IE630: Simulation Modelling & Analysis
Discrete Event Simulation

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Quick Recap

What is Discrete Event Simulation

- A discrete-event simulation
  - models a system whose state may change only at discrete point in time
- System - is composed of objects called entities that have certain properties called attributes
- State - a collection of attributes or state variables that represent the entities of the system.
- Event - an instantaneous occurrence in time that may alter the state of the system
- Resource - Things that entities compete for (space/ machine/ personal)
- An event initiates an activity, which is the length of time during which entities engage in some operations
- Entities, attributes, events, activities and the interrelationships between the model of the system

Components of a System: Definitions

- Entities
  - Dynamic objects in the simulation
  - Something moving throughout the system
  - Examples:
  - It may not be tangible, such as order for the company, requests, etc.
- Attributes
  - Local variable (tag) characterizing entities
  - Each entity can have same or different values
  - Most important thing is that their values are tied to specific entities
  - Examples:
  - Values can be real number, integer number, texts, or some other object types: different software supports different data types

Components of a System : Definitions

- Variables
  - Information that reflects some characteristics of the system, regardless how many or what kinds of entities are around
  - Variables are not tied in specific entity, rather pertain to the system at large
  - Examples:
  - System (Anylogic) built-in variables: TNOW (current simulation time)
  - User specified variables
- Resources
  - Entities compete with each other for service from resources such as personnel, equipment, or space in storage area of limited size
  - Seizing and releasing. Capturing and freeing. Changing of system variables
  - Capacity or units of a resource: a resource can represent a group of several individual and identical servers. If you want to differentiate them, you need to define another resource

Components of a System : Definitions

- Queues
  - A place where entities can wait when no resource is available
  - Queue also has a capacity
  - Example:
  - You must model how to handle an entity arriving at a queue that's already full
  - Should we allow this happen and do something? Balking!!
  - Should we model so that this does not occur?
  - This could be a logical queue or physical queue. This differs depending on different simulation packages
- Statistical Accumulators
  - Variables needed for storing statistical information necessary to estimate the desired performance measures (e.g., mean time in system)
  - Several simulation packages take care of statistical accumulation. But, in our hand simulation, we will do it manually to understand how it works
  - Examples:

Components of a System : Definitions

- Events
  - An event is something that happens at an instant of (simulated) time that might change a system state (such as attributes, variables, or statistical accumulators)
  - There are four kinds of events in our example:
    - Initialization to set things up: making all zeroes or initialization values
    - Arrival: A new part enters the system
    - Departure: A part finishes its service at the machine and leaves the system
    - The End: The simulation is stopped at time 15 minutes
    - Question: why not the following is not included to the event:

    "parts leave the queue and begin service at the machine, which changes the system"?
- Event calendar
  - Event calendar is the place where future events are stored
  - Event calendar has a set of records, where each record is associated with 1) id of an entity, 2) event time, and 3) the kind of event (init, arrive, depart, or end)

Components of a System : Definitions

- When it’s time to execute the next event, the top record is removed from the calendar and the information in this record is used to execute the appropriate logic, such as changing variables, showing message boxes, or placing new records to the event calendar
- Anylogic (or other commercial simulation packages) places newly scheduled events to the event calendar in increasing order of event times) => the top one is always the next event to occur
- The variables that describe the system don’t change between successive events
  - Example: machine start processing, machine end processing: no change of system states

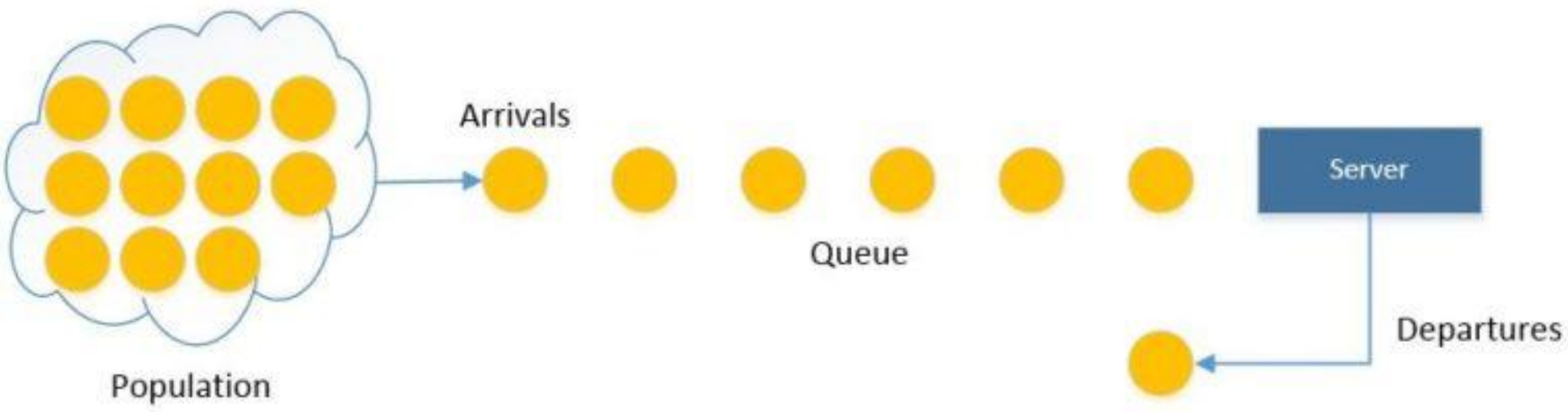
- Simulation clock:
  - The current value of simulation clock (TNOW)
  - Simulation does not take all values and flow continuously
  - Updated when events occur (the event time of the top record from the event calendar becomes the current simulation time before the associated logic is performed)

Components of a System

System	Entities	Attributes	Activities	Events	State Variables
Banking	Customers	Checking account balance	Making deposits	Arrival; departure	Number of busy tellers; number of customers waiting
Rapid rail	Riders	Origination; destination	Traveling	Arrival at station; arrival at destination	Number of riders waiting at each station; number of riders in transit
Production	Machines	Speed; capacity; breakdown rate	Welding; stamping	Breakdown	Status of machines (busy, idle, or down)
Communications	Messages	Length; destination	Transmitting	Arrival at destination	Number waiting to be transmitted
Inventory	Warehouse	Capacity	Withdrawing	Demand	Levels of inventory; backlogged demands

What is Discrete Event Simulation

- Events occur at discrete points of time, leading to changes in (   )
- Can customers go directly into the service?
- Waiting time for first customer?



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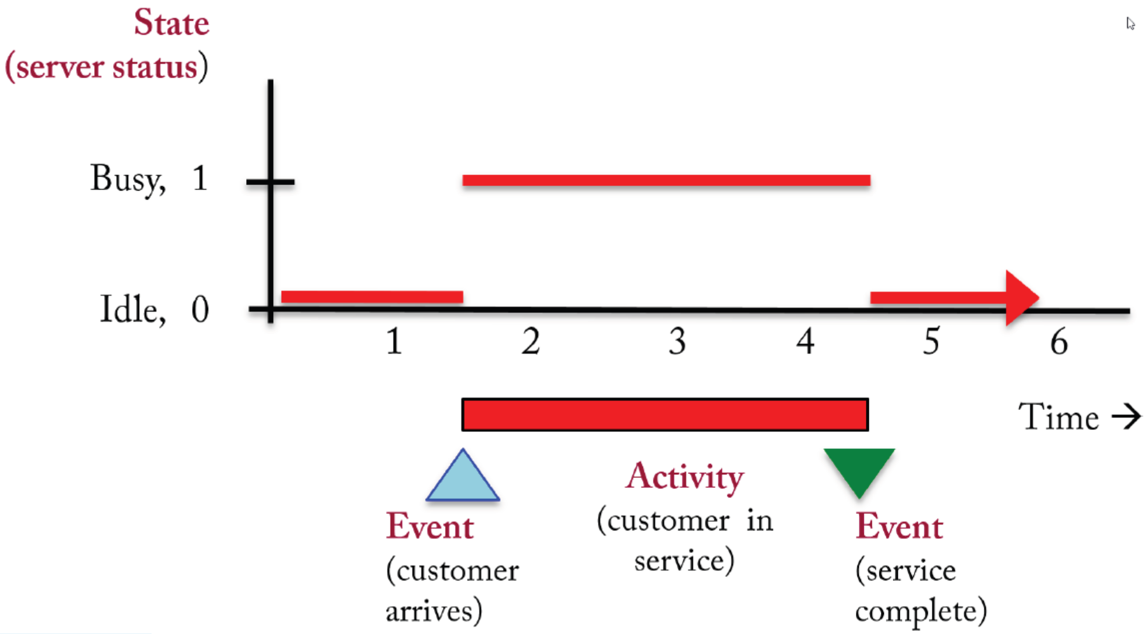
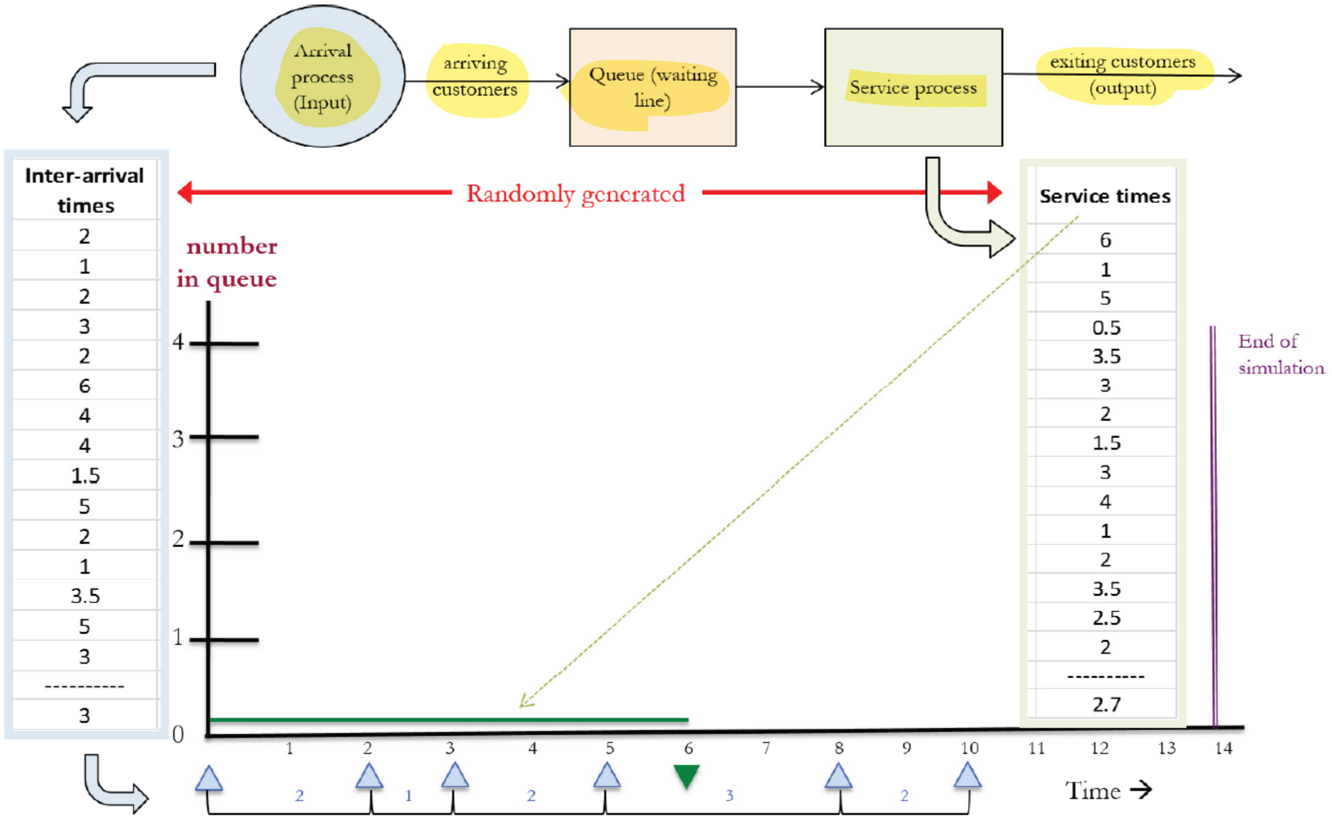
Simple Queuing System

- Let’s try to understand the functionality using simple example





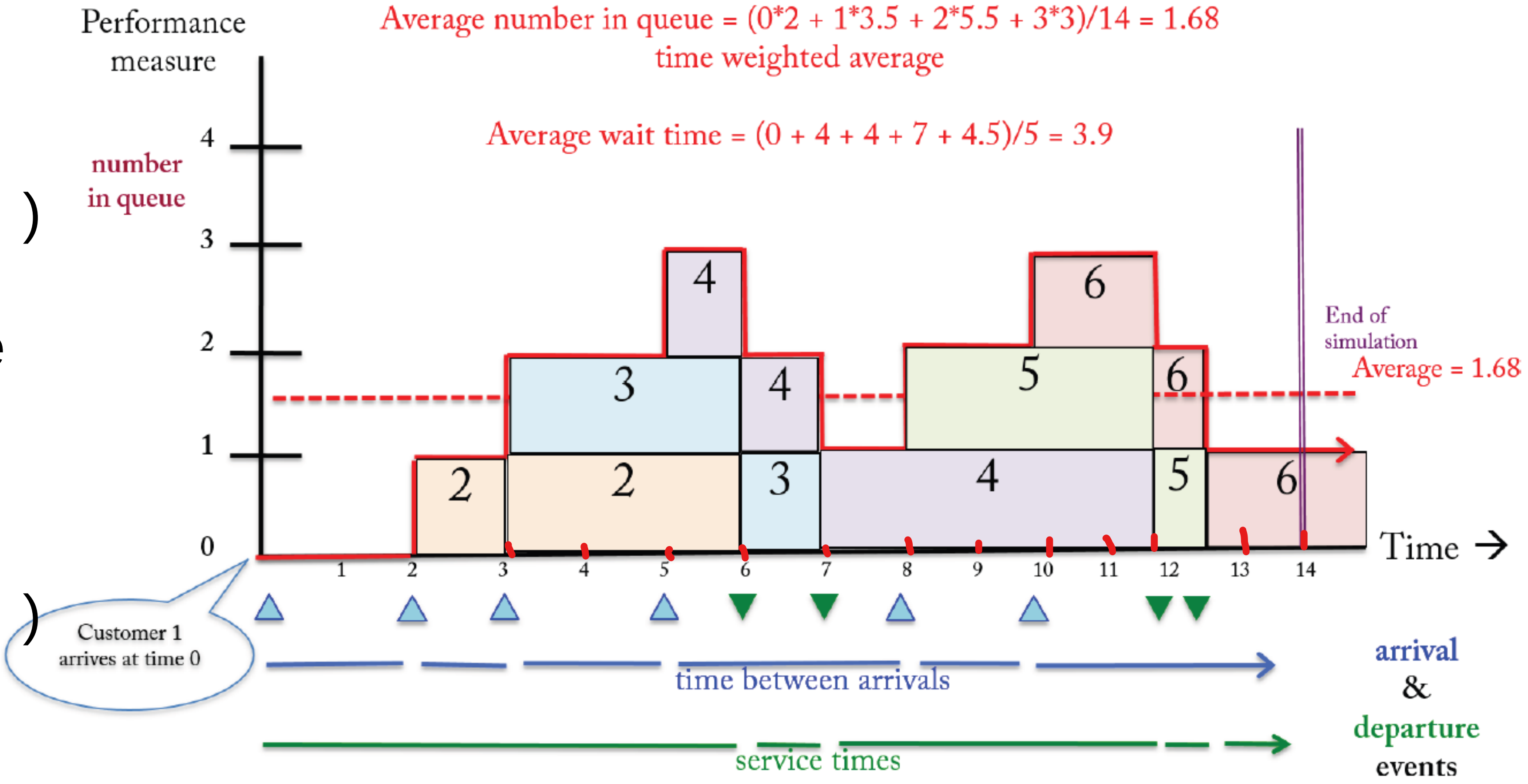
Simple Queuing System

- What does this graph denote?

( )

- Can we trace the events, and statistics?

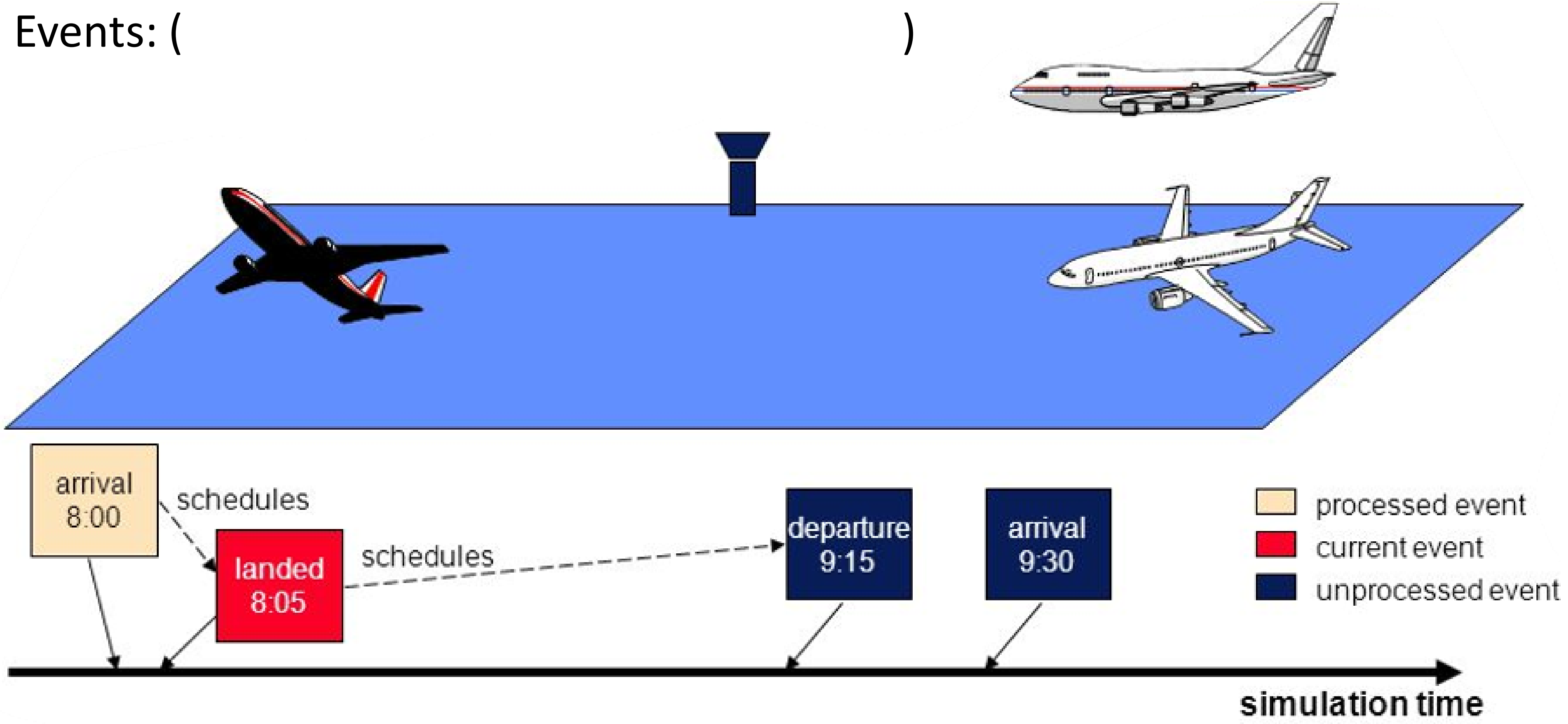
( )



Discrete Event Simulation

Example: Air traffic at an airport

Events: ( )



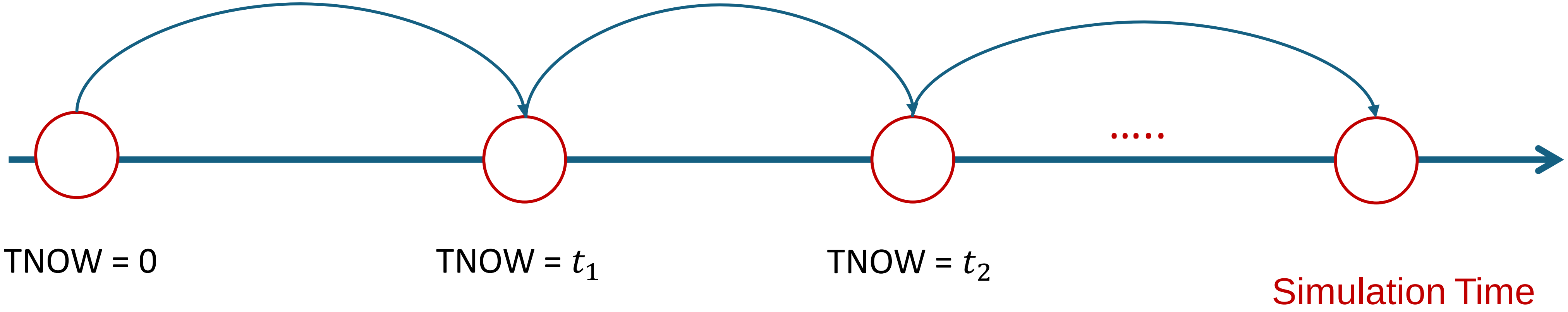
Discrete Event Simulation Framework

- Pick first event from Event Calendar.
- Advance simulation clock time
- Call the function associated with that event
  - Change the state variables
  - Schedule future events, which are then inserted into the event calendar
- Simulation ends when there are no more events, or reached end event

1. (   )

2. (   )

Discrete Event Simulation Framework



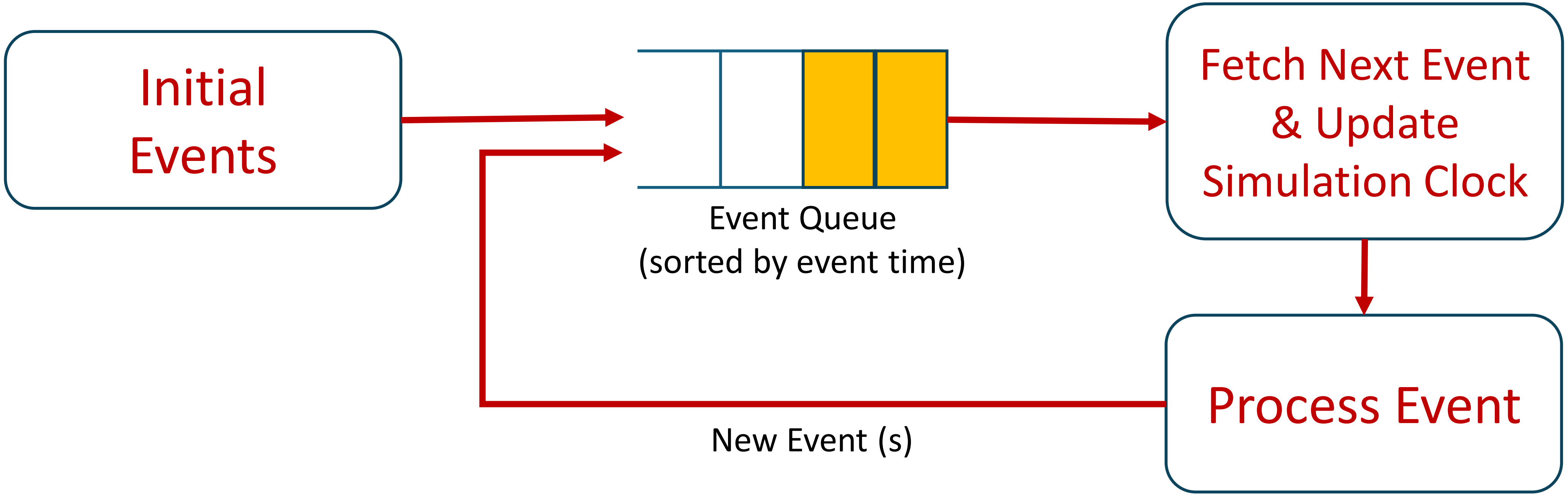
Entity ID	Time	Event Type
e_1	0	
e_2	t_1	
e_2	t_2	
……..	…….	…….
e_n	t_n	

Idle

Busy

Discrete Event Simulation Framework

- Simulated Operations are performed as discrete sequence of events in time
- “Time stops during event processing
- Time jumps -> next event time



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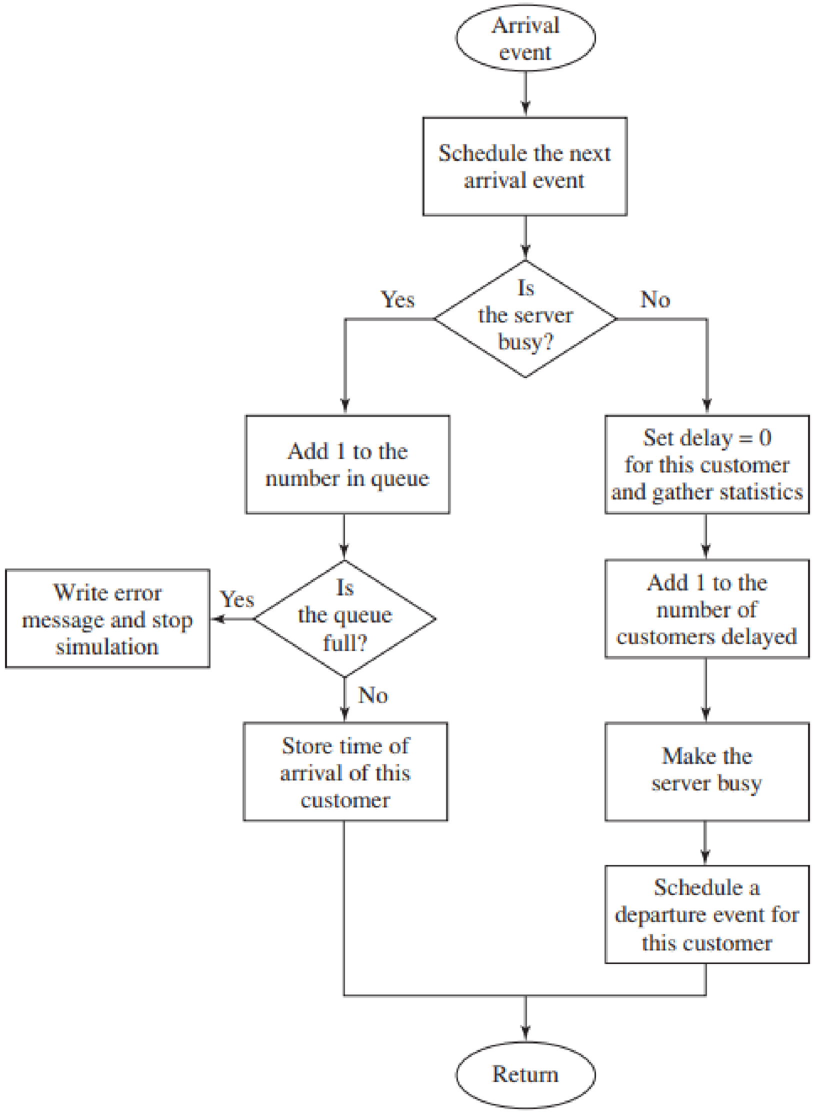
Discrete Event Simulation Framework

- “Initialize” event
  - Set simulation clock (“TNOW”) = 0
  - Initialize system state and statistical accumulators
  - Initialize event list (calendar)
  - Schedule any events that are supposed to occur at time zero
  - Schedule the end event (if any)

- “End” event
  - Update the time-persistent statistics to the end of the simulation
  - Compute the final summary output performance
  - Write report (in nice format)

Discrete Event Simulation Framework

- "Arrive" event
  - Update the time-persistent statistics (between the last event and now)
  - "Mark" the arriving part (entity) with an attribute giving its time of arrival (the current value of the clock) => this will be used to compute its flow time or the time it spends in the queue
  - If the machine is idle
    - The parts goes right into service, meaning
    - The machine is made busy
    - The end of this service is scheduled at (( ) + ( ))
    - Tally this part's "time in queue" zero

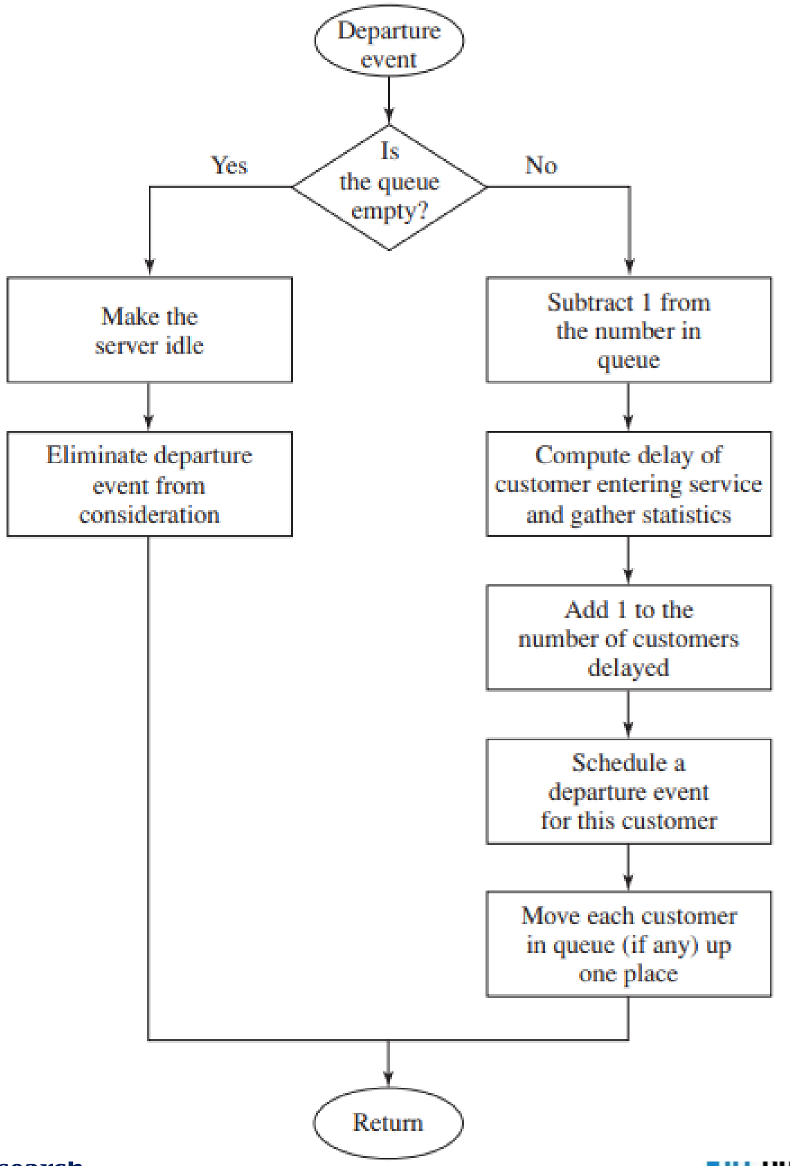


Discrete Event Simulation Framework

- Else If the machine is busy
  - The arriving part is put at the end of the queue and the queue-length variable is incremented
  - Finally, schedule ( ) at the next arrival time by placing a new event record for it onto the event calendar (inter-arrival time)

Discrete Event Simulation Framework

- “Depart” event
  - Increment the number produced statistical accumulator
  - Compute and tally the flow time of the departing entity (( ) – ( ))
  - Update the time-persistent statistics
  - If there are any parts in queue,
    - Take the first one out
    - Compute and tally its time in queue (using its arrival time)
    - Begin service by ( ) at ( )
    - Otherwise
    - Make the machine idle
    - Finally, schedule ( ) at the next arrival time by placing a new event record for it onto the event calendar (inter-arrival time)



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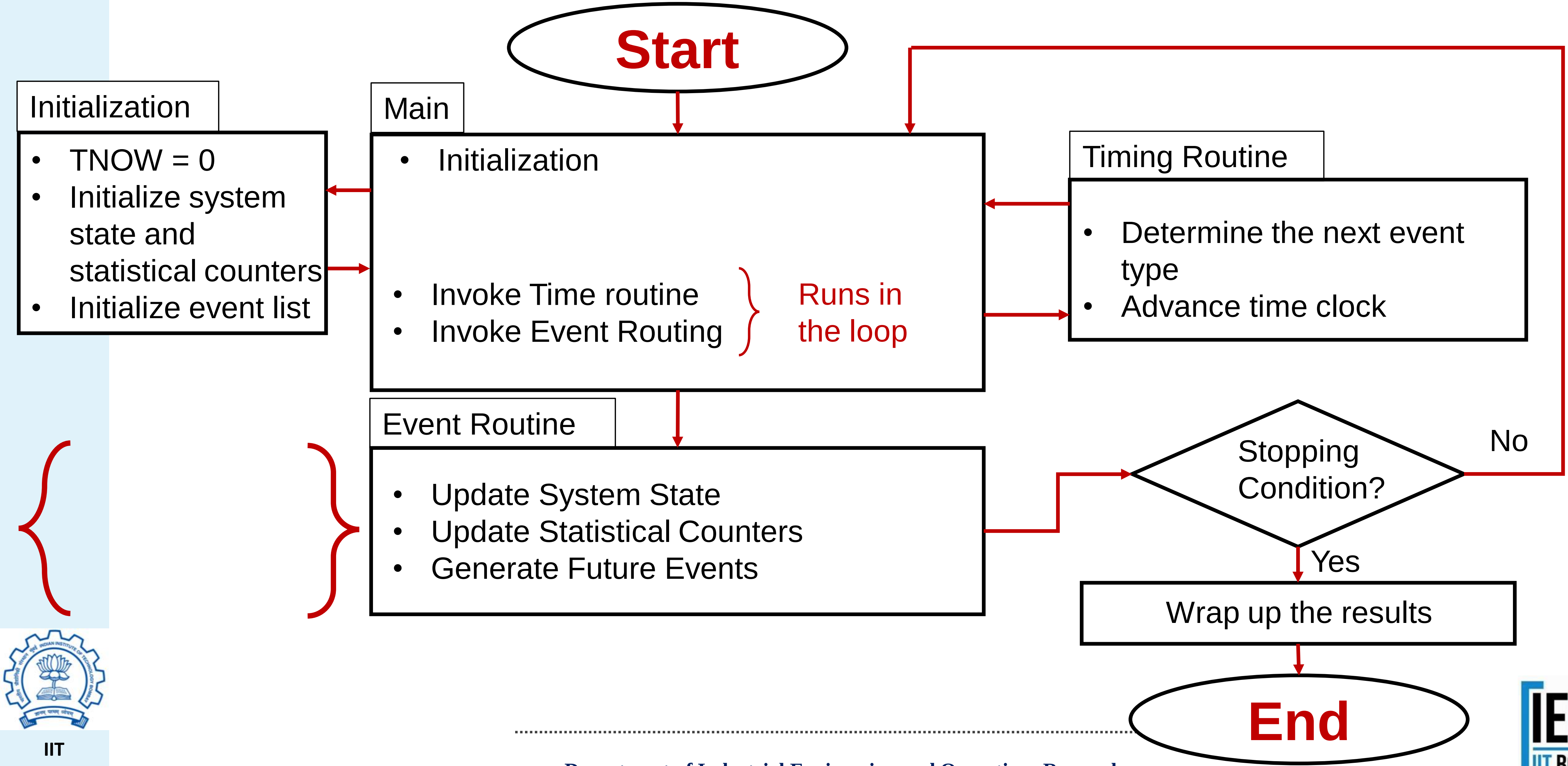
Discrete Event Simulation Framework



Hand Simulation

Run the Simulation for 20 minutes.

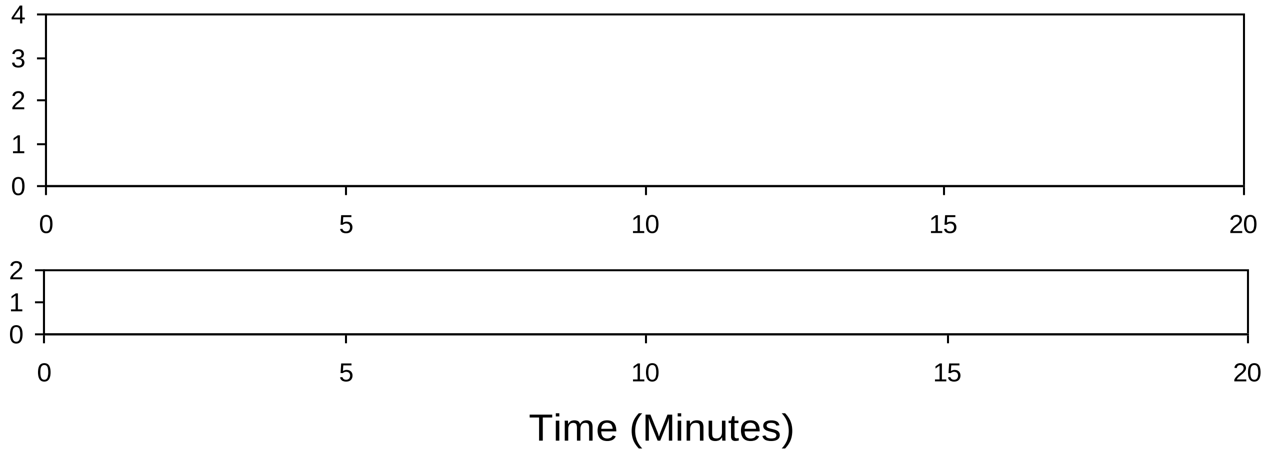
Part Number	Arrival Time	Interarrival Time	Service Time
1	0.00	1.73	2.90
2	1.73	1.35	1.76
3	3.08	0.71	3.39
4	3.79	0.62	4.52
5	4.41	14.28	4.46
6	18.69	0.70	4.36
7	19.39	15.52	2.07
8	34.91	3.15	3.36
9	38.06	1.76	2.37
10	39.82	1.00	5.38
11	40.82	.	.
.	.	.	.
.	.	.	.

Simulation by Hand: Setup

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
Number of completed waiting times in queue	Total of waiting times in queue		Area under $Q(t)$		Area under $B(t)$

$Q(t)$ graph

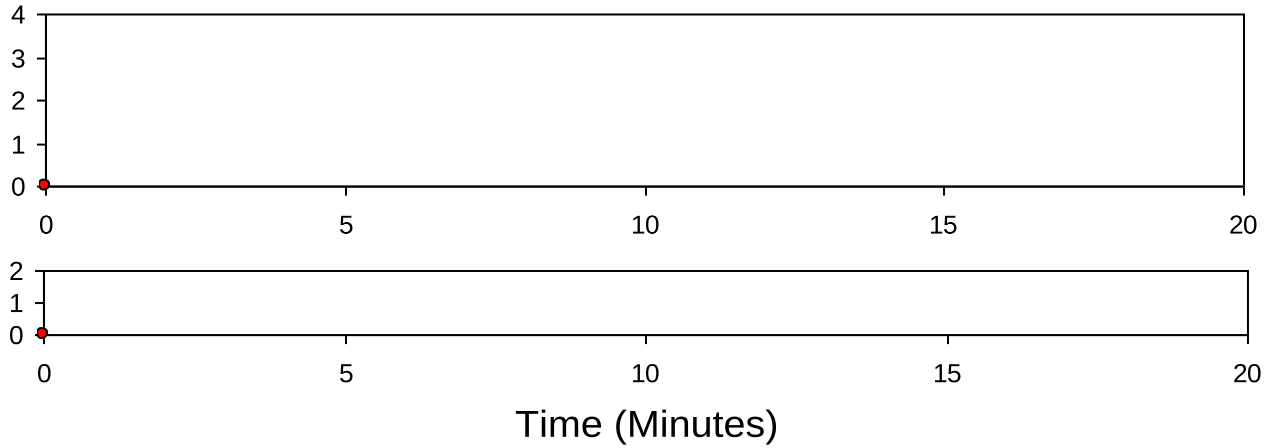
$B(t)$ graph



Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand: t = 0.00, Initialize

<table>
<tr><td>System</td><td>Clock
0.00</td><td>B(t)
0</td><td>Q(t)
0</td><td>Arrival times of custs. in queue
<empty></td><td>Event calendar
[1, 0.00, Arr]
[–, 20.00, End]</td></tr>
<tr><td>Number of completed waiting times in queue
0</td><td colspan="2">Total of waiting times in queue
0.00</td><td colspan="2">Area under Q(t)
0.00</td><td>Area under B(t)
0.00</td></tr>
<tr><td>Q(t) graph

B(t) graph</td><td colspan="5">
</td></tr>
<tr><td>Interarrival times</td><td colspan="5">1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="5">2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Simulation by Hand: t = 0.00, Arrival of Part 1

<table>
<tr><td>System
1</td><td>Clock

0.00</td><td>$B(t)$

1</td><td>$Q(t)$

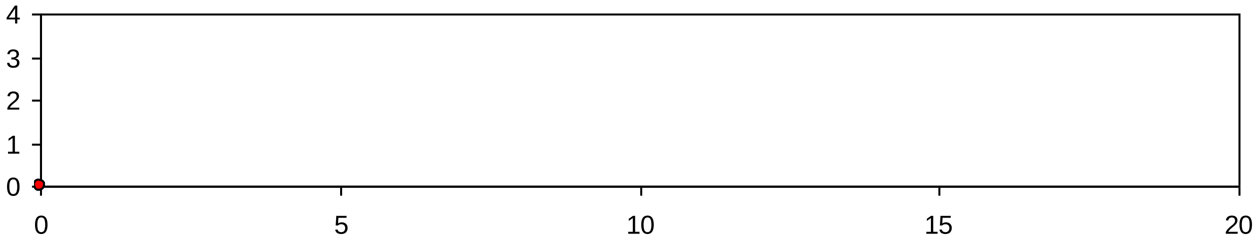
0</td><td>Arrival times of custs. in queue
<empty></td><td>Event calendar
[2, 1.73, Arr]
[1, 2.90, Dep]
[–, 20.00, End]</td></tr>
<tr><td>Number of completed waiting times in queue
1</td><td colspan="2">Total of waiting times in queue

0.00</td><td colspan="2">Area under $Q(t)$

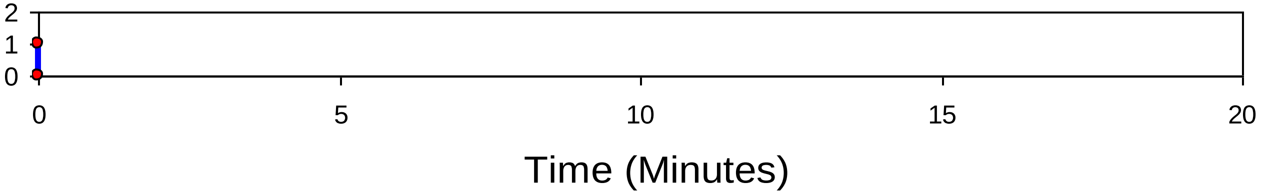
0.00</td><td>Area under $B(t)$

0.00</td></tr>
</table>

$Q(t)$ graph



$B(t)$ graph



Interarrival times	~~1.73~~, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

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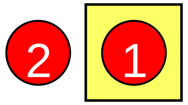
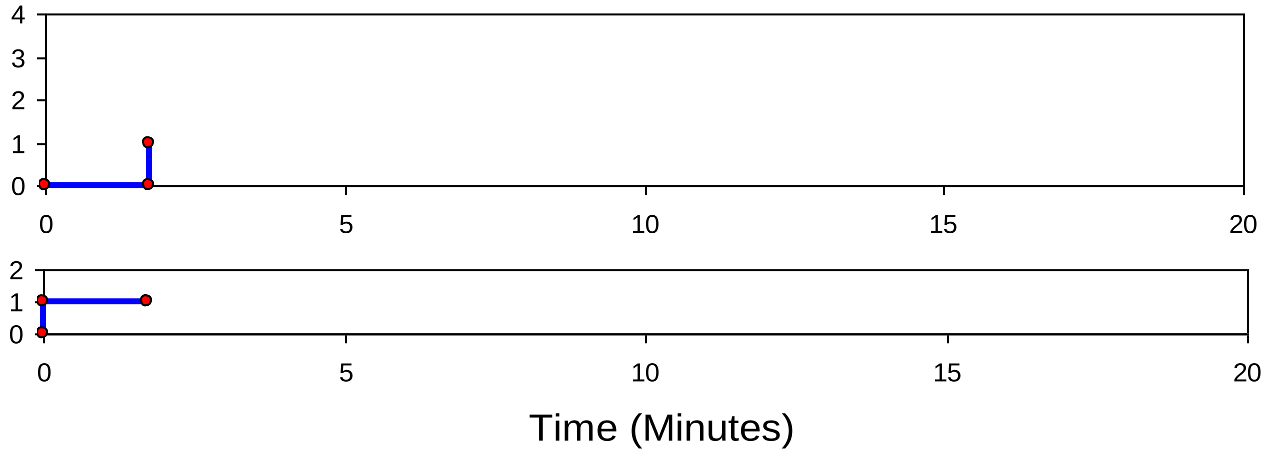
Simulation by Hand: t = 1.73, Arrival of Part 2

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
	1.73	1	1	(1.73)	[1, 2.90, Dep] [3, 3.08, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
1	0.00	0.00	1.73

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

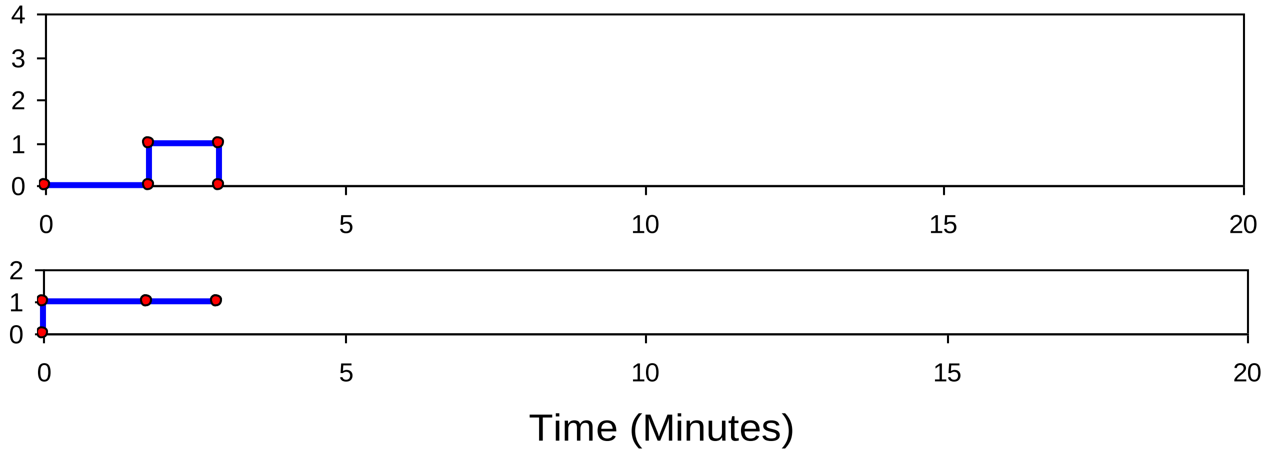
Simulation by Hand: t = 2.90, Departure of Part 1

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
	2.90	1	0	<empty>	[3, 3.08, Arr] [2, 4.66, Dep] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
2	1.17	1.17	2.90

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand: t = 3.08, Arrival of Part 3

<table>
<tr><td>System
3 2</td><td>Clock

3.08</td><td>B(t)

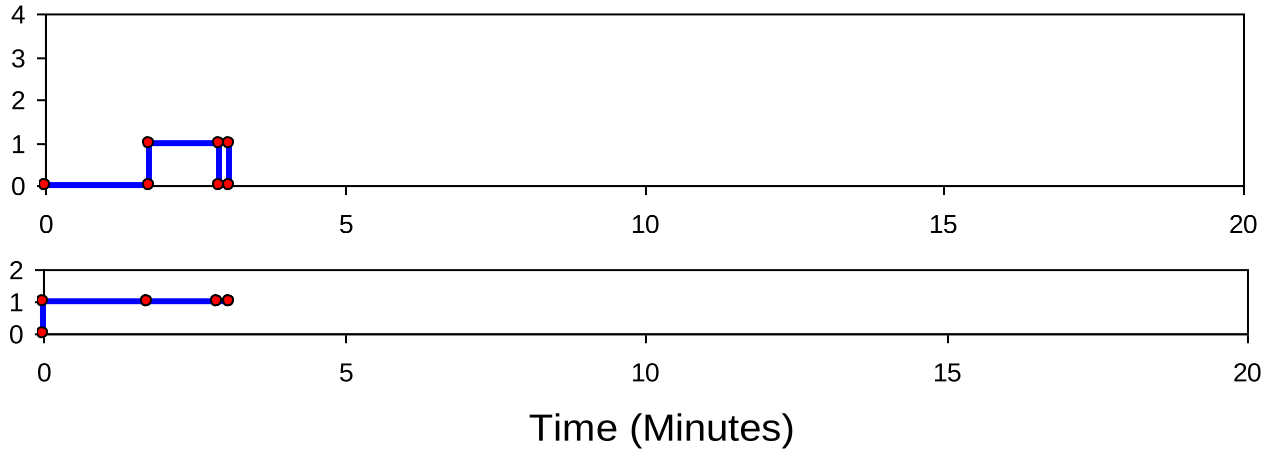
1</td><td>Q(t)

1</td><td>Arrival times of custs. in queue
(3.08)</td><td>Event calendar
[4, 3.79, Arr]
[2, 4.66, Dep]
[–, 20.00, End]</td></tr>
<tr><td>Number of completed waiting times in queue
2</td><td colspan="2">Total of waiting times in queue

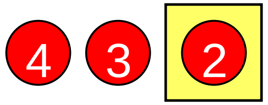
1.17</td><td colspan="2">Area under Q(t)

1.17</td><td>Area under B(t)

3.08</td></tr>
<tr><td>Q(t) graph

B(t) graph</td><td colspan="5">
</td></tr>
<tr><td>Interarrival times</td><td colspan="5">~~1.73~~, ~~1.35~~, ~~0.71~~, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="5">~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Simulation by Hand: t = 3.79, Arrival of Part 4

<table>
<tr><td>System

</td><td>Clock

3.79</td><td>B(t)

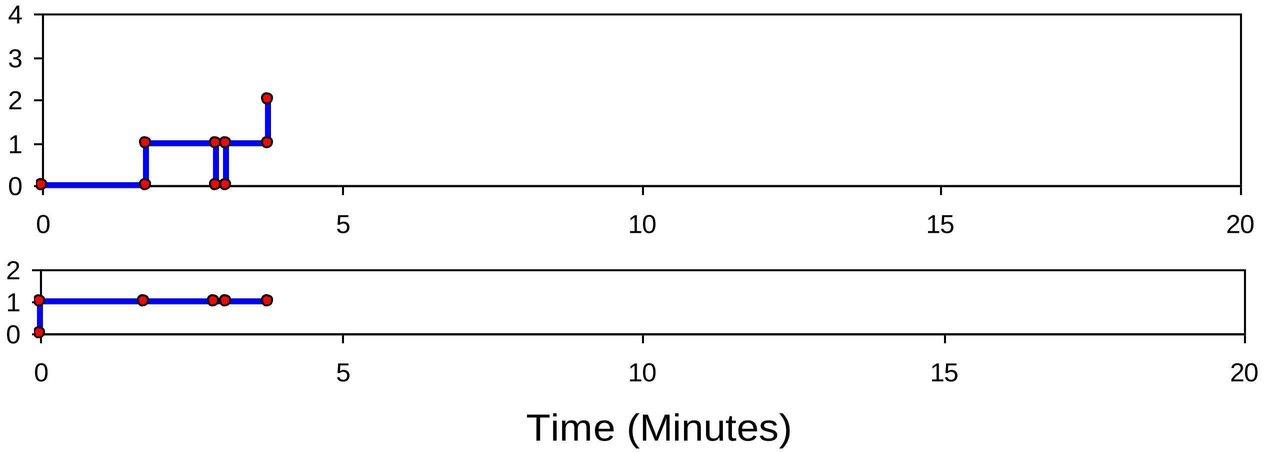
1</td><td>Q(t)

2</td><td>Arrival times of custs. in queue
(3.79, 3.08)</td><td>Event calendar
[5, 4.41, Arr]
[2, 4.66, Dep]
[–, 20.00, End]</td></tr>
<tr><td>Number of completed waiting times in queue
2</td><td colspan="2">Total of waiting times in queue

1.17</td><td colspan="2">Area under Q(t)

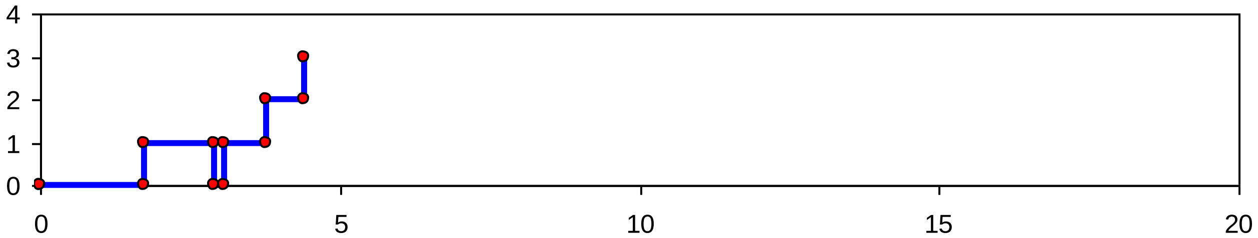
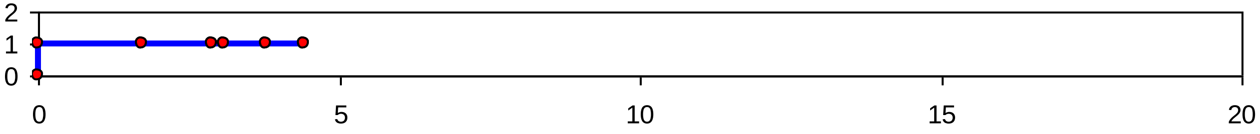
1.88</td><td>Area under B(t)

3.79</td></tr>
<tr><td>Q(t) graph

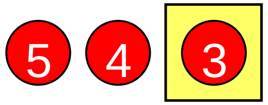
B(t) graph</td><td colspan="5">
</td></tr>
<tr><td>Interarrival times</td><td colspan="5">~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="5">~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Simulation by Hand: *t* = 4.41, Arrival of Part 5

<table>
<tr><td>System
5 4 3 2</td><td>Clock
4.41</td><td>B(t)
1</td><td>Q(t)
3</td><td>Arrival times of custs. in queue
(4.41, 3.79, 3.08)</td><td>Event calendar
[2, 4.66, Dep]
[6, 18.69, Arr]
[–, 20.00, End]</td></tr>
<tr><td>Number of completed waiting times in queue
2</td><td colspan="3">Total of waiting times in queue
1.17</td><td>Area under Q(t)
3.12</td><td>Area under B(t)
4.41</td></tr>
<tr><td>Q(t) graph

B(t) graph</td><td colspan="5"></td></tr>
<tr><td>Interarrival times</td><td colspan="5">~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="5">~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

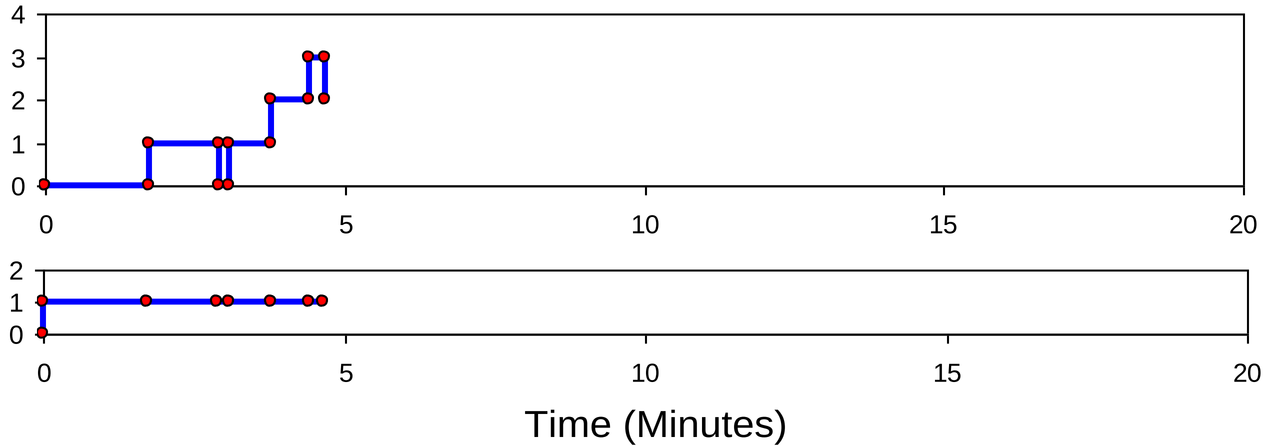
Simulation by Hand: t = 4.66, Departure of Part 2

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
	4.66	1	2	(4.41, 3.79)	[3, 8.05, Dep] [6, 18.69, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
3	2.75	3.87	4.66

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

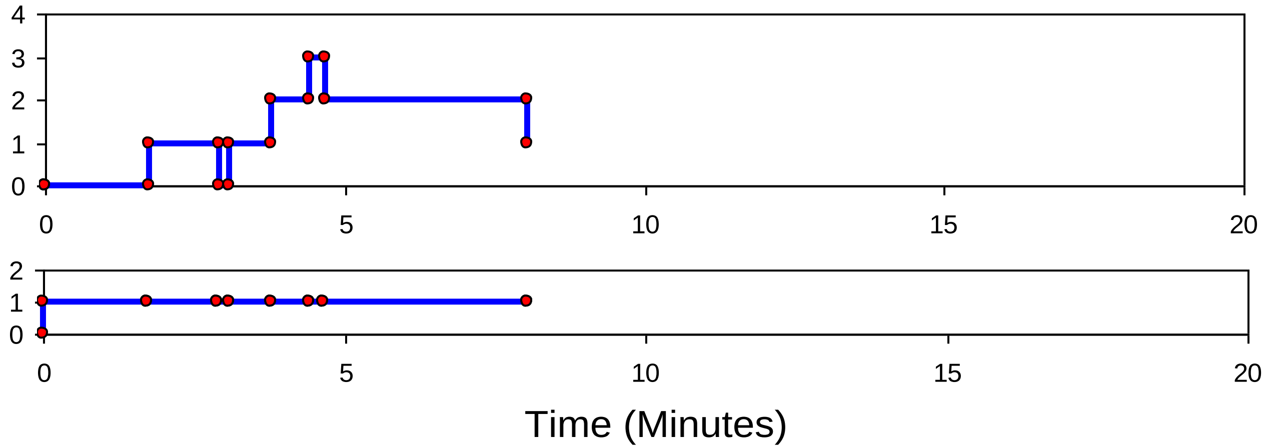
Simulation by Hand: t = 8.05, Departure of Part 3

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
5 4	8.05	1	1	(4.41)	[4, 12.57, Dep] [6, 18.69, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
4	7.01	10.65	8.05

$Q(t)$ graph

$B(t)$ graph



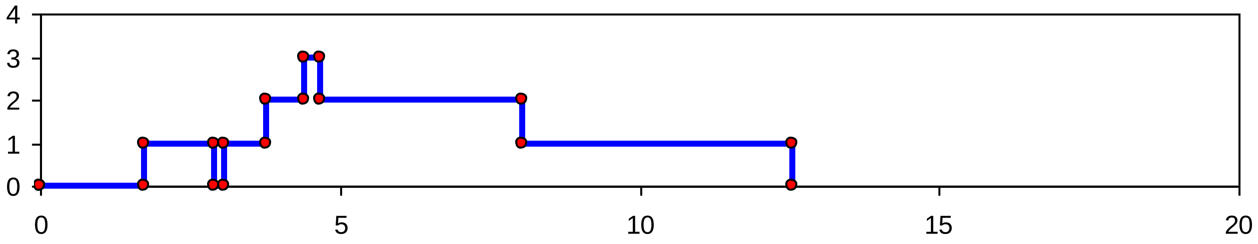
Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand: t = 12.57, Departure of Part 4

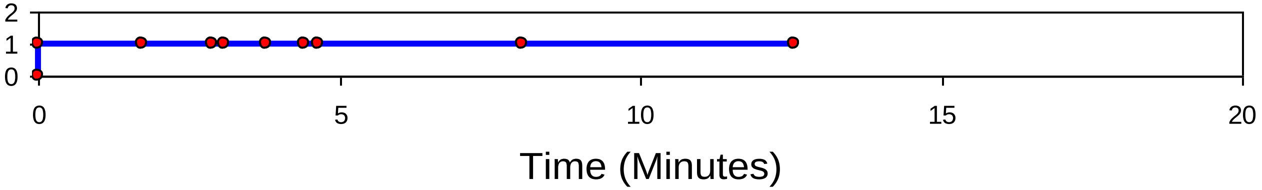
System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
5	12.57	1	0	()	[5, 17.03, Dep] [6, 18.69, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
5	15.17	15.17	12.57

Q(*t*) graph



B(*t*) graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...

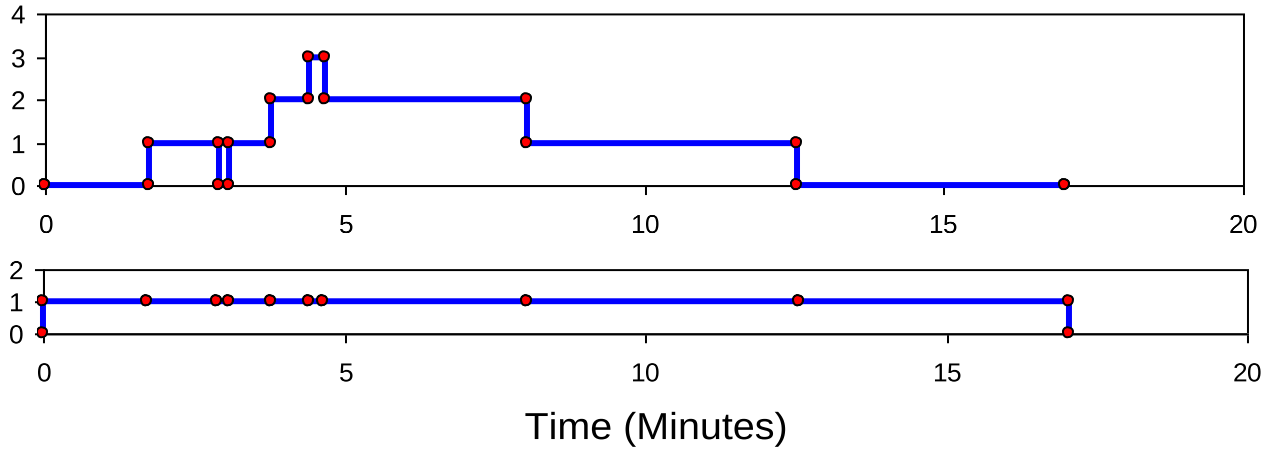
Simulation by Hand: t = 17.03, Departure of Part 5

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
	17.03	0	0	()	[6, 18.69, Arr] [–, 20.00, End]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
5	15.17	15.17	17.03

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...

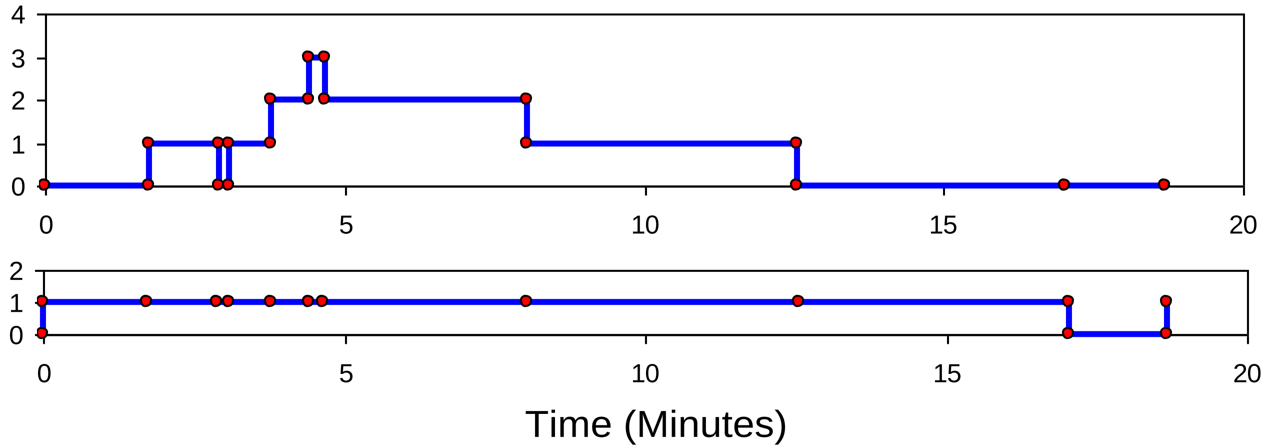
Simulation by Hand: t = 18.69, Arrival of Part 6

System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
	18.69	1	0	()	[7, 19.39, Arr] [–, 20.00, End] [6, 23.05, Dep]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
6	15.17	15.17	17.03

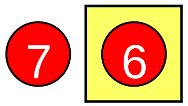
$Q(t)$ graph

$B(t)$ graph



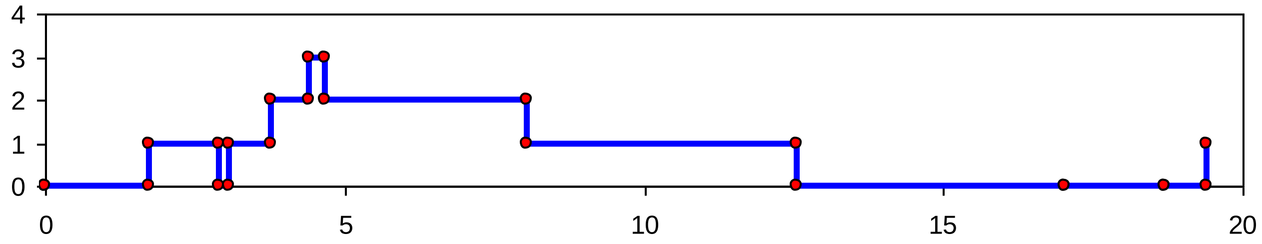
Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand: t = 19.39, Arrival of Part 7

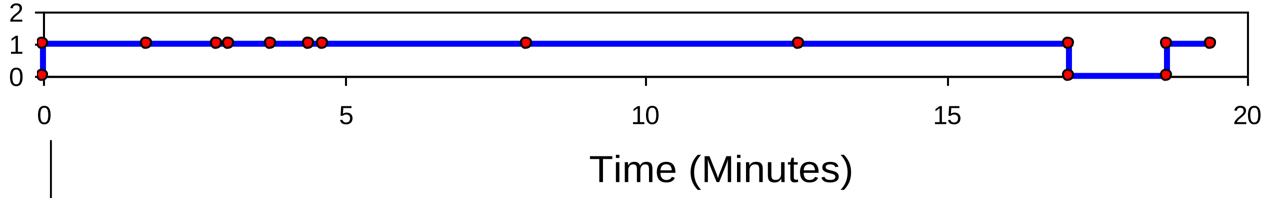
System	Clock	$B(t)$	$Q(t)$	Arrival times of custs. in queue	Event calendar
	19.39	1	1	(19.39)	[–, 20.00, End] [6, 23.05, Dep] [8, 34.91, Arr]

Number of completed waiting times in queue	Total of waiting times in queue	Area under $Q(t)$	Area under $B(t)$
6	15.17	15.17	17.73

$Q(t)$ graph



$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...

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Simulation by Hand: t = 20.00, The End

<table>
<tr><td>System
7 6</td><td>Clock

20.00</td><td>B(t)

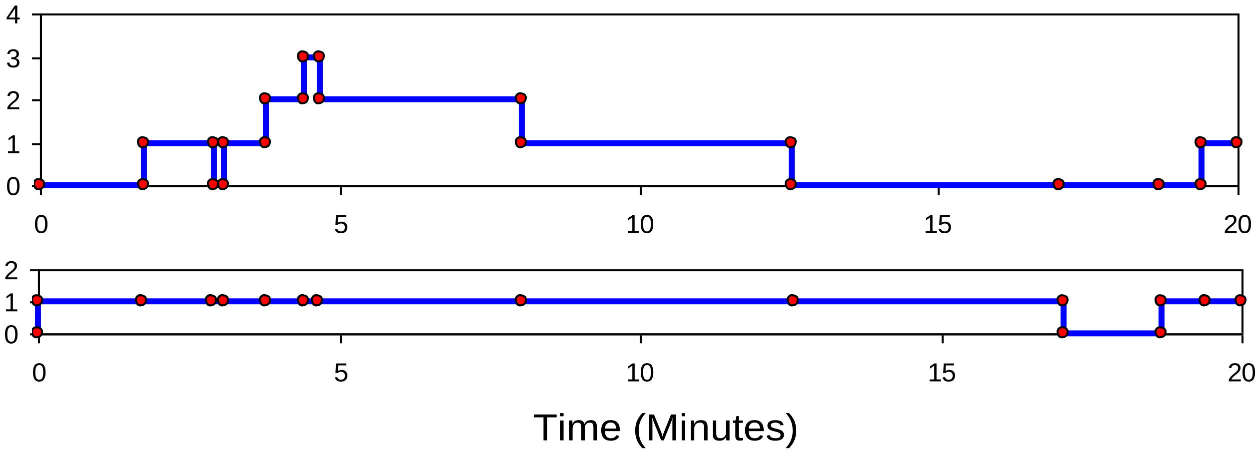
1</td><td>Q(t)

1</td><td>Arrival times of custs. in queue
(19.39)</td><td>Event calendar
[6, 23.05, Dep]
[8, 34.91, Arr]</td></tr>
<tr><td>Number of completed waiting times in queue
6</td><td colspan="2">Total of waiting times in queue

15.17</td><td colspan="2">Area under Q(t)

15.78</td><td>Area under B(t)

18.34</td></tr>
<tr><td>Q(t) graph

B(t) graph</td><td colspan="5">
</td></tr>
<tr><td>Interarrival times</td><td colspan="5">~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, ~~15.52~~, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="5">~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Simulation by Hand: Finishing Up

- Average waiting time in queue:

$$\frac{\text{Total of times in queue}}{\text{No. of times in queue}} = \frac{15.17}{6} = 2.53 \text{ minutes per part}$$

- Time-average number in queue:

$$\frac{\text{Area under } Q(t) \text{ curve}}{\text{Final clock value}} = \frac{15.78}{20} = 0.79 \text{ part}$$

- Utilization of drill press:

$$\frac{\text{Area under } B(t) \text{ curve}}{\text{Final clock value}} = \frac{18.34}{20} = 0.92 \text{ (dimensionless)}$$

Complete Record of the Hand Simulation

Just-Finished Event			Variables		Attributes		Statistical Accumulators									Event Calendar
Entity No.	Time t	Event Type	$Q(t)$	$B(t)$	Arrival Times: (In Queue)	In Service	P	N	ΣWQ	WQ^*	ΣTS	TS^*	$\int Q$	Q^*	$\int B$	[Entity No., Time, Type]
–	0.00	Init	0	0	( )	–	0	0	0.00	0.00	0.00	0.00	0.00	0	0.00	[1, 0.00, Arr] [–, 20.00, End]
1	0.00	Arr	0	1	( )	0.00	0	1	0.00	0.00	0.00	0.00	0.00	0	0.00	[2, 1.73, Arr] [1, 2.90, Dep] [–, 20.00, End]
2	1.73	Arr	1	1	(1.73)	0.00	0	1	0.00	0.00	0.00	0.00	0.00	1	1.73	[1, 2.90, Dep] [3, 3.08, Arr] [–, 20.00 End]
1	2.90	Dep	0	1	( )	1.73	1	2	1.17	1.17	2.90	2.90	1.17	1	2.90	[3, 3.08, Arr] [2, 4.66, Dep] [–, 20.00, End]
3	3.08	Arr	1	1	(3.08)	1.73	1	2	1.17	1.17	2.90	2.90	1.17	1	3.08	[4, 3.79, Arr] [2, 4.66, Dep] [–, 20.00, End]
4	3.79	Arr	2	1	(3.79, 3.08)	1.73	1	2	1.17	1.17	2.90	2.90	1.88	2	3.79	[5, 4.41, Arr] [2, 4.66, Dep] [–, 20.00, End]
5	4.41	Arr	3	1	(4.41, 3.79, 3.08)	1.73	1	2	1.17	1.17	2.90	2.90	3.12	3	4.41	[2, 4.66, Dep] [6, 18.69, Arr] [–, 20.00, End]
2	4.66	Dep	2	1	(4.41, 3.79)	3.08	2	3	2.75	1.58	5.83	2.93	3.87	3	4.66	[3, 8.05, Dep] [6, 18.69, Arr] [–, 20.00, End]
3	8.05	Dep	1	1	(4.41)	3.79	3	4	7.01	4.26	10.80	4.97	10.65	3	8.05	[4, 12.57, Dep] [6, 18.69, Arr] [–, 20.00, End]
4	12.57	Dep	0	1	( )	4.41	4	5	15.17	8.16	19.58	8.78	15.17	3	12.57	[5, 17.03, Dep] [6, 18.69, Arr] [–, 20.00, End]
5	17.03	Dep	0	0	( )	–	5	5	15.17	8.16	32.20	12.62	15.17	3	17.03	[6, 18.69, Arr] [–, 20.00 End]
6	18.69	Arr	0	1	( )	18.69	5	6	15.17	8.16	32.20	12.62	15.17	3	17.03	[7, 19.39, Arr] [–, 20.00, End] [6, 23.05, Dep]
7	19.39	Arr	1	1	(19.39)	18.69	5	6	15.17	8.16	32.20	12.62	15.17	3	17.73	[–, 20.00, End] [6, 23.05, Dep] [8, 34.91, Arr]
–	20.00	End	1	1	(19.39)	18.69	5	6	15.17	8.16	32.20	12.62	15.78	3	18.34	[6, 23.05, Dep] [8, 34.91, Arr]

Goals of the Study: Output Performance Measures

- *Total production* of parts over the run (*P*)
- *Average waiting time* of parts in queue:

$$\frac{\sum_{i=1}^{N} WQ_i}{N}$$

N = no. of parts completing queue wait
WQ_i = waiting time in queue of ith part
Know: $WQ_1 = 0$ (why?)
$N \geq 1$ (why?)

- *Maximum waiting time* of parts in queue:

$$\max_{i=1,\ldots,N} WQ_i$$

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Goals of the Study: Output Performance Measures (cont'd.)

- *Time-average number of parts in queue*:

$$\frac{\int_0^{20} Q(t)\, dt}{20}$$

$Q(t)$ = number of parts in queue at time t

- *Maximum number of parts in queue*: $\max_{0 \le t \le 20} Q(t)$

- *Average* and *maximum total time in system* of parts (a.k.a. *cycle time*):

$$\frac{\sum_{i=1}^{P} TS_i}{P}, \quad \max_{i=1,\ldots,P} TS_i$$

TS_i = time in system of part i

Goals of the Study:
Output Performance Measures (cont'd.)

- *Utilization* of the machine (proportion of time busy)

$$\frac{\int_0^{20} B(t)\,dt}{20}, \quad B(t) = \begin{cases} 1 & \text{if the machine is busy at time } t \\ 0 & \text{if the machine is idle at time } t \end{cases}$$

- Many others possible (information overload?)