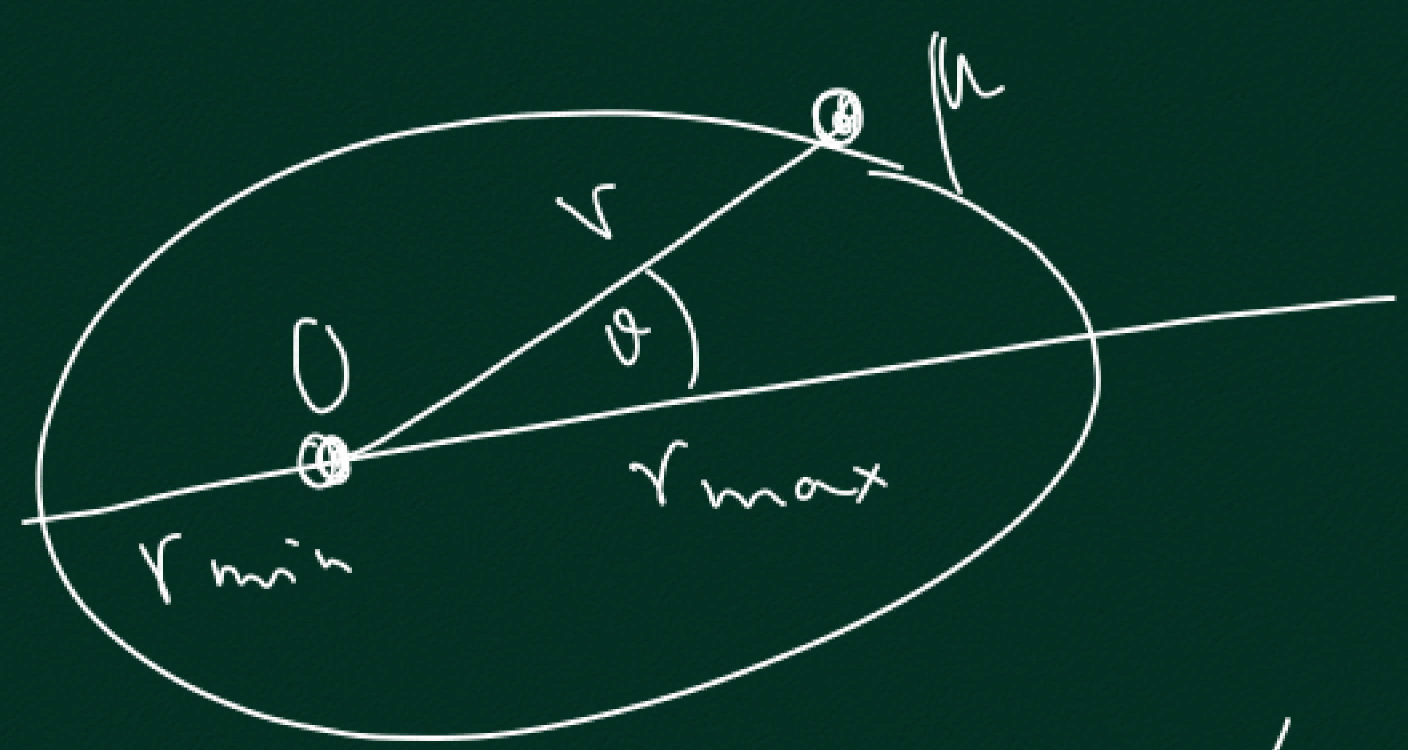
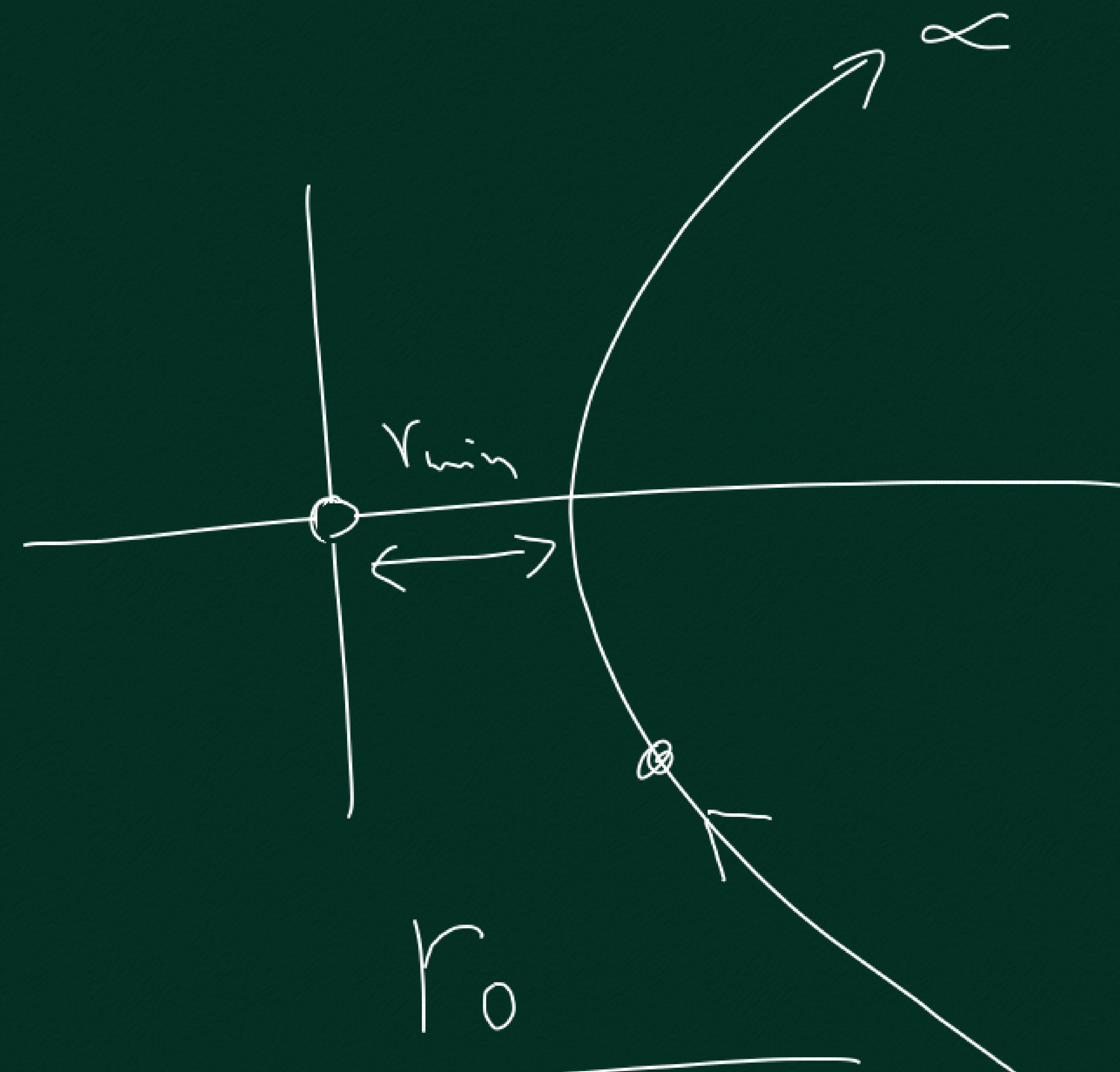




$$r = \frac{r_0}{1 - \epsilon \cos\theta}$$

const: E, l, $V(r) = -\frac{C}{r}$ C, μ

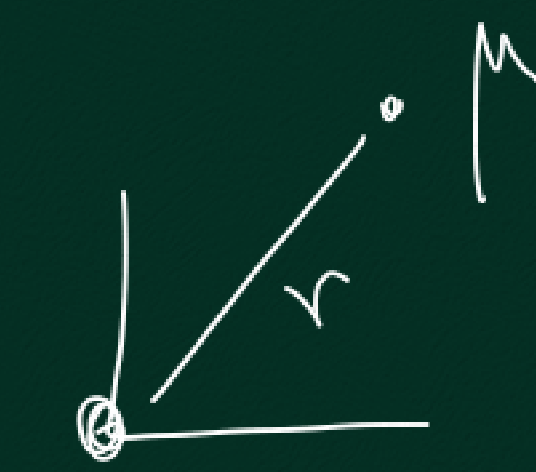
$$E = \frac{\mu \dot{r}^2}{2} + \frac{l^2}{2\mu r^2} - \frac{C}{r} \leftarrow \text{Grav. potnl}$$

$$\mu r^2 \dot{\theta} = l$$

$$\frac{d\theta}{dt} = \frac{l u^2}{\mu}$$

$$r(\theta) = ?$$

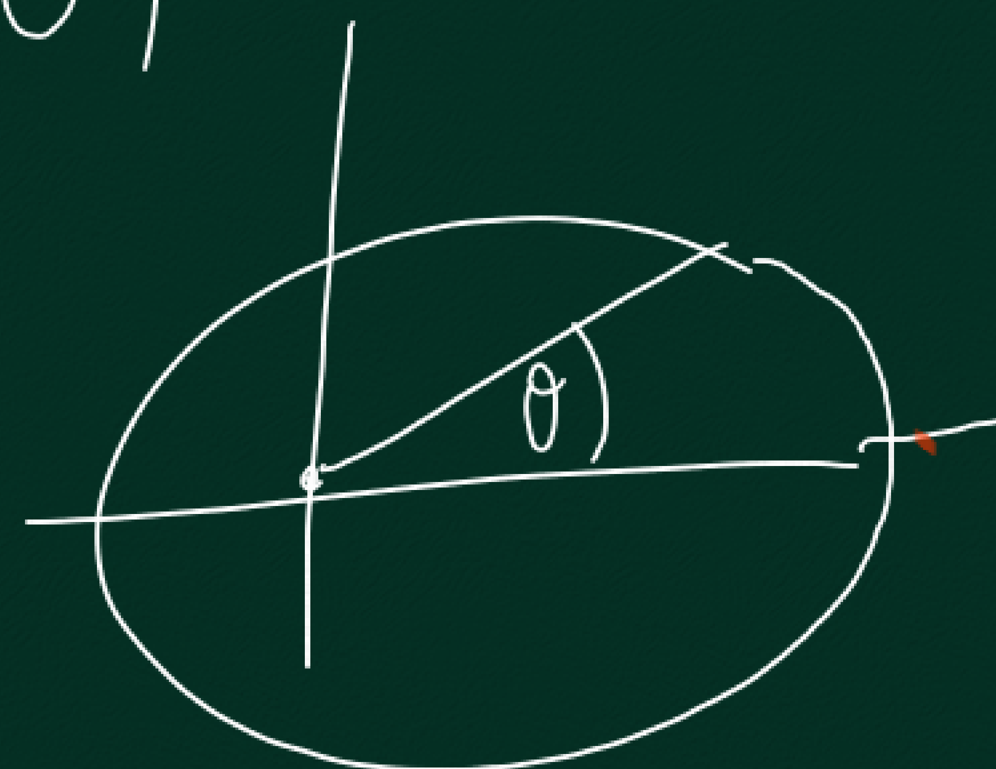
$$\dot{r} = -\frac{1}{u^2}\frac{du}{dt} = -\frac{1}{u^2}\frac{du}{d\theta}\frac{d\theta}{dt} = -\frac{du}{d\theta}\frac{l}{\mu}$$



$$u = \frac{1}{r}$$

$$E = \frac{l^2}{2\mu}\left[\left(\frac{du}{d\theta}\right)^2 + u^2\right] - Cu$$

$$\frac{dE}{dt} = 0 \Rightarrow \frac{dE}{d\theta}\frac{d\theta}{dt} = 0 \Rightarrow \frac{dE}{d\theta} = 0 \quad (\because \dot{\theta} \neq 0)$$



$$\frac{dE}{d\theta} = 0 \Rightarrow \frac{l^2}{2\mu}\left[2\left(\frac{du}{d\theta}\right)\frac{d^2u}{d\theta^2} + 2u\frac{du}{d\theta}\right] - C\frac{du}{d\theta} = 0$$

$$\boxed{\frac{d^2u}{d\theta^2} + u = \frac{\mu}{l^2}C}$$

$$u_1 = u - \frac{\mu C}{l^2}$$

$$\frac{d^2u_1}{d\theta^2} = \frac{d^2u}{d\theta}$$

$$\frac{d^2u_1}{d\theta^2} = -u_1 \Rightarrow \boxed{u_1 = \bar{u}_1 \cos(\theta - \theta_0)}$$

$$\Downarrow$$

$$\frac{du}{d\theta} = \frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{dr}{d\theta}$$

$$\frac{d^2u}{dt^2} = -ku + f$$

$$m\frac{d^2u}{dt^2} = -ku + f_0$$

$$u_1 = \frac{k}{m}\left(u - \frac{f_0}{k}\right)$$

$$\frac{d^2u_1}{dt^2} = -ku_1$$

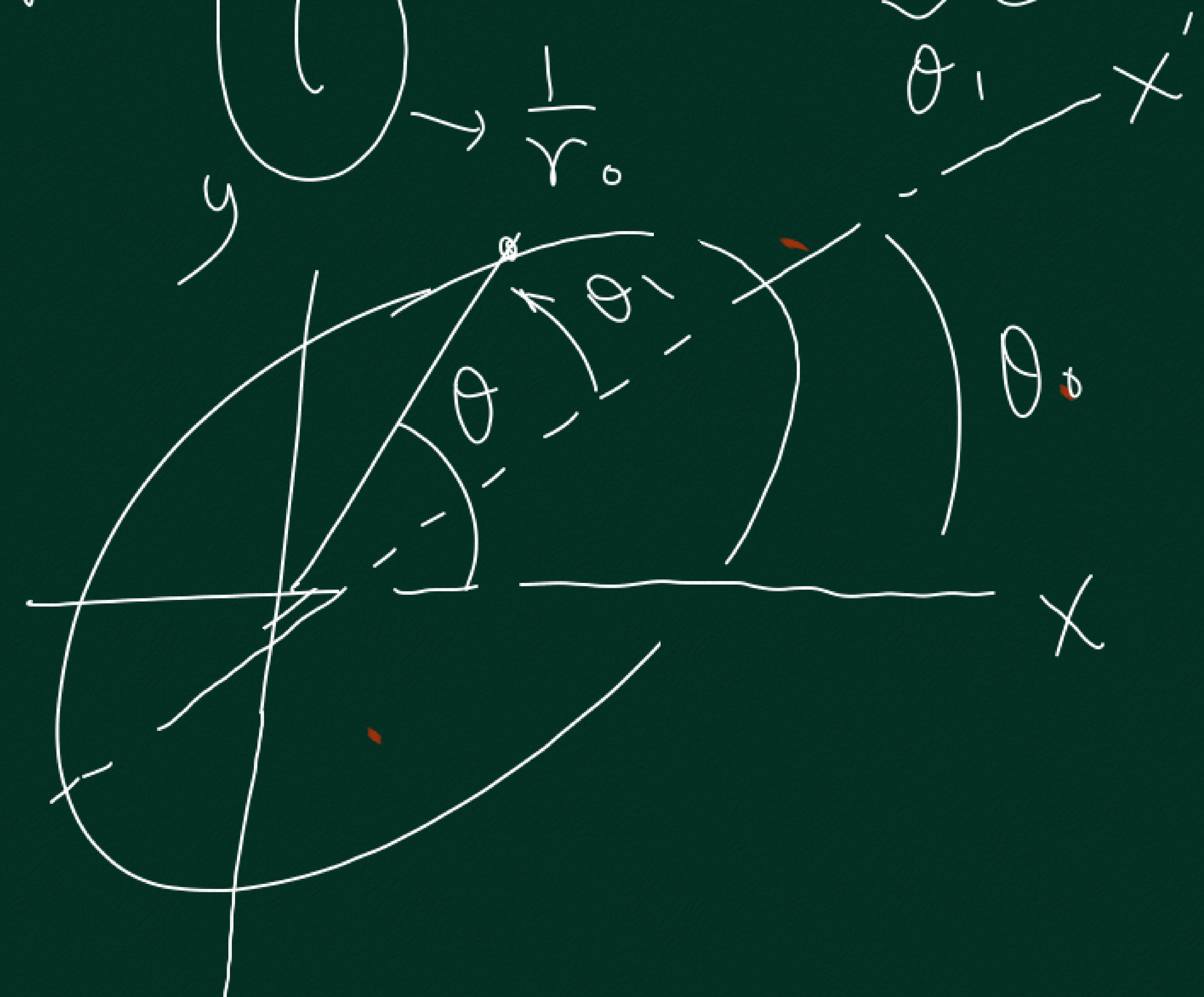
$$u_1 = u_1(0)\cos\omega(t - t_0)$$

$$u = \frac{m}{k}u_1(t) + \frac{f_0}{k}$$

$$u = \frac{\mu C}{l^2} \pm \bar{u}_1\cos(\theta - \theta_0)$$

$$\frac{1}{r} = \frac{\mu c}{l^2} - \bar{u}_1\cos\underbrace{(\theta - \theta_0)}_{\theta_1}$$

$$\frac{\mu c}{l^2} \rightarrow \frac{1}{r_0}$$



$$\frac{1}{r} = \frac{1}{r_0} - \bar{u}_1 \cos\theta \qquad \frac{\epsilon}{r_0} = \bar{u}_1$$

$$= \frac{1}{r_0}\left[1 - \epsilon\cos\theta\right]$$

$$\boxed{r = \frac{r_0}{1 - \epsilon\cos\theta}}$$

$$r_0 = \frac{l^2}{\mu C}, \quad \epsilon = \sqrt{1 + \frac{2El^2}{\mu C^2}}$$

Ellipse $\quad E = \frac{C}{2r_0}(\epsilon^2 - 1)^{(\text{gnrl})}$

$$= -\frac{C}{2a}(\text{ellps})$$

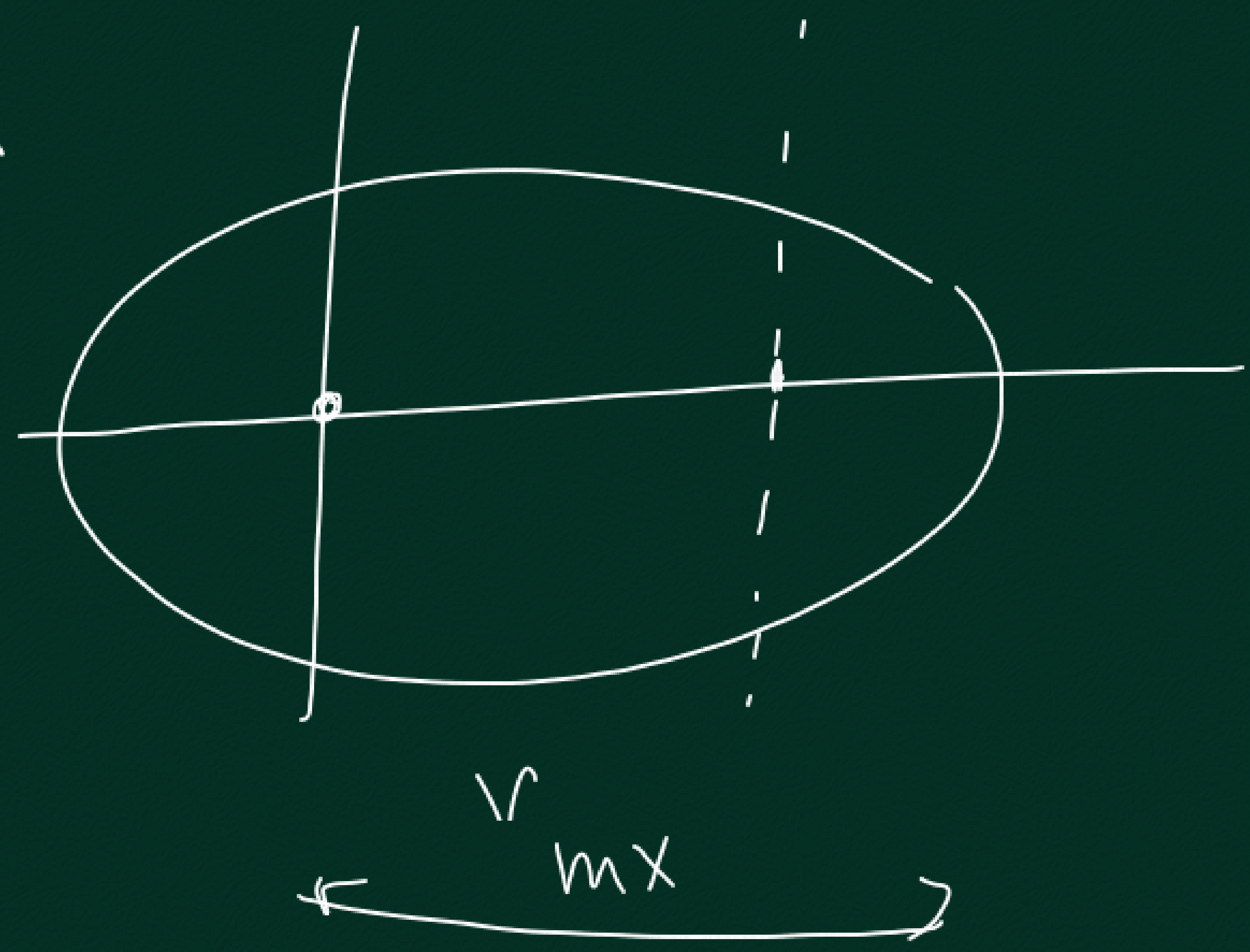
$$r = \frac{r_0}{1 - \epsilon \cos\theta}$$

$1 > \epsilon > 0$ Ellipse

$$\Rightarrow r_{mx}(\theta = 0) = \frac{r_0}{1-\epsilon}$$

$$\Rightarrow r_{mn}(\theta = \pi) = \frac{r_0}{1+\epsilon}$$

r_{mx}



$$r_{mx} + r_{min} = 2a$$

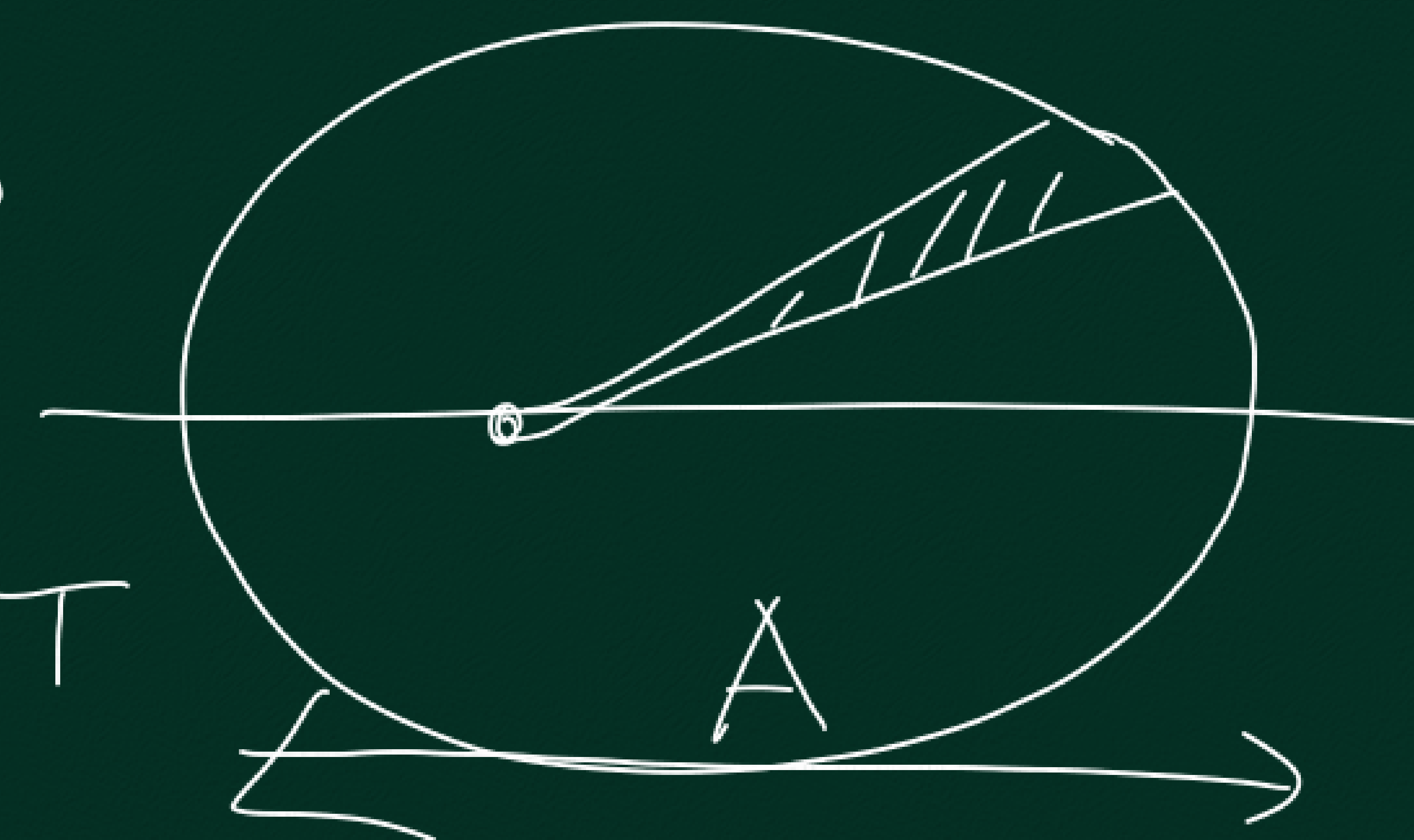
$$2a = r_0\left(\frac{1}{1-\epsilon} + \frac{1}{1+\epsilon}\right) = \frac{2r_0}{1-\epsilon^2}$$

$$\epsilon = \sqrt{1 - \frac{b^2}{a^2}}, \quad b = \frac{r_0}{\sqrt{1-\epsilon^2}}$$

$A = 2a$

$r(\theta)$

$r(t)$



$$\frac{dA}{dt} = \frac{l}{2\mu}$$

$$\int_0^T dA = \frac{l}{2\mu}\int_0^T dt$$

$$a = \frac{r_0}{1-\epsilon^2}$$

$$b = \frac{r_0}{\sqrt{1-\epsilon^2}}$$

$$A = \frac{l}{2\mu} T$$

$$\pi ab = \frac{l}{2\mu} T \Rightarrow T = \pi ab \frac{2\mu}{l} = \pi\sqrt{\frac{2\pi}{C}}A^{\frac{3}{2}}$$