PH110: QM : Tutorial Sheet 3

Notes:

1. * marked problems will be solved in the Wednesday tutorial class.
2. Please make sure that you do the assignment by yourself. You can consult your classmates and seniors and ensure you understand the concept. However, do not copy assignments from others.

Heisenberg Uncertainty Principle

1. Estimate the uncertainty in the position of (a) a neutron moving at 5×10^6 m s^{-1} and (b) a 50 kg person moving at 2 m s^{-1}. The error in the measurement of the velocity is 1%.

2. A lead nucleus has a radius 7×10^{-15} m. Consider a proton bound within nucleus. Using the uncertainty relation Δp.Δr $\geq \hbar/2$, estimate the root mean square speed of the proton, assuming it to be non-relativistic. (You can assume that the average value of p^2 is square of the uncertainty in momentum.)

3. * A π^0 meson is an unstable particle produced in high energy particle collisions. It has a mass-energy equivalent of about 135MeV, and it exists for an average lifetime of only 8.7×10^{-17} s before decaying into two γ rays. Using the uncertainty principle, estimate the fractional uncertainty $\Delta m/m$ in its mass determination.

4. * For a non-relativistic electron, using the uncertainty relation $\Delta x \Delta p_x = \hbar/2$
   (a) Derive the expression for the minimum kinetic energy of the electron localized in a region of size ' a '.
   (b) If the uncertainty in the location of a particle is equal to its de Broglie wavelength, show that the uncertainty in the measurement of its velocity is same as the particle velocity.
   (c) Using the expression in (b), calculate the uncertainty in the velocity of an electron having energy 0.2keV
   (d) An electron of energy 0.2keV is passed through a circular hole of radius 10^{-6} m. What is the uncertainty introduced in the angle of emergence in radians? (Given $\tan\theta \cong \theta$)

5. An atom in an excited state 1.8eV above the ground state remains in that excited state 2.0μs before moving to the ground state. Find (a) the frequency of the emitted photon, (b) its wavelength, and (c) its approximate uncertainty in energy.

6. * An electron microscope is designed to resolve objects as small as 0.14 nm. What energy electrons must be used in this instrument?

7. * Show that the uncertainty principle can be expressed in the form $\Delta L \Delta\theta \geq \hbar/2$, where θ is the angle and L the angular momentum. For what uncertainty in L will the angular position of a particle be completely undetermined?

   For circular motion $L = rp$ and so $\Delta L = r\Delta p$. Along the circle $x = r\theta$ and $\Delta x = r\Delta\theta$. Thus $\Delta p \Delta x = \frac{\Delta L}{r}(r\Delta\theta) = \Delta L \Delta\theta \geq \frac{\hbar}{2}$. For complete uncertainty $\Delta\theta = 2\pi$ and $\Delta L = \frac{\hbar/2}{2\pi} = \frac{\hbar}{4\pi}$