

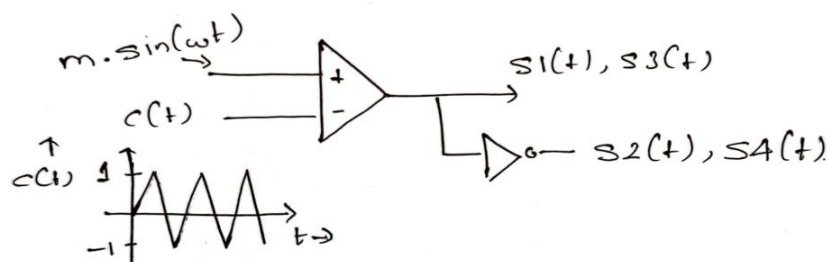
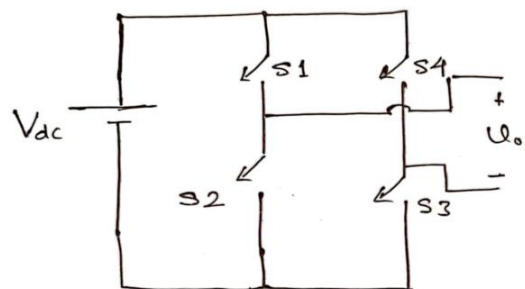
## Assignment 4 (EE 238)

1. A single phase full bridge inverter has a switching sequence that produces a square wave voltage across a series RL load. The switching frequency is 60 Hz,  $V_{dc}=100$  V,  $R=10\ \Omega$  and  $L=25$  mH. Determine (a) an expression for load current at steady state, (b) the power absorbed by the load, and (c) the average current in the dc source. **(Ans: (b) 441 W (c) 4.41 A )**
2. Find the relation between modulation signal  $m(t)$  and the duty ratio  $d(t)$ . Assume  $|m(t)| < 1$  and  $c(t)$  is a triangular wave swinging from -1 to +1. The frequency  $f_c$  of the  $c(t)$  is assumed to be very large such that  $m(t)$  is assumed to be constant over one period of  $c(t)$ .  $d(t)$  is the ratio of on time period and  $1/f_c$ , corresponding to the switching function  $s(t)$ . The switching function  $s(t)$  is defined as

$$s(t)=1, \text{ when } m(t)>c(t) \\ 0 \text{ otherwise .}$$

$$\text{(Ans: } m(t)=2.d(t)-1\text{)}$$

3. Consider the following single phase full bridge inverter. The switching is defined by the switching functions defined below. Prove that the average value of  $v_o$  over one time period of  $c(t)$  is  $m.V_{dc}.\sin(\omega t)$  where  $|m| < 1$ .



4. A three-phase full bridge inverter delivers power to a resistive load from a 450 V dc source. For a star connected load of  $10\ \Omega$  per phase, determine for  $180^\circ$  conduction mode, (a) rms value of load current, (b) rms value of switch current and (c) load power. **(Ans: (a) 21.213 A (b) 15 A (c) 13.5 kW)**
5. A 3-ph inverter is controlled in the 180 deg conduction mode for each switch, without PWM. The fundamental inverter output frequency is  $\omega = 100\pi$  radians per second. A balanced three phase star connected load is connected to the output. The load in each phase is made up of a series connection of resistor(R) and inductor(L), such that  $\omega L \gg R$ . If the amplitude of the 50 Hz component of the load current in each phase is 100 A, what is the amplitude of the 250 Hz current component?  
**(Ans: 4 A)**

## ASSIGNMENT-4 (EE-238)

Q1: a)  $T = 1/f = 1/60 = 0.0167s$ .

$\tau = L/R = 0.025/10 = 0.0025s$ .

$T/2\tau = 3.33$ .

$I_{\max} = -I_{\min} = \frac{V_{dc}}{R} \left( \frac{1 - e^{-T/2\tau}}{1 + e^{-T/2\tau}} \right)$

$= \frac{100}{10} \left( \frac{1 - e^{-3.33}}{1 + e^{-3.33}} \right) = 9.31A$ .

$i_o(t) = \frac{100}{10} + \left( -9.31 - \frac{100}{10} \right) e^{-t/0.0025}$

$\therefore i_o(t) = \begin{cases} \frac{V_{dc}}{R} + (I_{\min} - \frac{V_{dc}}{R}) e^{-t/\tau} & t < \frac{T}{2} \end{cases}$

$i_o(t) = 10 - 19.31 e^{-t/0.0025} \quad 0 \leq t \leq \frac{1}{120}$

$= -10 + 19.31 e^{-(t - 0.00835)/0.0025}$

$\frac{1}{120} \leq t \leq \frac{1}{60}$

$\begin{cases} -\frac{V_{dc}}{R} + (I_{\max} + \frac{V_{dc}}{R}) e^{-(t-T/2)/\tau} & T/2 \leq t < T \end{cases}$

b)  $I_{rms} = \sqrt{\frac{1}{120} \int_0^{1/120} [10 - 19.31 e^{-t/0.0025}]^2 dt}$

$= 6.64A$ .

$P = I_{rms}^2 R = (6.64)^2 \times 10 = 441W$ .

c) Average source current can be computed by equating source and load power.

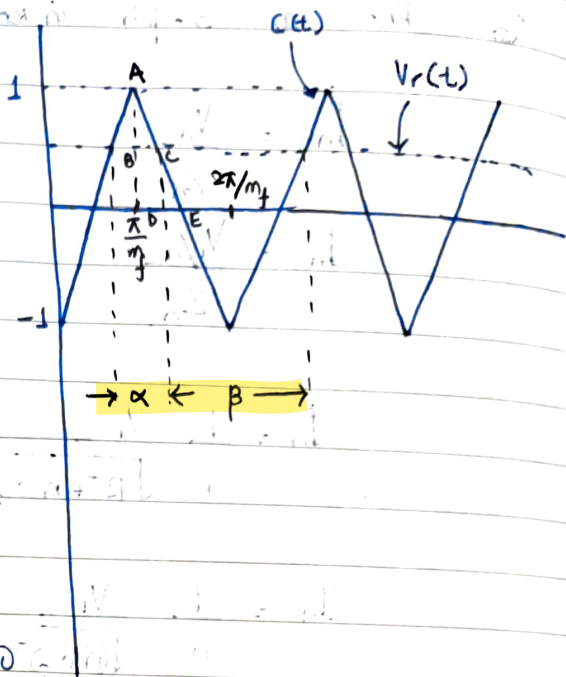
$I_s = \frac{P_{dc}}{V_{dc}} = \frac{441}{100} = 4.41A$ .

Q2:-

$$m_a(t) = \frac{V_r(t)}{C_t}$$

$$m_f = \frac{f_c}{f_r}$$

$m_f$  is large.  
 $m_a \leq 1$ .



Using similarity property of triangles, in triangle  $\triangle ABC$  and  $\triangle ADE$

$$\frac{\alpha/2}{\pi/(2m_f)} = \frac{C_t - V_r(t)}{C_t} = \frac{C_t - m_a C_t}{C_t}$$

$$\alpha = (1 - m_a) \frac{\pi}{m_f}$$

$$\text{and } \beta = \frac{2\pi}{m_f} - \alpha = (1 + m_a) \frac{\pi}{m_f}$$

The duty ratio  $d(t)$  is

$$d(t) = \frac{\beta}{2\pi/m_f} = (1 + m_a) \frac{\pi}{m_f} \times \frac{m_f}{2\pi}$$

$$d(t) = \frac{1}{2} (1 + m_a(t))$$

$$m_a(t) = 2d(t) - 1$$

Q8:-  $V_o = +V_{dc}$   
 $V_o = -V_{dc}$

$S_1 - S_2$  ON  
 $S_3 - S_4$  ON

The average value of  $V_o$  over one carrier cycle  $T_c$  is

$$\bar{V}_o = \frac{1}{T_c} \int_0^{T_c} V_o(t) dt$$

Since  $V_o = +V_{dc}$  for  $DT_c$   
 $= -V_{dc}$  for  $(1-D)T_c$ .

$$\bar{V}_o = \frac{1}{T_c} \left[ \int_0^{DT_c} V_{dc} dt + \int_{DT_c}^{T_c} (-V_{dc}) dt \right]$$

On simplifying,

$$\bar{V}_o = (2D-1)V_{dc}$$

In Sinusoidal PWM, the duty cycle  $D$  is varied sinusoidally.

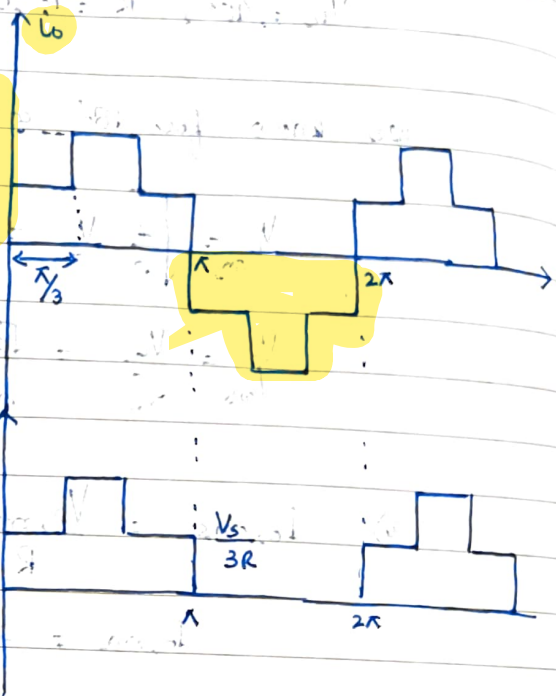
$$D = \frac{1}{2} (1 + m \sin(\omega t))$$

Substituting this into  $V_o$ ,

$$\bar{V}_o = \left( 2 \times \frac{1}{2} (1 + m \sin \omega t) - 1 \right) V_{dc}$$

$$\Rightarrow \bar{V}_o = m V_{dc} \sin \omega t$$

Q4:- For a resistive load, waveforms of phase-load current and thyristor current are shown.



$$I_{o,rms} = \left[ \frac{1}{\pi} \left[ \left( \frac{V_s}{3R} \right)^2 \frac{\pi}{3} + \left( \frac{2V_s}{3R} \right)^2 \times \frac{\pi}{3} + \left( \frac{V_s}{3R} \right)^2 \frac{\pi}{3} \right] \right]^{1/2}$$

$$= \left[ \left( \frac{450}{3 \times 10} \right)^2 \times \frac{2}{3} + \left( \frac{2 \times 450}{3 \times 10} \right)^2 \times \frac{1}{3} \right]$$

$$= \sqrt{450} = 21.213 \text{ A.}$$

$$I_{T,rms} = \left[ \frac{1}{2\pi} \left\{ \left( \frac{450}{3 \times 10} \right)^2 \times \frac{2\pi}{3} + \left( \frac{2 \times 450}{3 \times 10} \right)^2 \times \frac{\pi}{3} \right\} \right]^{1/2}$$

$$= \sqrt{225} = 15 \text{ A.}$$

Power delivered to load

$$= 3 I_{o,rms}^2 R$$

$$= 3 \times 21.213^2 \times 10$$

$$= 13.5 \text{ kW.}$$



Qs: For a 3-ph inverter without PWM.

$$I_n = \frac{V_n}{Z_n}$$

$$I_n = \frac{V_i/n}{Z_n}$$

$$I_n = \frac{1}{n} \frac{V_i}{\sqrt{R^2 + (n\omega L)^2}}$$

$$I_n = \frac{1}{n} \frac{V_i}{\sqrt{(n\omega L)^2}}$$

$$\because \omega L \gg R$$

$$(n\omega L)^2 + R^2 \approx (n\omega L)^2$$

$$= \frac{1}{n^2} \frac{V_i}{\omega L}$$

$$= \frac{1}{n^2} \times I_1$$

$$I_n = \frac{1}{n^2} \times 100$$

$$\therefore I_5 = \frac{1}{5^2} \times 100$$

$$I_5 = 4A$$