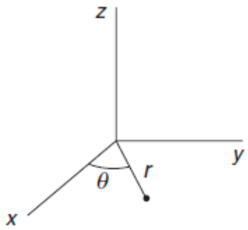
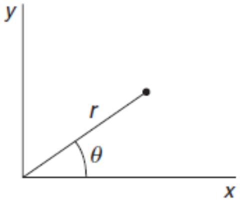
Coordinate System: Plane Polar Coordinates

- Whenever we are dealing with a system with circular symmetry, such as a particle executing circular motion, it is much more convenient to use a different set of coordinates called “Plane Polar Coordinates.



- It is a 2D coordinate system equivalent to Cartesian 2D:(x, y)
- Location of a point specified by (r, θ)
- r is distance from the origin
- θ is the angle which line joining the point to the origin, makes with the x axis.

Plane polar coordinates contd.

- 

  Clearly

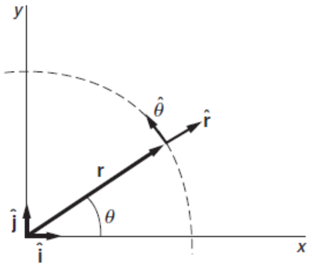
$$x = r\cos\theta$$
$$y = r\sin\theta$$

- Easy to deduce from above

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Plane polar coordinates ...

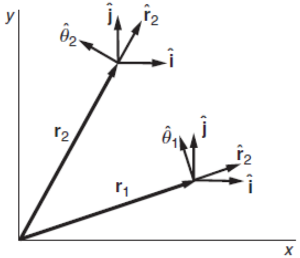
- Unit vectors denoted as $\hat{r}$ and $\hat{\boldsymbol{\theta}}$ are shown below



- Direction of $\hat{r}$ is the one in which r increases, but θ is held fixed.
- Similarly $\hat{\boldsymbol{\theta}}$ is in the direction in which θ increases, but r is held fixed
- Yet $\hat{r}$ and $\hat{\boldsymbol{\theta}}$ are mutually perpendicular, just like $\hat{i}$ and $\hat{j}$.
- Also note that unlike Cartesian coordinates, (r, θ) have different dimensions.
- r has dimensions of length, while θ is dimensionless

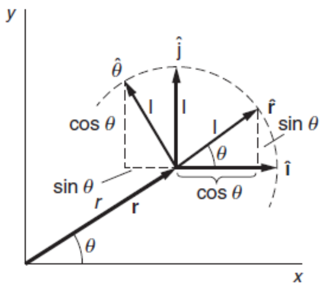
Plane Polar Coordinates....

- In Cartesian coordinates, directions of unit vectors $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$ and $\hat{\mathrm{k}}$ are fixed in space, and same everywhere
- This is not true in plane polar coordinates



Relation between plane polar and Cartesian unit vectors

- Consider the figure below



- From above, it is easy to derive the relationship between two sets of unit vectors

$$\hat{\mathrm{r}} = \cos\theta\hat{\mathrm{i}} + \sin\theta\hat{\mathrm{j}}$$
$$\hat{\boldsymbol{\theta}} = -\sin\theta\hat{\mathrm{i}} + \cos\theta\hat{\mathrm{j}}$$

Polar-Cartesian Relationship contd...

- And, the inverse relationship

$$\hat{\mathrm{i}} = \cos\theta\,\hat{\mathrm{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$$
$$\hat{\mathrm{j}} = \sin\theta\,\hat{\mathrm{r}} + \cos\theta\,\hat{\boldsymbol{\theta}}$$

Polar-Cartesian Comparison

- Position vector of an arbitrary point P in two coordinate systems is given by

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$
$$\mathbf{r} = r\hat{\mathbf{r}}$$

- Infinitesimal displacement $d\mathbf{r}$ is given by

$$d\mathbf{r} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}$$
$$d\mathbf{r} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}}$$