Tutorial - 5

☆ 2D/3D systems —

1.) a.) $$\hat{H} = \frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{1}{2}m\omega^2(x^2+y^2)$$

$$= \left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2x^2\right] + \left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + \frac{1}{2}m\omega^2y^2\right]$$

$\Rightarrow$ Splittable into two 1D SHOs.

So $E_{n_x, n_y} = E_{n_x} + E_{n_y}$

$$= \left(n_x + \frac{1}{2}\right)\hbar\omega + \left(n_y + \frac{1}{2}\right)\hbar\omega$$

$$= (n_x + n_y + 1)\hbar\omega$$

$[n_x, n_y \in W]$

b.) Let $n_x + n_y = n \quad (n \in W)$

$\Rightarrow$ Degeneracy level $= {}^{n+2-1}C_{2-1}$

$= n+1$

2.) a.) (Consider $P_1 \equiv P_x$, $\omega_1 \equiv \omega_x$, $q_1 \equiv x$
$P_2 \equiv P_y$, $\omega_2 \equiv \omega_y$, $q_2 \equiv y$)

Using seperation of variables, we have the following 2 eqn's:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_x}{\partial x^2} + \frac{1}{2} m\omega_x^2 x^2 \psi_x = E_x \psi_x$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_y}{\partial y^2} + \frac{1}{2} m\omega_y^2 y^2 \psi_y = E_y \psi_y$$

$\Rightarrow \psi = \psi_x \psi_y$ (written as $\phi_{n_x}\phi_{n_y}$)

$$= \frac{1}{2^n n!}\sqrt{\frac{m}{\pi\hbar}}\,(\omega_x\omega_y)^{1/4} H_n\left(\sqrt{\frac{m\omega_x}{\hbar}}\right) H_n\left(\sqrt{\frac{m\omega_x}{\hbar}}\right) \times e^{-m/\hbar\,(\omega_x x^2 + \omega_y y^2)}$$

Eigen values of ψ

$$= E_{n_x} + E_{n_y}$$

$$= \hbar\left[(n_x + 1/2)\,\omega_x + (n_y + 1/2)\,\omega_y\right]$$

b.) $\dfrac{\omega_1}{\omega_2} \equiv \dfrac{\omega_x}{\omega_y} = \dfrac{3}{4}$

$$\Rightarrow \omega_y = \frac{4}{3}\omega_x$$

So $E = \hbar\omega_x\left[(n_x + 1/2) + 4/3\,(n_y + 1/2)\right]$

$$= \hbar\omega_x\left[n_x + \frac{4}{3}n_y + \frac{7}{6}\right]$$

$$= \hbar\omega_x\left[\frac{3n_x + 4n_y}{3} + \frac{7}{6}\right]$$

let $3n_x + 4n_y = n$

Put $n = 12 \Rightarrow n_x = 4,\ n_y = 0$ and $n_x = 0,\ n_y = 3$

Put $n = 19 \Rightarrow n_x = 5,\ n_y = 1$ and $n_x = 1,\ n_y = 4$

If $\omega_1 \notin Q \Rightarrow$ No degeneracy.

ω_2

3.) $E_{n_x,n_y,n_z} = (n_x + n_y + n_z + 1.5)\hbar\omega$

Put $n_x + n_y + n_z = n$ (all $\in W$)

$$g_n = {}^{n+3-1}C_{3-1} = {}^{n+2}C_2 = \frac{(n+2)(n+1)}{2}$$

[Beggar's method]

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