PH110: QM : Tutorial Sheet 1

Notes:

1. * marked problems will be solved in the Wednesday tutorial class.
2. Please make sure that you do the assignment by yourself. You can consult your classmates and ensure you understand the concept. However, do not copy assignments from others.

Compton Scattering

1. * A photon of energy $h\nu$ is scattered through 90° by an electron initially at rest. The scattered photon has a wavelength twice that of the incident photon. Find the frequency of the incident photon and the recoil angle of the electron. tanh(0.5)

2. Derive the relation for the recoil kinetic energy of the electron and its recoil angle ϕ in Compton scattering. Show that

$$\boxed{\begin{aligned} \text{K.E. (electron)} &= \frac{\Delta\lambda/\lambda}{1+(\Delta\lambda/\lambda)} hf \\ \cot\phi &= \left(1+\frac{hf}{mc^2}\right)\tan\frac{\theta}{2} \end{aligned}}$$

3. Show that a free electron cannot absorb a photon so that a photoelectron requires bound electron. However, the electron can be free in Compton Effect. Why?

4. Two Compton scattering experiments were performed using x-rays (incident energies E_1 and $E_2 = E_1/2$). In the first experiment, the increase in wavelength of the scattered x-ray, when measured at an angle $\theta = 45°$, is 7×10^{-14} m. In the second experiment, the wavelength of the scattered x-ray, when measured at an angle $\theta = 60°$, is 9.9×10^{-12} m.

   (a) Calculate the Compton wavelength and the mass (m) of the scatterer. $\frac{h}{mc}$

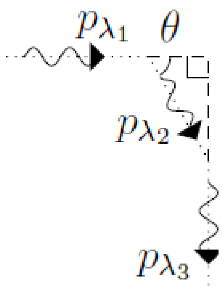
   (b) Find the wavelengths of the incident x-rays in the two experiments.

5. Find the smallest energy that a photon can have and still transfer 50% of its energy to an electron initially at rest. $\frac{m_e c^2}{2}$

6. *γ-rays are scattered from electrons initially at rest. Assume the it is back-scattered and its energy is much larger than the electron's rest-mass energy, $E \gg m_e c^2$.
   (a) Calculate the wavelength shift
   (b) Show that the energy of the scattered beam is half the rest mass energy of the electron, regardless of the energy of the incident beam
   (c) Calculate the electron's recoil kinetic energy if the energy of the incident radiation is 150MeV

7. In Compton Scattering, show that the maximum energy of the scattered photon will be $2m_0c^2$, irrespective of the energy of the incident photon. Find the value of θ_0, the angle at which the maximum energy occurs.

8. * In a Compton scattering experiment (see figure), X-rays scattered off a free electron initially at rest at an angle $\theta(> \pi/4)$), gets re-scattered by another free electron, also initially at rest.



(a) If $\lambda_3 - \lambda_1 = 1.538 \times 10^{-12}$ m, find the value of θ. 63.41757498 degrees

(b) If $\lambda_2 = 68 \times 10^{-12}$ m , find the angle at which the first electron recoils due to the collision.

de Broglie hypothesis

1. Calculate the wavelength of the matter waves associated with the following:
   (a) A 2000 kg car moving with a speed of 100 km/h.
   (b) A 0.28 kg cricket ball moving with a speed of 40 m/s.
   (c) An electron moving with a speed of 10^7 m/s.

   Compare in each case the result with the respective dimension of the object. In which case will it be possible to observe the wave nature.

2. Show that the Bohr's angular momentum quantization leads to the formation of standing waves by the electrons along the orbital circumference in hydrogen atom.

3. Determine the de Broglie wavelength of a particle of mass m and kinetic energy K. Do this for both (a) a relativistic and (b) a non-relativistic particle.

4. *Thermal kinetic energy of a hydrogen atom is $\sim k_B T$ and the radius is $\sim r_1$ (= 0.53 Å, radius of the $n = 1$ Bohr orbit). Find the temperature at which its de Broglie wavelength has a value of $2r_1$. Take the mass of the hydrogen atom to be that of a proton. 844.486 K

(a) $\lambda = \dfrac{h}{\sqrt{2mK}}$

(b) $\lambda = \dfrac{h}{\sqrt{2mK + \frac{K^2}{c^2}}}$