PH110: Tutorial Sheet 3

This tutorial sheet contains problems related to vector nature of angular velocity, non-inertial frames of reference, and pseudo forces.

1. * A particle is rotating in the xy-plane, along a circular path in counter-clockwise direction, with angular speed ω, about the z-axis.

   (a) Write down the angular velocity of the particle in the vector form, i.e., in terms of components and unit vectors in the inertial frame.

   (b) If the particle is moving along a circle of radius a, write down its position vector $\mathbf{r}(t)$, as a function of time, assuming that $\mathbf{r}(0) = a\hat{\mathbf{i}}$

   (c) Express its velocity both in Cartesian, and plane polar coordinates

   (d) Compute the acceleration of the particle both in Cartesian, and plane polar coordinates

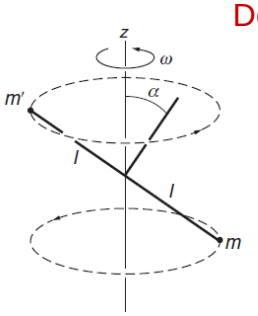
2. A vector $\mathbf{A}$ of magnitude a is rotating in the yz plane in a counter-clockwise manner, with a uniform angular velocity ω. It is given that $\mathbf{A}(t = 0) = a\hat{\mathbf{j}}$.

   (a) Obtain $\mathbf{A}(t)$, as a function of time.

   (b) Show that $\frac{d\mathbf{A}}{dt}$ calculated directly, and computed using $\boldsymbol{\omega} \times \mathbf{A}$, are the same.

3. * Consider a simple rigid body consisting of two particles of mass m separated by a massless rod of length $2l$. The midpoint of the rod is attached to a vertical axis that rotates at angular speed ω around the z axis. The rod is skewed at angle α, as shown in the figure.

   Don't assume the masses to be at some specific positions.



   (a) Calculate the angular momentum $\mathbf{L}(t) = \mathbf{r} \times \mathbf{p}$ of the system, in Cartesian coordinates.

   (b) Verify that $\frac{d\mathbf{L}}{dt}$ is same as $\boldsymbol{\omega} \times \mathbf{L}$.

4. * A cylinder of mass M and radius R rolls without slipping on a plank which is moving with an acceleration $\mathbf{A}$. Calculate the acceleration of the cylinder by analyzing the problem both in the inertial frame and the non-inertial frames. You can use the fact that moment of inertial of a cylinder about its axis is $\frac{1}{2}MR^2$.

5. * A bead of mass m slides without friction on a horizontal rigid wire rotating at constant angular speed ω about the z axis.

(a) Find the distance of the bead from the axis of rotation $r(t)$, as a function of time given that $r(0) = 0$, and $\dot{r}(0) = v_0$.

(b) What is the force exerted on the bead by the wire.