Assignment #1
Announced on February 10
Due by February 21 (5PM via Moodle)

This assignment is composed of 8 problems, where some problems have sub-problems. You need to submit a word (or PDF) document containing your answers into Moodle. Alternately, if you solve the problems by hand, you will need to scan (in PDF file) your answers and submit it into Moodle.

Total Points: 50

1. Problem 1 (10 marks)

This problem is composed of six questions (a, b, c, d, e, f). For the system described below, answer those questions.

<System>
Three types (gold, silver, and regular) of customers arrive at a single server John Jr burger shop where they serve one customer at a time. Gold type customers have higher priority than Silver type customers, and Silver type customers have higher priority than regular type customers. No pre-emption occurs; in other words, if the server is serving a regular type customer when a gold type customer arrives, the server must finish serving the regular customer first and then serve the gold type customer.

<u>Customer Arrival and Service Information</u>

- Gold customers: Inter-arrival time of 50 minutes
- Silver customers: Inter-arrival time of 30 minutes
- Regular customers: Inter-arrival time of 10 minutes
- Assumption: At time “zero”, one Regular customer (the first Regular customer) arrives at the system; At time “5”, one Silver customer (the first Silver customer) arrives at the system; at time “10” one Gold customer (the first Gold customer) arrives as the system
- It takes 20 minutes (service time) for all the customers

<u>Queues and their capacity</u>

- There are three queues (waiting lines) in the system, each of which is dedicated for Gold customers, Silver customers, and Regular customers, respectively
- Assumption: their capacity is “Infinite”

<u>Ranking rule in the Queue</u>

- Customers are serviced in FCFS (first come first service) order.

<u>Performance measures to be calculated (goals of simulation)</u>

- Average times that each customer types (Gold, Silver, Regular) spend in the system
- Total number of customers (sum of Gold, Silver, and Regular) served at the end of the simulation run;
- Average waiting times of each customer types (Gold, Silver, Regular) in the queue

<u>Simulation replication (ending) time</u>

- “100” minutes
- Note: If any other event is scheduled at time 100 in addition to “End” event, assume that those events are executed before the “End” event

(a)

For the system described above, define components for a discrete event simulation model. Components should include 1) entities, 2) resources, 3) attributes of the entities, 4) variables, 5) statistical accumulators (variables used to calculate statistics), 6) queues, 7) event types, and 8) conceptual logic (pseudo code) for each event type.

- Entities:
- Resources:
- Attributes:
- Variables:
- Statistical accumulators:
- Queues:
- Event types: type A, type B, type C …
- Event type *A* Logic {
  --- (pseudo code goes here)
  }
- Event type *B* Logic {
  --- (pseudo code goes here)
  }

(b)
Where in your simulation logic does it guarantee that the correct priority logic (gold customers are served before silver customers; silver customers are served before regular customers) is implemented? Answer very briefly.

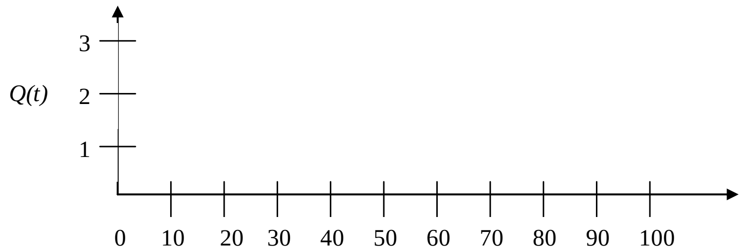
(c)
Construct a table containing the sample path of the simulation that you have developed conceptually in Problem (a). In Problem 1(a), performance measures to be calculated were given. In your table (sample path), you must include only those variables, attributes, and statistical accumulators that are necessary to calculate the requested performance measures (do not include unnecessary information).

- In Table below, you need to fill in only the first 5 rows for the "Attributes" column; and you can leave blank from the 6th row. However, you must fill in all the rows until the simulation end time for all the other columns (e.g. just finished event, variables, statistical accumulators, and event calendar).
- Note that you do **not** have to actually build a simulation model.

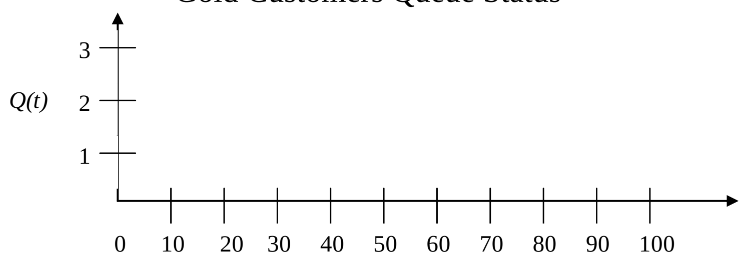
Sample path (record) Table

Just Finished Event	Variables	Attributes (arrival_t) – **show only the first 5 rows**	Statistical Accumulators	Event Calendar
E# Time Evt				E# Time Evt

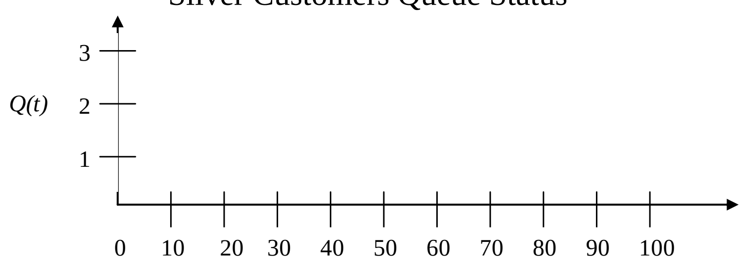
(d)
Based on your simulation results obtained in Problem 1(c), plot the number of customers in the queue (Q(t)) and the machine status (B(t)) over time.



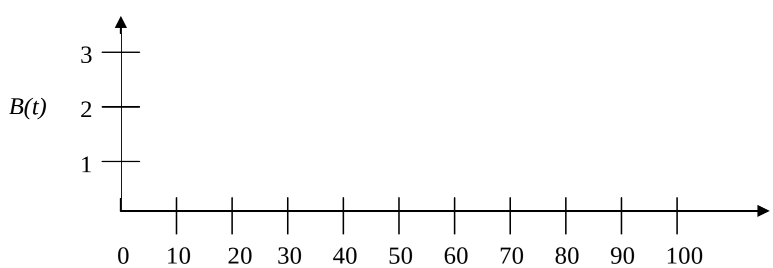
Gold Customers Queue Status



Silver Customers Queue Status



Regular Customers Queue Status



Machine Status

(e)
What is the average numbers of customers in the Gold queue, Silver queue, and Regular queue, respectively?

- Average numbers of customers in the Gold customer queue: ( )
- Average numbers of customers in the Silver customer queue: ( )
- Average numbers of customers in the Regular customer queue: ( )

(f)
What is the machine utilization (% of time when the machine is busy)?

2. Problem 2 (5 marks)

The system considered in Problem 2 is almost same with the one considered in **Problem 1**, where the only difference is that there is only **one** queue shared by all three types of customers. For this system, answer the following two questions.

(a)
For this system, your conceptual logic (pseudo code) for each event type will be slightly different from that of Problem 1. Show your conceptual logic (pseudo code) for each event type.

- Event types: type A, type B, type C …
- Event type *A* Logic {
  --- (pseudo code goes here)
  }
- Event type *B* Logic {
  --- (pseudo code goes here)
  }

(b)
Where in your simulation logic does it guarantee that the correct priority logic (gold customers are served before silver customers; silver customers are served before regular customers) is implemented? Answer very briefly.

3. Problem 3 (5 marks)

The system considered in Problem 3 is almost same with the one considered in **Problem 1**, where the only difference is on the "Customer Arrival Information" (see below). Note that we have three explicit queues. For this system, answer the following one question.

Customer Arrival Information

- Customers: Inter-arrival time of 30 minutes
- Percentages of arrived customers: 20% for Gold customers, 30% for Silver customers, 50% for Regular customers
- Assumption: At time "zero", one customer (whose type is not known at that moment) arrives at the system

(a)
For the system, event types and your conceptual logic (pseudo code) for those event types will be slightly different. Show me your conceptual logic (pseudo code) for each event type.

- Event types: type A, type B, type C …
- Event type *A* Logic {
  --- (pseudo code goes here)
  }
- Event type *B* Logic {

--- (pseudo code goes here)
}

4. Problem 4 (5 marks)

Answer the following “TRUE” or “FALSE” questions.

(a) The simulation model considered in Problem 3 is a stochastic model.
1) TRUE ( ) or 2) FALSE ( )

(b) The above simulation models considered in Problems 1~3 are static model.
1) TRUE ( ) or 2) FALSE ( )

(c) We understand that the above problems could have been solved without using simulation. Even if an analytical model can solve the above problems, we still must use simulation tool.
1) TRUE ( ) or 2) FALSE ( )

(d) A continuous time modeling approach is preferred to a discrete event simulation modeling approach for the above systems.
1) TRUE ( ) or 2) FALSE ( )

(e) In the above simulations, we must consider how much money each customer spends in each visit.
1) TRUE ( ) or 2) FALSE ( )

5. Problem 5 (5 marks)

The following table contains a sample path of a simulation (spreadsheet-based simulation involving recursive equations) run for a one machine queueing system. In our class, we have obtained A_i, D_i, E_i, and F_i using B_i and C_i (input to the system) and their (A, B, E, F) recursive equations. Fill in the blanks in (a) to (d). Note that you have to fill in the table below.

(a) A_{i+1} = ( ) + ( )

(b) D_{i+1} = Max (( ) + ( ) – ( ), 0)

(c) E_i = ( ) + ( )

(d) F_i = ( ) + ( )

Part #	Arrival Time (A_i)	Inter-arrival Time (B_i)	Service Time (C_i)	Time_in_Q (D_i)	Time_in_Sys (E_i)	Depart Time (F_i)
1	0.00	1.73	2.90			
2		1.35	1.76			
3		0.71	3.39			
4		0.62	4.52			
5		14.28	4.46			

6. Problem 6 (5 marks)

Consider the shifted (two-parameter) exponential distribution, which has density function

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-(x-\gamma)/\beta} & \text{if } x \geq \gamma \\ 0 & \text{otherwise} \end{cases}$$

For $\beta > 0$ and any real number γ. Given the sample $X_1, X_2, \ldots X_n$ of IID random values from this distribution, find formulas for the joint MLEs $\hat{\gamma}$ and $\hat{\beta}$.

7. Problem 7 (10 marks)

You are given a dataset representing service times (in minutes) at a customer service centre:

10.3	10.9	12	11.5	15.2	14	13.6	10.3
8.9	7.8	11.3	9.5	16.5	17.5	17.8	12.1
5.5	8.6	9.5	12	13.7	12.1	11.6	4.5

Perform the following analysis:
a. Exploratory Data Analysis
b. Identify Candidate Distributions
- Based on the histogram and summary statistics, suggest at least two candidate discrete distributions that may fit the data

c. Estimate Parameters using Maximum Likelihood Estimation (MLE)
- Compute the MLE for each candidate distribution.

d. Perform Hypothesis Testing
e. Conclude which distribution best fits the data and justify your reasoning.

8. **Problem 8 (5 marks)**

Let's suppose we performed data collection of the time that customers spend at a local post office. Twenty (20) data have been collected, and they are shown in the following table.

0.1	0.3	0.9	1.1	1.2	1.4	1.8	1.9	2.3	2.5
2.8	3.1	3.2	3.2	3.4	3.5	3.6	3.7	3.8	3.9

Let's also suppose that the Software/Input Analyzer suggested the linear PDF shown below as the best-fitted distribution for the collected data. Using the "Chi-square test", **determine** whether the collected data are independent samples from this fitted distribution or not at level $\alpha = 0.05$. Let's assume the number of interval (k) used in the "Chi-square test" is **4**. Let's also assume that the range of the values is 0 to 4. Note: $\chi^2_{3,\,0.05}$ is 7.81.

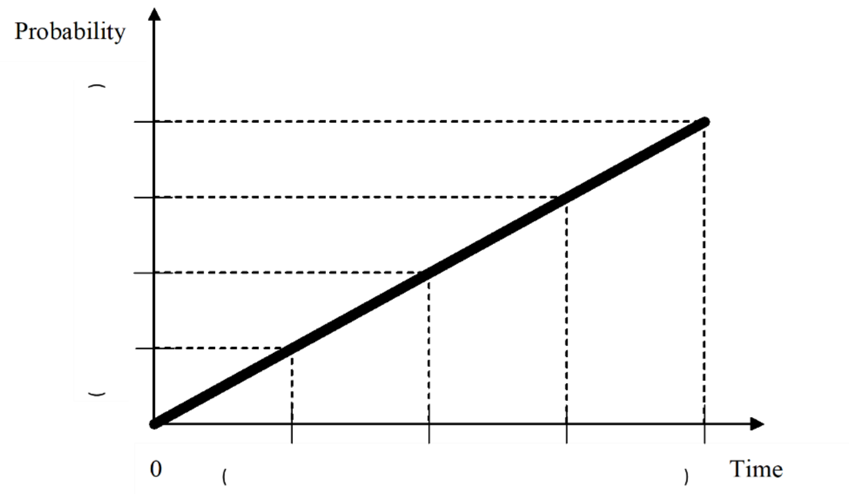


Figure. Fitted linear distribution for Problem 8