$h\nu$

Ex. 1

400 gm, 200 km/h

$$\lambda = \frac{h}{p} = \frac{h}{\dfrac{m_0 u}{\sqrt{1-u^2/c^2}}} \simeq \frac{h}{m_0 u}$$

$h = 6.63 \times 10^{-34}$ J.s

$\lambda \simeq 10^{-35}$ m

$E = h\nu$ Einstine

$E^2 = p^2c^2 + m_0^2 c^4$ Relativity

for $m_0 = 0$ (photon) $E = pc = p\lambda f \Rightarrow p = E/c$

De Broglie, for $m_0 \neq 0$ ptcls $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

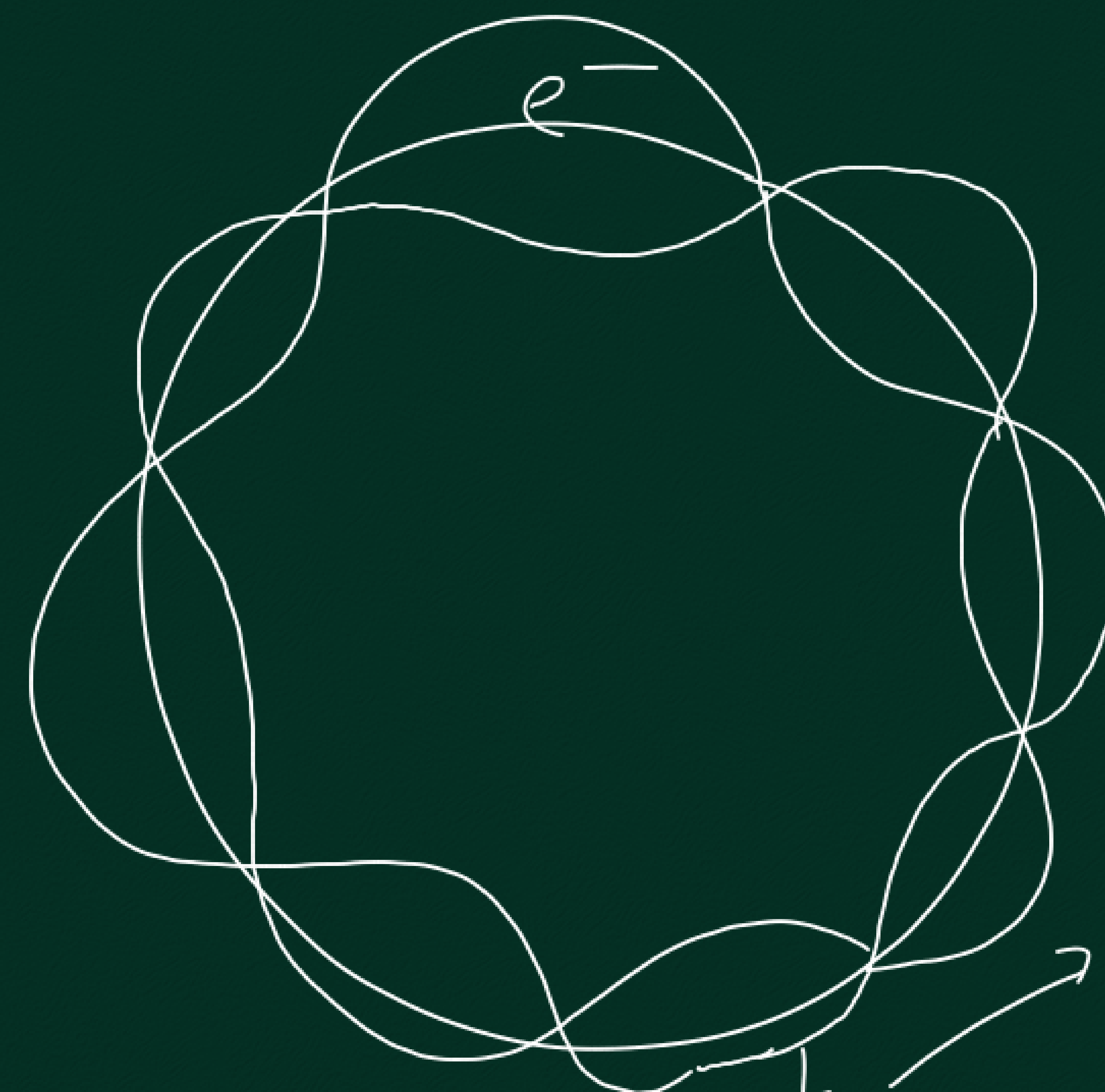
(1923)

$$\lambda = \frac{h}{p} \rightarrow \lambda_e \simeq 3.3\ \text{Å}$$

$$m_0 \sim 9.1 \times 10^{-31}\ \text{kg}$$

$$v \sim 2.2 \times 10^{6}\ \text{m/s.}$$

$$\sqrt{1 - u^2/c^2} \rightarrow \left(\frac{u}{c}\right)^2 \sim 10^{-4}$$

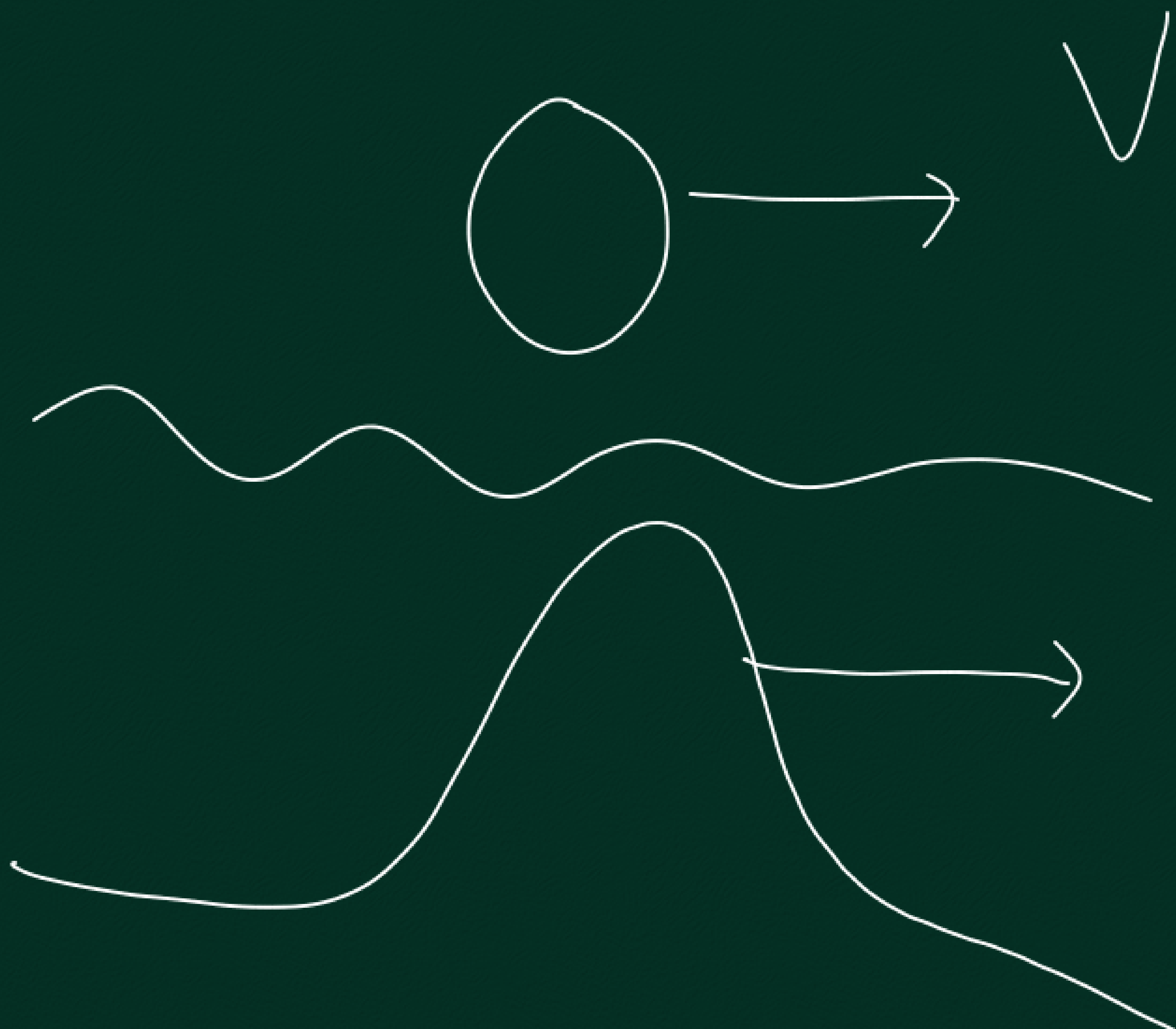


$h/2\pi$

Ang mom is quantized

$$mvr = n\hbar$$

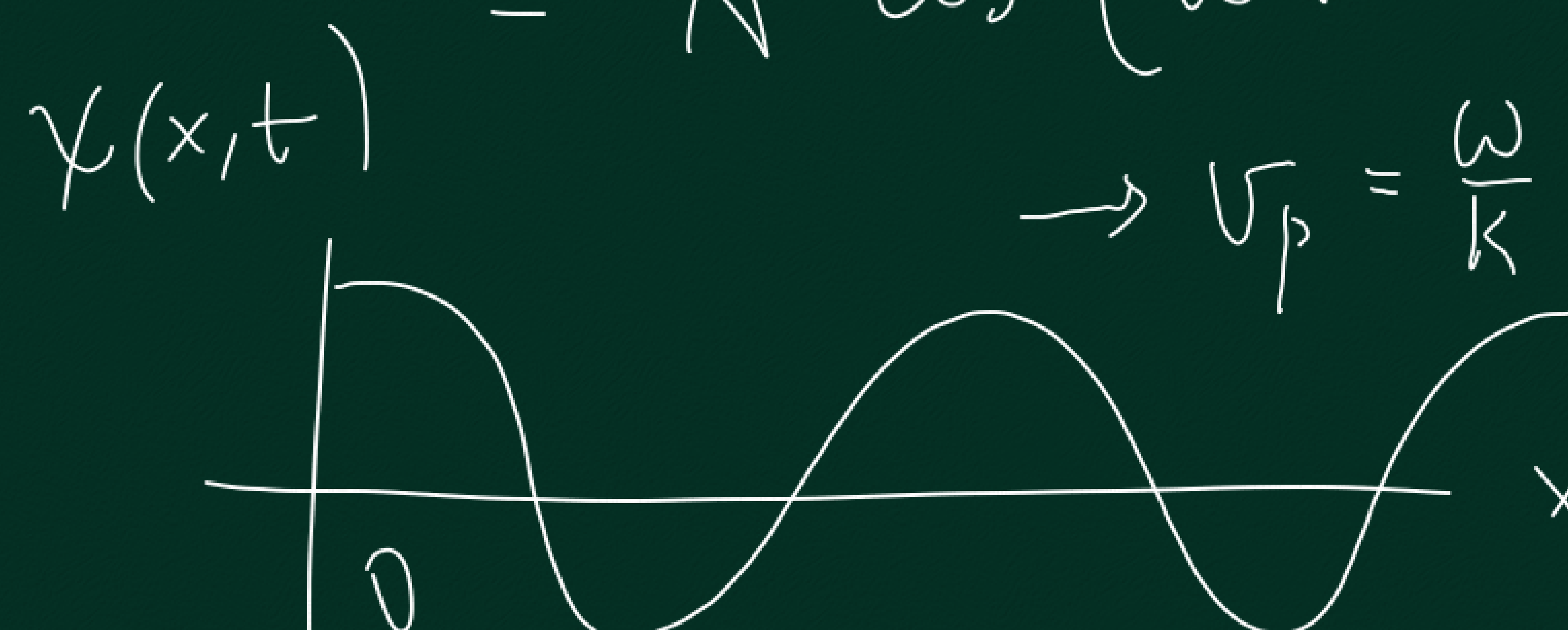
$$2\pi r = n\underbrace{\left(\frac{h}{mv}\right)}_{\lambda}$$

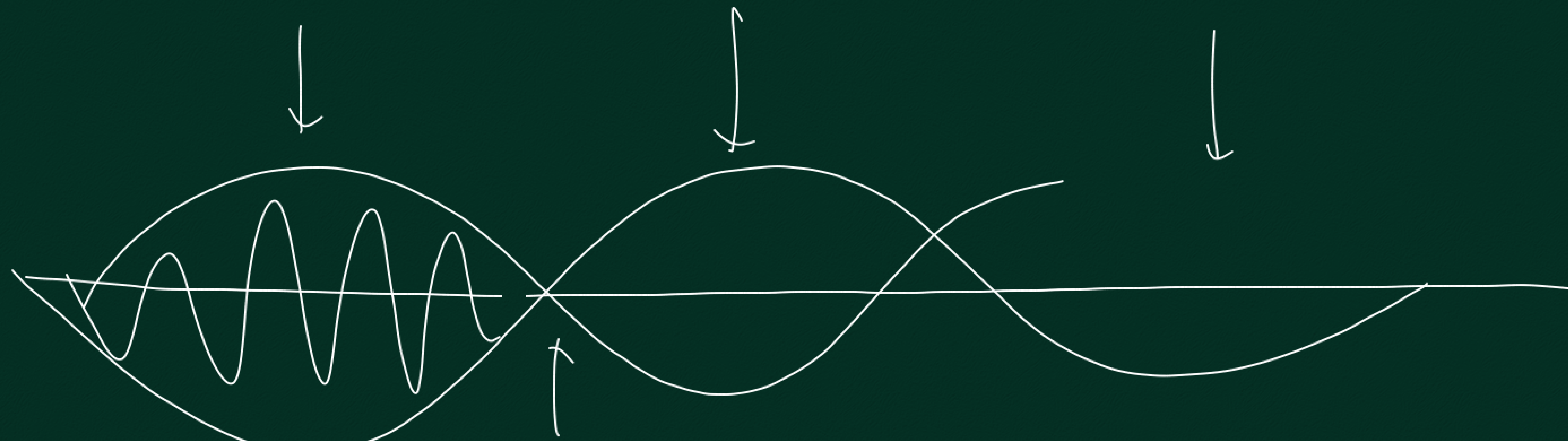


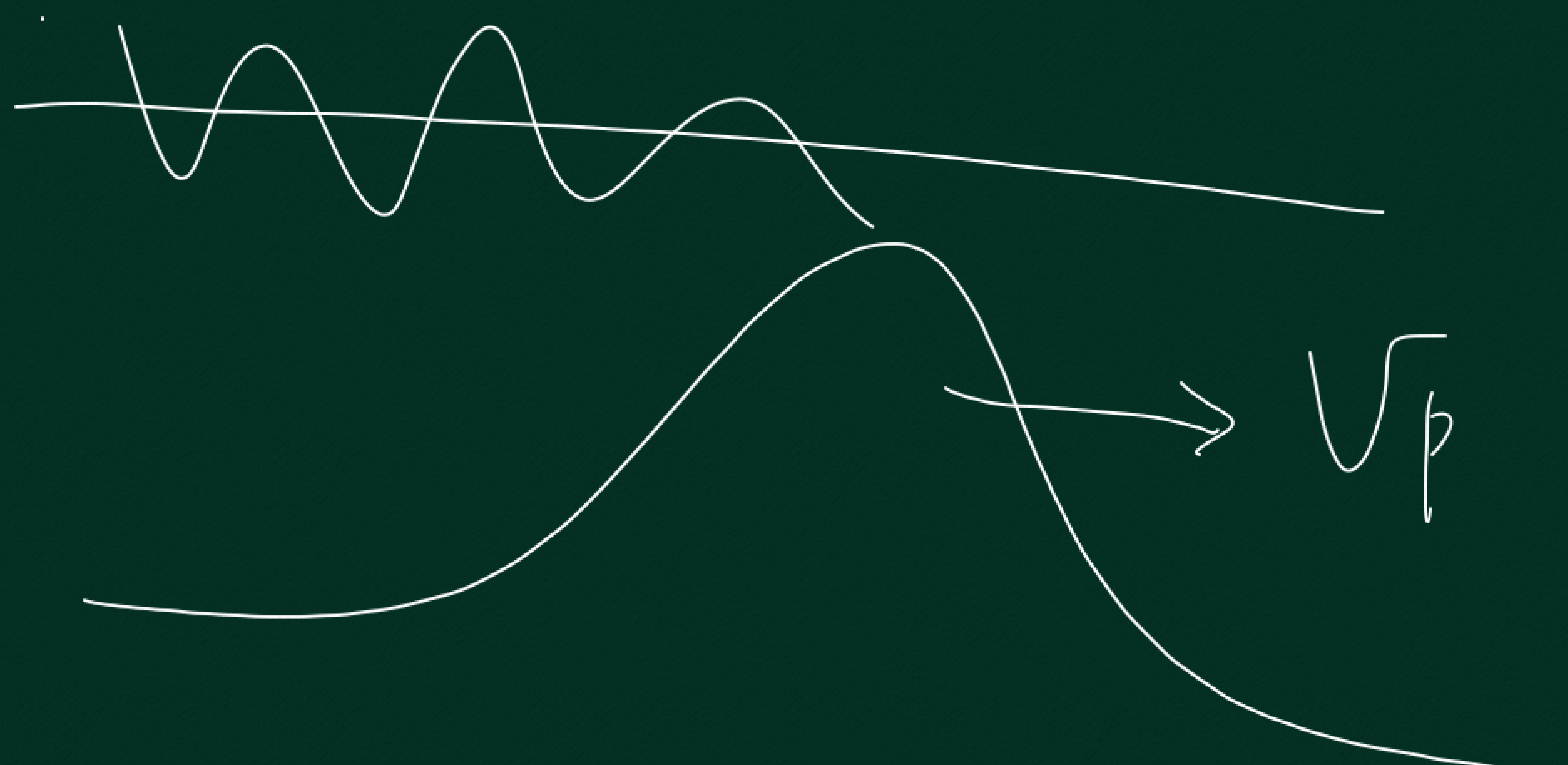
$$\psi(x,t) = A\,\mathrm{Re}\left[e^{i(\omega t - kx + \cancel{\varphi_0})}\right]$$

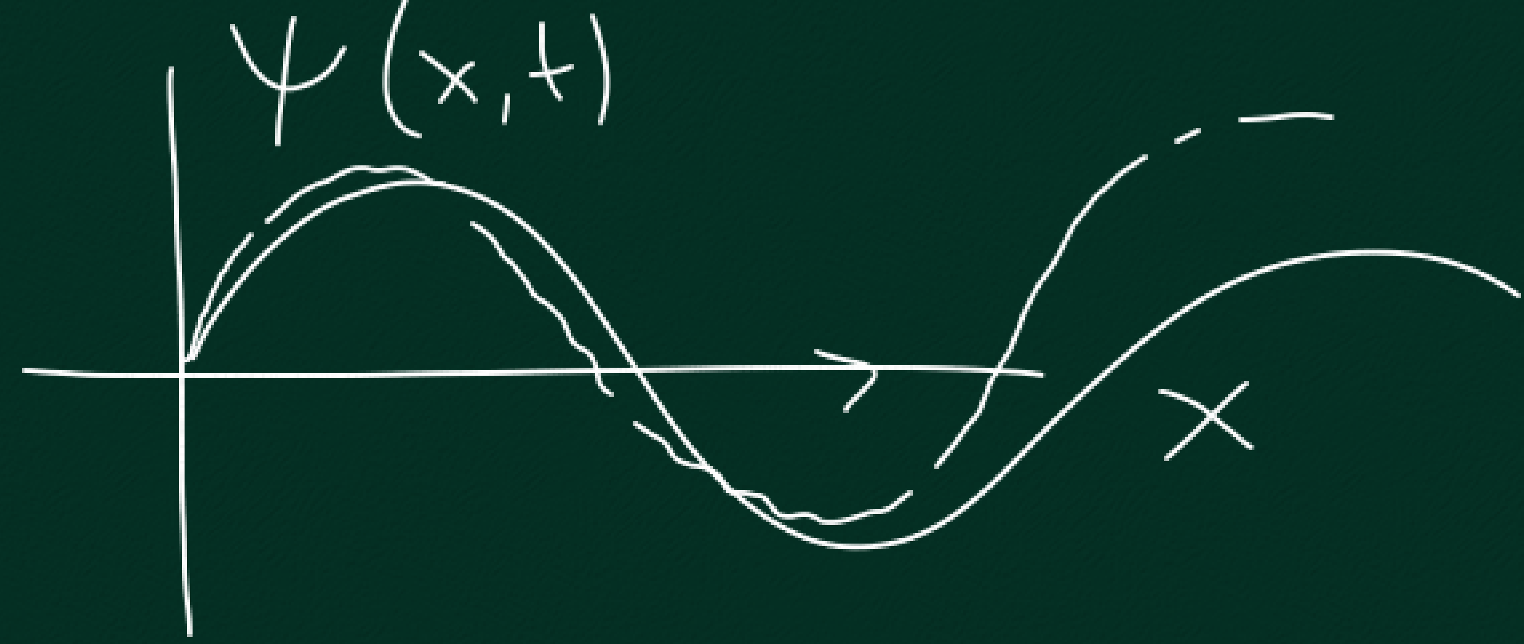
$$= A\cos(\omega t - kx + \cancel{\varphi_0})$$

$\rightarrow v_p = \frac{\omega}{k}$






V_p



$$\Psi = A\,\mathrm{Re}\left[e^{i(\omega_1 t - k_1 x)} + e^{i(\omega_2 t - k_2 x)} \right]$$

$$e^{i(vt-x)}$$

$$\omega_1, k_1 \rightarrow \omega_0 = \frac{\omega_1+\omega_2}{2}$$

$$\omega_2, k_2 \rightarrow k_0 = \frac{k_1+k_2}{2}$$

$$= A\,\mathrm{Re}\left\{ e^{i(\omega_0 t - k_0 x) - i(\delta\omega t - \Delta k x)} + e^{i(\omega_0 - k_0 x) + i(\delta\omega t - \delta k x)} \right\}$$

$$\omega_1 = \omega_0 - \delta\omega$$

$$\omega_2 = \omega_0 + \delta\omega$$

$$k_1 = k_0 - \delta k$$

$$k_2 = k_0 + \delta k$$

$$= A\,\mathrm{Re}\left\{ e^{i(\omega_0 t - k_0 x)} \left\{ e^{-i(\delta\omega t - \delta k x)} + e^{i(\delta\omega t - \delta k x)} \right\} \right\}$$

$$= A\,\mathrm{Re}\, e^{i(\omega_0 t - k_0 x)} \cos(\delta\omega t - \delta k x)$$

$$\cos \delta k (x - Vt)$$

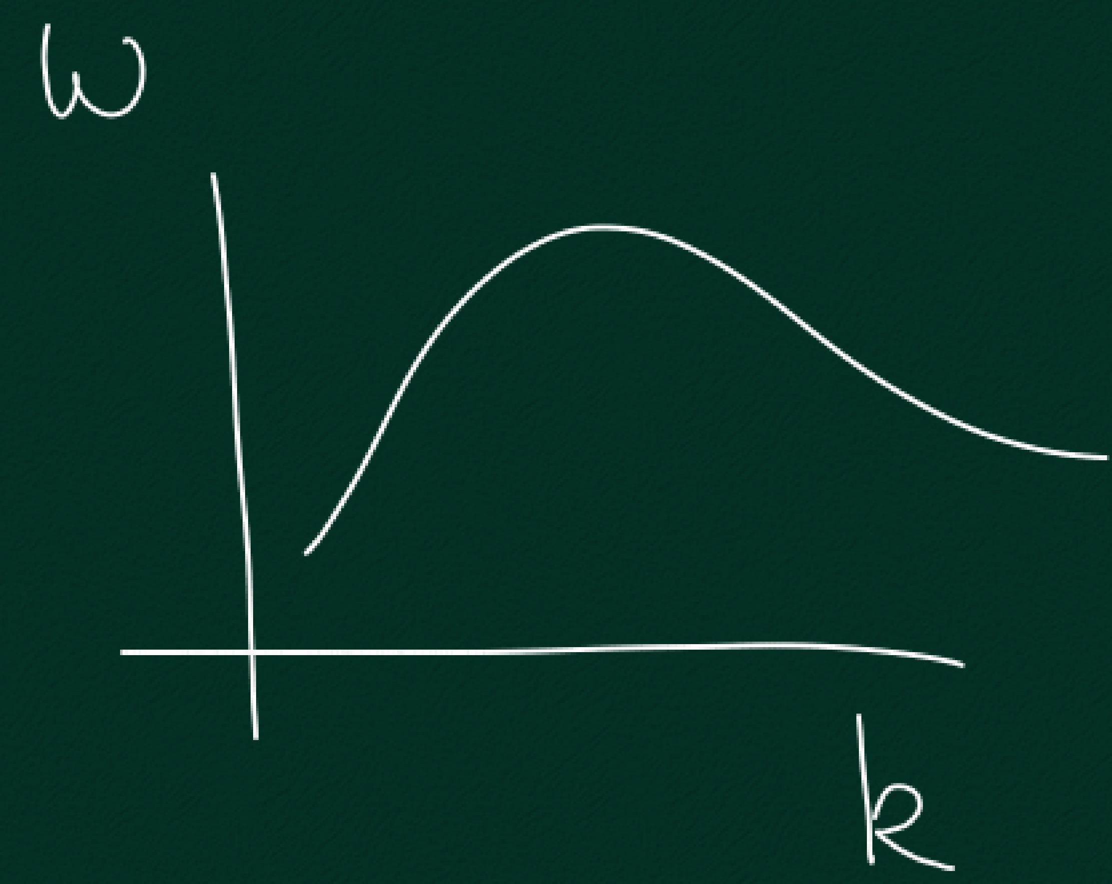
$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\theta} = \cos\theta - i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$v_g = \frac{\Delta \omega}{\Delta k} \to \frac{d\omega}{dk}$$



$$\omega(k)$$

$$e^{i(\omega t - kx - \varphi_0)}$$

$$\omega t - kx - \varphi_0 = \alpha$$

$$\frac{d}{dt} \to \omega - k\frac{dx}{dt} = 0 \Rightarrow v_p = \frac{\omega}{k}$$

$$v_g \longleftrightarrow v_p$$

$$v_g = \frac{d\omega}{dk} = \frac{d\omega}{d\left(\frac{\omega}{v_p}\right)} = \frac{1}{\frac{d}{d\omega}\left(\frac{\omega}{v_p}\right)}$$

$$= \frac{1}{\frac{1}{v_p} - \frac{\omega}{v_p^2}\frac{dv_p}{d\omega}}$$

$$= \frac{v_p}{1 - \frac{\omega}{v_p}\frac{dv_p}{d\omega}} = \frac{v_p}{1 - k\frac{dv_p}{d\omega}}$$

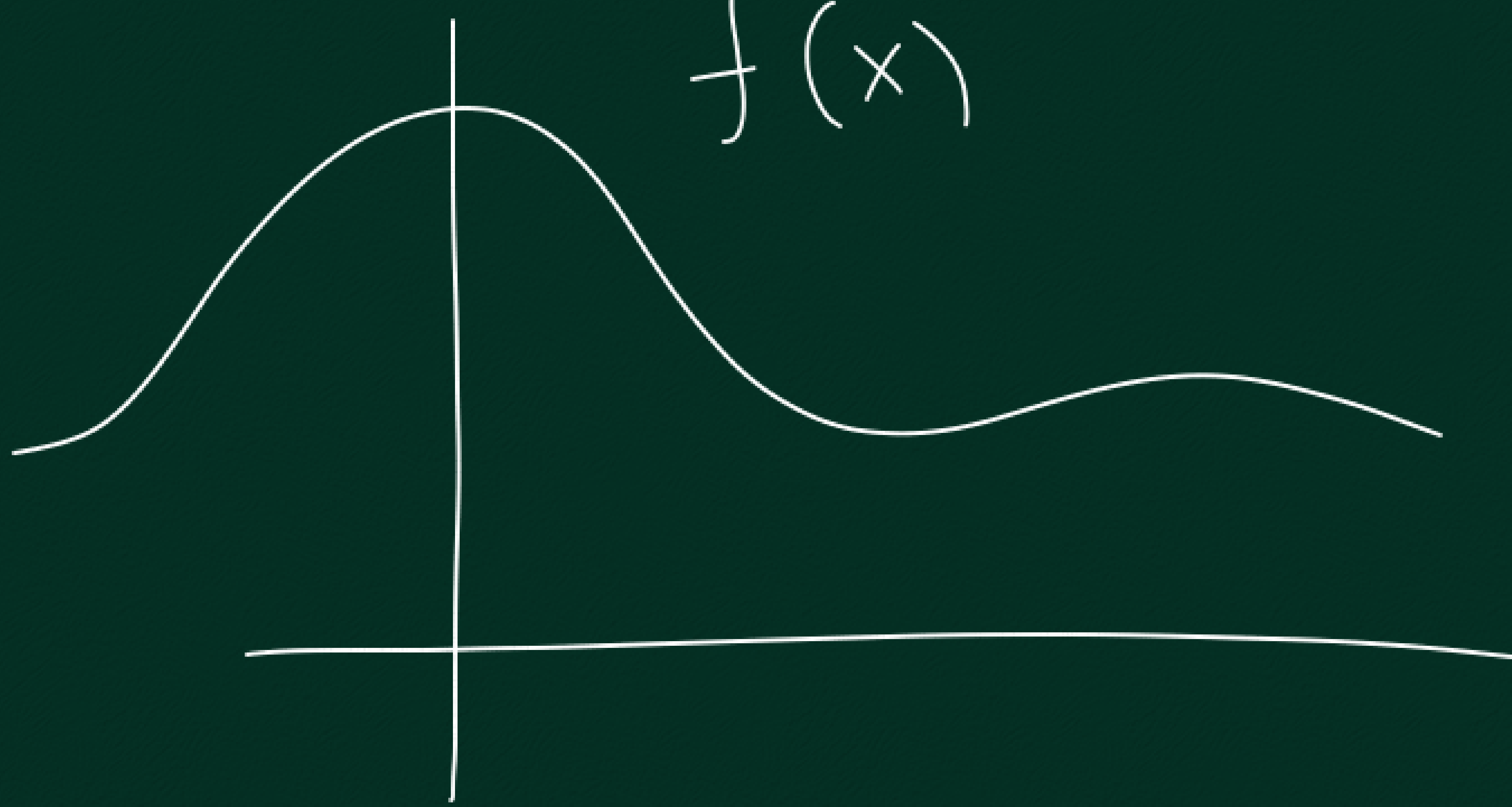
$$= v_g$$

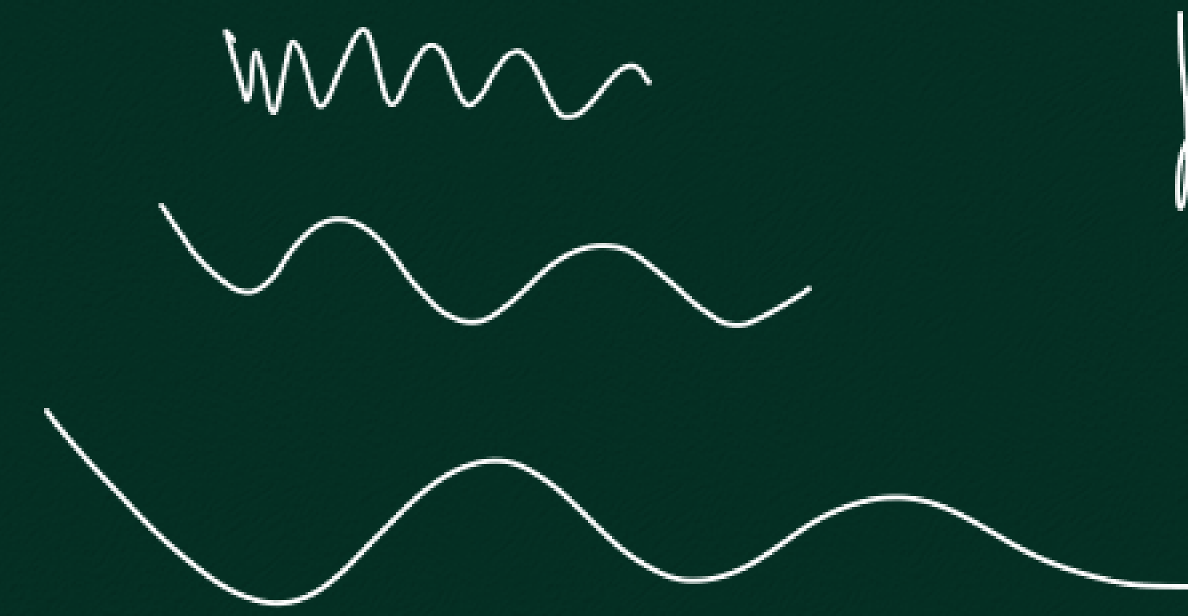
$$v_g = \frac{v_p}{1 - \underbrace{k\,\frac{dv_p}{d\omega}}_{B}}$$

$B < 0 \Rightarrow v_g < v_p$

$B > 0 \Rightarrow v_g > v_p$

$B > 1 \Rightarrow v_g$ has opposite sign wrt v_p



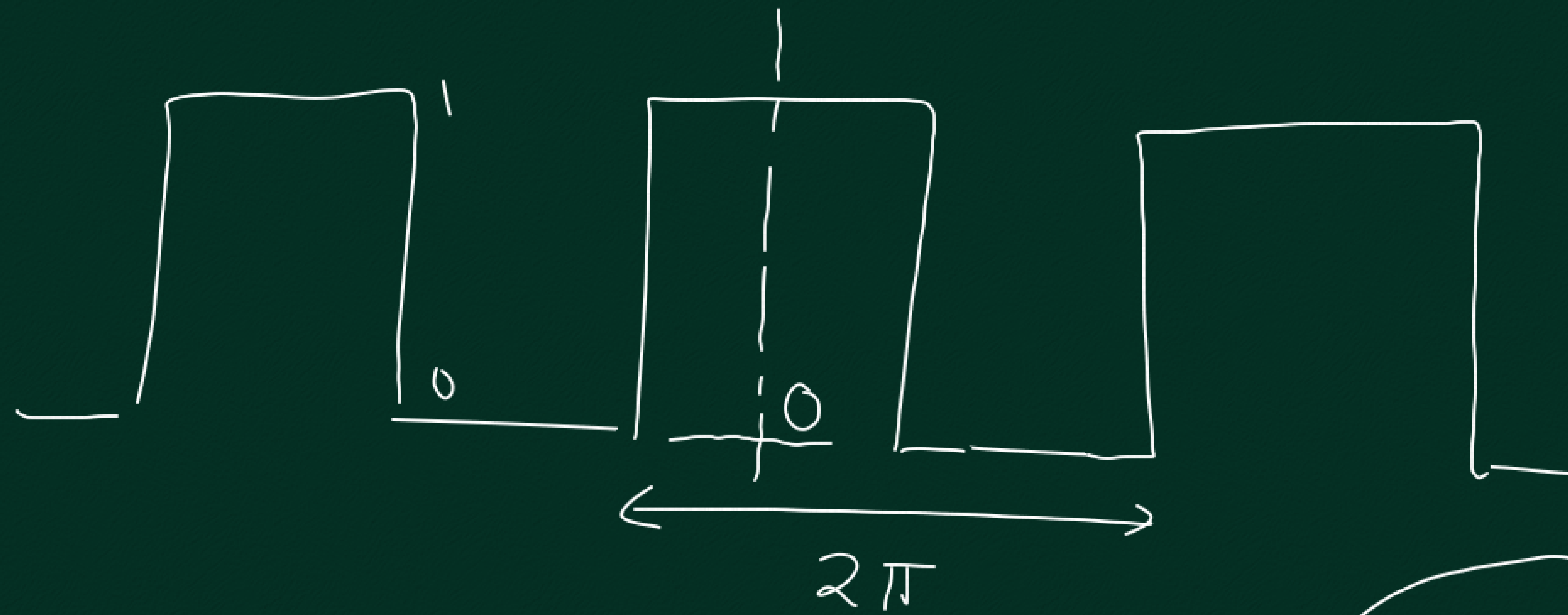


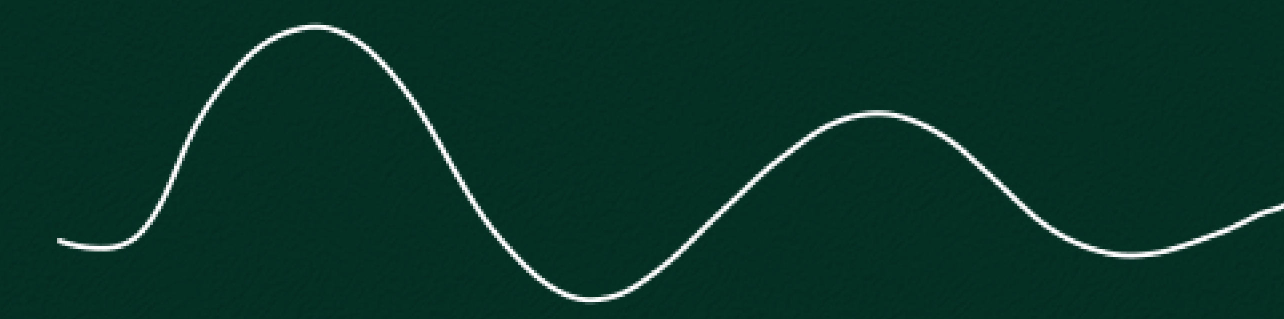
$k = \frac{2\pi}{\lambda}$

$$f(x) = \int_{-\infty}^{\infty} \underbrace{\tilde{f}(k)}_{A(k)} e^{ikx} \, dk$$

$\tilde{f}(k)$ is Fourier transform of $f(x)$

$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{ik_n x}$$ discrete form





$$f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=0}^{\infty} \underbrace{\frac{(-1)^n}{2n+1}}_{a_n} \cos \underbrace{(2n+1)}_{k_n} x$$

$$k_n = \frac{2\pi}{\lambda_n}$$