PH110: Tutorial Sheet 2

This tutorial sheet contains problems related to plane-polar coordinate system & work-energy conservative forces etc.

1. Using known results from the Cartesian coordinate system, and the relation between plane-polar and Cartesian basis vectors, calculate: (a) $\hat{\mathbf{r}} \times \hat{\boldsymbol{\theta}}$, (b) $\hat{\boldsymbol{\theta}} \times \hat{\mathbf{k}}$, and (c) $\hat{\mathbf{k}} \times \hat{\mathbf{r}}$.

2. A particle is moving along a circular path of radius a, with angular velocity given by $\omega(t) = \omega_0 + \alpha t$, where ω_0 and α are constants. Obtain the radial and tangential components of its velocity and acceleration.

3. A particle is moving along the line $y = a$, with the velocity $\mathbf{v} = u\hat{\mathbf{i}}$, where u is a constant. Express its velocity in plane polar coordinates.

4. A particle moves in such a way that $\dot{\theta} = \omega$ (constant), and $r = r_0 e^{\beta t}$, where r_0 and β are constants. Write down its velocity and acceleration in plane polar coordinates. For what values of β will the radial acceleration of the particle by zero?

5. Consider a circle of radius a, with the origin of the plane polar coordinate system placed at a point on the circumference. The particle is moving along the circle with a constant speed u.
   (a) What is the equation of the circle in this coordinate system?
   (b) What is the value of $\dot{\theta}$ in terms of u and a?
   (c) Write down the velocity of the particle in plane-polar coordinate system.
   (d) What is the acceleration of the particle in plane-polar coordinate system?

6. A particle moves along the curve $r = A\theta$, with $A = 1/\pi$ m/rad, and $\theta = \alpha t^2$, where α is a constant. Obtain the expressions for the velocity and acceleration of this particle in plane polar coordinates.
   (a) Show that the radial acceleration is zero when $\theta = 1/\sqrt{2}$ rad.
   (b) At what angles do radial and tangential components of the acceleration have equal magnitude?

7. * Mass m rotates on a frictionless table, held to circular path by a string which passes through a hole in the table. The string is slowly pulled through the hole so that the radius of the circle changes from R_0 to R_1. Show that the work done in pulling the string equals the increase in kinetic energy of the mass.

8. * A particle of mass m moves in one dimension along the x axis, such that $0 < x < \infty$. It is acted on by a constant force directed towards the origin with the magnitude B, and and inverse law repulsive force of magnitude A/x^2.
   (a) Find the potential energy function $V(x)$
   (b) Plot the potential energy as a function of x, and the total energy of the system, assuming that the maximum kinetic energy is $K_0 = \frac{1}{2}mv_0^2$.
   (c) What is the point of equilibrium, i.e., the point where net force acting on the particle is zero.

9. * A particle of mass M is held fixed at the origin. The gravitational potential energy of another particle of m, in the field of the first mass, is given by

$$V(\mathbf{r}) = -\frac{GMm}{r},$$

where G is the gravitational constant, and r is the distance of mass m from the origin.

   (a) What is the force acting on the particle of mass m?

   (b) Calculate the curl of this force.

10. * Consider a 2D force field $\mathbf{F} = A(y^2\hat{\mathbf{i}} + 2x^2\hat{\mathbf{j}})$. Calculate the work done by this force in going around a closed path which is a square made up of sides of length a, lying in the xy-plane, with two of its vertices located at the origin, and point (a, a). Find the answer by doing the line integral, as well as by using the Stokes' theorem. The path is traversed in a counter clockwise manner.

11. Find the forces for the following potential energies

   (a) $V(x, y, z) = Ax^2 + By^2 + Cz^2$

   (b) * $V(x, y, z) = A\ln(x^2 + y^2 + z^2)$

   (c) * $V(r, \theta) = A\cos\theta/r^2$ (r and θ are plane polar coordinates)

   Above, A, B, and, C are constants.

12. Determine whether each of the following forces is conservative. Find the potential energy function if it exists. A, α, β are constants.

   (a) * $\mathbf{F} = A(3\hat{\mathbf{i}} + z\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

   (b) * $\mathbf{F} = Axyz(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

   (c) $F_x = A\sin(\alpha y)\cos(\beta z)$, $F_y = -Ax\alpha\cos(\alpha y)\cos(\beta z)$, $F_z = Ax\sin(\alpha y)\sin(\beta z)$