PH110: Tutorial Sheet 4

This tutorial sheet contains problems related to the central force motion

1. * The effective potential for the central force problem is

$$V_{eff}(r) = \frac{L^2}{2\mu r^2} + V(r),$$

where $V(r)$ is the potential energy corresponding to the central force, and L is the angular momentum. Consider the case of gravitational motion so that $V(r) = -\frac{C}{r}$, with $C > 0$. Plot the effective potential as a function of r, and argue based upon the plot that for $E \geq 0$, orbits will be unbound, while for $E < 0$, we will obtain bound orbits, where E is the total energy of the system.

2. * Suppose a satellite is moving around a planet in a circular orbit of radius r_0. Due to a collision with another object, satellite's orbit gets perturbed. Show that the radial position of the satellite will execute simple harmonic motion with $\omega = \frac{L}{mr_0^2}$, where L is the initial angular momentum of the satellite.

3. * In this problem we will explore an alternative way of obtaining the equation of the curve corresponding to the central force orbits.

   (a) Make a change of variable $u = \frac{1}{r}$ and show that the $u - \theta$ differential equation for a central force $\mathbf{F}(\mathbf{r}) = f(r)\hat{\mathbf{r}}$ is

$$\frac{d^2u}{d\theta^2} + u = -\frac{\mu}{u^2L^2} f(\frac{1}{u})$$

   (b) Integrate this differential equation for the case of gravitational force ($f(r) = -\frac{C}{r^2}$), and show that it leads to the same orbit as obtained in the lectures

$$r = \frac{r_0}{1 - \epsilon\cos\theta}$$

4. A particle of mass m is moving under the influence of a central force $\mathbf{F}(\mathbf{r}) = -\frac{C}{r^3}\hat{\mathbf{r}}$, with $C > 0$. Find the nonzero values of angular momentum L for which the particle will move in a circular orbit.

5. A geostationary orbit is one in which a satellite moves in a circular orbit at the given height in the equatorial plane, so that its angular velocity of rotation around earth is same as earth's angular velocity, thereby, making it look stationary when seen from a point on equator. Assuming that the earth's rotational velocity, and radius, respectively, are $\Omega_e = \frac{2\pi}{86400}$ rad/s, and$R_e = 6400$ km, calculate the altitude of the satellite, and its orbital velocity.

6. Suppose the unit of mass is taken to be the mass of the Earth, the unit of length is taken to be the mean distance between the Earth and the Sun (called the "astronomical unit" of distance) and the unit of time is taken to be one year. In this new system what would be be numerical value of the universal Gravitational constant ?