Optional hardware project for EE309(S2) class [Spring 2025]

Passive devices such as resistors, capacitors and inductors, are mostly linear in nature. The simplified response of a linear system with input $x(t)$ can be written as:

$$y_{LIN}(t) = \alpha_1 x(t)$$

Note that if the input has two frequency tones, i.e. $x(t) = A[\cos(2\pi f_1 t) + \cos(2\pi f_2 t)]$, the output will also have the same two frequency tones, i.e.

$$y_{LIN}(t) = \alpha_1 A[\cos(2\pi f_1 t) + \cos(2\pi f_2 t)]$$

On the other hand, active circuit components that include semiconductor devices such as diodes and transistors exhibit non-linear behaviour, which becomes apparent as the input signal amplitude increases. The simplified response of a system exhibiting the 2nd and 3rd order non-linearities, for an input $x(t)$, can be written as:

$$y_{NL}(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

In this case, if the input has two frequency tones, i.e. $x(t) = A[\cos(2\pi f_1 t) + \cos(2\pi f_2 t)]$, multiple new frequency tones will be produced.

Use trigonometric relationships to find that non-zero α_2 (i.e. the 2nd order non-linear response), i.e. $y(t) = \alpha_2 x^2(t)$ results in the following tones that have amplitudes that are proportional to $\alpha_2 A^2$:

$f_1 - f_1$ & $f_2 - f_2$ (producing a DC component), $f_2 - f_1$, $f_2 + f_1$, $2f_1$ and $2f_2$

Note that all these terms involve "two frequencies". Similarly, non-zero α_3 (i.e. the 3rd order non-linearity) result in the following tones that have amplitudes that are proportional to $\alpha_3 A^3$:
$2f_1 - f_2,\ 2f_1 - f_2,\ 2f_2 - f_1,\ 3f_1,\ 2f_1 + f_2,\ 2f_2 + f_1,$ and $3f_2$.
In addition, it also produces $f_1 = 2f_1 - f_1,$ and $f_2 = 2f_2 - f_2$ components.

In this project you need to generate a signal $x(t) = A[\cos(2\pi f_1 t) + \cos(2\pi f_2 t)]$, with frequencies f_1 = 4.1 kHz, f_2 = 4.7 kHz using a microcontroller (on PT-51 board) and a D-to-A converter. Give it to a circuit (for example, a simple BJT based amplifier or diode) that shows non-linear response. In addition to other frequencies, non-zero α_2 will result in "inter-modulation" tone at $f_2 - f_1 =$ 600 Hz. Similarly, non-zero α_3 produces another intermodulation tone at $2f_1 - f_2 =$ 3.5 kHz.

Use the same microcontroller and an A-to-D converter to sample the resultant signal, and find the amplitudes of the tones at $f_1 - f_2$, $2f_1 - f_2$, f_1 and f_2. Compare them with the relative amplitudes of the tones observed on the oscilloscope displayed in the FFT (spectrum) mode.

Note that if the signal is sampled coherently at a certain frequency, with an appropriate timing, you can get the amplitude of a specific tone relatively. For example, if you sample $\cos(2\pi f_m t)$ at 1.2kHz, where $f_m =$ 600 Hz, the first sample is at t=0, multiply alternate samples with +1 and -1, add and average them, you can find the amplitude of the 600 Hz tone. This assumes that no other tones at integer multiple frequency of 600 Hz are present in the signal. You can choose your own frequencies f_1 and f_2 (that make the project easy for you). If it is difficult to align the phase, you have to sample the signal four times in one tone period and do some additional math (i.e. you need to get sine and cosine component amplitudes of the tone, and find the overall amplitude from it.