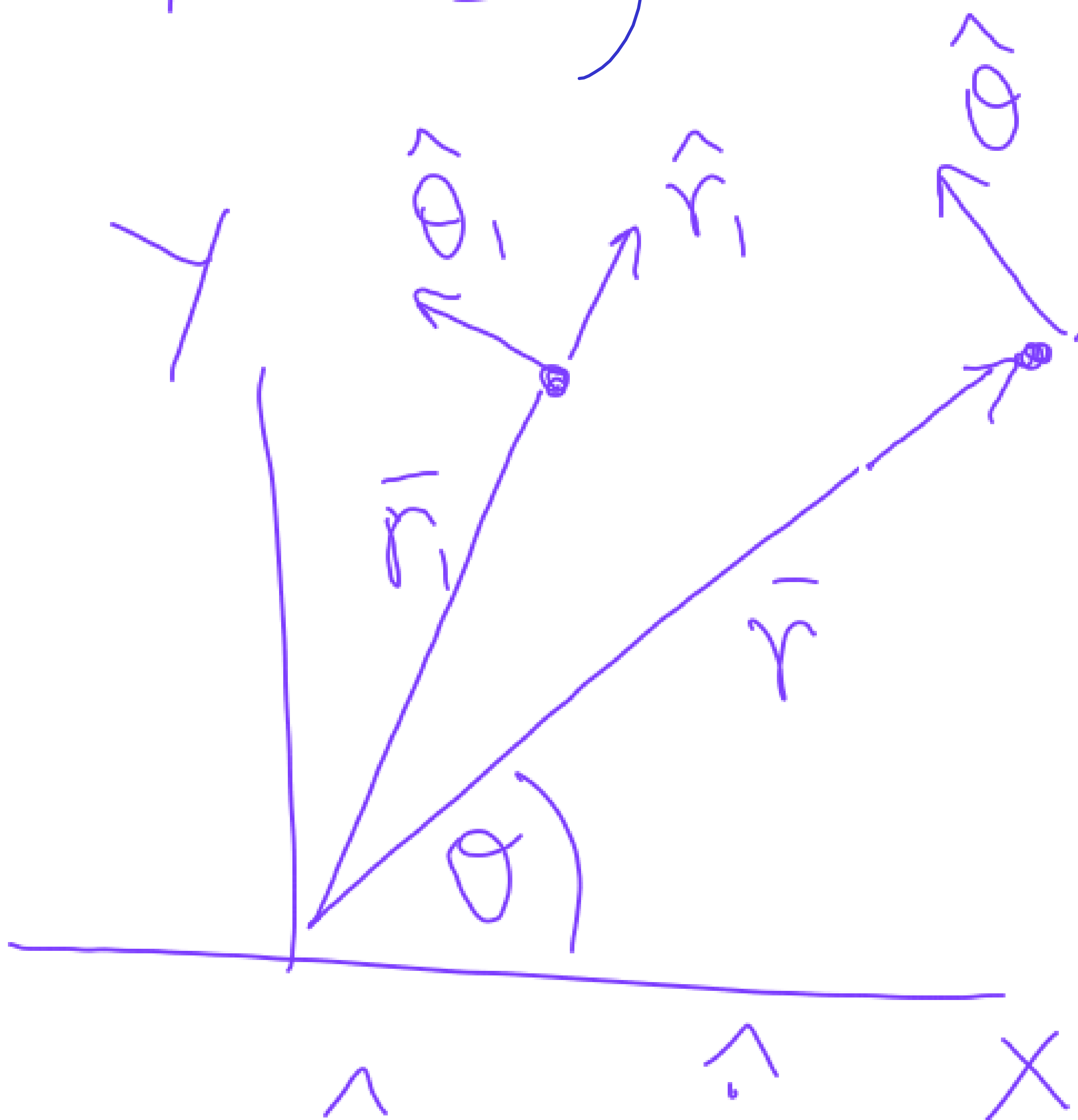
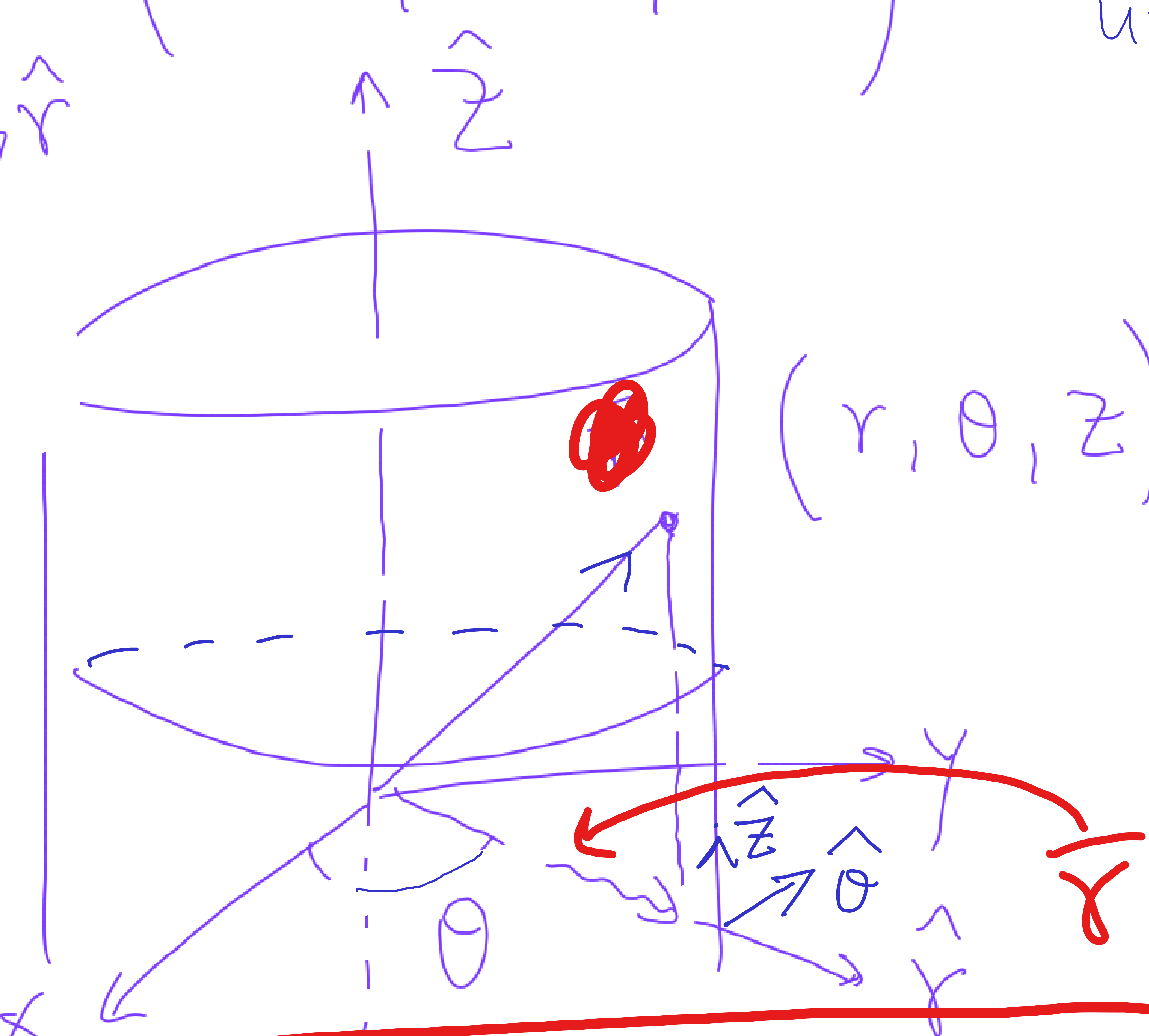
one orthogonal set to another orthogonal set of co-ordinates

$$(\hat{X} \perp \hat{Y} \perp \hat{Z}) \longrightarrow (\hat{r}, \hat{\theta}, \hat{z})$$

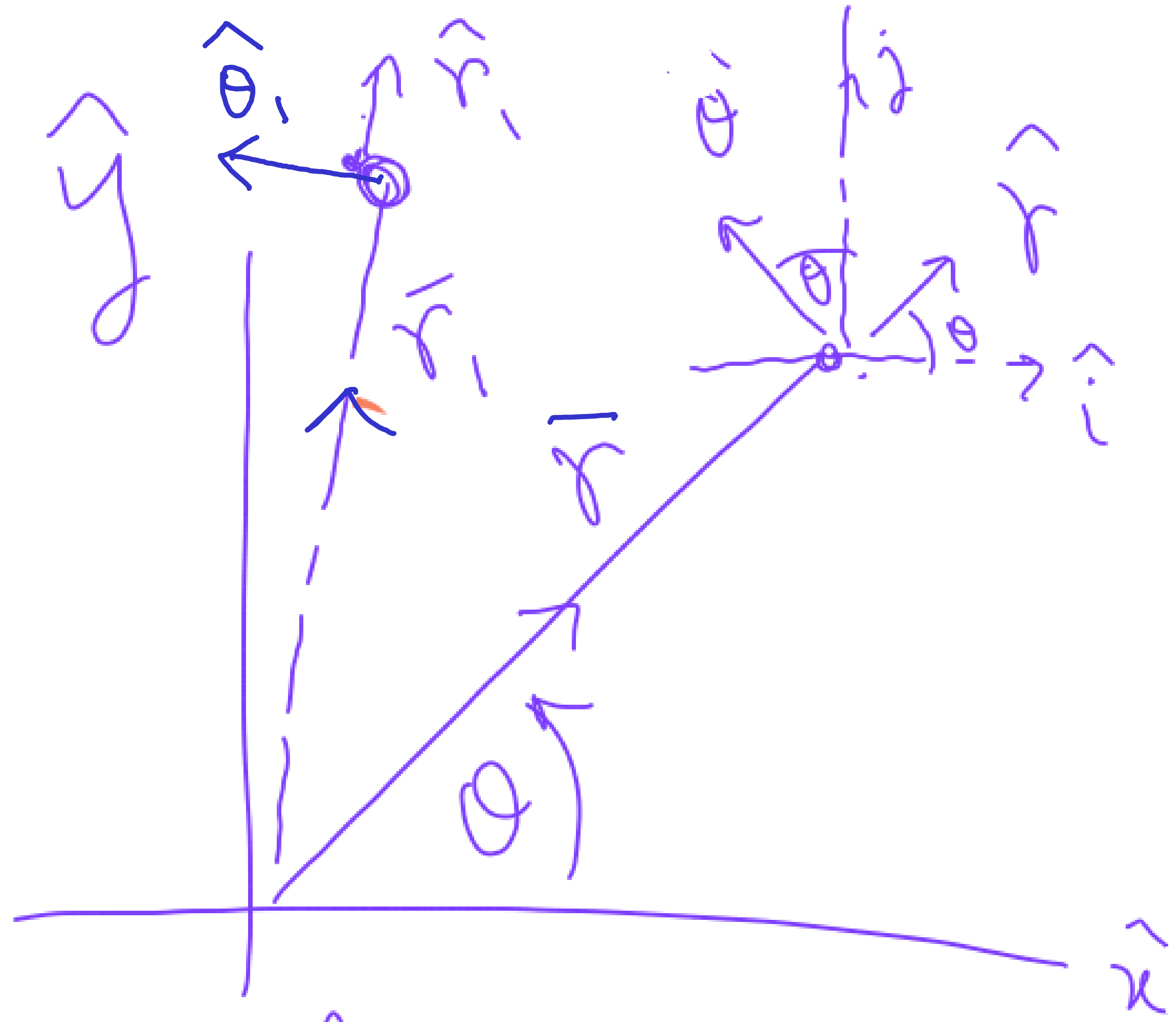
Hat ^ implies unit vector



$$\bar{r} = x\hat{i} + y\hat{j}$$
$$= r\hat{r}$$



While $(\hat{x}, \hat{y}, \hat{z})$ are fixed in space, $(\hat{r}, \hat{\theta})$ keeps changing depending on $\bar{r}$



$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r = \sqrt{x^2+y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\hat{i} = \hat{x}$$

$$\hat{j} = \hat{y}$$

$$\hat{k} = \hat{z}$$

$$\hat{i} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

$$\hat{j} = \sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}$$

$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

Relation among (r,θ,z) & $(\hat{i},\hat{j},\hat{k})$

$$\bar{r} = r\,\hat{r}$$

$$\bar{r} = x\,\hat{i} + y\,\hat{j}$$

$$d\bar{r} = dx\,\hat{i} + dy\,\hat{j} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

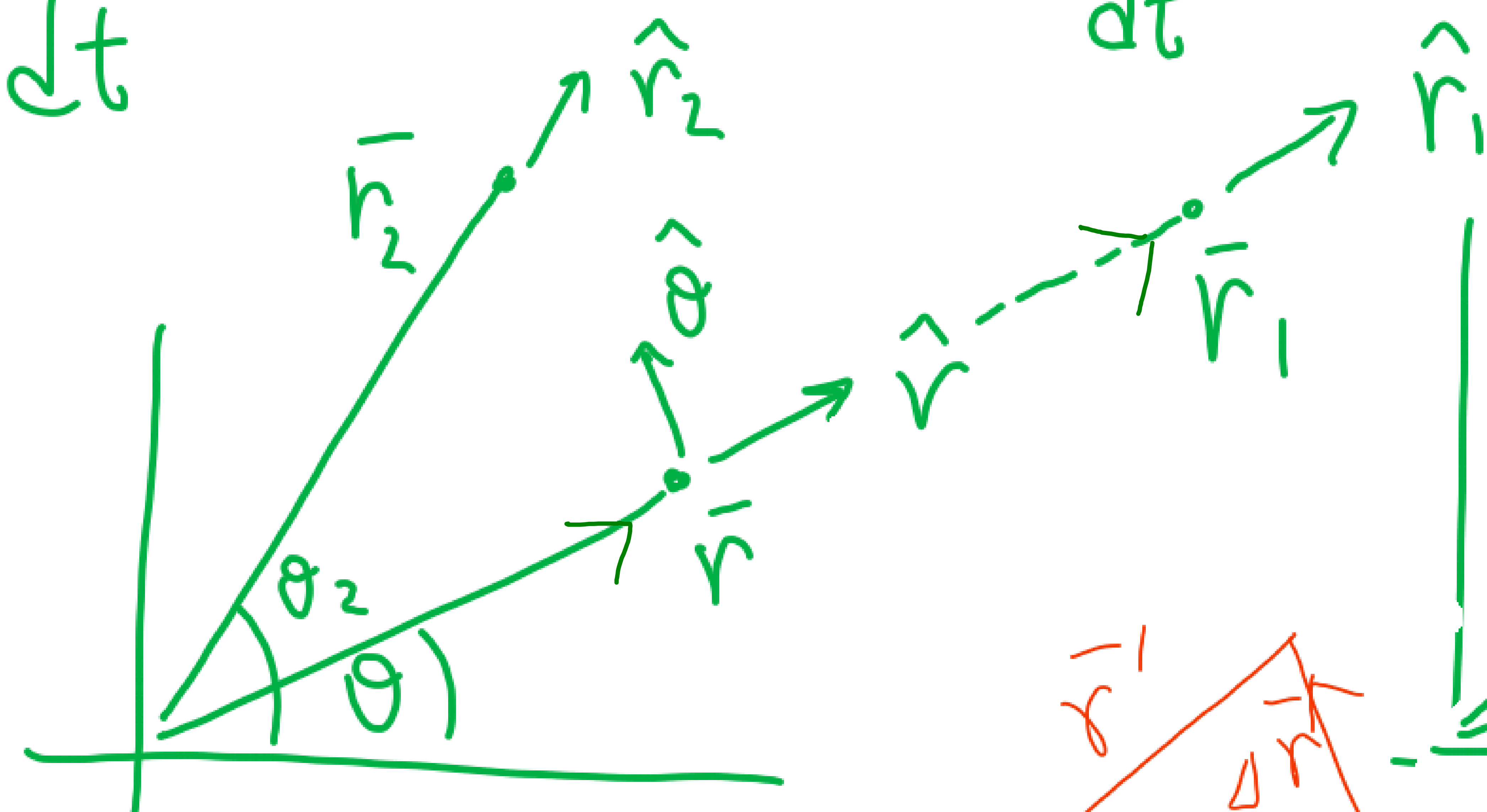
(prove this)

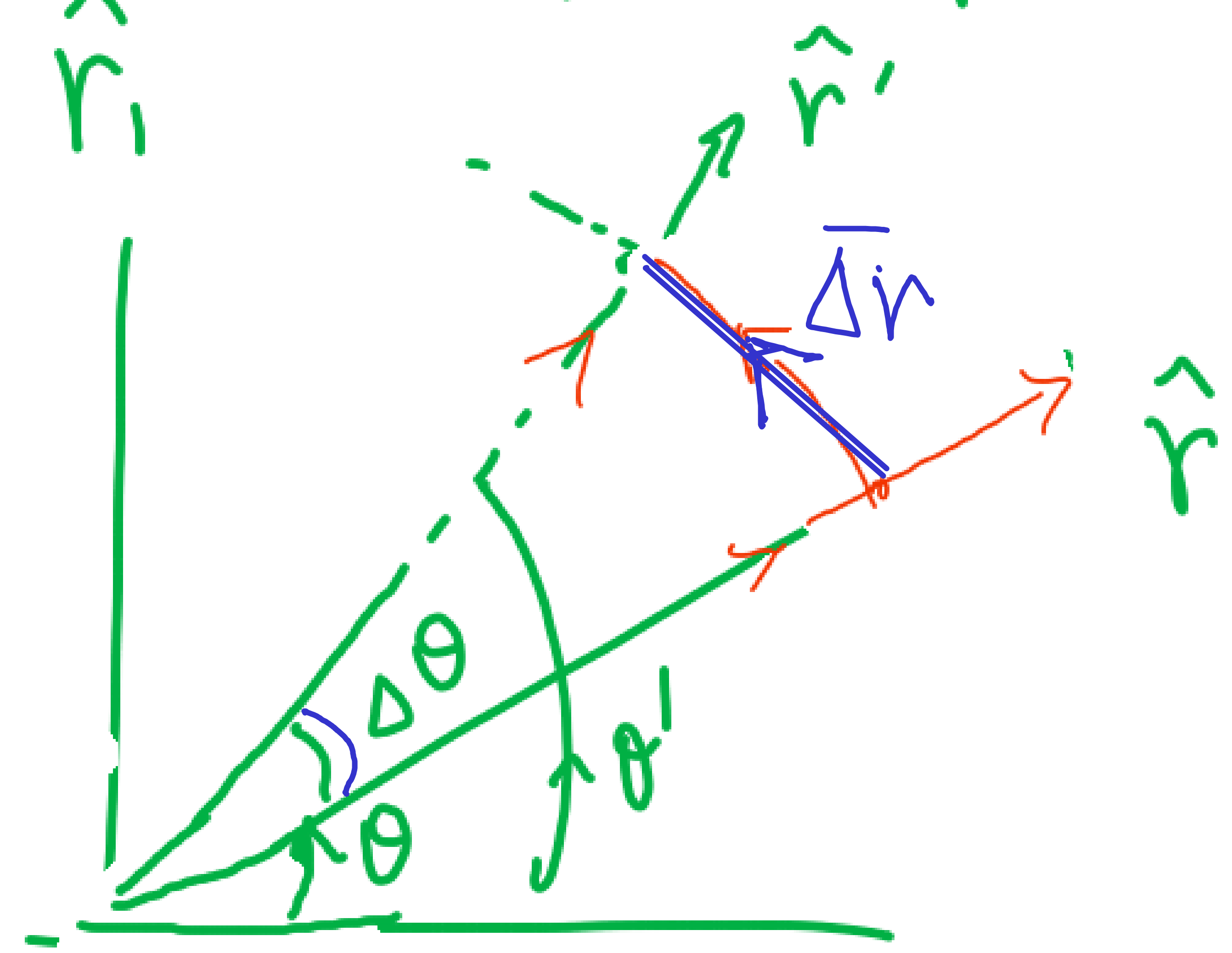
Compute velocity $\frac{d\bar{r}}{dt}$:

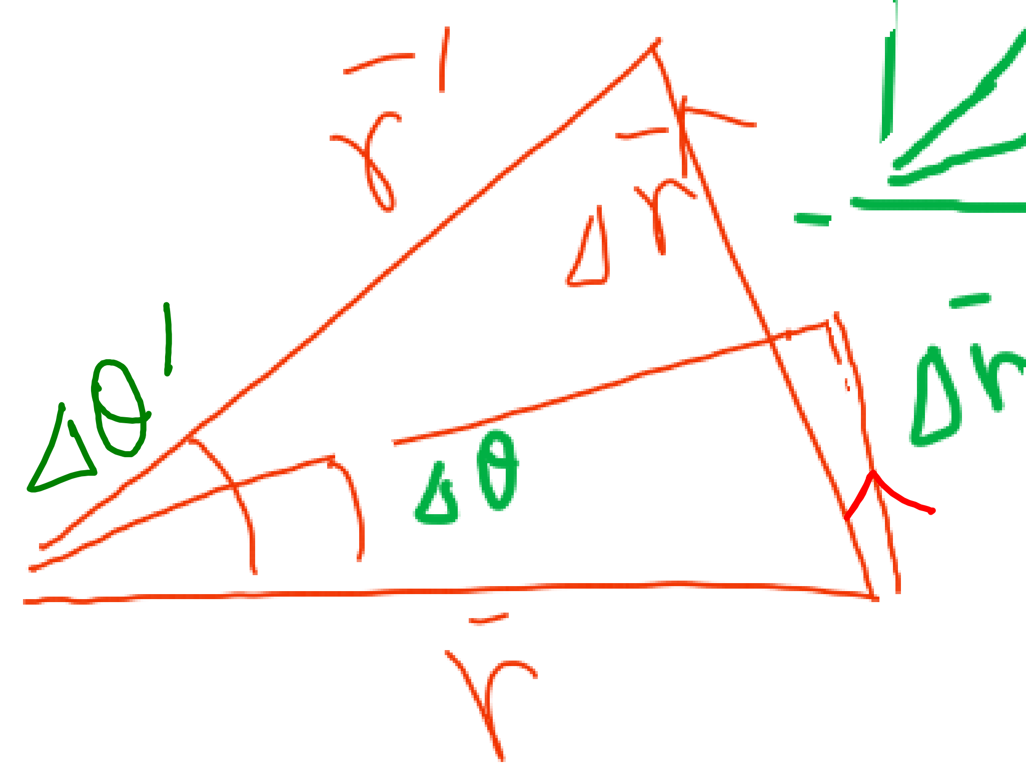
$$\bar{r} = r\hat{r}$$

$$\frac{d\hat{r}(r,\theta)}{dt} = \cancel{\frac{\partial \hat{r}}{\partial r}}\,\dot{r} + \frac{\partial \hat{r}}{\partial \theta}\,\dot{\theta}$$

$$\frac{d\bar{r}}{dt} = \dot{r}\hat{r} + r\frac{d\hat{r}}{dt} = \dot{r}\hat{r} + r\frac{\partial \hat{r}}{\partial \theta}\dot{\theta} \boxed{= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}}$$







$r' = r$

because $\partial\hat{r}/\partial\theta$ is partial derivatv

$$\frac{\partial \hat{r}}{\partial \theta} = \lim_{\Delta\theta \to 0} \frac{\hat{r}' - \hat{r}}{\Delta\theta} = \left(\frac{\bar{r}'}{r'} - \frac{\bar{r}}{r}\right)\frac{1}{\Delta\theta} = \left(\bar{r}' - \bar{r}\right)\frac{1}{r\Delta\theta} = \cancel{\frac{r\Delta\theta}{r\Delta\theta}}\,\hat{\theta}$$

As $\Delta\theta \to 0$, $\Delta\bar{r}$ becomes $\perp$ to $\bar{r}$

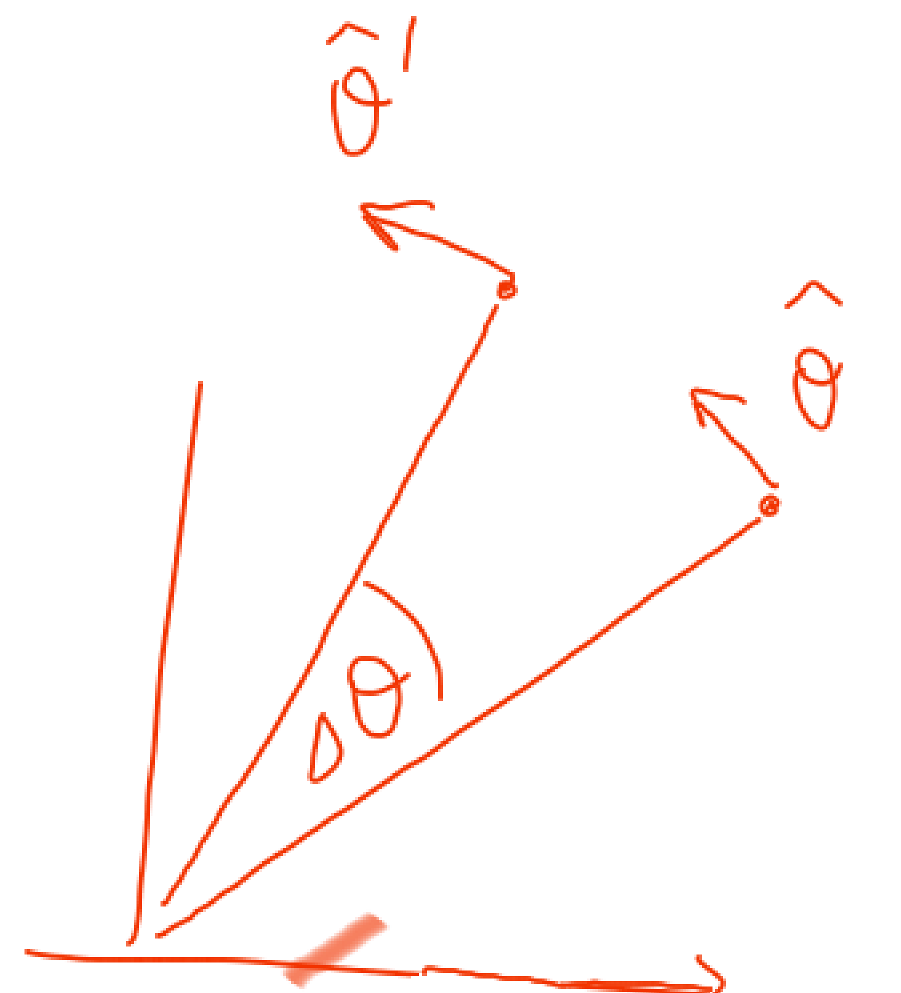
All the partial derivatives of the unit vectors.

$$\frac{\partial \hat{r}}{\partial \theta} \approx \frac{\hat{r}' - \hat{r}}{\Delta\theta} = \frac{\Delta\theta\, \hat{\theta}}{\Delta\theta} = \hat{\theta}$$

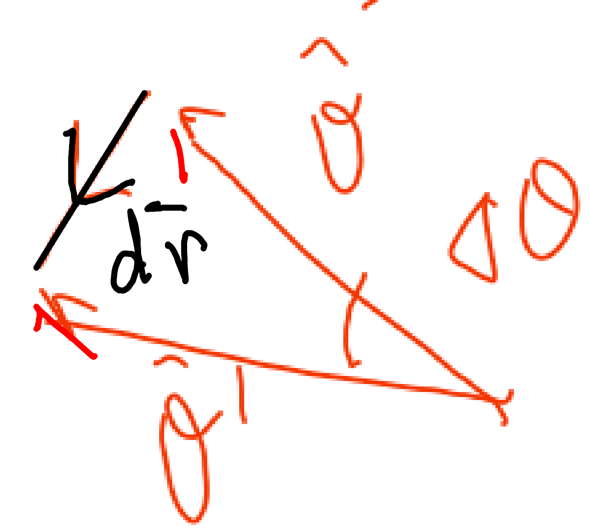
$$\frac{\partial \hat{r}}{\partial r} = 0$$

$$\frac{\partial \hat{\theta}}{\partial r} = 0$$

$$\frac{\partial \hat{\theta}}{\partial \theta} = -\hat{r}$$



$-\hat{r}\,\Delta\theta = d\bar{r}$



Note: as $\Delta\theta \to 0$, $d\bar{r}$ becomes $\perp$ to $\hat{\theta}$

Calculate Acc^n.

$$\frac{\partial \hat{r}}{\partial \theta} = \hat{\theta}$$

$$\bar{v} = \frac{d\bar{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\frac{d\hat{\theta}}{dt} = \frac{\partial \hat{\theta}}{\partial \theta}\dot{\theta} = -\hat{r}\dot{\theta}$$

$$\frac{\partial \hat{\theta}}{\partial \theta} = -\hat{r}$$

$$\bar{a} = \frac{d^2\bar{r}}{dt} = \ddot{r}\hat{r} + \dot{r}\frac{d\hat{r}}{dt} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\frac{d\hat{\theta}}{dt}$$

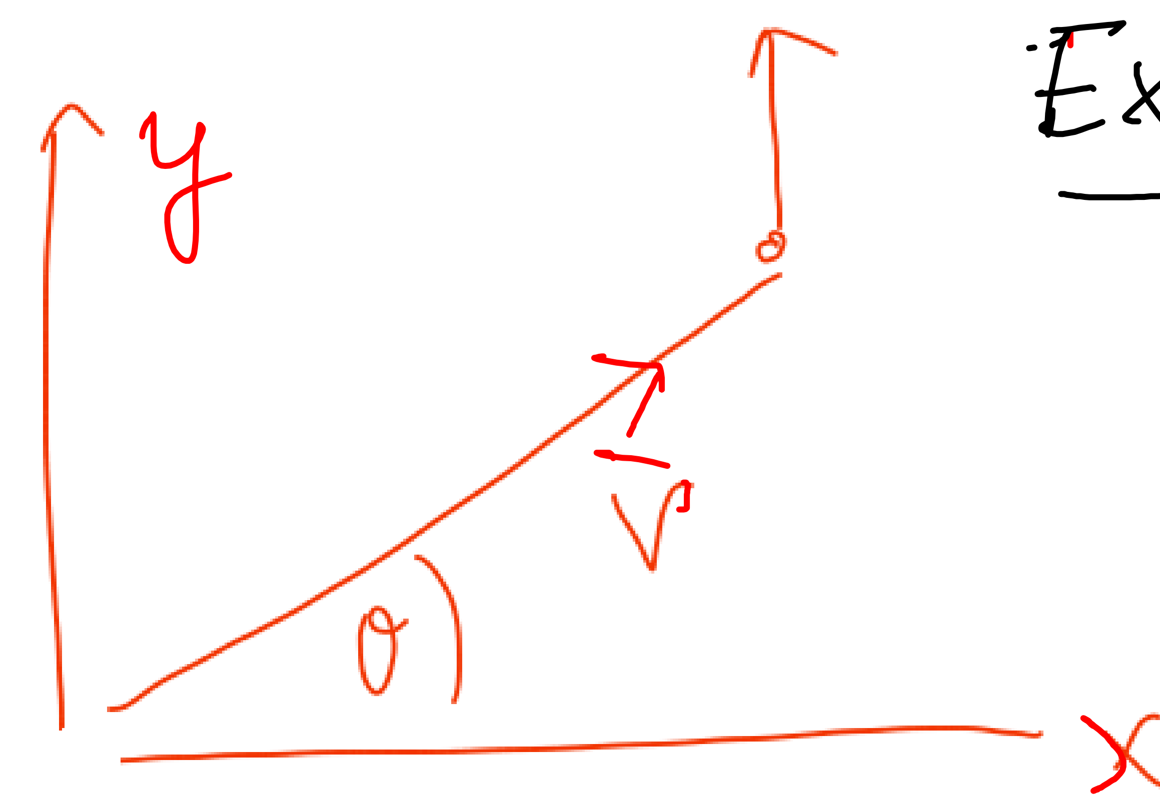
$$= \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$$

$$\frac{\partial \hat{r}}{\partial r} = \frac{\partial \hat{\theta}}{\partial r} = 0$$

$$= \hat{r}\left(\ddot{r} - \underline{r\dot{\theta}^2}\right) + \hat{\theta}\left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)$$

$r(t)$

$\theta(t)$



Examples 1) Stdy circular motn $\dot{r} = 0$

only $-\hat{r}\, r\dot{\theta}^2 \neq 0$

centripetal accl^n

2) $\dot{\theta} = 0, \ddot{\theta} = 0$

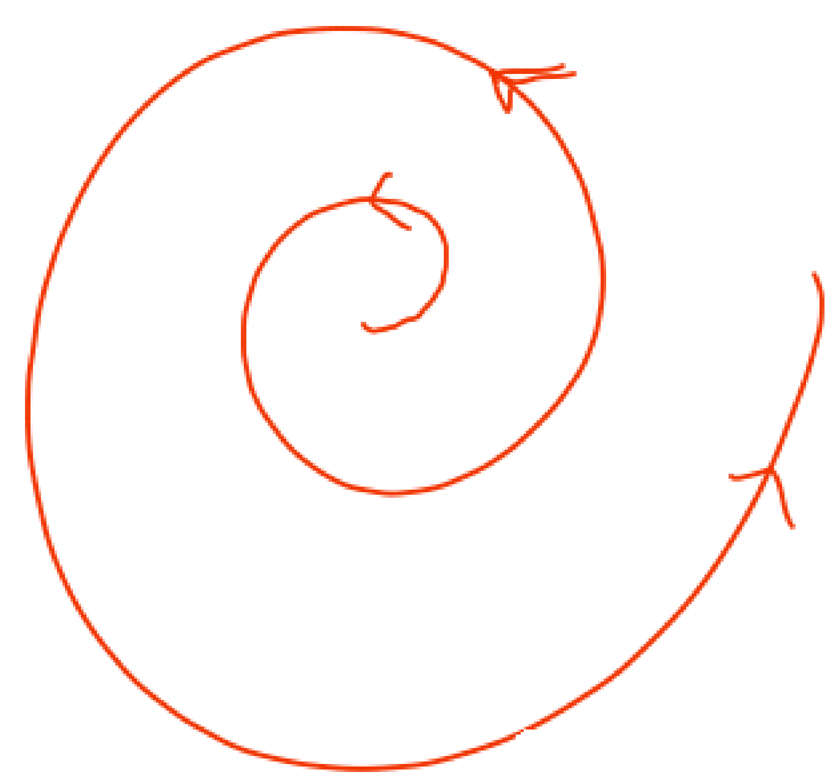
1-D motn along $\hat{r}$

3) $\dot{\theta} = \omega_0$, $\dot{r} = \alpha$

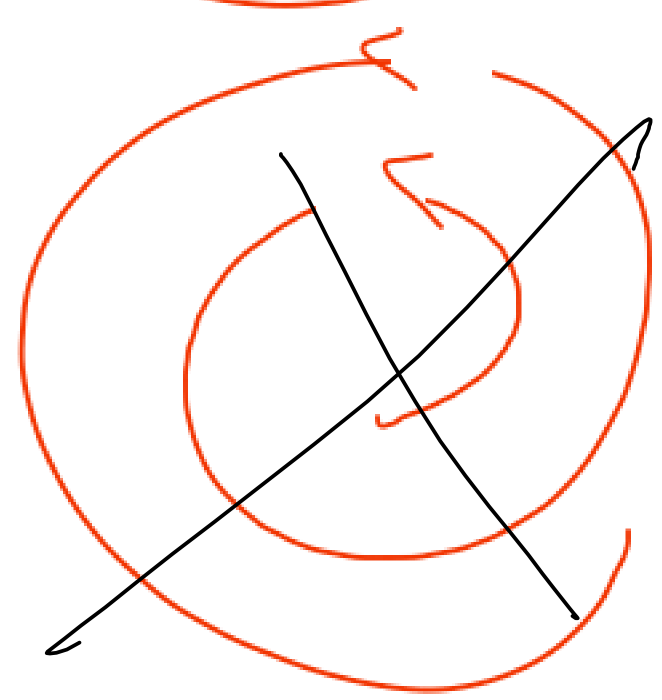
$\downarrow$

$r(t) = \alpha t + \alpha_0$

$\alpha > 0$

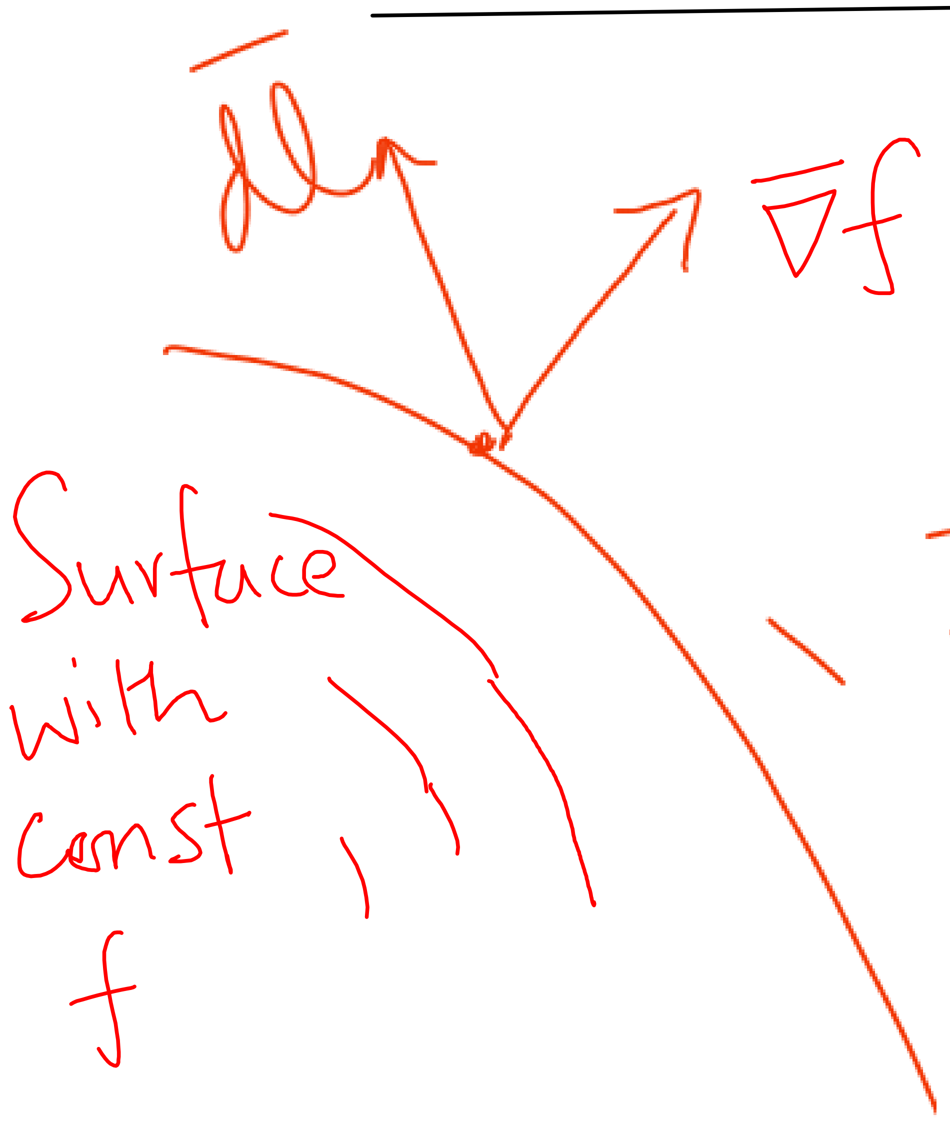


$\alpha < 0$



Since $\dot{\theta} = \omega_0 > 0$

θ keeps increasing while $r(t)$ decreases grad, div, curl : all are certain types of space derivatives, but what's the difference?



$\vec{\nabla} f$ is $\perp$ to the surface

$f(x,y,z) = c_1$

$df = \bar{\nabla} f \cdot \bar{dl}$ (change in f along $\bar{dl}$)

$\bar{\nabla} \times \bar{A}(x,y,z)$

Fields with nonzero curl

$\overline{\text{div}} \cdot \bar{A}(x\, y\, z) = c_0 \hat{r}$