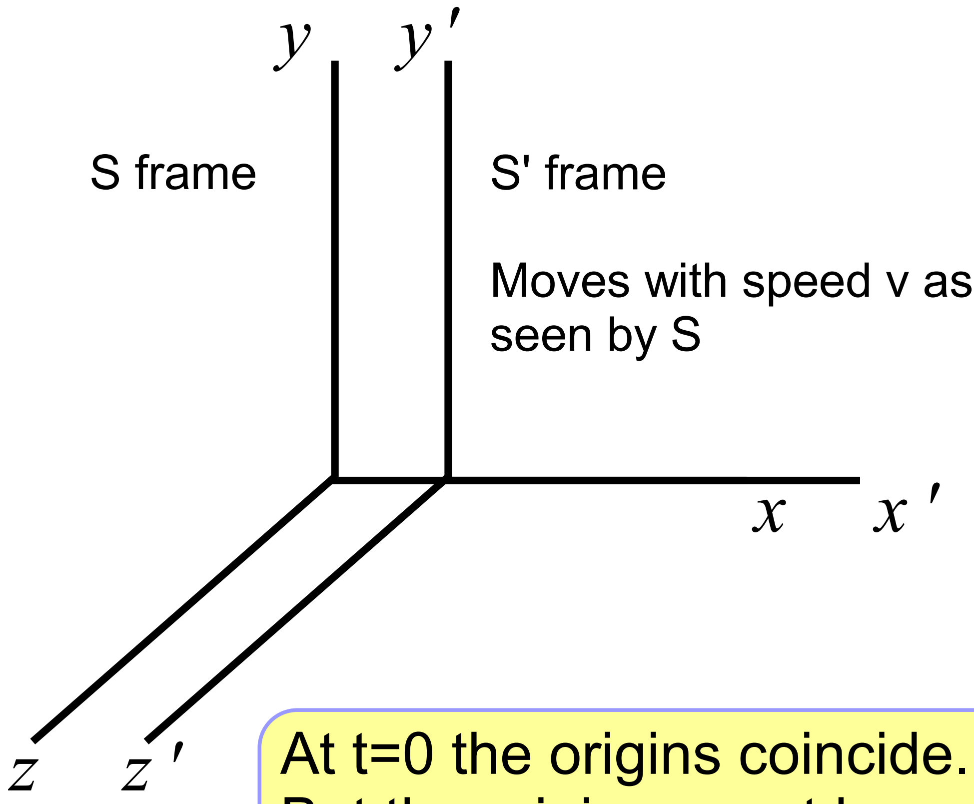
The postulates of special relativity & Lorenz transformation

1. The laws of physics are same in all intertial frames.
2. The speed of light in vacuum (c) is same in all inertial frames.



At t=0 the origins coincide.
But the origin cannot be a special point.
This can be ensured only if the transformation is LINEAR

The postulates of special relativity & Lorenz transformation

$$
\begin{array}{ccccccccc}
x' & = & a_{11}x & + & \cancel{a_{12}}y & + & \cancel{a_{13}}z & + & a_{14}t \\
y' & = & \cancel{a_{21}}x & + & a_{22}y & + & \cancel{a_{23}}z & + & \cancel{a_{24}}t \\
z' & = & \cancel{a_{31}}x & + & \cancel{a_{32}}y & + & a_{33}z & + & \cancel{a_{34}}t \\
t' & = & a_{41}x & + & \cancel{a_{42}}y & + & \cancel{a_{43}}z & + & a_{44}t
\end{array}
$$

A point on the x-z plane goes over to a point on the x'z' plane

A point on the x-y plane goes over to a point on the x'y' plane

Clocks placed symmetrically below and above the x'y' plane should not give different time. Up and down can not be different!

Clocks placed symmetrically below and above the x'z' plane should not give different time. Up and down can not be different!

x=vt and x'=0 must coincide at all times

Different colours are used to indicate which argument makes which matrix element zero.

The postulates of special relativity & Lorenz transformation

A rod of length L stands along the y axis in S frame.
S' measures the length to be a_{22}
Bring the rod to rest in S'.
S measures the length to be $1/a_{22}$

equivalence of reference frames requires that $a_{22} = \frac{1}{a_{22}} = 1$

Same argument holds for $a_{33} = 1$

The equations reduce to

$$
\begin{aligned}
x' &= a_{11}(x - vt) \\
y' &= y \\
z' &= z \\
t' &= a_{41}x + a_{44}t
\end{aligned}
$$

Three unknowns, so we need three equations.

The postulates of special relativity & Lorenz transformation

1. Consider a light pulse sent along +x at time t=0 by S
   At time t, the pulse has co-ordinates (ct, t)

$$x' = a_{11}(ct - vt)$$

$$t' = a_{41}ct + a_{44}t$$

$$\text{But } \frac{x'}{t'} = c \Rightarrow \boxed{\frac{a_{11}}{a_{41}c + a_{44}} = \frac{c}{c - v}}$$

2. Consider a light pulse sent along -x at time t=0 by S
   At time t, the pulse has co-ordinates (-ct, t)

$$x' = a_{11}(-ct - vt)$$

$$t' = -a_{41}ct + a_{44}t$$

$$\text{But } \frac{x'}{t'} = -c \Rightarrow \boxed{\frac{a_{11}}{-a_{41}c + a_{44}} = \frac{c}{c + v}}$$

3. Consider a light pulse sent along +y by S
   At time t the pulse has co-ordinates (0,ct,0,t)

$$
\begin{aligned}
x' &= a_{11}(0 - vt) \\
y' &= ct \\
z' &= 0 \\
t' &= a_{41}.0 + a_{44} t
\end{aligned}
$$

Speed measured by S'

$$\boxed{\frac{\sqrt{a_{11}^2 v^2 + c^2}}{a_{44}} = c}$$

Solving for the 3 unknowns

$$a_{11} = a_{44} = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$a_{41} = -\frac{v/c^2}{\sqrt{1 - v^2/c^2}}$$

$$
\begin{aligned}
x' &= \frac{x - vt}{\sqrt{1 - v^2/c^2}} \\
y' &= y \\
z' &= z \\
t' &= \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}
\end{aligned}
$$

Invert these equations. What result should you expect?
Do you see that happening?

How does one measure length ? Why does time get involved in this ?

Measure the co-ordinates of two ends of a stick at the same instant of time.

Is "at the same instant" (simultaneous) an unambiguous notion?

In an inertial frame, yes . But NOT across different frames.

Two clocks may be synchronised in frame S. To S' they will not be so.

A rod of length L_0 is at rest in S' : between x_2 and $x_1{}'$

S' measures the rest frame length $L_0 = x_2{}' - x_1{}'$

S tries to measure the length

he must measure the co-incience of the endpoints at the same instant.

$$\left.\begin{aligned} x_1{}' &= \frac{x_1 - vt}{\sqrt{1-\beta^2}} \\ x_2{}' &= \frac{x_2 - vt}{\sqrt{1-\beta^2}} \end{aligned}\right\} \quad L = (x_2{}' - x_1{}')\sqrt{1-\beta^2} = L_0\sqrt{1-\beta^2}$$

Let's do it another way....

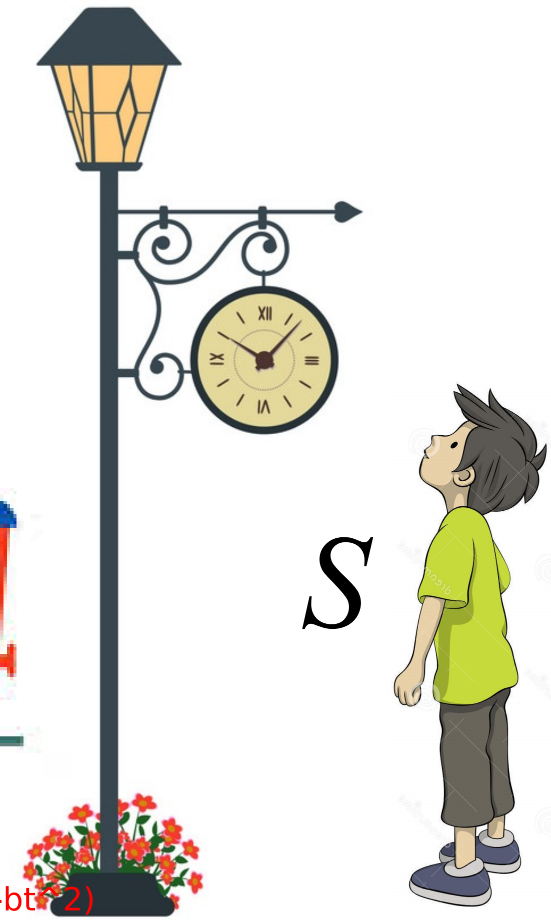
A train moving with a speed v crosses a lamp-post in time t.

Q: What is its length ?

A: classically it is vt

S'

S



In the boy's frame S, which is moving right wrt to S' at speed v, the 2 events are occuring at the post (same point), So he sees the Del t shortened. S' sees it dialated. So S sees, L=v.delt=v. delt' sqrt(1-beta^2) and delt'=Lo/v. So L=Lo*sqrt(1-bt^2)

In S' tip & rear ends are at $x'=0$ and $x'=-L_0$ always

Q: At what time in S does $x'=-L_0$ give $x=0$?

A: $-L_0=\dfrac{0-vt}{\sqrt{1-\beta^2}}$ $\Rightarrow$ for S train's length : $vt=L_0\sqrt{1-\beta^2}$

How does one compare measurement of time intervals ?

Consider a clock at rest in the S' frame at the origin of S' (so x'=0 always)

At t'=0 we have t=0 : What happens when t' =1 (say)

$$t = \frac{t' + vx'/c^2}{\sqrt{1-\beta^2}}$$

$$x' = 0 \quad \text{and} \quad t' = 1$$

$$t = \frac{1}{\sqrt{1-\beta^2}}$$

A clock at rest in S' will appear to run slow when timed by clocks in S.

If the "clock" is a particle with a 1 second lifetime it will appear to live longer when seen by S

This is why many fast moving particles in cosmic rays manage to reach the earth's surface.

The universe is a clock.. ☑
The universe has a clock ☒

"Length contraction" and "time dilation" are consequences of the two postulates of relativity.