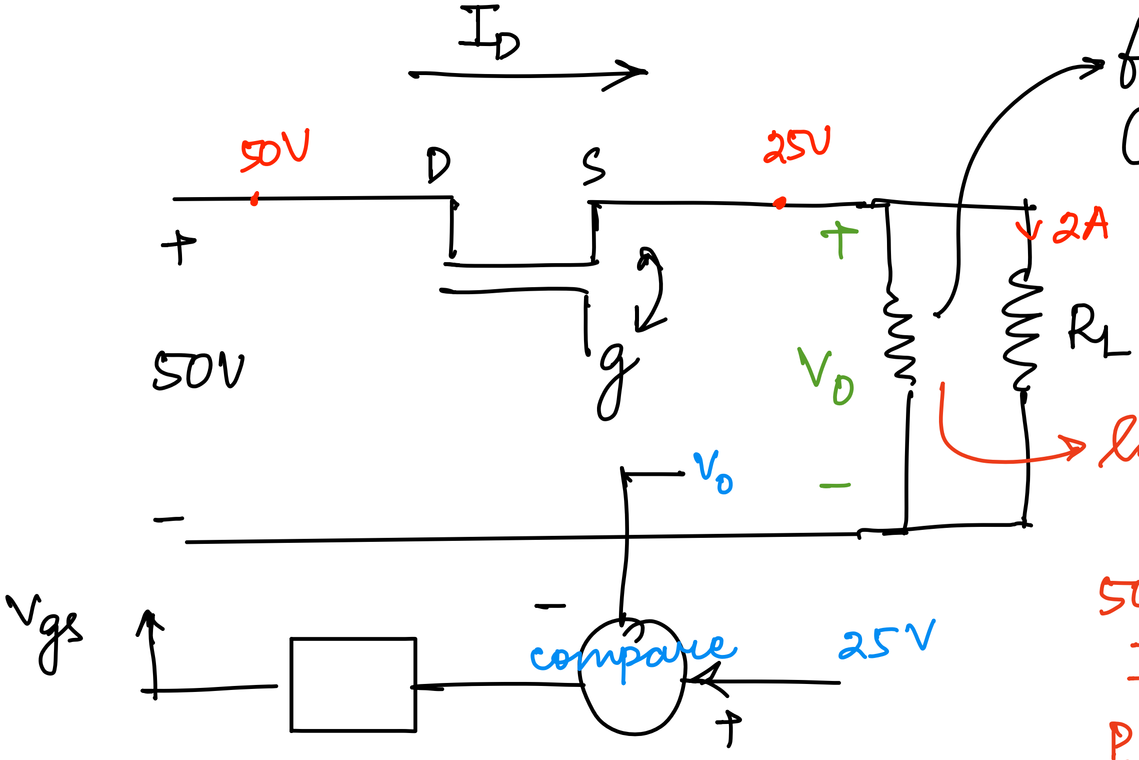
Disadvantages

→ Poor η $\left(\frac{P_{out}}{P_{in}}\right)$
→ Heat loss
→ No step up
→ Output voltage drastically depends on output load. Unacceptable voltage regulation.

3) Linear Regulators



MOSFET curve



fixed load (useful at no load)

large (10s of kΩ)

50V → 25V

$I_o = 2A$

$P_o = 50W$

$P_{loss} = (50-25) \times 2$

$= 50\ W$

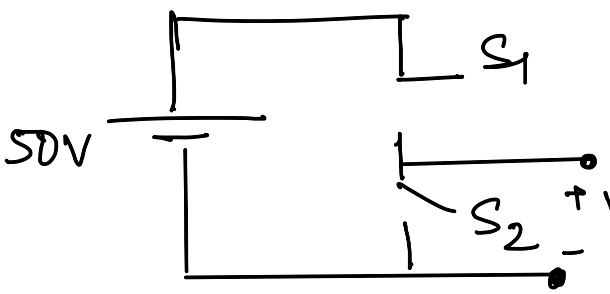
(operation, efficiency)

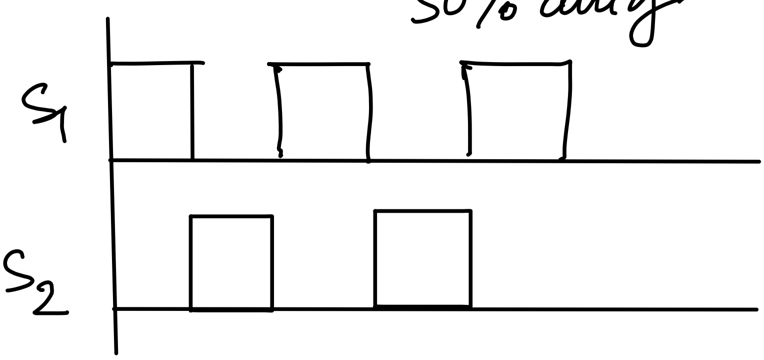
↳ still have scope of improvement.

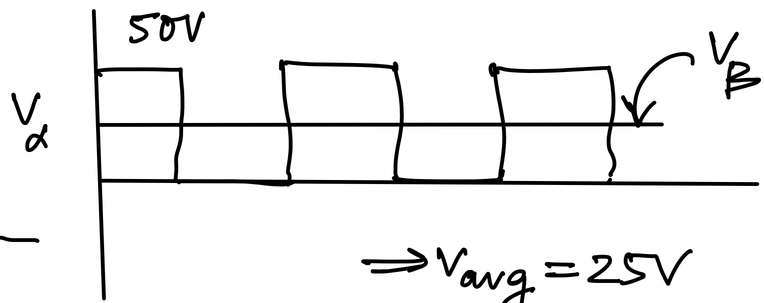
↳ good load voltage regulation.

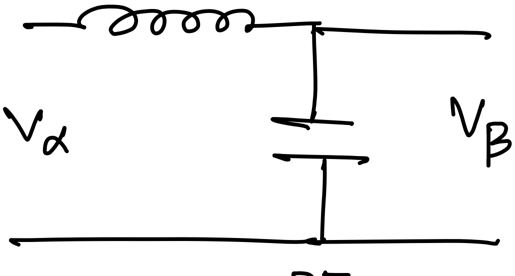
4) Switching Regulators

50% duty



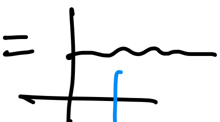






LPF

$V_B =$



very small ripple.

$\Rightarrow V_{avg} = 25V$

Step 1 → PWM
Step 2 → Low pass filter.
(To filter out high frequency comp and retain only the avg)

no load ⇒ infinite resistance
heavy load ⇒ low resistance.

S_1, S_2 → can't be mechanical
↳ could be MOSFET (different operating region)

When using MOSFET as Linear Regulator → operate in saturation region.

When using as Semiconductor Switch → operate in ohmic region and cut-off region.

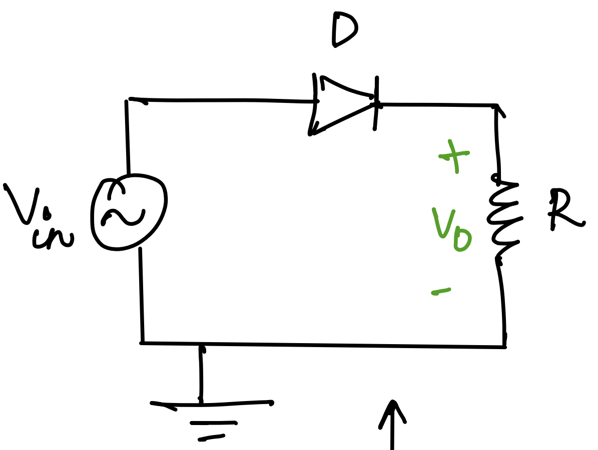
Mercury Arc Valve

→ Grid (Mesh) with a −ve charge is placed for turning off

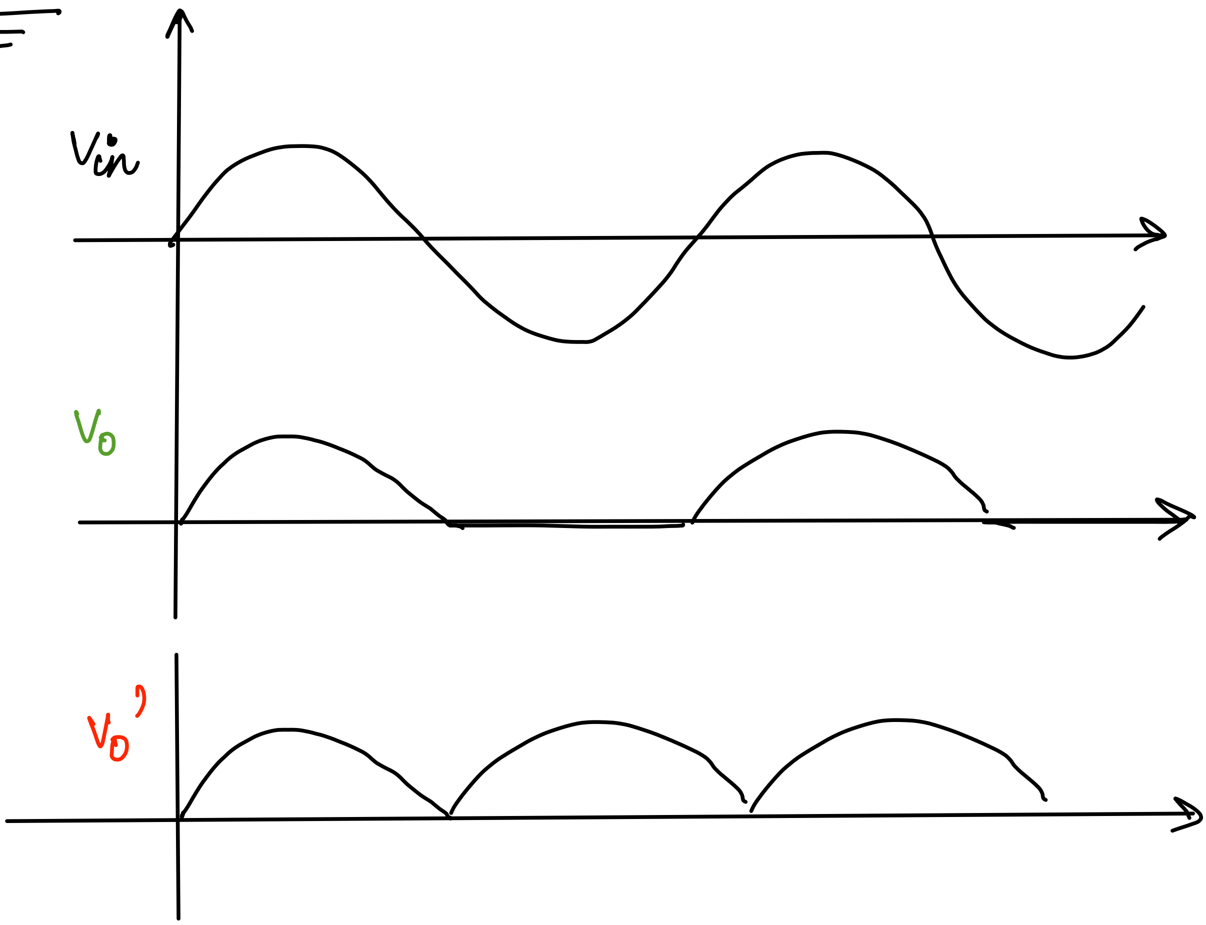
→ Can turn it on, cannot turn it off through grid voltage.

↳ Semi Controlled Devices

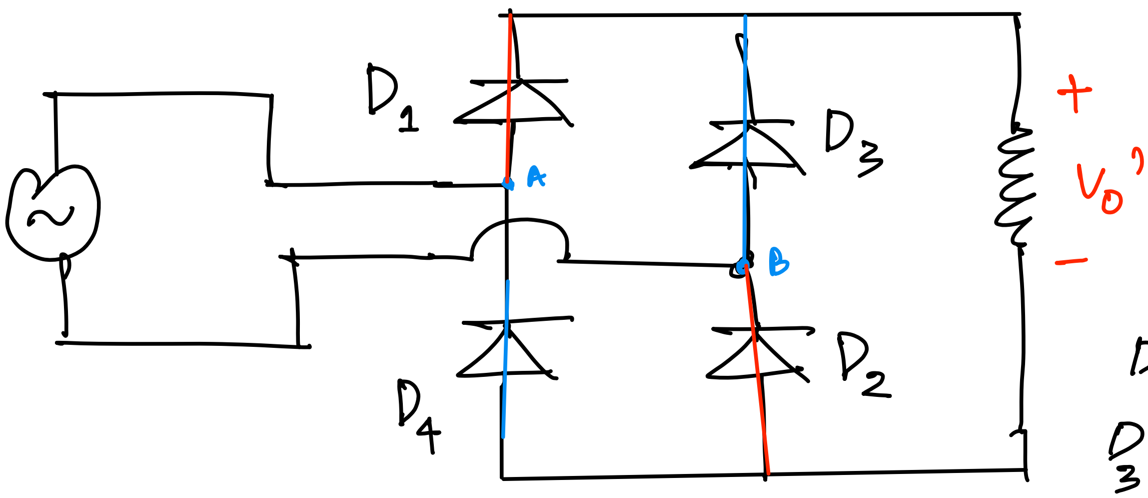
Uncontrolled Rectifier



• Half wave with R load.



Full Bridge



$D_1 - D_2$

$D_3 - D_4$

Negative half cycle:

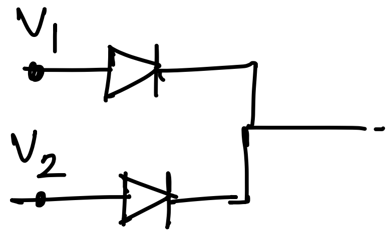
$V_{D_1} = V_A - V_B = V_{in}$ $V_{D_2} = V_A - V_B = V_{in}$

$V_{D_3} = 0$ $V_{D_4} = 0$

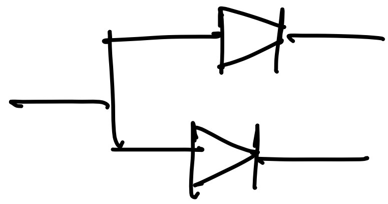
Positive half cycle

$V_{D_3} = V_B - V_A = -V_{in}$ $V_{D_4} = V_B - V_A = -V_{in}$ $V_{D_1} = 0,\ V_{D_2} = 0$

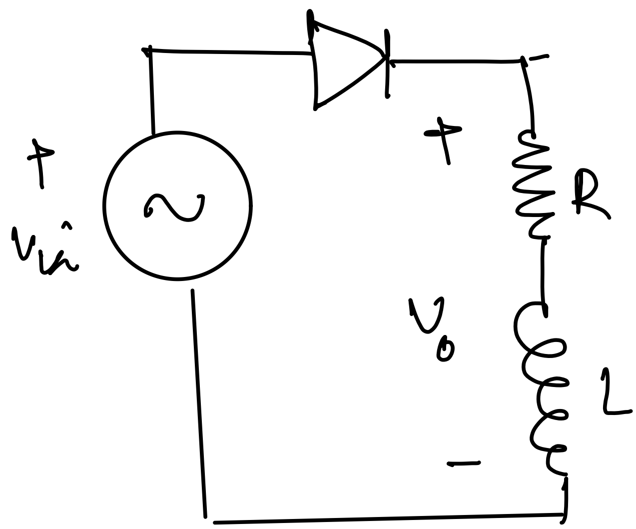
Common Cathode Configuration



Common Anode Configuration

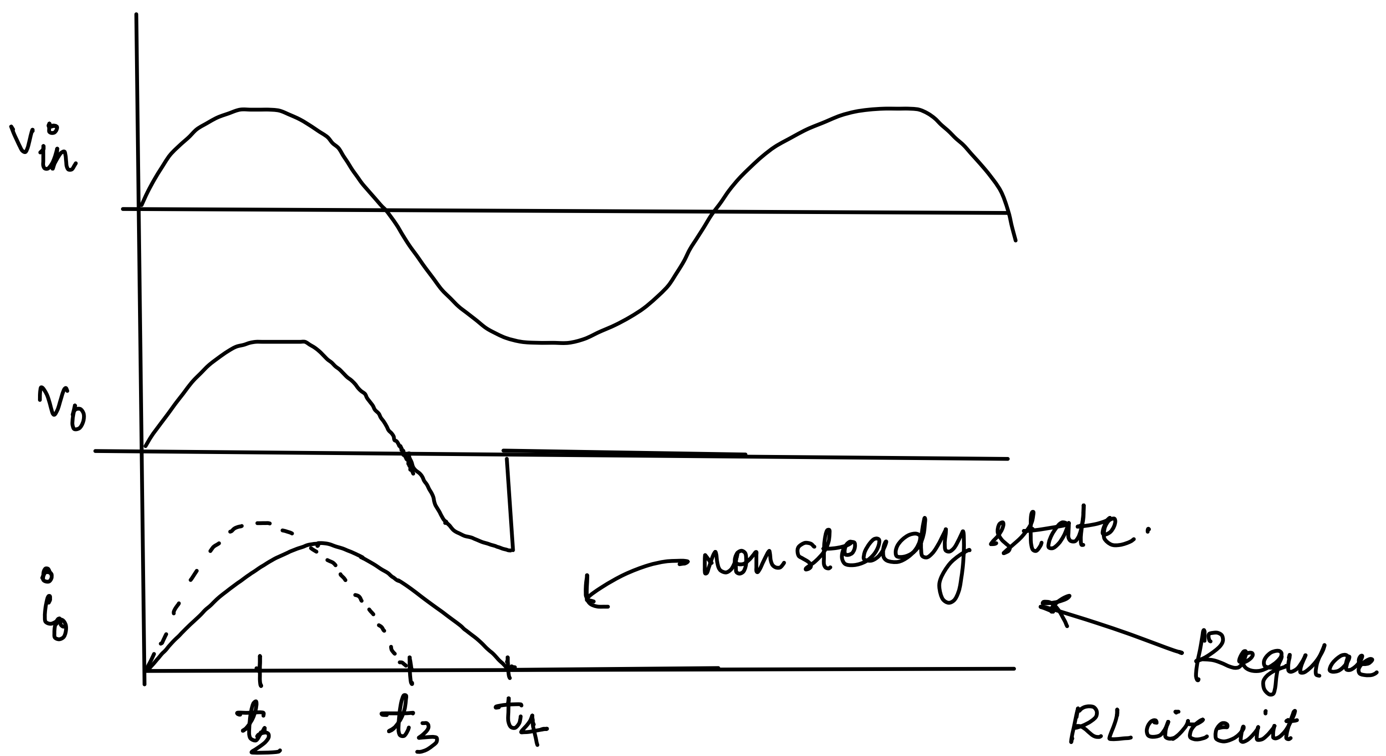


Half wave with R-L load.

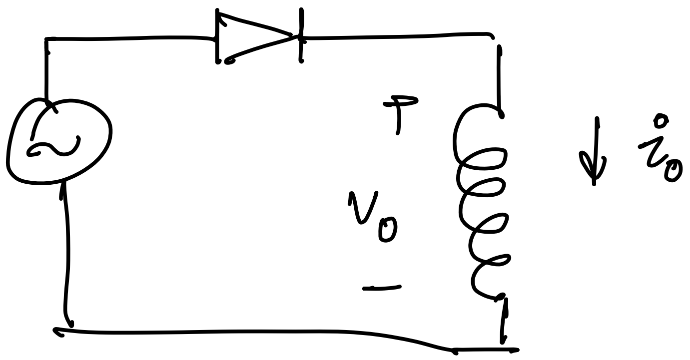


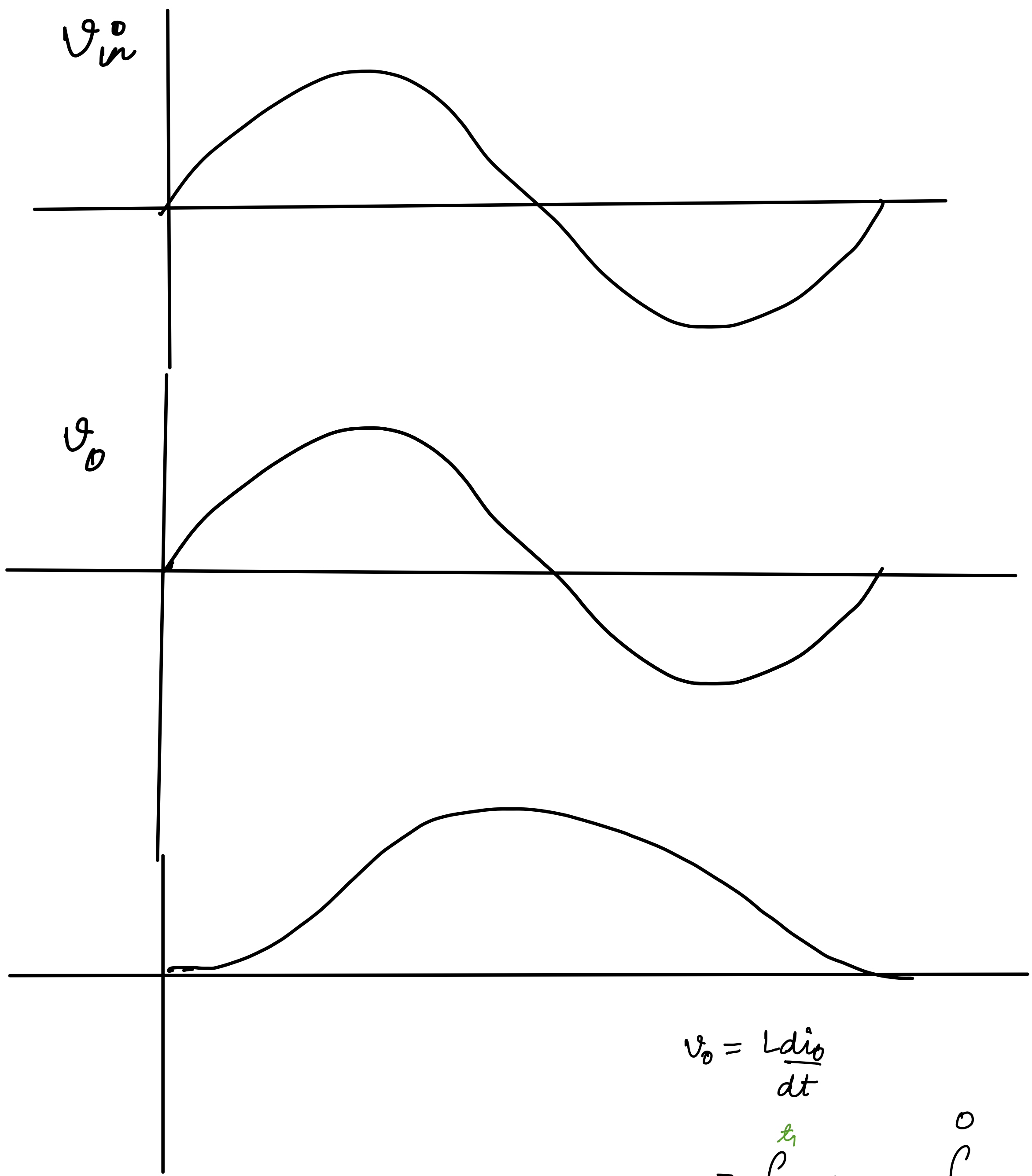
$$v_0 = i_0 R + \frac{L di_0}{dt}$$

$$v_0 - i_0 R = \frac{L di_0}{dt}$$



(lagging peak)

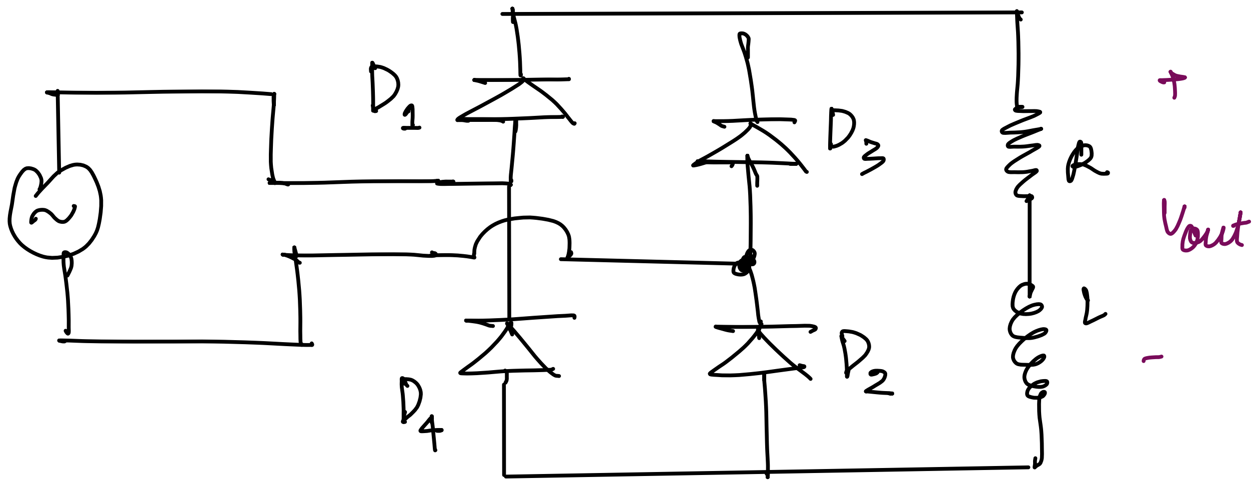


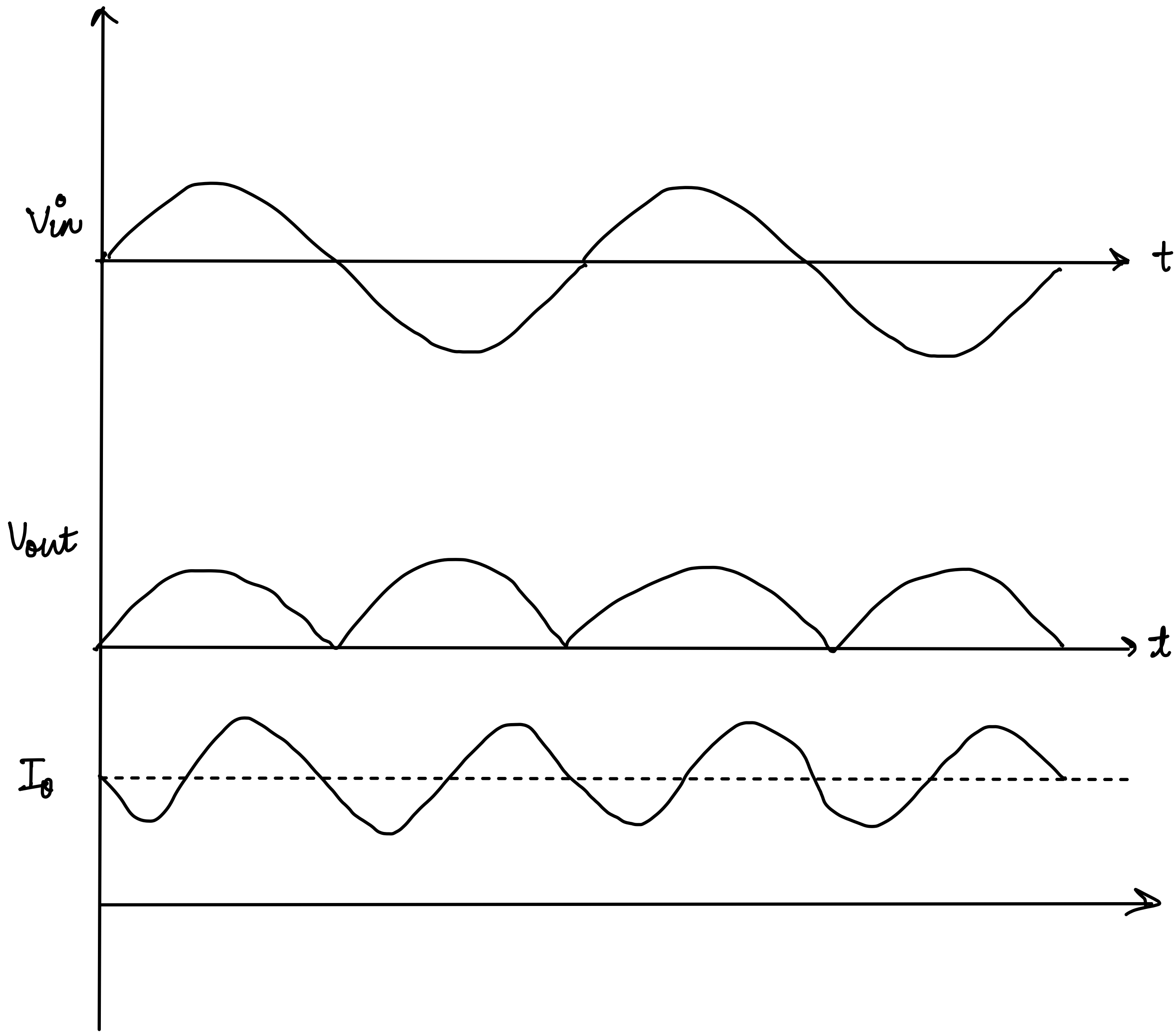


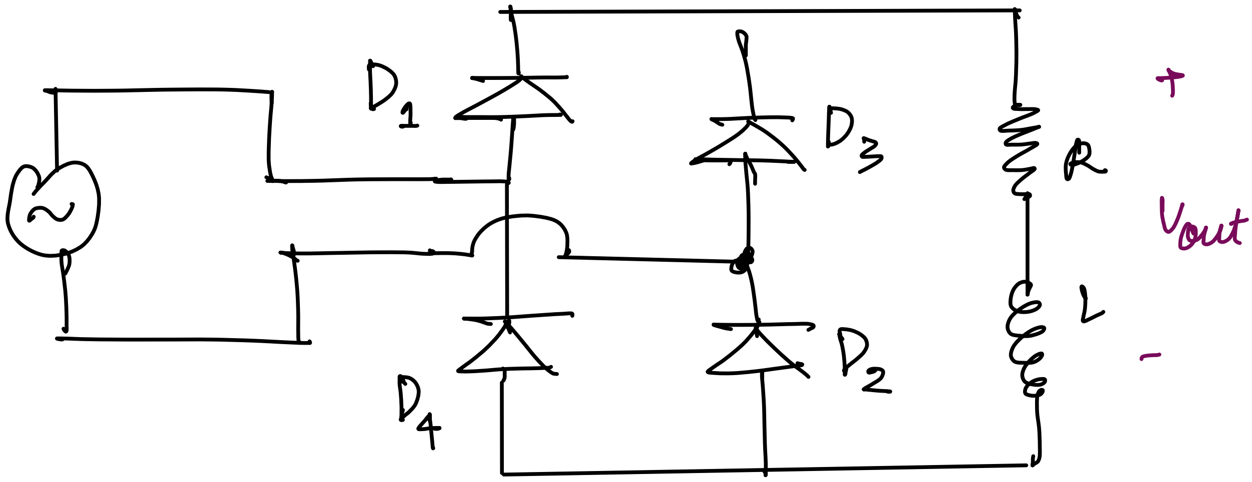
$$v_0 = L\frac{di_0}{dt}$$

$$= \int_0^{t_1} v_0\,dt = L\int_0^0 di_0$$

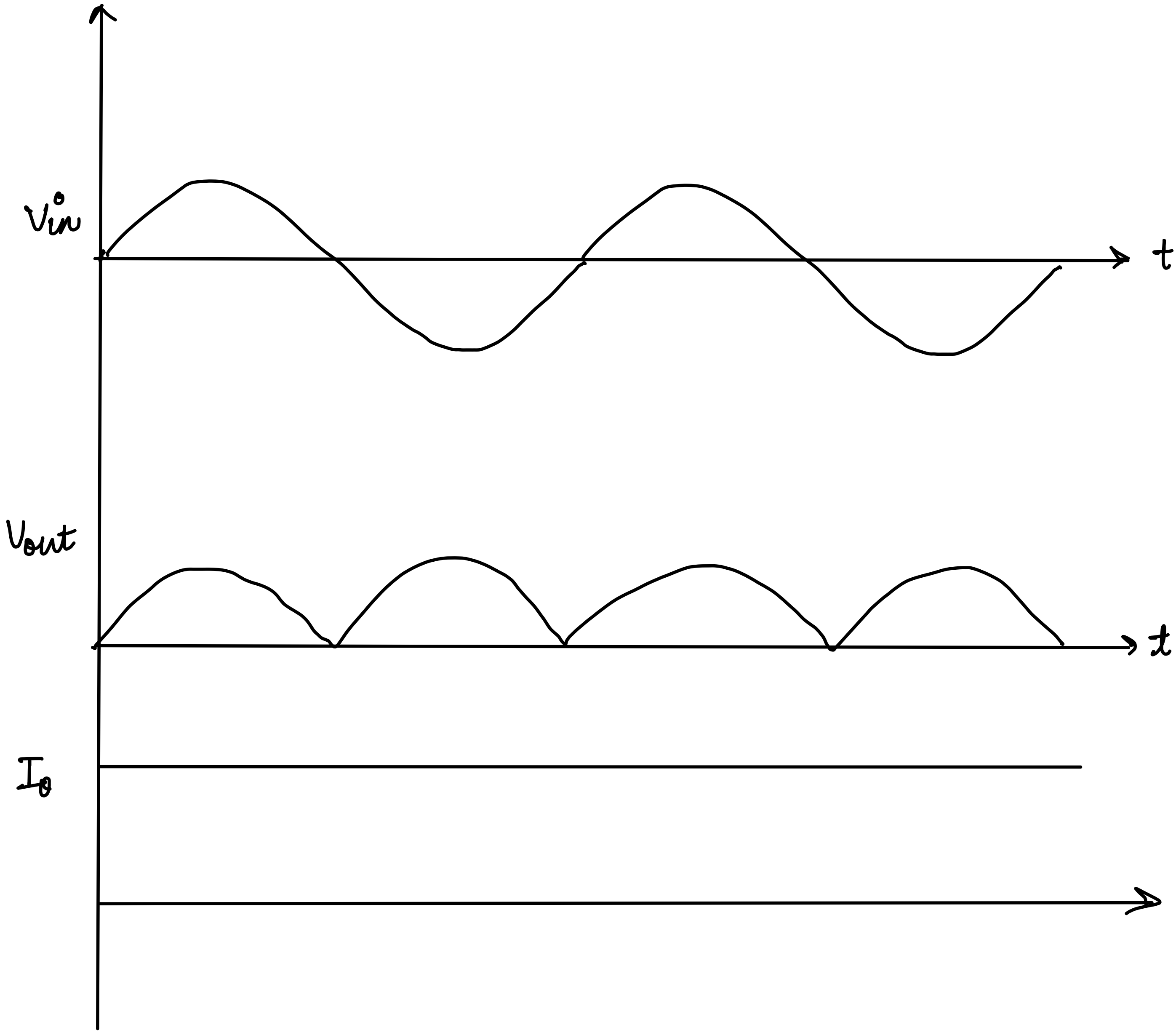
Q Do both these for full bridge rectifier





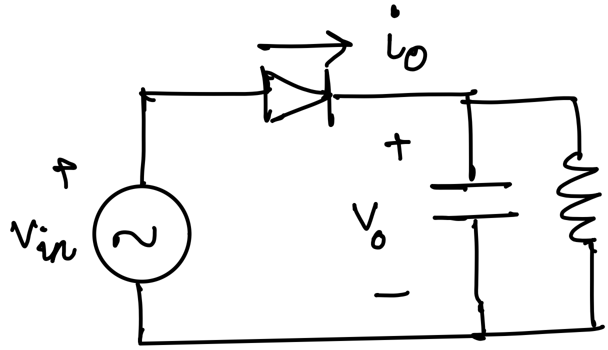


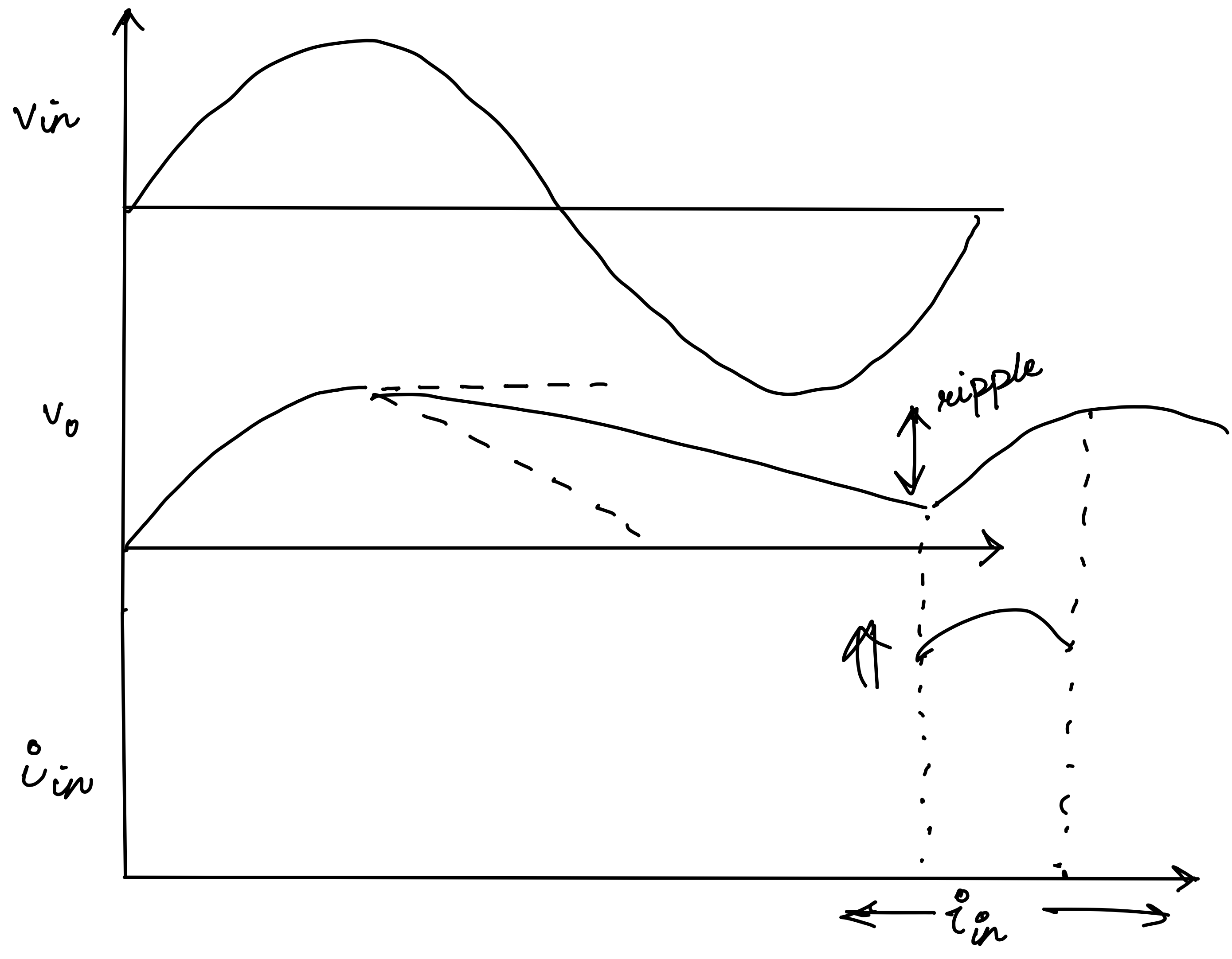
D1
D3
R
+
Vout
L
-
D2
D4



Vin
t
Vout
t
Io

Half-Wave, Full Bridge with C-R Load.

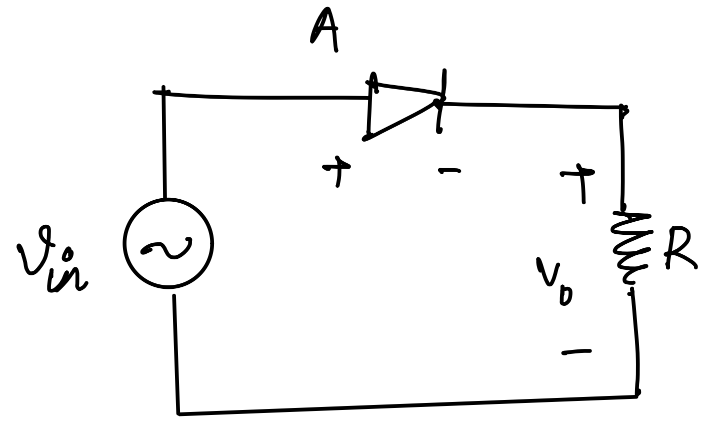


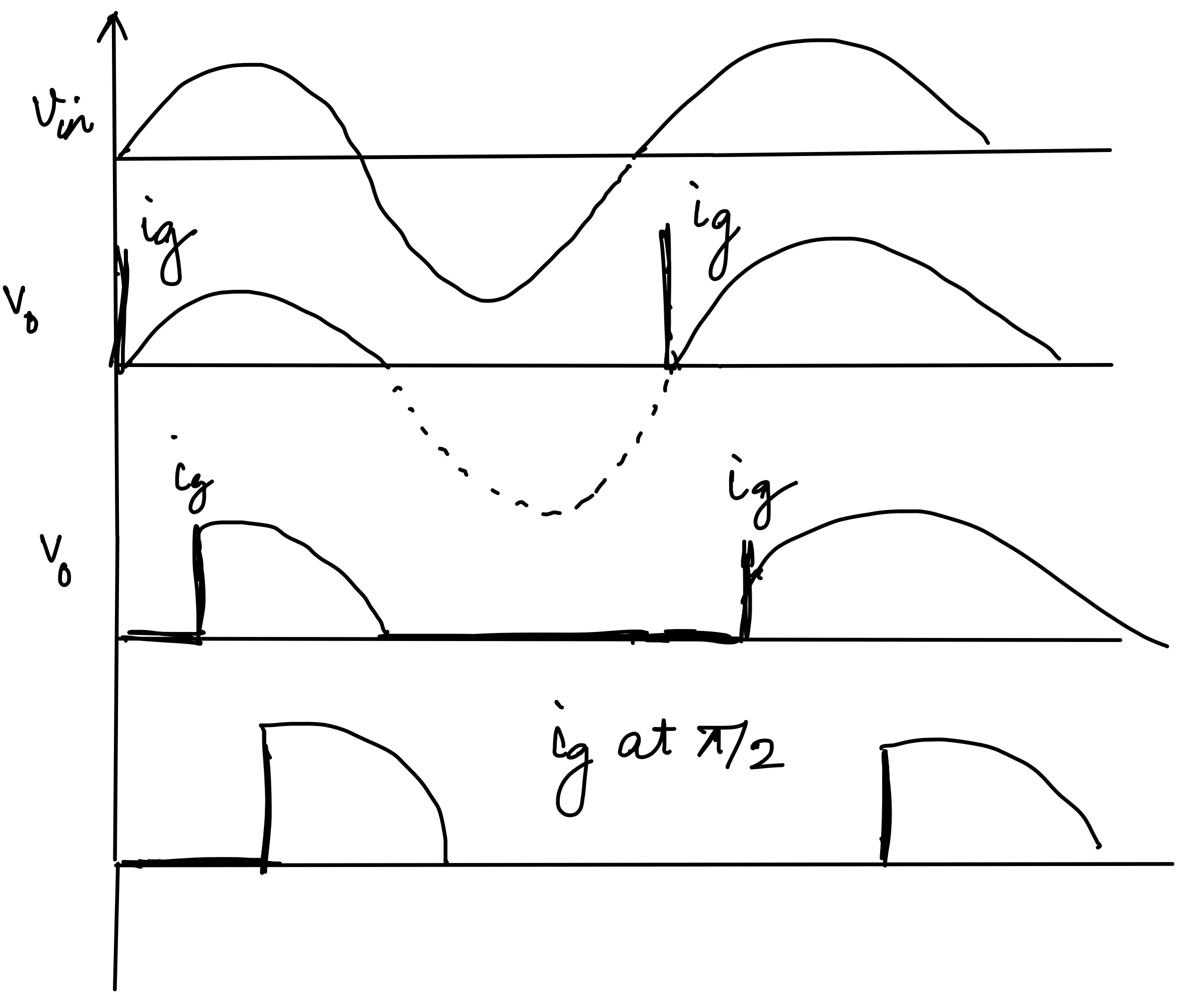


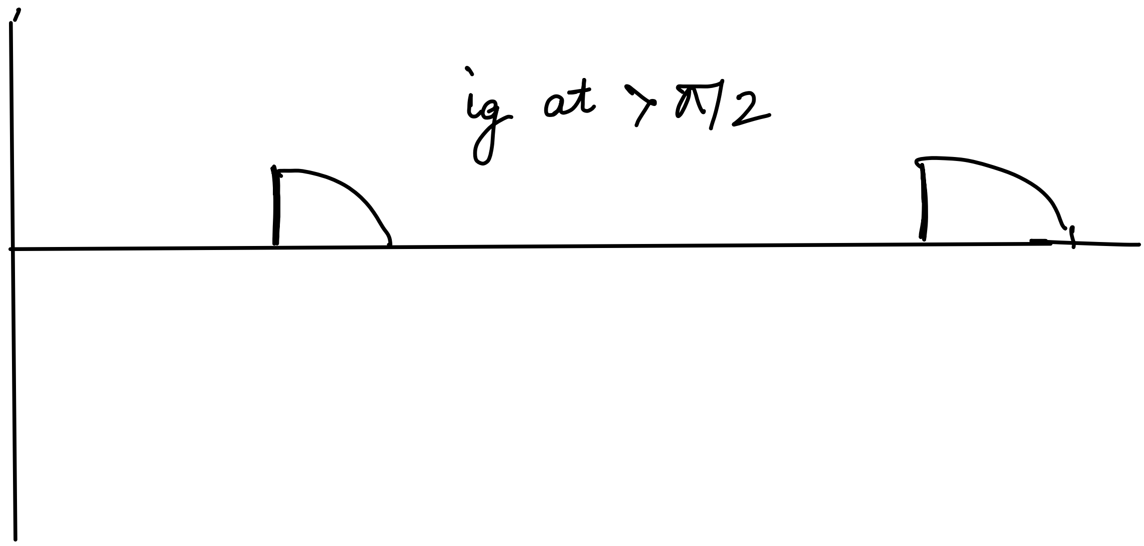
If C is large, diode may be damaged.
, power quality drops.

$$\langle V_0 \rangle \approx 325\ V$$

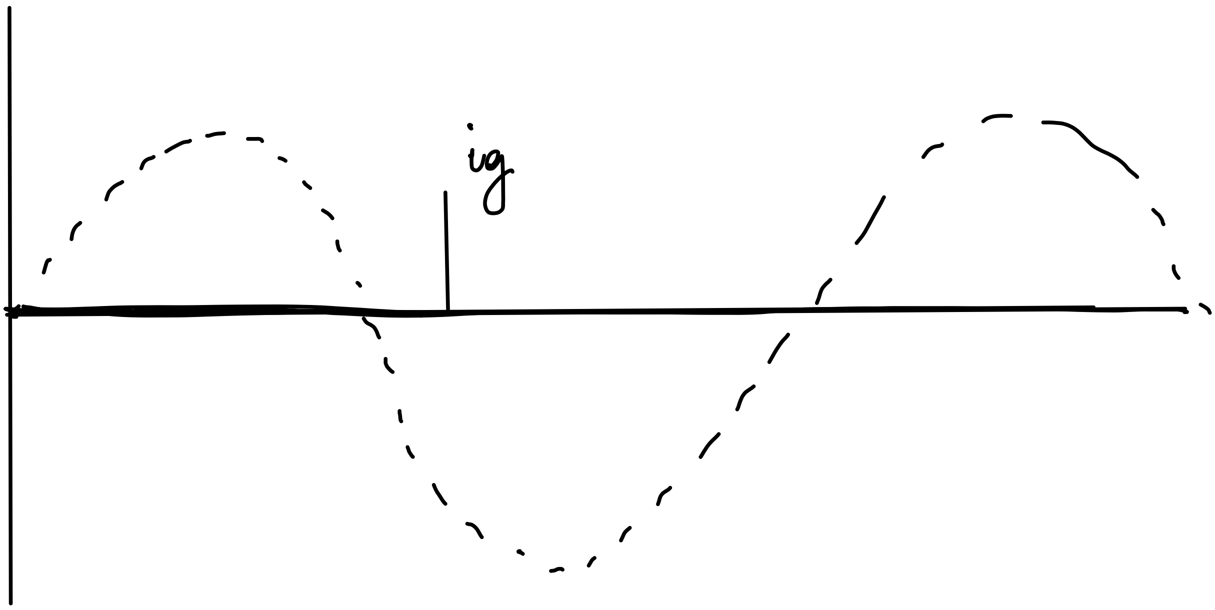
- Semi-Controlled Device Based Rectifier.







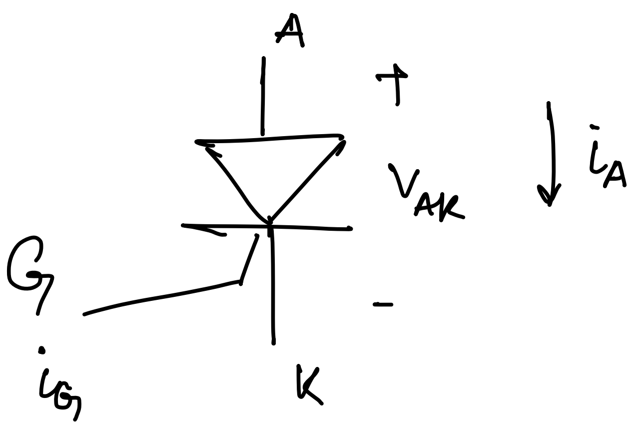
By varying ig avg. output voltage can be varied.



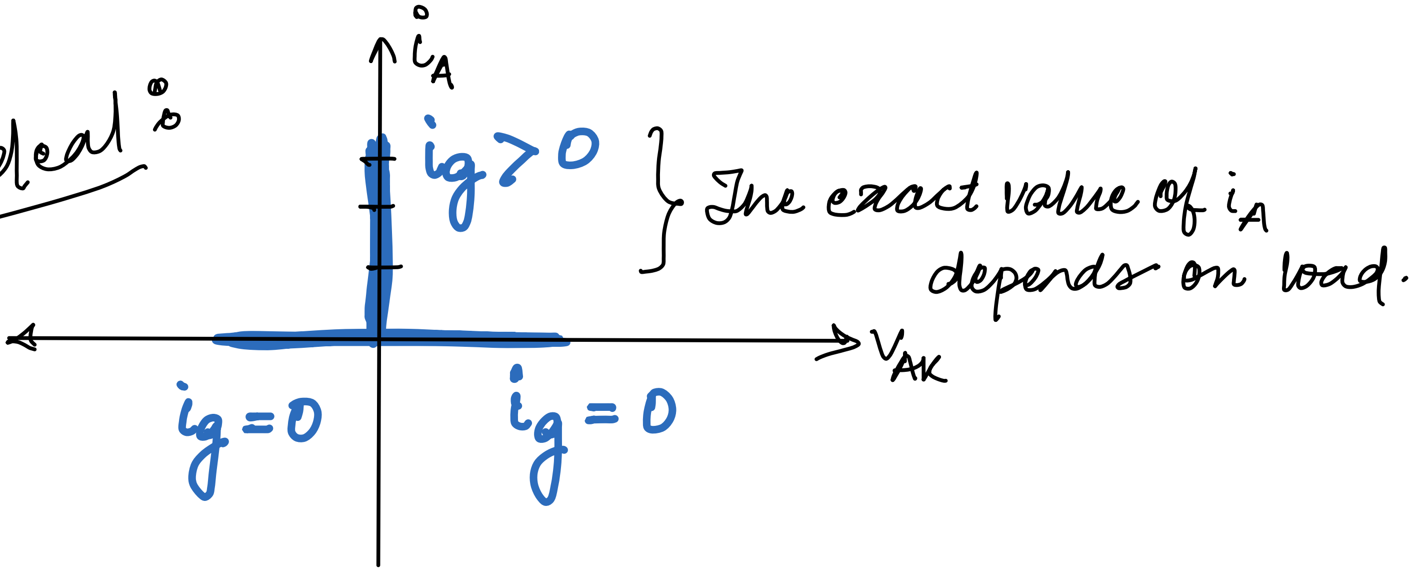
For arc to be formed

- anode - cathode potential should be +ve ($V_{AK} > 0$)
- $i_g > 0$

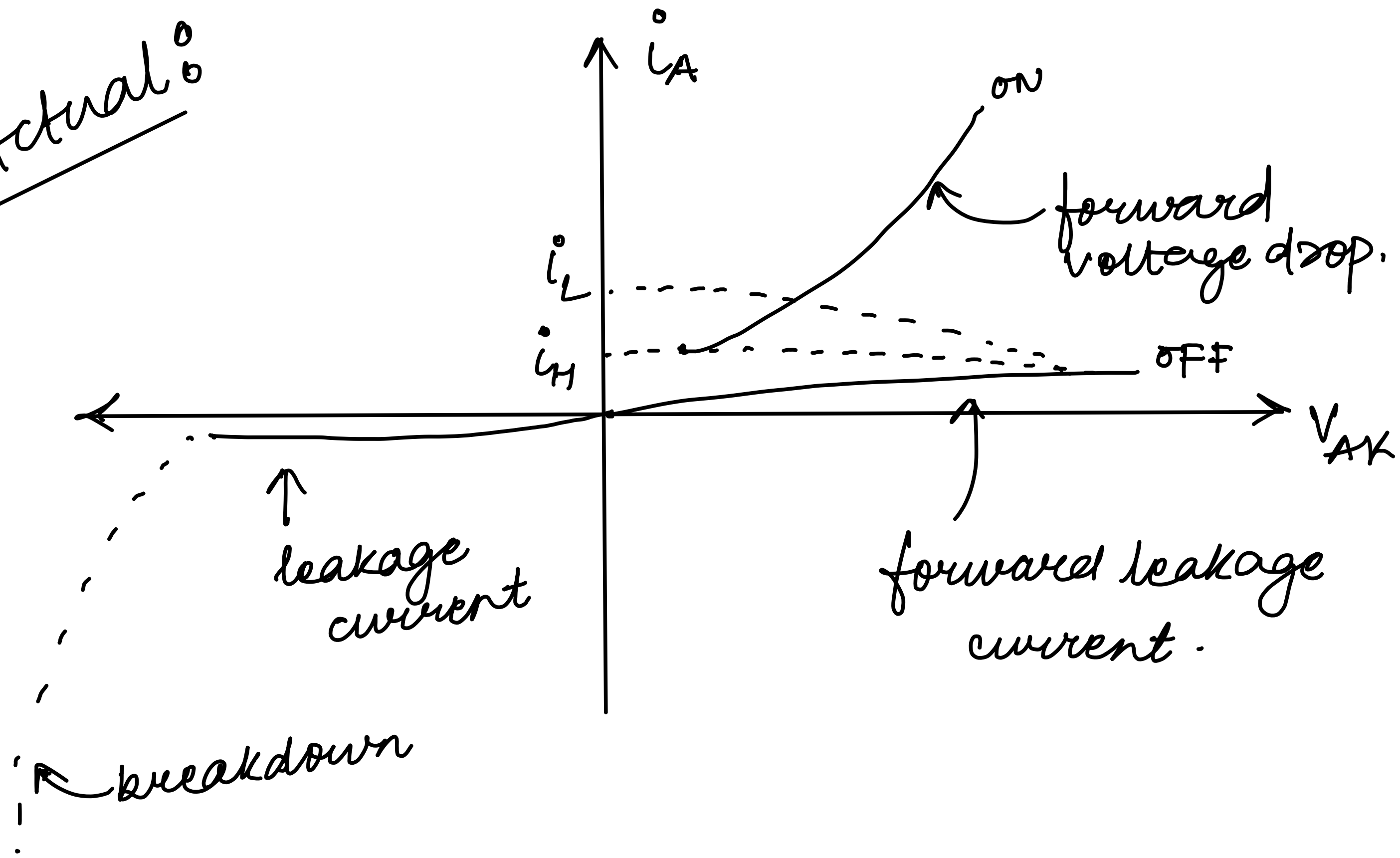
Thyristor (aka Silicon Controlled Rectifiers)



Ideal:



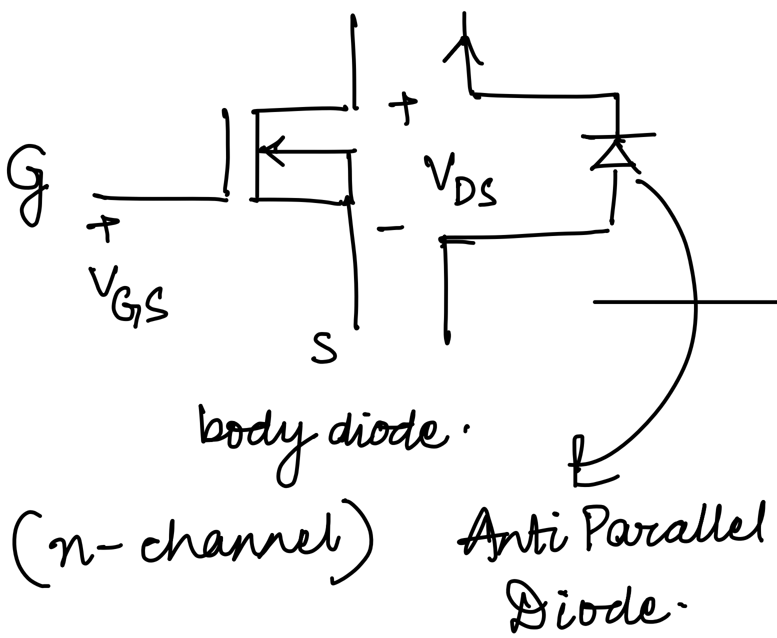
Actual:

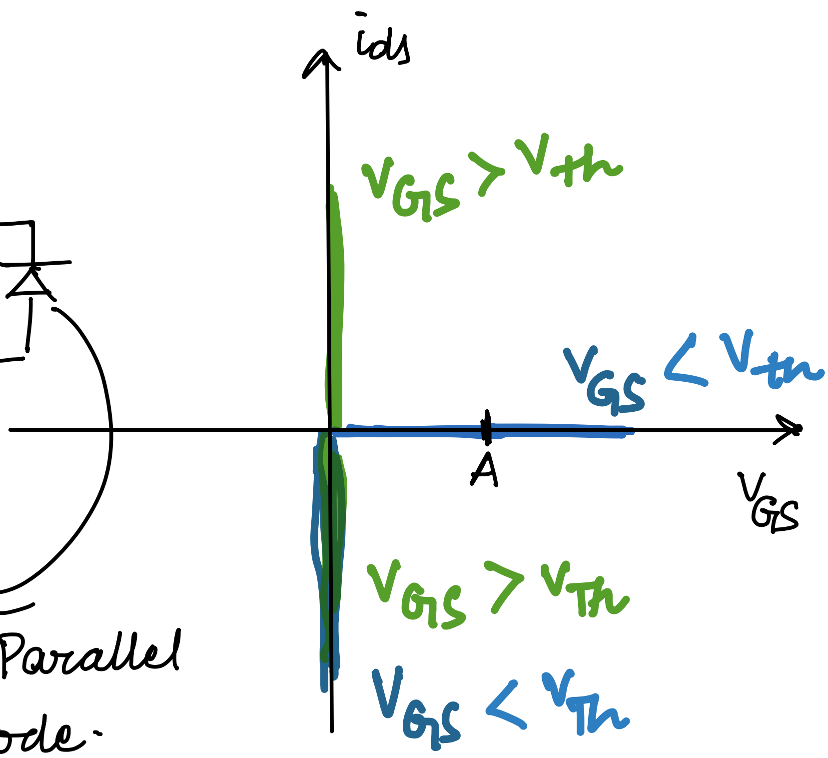


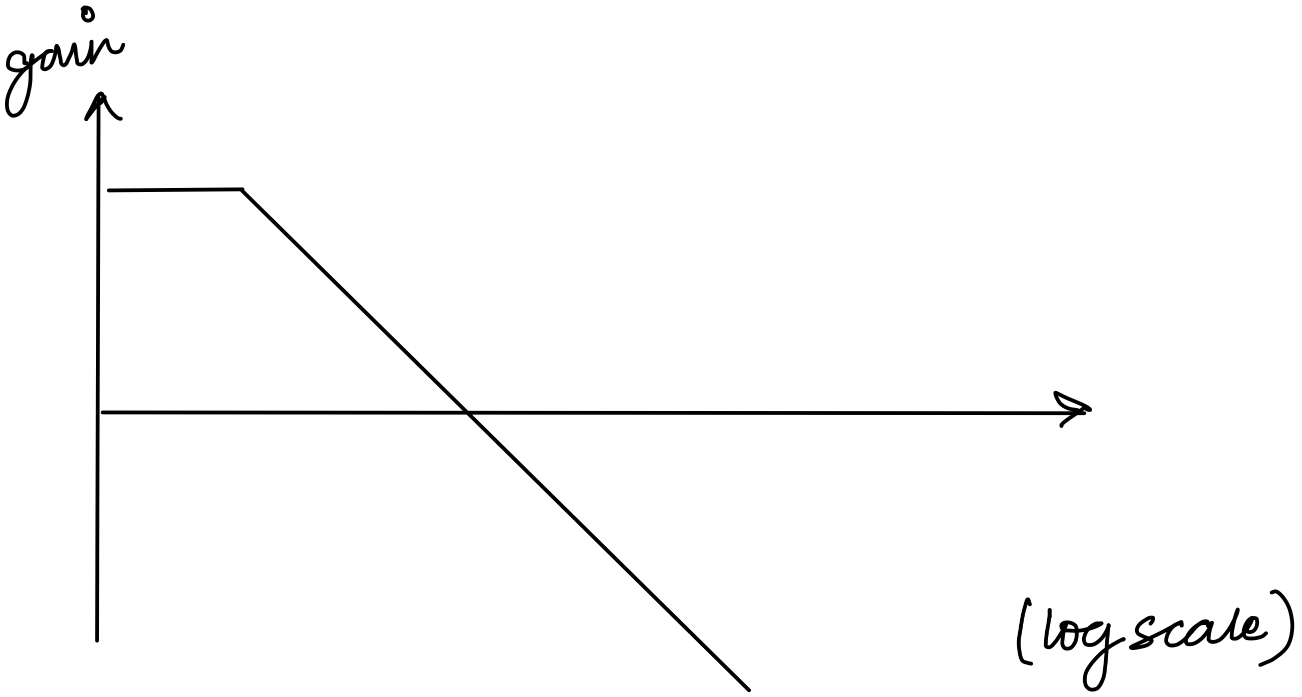
i_L : Latching Current for device to turn on

i_H : Holding Current for device to remain on.

Power MOSFET







We want higher frequency to filter out the high frequency component easily

Higher frequency ⇒ L, C.

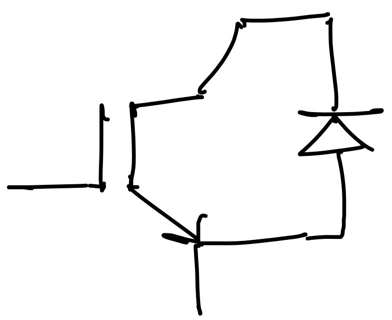
MOSFET ⇒ 100s KHz
IGBT ⇒ 10s KHz
IGCT ⇒ KHz
GTO ⇒

Increasing Blocking Voltage.

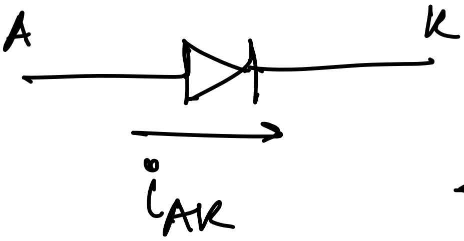
→ Trade off between efficiency & size due to F_s.

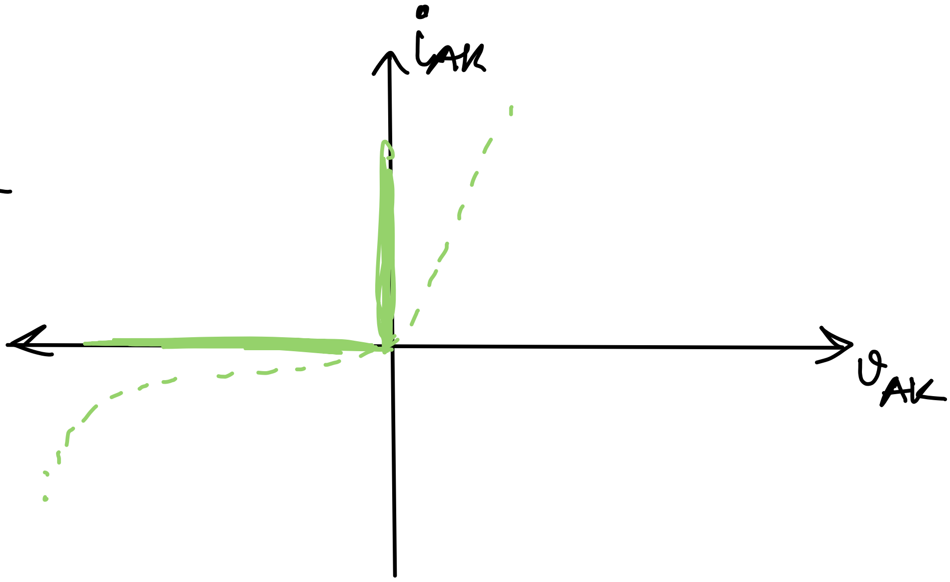
↪ (switching frequency)

IGBT



Diode





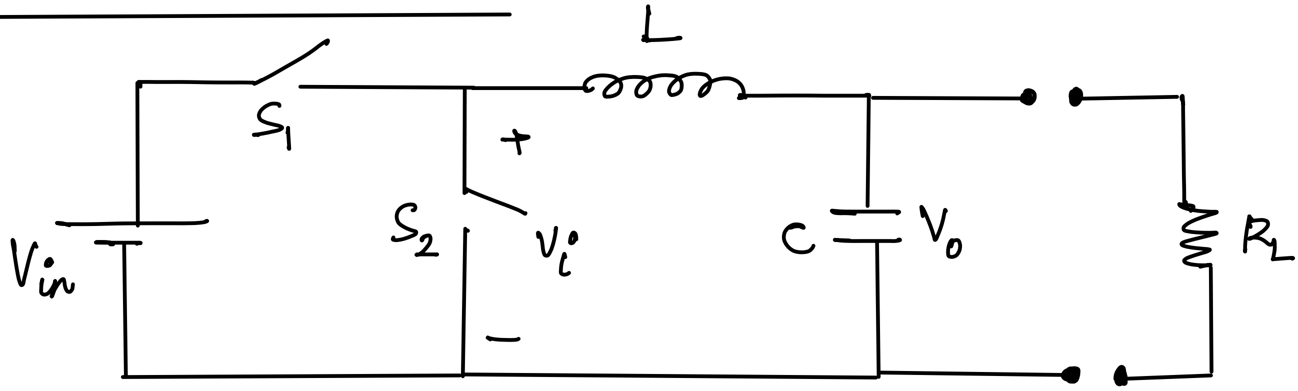
Uncontrolled → Diode.

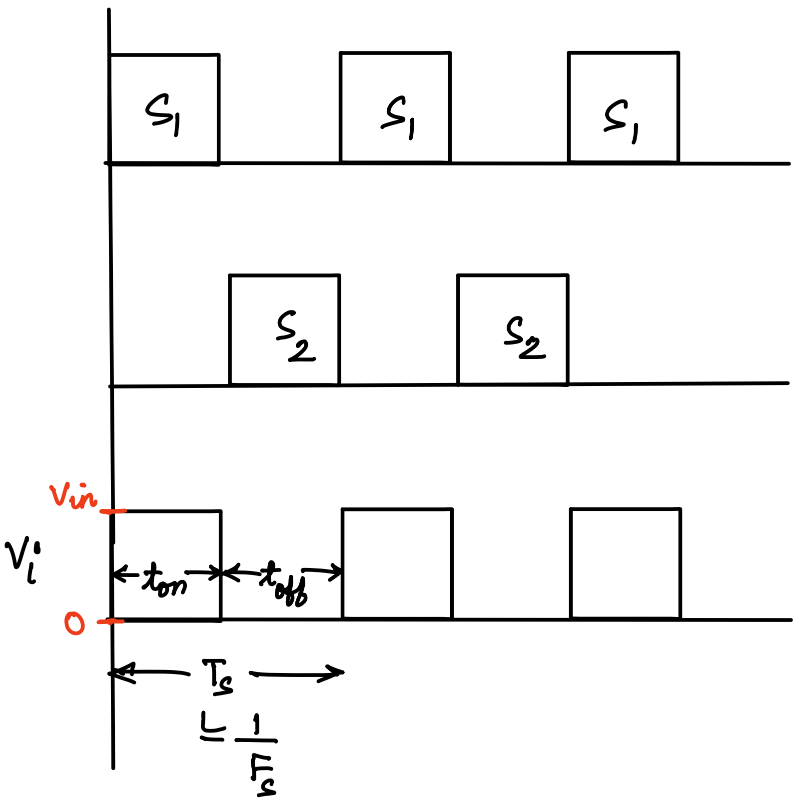
Semi controlled → Thyristor, Mercury Arc.

Fully controlled → MOSFET

Chargers → Si → GaN, SC

Buck Converters





$$\langle v_i \rangle = \frac{1}{T_s} \int_0^{T_s} v_i \, dt$$

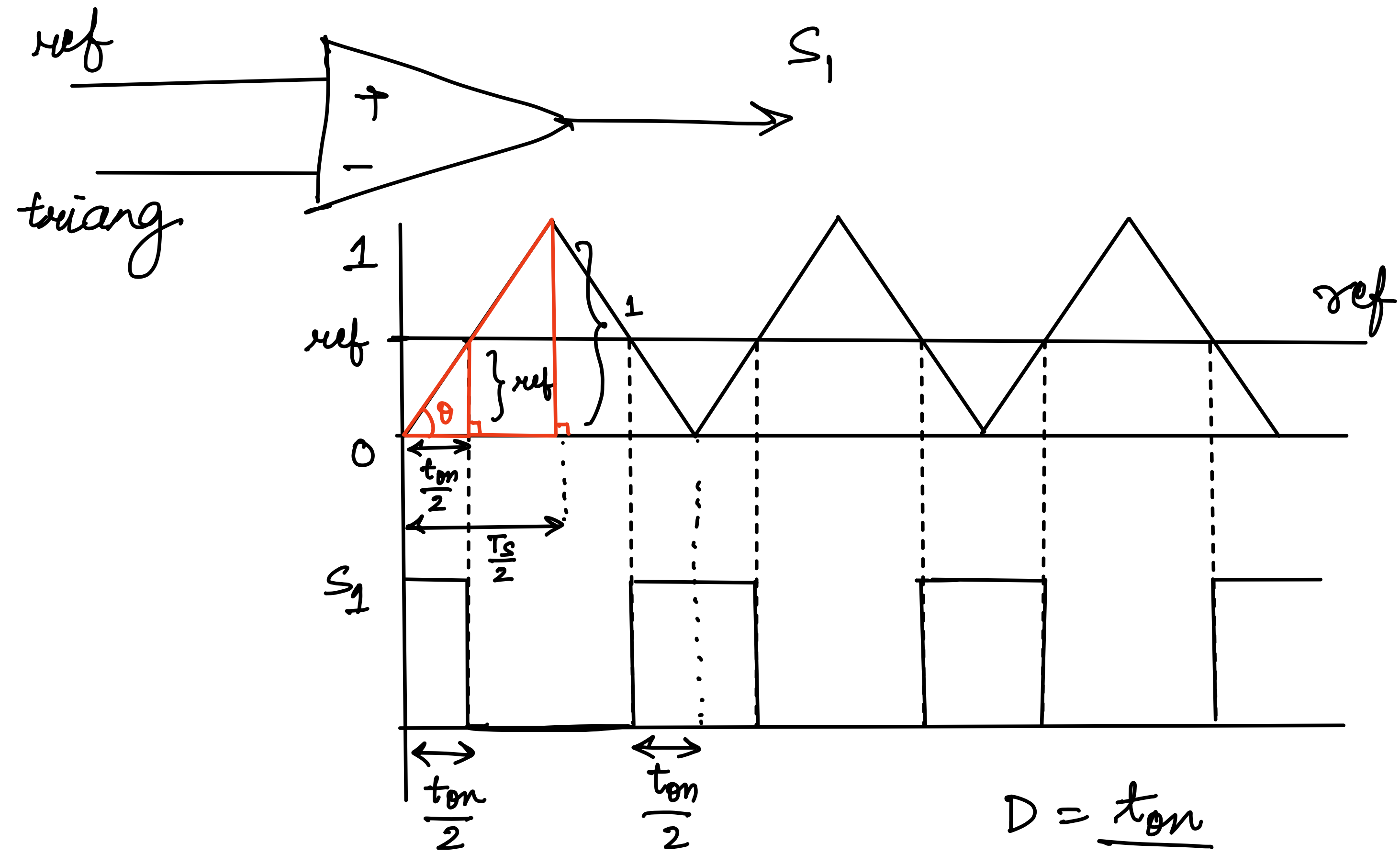
$$= \frac{1}{T_s} \left[\int_0^{t_{on}} V_{in} \, dt + \int_{t_{on}}^{T_s} 0 \, dt \right]$$

$$= \frac{t_{on}}{T_s} V_{in} = D V_{in} \quad (< V)$$

$$\boxed{D = \frac{t_{on}}{T_s} = \text{Duty Cycle.}}$$

Fundamental def^n of duty cycle.

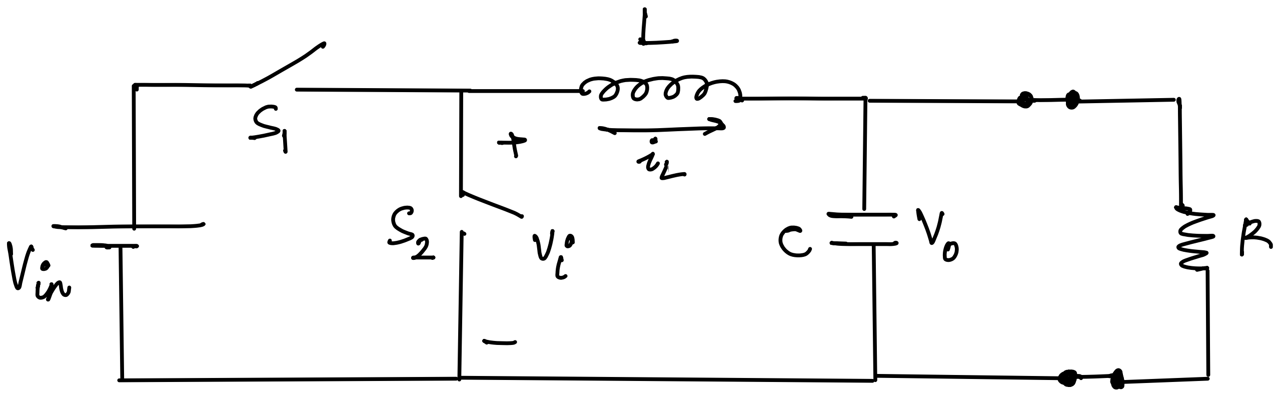
Modulation Signal Generation

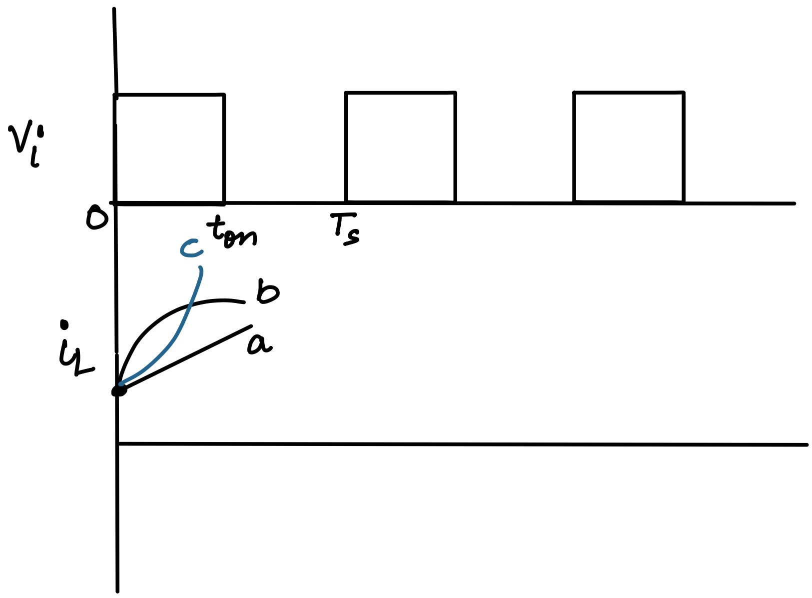


$$D = \frac{t_{on}}{T_s}$$

$$\tan\theta = \frac{1}{T_s/\cancel{2}} = \frac{(ref)}{(t_{on}/\cancel{2})}$$

$$\Rightarrow D = ref.$$

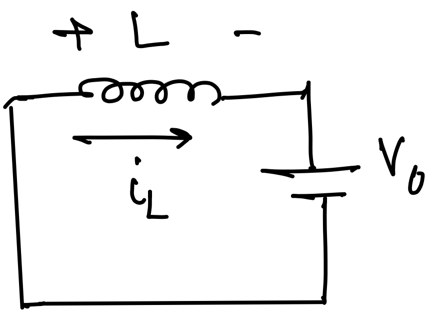




a → Ideal
b → Real (R+L)
c → If core is Saturable

For first part : $V_{in} - V_0 = L\dfrac{di_L}{dt}$

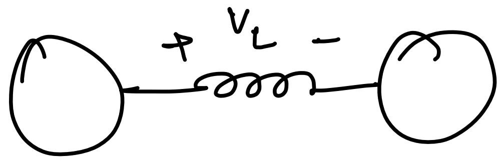
For second part :



$$-V_0 = L\frac{di_L}{dt}$$

Average Output Voltage

Define voltage second balance



At sinusoidal steady state (S-SS)

$f(x) = f(x+T)$

Periodic Steady State

$$V_L = L\frac{di}{dt}$$

$$\int_{t=x}^{t=x+T} V_L\,dt = L\int_{i_L(x)}^{i_L(x+T)} di$$

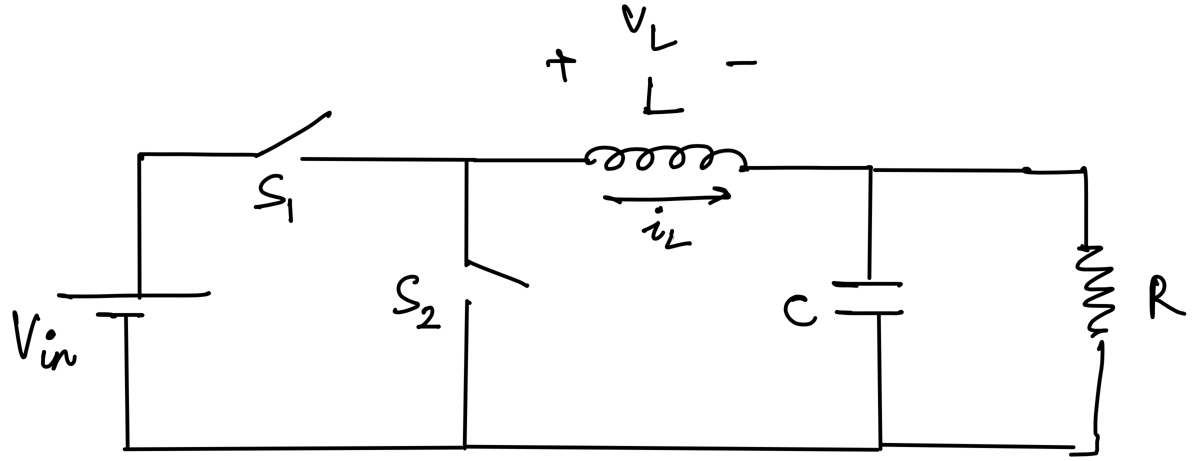
$$\Rightarrow \frac{1}{T_S}\int_0^{T_S} V_L\,dt = L\left(i_L(x+T) - i_L(x)\right)$$

$$= 0$$

$$\Rightarrow \boxed{\langle V_L \rangle_{T_S} = 0} \rightarrow \text{voltage second balance}$$

$T_S \rightarrow$ switching time period

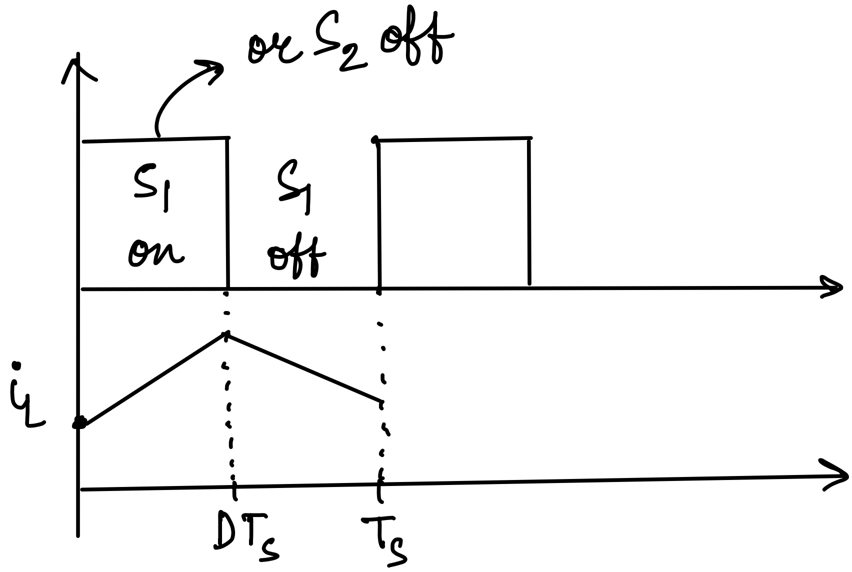
For capacitor : $\boxed{\langle i_C \rangle_{T_S} = 0}$



1) $0 \le t \le DT_s$

$V_L = V_{in} - V_0$

$i_L(0) = i_L(T_s)$



2) $DT_s \le t \le T_s$

Voltage second balance:

$$\int_0^{T_s} V_L \, dt = 0$$

$$\int_0^{DT_s} (V_{in} - V_0)\, dt + \int_{DT_s}^{T_s} (-V_0)\, dt = 0$$

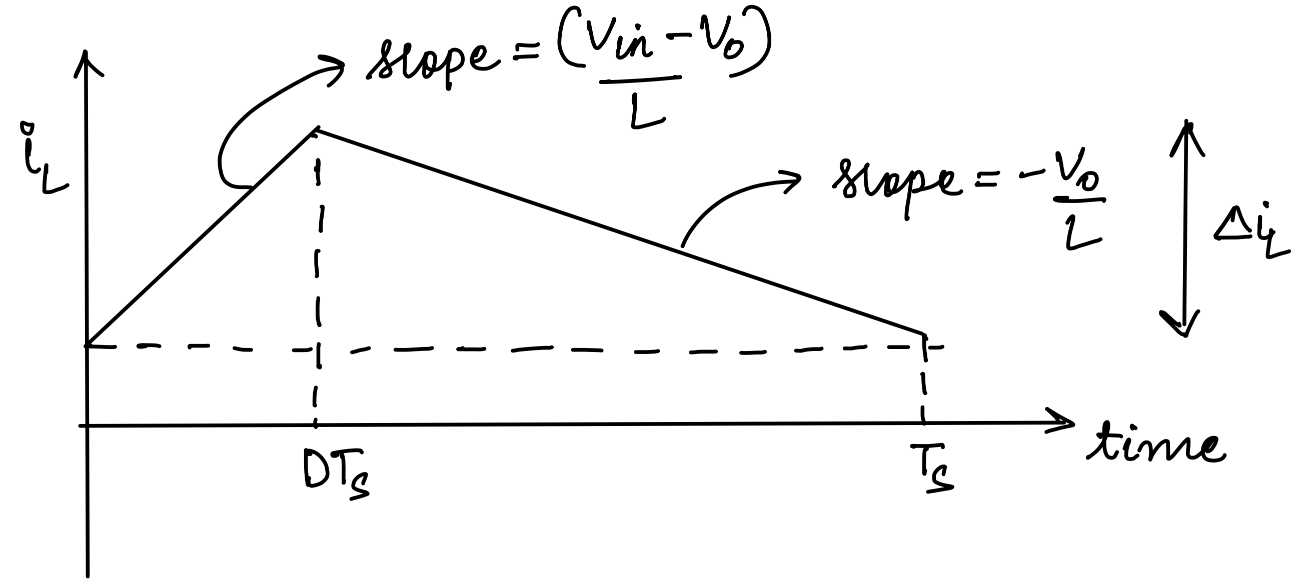
$$\Rightarrow (V_{in} - V_0)\, DT_s - V_0 (T_s - DT_s) = 0$$

$$\Rightarrow \boxed{V_o = V_{in} D}$$ for buck in CCM

(graph of i_L is above time axis)

Current Ripple

Inductor ripple current



Δi_L : Inductor ripple current

$$V_L = L\frac{di_L}{dt}$$

$$\Rightarrow \int_0^{DT_s} v_L \, dt = \int_{i_L min}^{i_L max} L \, di_L$$

$$\Rightarrow (v_{in} - v_o) DT_s = L \Delta i_L$$

$$\Rightarrow \boxed{\Delta i_L = \frac{(V_{in} - V_o) DT_s}{L}}$$

$$\Rightarrow \Delta i_L = \frac{(V_{in} - DV_{in}) DT_s}{L} \quad \text{(only for CCM)}$$

$$\Rightarrow \boxed{\Delta i_L = \frac{(1-D) D V_{in} T_s}{L}} \rightarrow \text{for CCM}$$

↑ inductance, ripple ↓

↑ $F_s \Rightarrow T_s \downarrow \Rightarrow$ ripple ↓

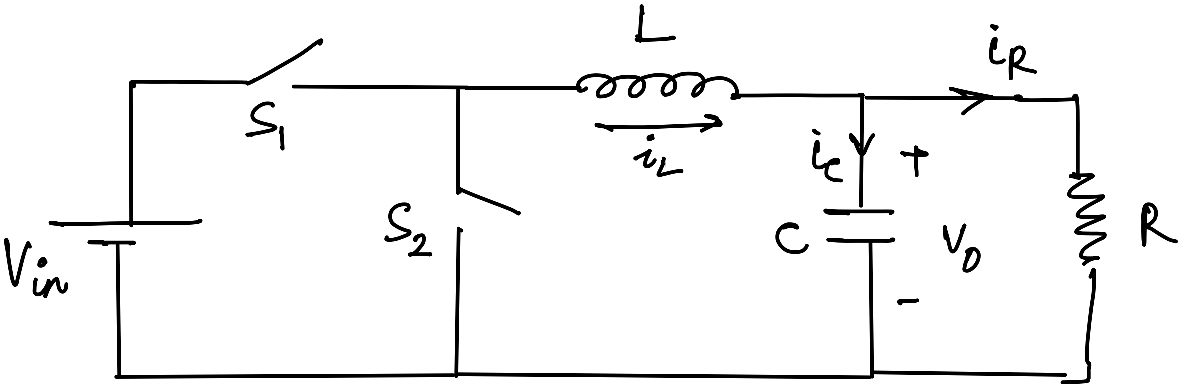
For a given Δi_L, how to reduce L (because we want to reduce size of L)

$F_s \uparrow\uparrow$ (but switching losses increase)

- Δi_L max happens at $D = 0.5$

D → usually fixed by V_{in} and V_o

Capacitor Voltage Ripple



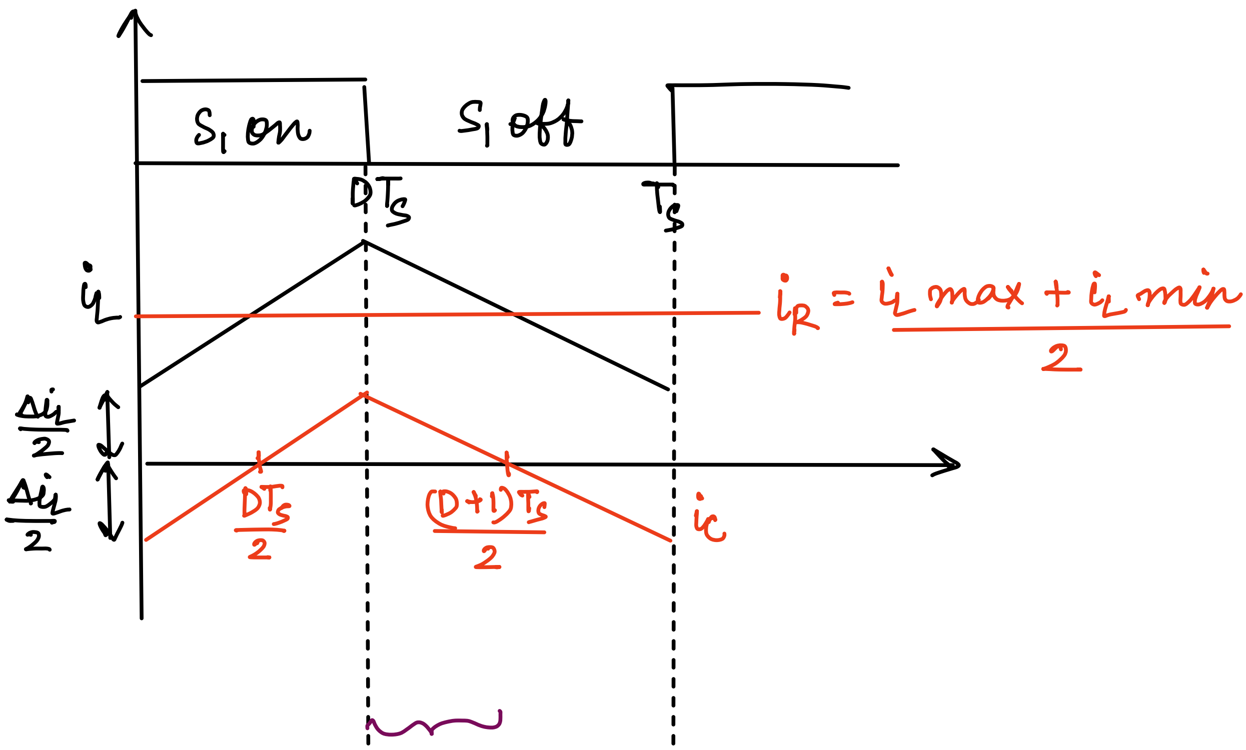
$i_L = i_C + i_R$

ripple + avg.

ripple

avg

(Because avg. current of capacitor is zero and as i_R is dc no ripple will go to i_R)



$$\frac{(D+1)T_s}{2} - \frac{2DT_s}{2} = \frac{T_s - DT_s}{2}$$

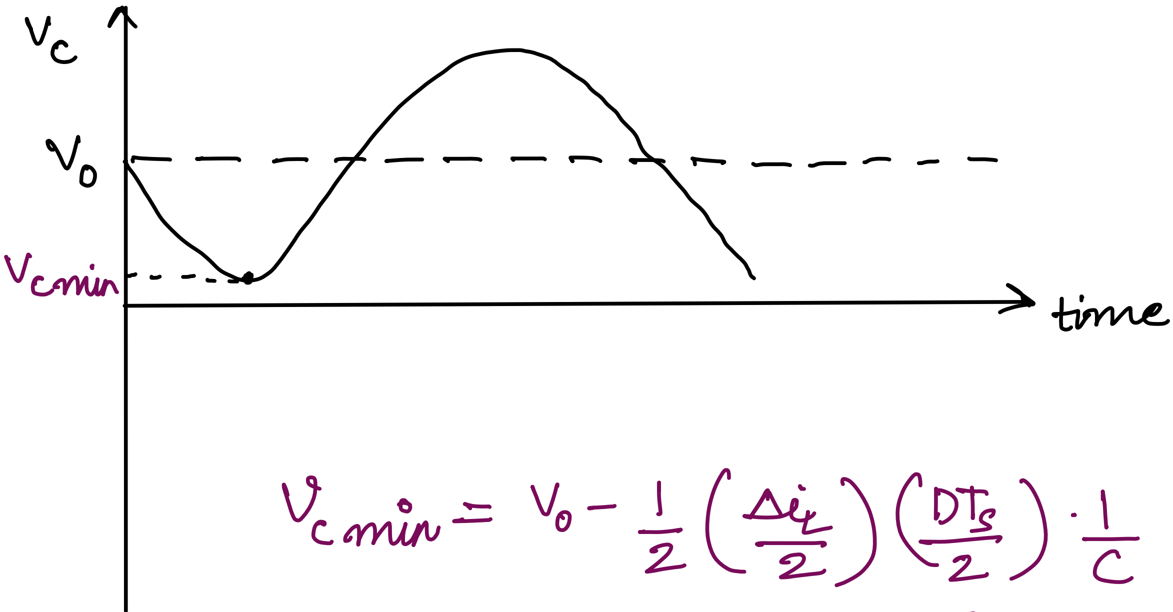
"Ampere-Second Law"

$$i_c = c\frac{dV_c}{dt}$$

$$\int_0^{T_s} i_c\,dt = \cancel{\int_{V_c(0)}^{V_c(T_s)} c\,dV_c} \rightarrow 0 \text{ assuming S-SS}$$

$$\Rightarrow \int_0^{T_s} i_c\,dt = 0 \quad \Rightarrow \text{Average capacitor current} = 0$$

This circuit is linear $\quad V_c = \dfrac{\int i_c\,dt}{c}$



$$V_{c\,min} = V_0 - \frac{1}{2}\left(\frac{\Delta i_L}{2}\right)\left(\frac{DT_s}{2}\right)\cdot\frac{1}{C}$$

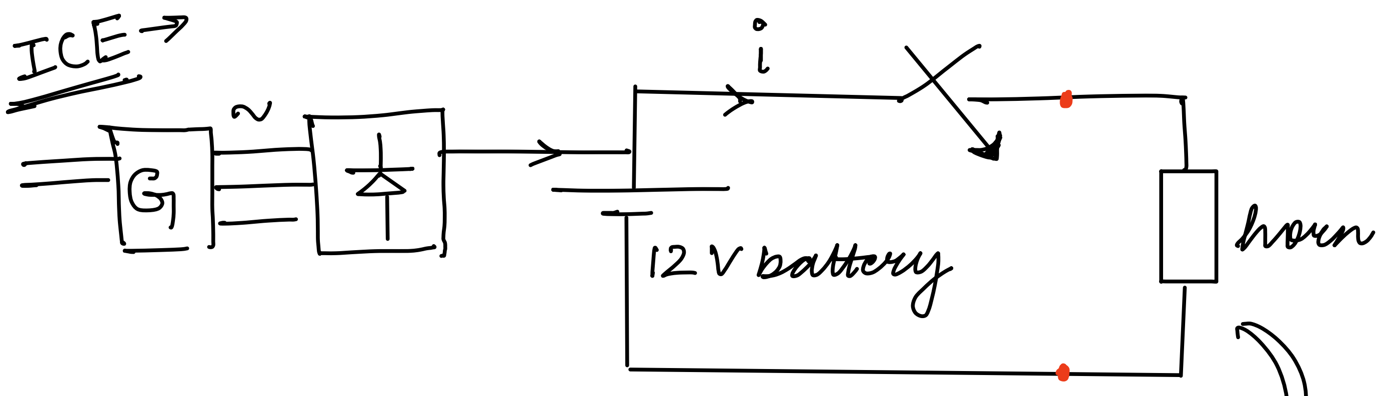
$$= V_0 - \frac{(1-D)D^2 V_{in} T_s^2}{8LC}$$

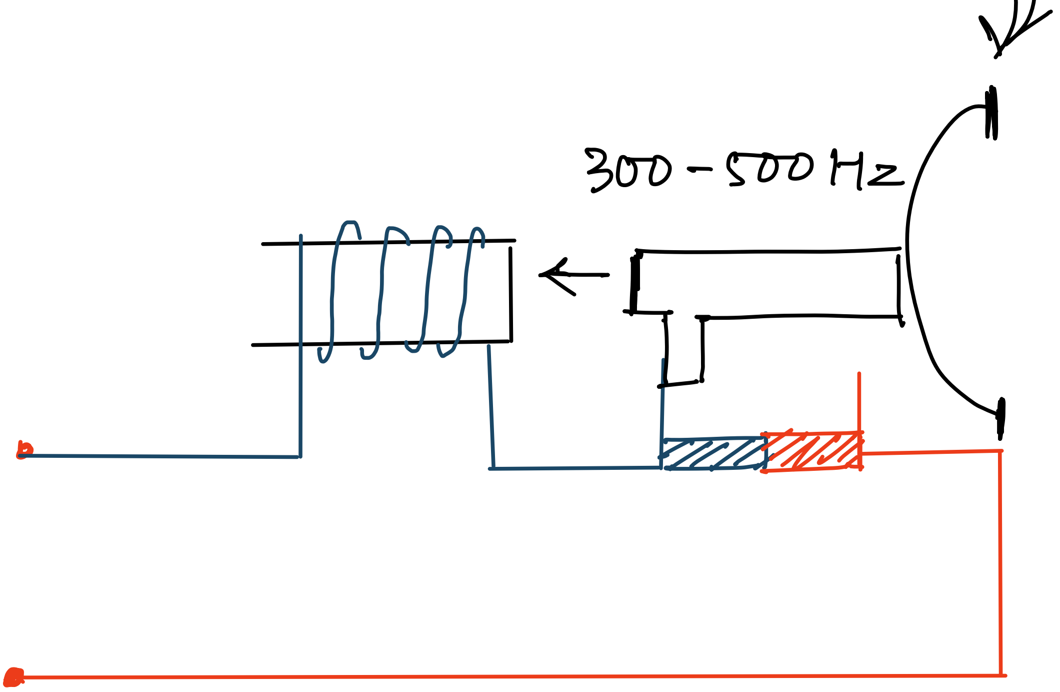
$$V_{cmax} = V_o + \frac{1}{2}\left(\frac{\Delta i_L}{2}\right)\frac{T_s(1-D)}{2}\cdot\frac{1}{C}$$

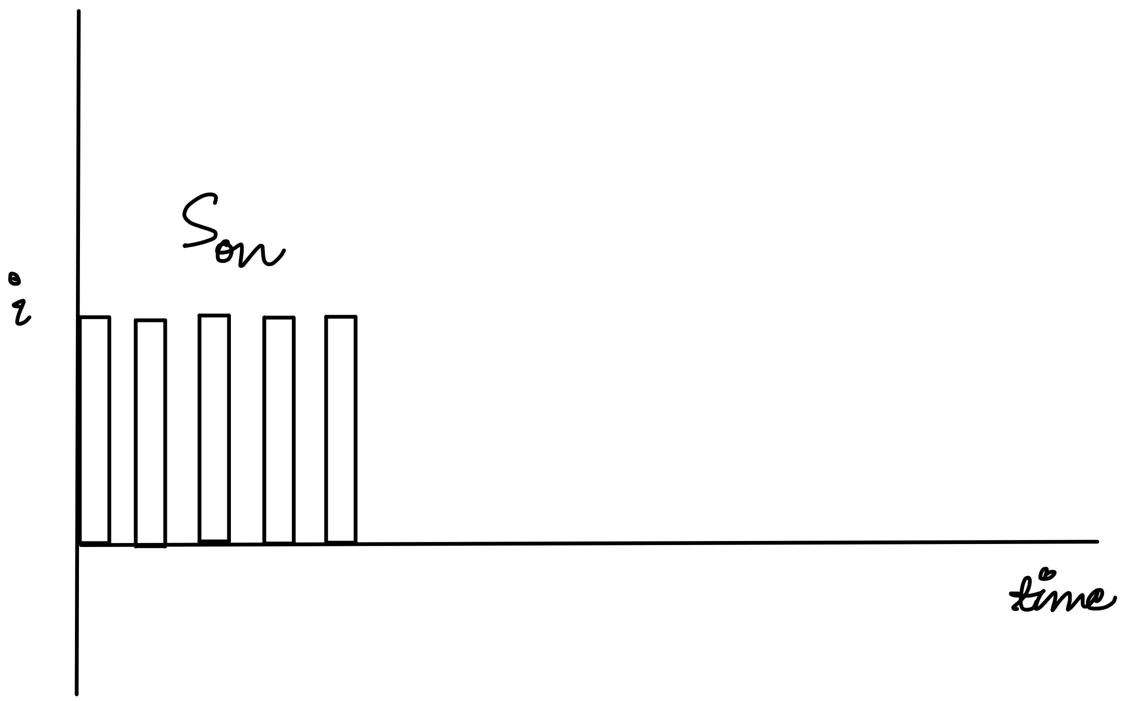
$$= V_o + \frac{(1-D)^2 D V_{in} T_s^2}{8LC}$$

$\therefore$ Voltage ripple of capacitor $= \dfrac{(1-D) D V_{in} T_s^2}{8LC}$

Q. How do the horn of vehicles work?



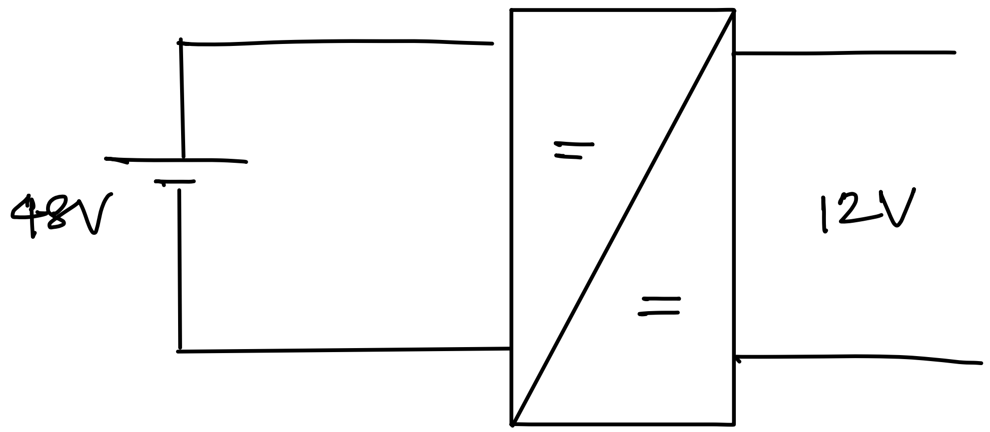




EV

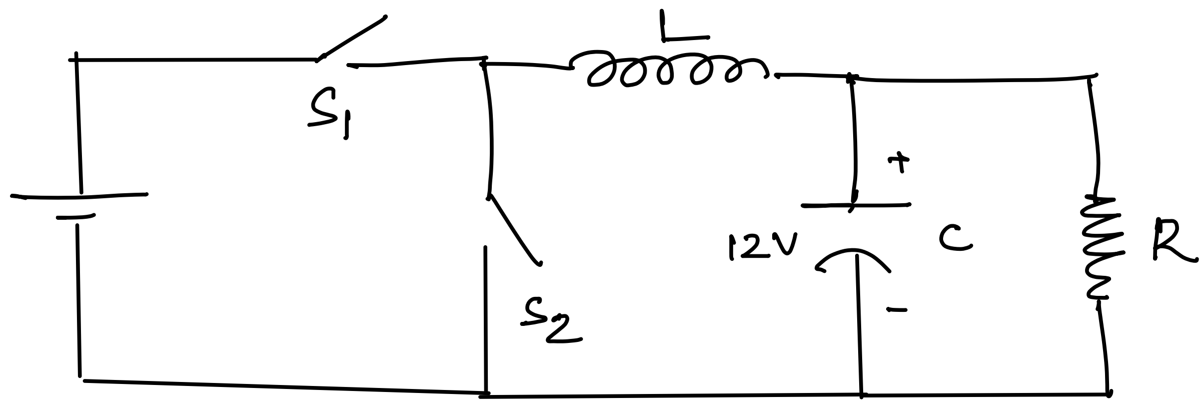
48 V battery - EV two wheeler
400 V battery - EV four wheeler

EV 2 wheeler

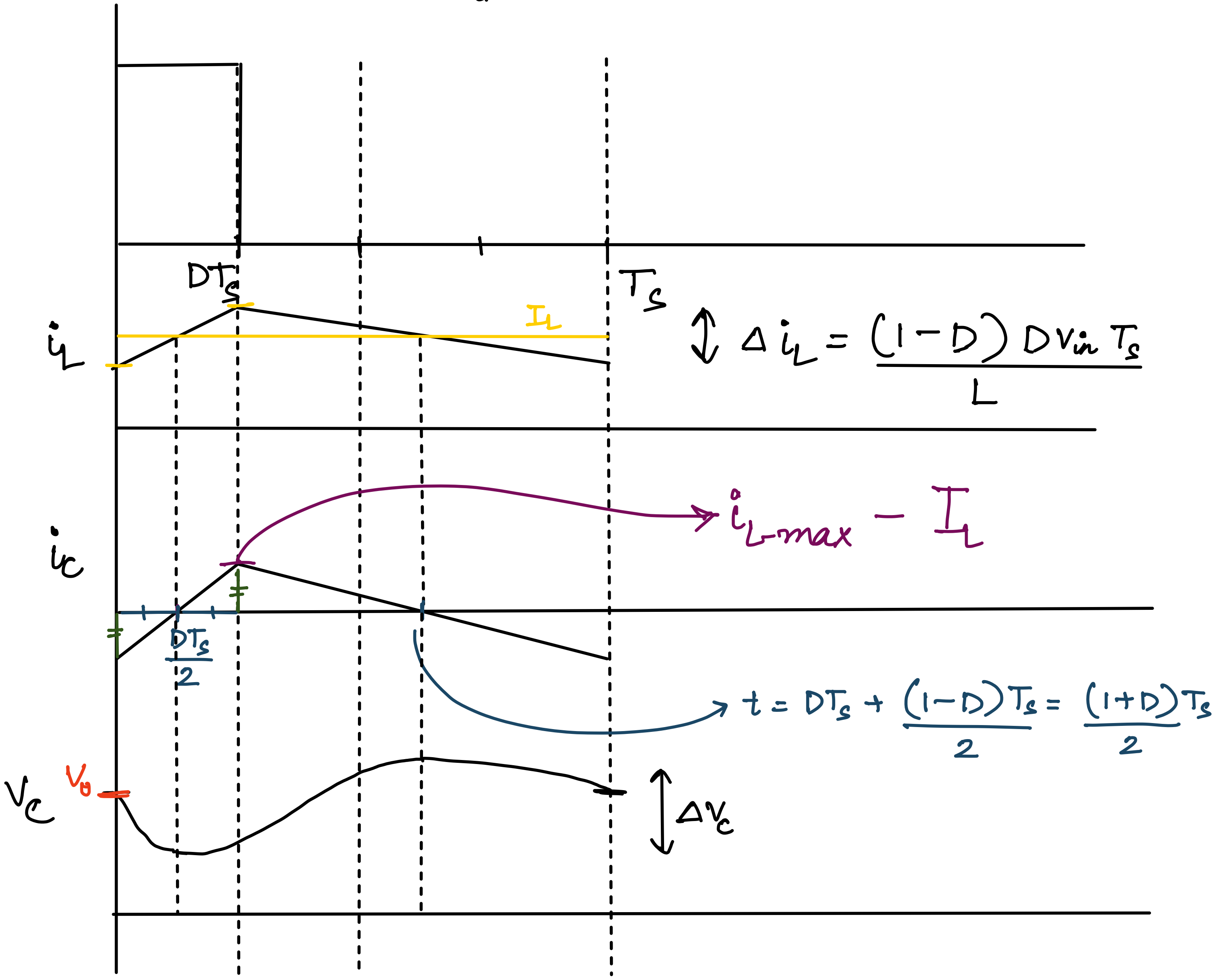


We could have changed other parameters (n of solenoid) to make the horn work with 48V. But that is not cost economic.

So we use DC-DC buck converter.



Duty cycle of $S_1 = \frac{V_o}{V_{in}} = 0{\cdot}25$



$$i_c = i_L - i_R$$

$$i_c = i_L - I_L$$

$$i_c = C\frac{dV_c}{dt}$$

$$i_c = C\frac{dV_c}{dt}$$

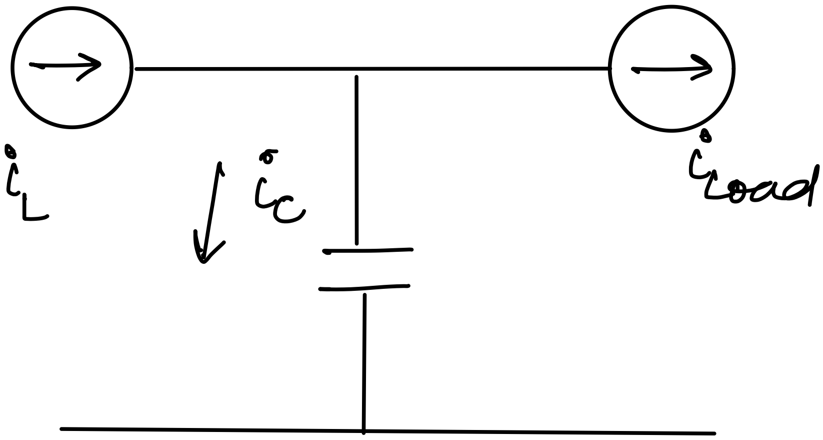
$$\int_{\frac{DT_s}{2}}^{\frac{(1+D)T_s}{2}} i_c\,dt = C\int_{V_{c\,min}}^{V_{c\,max}} dV_c = \frac{1}{2}\underbrace{\left(\frac{T_s}{2}\right)}_{\text{base}}\underbrace{\left(\frac{\Delta i_L}{2}\right)}_{\text{height}}$$

$$\Delta V_c = \frac{\Delta I_L T_s}{8C} = \frac{(1-D)\,D\,V_{in}\,T_s^{\,2}}{8LC}$$

Reduce ripple : 2% → 1%

- $C\uparrow$] size issue
- $L\uparrow$
- $T_s\downarrow = \dfrac{1}{F_s\uparrow}$] Efficiency issue

→ $\cancel{D}$



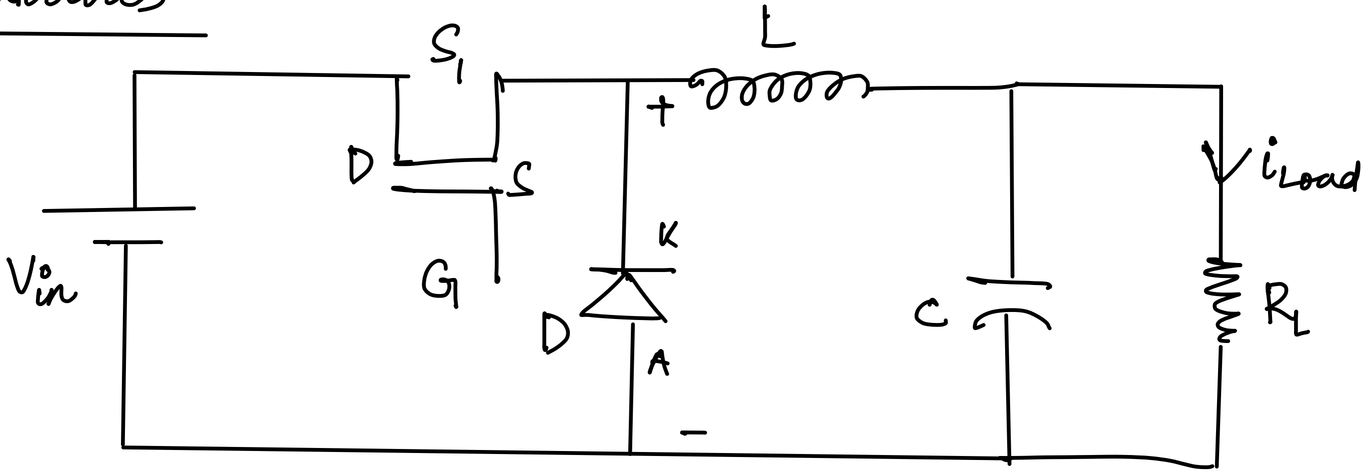
$$i_C = i_L - i_{Load}$$

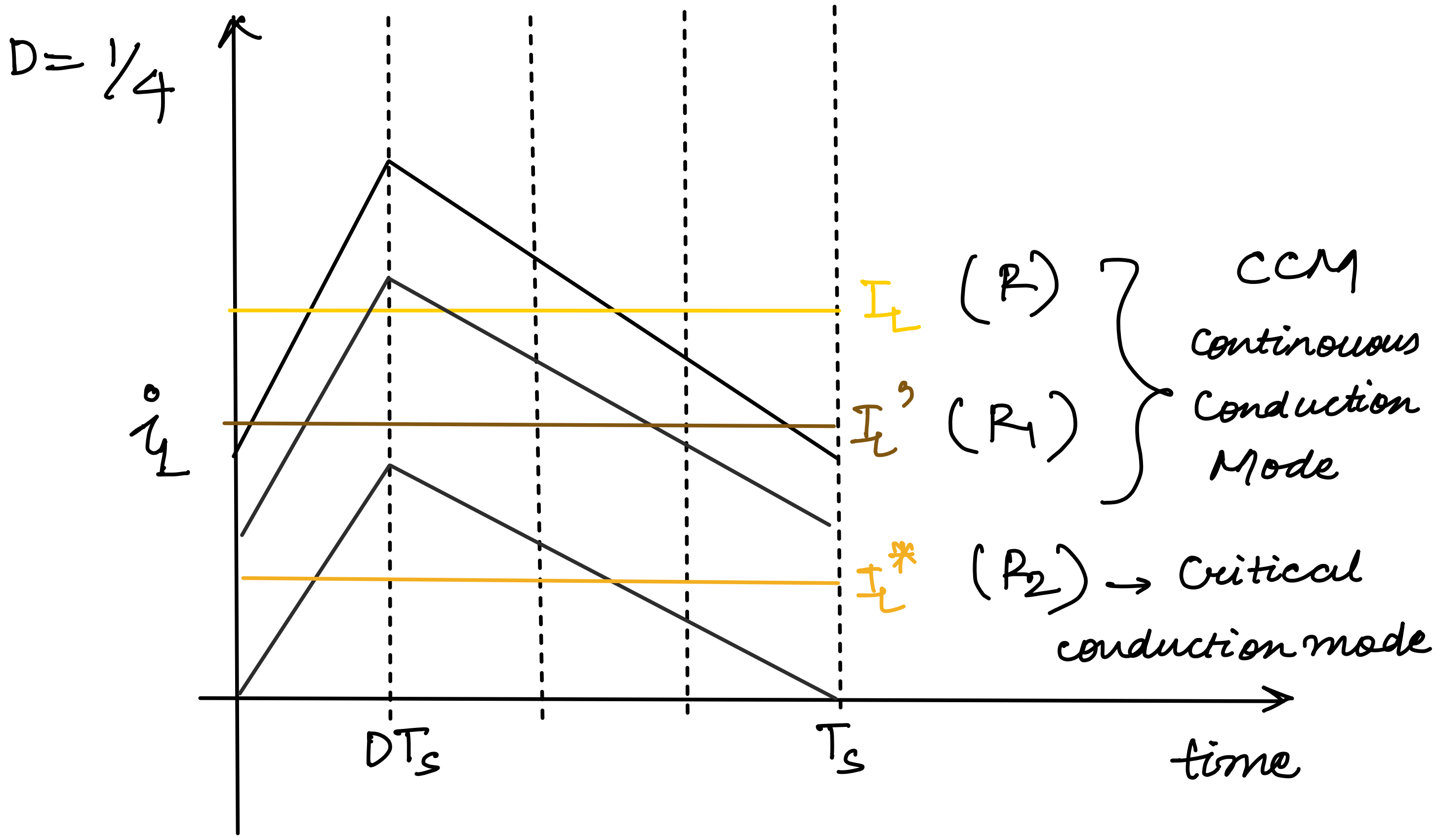
$$= \left[\cancel{I_L} + \Delta i_L(t)\right] - \left[\cancel{I_{Load}} + \Delta i_{load}(t)\right]$$

$$\langle i_C \rangle = 0$$

Horn will actually increase the ripple.

Switches



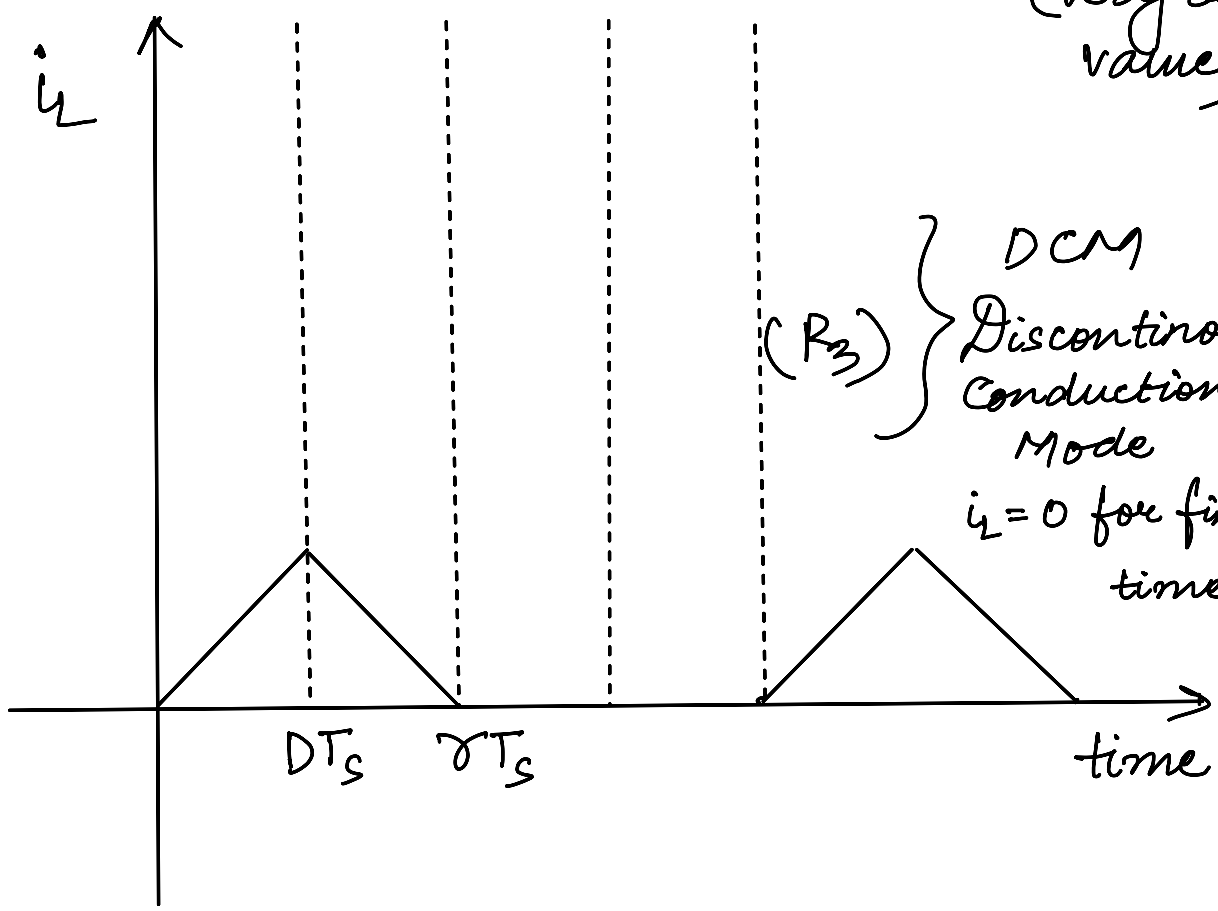


Q What would happen if R_L increases?

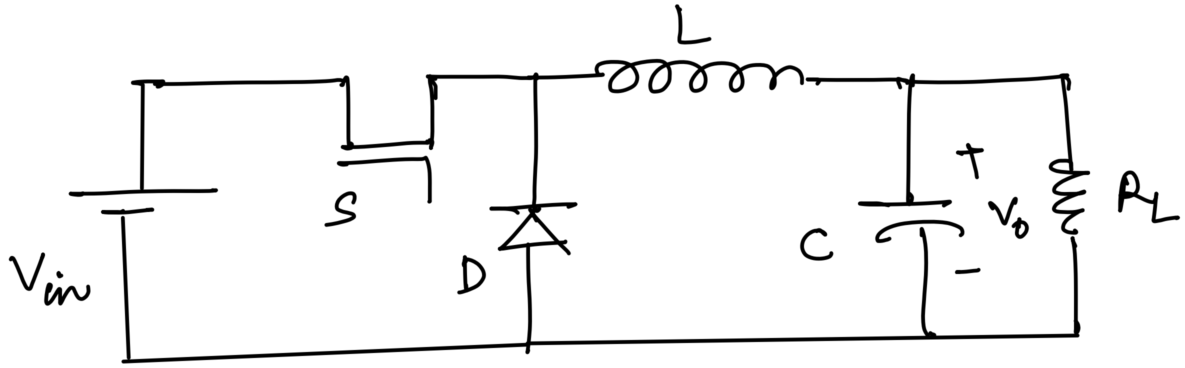
$i_{load} \downarrow \Rightarrow I_{Load} \downarrow \Rightarrow I_L \downarrow$ ($V_o \rightarrow$ constant)

Reduce Load → Reduce Load Current.
(Increase Resistance)

$R \longrightarrow R_1 \longrightarrow R_2 \longrightarrow R_3$ (very large value)



$\left.\right\}$ DCM
Discontinous Conduction Mode
$i_L = 0$ for finite time



$$I_L > \frac{\Delta i_L}{2} \longrightarrow \text{CCM}$$

$$\Rightarrow \frac{V_0}{R_L} > \frac{(V_{in} - V_0) D T_s}{2L}$$

$$\Rightarrow \boxed{R_L < \frac{2L}{(1-D)T_s}} \Longrightarrow \text{CCM}$$

$$R_{critical} = \frac{2L}{(1-D)T_s}$$

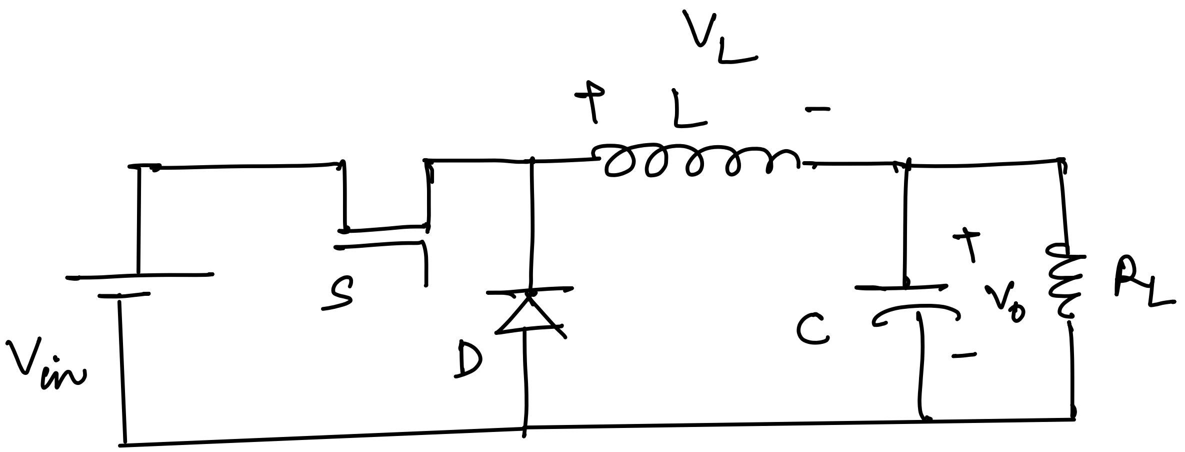
DCM

$0 - DT_s$: Switch S is on

$$V_L = V_{in} - V_o$$

$DT_s \rightarrow \gamma T_s$: S "OFF" ; D "ON"

$$V_L = -V_o$$



$\gamma T_s \rightarrow T_s$: S "OFF" ; D "OFF"

$$V_L = 0$$

$\langle V_L \rangle = 0$ (∵ circuit is in S-SS)

$$\Rightarrow (V_{in} - V_o) DT_s + (-V_o)(\gamma - D)T_s + 0\,(1-\gamma)T_s = 0$$

$$\Rightarrow \boxed{\frac{V_o}{V_{in}} = \frac{D}{\gamma}} \quad \text{———} \quad (1)$$

O/P Voltage does not remain constant on varying R_L

$$\langle I_L \rangle = \frac{V_o}{R_L}$$

$$\Rightarrow \frac{1}{2} \times \frac{\gamma \cancel{T_s} \times i_{Lpeak}}{\cancel{T_s}} = \frac{V_o}{R_L} \quad \text{———} \quad (2)$$

$$(V_{in} - V_o) = L \left(\frac{i_{Lpeak} - 0}{DT_s} \right) \quad \text{———} \quad (3)$$

$\Rightarrow$ From (2) we have: $V_o = \dfrac{R_L \gamma i_{Lpeak}}{2} = \underbrace{\dfrac{DV_{in}}{\gamma}}_{\text{from (1)}}$

$$\frac{1}{2}\left(\left(\cancel{V_{in}} - \frac{D\cancel{V_{in}}}{\gamma}\right)\frac{\cancel{D}T_s}{L}\right) R_L \gamma = \frac{\cancel{D}\cancel{V_{in}}}{\gamma}$$

$$\Rightarrow \frac{(\gamma - D) T_s R_L \gamma}{2L} = 1$$

$$\Rightarrow (\gamma - D)\gamma = \frac{2L}{R_L T_s}$$

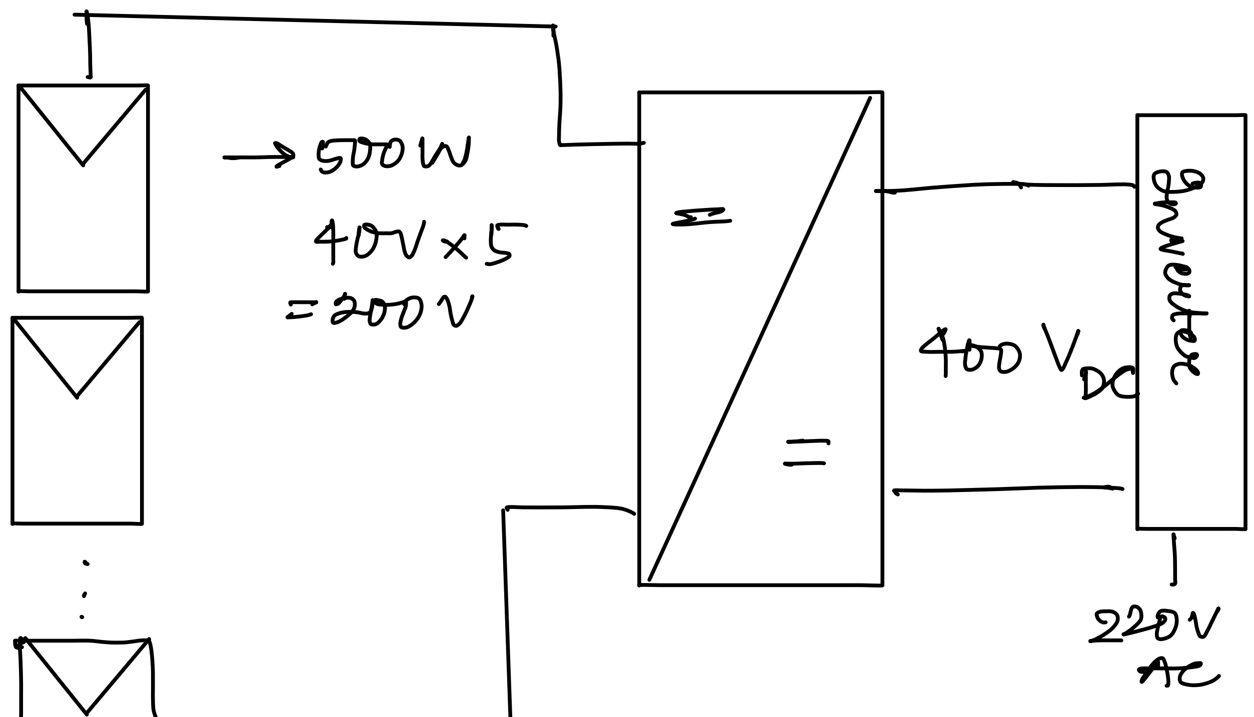
$$\Rightarrow \gamma^2 - D\gamma - \frac{2L}{R_L T_s} = 0$$

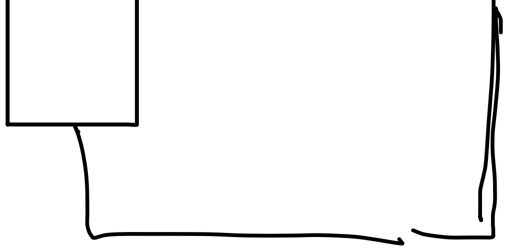
$$\gamma = \frac{D \pm \sqrt{D^2 + \frac{8L}{R_L T_s}}}{2}$$

reject γ which is more than D

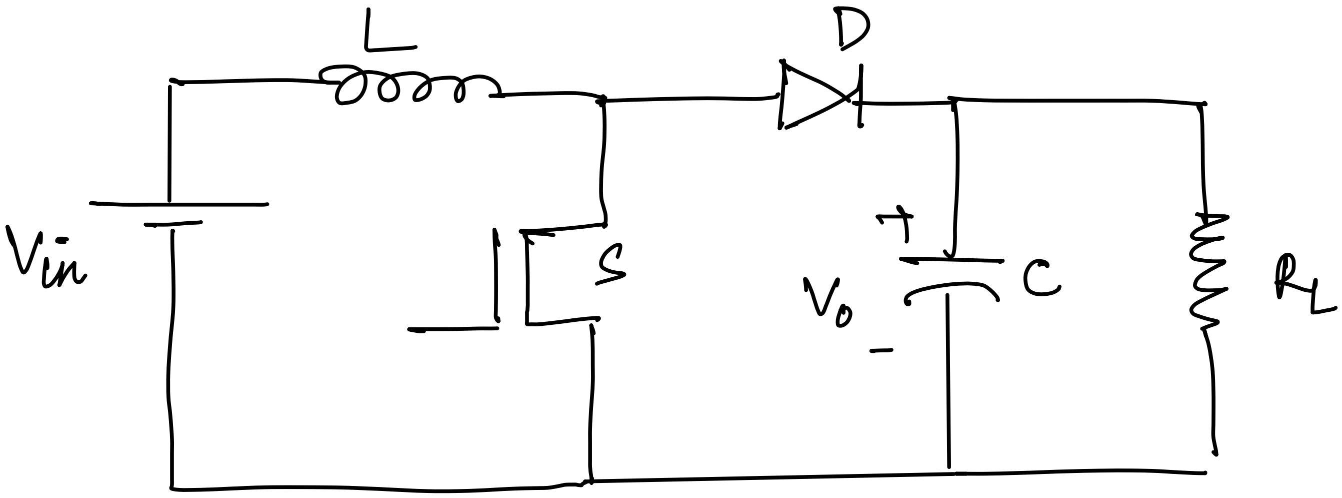
$$\Rightarrow \boxed{\gamma = \frac{D - \sqrt{D^2 + \frac{8L}{R_L T_s}}}{2}}$$

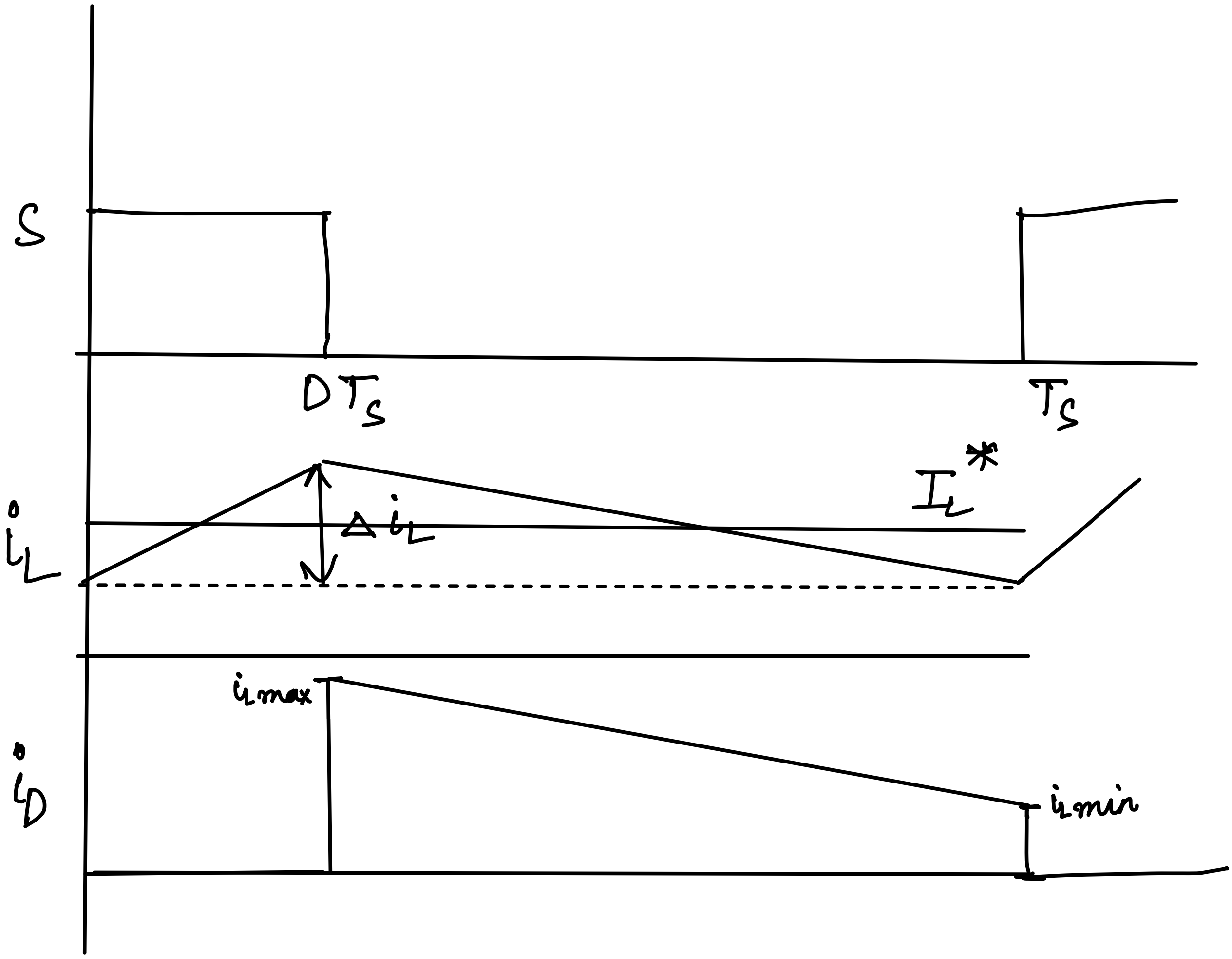
$\underline{\underline{V_o > V_{in}}} \longrightarrow$ Boost Converter.





DC-DC Boost Convertor





$$\frac{0 \to DT_s}{V_L = V_{in}} \qquad \frac{DT_s \to T_s}{V_L = V_{in} - V_o}$$

$$\langle V_L \rangle = 0$$

$$\Rightarrow V_{in}(DT_s) + (V_{in} - V_o)\,T_s(1-D) = 0$$

$$\Rightarrow \cancel{V_{in} DT_s} + V_{in} T_s - \cancel{V_{in} DT_s} - V_o T_s + V_o DT_s = 0$$

$$\Rightarrow V_{in} \cancel{T_s} = V_o(\cancel{T_s} - D\cancel{T_s})$$

$$\Rightarrow \boxed{\frac{V_o}{V_{in}} = \frac{1}{1-D}}$$

$$\langle i_D \rangle = \frac{V_o}{R_L} \qquad \langle i_L \rangle = \frac{i_{L max} + i_{L min}}{2}$$

$$\Rightarrow \left(\frac{i_{L-max} + i_{L\,min}}{2}\right)(1-D) = \frac{V_o}{R_L}$$

$$\Rightarrow \boxed{(1-D)\langle i_L \rangle = \langle i_D \rangle = \frac{V_o}{R_L} = \langle I_o \rangle}$$

Ideal Converter $\Rightarrow$ no Losses.

$$\Rightarrow \langle V_{in} \rangle \times \langle I_{in} \rangle = V_0 \langle I_0 \rangle$$

$$\langle I_{in} \rangle = \langle i_L \rangle$$

CCM and DCM are always defined in context of inductor current

AC $\xrightarrow[\text{rectifier}]{\text{full wave}}$ 325 V $\longrightarrow$ 5V
$(230\sqrt{2})$

$$\frac{325}{5} = 65 \text{ times} \downarrow$$

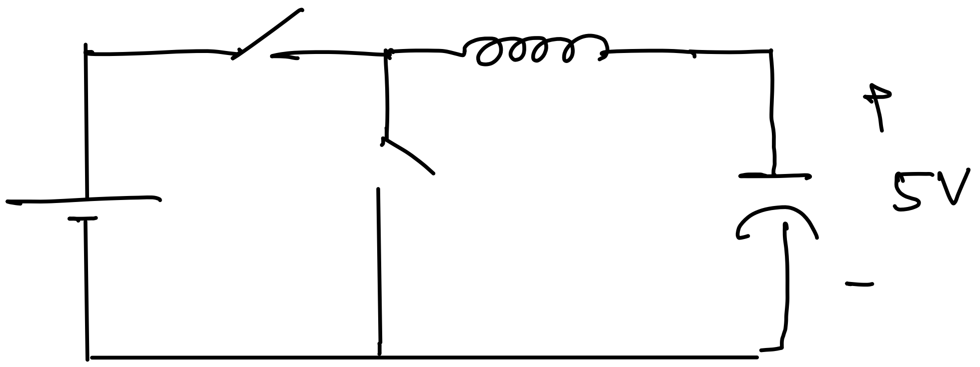
$$\Rightarrow D = \frac{5}{325} = 0.0154$$

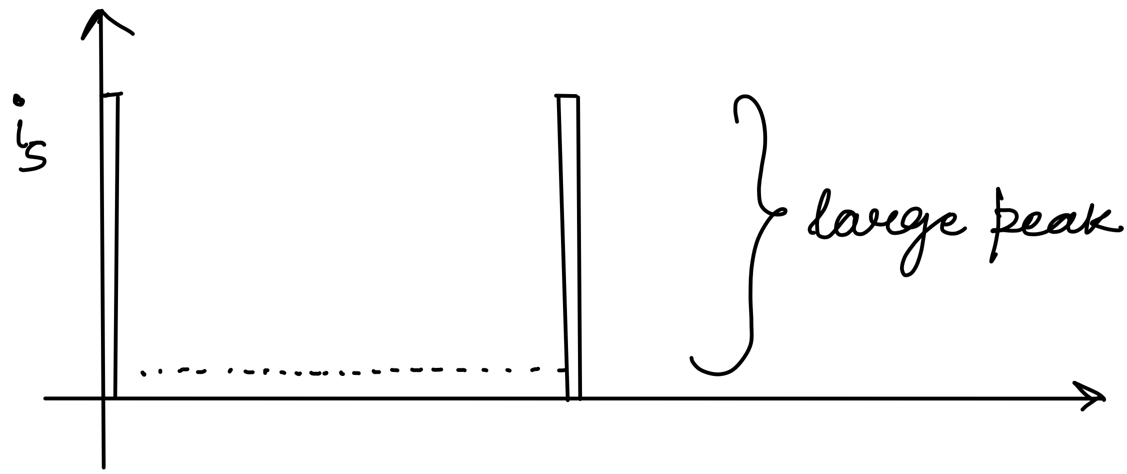
$$f_1 = 1 MHz$$

$$DT_s = 15.4 \text{ ns}$$

Resolution of PWM is a problem.

2)





peak current requirement of switch is high
I_{rms} of switch (semi-conductor) is also high

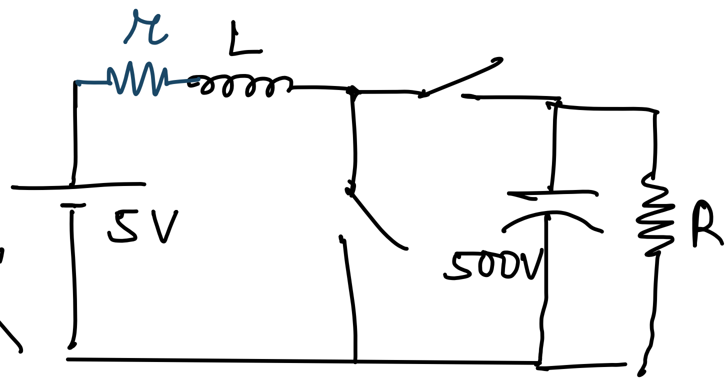
3) Efficiency reduces $\eta \downarrow$ (heat dissipation $\uparrow$)

Similar problems with boost converter.

Boost

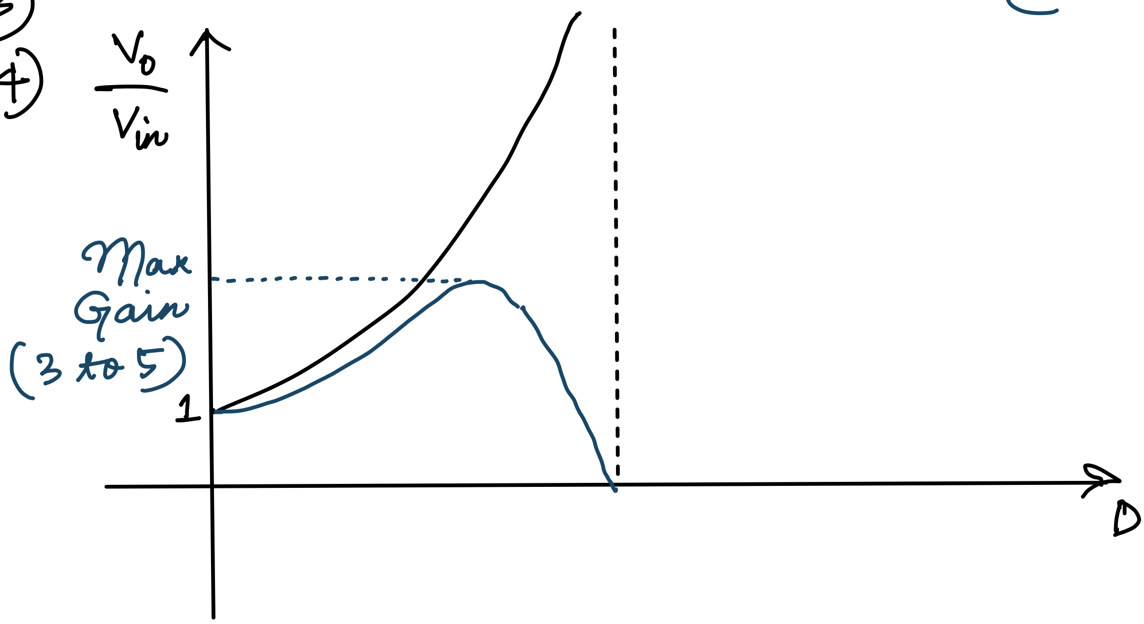
$5V \longrightarrow 500V$

DC-DC Boost converter.



$$V_o = \frac{V_{in}}{\left[1 - D + \frac{r}{R(1-D)}\right]}$$

1)
2)
3)
4)



∴ In real life, gain of boost converter is limited

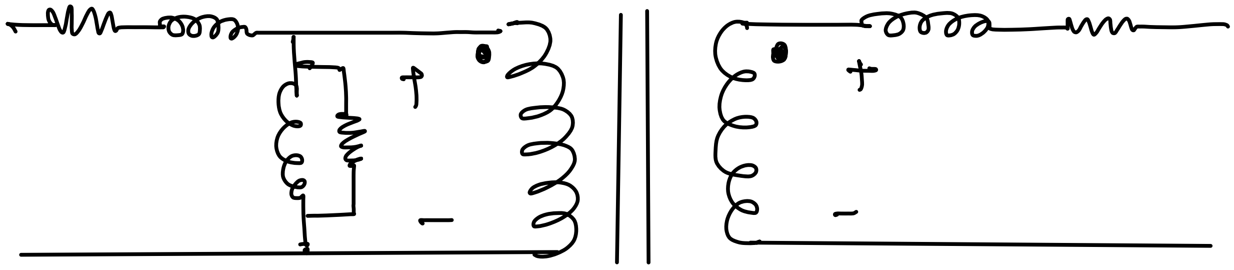
Solution - 1

1) Use transformer in i/p AC

$$E = 4.44 N f \phi_m$$

Very big, flux ↑↑

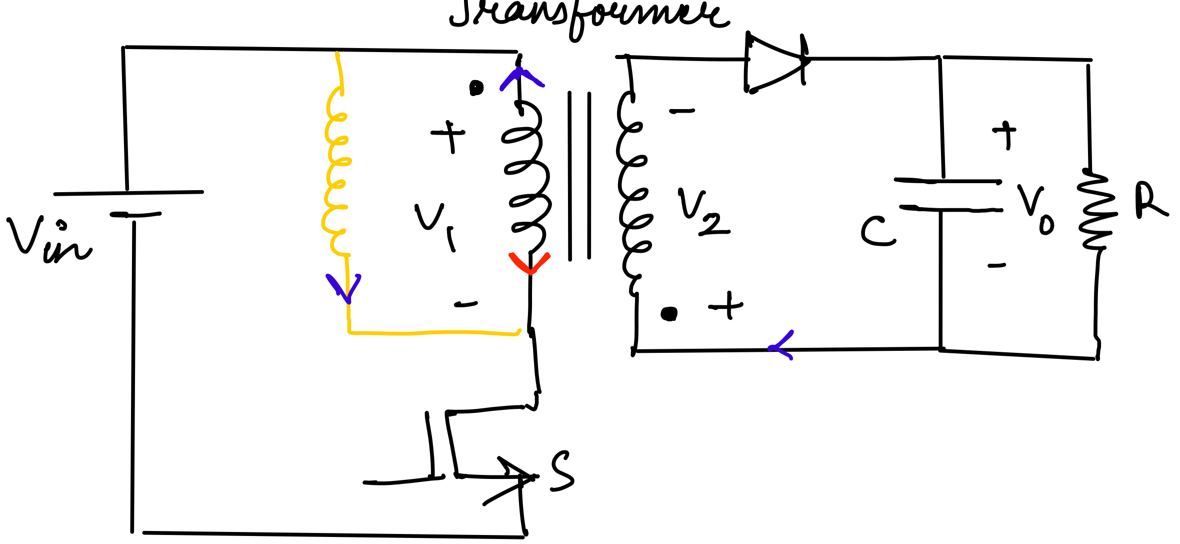
2) High frequency transformer



Ideal

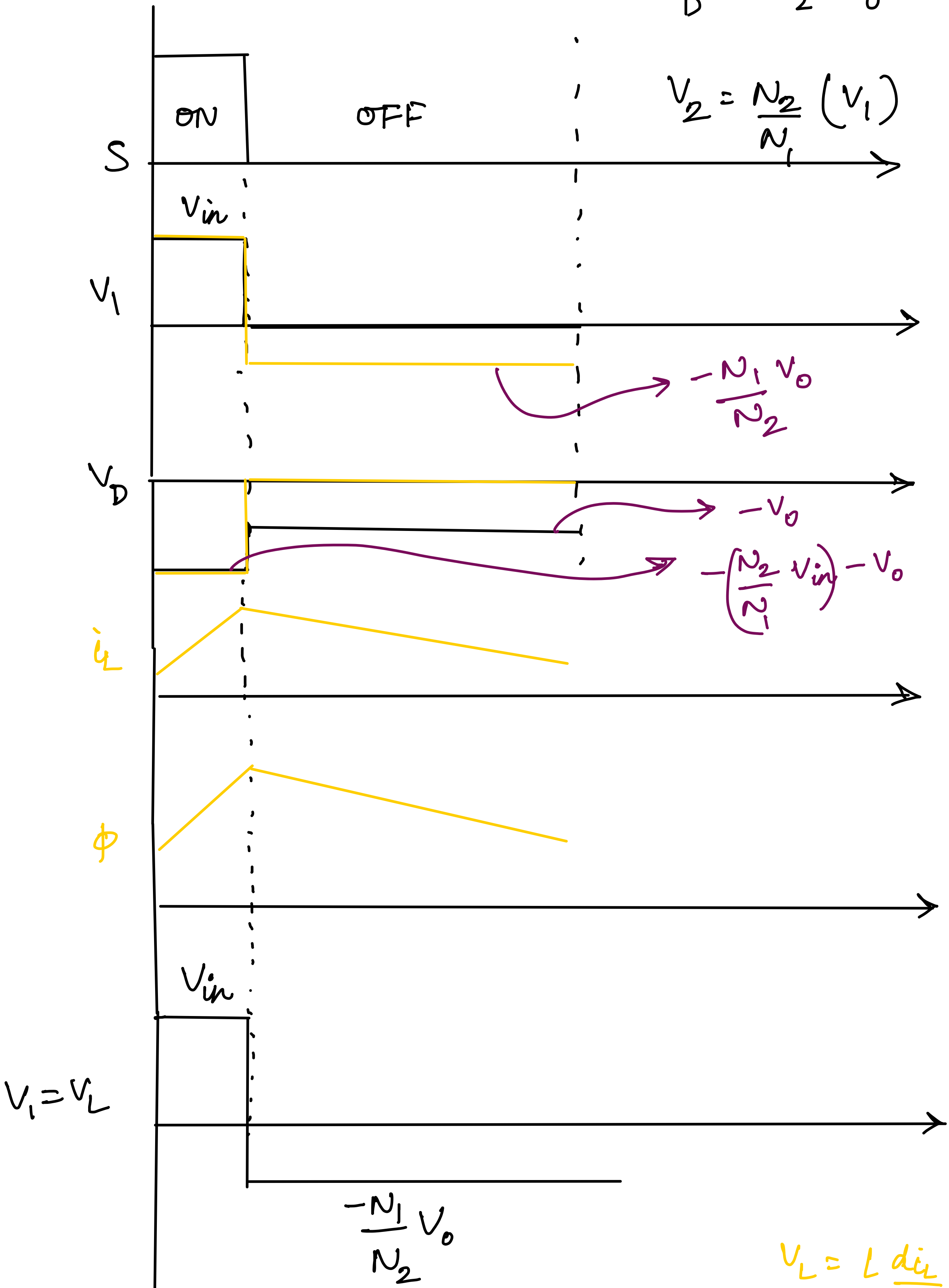
Flyback Converter

S on
S off



$$V_D = -V_2 - V_0$$

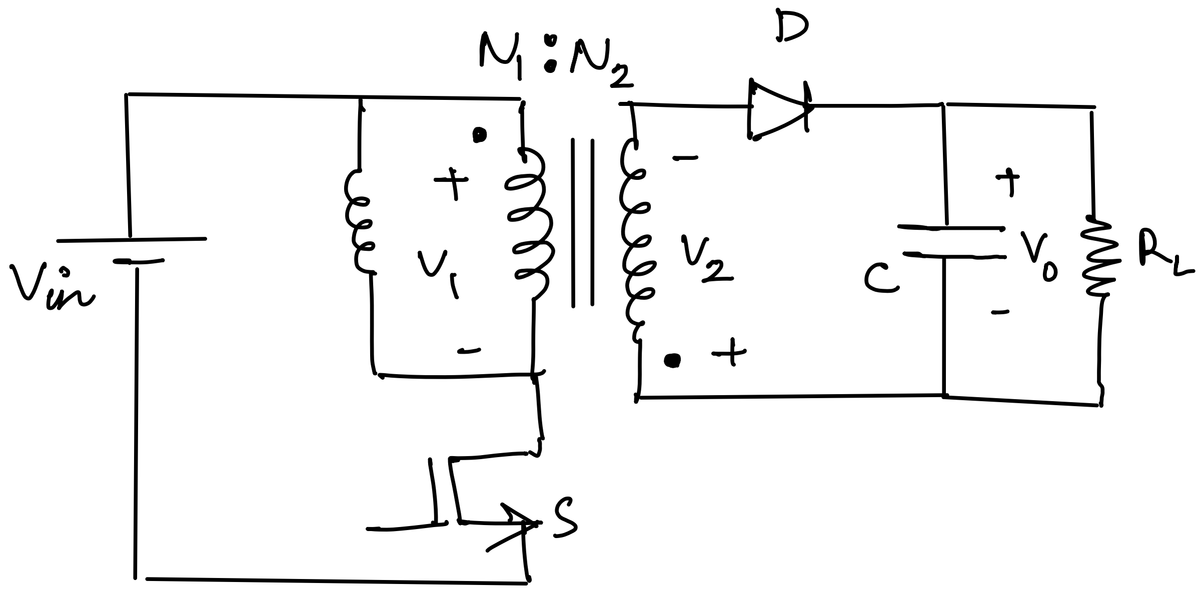
$$V_2 = \frac{N_2}{N_1}(V_1)$$



$$V_L = L\frac{di_L}{dt}$$

$$= N_1\frac{d\phi}{dt}$$

$$\frac{Li_L}{N} = \phi$$



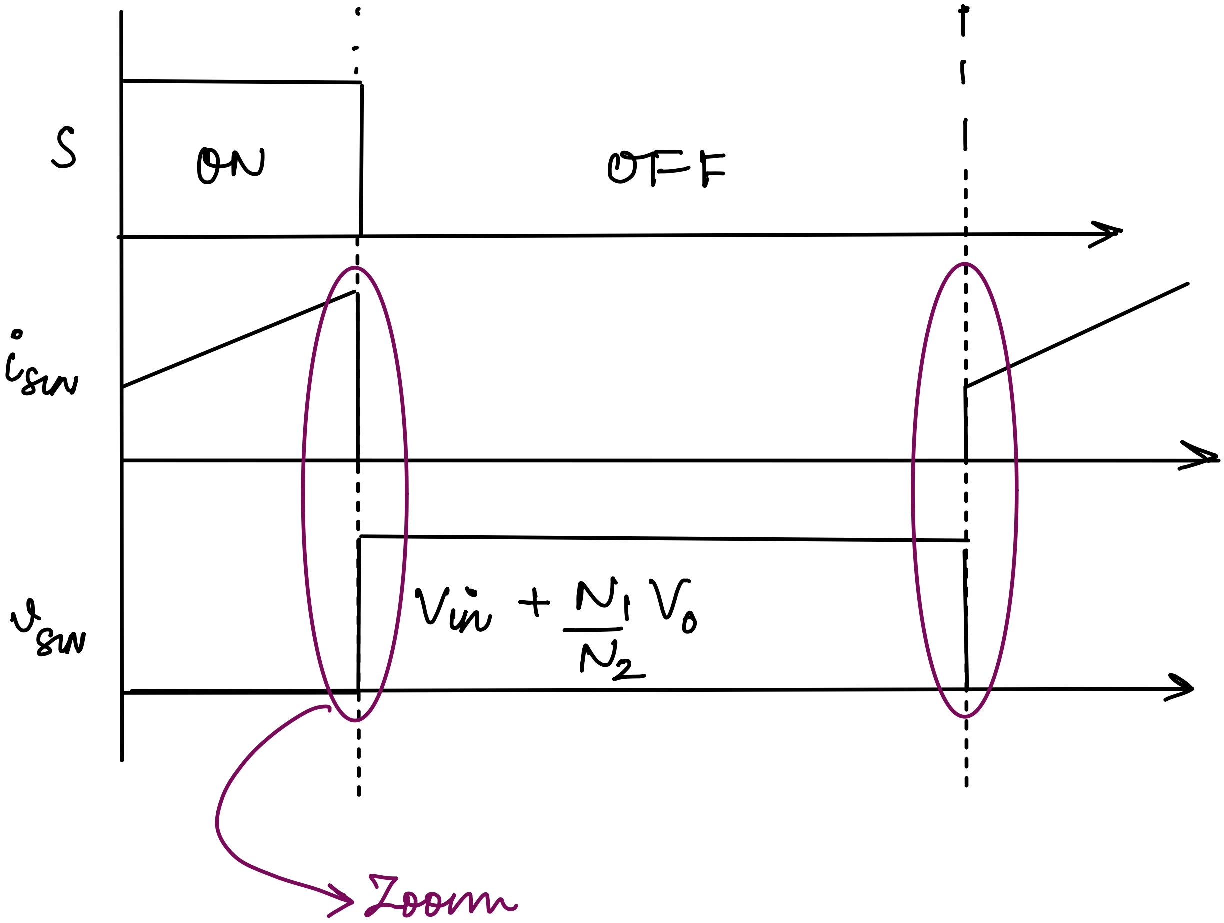
Appey $\int_0^{T_s} v_L \, dt = 0$ (voltage second balance)

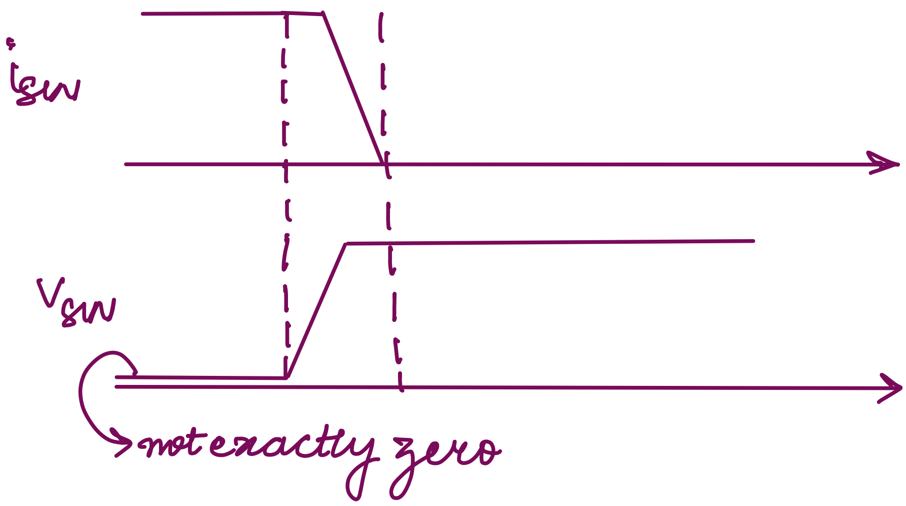
$$\Rightarrow \int_0^{T_s} V_1 \, dt = 0$$

$$\Rightarrow V_{in} (D\cancel{T_s}) - \frac{N_1}{N_2} v_o (1-D)\cancel{T_s} = 0$$

$$\Rightarrow \boxed{\frac{V_o}{V_{in}} = \frac{N_2}{N_1} \frac{D}{(1-D)}}$$

In DCM, V_o has to ↑↑ to keep avg = 0 since the second half will shrink.





Switch is on:

1) Conduction loss: $\frac{1}{T}\int V_{sw}\, i_{sw}\, dt$

Switch is off:

2) Very small leakage current $\Rightarrow$ Negligible loss

Switch in transition period:

3) Switching loss (larger than conduction loss)
→ depends directly on switching frequency
(In diode → reverse recovery losses)

Inductor Loss / Transformer loss

→ Conduction loss
→ Core loss

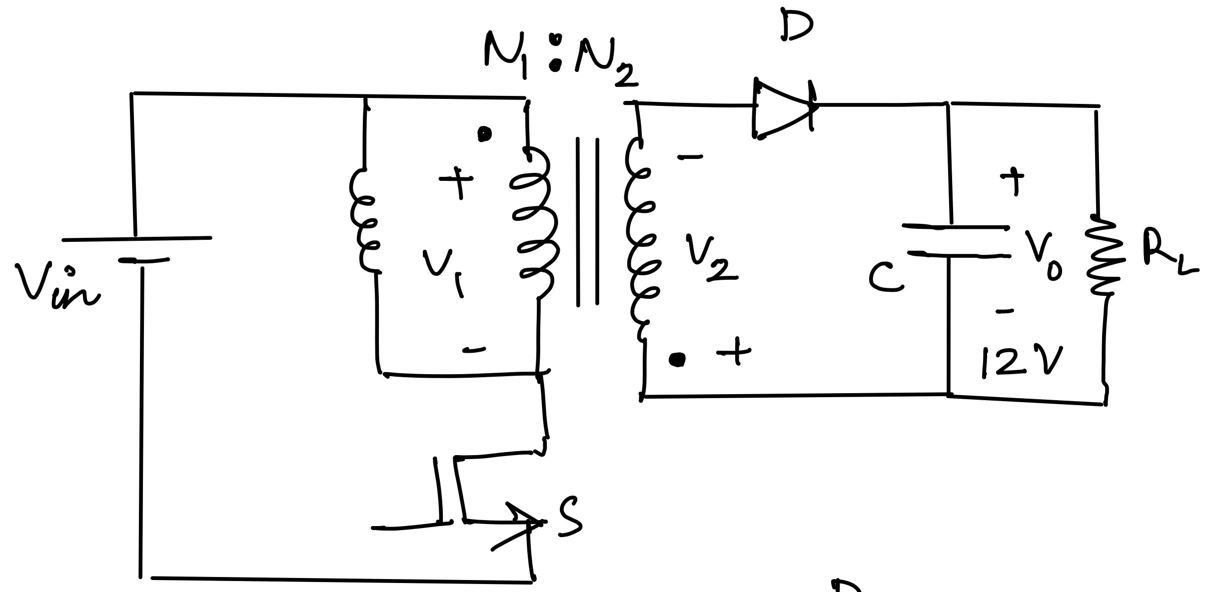
These are the predominant losses

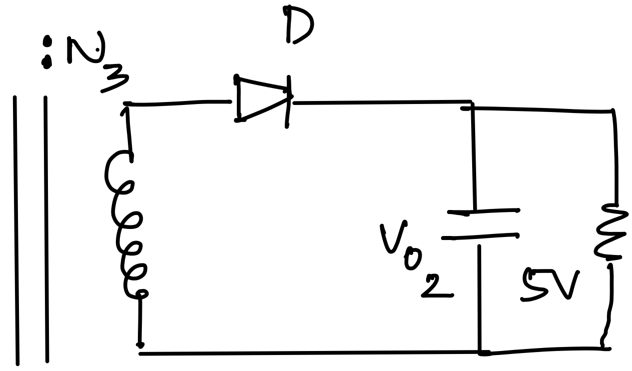
Basic details which define a MOSFET.

→ Blocking Voltage (A buffer of 40-50% over max voltage is kept)
→ Peak current
→ Continuous Current (RMS current)
→ Maybe thermal constraints

Advantages of flyback

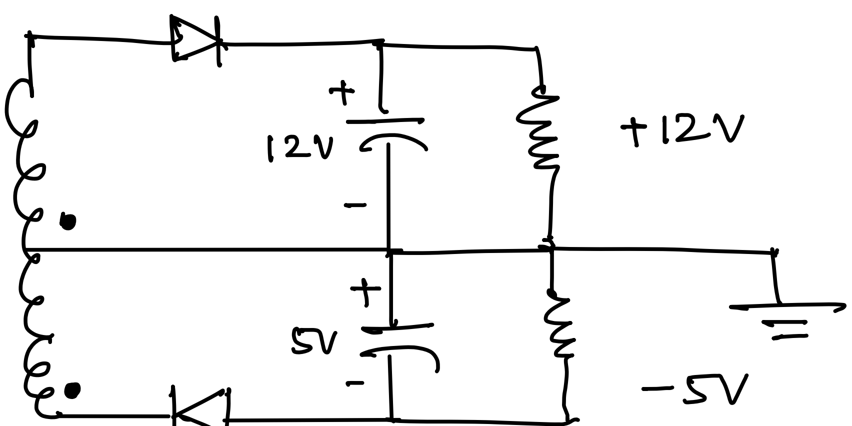
→ Can vary turns ratio for large attenuation.
No stress on duty cycle.





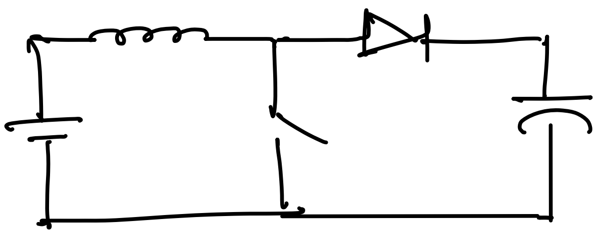
$$\frac{12}{5} = \frac{N_2}{N_3}$$

Can use a tapped transformer here as well.

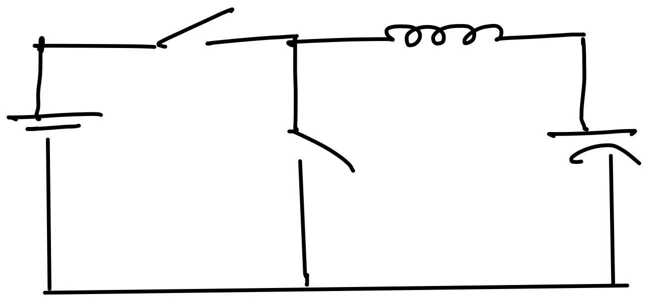


Limitations of flyback

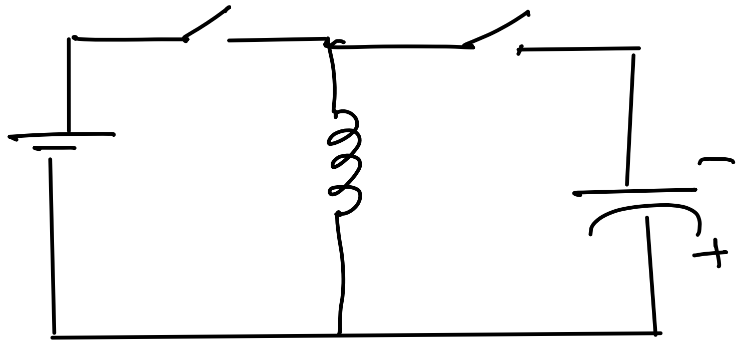
1) Output Ripple Voltage.
2) Large L_m ⇒ Small Energy Transfer.
3)



Boost



Buck

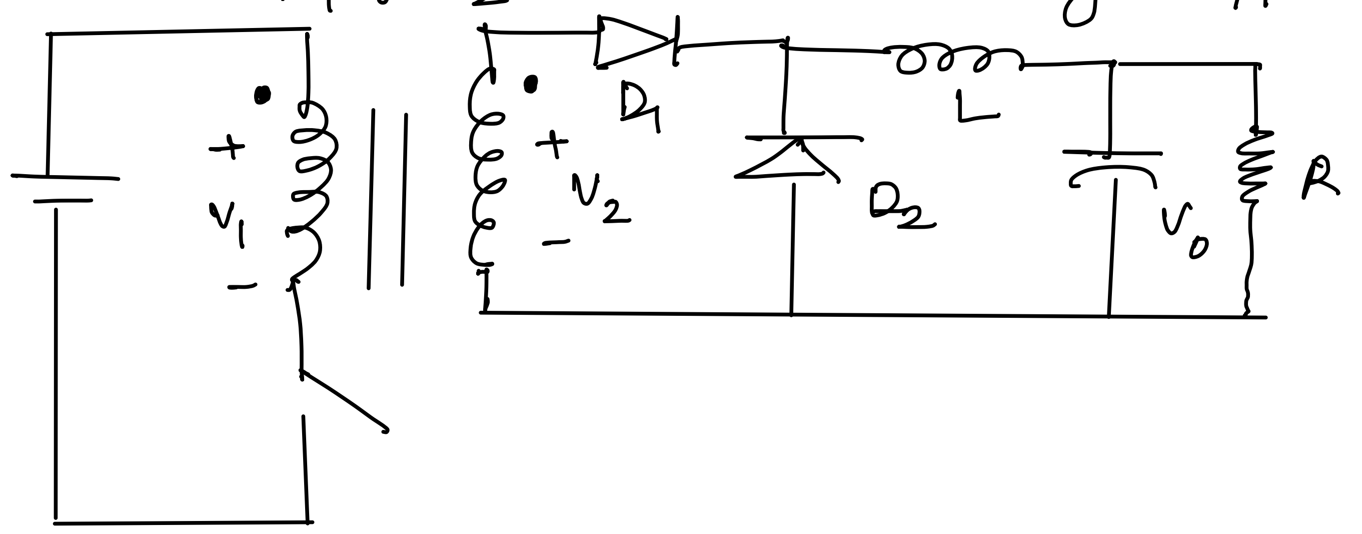


Buck - Boost
(Large ripple current at both i/p & o/p)

Forward Convertor (Buck Derived)

$\Downarrow$

Small o/p voltage ripple

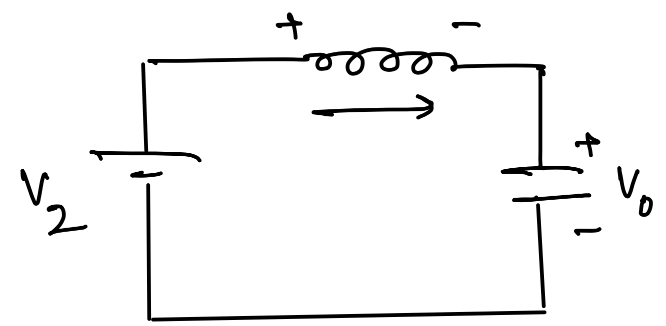


When S is on

$$V_1 = V_{in} \Rightarrow V_2 = \frac{V_{in} N_2}{N_1}$$

$V_2 > 0 \Rightarrow D_1$ conducts, D_2 remains reverse biased

$\Rightarrow$ Eq circuit on right side $\Rightarrow$



$$\frac{L\,di_L}{dt} = V_2 - V_0 = \frac{V_{in} N_2}{N_1} - V_0$$

S is off

Inductor current has two possibilities, thru D_1 or D_2

Consider D_1

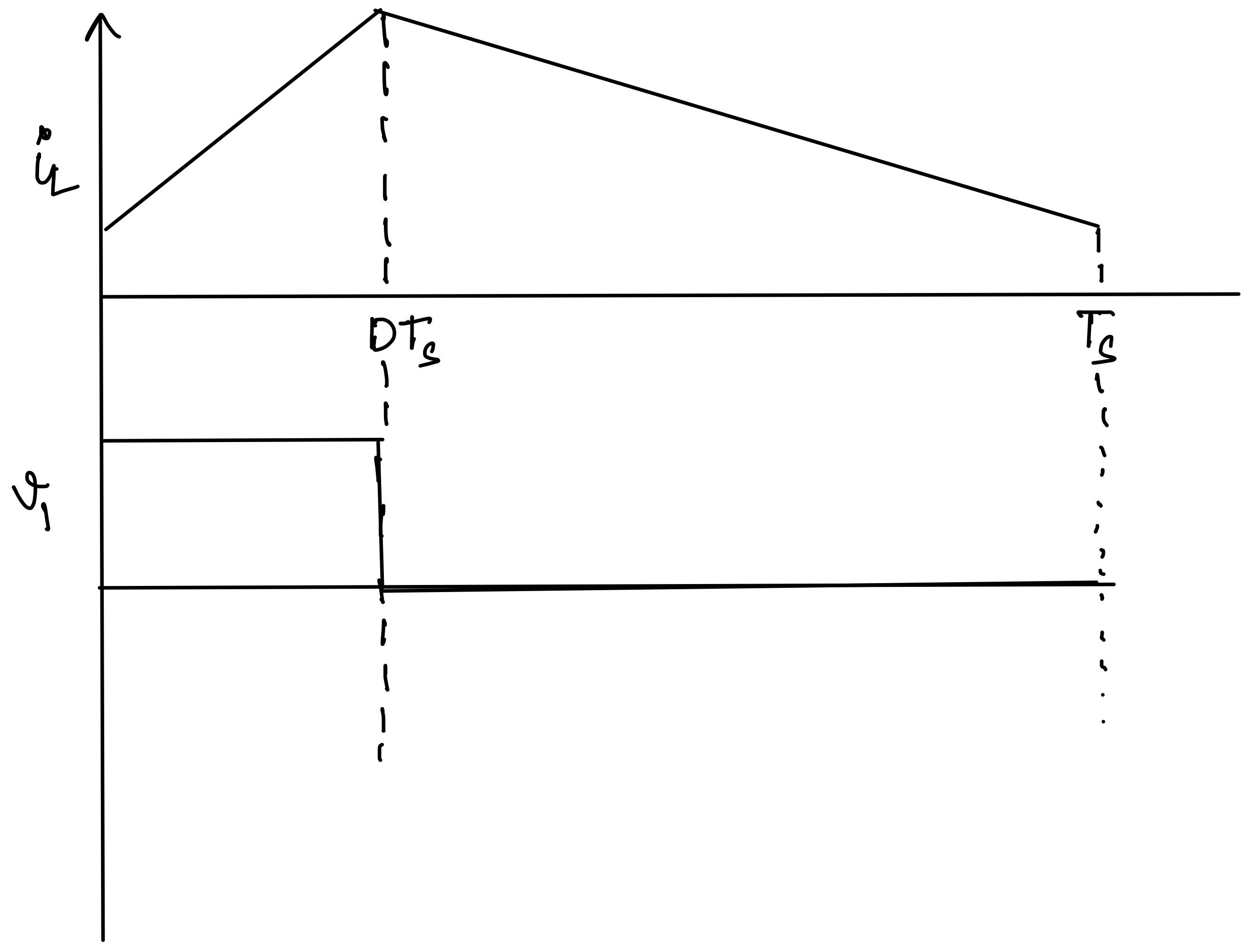
⇒ current is leaving the dot on right side

⇒ Current will have to enter the dot on left side

⇒ Not possible

∴ D_2 conducts

$$v_L = L\frac{di_L}{dt} = -V_o$$



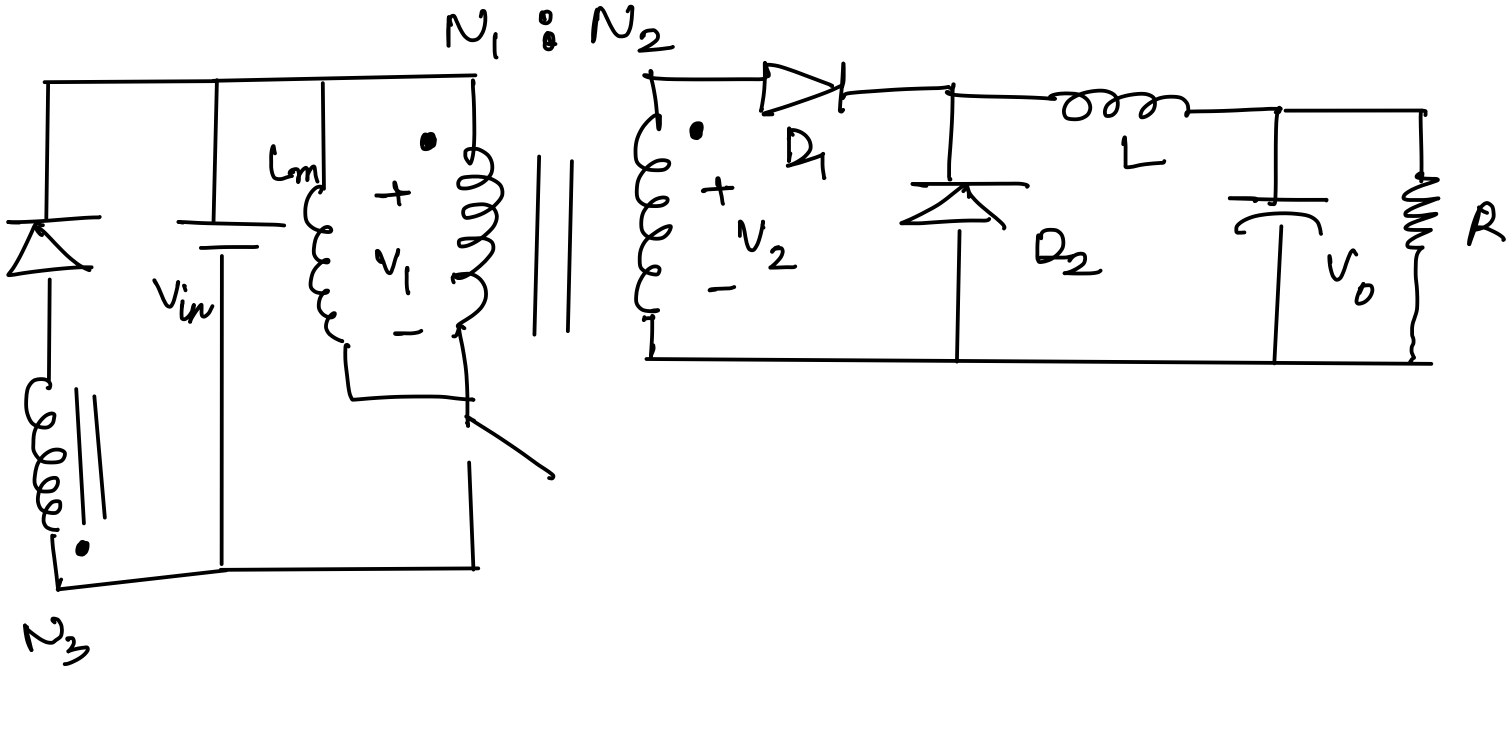
$$\boxed{V_0 = D\,V_{in}\,\frac{N_2}{N_1}}$$

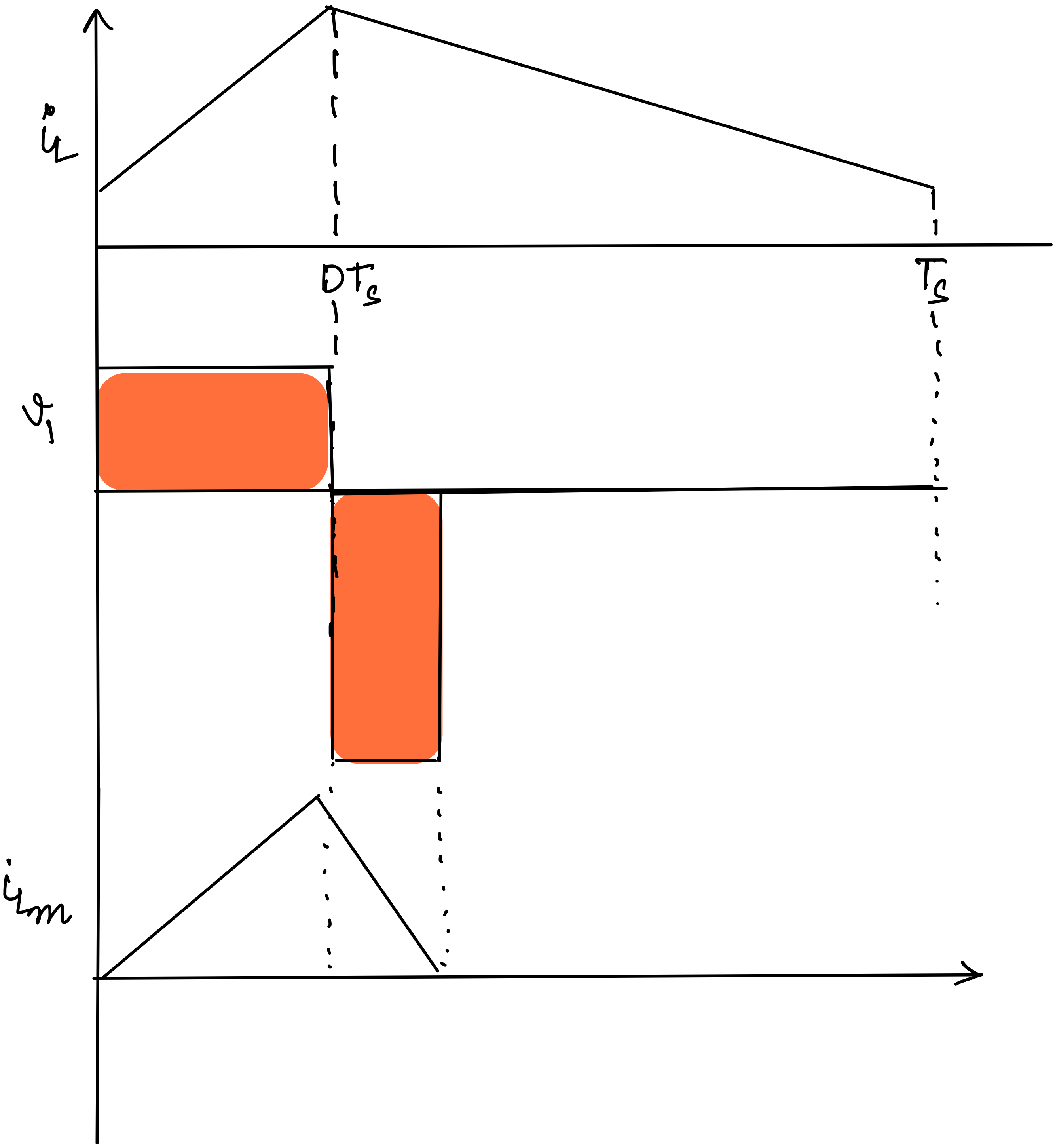
Although it is buck derived, we can still get a gain $\left(\frac{N_2}{N_1}\uparrow\uparrow\right)$

$$\Rightarrow \boxed{V_0 \gtrless V_{in}}$$

Here v_1 avg value > 0 $\Rightarrow$ transformer flux will keep on rising $\Rightarrow$ current rises exponentially.

$\therefore$ This circuit can't work in practical conditions





i_L
DT_s
T_s
v_1
i_m

$\underline{S_{off}}$

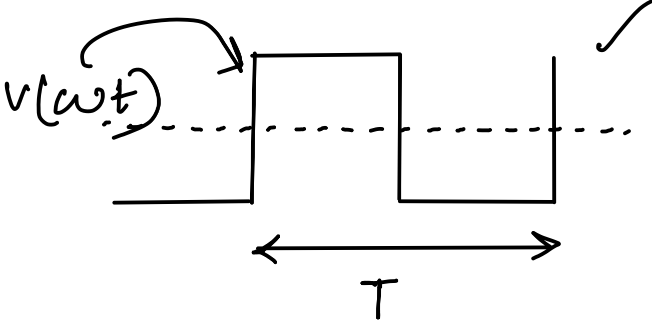
$$V_3 = V_1 \times \frac{N_3}{N_1}$$

$$V_3 + V_{in} = 0$$

$$V_3 = -V_{in}$$

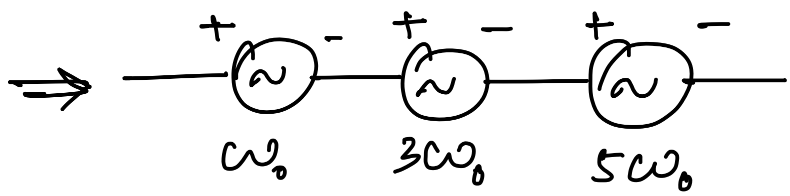
$$\boxed{V_1 = \frac{-N_1}{N_3} V_{in}}$$

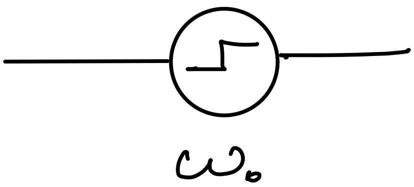
Fourier Series Revision



half wave symmetric

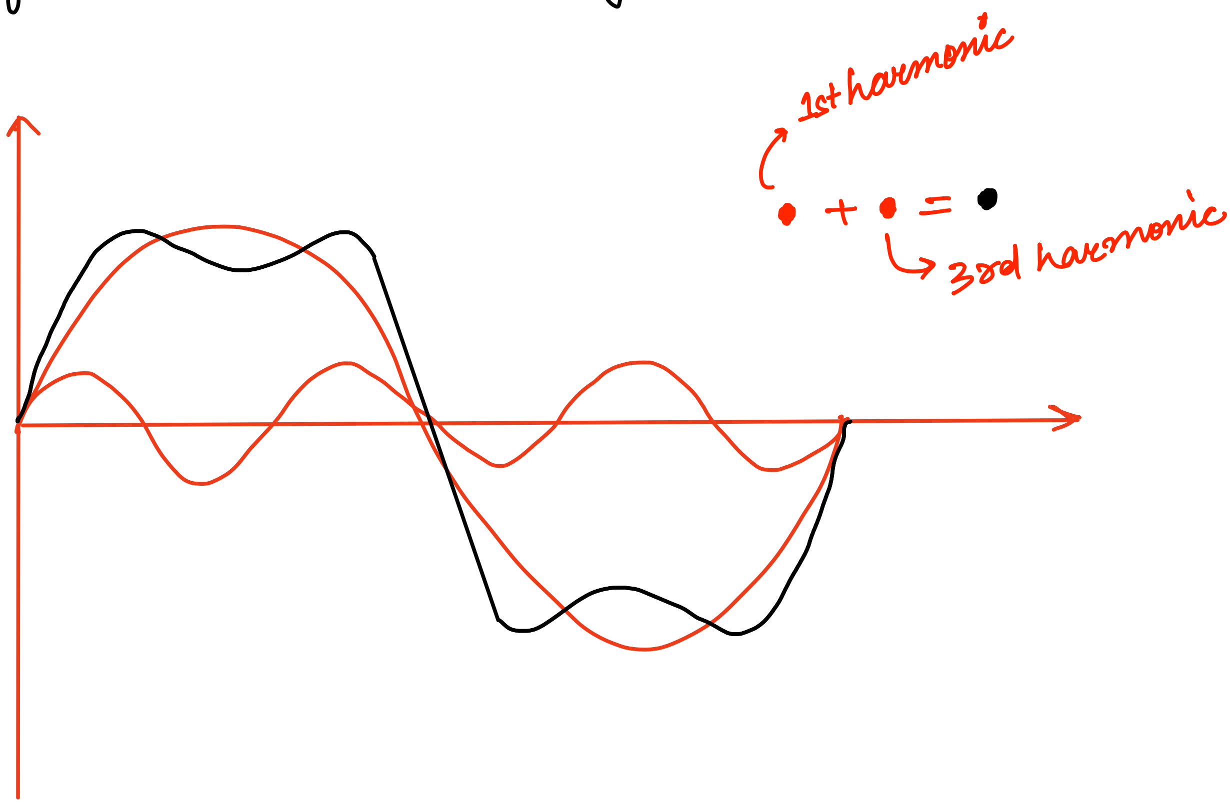
$f(\omega t) = -f(-\omega t)$





fundamental frequency

$$f(x) = -f\left(x + \frac{T}{2}\right)$$



$$v(\omega t) = \left(\frac{1}{2}a_0\right) + \sum_{n=1}^{\infty} a_n \cos(\omega_0 n t) + \sum_{n=1}^{\infty} b_n \sin(\omega_0 n t)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} v(\underbrace{\omega t}_{\theta}) \cos(n\theta)\, d\theta$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} v(\theta) \sin(n\theta)\, d\theta$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} v(\theta)\, d\theta$$

$a_0 \rightarrow$ avg. dc value.

There is no j anywhere here because the waveform is real.

$$V^{rms^2} = V_1^{rms^2} + V_2^{rms^2} + V_3^{rms^2} + \dots + V_{dc}^{rms^2}$$

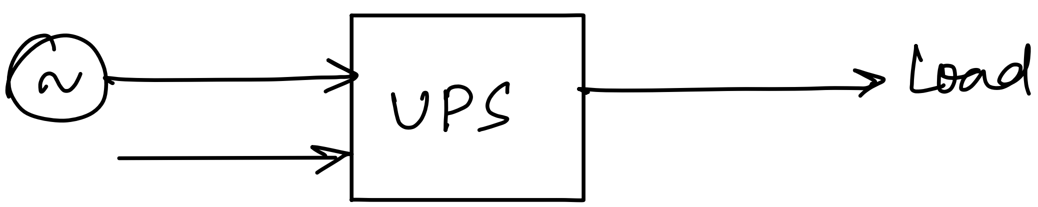
Total Harmonic Distortion (THD)

$$THD = \sqrt{\frac{V_2^{rms^2} + V_3^{rms^2} + \dots}{V_1^{rms^2}}}$$

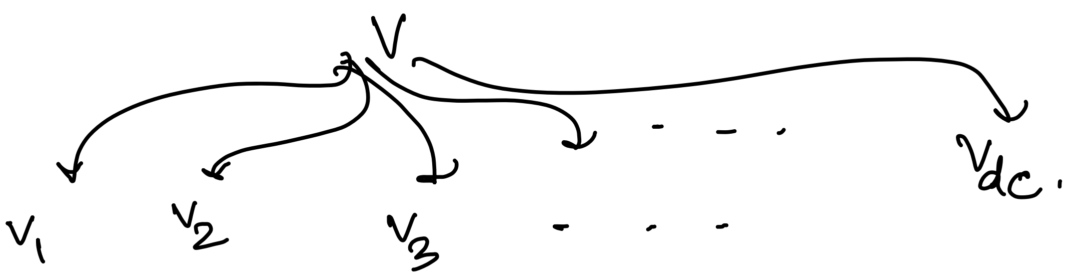
$$V_{rms}^2 - V_{1rms}^2 - V_{dc}^2$$

(can be greater than 1)

For a pure sine wave, THD = 0%



$$V_2^{rms} = \sqrt{\left(\frac{a_2}{\sqrt{2}}\right)^2 + \left(\frac{b_2}{\sqrt{2}}\right)^2}$$



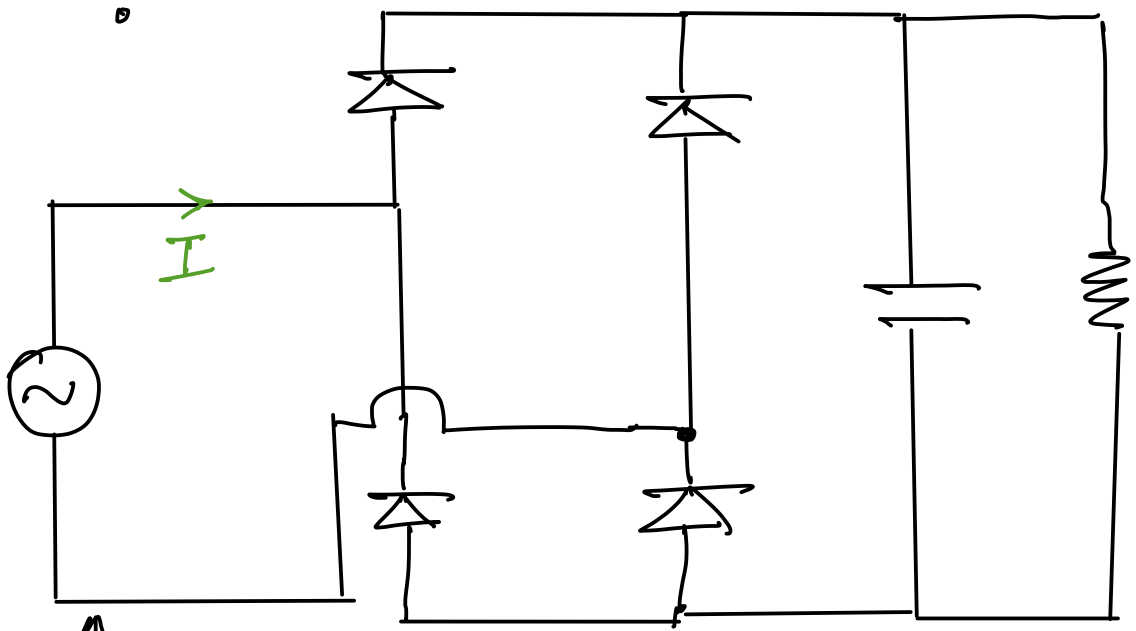
$$V_{dc} = \frac{a_0}{2}$$

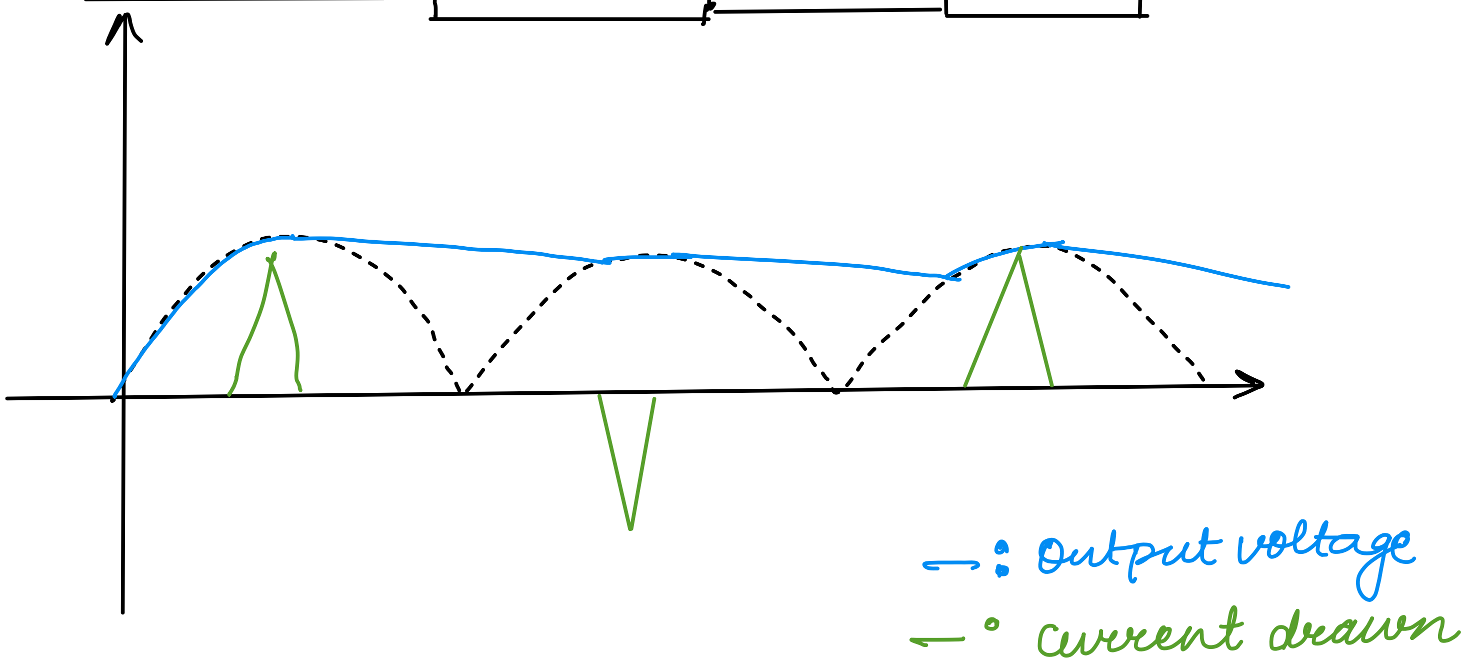
$$V_1 = a_1 \cos(\omega_0 t) + b_1 \sin(\omega_0 t)$$

$$V_2 = a_2 \cos(2\omega_0 t) + b_2 \sin(2\omega_0 t)$$

$$V_3 = a_3 \sin(3\omega_0 t) + b_3 \sin(3\omega_0 t)$$

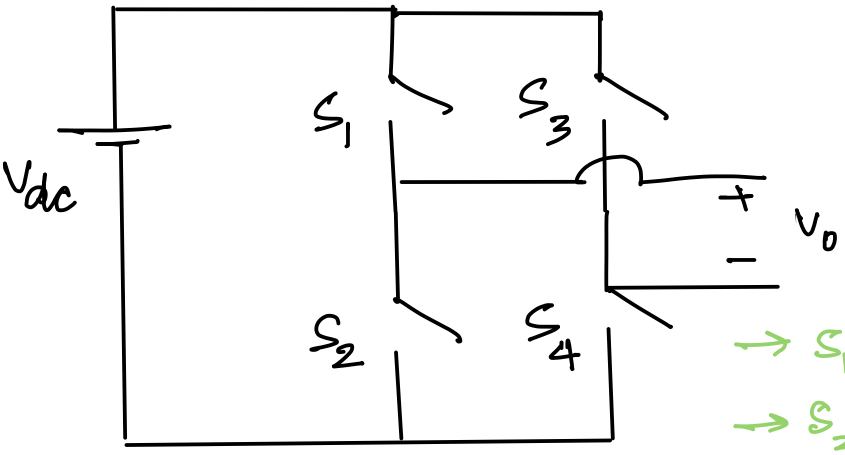
$\vdots$





I is half wave symmetric $\therefore$ Grid rarely contains even harmonics.

Inverters (aka DC–AC)



(in further ckts)

↳ cant keep

→ S_1 and S_2 both open

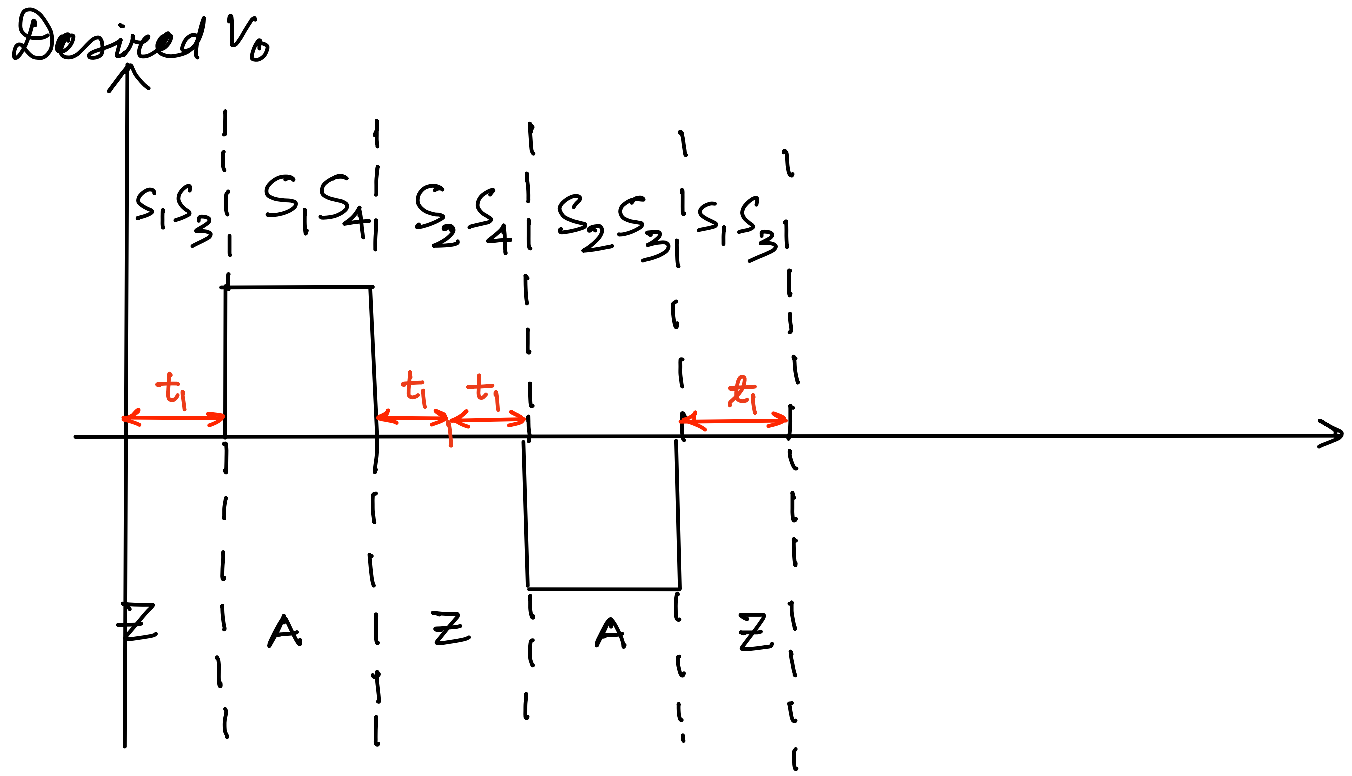
→ S_3 and S_4 both open

1 ϕ inverter

H-bridge inverter or full bridge inverter

	S_1	S_2	S_3	S_4	v_0	
	1	0	1	0	0	Zero states
Active states	1	0	0	1	V_{dc}	
Active states	0	1	1	0	$-V_{dc}$	
	0	1	0	1	0	Zero states

S_1 S_2 S_3 S_4 combos → popularly called states



If we use say only S_1S_3 as zero state, RMS current ↑↑ and ~~too~~ much load on S_1, S_3
(We try to divide the conduction losses)

To change the magnitude ($V_{o\,rms}$), change the width of states.

$$V_{o\,rms\,max} = V_{dc}$$

$$\boxed{V_{o\,rms} \leq V_{dc}}$$

Q Why is the waveform drawn to be so symmetric?

Ans: To keep half wave symmetry and eliminate the even frequencies.

(we want as less harmonics as possible)

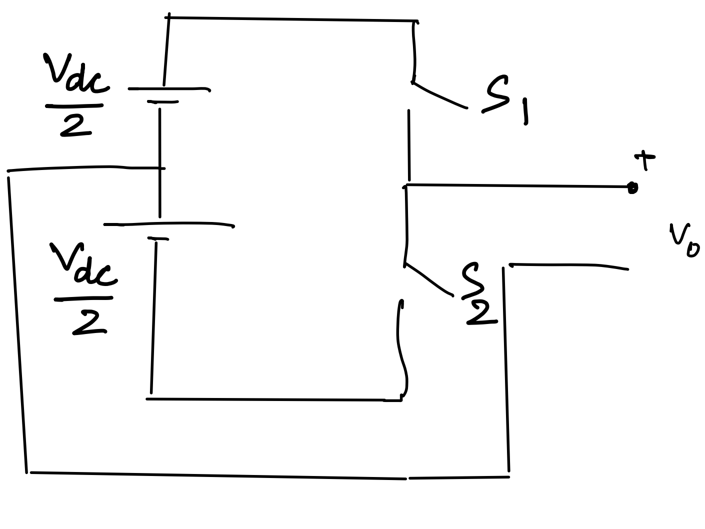
Say we use this inverter to power an induction motor. It would draw some fundamental current and harmonic currents and the motor will not work as smoothly as we want.

Pure sinusoid would have created a rotating flux field perfectly, but now these harmonics mess up a bit with the rotating flux field.

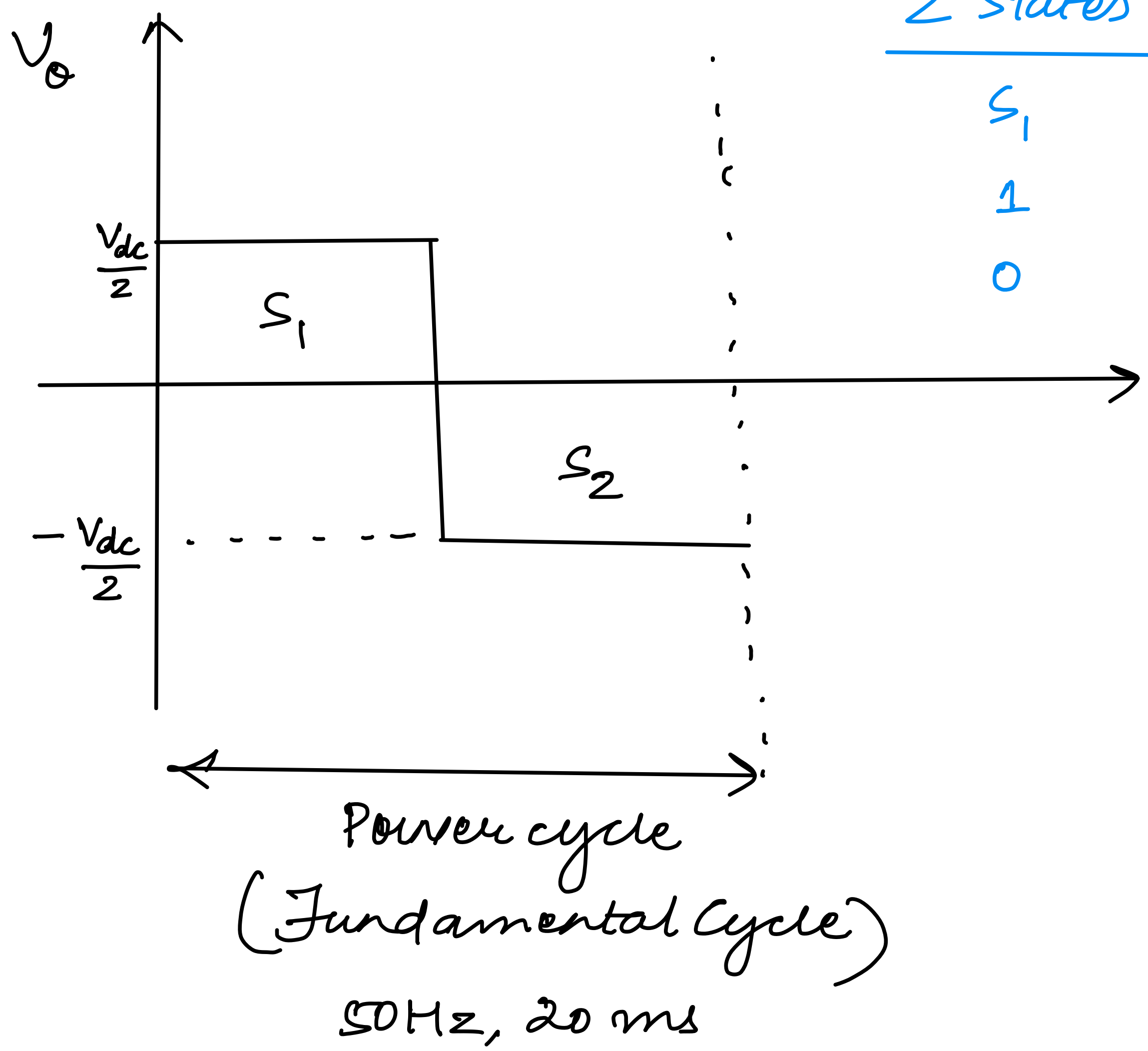
So why not at least eliminate the even harmonics :)

Keeping the midway symmetry also reduces fundamental frequency rms.

Half Bridge



Here $V_{o\,rms} = \frac{V_{dc}}{2}$ (non-adjustable)

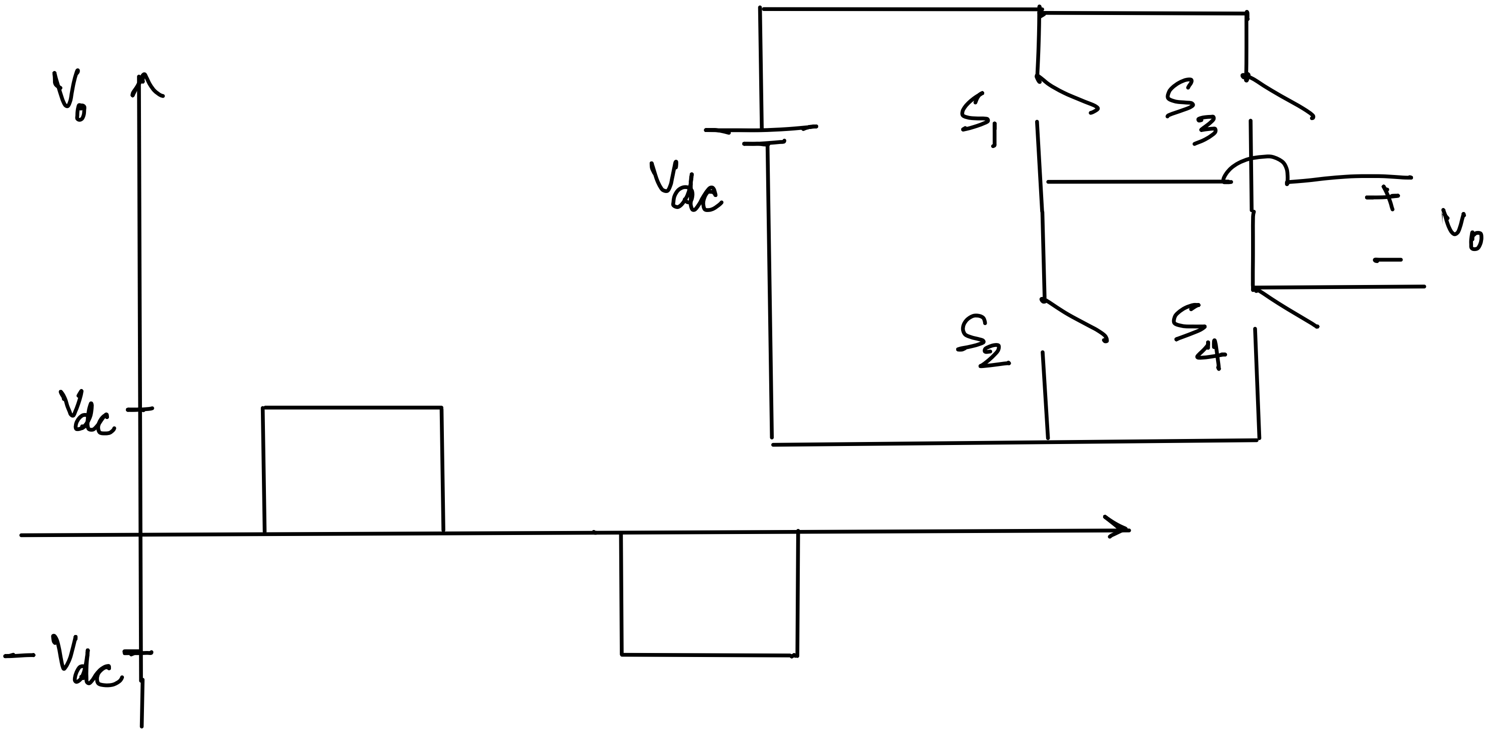


2 States Only.

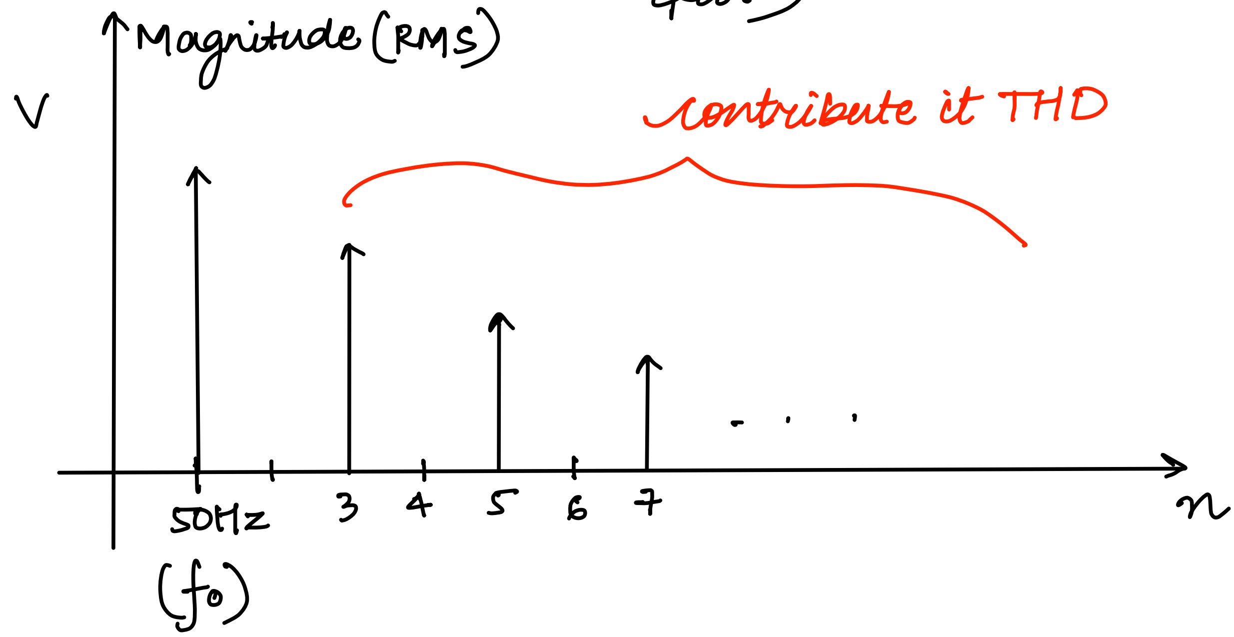
S_1	S_2
1	0
0	1

What we have looked at till now $\Rightarrow$ fundamental modulation

Each switch is switched only once in power cycle.



Frequency Spectrum (Find fourier series and then plot)



$$THD = \frac{\sqrt{\sum_{i=2}^{\infty} V_i^2}}{V_1} = \frac{\sqrt{\sum_{i=2}^{50} V_i^2}}{V_1}$$

(standard)

Problems with THD:

→ Oscillatory torque in motors
→ Less utilisation of cables/wires

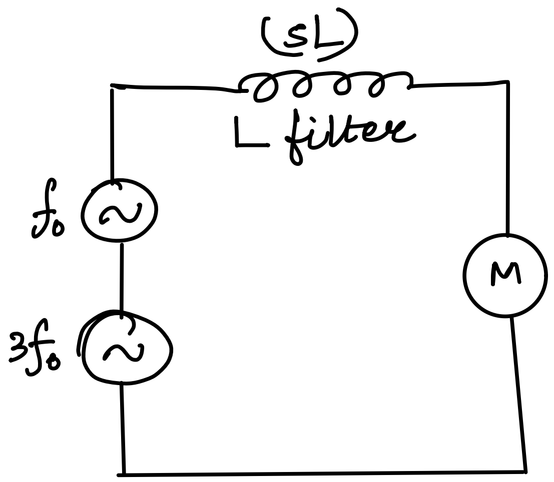
$$I^{RMS} = \sqrt{I_1^2 + \underbrace{I_2^2 + I_3^2 + \cdots}_{\text{unecessary.}}}$$

→ Losses (Harmonic currents causes additional losses) leading to heating of cables, motors, transformer

→ Resonance (Some harmonic may excite LC component due to resonance)

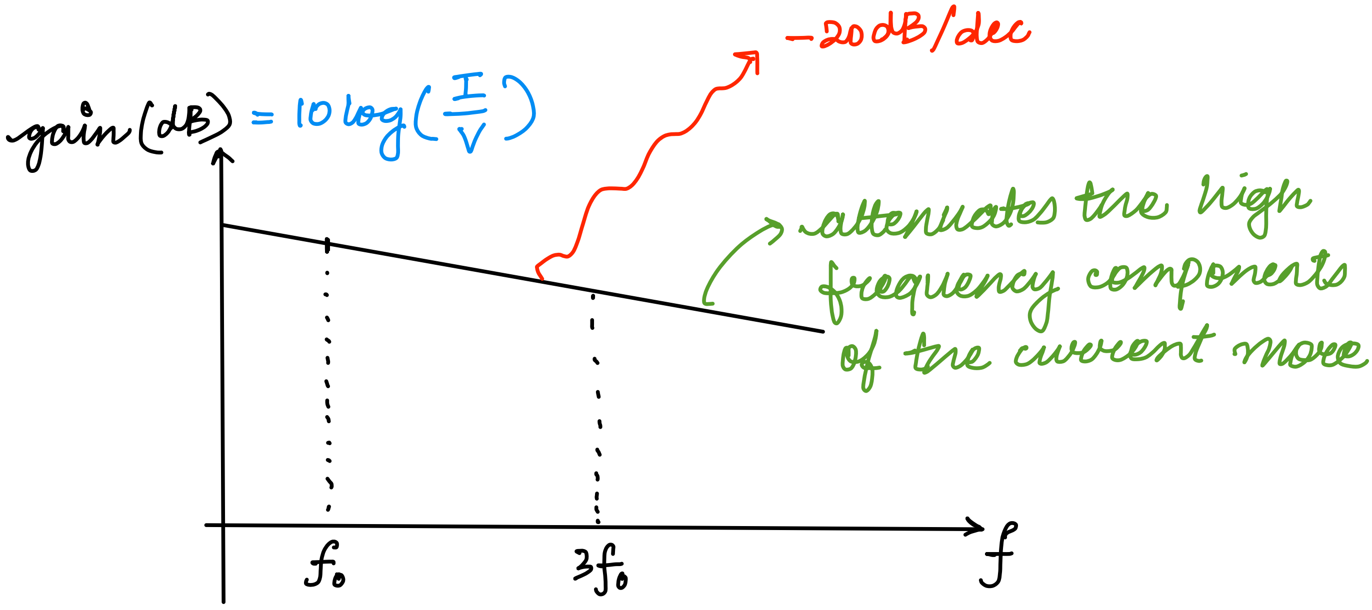
→ Interference (to other appliances)

↳ Because wire is acting kinda like an antenna (due to high frequency components)



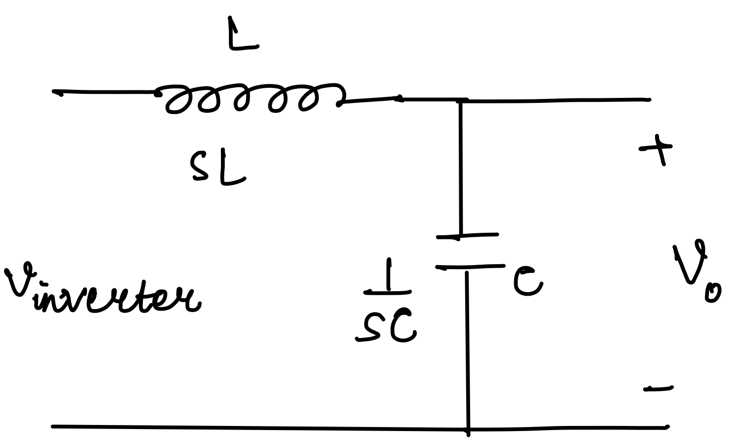
$$\bar{I} = \frac{\bar{V}}{sL}$$

$$H(s) = \frac{1}{sL}$$



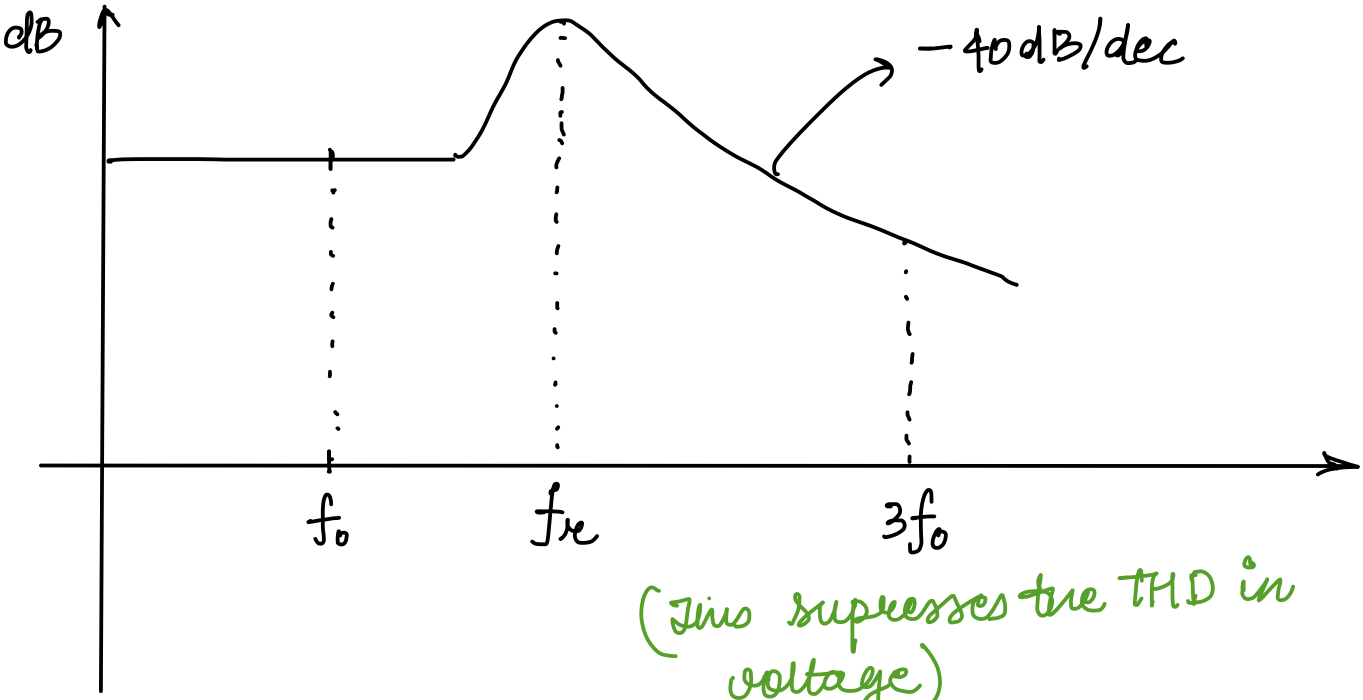
Using this inductor reduces THD in current as compared to THD in voltage.

(Say voltage has 50% distortion, this L filter would cause current distortion to be < 50%)



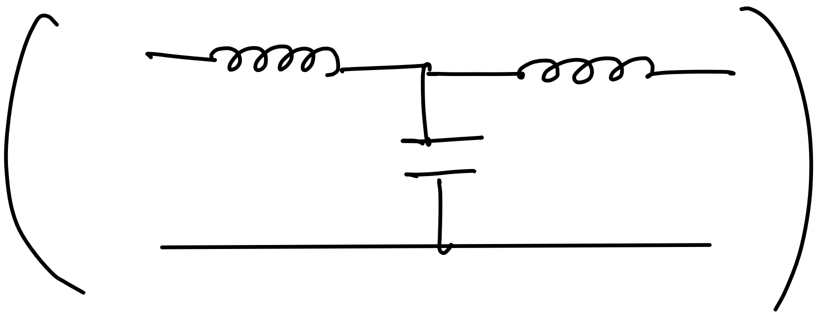
$$V_0 = \frac{V_{in}}{sL + \frac{1}{sC}} \times \frac{1}{sC} = \frac{V_{in}}{s^2LC + 1}$$

$$G(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{s^2 LC + 1}$$



(This supresses the THD in voltage)

We can have LCL filter, LCLC network filter. Keeps giving us better attenuation.



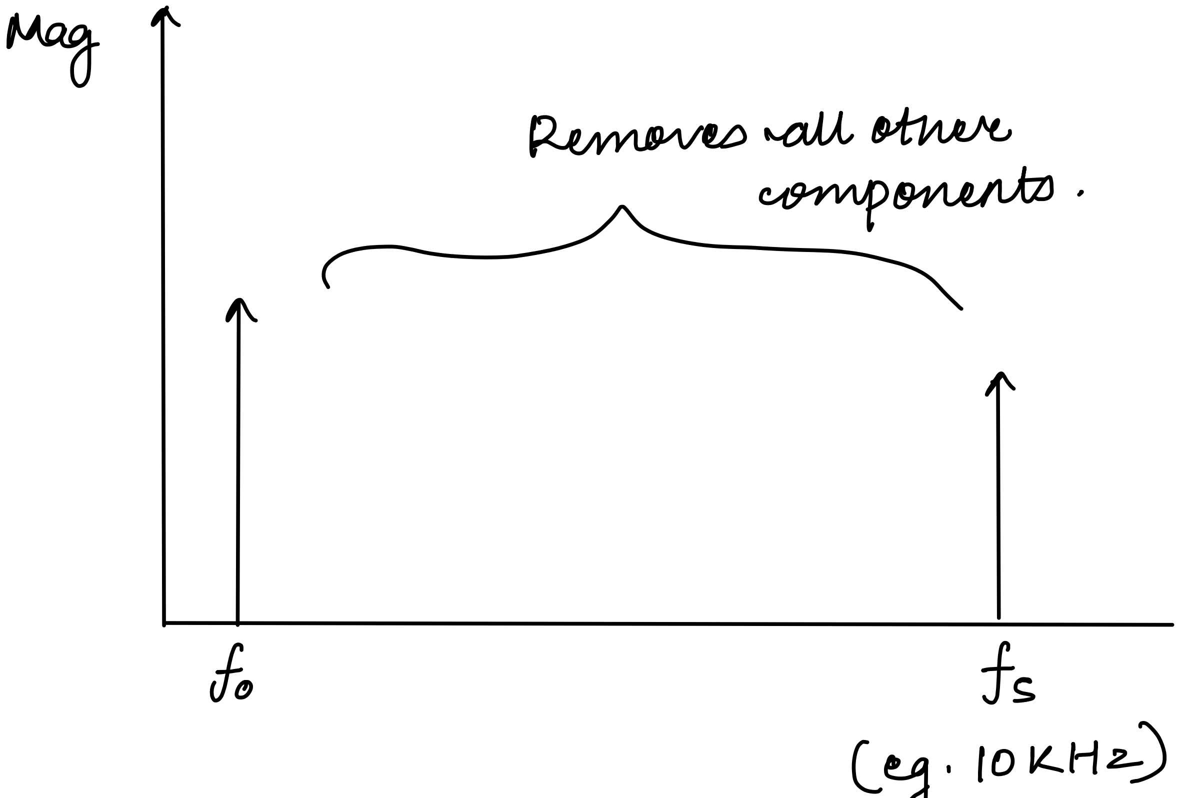
eg $f_r = \frac{1}{2\pi\sqrt{LC}} \approx 100$ Hz. Two consequences:

1) f_r is relatively low $\Rightarrow$ Large L, C

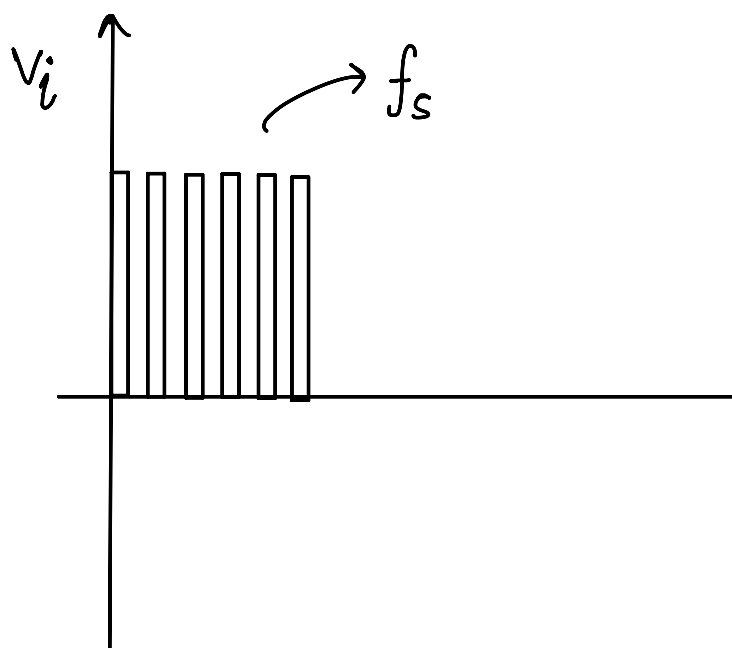
2) Difference between f_3 (150 Hz) and f_r (100 Hz) is small $\Rightarrow$ Less attenuation

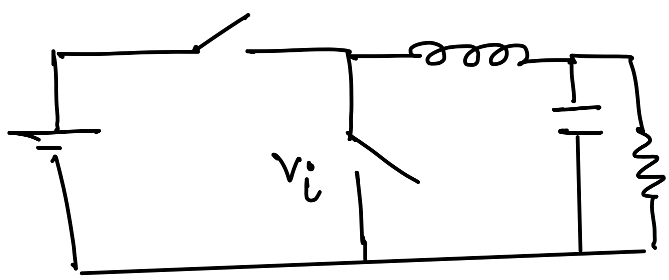
Sine Triangle PWM

Harmonic Spectrum



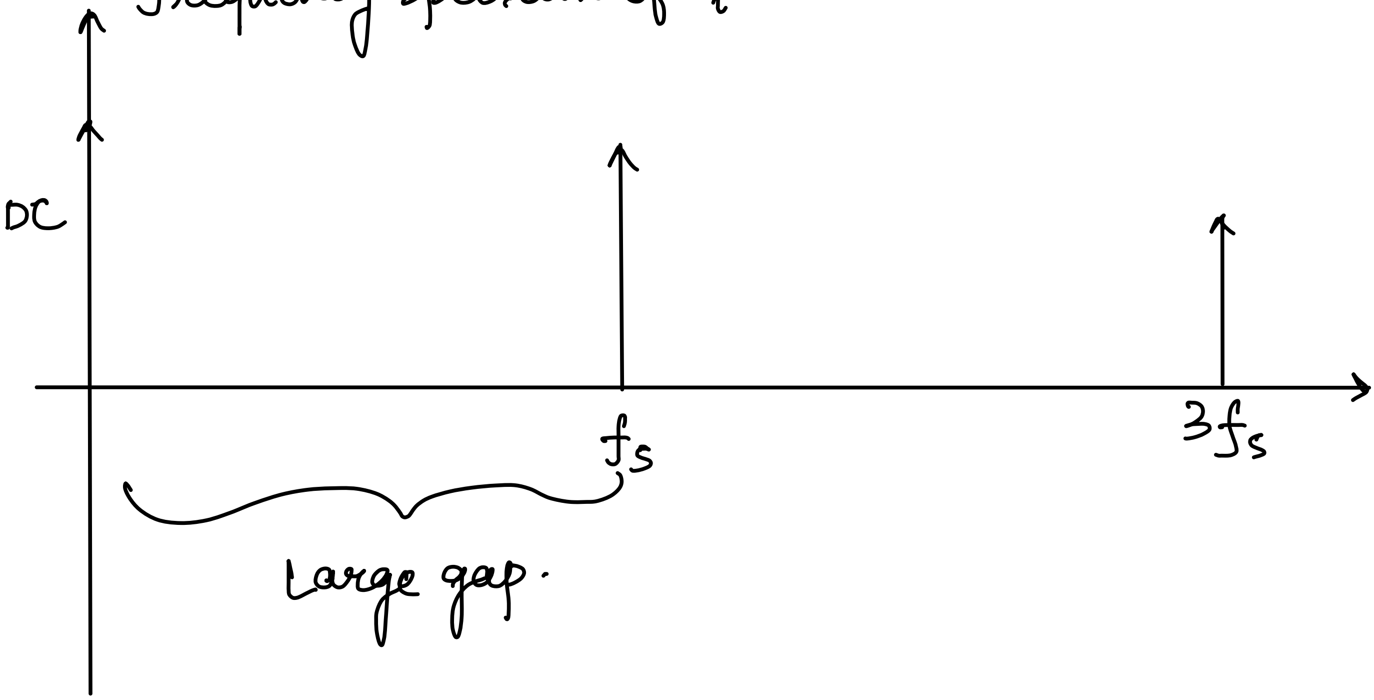
∴ Now if I want to use a LC filter, f_r can be anywhere between f_o and f_s. Thus smaller values of L and C. Also since f_s is crazy far away, we get crazy attenuation for f_s on using say a LC filter (−40 dB/dec)





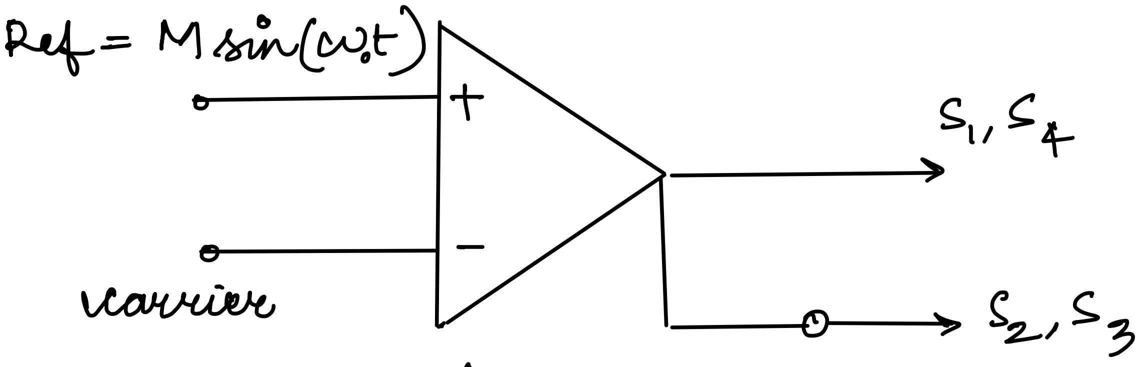
DC−DC buck

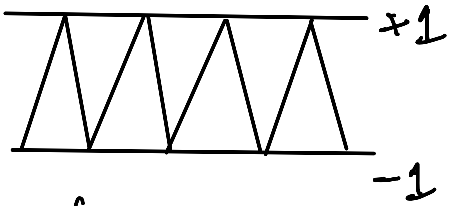
Frequency spectrum of V_i



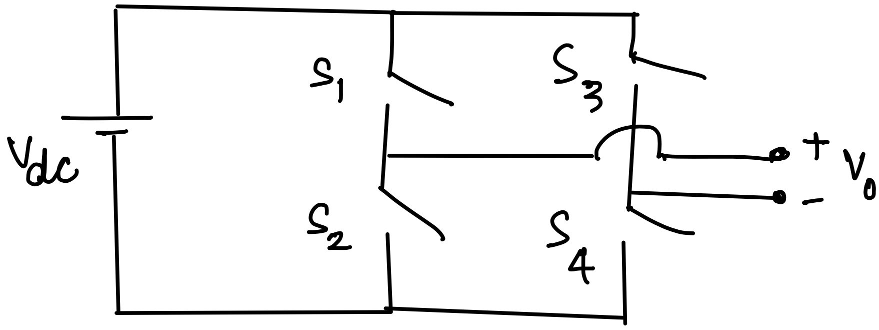
$0 \leq M \leq 1$

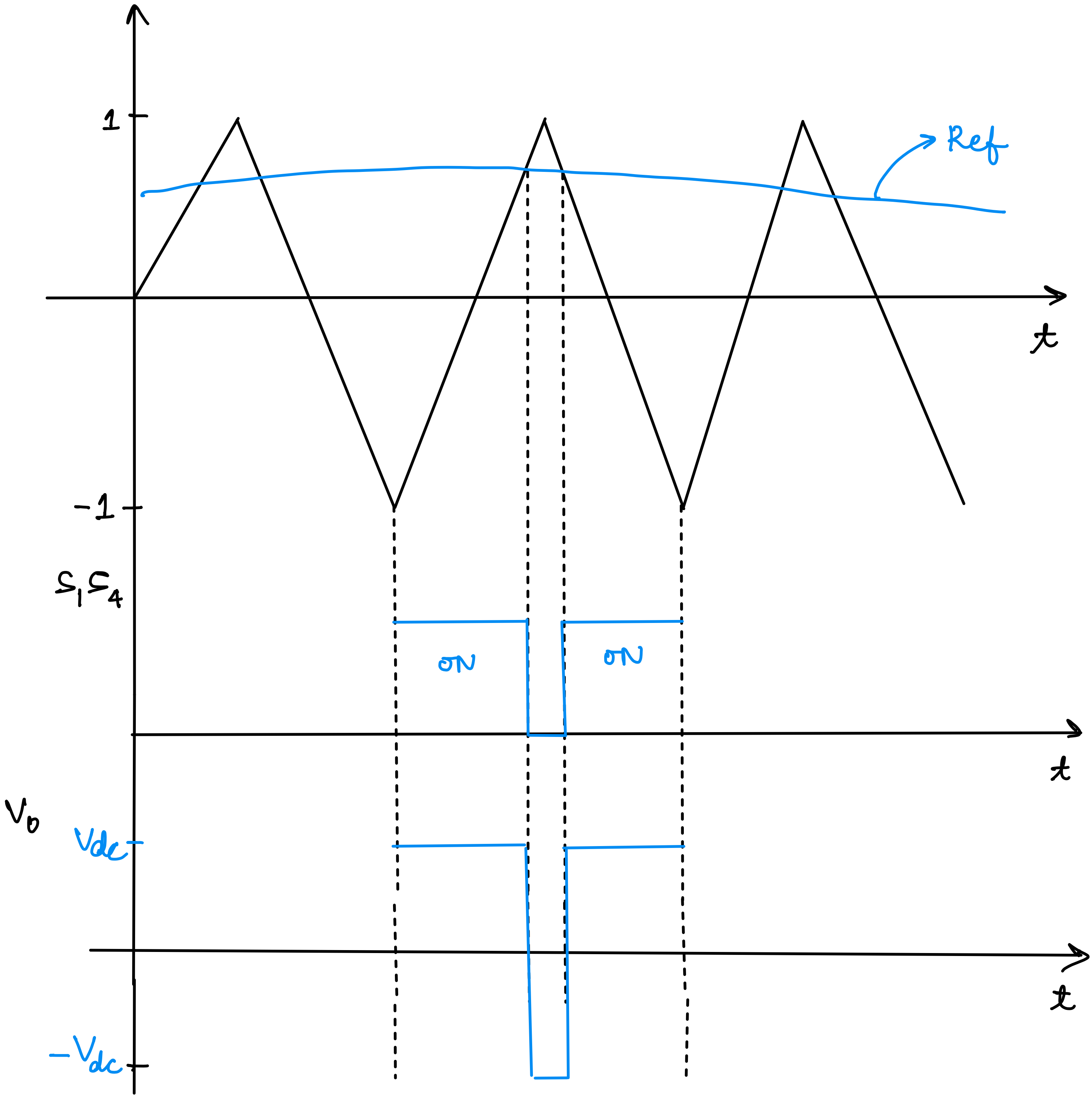
These are signals (say written in Microcontroller)

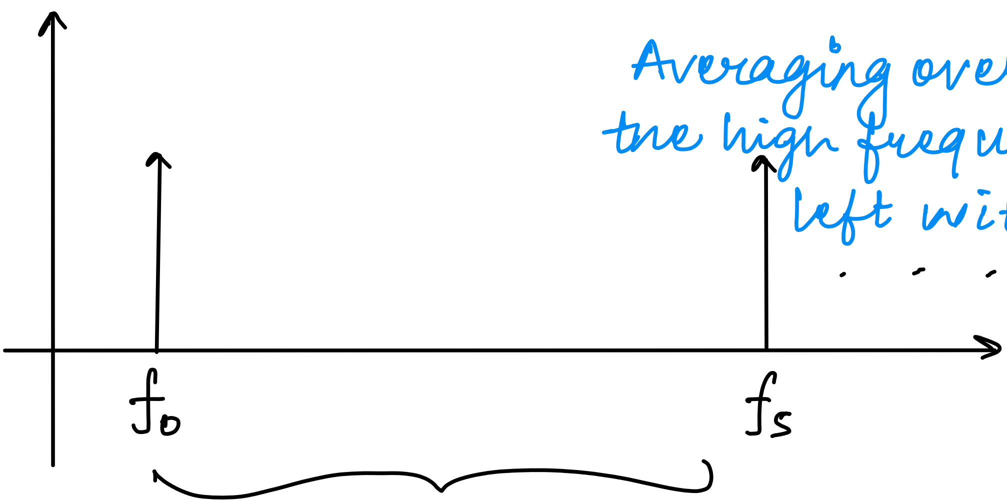




$f_s = 10KHz$





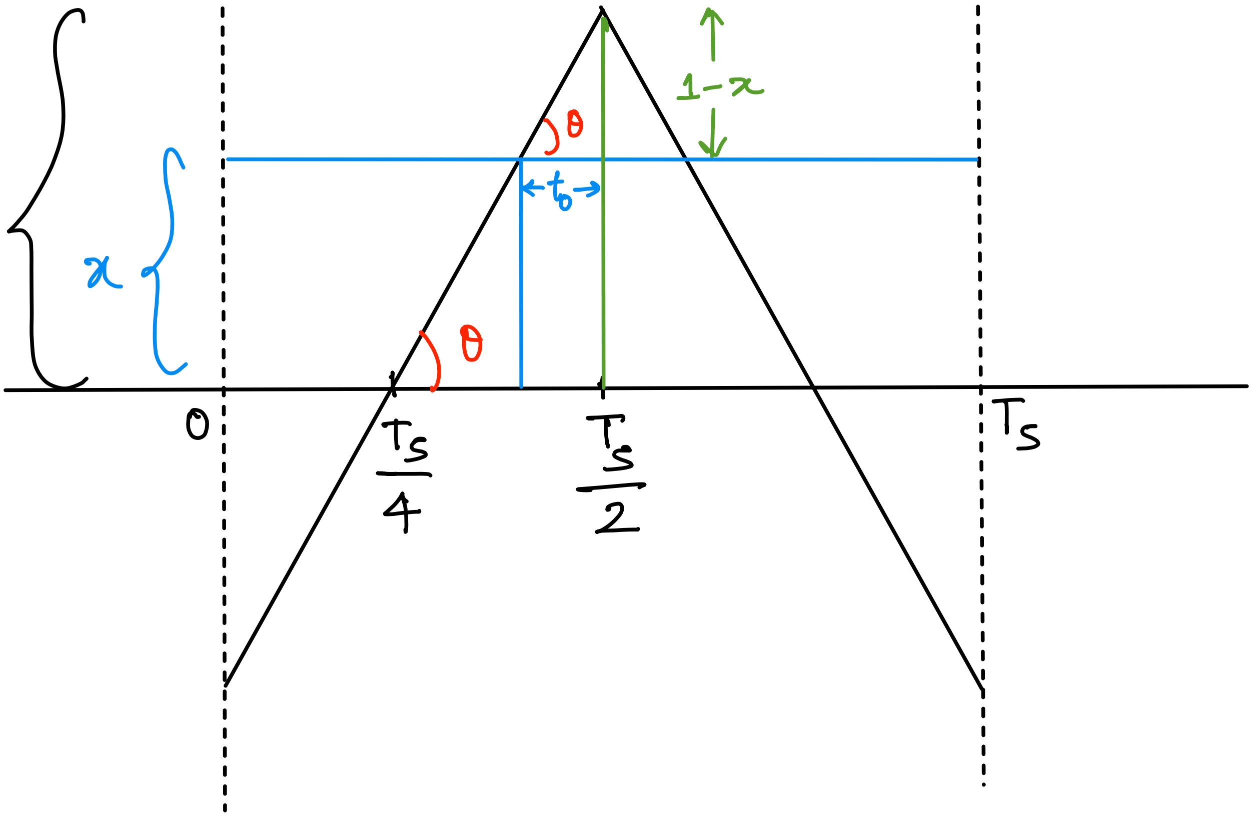


Averaging over T_s removes the high frequency. We are left with fundamental freq. only.

nothing in between.

Q. Derive $\langle V_0 \rangle_{T_s} = M V_{dc} \sin(\omega_0 t)$

Sol^n



Reference signal is assumed to be constant over T_s

$$\tan\theta = \frac{1-x}{t_0} = \frac{1}{(T_s/4)}$$

$$\Rightarrow 2t_0 = (1-x)\left(\frac{T_s}{2}\right)$$

$$\therefore \langle V_0 \rangle_{T_s} = \frac{2t_0(-V_{DC}) + (T_s - 2t_0)V_{DC}}{T_s}$$

$$= \frac{T_s V_{DC} - 4t_0 V_{DC}}{T_s}$$

$$= \frac{V_{DC}}{T_s}\left(T_s - (1-x)T_s\right)$$

$$= V_{DC}(1-1+x)$$

$$= xV_{DC}$$

$$= MV_{DC}\sin(\omega_0 t)$$

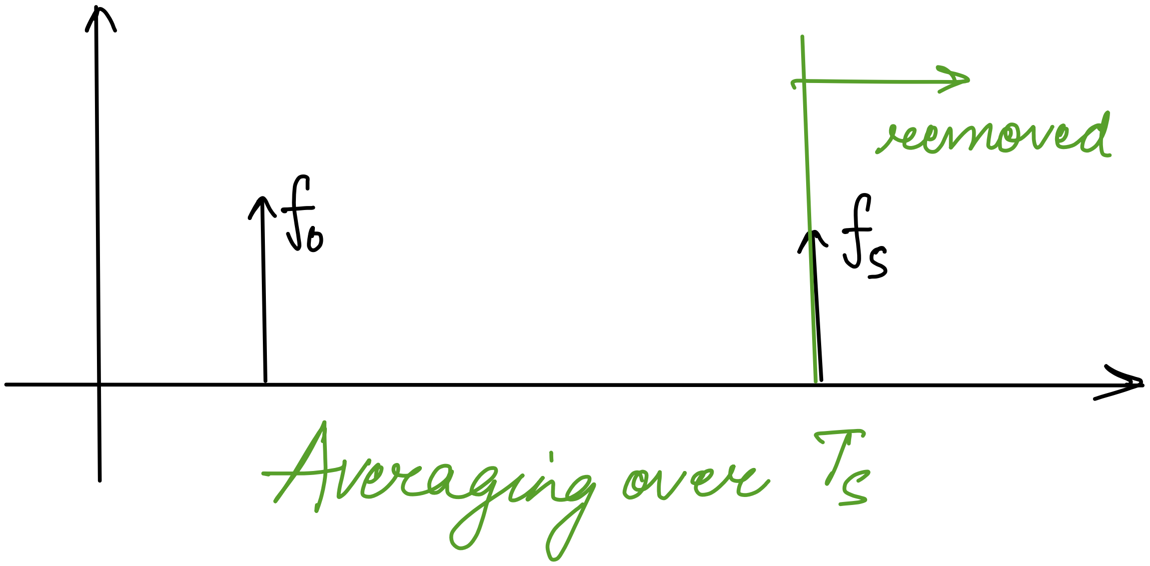
Say our reference is a constant signal M.
Then on averaging over T_s, we would have got
$MV_{dc} \Rightarrow$ Buck convertor.

Q How do we physically average over T_s?

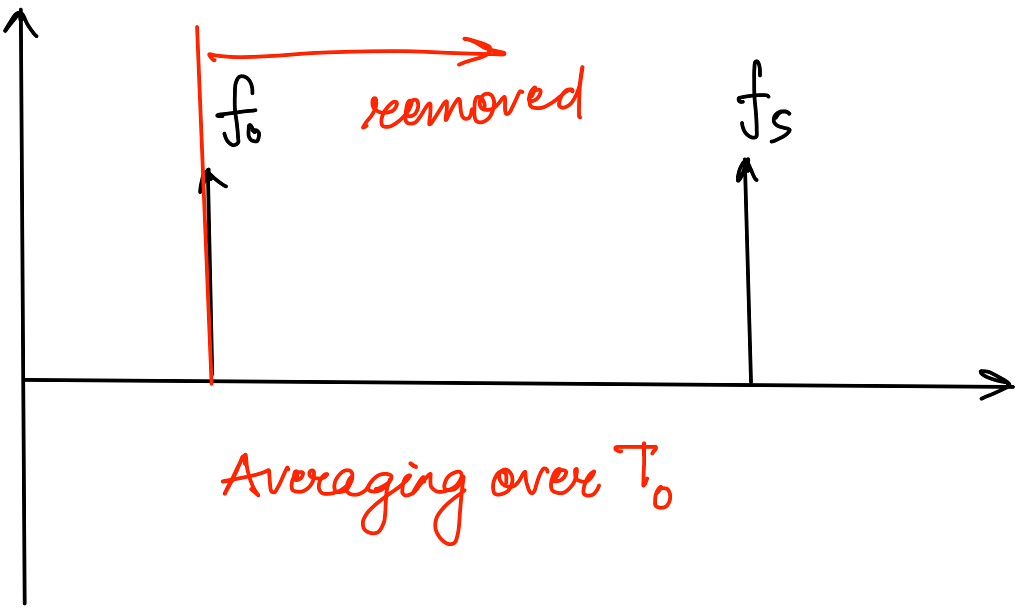
$\Rightarrow$ Low pass filter.

Q. Why are we getting $MV_{dc} \sin(\omega_0 t)$ instead of 0 after averaging?

Ans: We are averaging over the switching period not the fundamental period



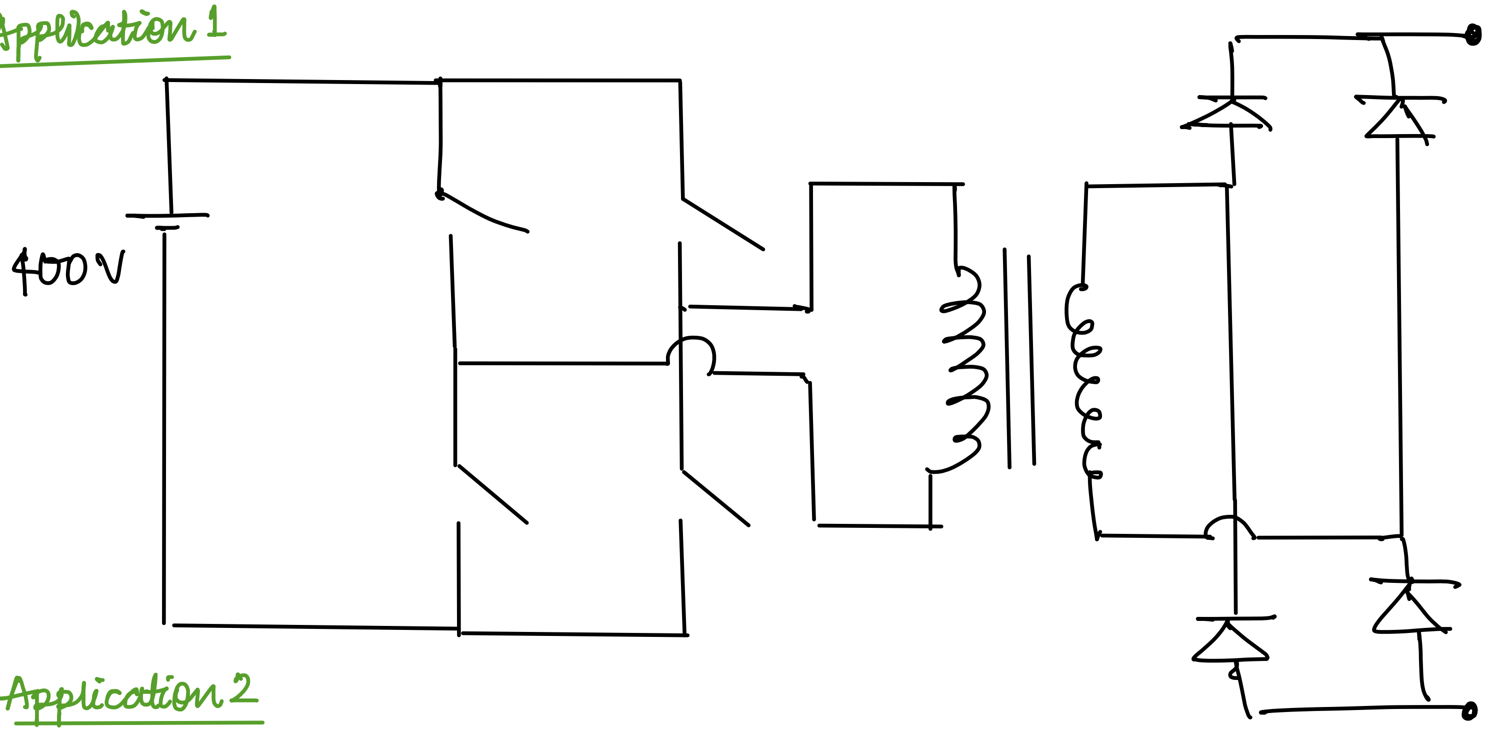
If we average over fundamental cycle, we get 0 because there is no DC component.



Inverter

Using 1-ϕ inverter for DC-DC

Application 1



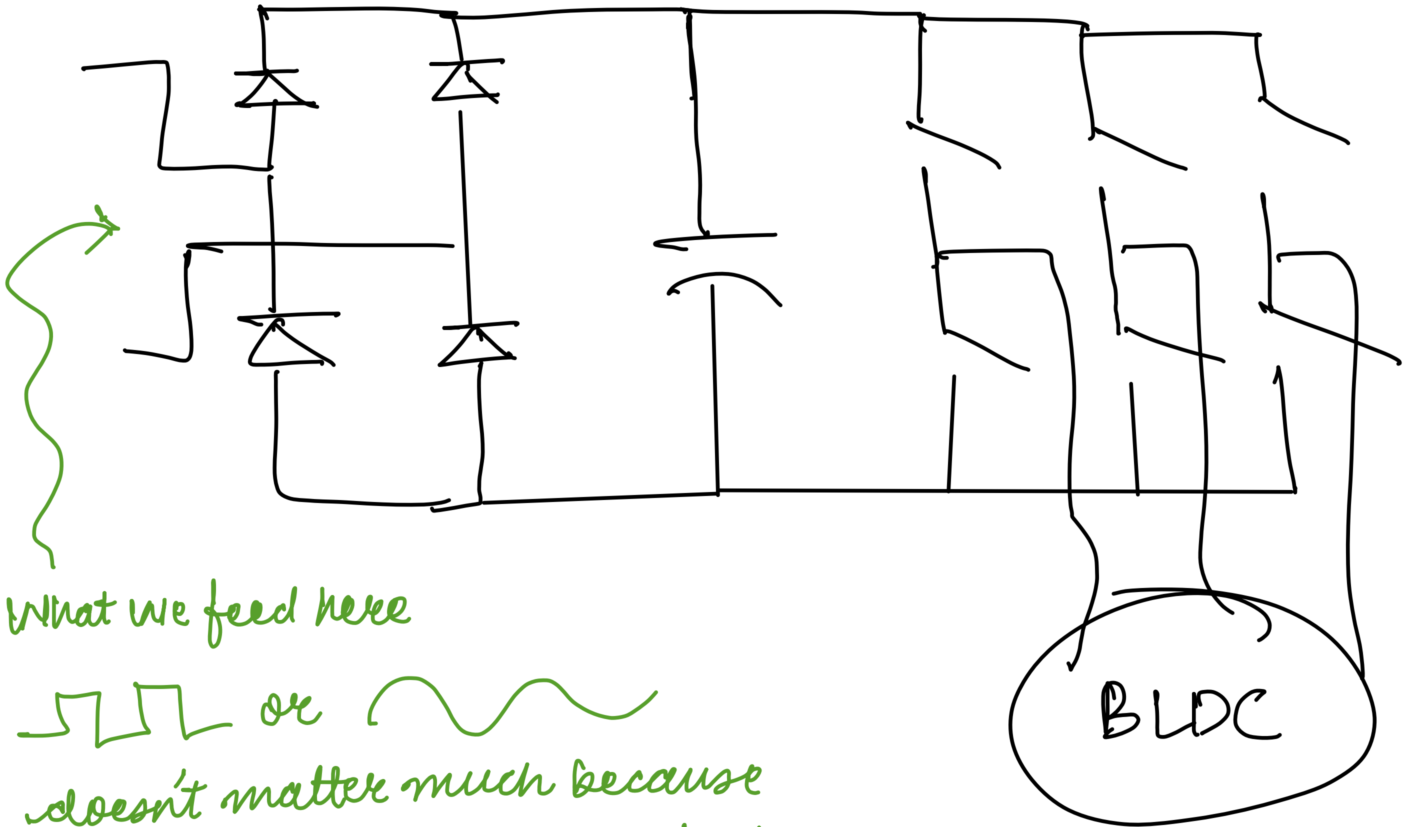
Application 2

In a house UPS system, because house would mostly have a single ϕ load

If we use SPWM → no humming sound from fan
If we use fundamental modulation → humming sound from fan due to other frequencies.

Only for IM fans.

BLDC fans → no difference whether it gets square or sine wave.

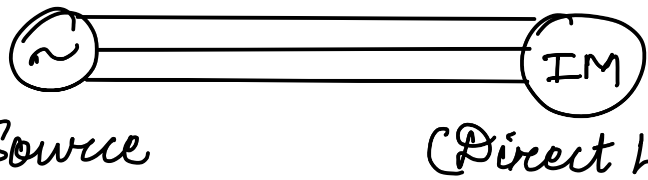


What we feed here

or

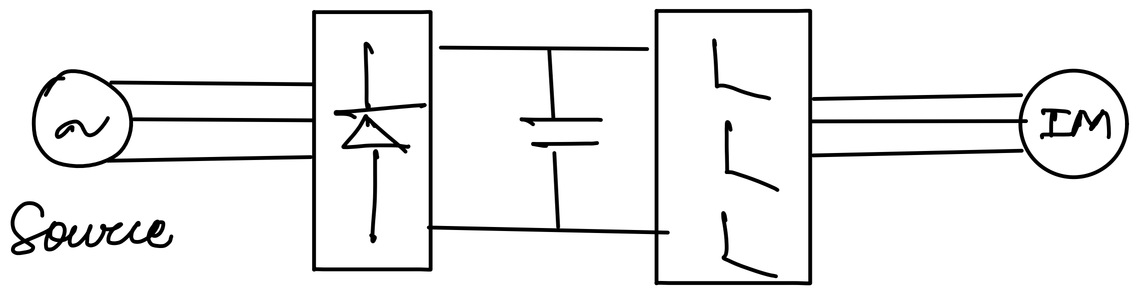
doesn't matter much because it is anyways getting rectified. The torque production happens on the right side not via the inverter in UPS.

(a)



(Direct Line Start)

(b)



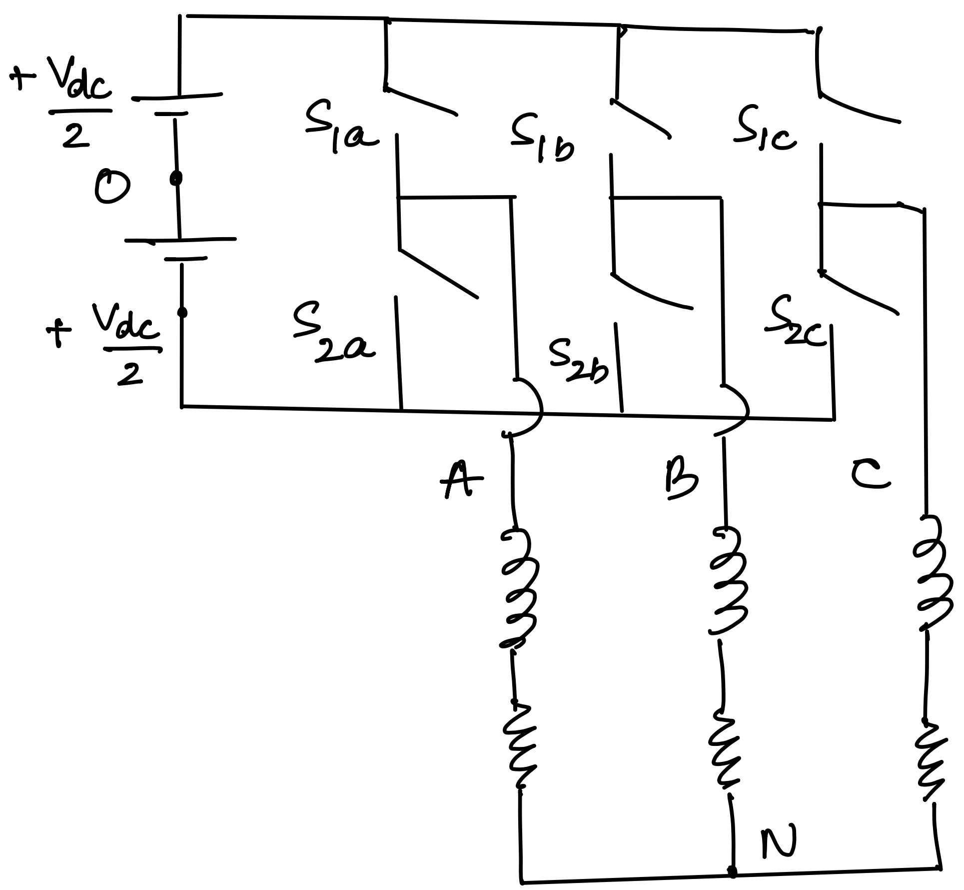
(Speed Control)

(b) → Smooth Speed/Torque control
→ Field weakening ($\uparrow \omega$)
→ High τ_{start}
→ Input current is always limited ∴ Low starting current.

Revise:

- τ-speed characteristics
- τ_{max} formula
- Eq. ckt of IM.

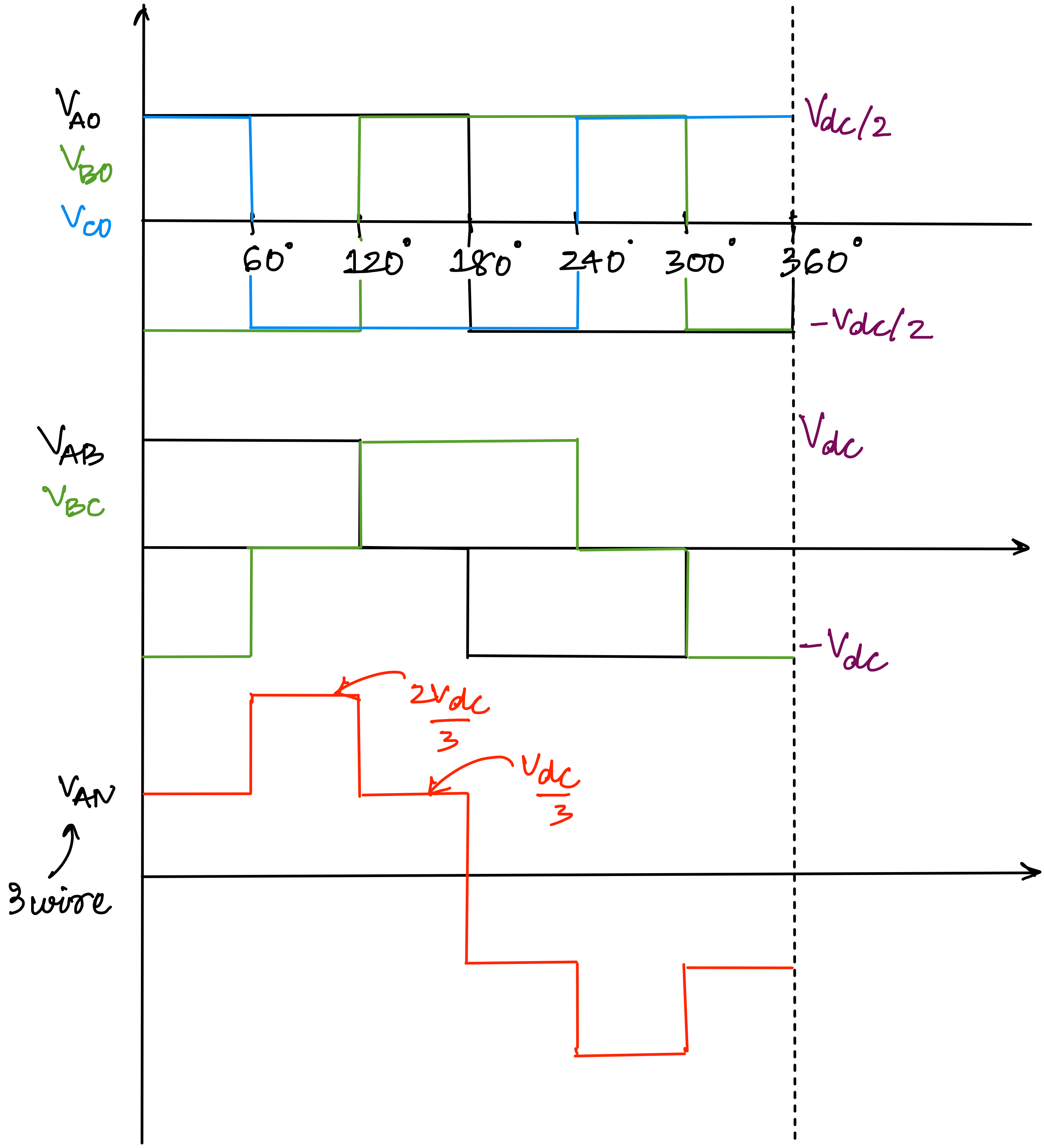
3ϕ Inverter



(★ connected load)

(Balanced Load) → $R_a = R_b = R_c (= R)$
$L_a = L_b = L_c (= L)$

Fundamental Modulation



For 4 wire: Phase voltage = Pole voltage.

$$\boxed{V_{AN} = V_{AO} - V_{NO}}$$

V_{AO}: Pole Voltage for A.

V_{BO}: Pole Voltage for B

V_{CO}: Pole Voltage for C

V_{AB}, V_{BC}, V_{CA} : Line to Line voltages

V_{AN}, V_{BN}, V_{CN}: Phase voltages

$$V_0 + V_{AO} - V_N = i_a R_a + L_a \frac{di_a}{dt}$$

$$V_0 + V_{BO} - V_N = i_b R_b + L_b \frac{di_b}{dt}$$

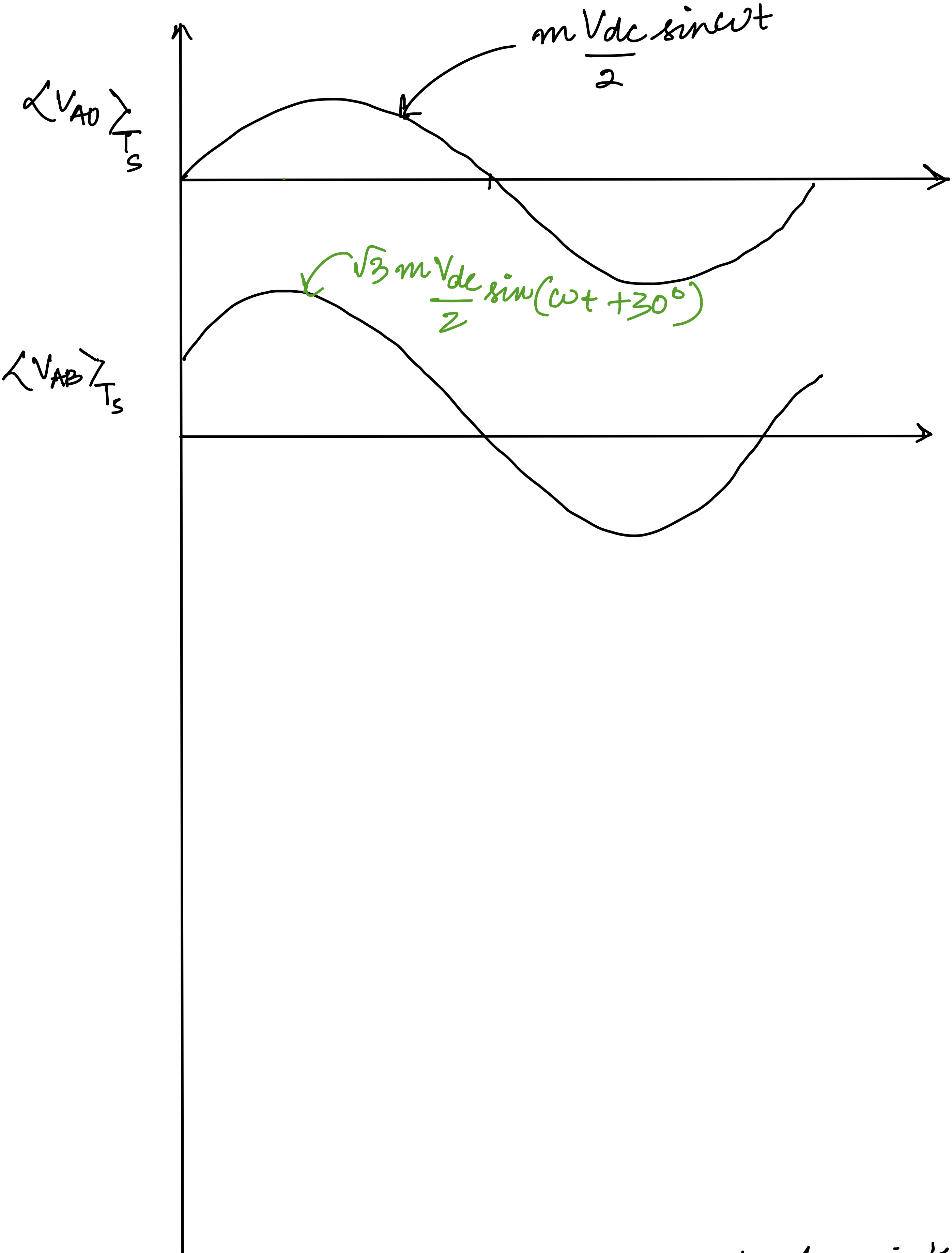
$$+ \quad V_0 + V_{CO} - V_N = i_c R_b + L_c \frac{di_c}{dt}$$

$$\Rightarrow 3V_0 + V_{AO} + V_{BO} + V_{CO} - R\cancelto{0}{\sum i_a} - L\frac{d}{dt}\left(\cancelto{0}{\sum i_a}\right) = 3V_N$$

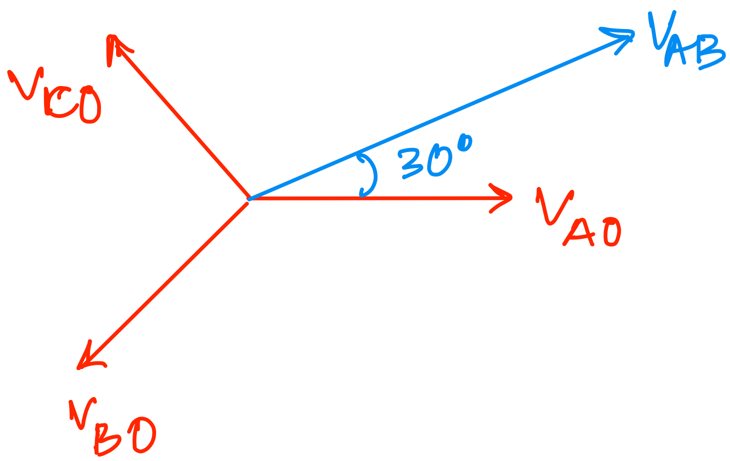
$$\Rightarrow \boxed{V_{NO} = \frac{1}{3}\left(V_{AO} + V_{BO} + V_{CO}\right)}$$

m: modulation index $[m \in (0,1)]$

SPWM



The averaging of I will be done by the inductor in the load itself. Need not worry about it.



$$V_{AB} = \frac{\sqrt{3}\, m V_{dc}}{2} \sin(\omega t + 30^\circ)$$

Q. What would be the phase and magnitude of fundamental component in phase voltage in 3-wire system?

Ans: $V_{AN} = V_{AO} - V_{NO}$

$$= V_{AO} - \cancelto{0}{\frac{1}{3}(V_{AO} + V_{BO} + V_{CO})}$$

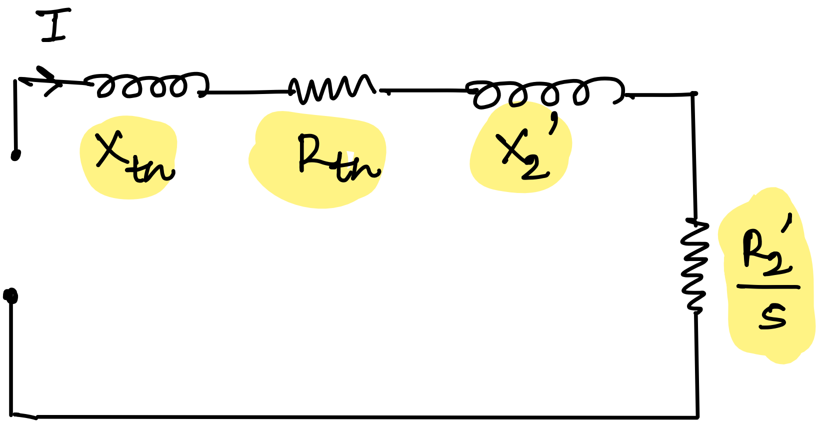
$$= V_{AO}$$

∴ Same as pole voltage.

In coming years, data centres as a load would become popular enormously.

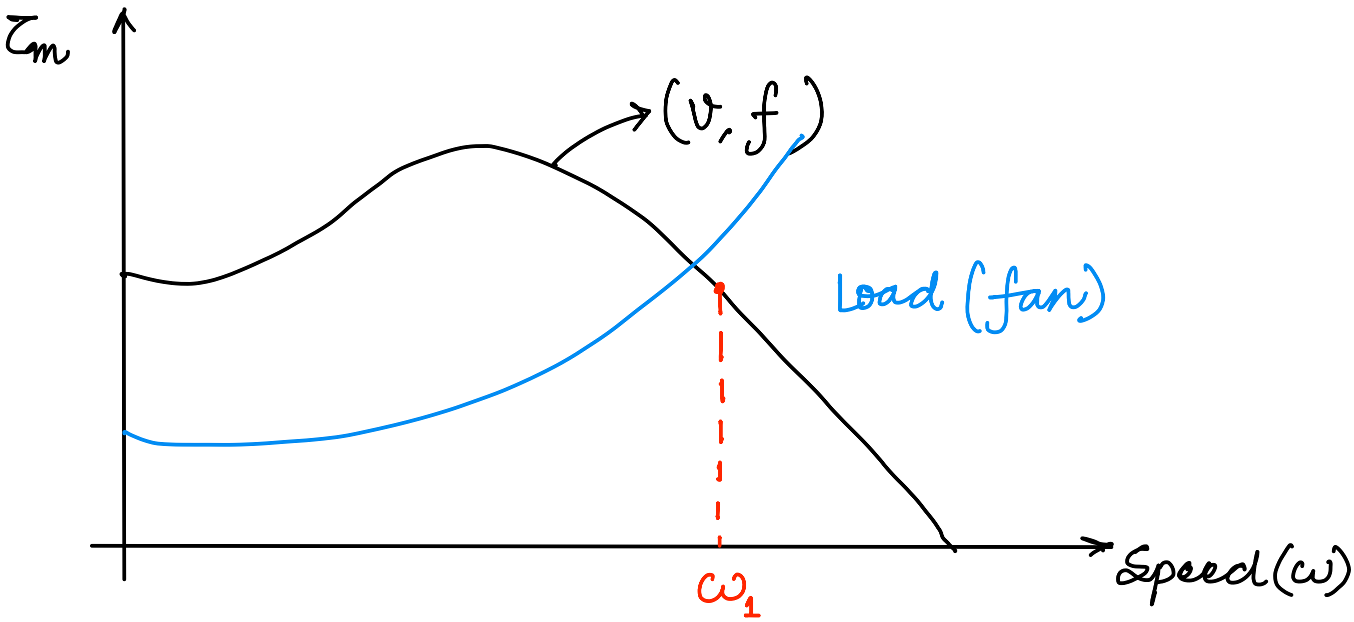
Induction Motors

Equivalent ckt :-



$$I = \frac{V_{th}}{\sqrt{(X_{th} + X_2')^2 + \left(R_{th} + \frac{R_2'}{s}\right)^2}}$$

$$\tau_{max} = 3 \times \frac{V_{th}^2}{(X_{th} + X_2')^2 + \left(R_{th} + \frac{R_2'}{s}\right)^2} \times \frac{R_2'}{\omega_{syn}} \times \frac{1}{s}$$



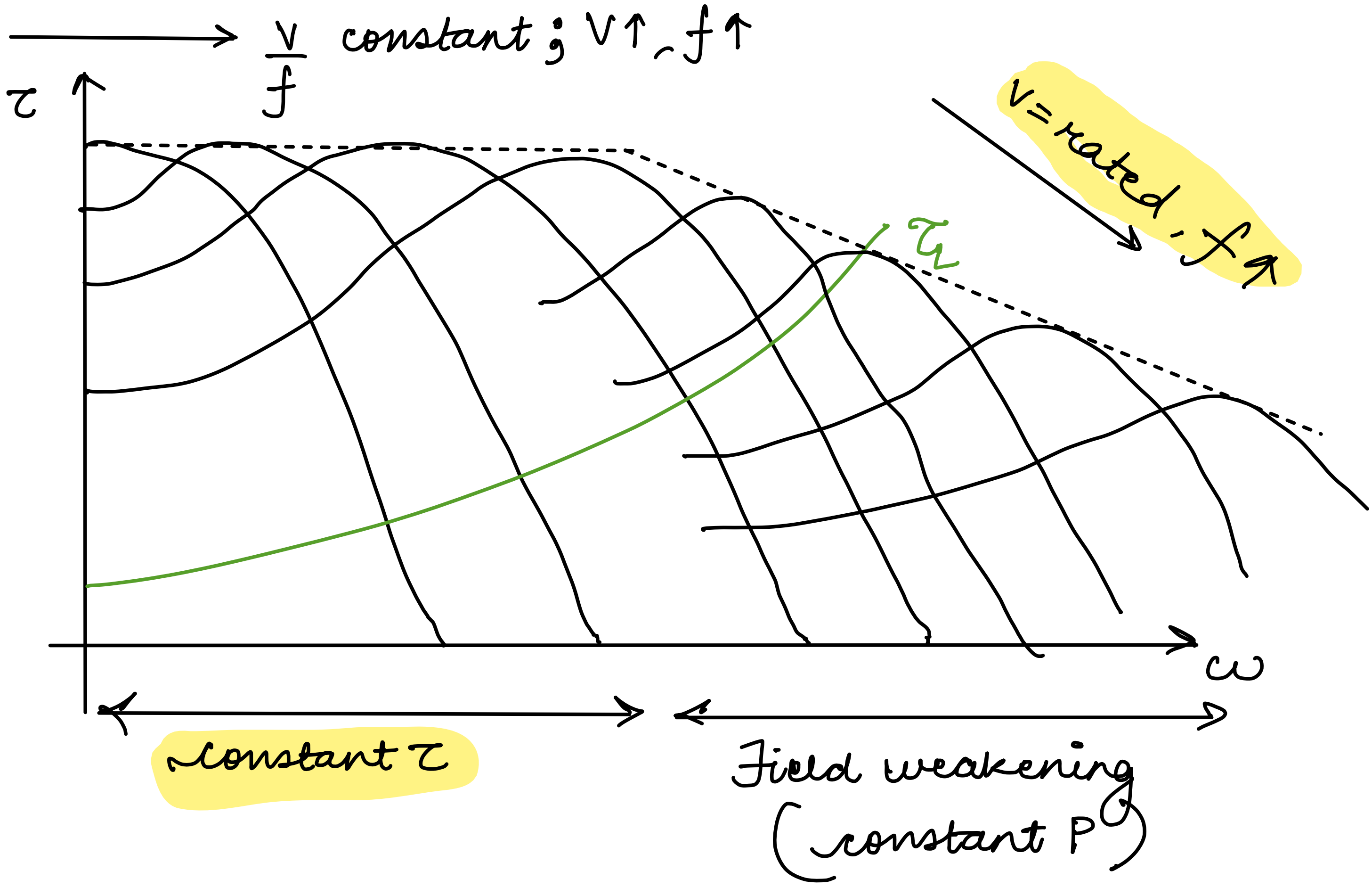
ω_1: speed at which the fan will rotate.

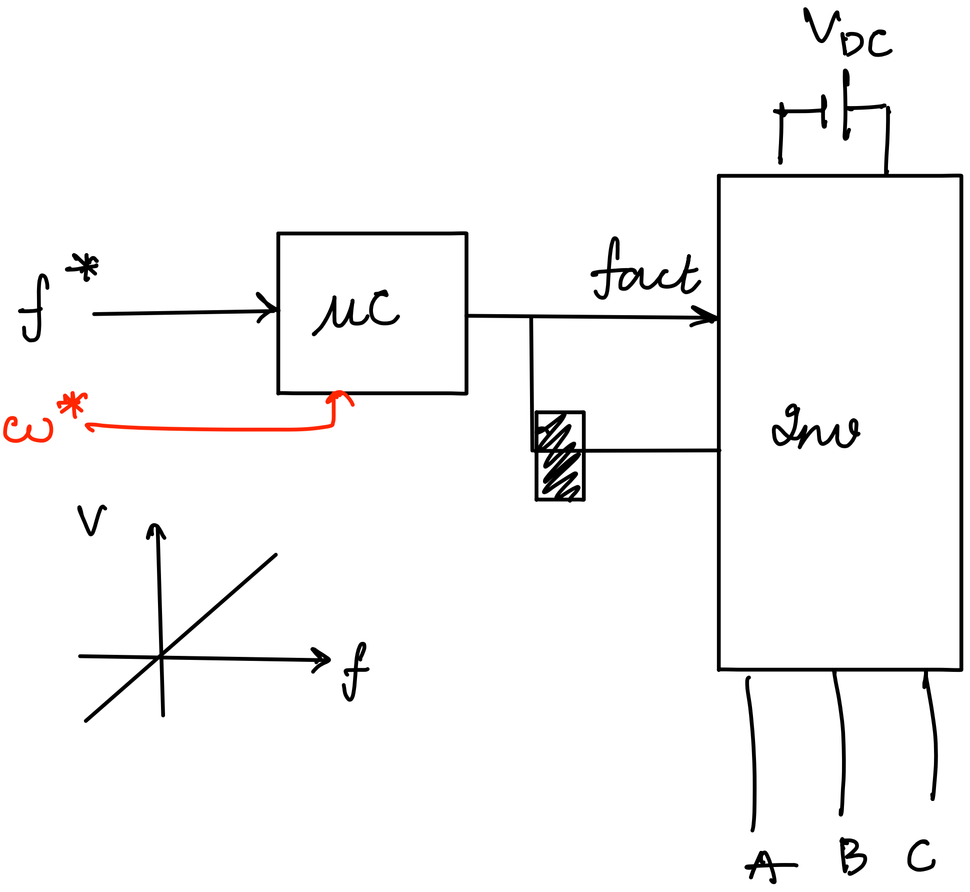
Line start limitations

→ Fixed speed

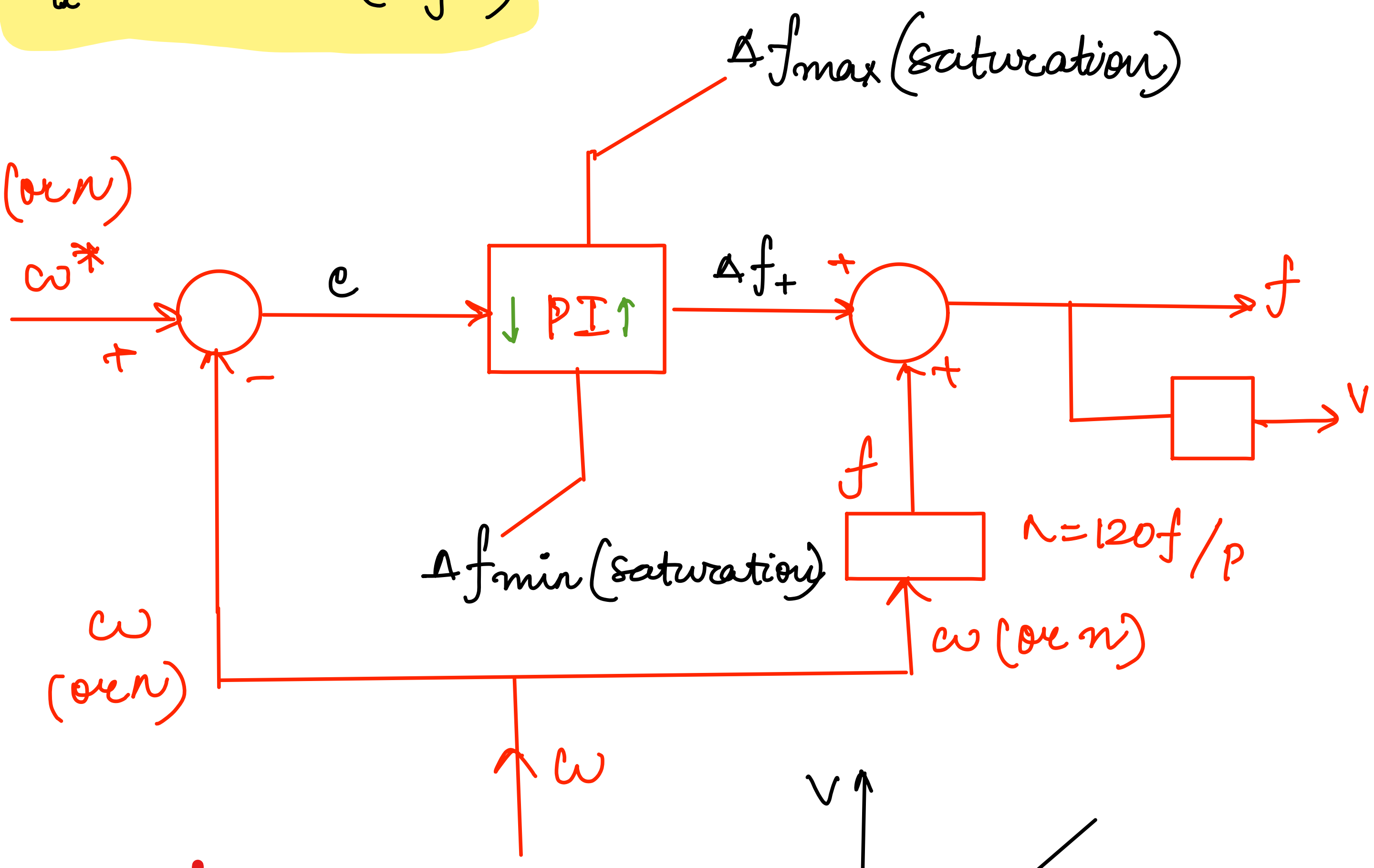
→ $\tau_{start} < \tau_{max}$ (so it takes a long time to settle to desired speed)

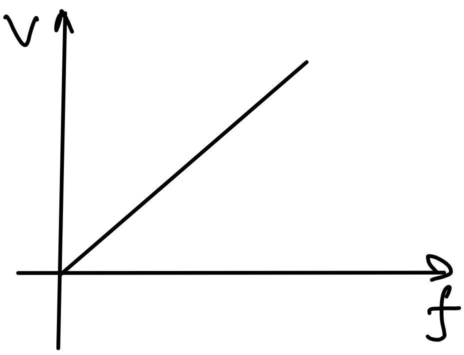
→ High starting current.

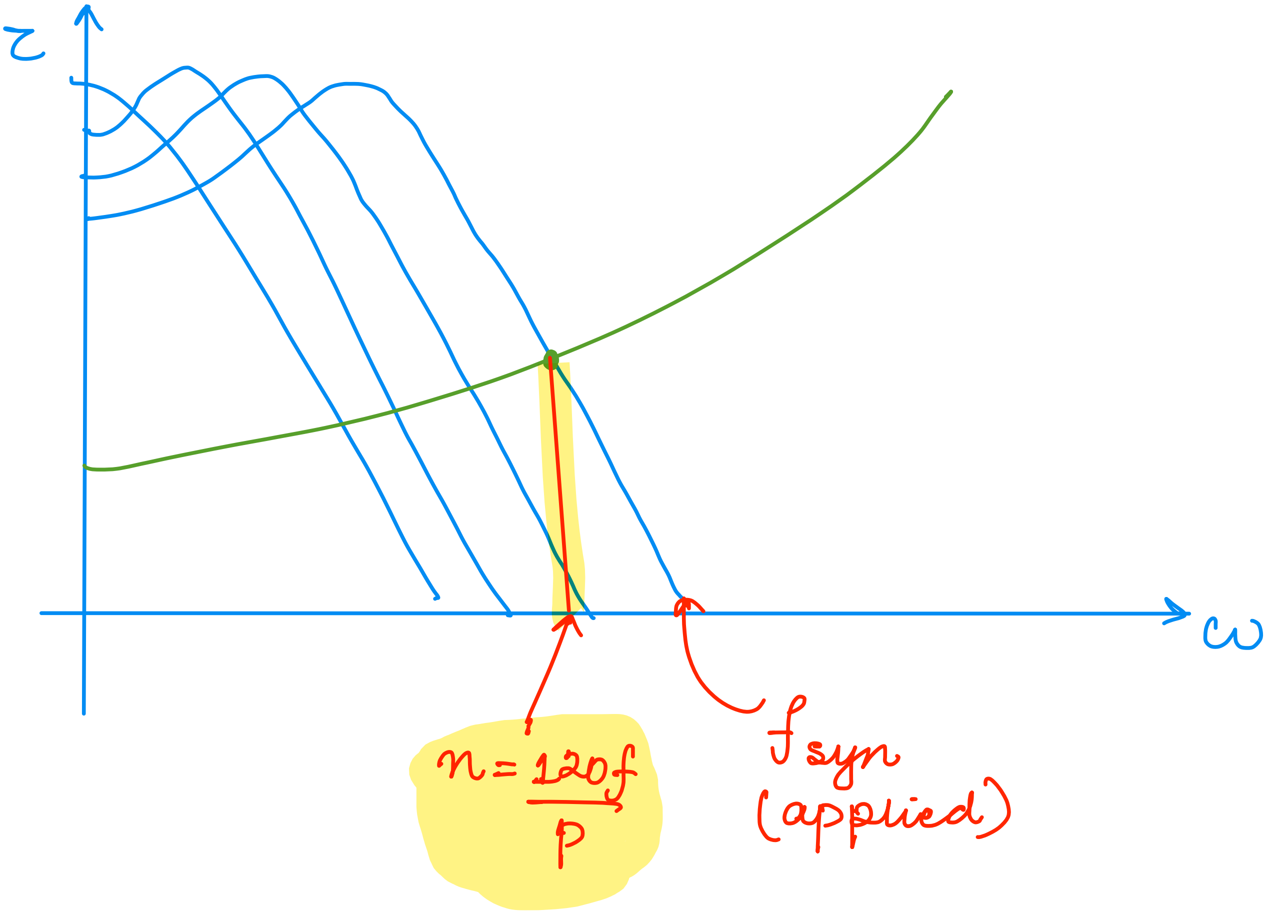




$$V_a^* = V\sqrt{2}\cos(2\pi f t)$$





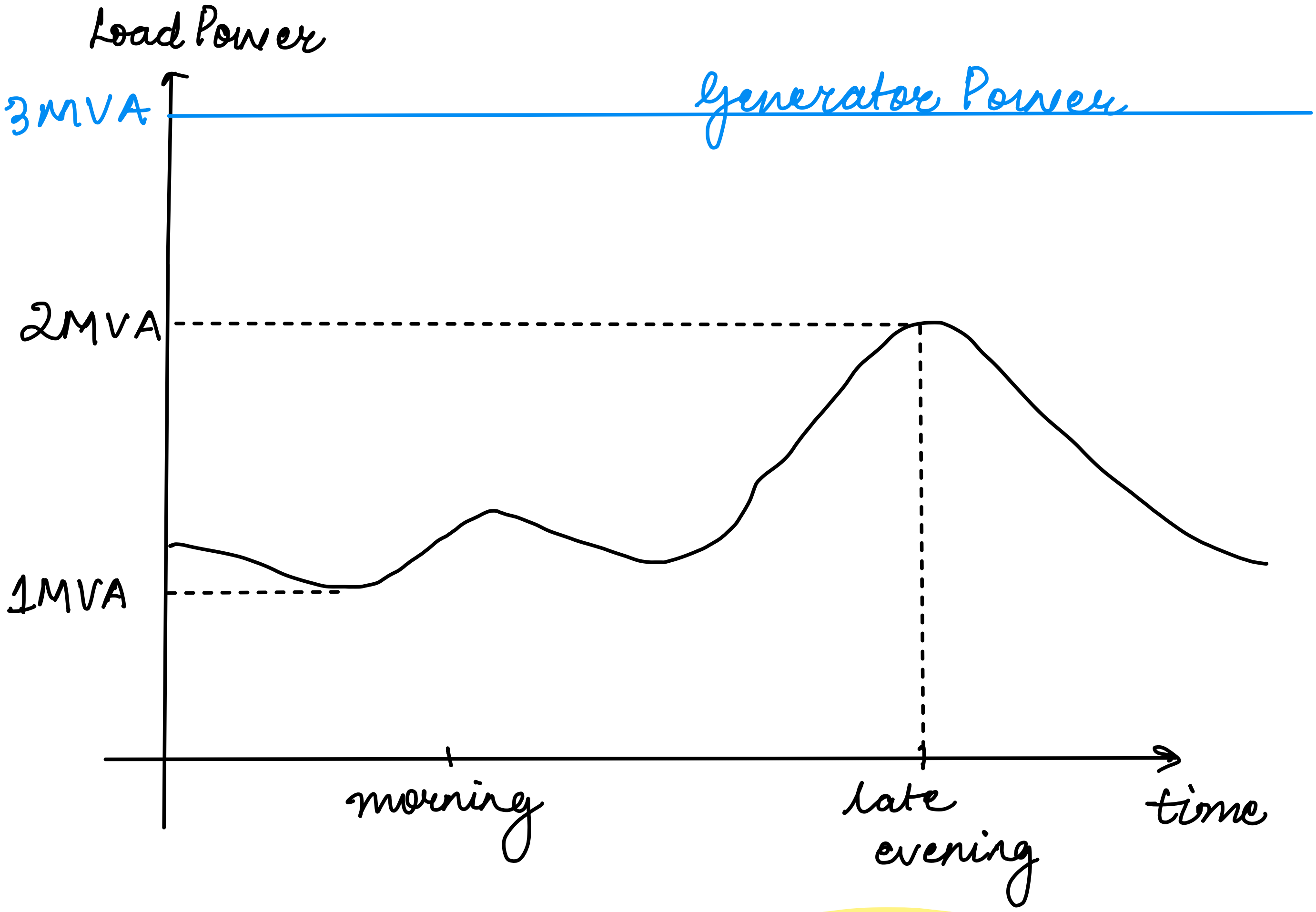


τ
ω
n = 120f / P
f syn (applied)

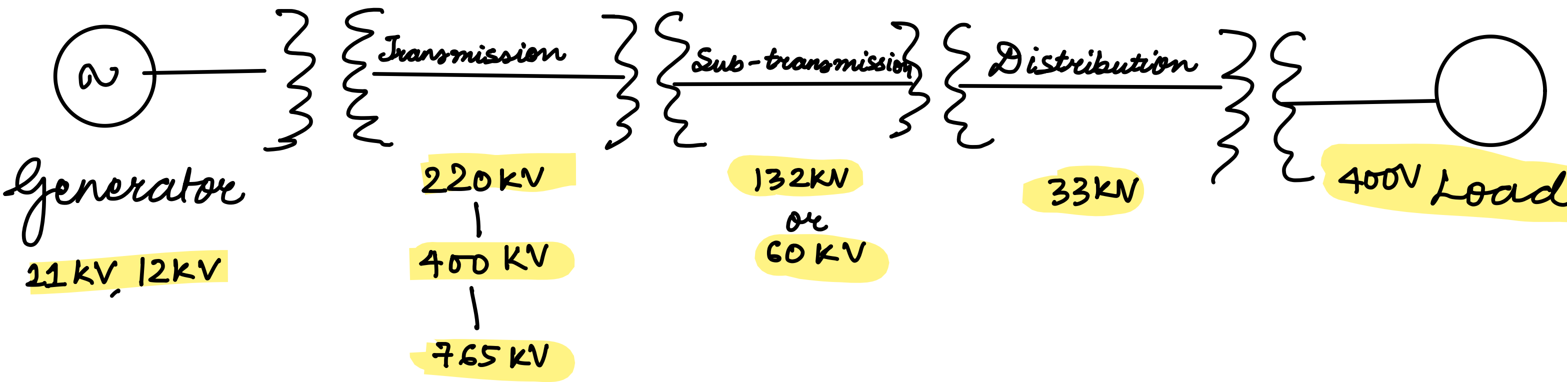
Power Systems.

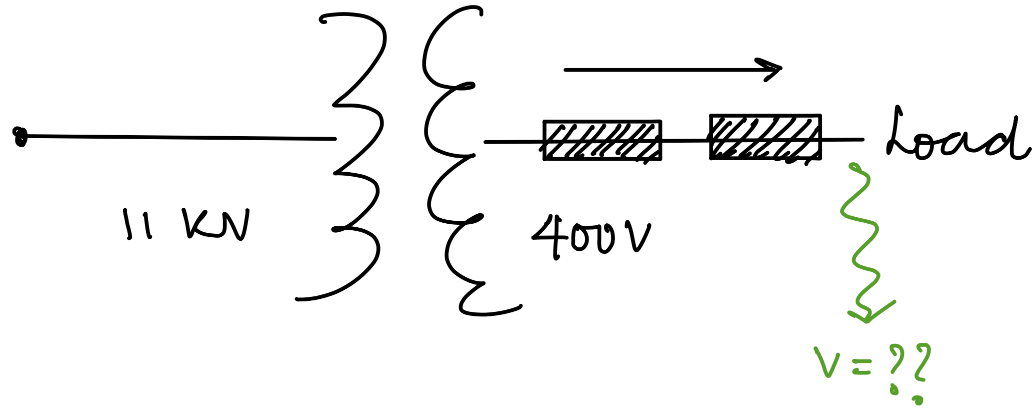
Limitations of 1 Generator

→ Reliability.
→ Over sizing / capacity generator
→ Poor efficiency at light load.



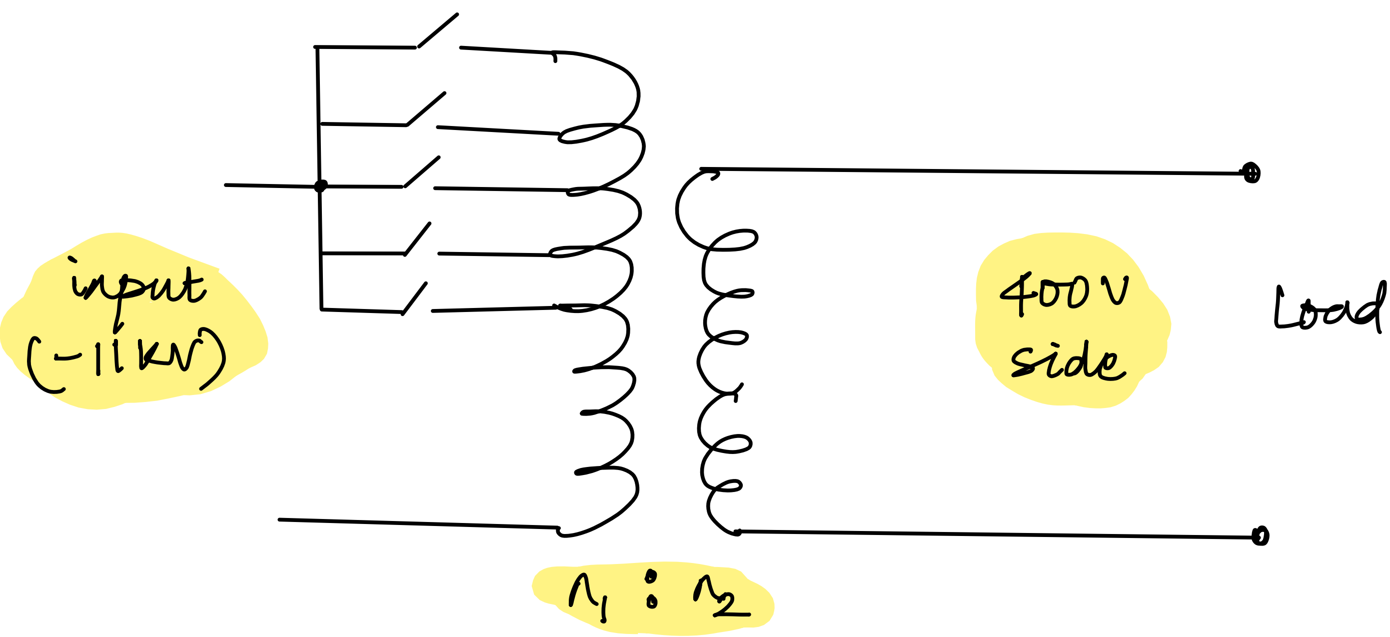
These three problems can be solved by interconnecting multiple loads & generator systems together.





→ Auto transformer can be used, but in a small scale (such as a room), it is highly impractical

Thus we do this in a substation level

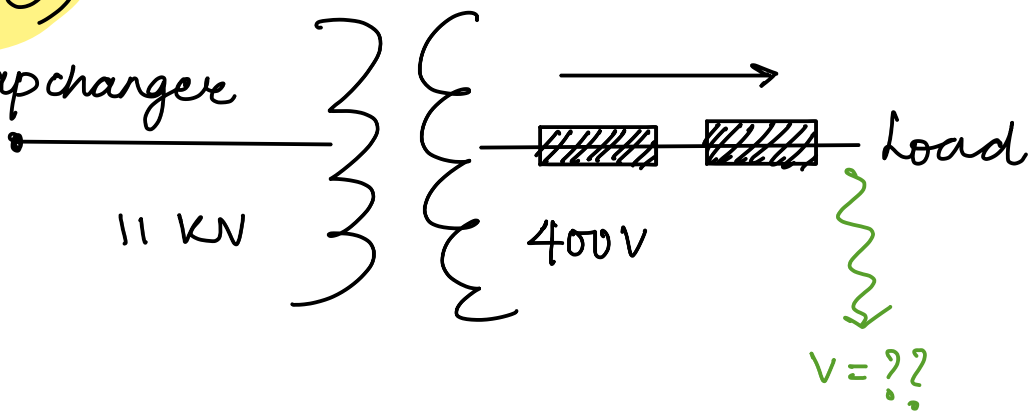


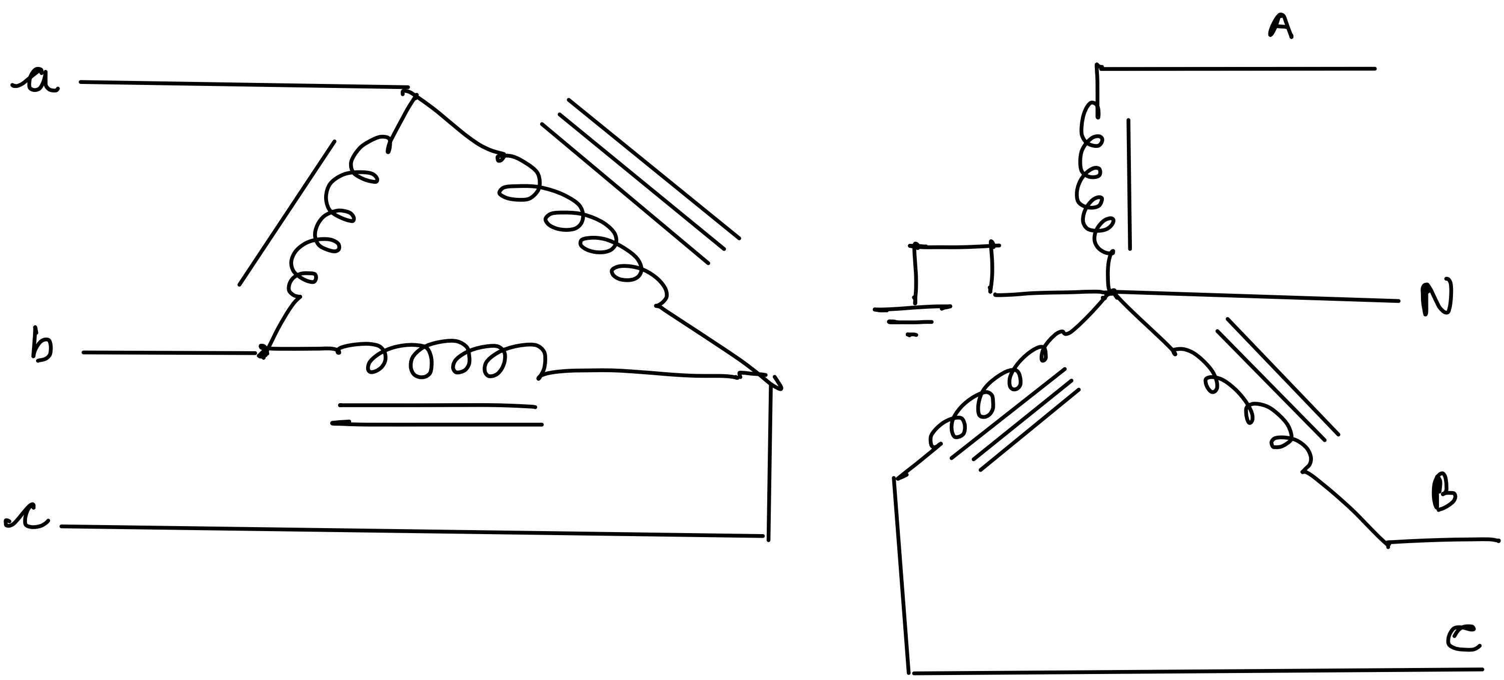
η can be changed

Tappings typically on HV side, because less current on HV side, so easier to make and break circuit

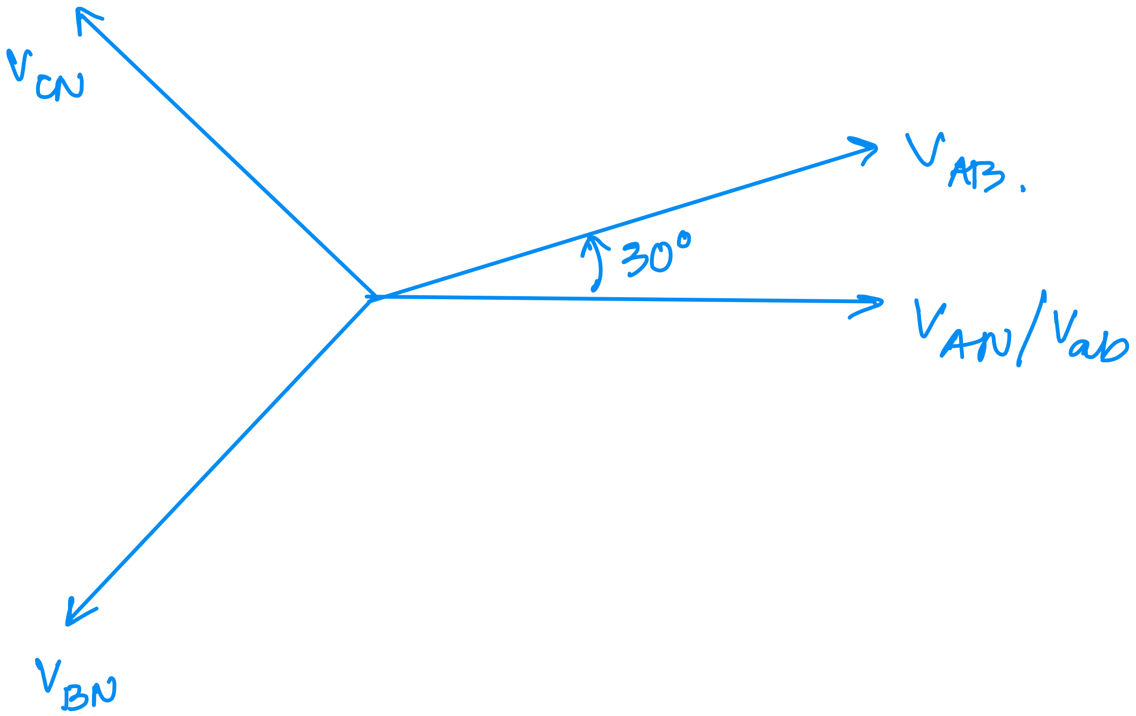
(OLTC)

On Load tapchanger





$\Delta Y \perp$



Line to Line of Y leads line to line of Δ

We have to do an earth at the Neutral on the load side, so that there is a well defined phase voltage that gets applied.

Δ on source? Because if we have a 3rd harmonic on source, we can chop it off or not send that 3rd harmonic to the load side.

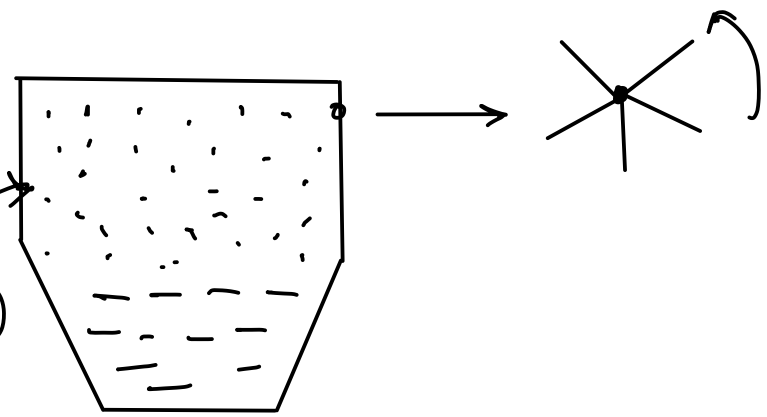
Synchronous Generator

→ High speed (Less poles)

→ Low speed (More poles)

Ways of electricity production → Coal

→ Nuclear



steam (pressure↑)

Boiler

→ Gas

→ Wind

→ Diesel

→ Hydro

→ Solar

- → Photo voltaics (PV)
- → Thermal (Based on sun's heat after concentration of sunlight using reflectors/concentreators)

All can rotate the sync. generator, but not all of them rotate with same speed.

$$n_s = \frac{120 f}{P}$$

when n_s is low → large number of poles
n_s is high → low number of poles

(We always want f to be 50 Hz in India)

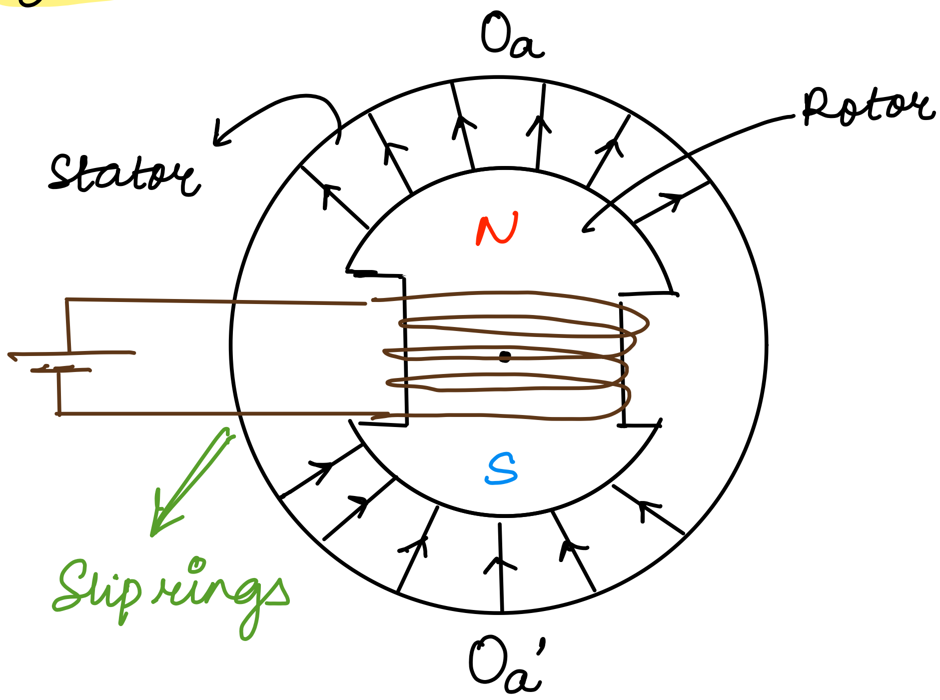
We kept some systems running even if there are no loads → Spinning Reserves.

(But we can't keep spinning reserves for all day)
⇒ ∴ we use a combo of spinning reserves and scheduling.

Photo Voltaics → V. Fast but no control over power
Diesel → Fast but not connected to grid
Hydro → Fast
Wind → Fast but again no control over power
Gas → Fast

The substitute to spinning reserves is storage.

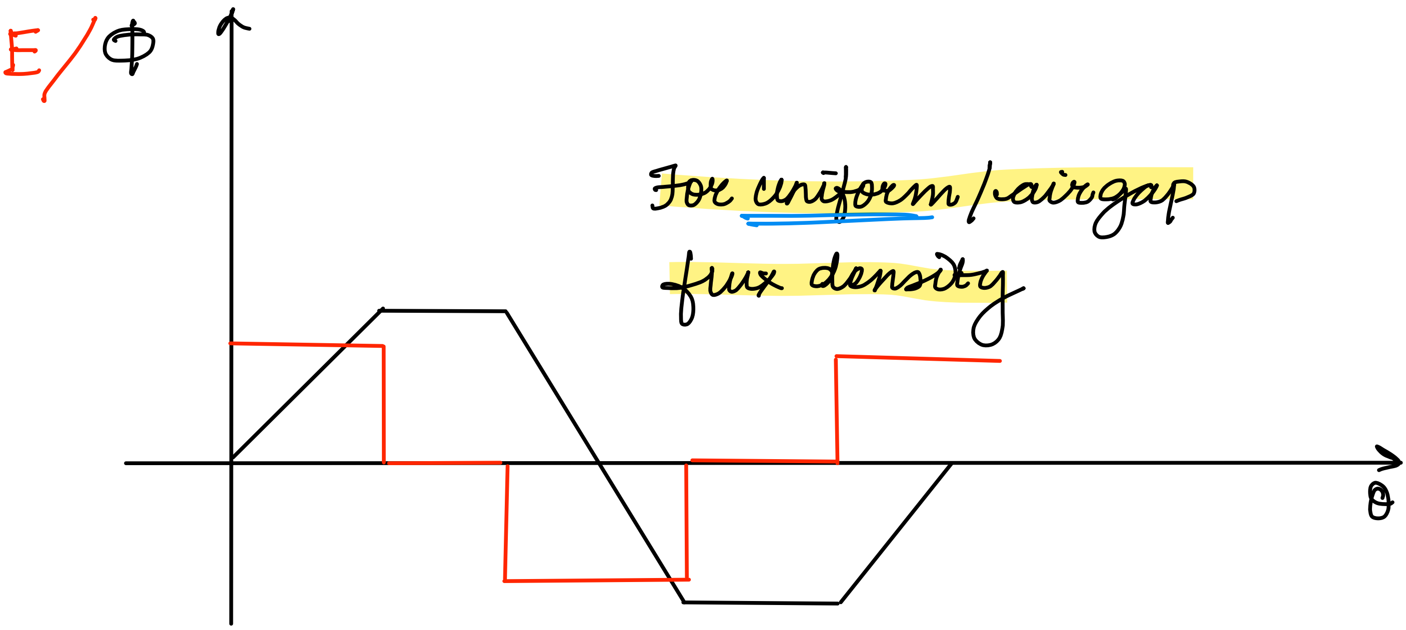
Synchronous Generator



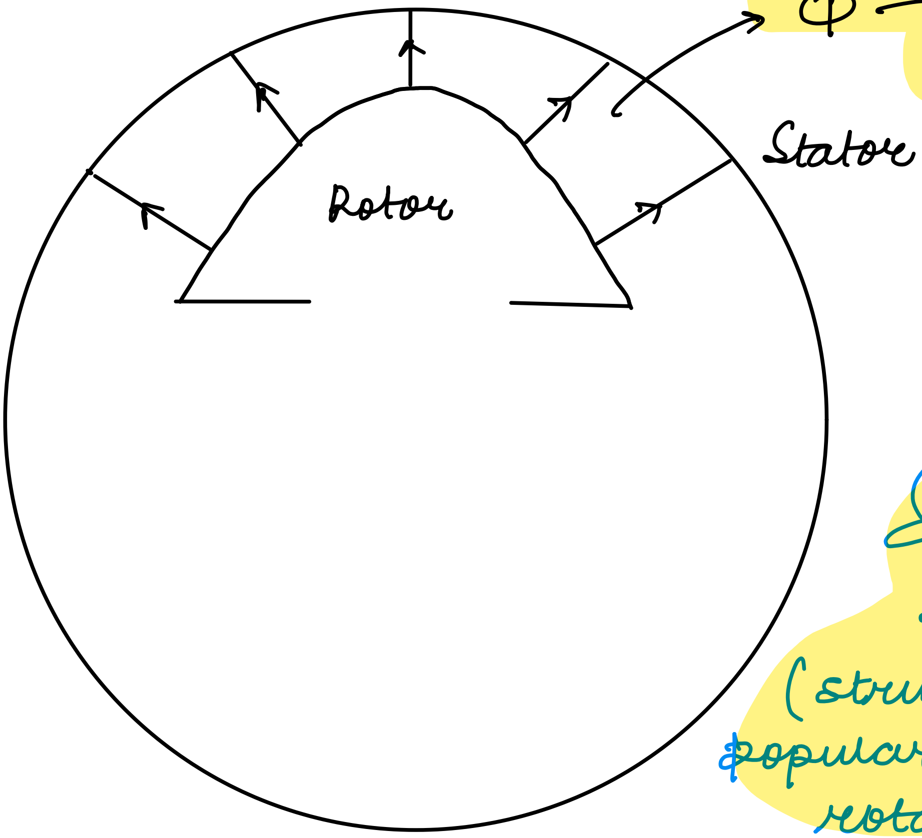
→ Wound rotor v/s Permanent magnet rotor

There is a net flux change in Oa Oa' coil when the rotor is rotating

(Dont confuse with brush commutator arrangement)



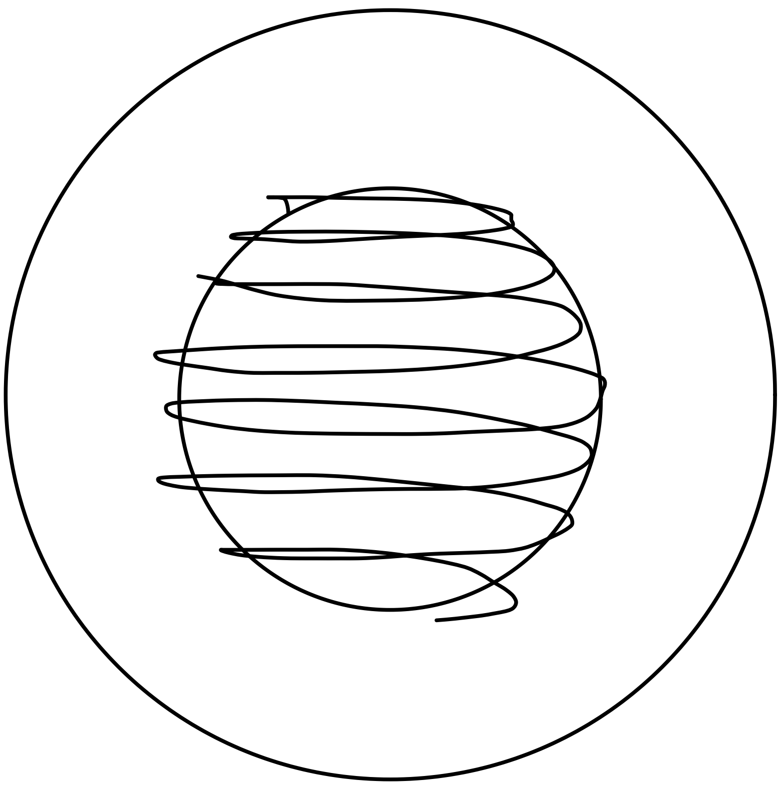
$\Phi \rightarrow$ sinusoidal flux distribution



Salient Rotor (structure not popular for fast rotation)

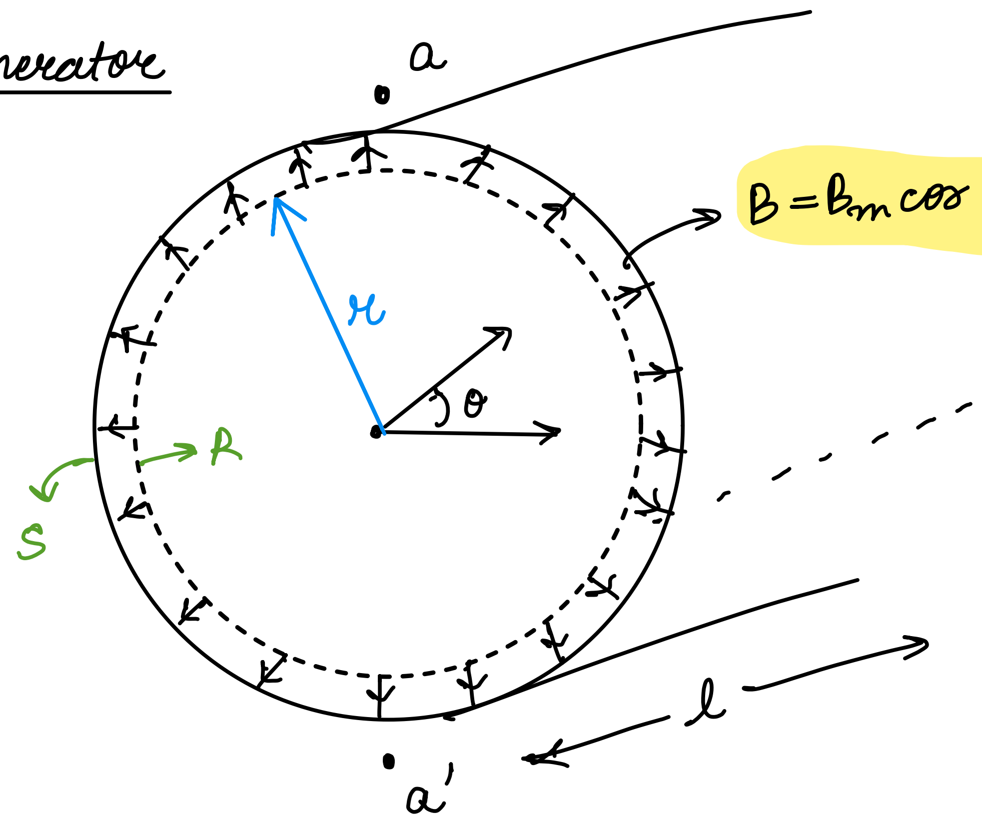
→ Shape of voltage induced = Sinusoidal

Other way of getting a sinusoidal flux :~



Cylindrical Rotor.

Sync Generator



Rotor winding a.k.a. field winding
Stator winding a.k.a. armature winding

(Used to be other way round in DC machine)

$$\phi_{a-a'} = \int_{\pi/2}^{3\pi/2} B l r \, d\theta$$

$$= \int_{\pi/2}^{3\pi/2} B_m l r \cos(\omega_s t - \theta) \, d\theta$$

$$= 2 B_m l r \cos(\omega_s t)$$

$$= \phi_m \cos(\omega_s t)$$

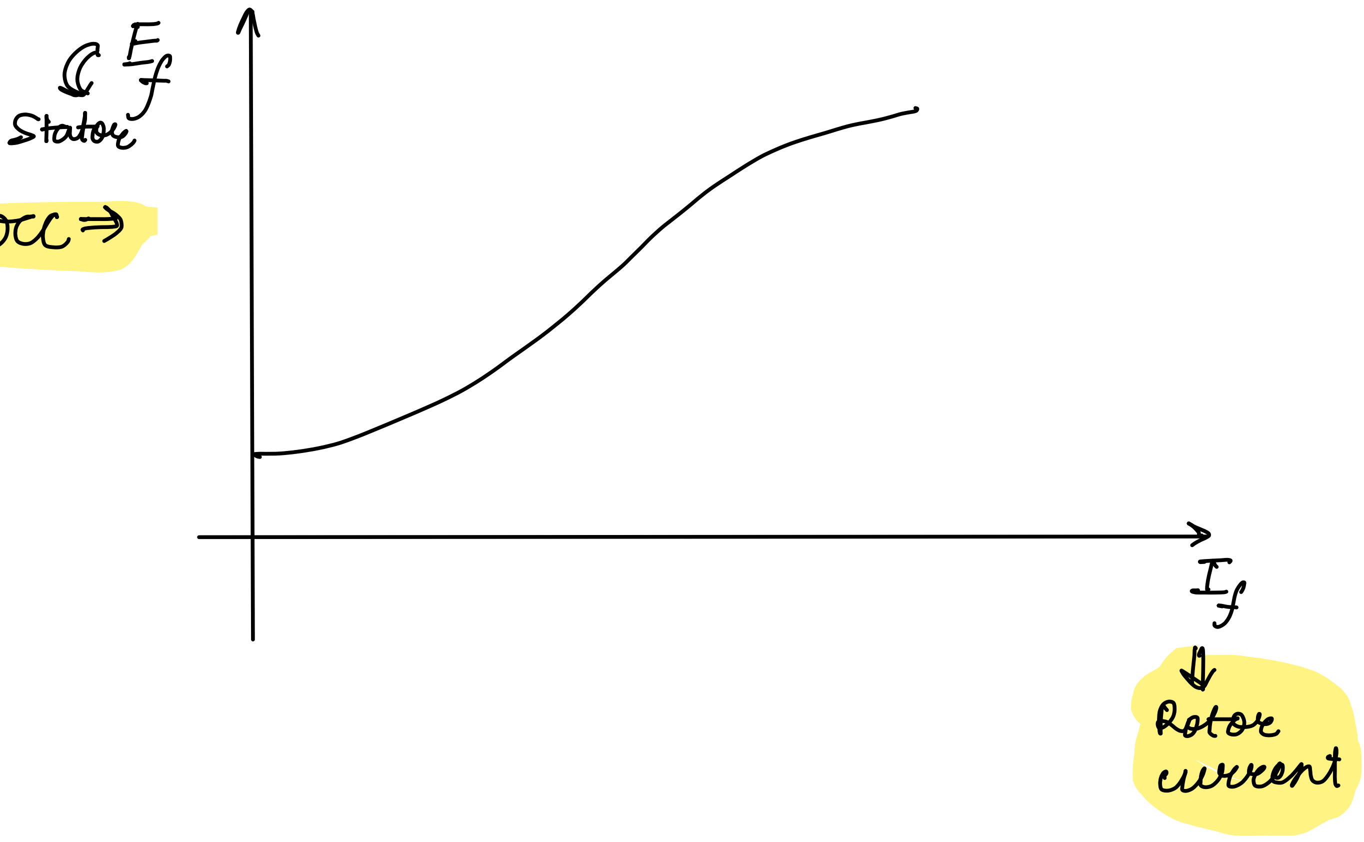
$$e_a = -N\frac{d\phi}{dt} = N\phi_m \omega_s \sin(\omega_s t)$$

In reality, we can't put N turns in a single slot.

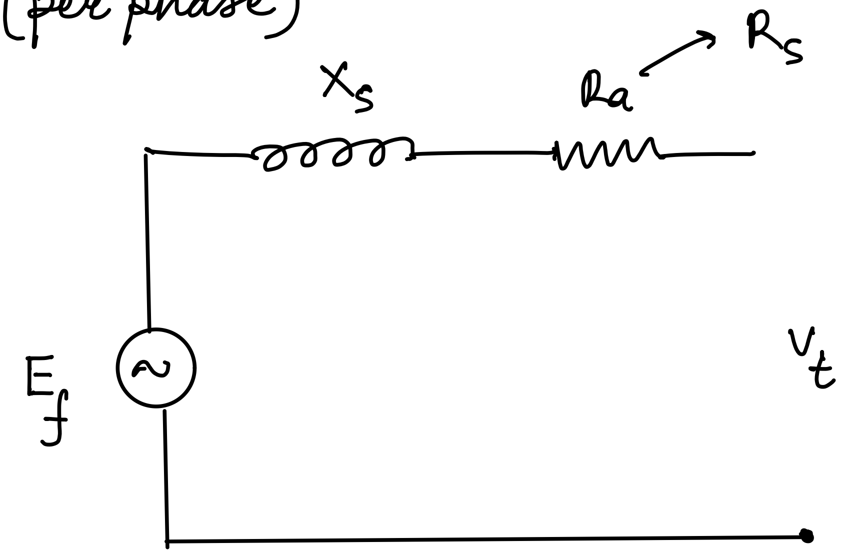
$$\therefore\ e_a = K_w N \phi_m \omega_s \sin(\omega_s t)$$

$$\boxed{E = 4.44 N f \phi_m K_w}$$

OCC ⇒

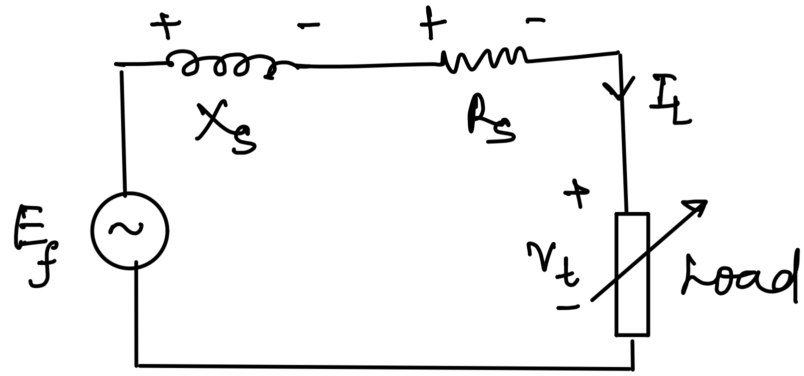


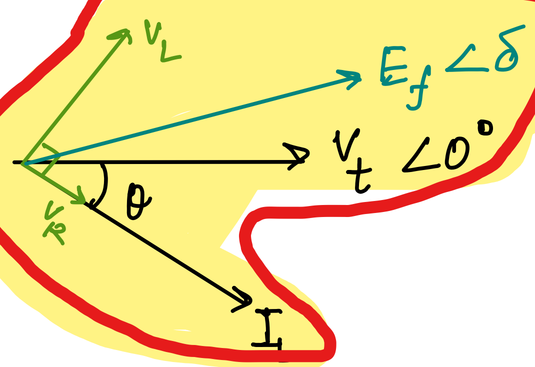
Eq ckt:
(per phase)



$X_s = X_{ar} + X_l$
(Synchronous Reactance)

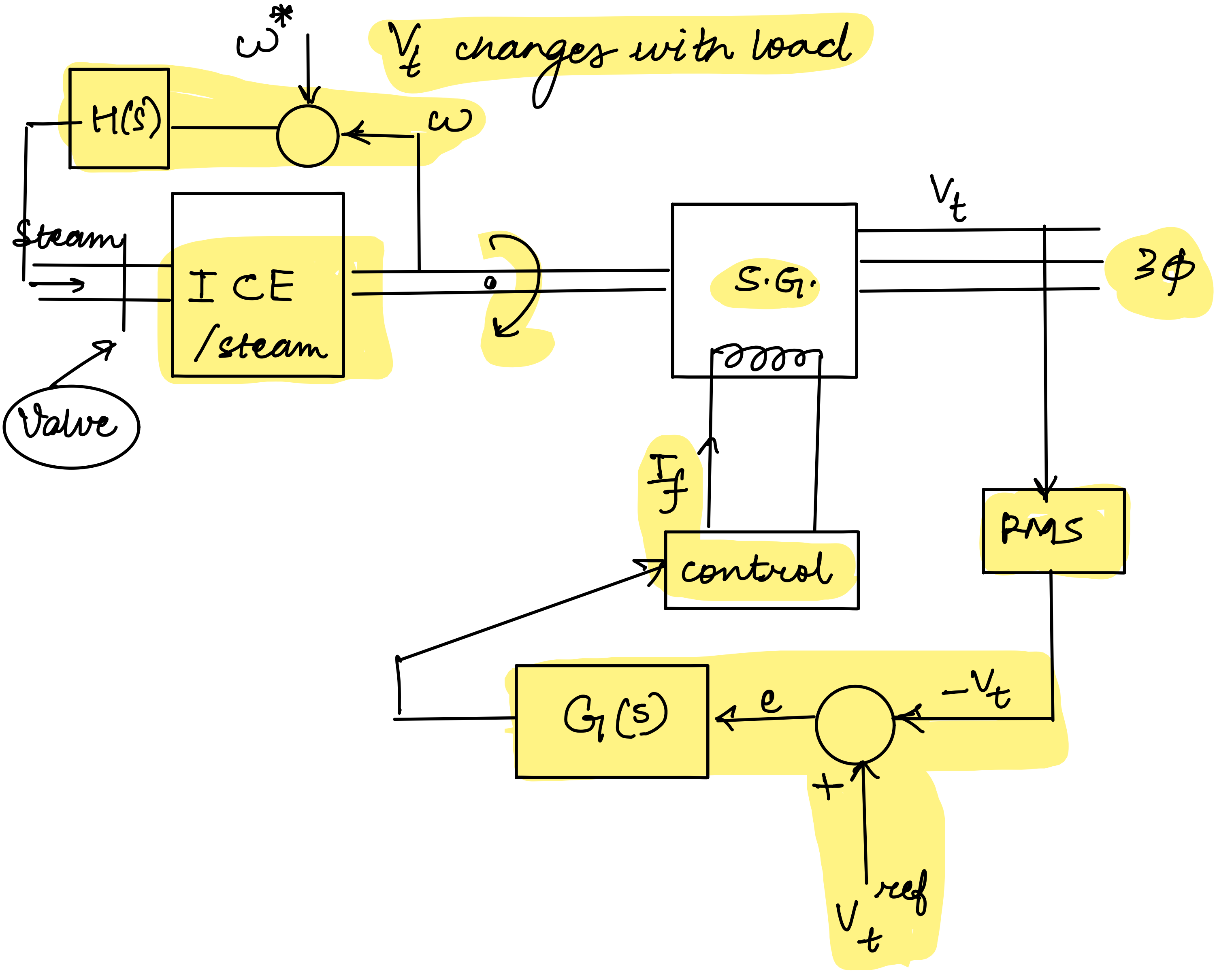
1 Generator + 1 Load





$$\vec{E_f} = \vec{V_t} + \vec{V_R} + \vec{V_x}$$

Assuming, rotation speed = constant
and current I_f = constant.

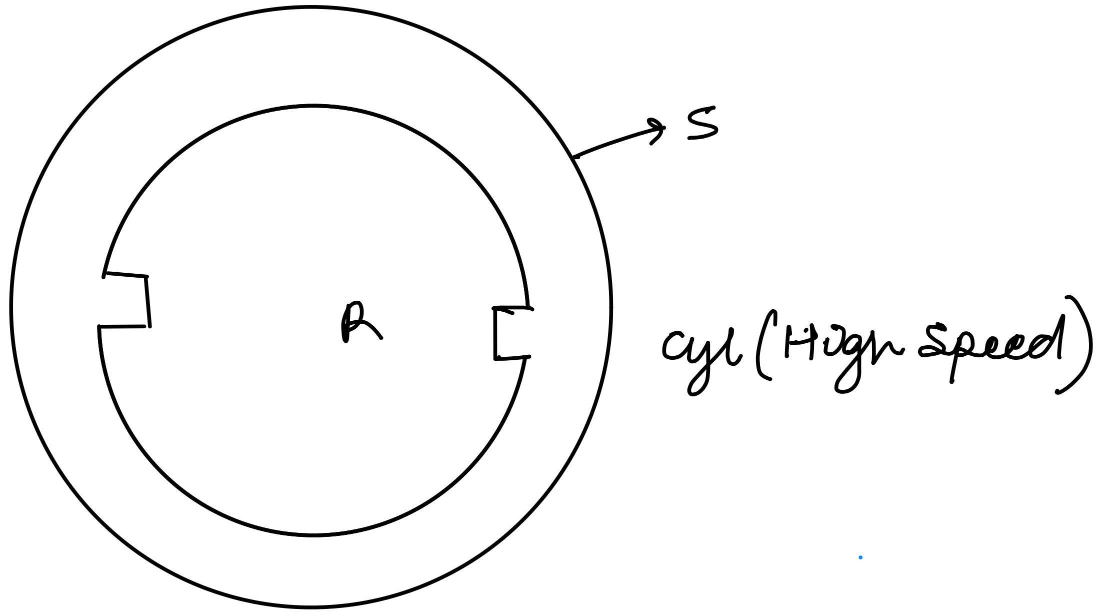


Objective

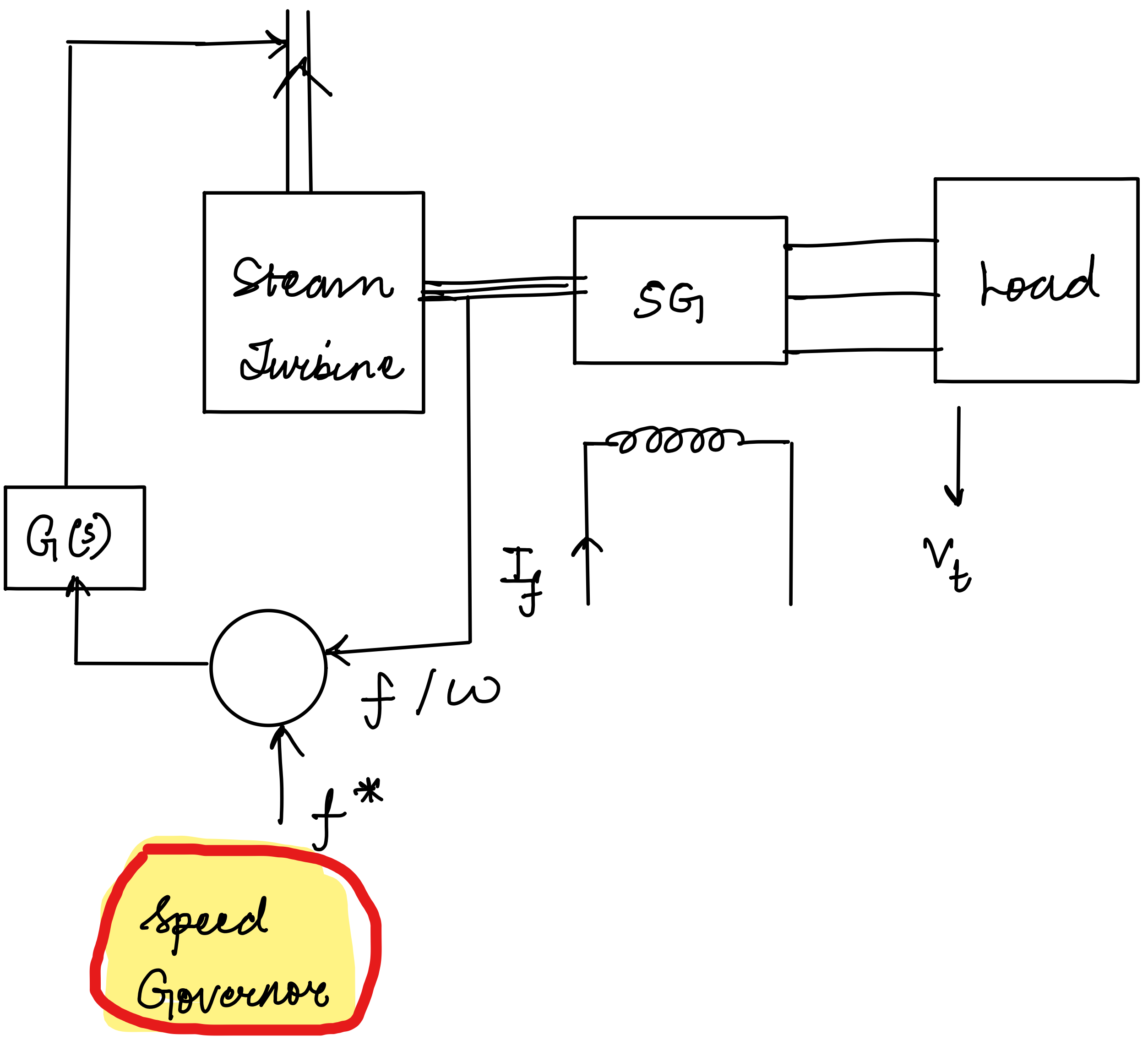
Fixed f
Fixed V

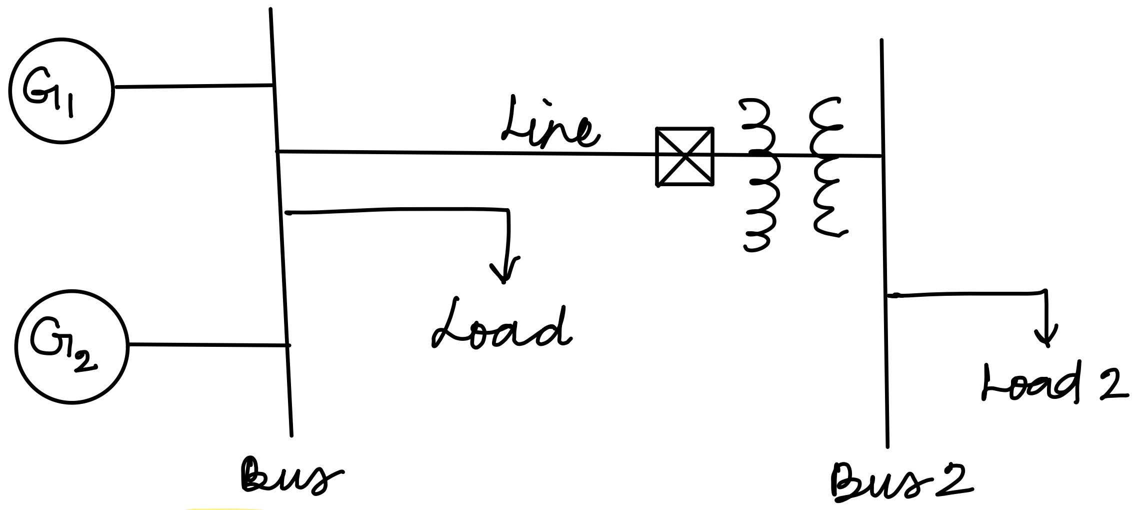
AVR
Automatic Voltage Regulator

We control I_f not valve (ω_s) because ω_s affects both f and V_t.



1 Generator - 1 Load

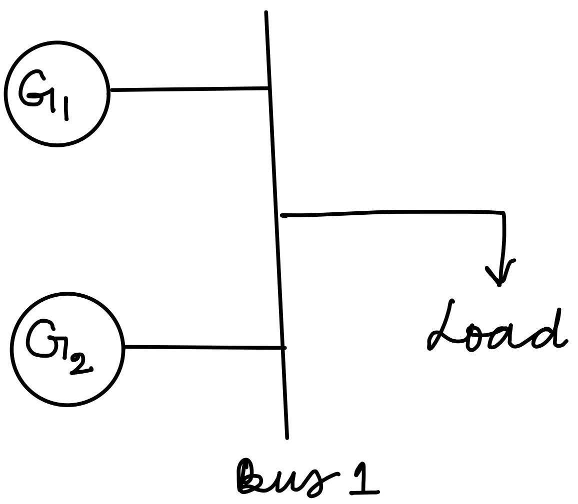




Single Line Diagram. (SLD)

Busbar → no impedance
Line → Some finite impedance.

2 Generators Feeding 1 Load.



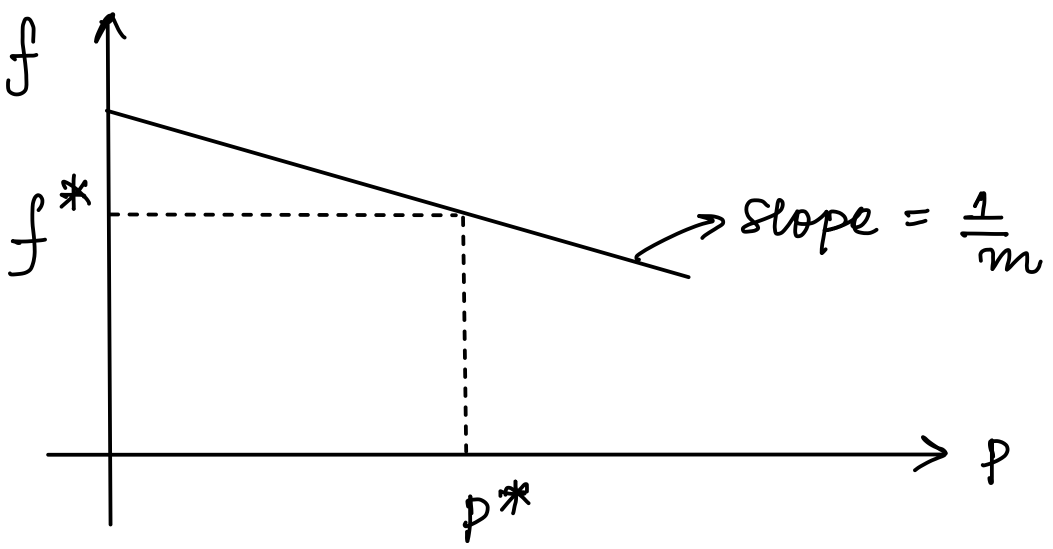
For equal work distrubution in G_1, G_2, we use only P instead of PI control.

"Droop Control"

$$P = P^* - m(f - f^*)$$

P^* ← Rated power.

→ $f \neq f^*$ ($f - f^*$ → small but not zero)



If m is large → $f - f^*$ ⇒ small

$$P_1 = P_1^* - m_1(f_1 - f^*)$$

$$\frac{P_1}{P_1^*} = 1 - \left(\frac{m_1}{P_1^*}\right)(f_1 - f^*) \qquad \text{——— (1)}$$

$$\frac{P_2}{P_2^*} = 1 - \left(\frac{m_2}{P_2^*}\right)\left(f_2 - f^*\right) \qquad (2)$$

$f^* = 50\,Hz$

$f_1 = f_2 = f$ (Because connected in a grid)

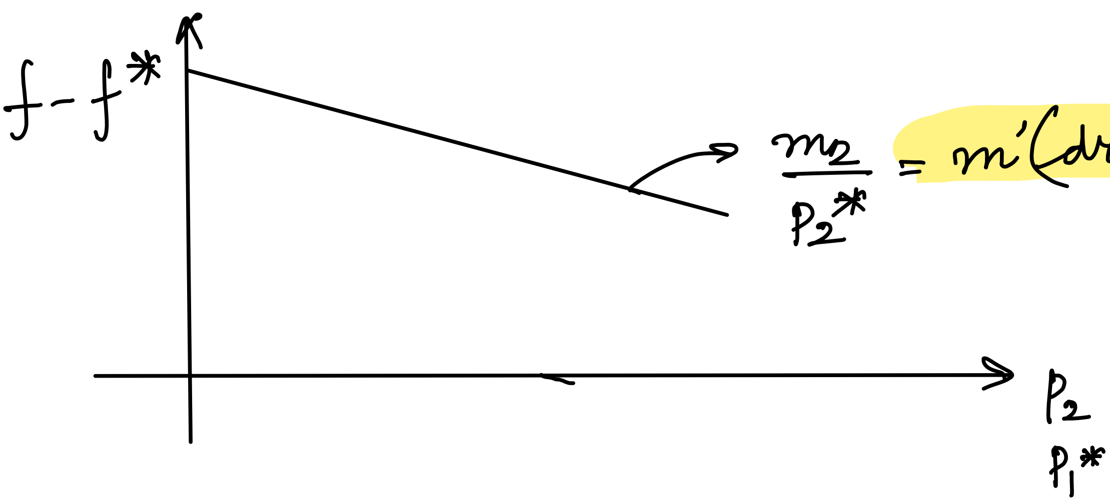
$$\boxed{\text{If } \frac{m_1}{P_1^*} = \frac{m_2}{P_2^*} \Rightarrow \frac{P_1}{P_2} = \frac{P_1^*}{P_2^*}}$$

Eg. 100 MW generator, 200 MW generator

If load is 30 MW → 10, 20

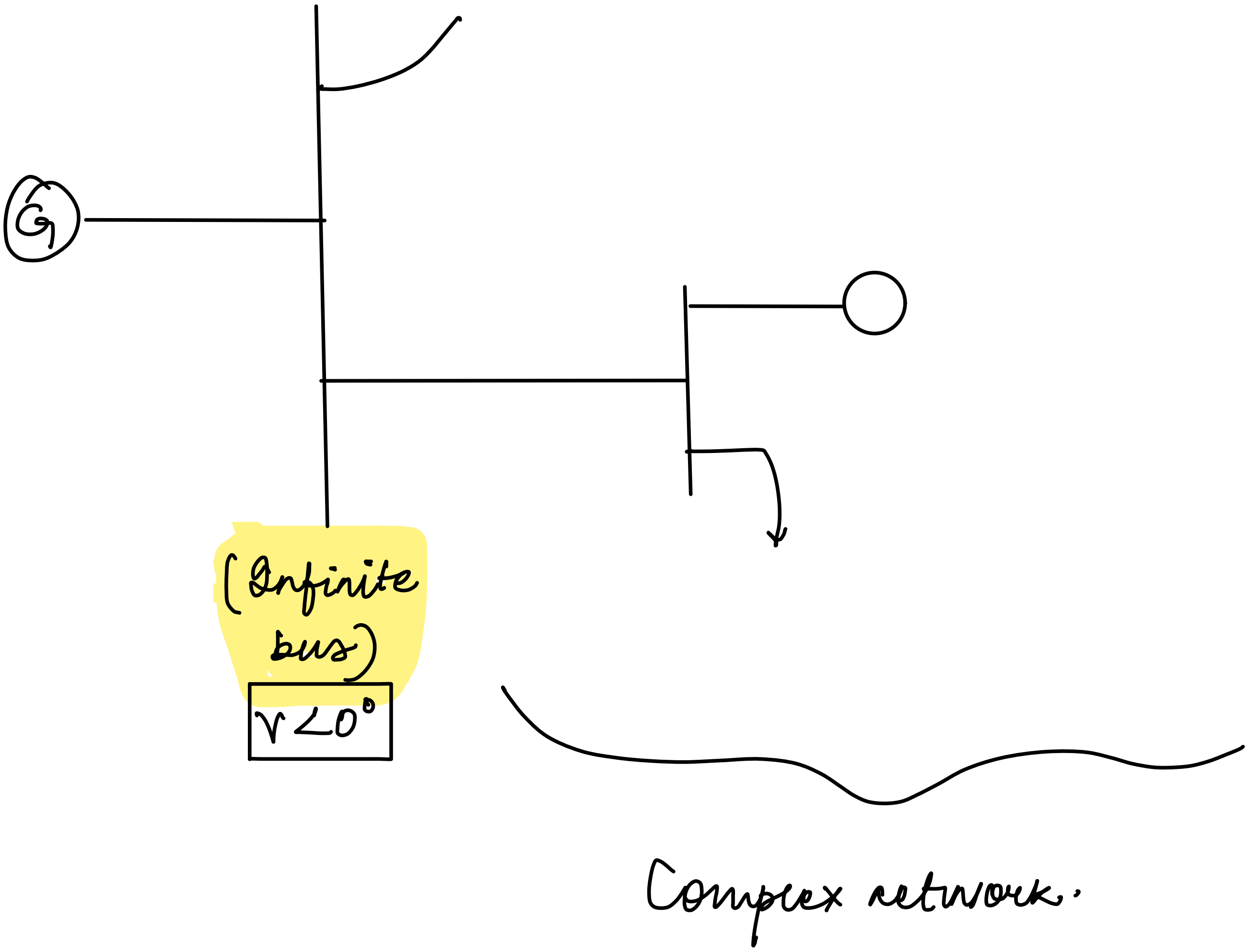
150 MW → 50, 100.

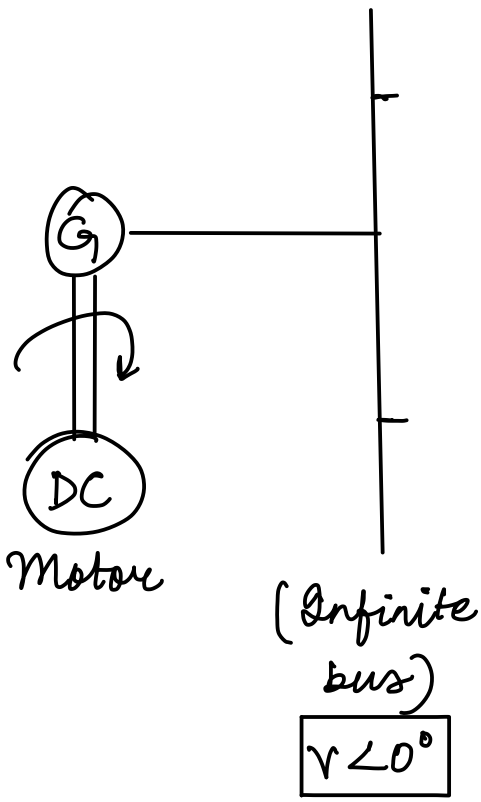
Eg. If G_1, G_2 are far away, we wouldn't want to divide power proportionally if there is heavy power loss in the line. Set m_1, m_2 accordingly.

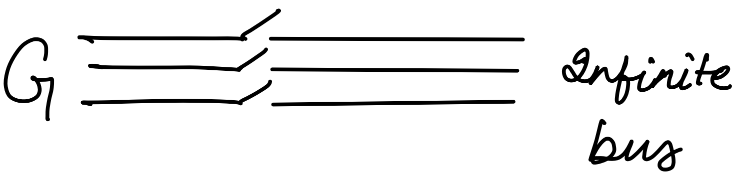


$\frac{m_2}{P_2^*} = m'$ (droop coeff.)

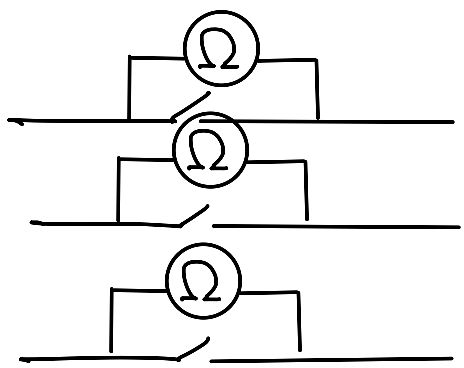
Multiple Generators.







Close - Same phase sequence
- Same phase angle
- Same voltage magnitude
- Same frequency.



when all conditions are met → bulbs become dark.

$$V_1^A = V_{m-1} \sin(\omega_1 t - \delta_1)$$

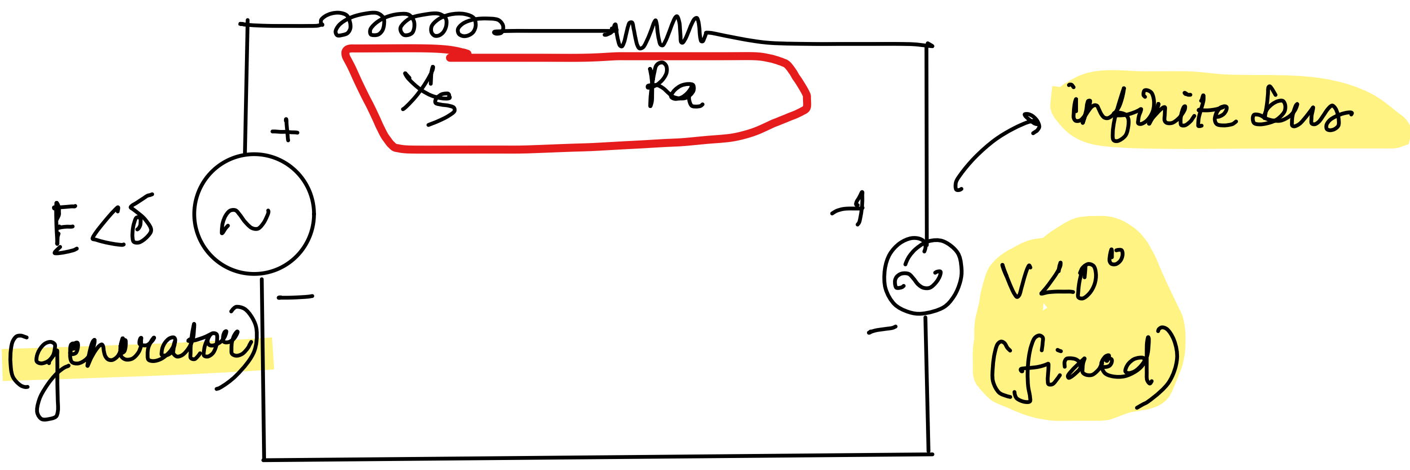
$$V_1^C = V_{m-1} \sin(\omega_1 t - \delta_2)$$

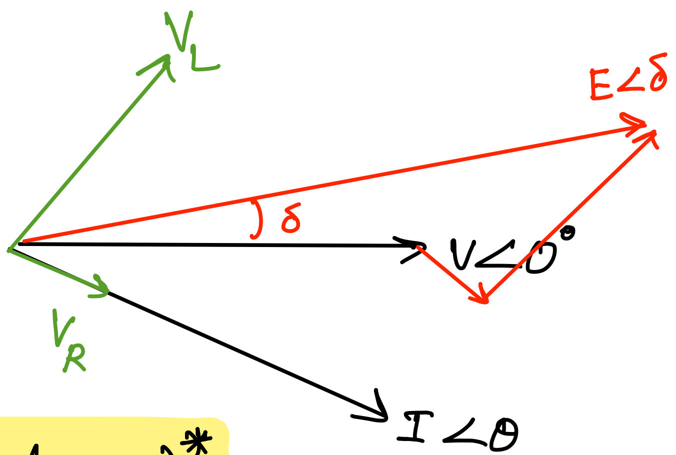
$\vdots$

$$V_2^A = V_{m-2} \sin(\omega_2 t - \delta_2)$$

→ Eyes cannot perceive 50Hz, it appears as if we are only seeing the 0.5Hz component.

Once generator is connected to infinite bus





$$S = 3\times(V\angle 0)\times(I\angle\theta)^*$$

$$S_{1\phi} = V I^*$$

$$= V\angle 0^\circ \left(\frac{E\angle\delta - V\angle 0}{X_s\angle 90^\circ + R_a\angle 0^\circ} \right)^*$$

negligible

$$= V \left(\frac{E\cos\delta + E\sin\delta\, j - V}{X_s\, j} \right)^*$$

$$= \frac{V}{X_s}\left(E\sin\delta + Vj - E\cos\delta j \right)^{*}$$

$$P = \frac{V}{X_s}\left(E\sin\delta\right)$$

$$Q = \frac{V}{X_s}\left(E\cos\delta - V\right)$$

$$\left.\right\} S = P + Qj$$

Per-unit

$$x_{pu} = \frac{x}{x_{base}} \times 100\,\%$$

$x = V, I, Z, \underbrace{P, Q, S}_{\text{all use } S_{base}}$

1-ϕ

P

V_{nom}, S_{rated}

N

$$\boxed{V_{base} = V_{nom}}$$

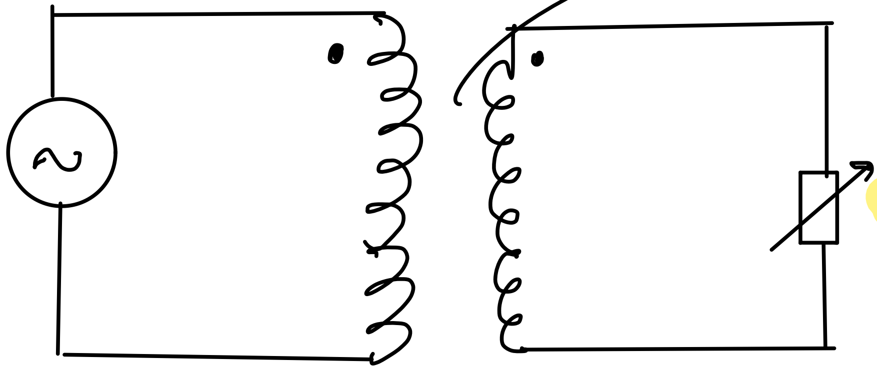
$$I_{base} = \frac{S_{rated}}{V_{nom}}$$

$$\boxed{S_{base} = S_{rated}}$$

$$\boxed{Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}}$$

$$P_{pu} = \frac{5\,kW\ (P)}{10\,kVA\ (S_{base})} = 0.5$$

$S_{rated} = 50\ kVA$



$R = 10\,\Omega$

$\frac{11}{\sqrt{3}}K : 230$

$V_{base} = \frac{11}{\sqrt{3}}\ kV$

$S_{base} = 50\ kVA$

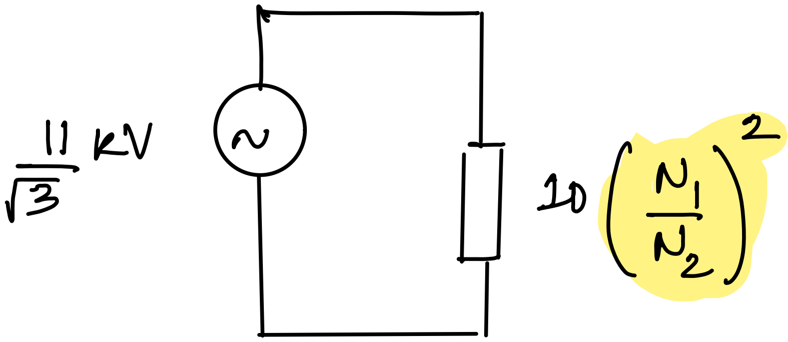
$V_{base} = 230\,V$

$S_{base} = 50\ kVA$

$$Z_{base} = \frac{V_{base}}{\left(\frac{S_{rated}}{V_{nom}}\right)} = \frac{V_{nom}^2}{S_{rated}} = 1.058\,\Omega$$

$$Z_{pu} = \frac{10}{1.058} = 9.45$$

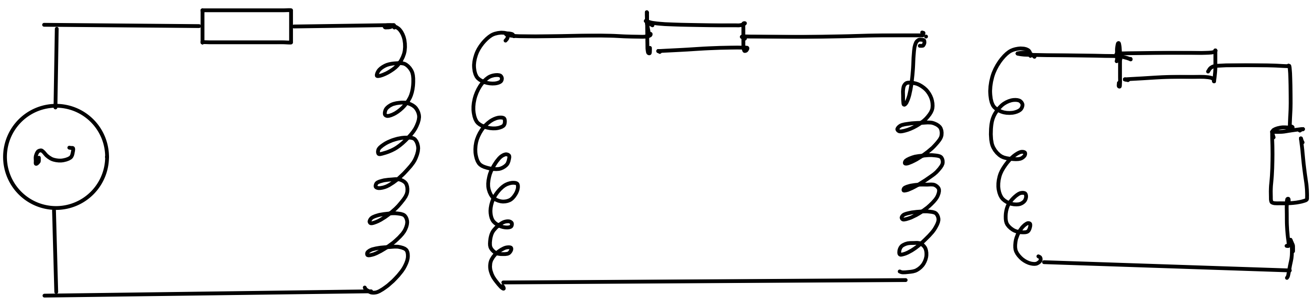
If we referred this to the primary side:



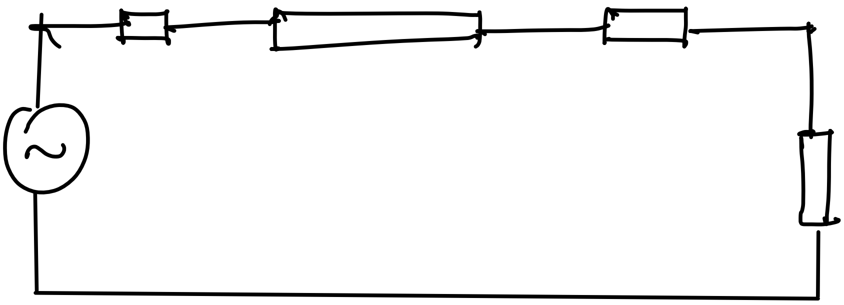
$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{\left(\frac{11}{\sqrt{3}}\right)^2 \times 10^6}{50 \times 10^3}$$

?

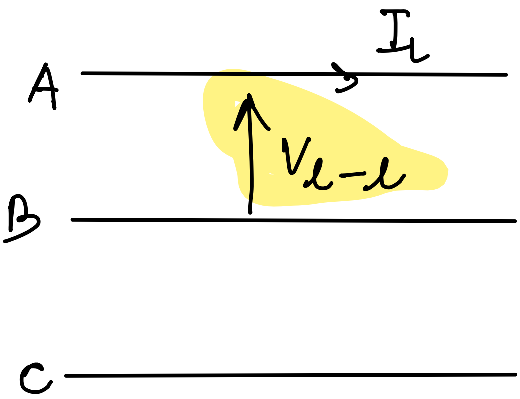
$$Z_{pu} = \frac{10 \cancel{\left(\frac{11}{\sqrt{3}}\right)^2} \left(\frac{1}{230}\right)^2}{\cancel{\left(\frac{11}{\sqrt{3}}\right)^2} \times 10^3} \times 50 \times 10^6 = 9.45\,\Omega$$



$\Downarrow$ pu



PU in 3ϕ



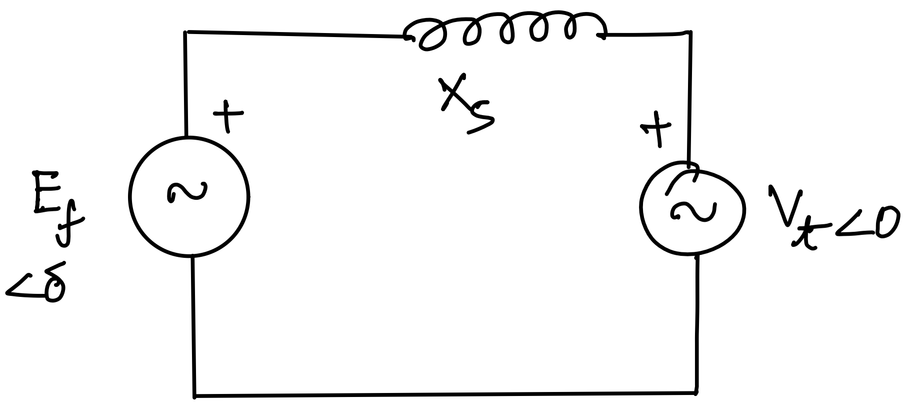
$$V_{base} = V_{l-l}^{nom}$$

$$I_{base} = I_L^{rated}$$

$$S_{base} = S^{rated}$$

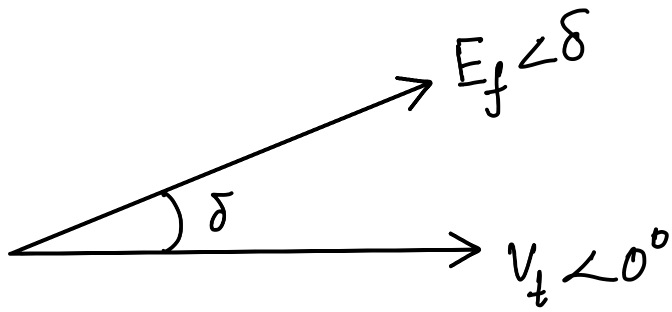
$$S^{rated} = \sqrt{3}\, V_{l-l}^{nom} I_L^{rated}$$

$$\boxed{Z_{base} = \frac{{V_{l-l}^{nom}}^2}{S^{rated}}} \left(= \frac{V_{ph}^{rated}}{I_L^{rated}} = \frac{V_{l-l}^{rated}}{\sqrt{3}\, I_L^{rated}} \right)$$



$$P = \frac{E_f V_t}{X_s} \sin\delta$$

$$Q = \frac{V_t}{X_s}\left(E\cos\delta - V_t\right)$$



In SPWM, if we have ref $\rightarrow m \sin(\omega t + \delta)$

then we would have got $m V_{dc} \sin(\omega t + \delta)$

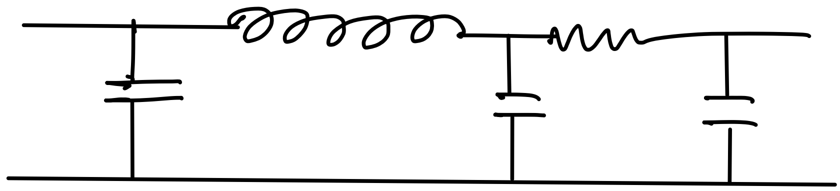
$\therefore$ changing $V_{dc} \rightarrow$ changes E_f
changing $\delta \rightarrow$ changes $\angle E_f$

δ : Power angle

↳ Angle btw 2 voltages. Different from Power factor

Transmission Line

- Short
- Medium
- Long

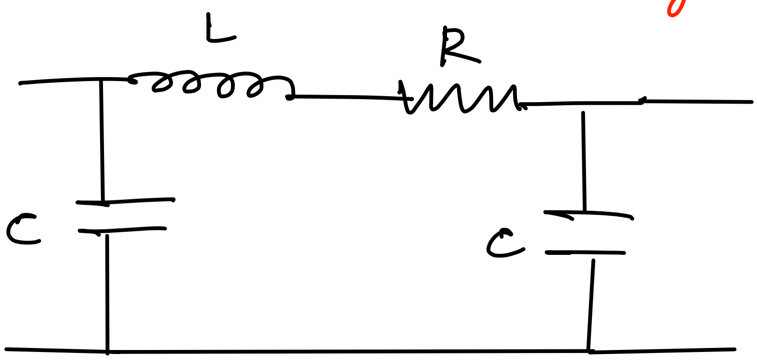


If short transmission line : forget capacitor.

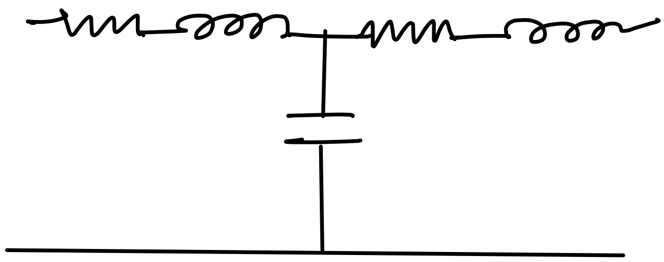
L, R : lumped parameters.

Medium

($\frac{1}{j\omega C}$ in shunt can be ignored if C is small)



π-model



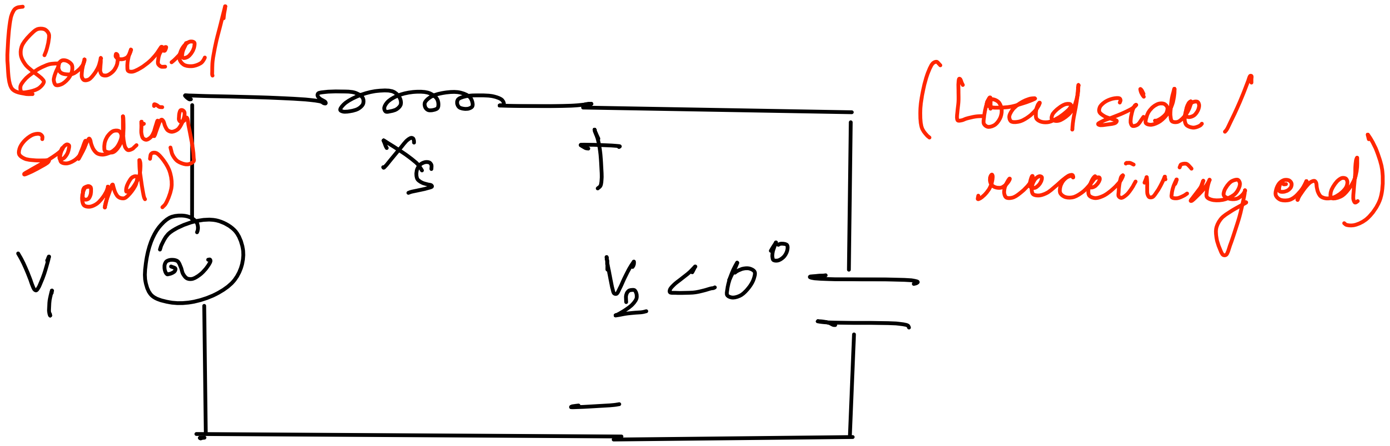
T-model

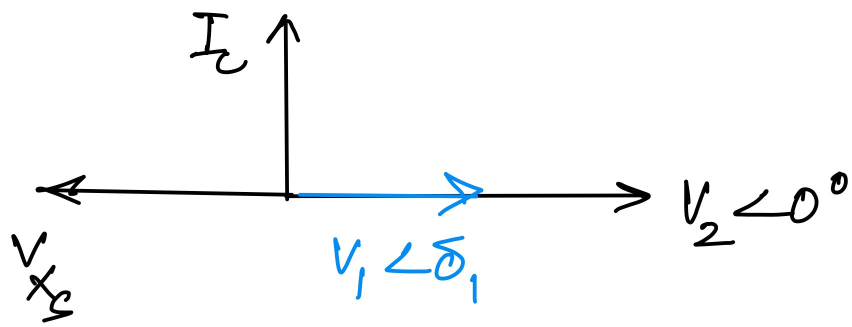
Long

Series combo of π model and T-model

Cable → Large C
Line → ×

Line is preferred over cables while transmitting over large distances.





Ferranti Effect : Load side has more voltage than source side.

HV AC transmission

$\Downarrow$

HV DC transmission (for very long distance transmission)