Tut 4

8)

$$\psi(x) = A\left[\sin\left(\frac{\pi x}{L}\right) + \sin\left(\frac{2\pi x}{L}\right)\right]$$

1) $A = ?$

2) $\langle x \rangle$, $\langle x^2 \rangle$, $\Delta x = \sqrt{\langle x^2\rangle - \langle x \rangle}$

3) $\langle p \rangle$, $\langle p^2 \rangle$, $\delta p = \cdots$

4) What's the problty of finding the ptcl in 1st excited st?

$$P(x) = |\psi|^2(x)$$

1) $\int_0^L |\psi|^2 dx = 1$

$\psi \psi^*$

$$|\psi|^2 = A^2\left[\sin\frac{kx}{L} + \sin\frac{2kx}{L}\right]^2$$

$k = \pi$

$$\int \psi_1^* \psi_2 \, dx = 0$$

$$\int_0^L |\psi|^2 dx = A^2 \int_0^L \left[\sin^2\frac{kx}{L} + \sin^2\frac{2kx}{L} + 2\sin\frac{kx}{L}\sin\frac{2kx}{L}\right]$$

$$\frac{1 - \int_0^L \cos\frac{2\pi x}{L}\, dx}{2}$$

$$A^2\left[\frac{2L}{2}\right] = 1$$

$$A = \frac{1}{\sqrt{L}}$$

$$A^2 [L$$

$$\boxed{\langle O\rangle = \frac{\int \psi^* \hat{O}\psi \, dx}{\underbrace{\int \psi^* \psi \, dx}_{1}}}$$

$$\langle x \rangle$$

$$\langle x\rangle = \int_0^L x\,\psi^2(x)\,dx = L\left[\frac{1}{2} - \frac{16}{9\pi^2}\right] \simeq 0.32\,L$$

$$\psi(x) = \frac{1}{\sqrt{L}}\left(\overbrace{\sin\frac{\pi x}{L}}^{\psi_1} + \overbrace{\sin\frac{2\pi x}{L}}^{\psi_2}\right)$$

$$\langle x^2\rangle = \int_0^L x^2\,\psi^2(x)\,dx = L^2 \times 0.12$$

$$x\left(\psi_1^2 + \psi_2^2 + 2\psi_1\psi_2\right)$$

Trick

$$\int x\cos\alpha x\,dx$$

$$= \frac{d}{d\alpha}\int \sin\alpha x\,dx = \frac{d}{d\alpha}\left[-\frac{\cos\alpha x}{\alpha}\right]$$

$$= \frac{x\sin\alpha x}{\alpha} + \frac{1}{\alpha^2}\cos\alpha x$$

$$\int x^2 \cos\alpha x \, dx$$

$$= -\frac{d}{d\alpha^2} \int \cos\alpha x \, dx$$

$$-\frac{d}{d\alpha^2}\left[\frac{\sin\alpha x}{\alpha}\right]$$

$$\psi_1 = \frac{1}{\sqrt{L}} \sin\frac{\pi x}{L}$$

$$\psi_2 = \frac{1}{\sqrt{L}} \sin\frac{2\pi x}{L}$$

$$\psi = c_1\psi_1 + c_2\psi_2 + - c_n\psi_n$$

$(\psi_1, \psi_2, \cdots \psi_n)$ eigen fns of Sdg eqn

$$\int \psi_n^* \psi_m \, dx = \delta_{mn}$$

$$\int |\psi|^2 dx = 1 \quad , \quad \psi = \sum_n c_n \psi_n$$

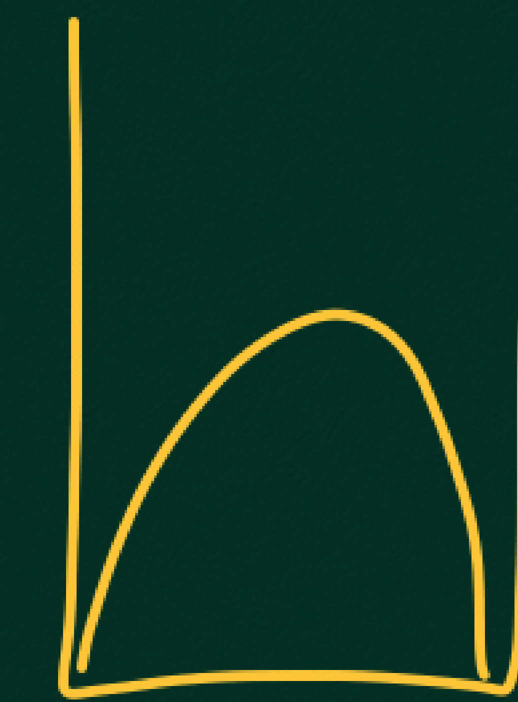
$$\int \left(\sum_n c_n^* \psi_n^*\right)\left(\sum_m c_m \psi_m\right) dx = 1$$

$$= |c_n|^2 \int |\psi_n|^2 = 1 \Rightarrow \sum_n |c_n|^2 = 1$$

if $\int |\psi_n| dx = 1$

$$\langle p \rangle = \int \psi^* \left(-i\hbar \frac{d}{dx}\right) \psi \, dx$$

$$\int x^2 \cos \alpha x$$

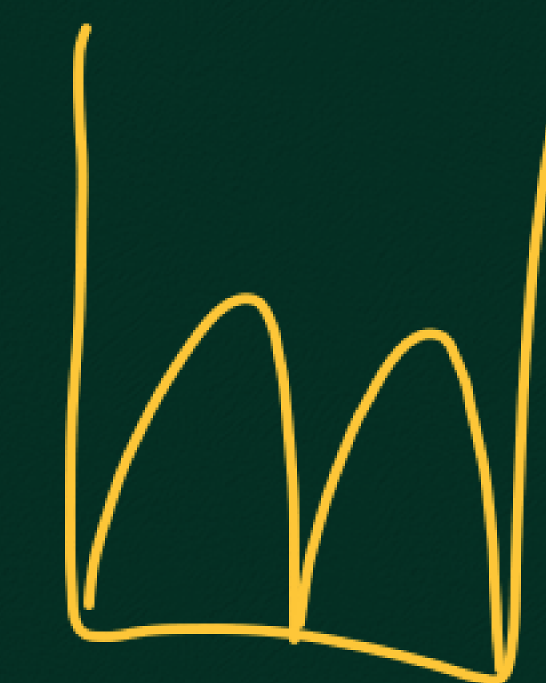


$\langle p \rangle$

$-i\hbar \nabla$

$$\frac{1}{L} \int (\quad) \left(-i\hbar \frac{d}{dx}\right) \left(\sin \frac{\pi x}{L} + \sin \frac{2\pi x}{L}\right)$$

$$-i\hbar \int (\sin + \sin) \left(\cos(\,) + \cos(\,)\right) = 0$$



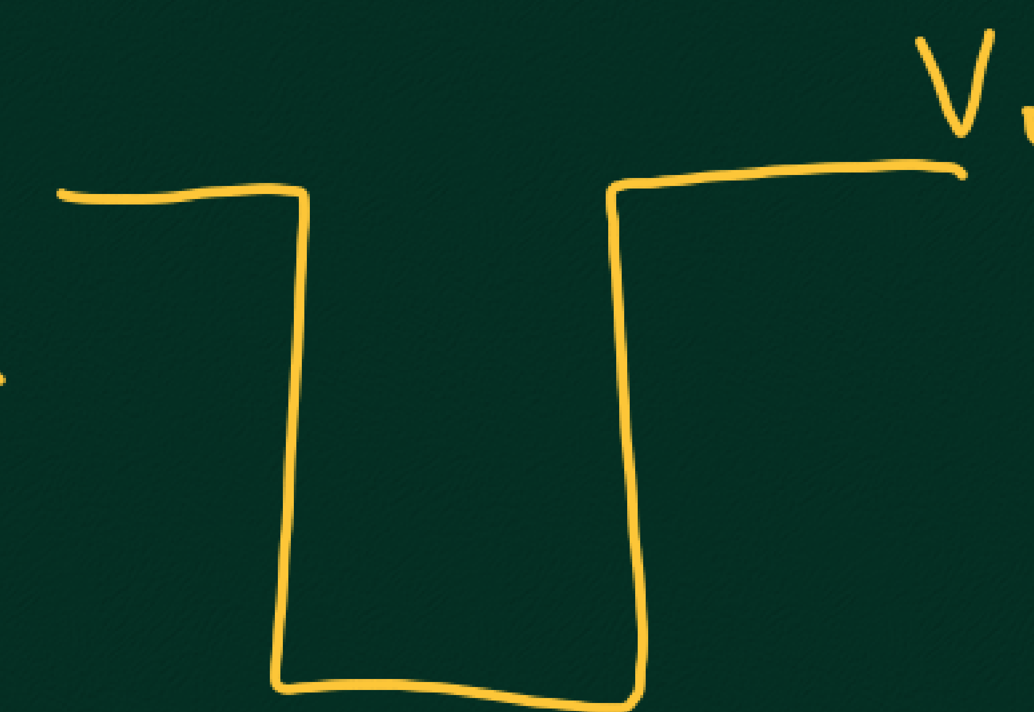
$$\int^{L} \sin(\,) \cos(\,) = 0$$

$$\langle p^2 \rangle = \int (\quad + \quad)^{0} \left(-i\hbar \frac{d}{dx}\right) \left(-i\hbar \frac{d}{dx}\right) (\quad + \quad)$$

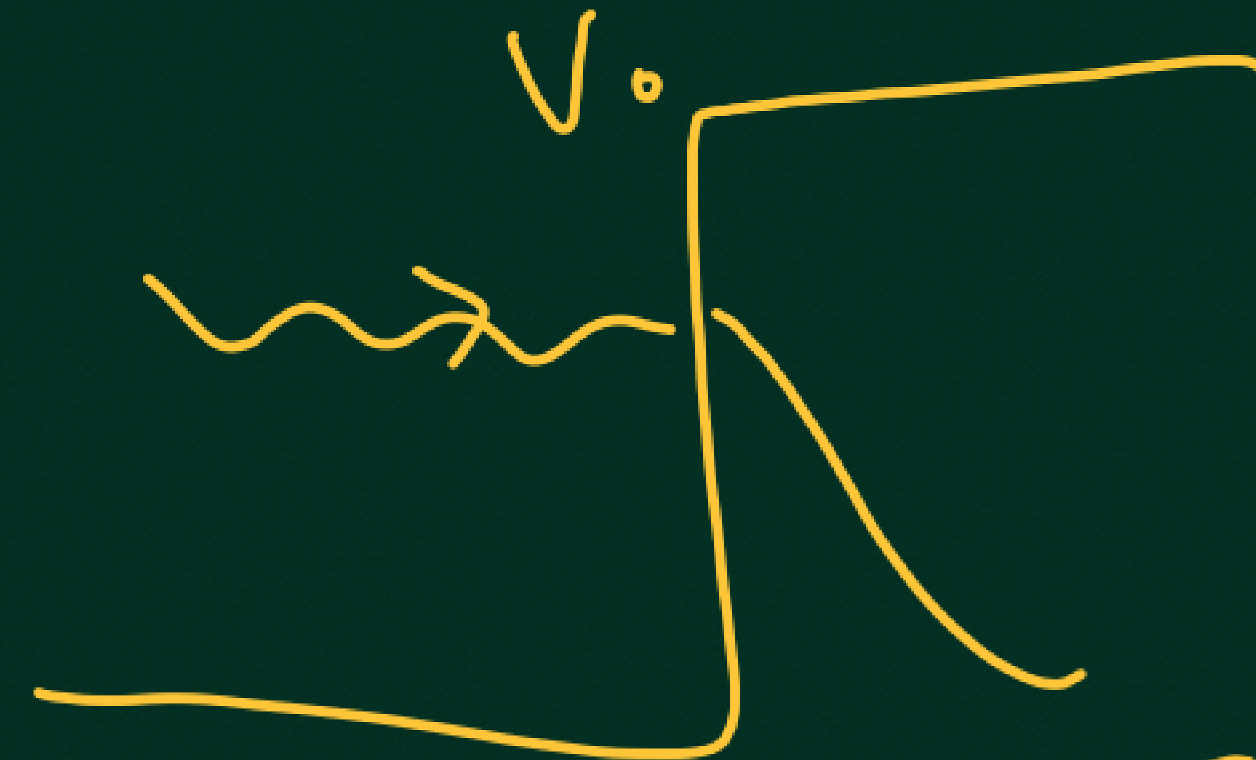
$$\langle p^2\rangle = \frac{\hbar^2\pi^2}{L^2}\cdot\frac{5}{2}$$

$$\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \sqrt{\frac{5}{2}}\,\frac{\hbar\pi}{L}$$

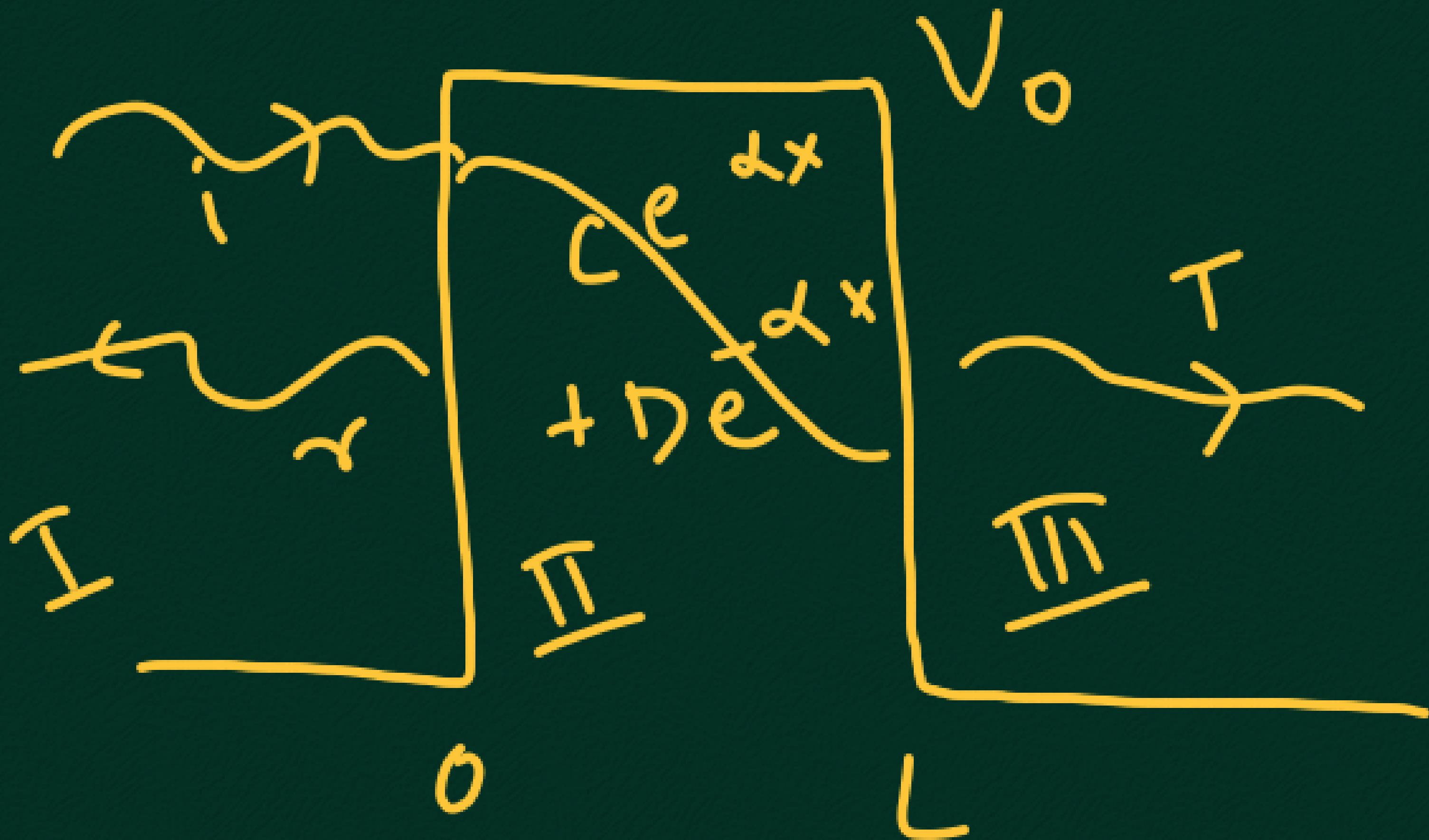
$$\Delta x \cdot \Delta p = 0.64\,\hbar$$



$V_0 > E$



No discrete energy levels, all E allowed

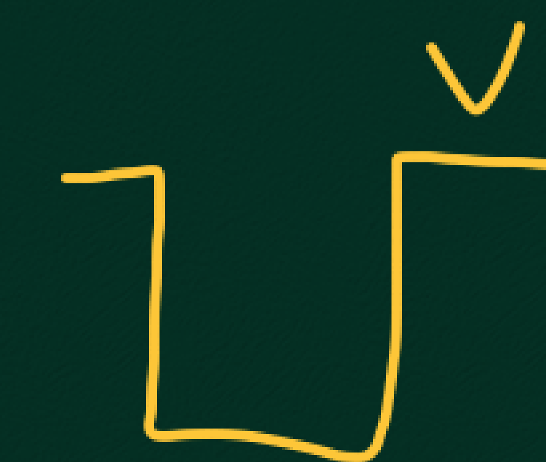


$T(E)$, $R(E)$

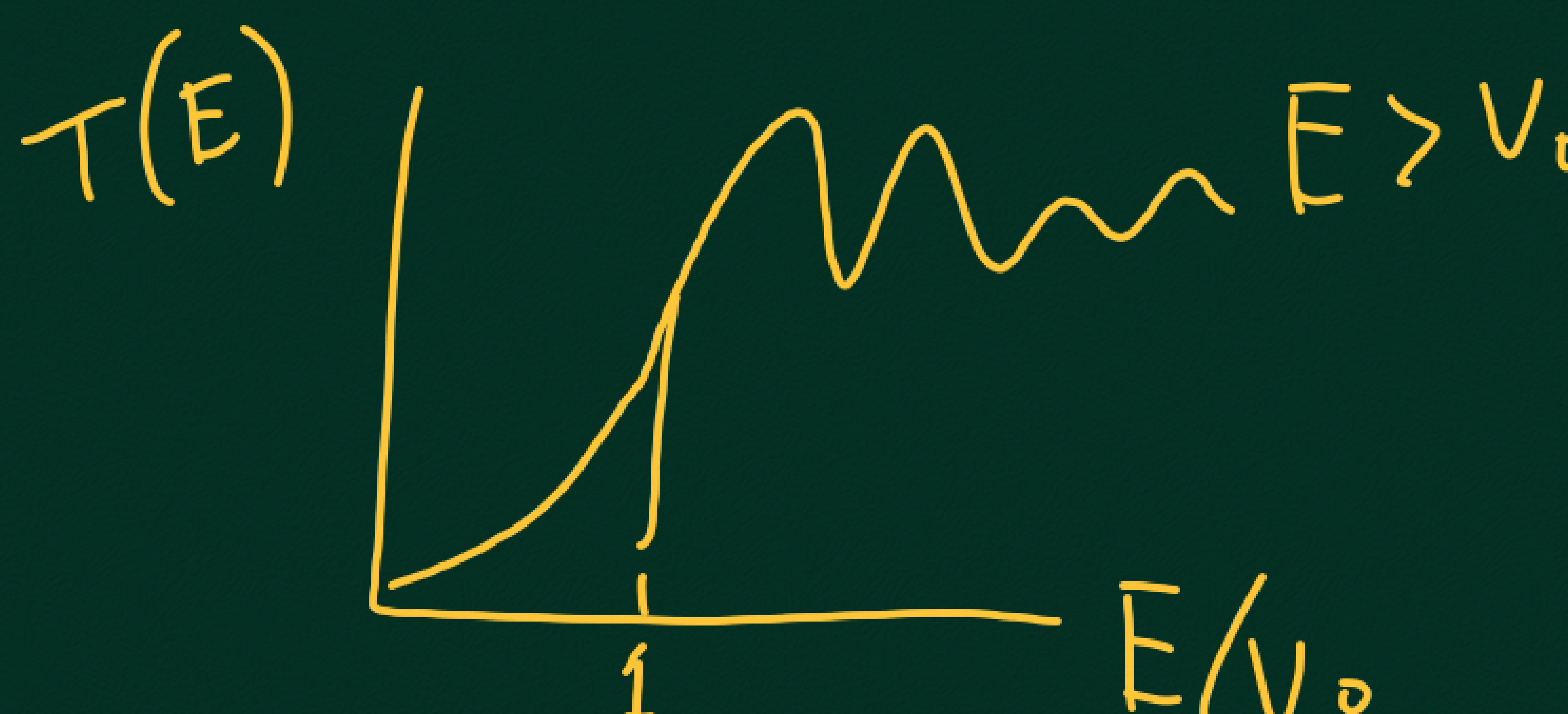
$= \dfrac{J_T}{J_i}$ $\qquad$ $= \dfrac{J_R}{J_i}$

A, B $\qquad$ C, D $\qquad$ G

$$\boxed{k_2^2 = \frac{2m}{\hbar^2}\left(E - V_0\right)} \qquad E > V_0$$



$$T(E) = \frac{|G|^2}{|A|^2}$$



$$k_2 L = n\pi$$

$$k_2 \sim \sqrt{E - V_0}$$

$$\Delta k_2 = \pi / L$$

Amplitude of oscillation $\downarrow$ with E

$E \gg V_0 \Rightarrow E - V_0 \simeq E$

$$T(E) = \frac{1}{1 + \frac{V_0^2}{4E\underbrace{(E - V_0)}_{\simeq E}} \sin^2 k_2 L}$$

$$\simeq \quad \text{Ampl} = \left(\frac{V_0}{E}\right)^2$$

if $x \to 0$

$\sinh x \simeq x$

$\sin x \simeq x$

$$\frac{e^x - e^{-x}}{2} \simeq \frac{(1 + x + x^2 + \cdots) - (1 - x + x^2 - \cdots)}{2}$$

$$\simeq x + O(x^3)$$

for x large

$\sinh x \sim \frac{e^x}{2}$

$$\alpha^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$T(E) \ll 1$

$E \ll V_0$

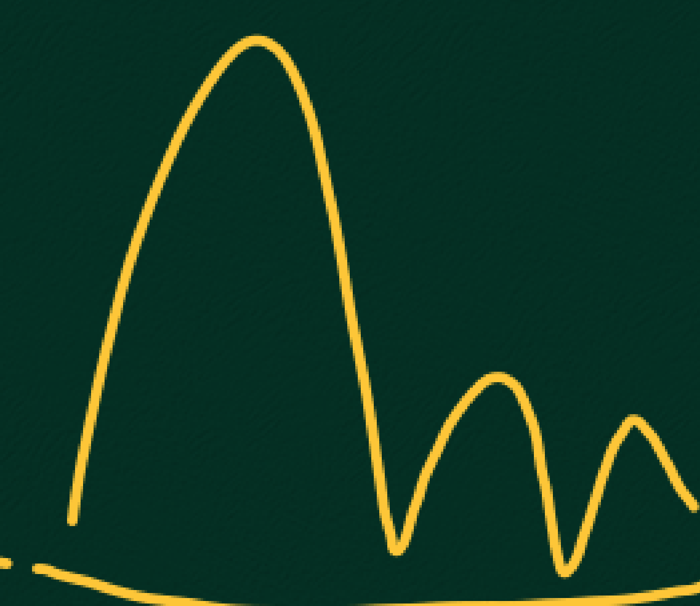
$|f(x)|$

$$\alpha^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$\cos x \to \sinh x$

$$\sum a_k (e^{ikx})$$

$$= e^{ix}$$



$E < V_0$

$$\boxed{\psi_{II} = C e^{\alpha x} + D e^{-\alpha x}}$$

complex

$$J_{II} = \frac{\hbar}{m} \operatorname{Im}\left(\psi^* \nabla \psi\right)$$

$$c^* e^{\alpha x} \frac{\partial}{\partial x}\left(c e^{\alpha x}\right) = |c|^2$$