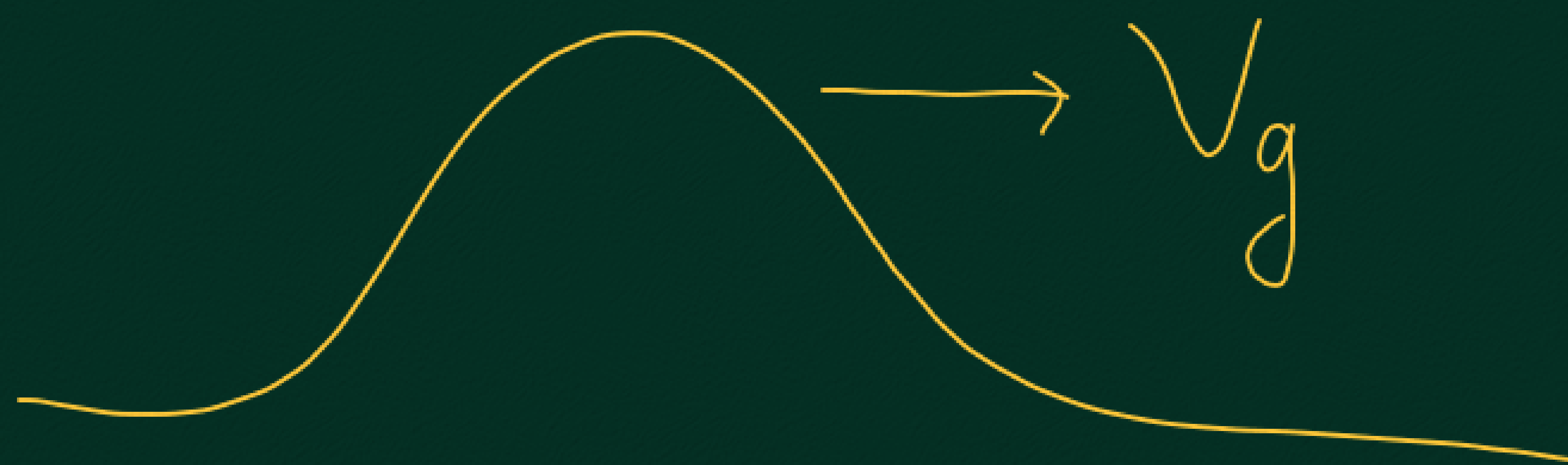
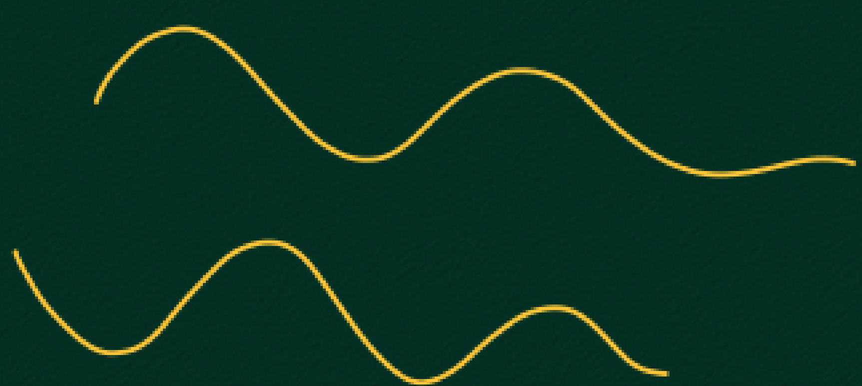




$$f(x) = f(x+L)$$

period L

$$= \sum_{n=0}^{\infty} \left(A_n \cos \frac{2\pi n}{L} + B_n \sin \frac{2\pi n}{L} \right)$$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} f(x) \cos \frac{2\pi m}{L} x \, dx$$

$$= \sum_{n=0}^{\infty} \int \left(A_n \cos \frac{2\pi n}{L} x \cos \frac{2\pi m}{L} x \right.$$

$$=$$

$$\sum \frac{1}{2}\int_{-L/2}^{L/2} dx \left[\cos \frac{2\pi(n+m)}{L} x + \cos \frac{2\pi}{L}(n-m) x\right]$$

$$\frac{1}{2\pi/L}\left[\sin \frac{2\pi(n+m)}{L} x\right]_{-\frac{L}{2}}^{L/2} + \sin \frac{2\pi}{L}(n-m)x$$

$n, m > 0$

$$\sin\left(\frac{L}{2}\frac{2\pi}{L}(n+m)\right)$$

$$\sin(\pi(n+m))$$

$= 0$

$n = m$

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} dx \, \frac{A_n}{2} = A_n \frac{L}{2}$$

$$\int \sin \frac{2\pi n}{L} x \, \cos \frac{2\pi m}{L} x$$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

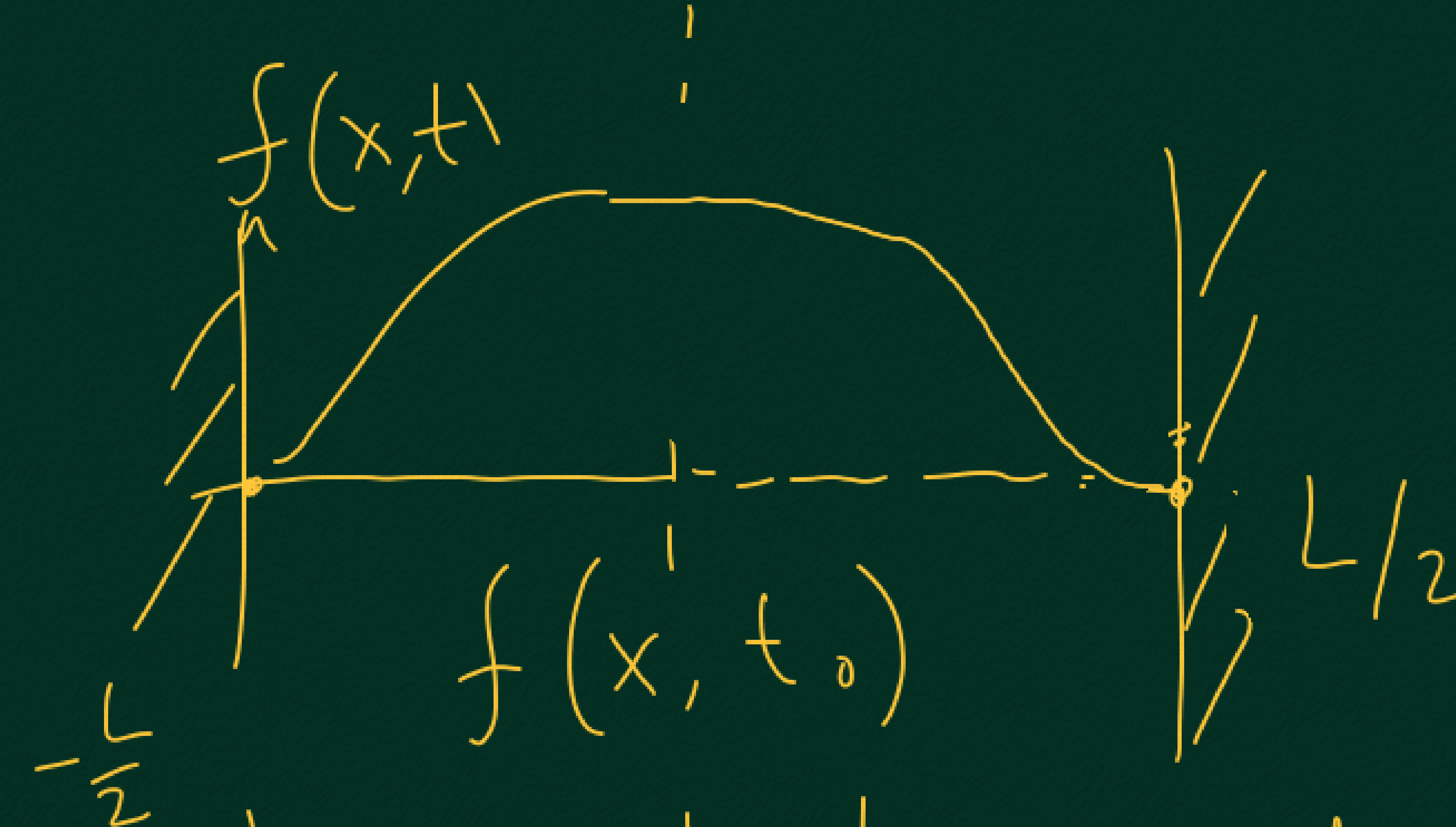
$$\frac{B_n}{2}\int dx \left[\sin \frac{2\pi}{L}(n+m)x + \sin \frac{2\pi}{L}(n-m)x\right]$$

$$\cos \frac{2\pi}{L}(n+m)x \Big|_{-\frac{L}{2}}^{L/2} \to 0 \qquad 0$$

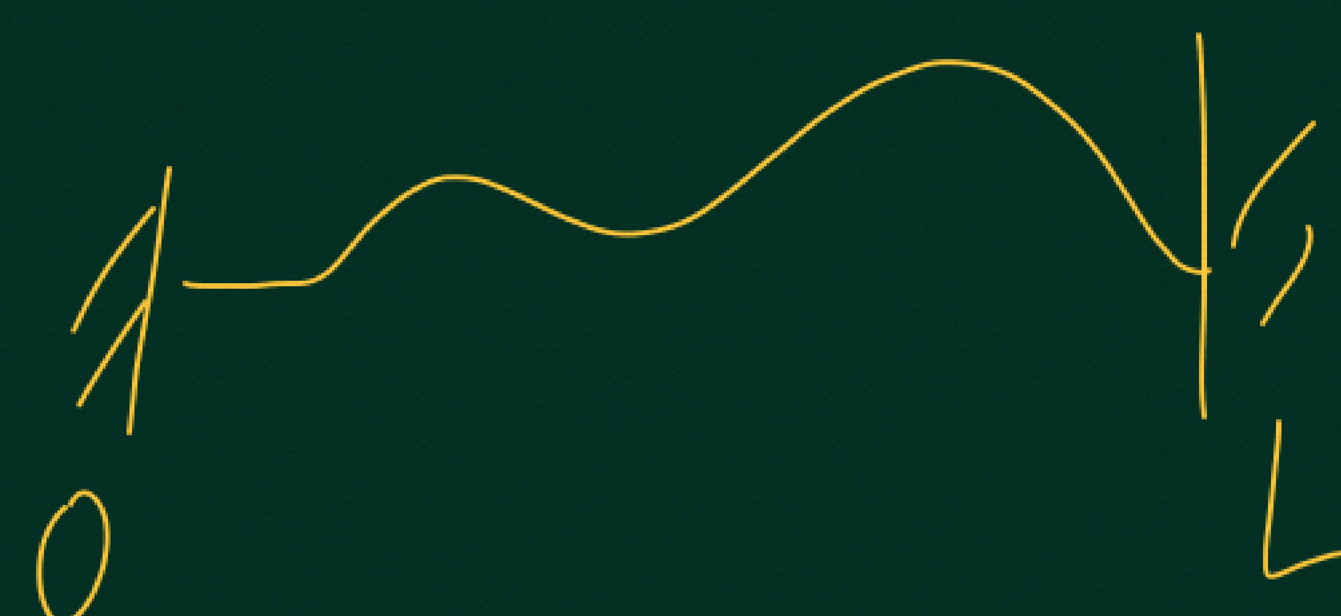
$$\int_{-\frac{L}{2}}^{L/2} f(x)\cos\frac{2\pi m}{L}x = A_m \frac{L}{2}$$

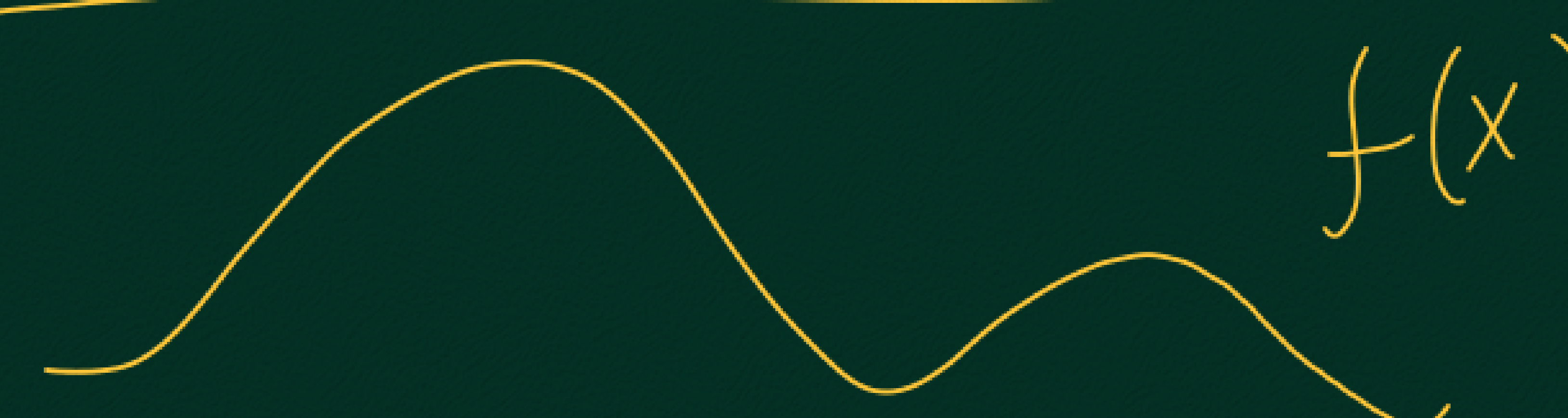
$$\frac{2}{L}\int_{-\frac{L}{2}}^{L/2} f(x)\cos\frac{2\pi m}{L}x = A_m$$

$$\frac{2}{L}\int_{-\frac{L}{2}}^{L/2} f(x)\sin\frac{2\pi m}{L}x \quad B_m$$

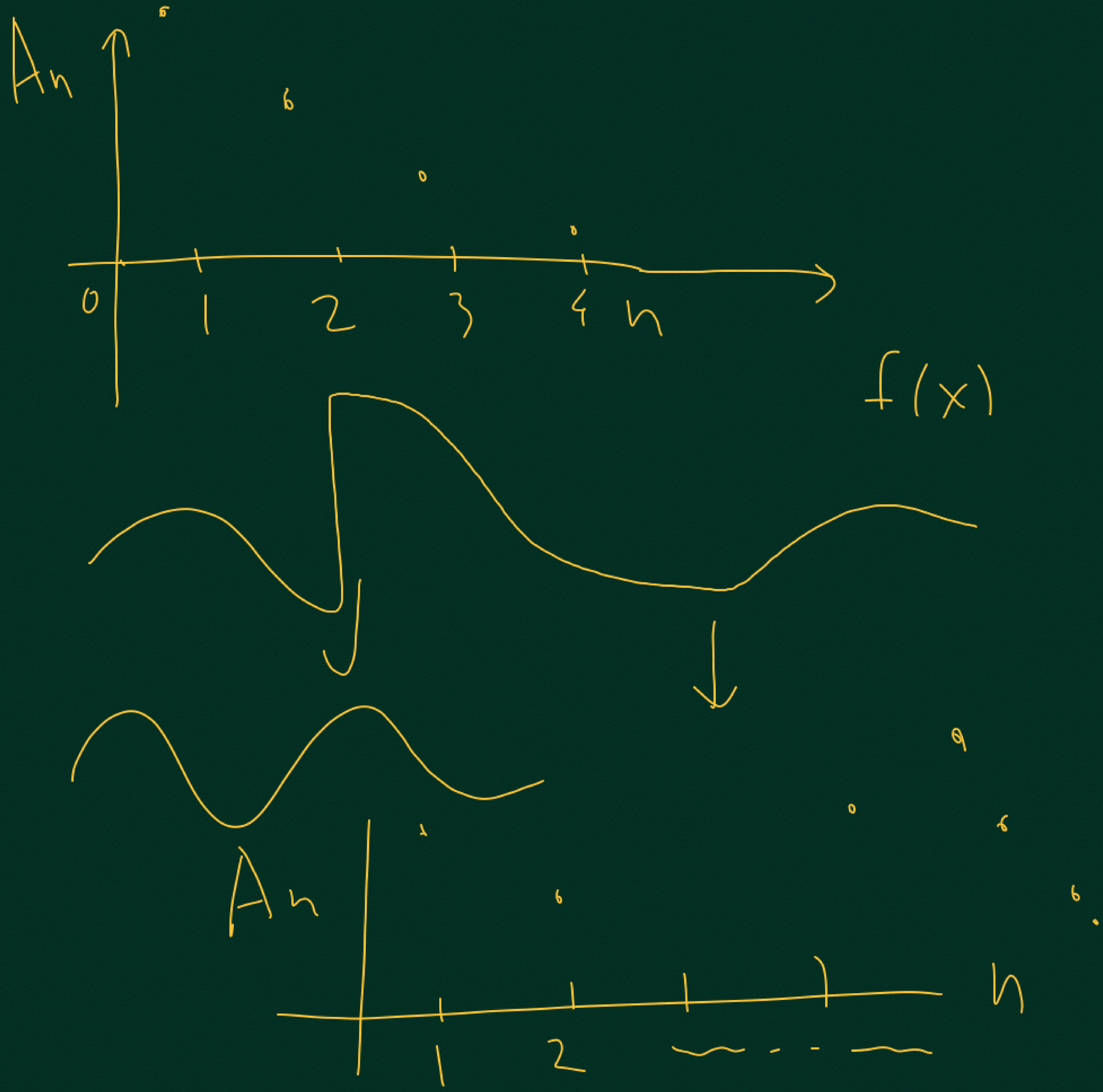


string betwn walls





A_n, B_n very small for short λ mode



An
0
1
2
3
4
n
f(x)
An
1
2
n

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \qquad i = \sqrt{-1}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$f(x) = \sum_{n=0}^{\infty} \left[\frac{A_n}{2}\left(e^{\frac{i2\pi n}{L}x} + e^{-\frac{2\pi}{L}nx} \right) + \frac{B_n}{2i}\left(e^{\frac{i2\pi n}{L}x} - e^{\frac{-i2\pi n x}{L}} \right) \right]$$

$$\sum_{n=0}^{\infty} \left[e^{\frac{i2\pi n}{L}x} \underbrace{\left(\frac{A_n + B_n}{2} \right)}_{a_n} + e^{-\frac{i2\pi}{L}nx} \underbrace{\left(\frac{A_n - B_n}{2i} \right)}_{= a_{-n}} \right.$$

$$= \sum_{n=-\infty}^{\infty} a_n e^{\frac{i2\pi n x}{L}}$$

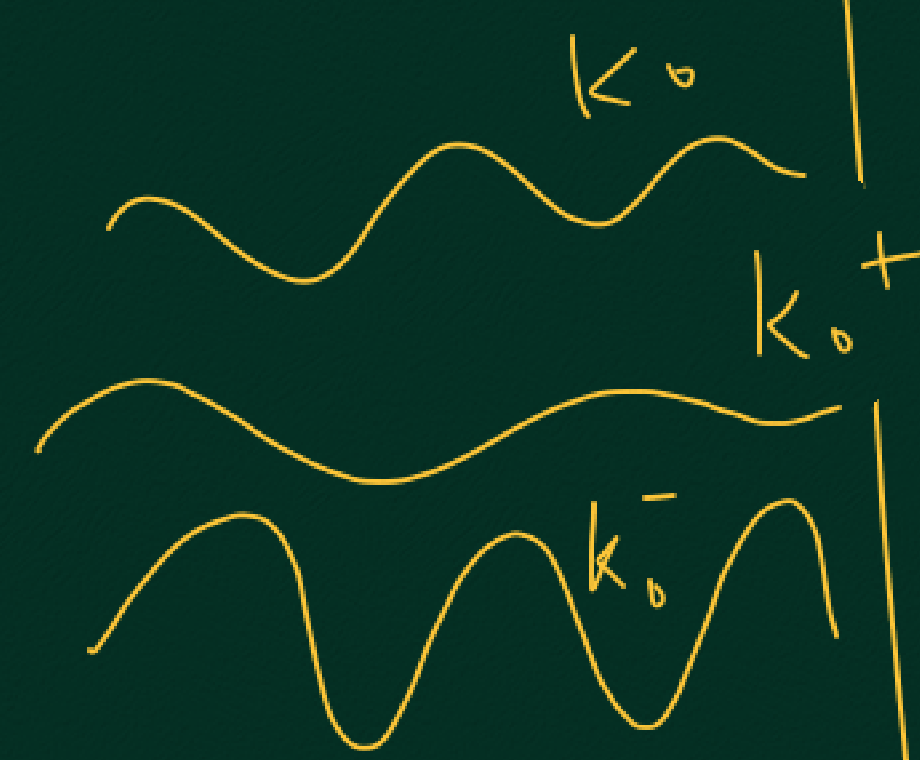
$$k = \underbrace{\frac{2\pi}{L}}_{=\Delta k} n = \Delta k \, n \qquad \left(\begin{array}{c}\text{Cntns} \\ \text{var } k\end{array}\right)$$

if $L \to \infty$

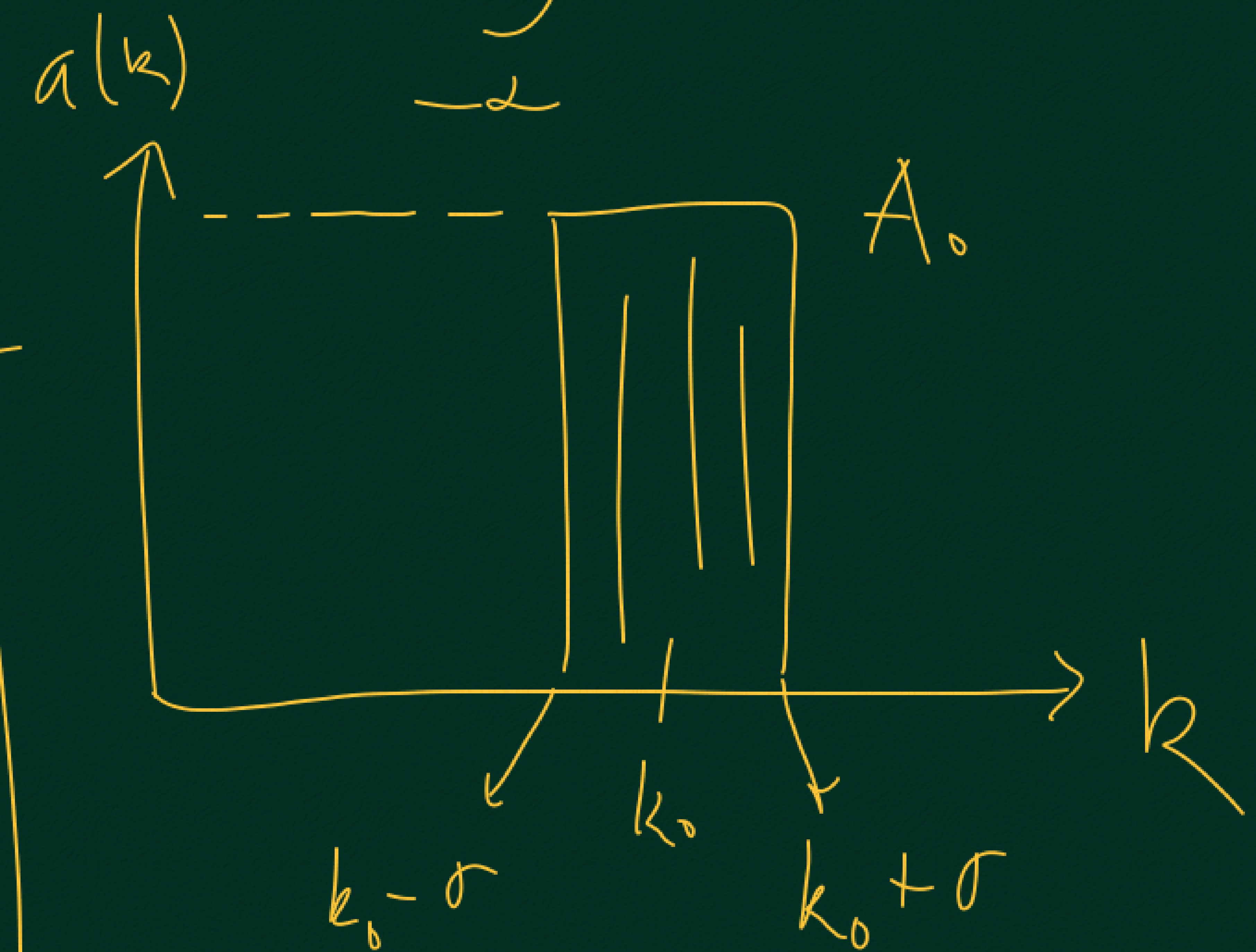
$$k_n = \Delta k \, n$$

$$k_{n+1} = \Delta k \,(n+1)$$

$$k_{n+1} - k_n = \Delta k \to 0 \qquad (\text{for } L \to \infty)$$



$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} a(k)\, e^{ikx}\, dx$$



$$q = k - k_0$$

$\omega(k)$: Dispersion

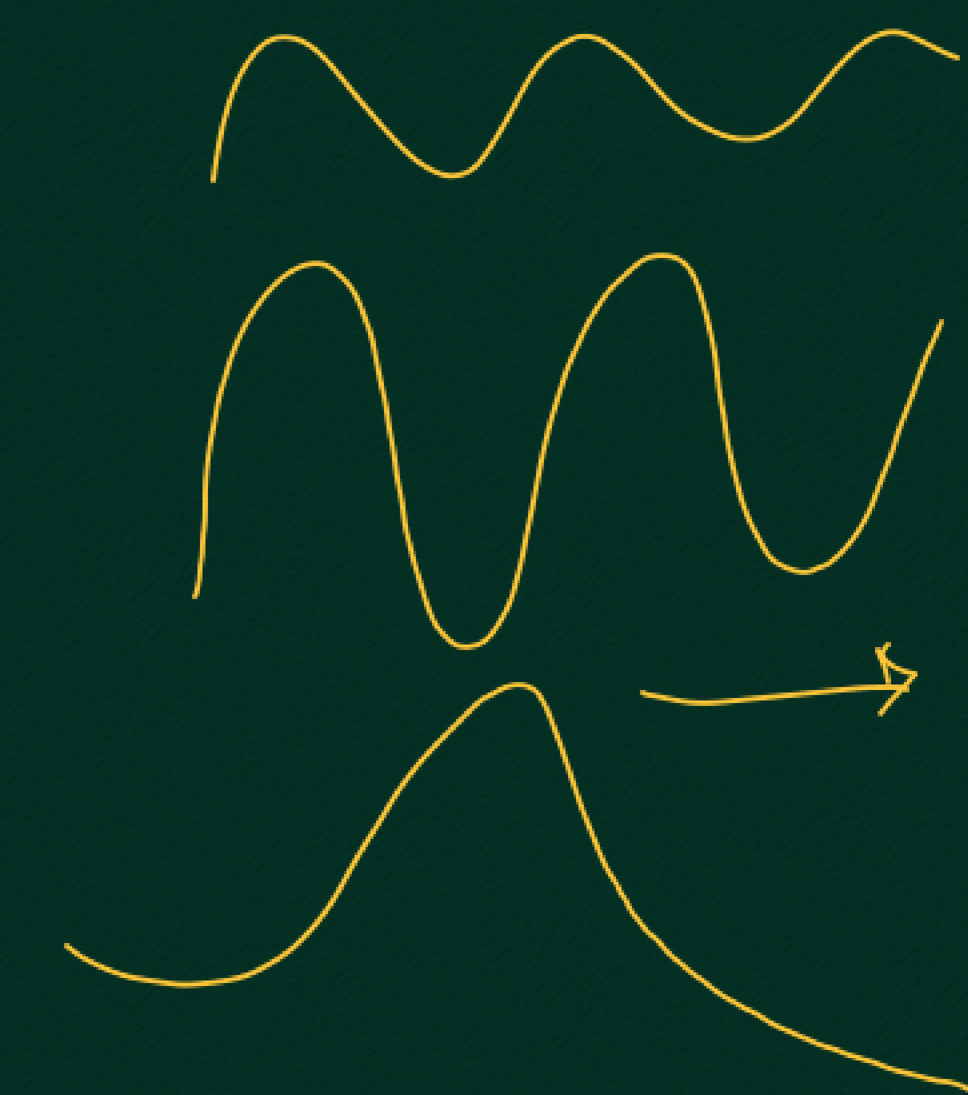
$\rightarrow v_g = d\omega / dk$

$v_k = \dfrac{\omega}{k}$

$e^{i(k_0 x - \omega_0 t)}$ A_0

if $\omega \propto k \Rightarrow v_k = \text{const}$

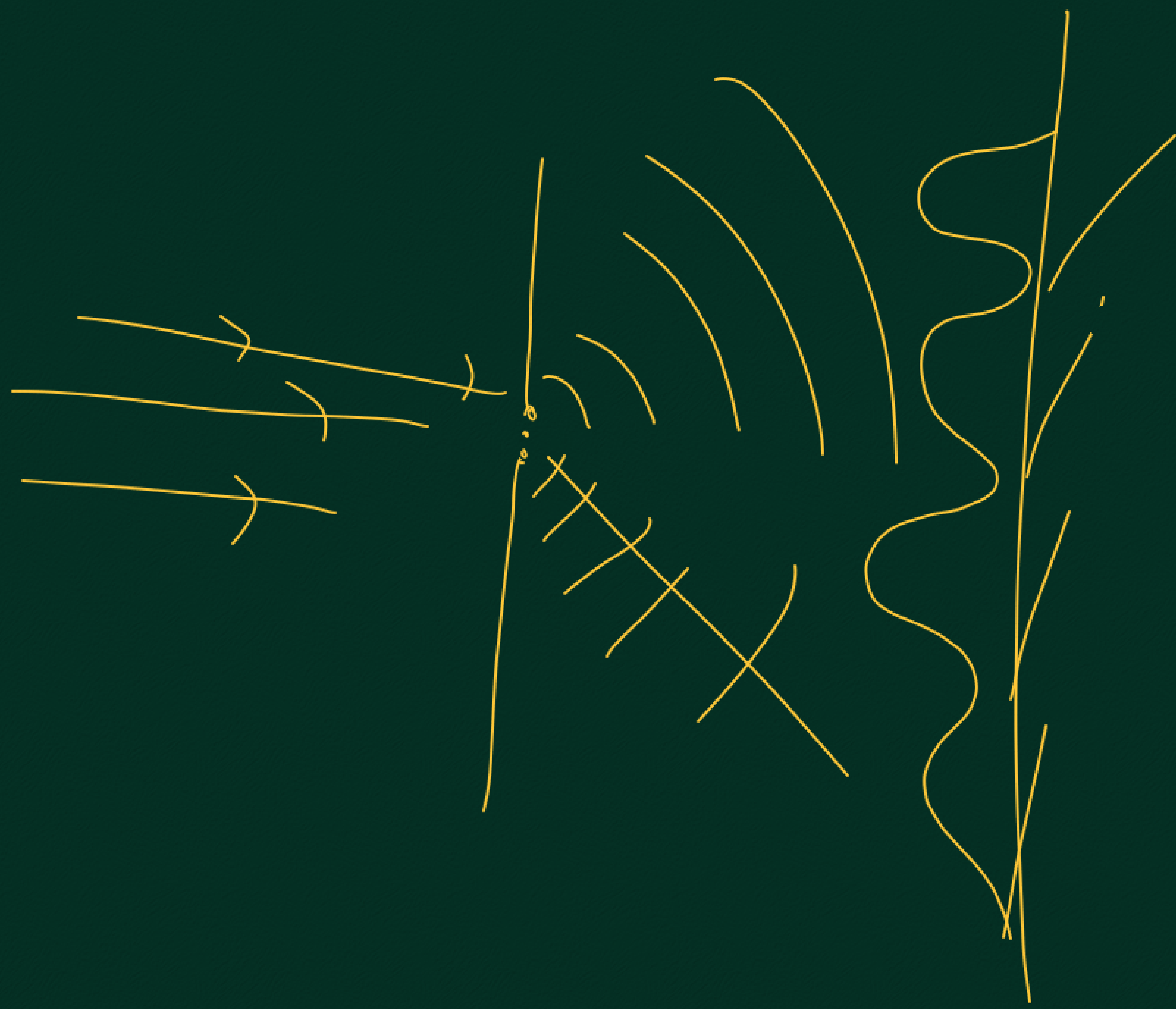
$e^{-\alpha x^2}$



$kx - \omega(k)\, t$

$= (k_0 + q)\, x - t\left[\underbrace{\omega(k_0)}_{\omega_0} + \underbrace{\dfrac{d\omega}{dk}\Big|_{k_0}}_{= \frac{d\omega(k_0)}{dk_0}} + \cdots\right]$

$\rightarrow v_g$



$$\Delta x \, \Delta p \sim \hbar$$

$$10^{-34}$$