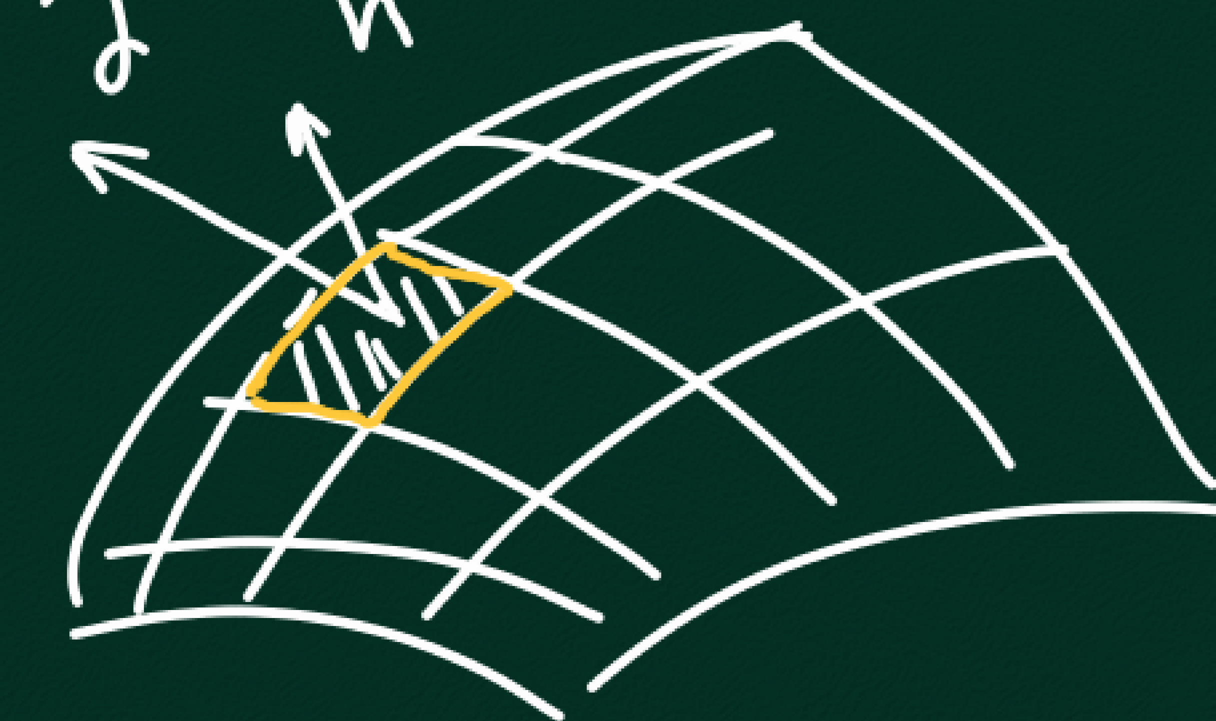
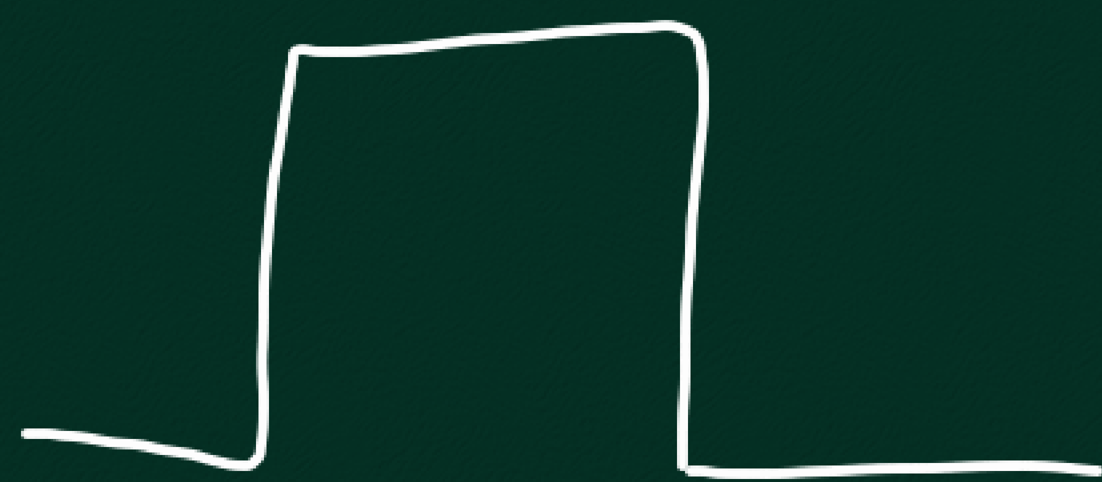
$$\bar{j} = \frac{-i\hbar}{2m}\left(\psi^* \bar{\nabla}\psi - \psi\bar{\nabla}\psi^*\right)$$





$\rho(\bar{x}, t)$

$$\frac{\partial \rho(\bar{x})}{\partial t} + \bar{\nabla}\cdot\bar{j} = 0$$

Continuity Eqn

# of ptcls inside a closed vol.

$$N_{\text{inside}} = \oint_{\text{vol}} \rho(\bar{x})\, d\tau$$

$d\tau \rightarrow$ vol. element

$$-\frac{\partial}{\partial t} N_{inside} = \oint_{surfc} \bar{J} \cdot \bar{ds}$$

$$-\frac{\partial}{\partial t}\int \rho \, d\tau = \oint_{vol} \bar{\nabla} \cdot \bar{j} \; d\tau$$

$$\oint_{vol} \left(\frac{\partial \rho}{\partial t} + \bar{\nabla} \cdot \bar{j} \right) d\tau = 0$$

Integrand = 0

$$\int_a^b f(x) \, dx = 0$$

$$\not\Rightarrow f(x) = 0$$

for arbitrary shape of vol element $\Rightarrow$ Integrand = 0

$$\psi^* \left[-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi \right] = i\hbar \psi^* \frac{\partial \psi}{\partial t}$$

$$\psi \left[-\frac{\hbar^2}{2m} \nabla^2 \psi^* + V\psi^* \right] = -i\hbar \psi \frac{\partial \psi^*}{\partial t}$$

$$-\frac{\hbar^2}{2m} \Big[\underbrace{\psi^* \nabla^2 \psi - \psi \nabla^2 \psi^*}_{B} \Big] = i\hbar \frac{\partial}{\partial t} \left(\psi^* \psi \right)$$

$$\bar{\nabla} \cdot \left[-\frac{\hbar^{\cancel{2}}}{2m} \left(\psi^* \bar{\nabla} \psi - \psi \bar{\nabla} \psi^* \right) \right] = \cancel{i\hbar} \frac{\partial}{\partial t} \underbrace{|\psi|^2}_{\rho}$$

$$\bar{\nabla} \cdot \bar{j} + \frac{\partial \rho}{\partial t} = 0$$

$$\bar\nabla\cdot(\varphi\bar A) = \bar\nabla\varphi\cdot\bar A + \varphi\bar\nabla\cdot\bar A$$

$$\bar A = \bar\nabla\psi, \quad \varphi = \psi^*$$

$$\bar\nabla\cdot(\psi^*\bar\nabla\psi) = \bar\nabla\psi^*\cdot\bar\nabla\psi + \psi^*\nabla^2\psi$$

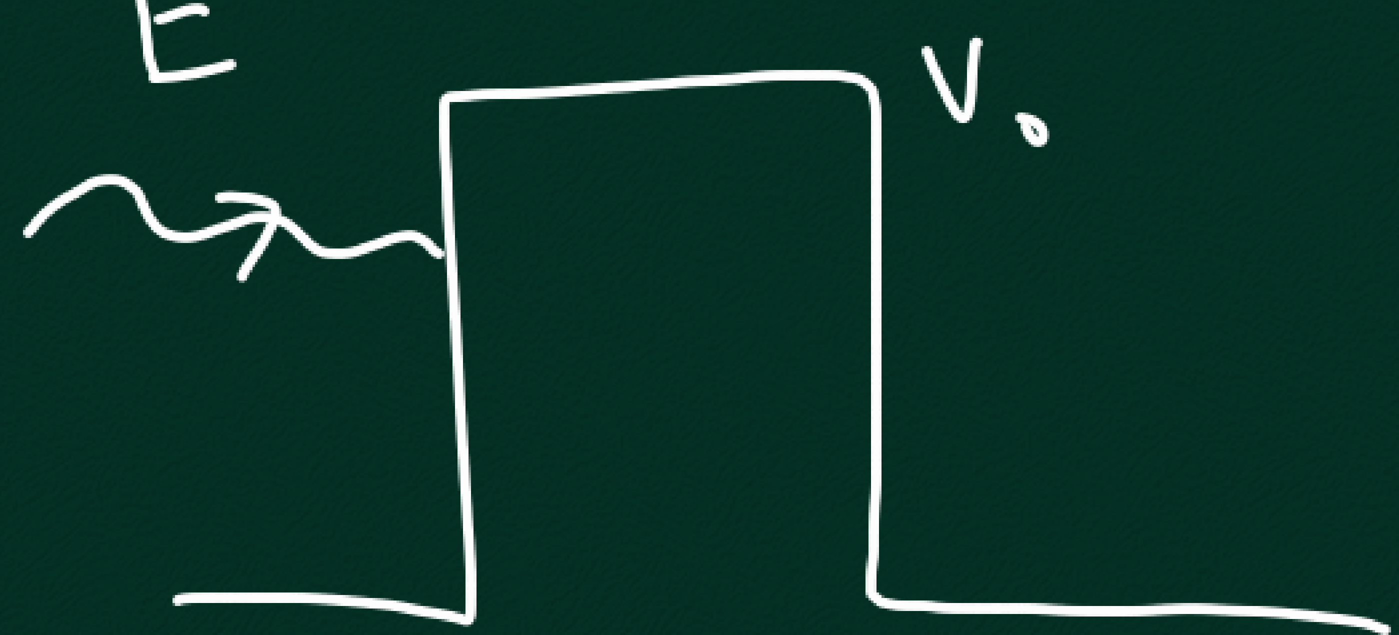
$$A = \bar\nabla\psi^*, \quad \varphi = \psi$$

$$\bar\nabla\cdot(\psi\bar\nabla\psi^*) = \bar\nabla\psi^*\cdot\bar\nabla\psi + \psi\nabla^2\psi^*$$

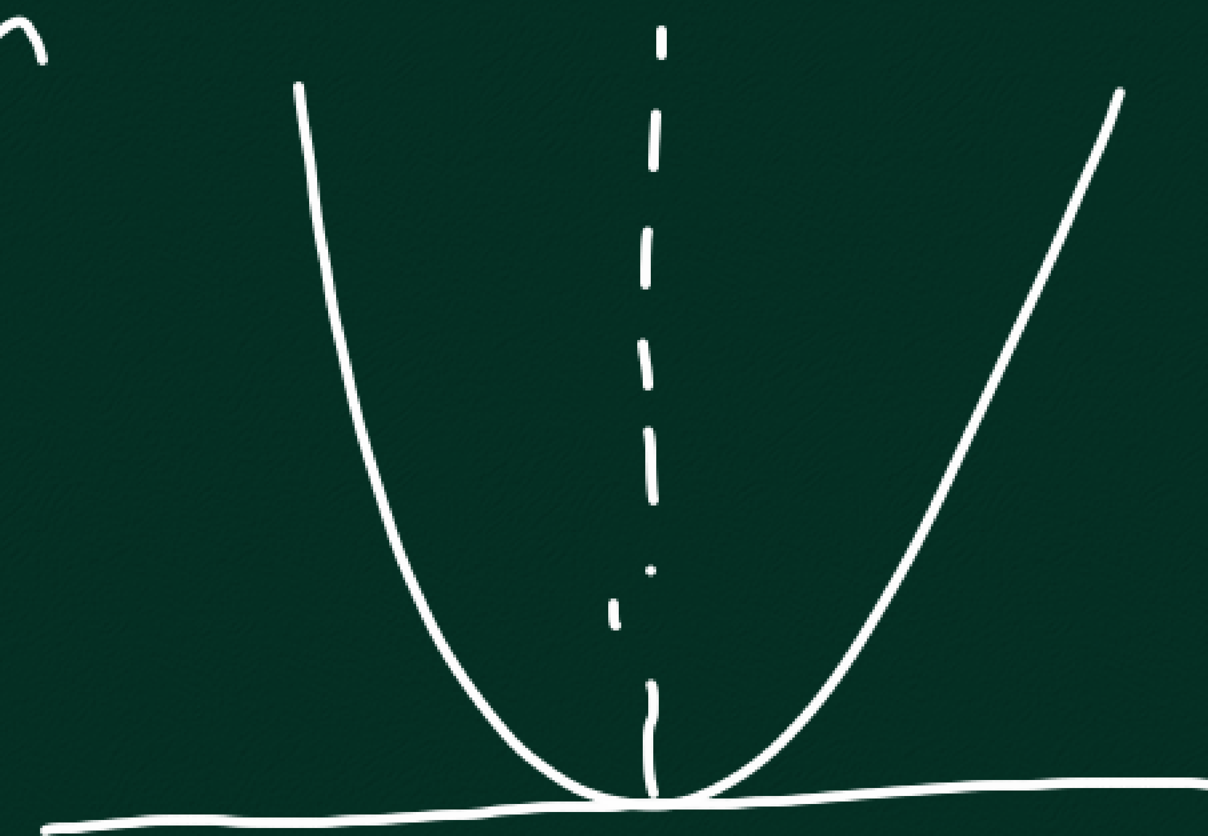
$$\bar\nabla\cdot(\psi^*\bar\nabla\psi - \psi\bar\nabla\psi^*) = \qquad B$$

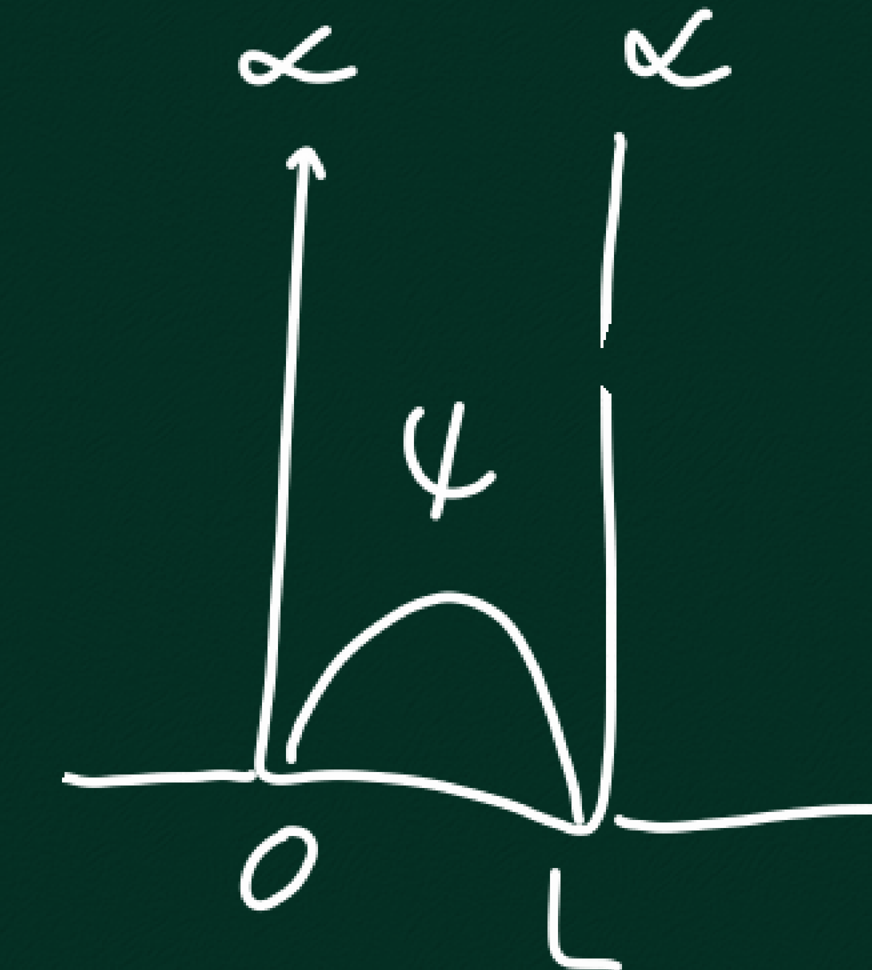
$$\boxed{\bar j = -\frac{i\hbar}{2m}\left(\psi^*\bar\nabla\psi - \psi\bar\nabla\psi^*\right)}$$

$E > V_0$



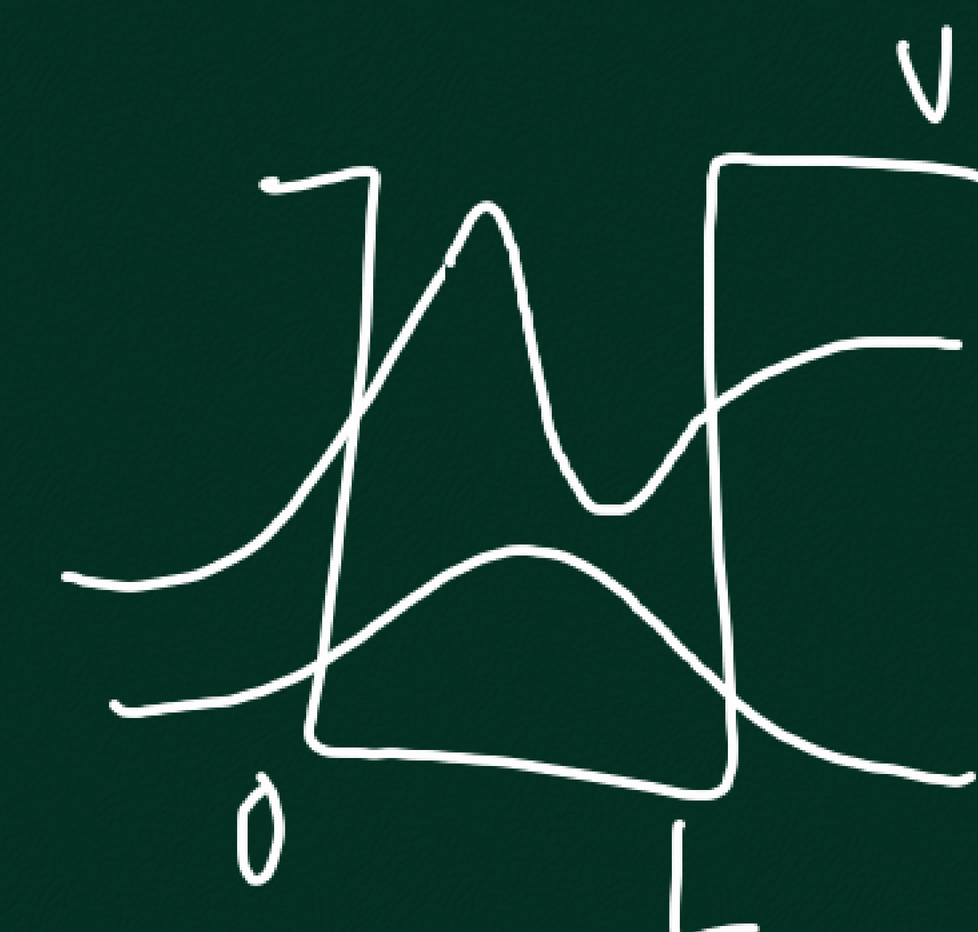
$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(x)\,\psi = E\psi$$





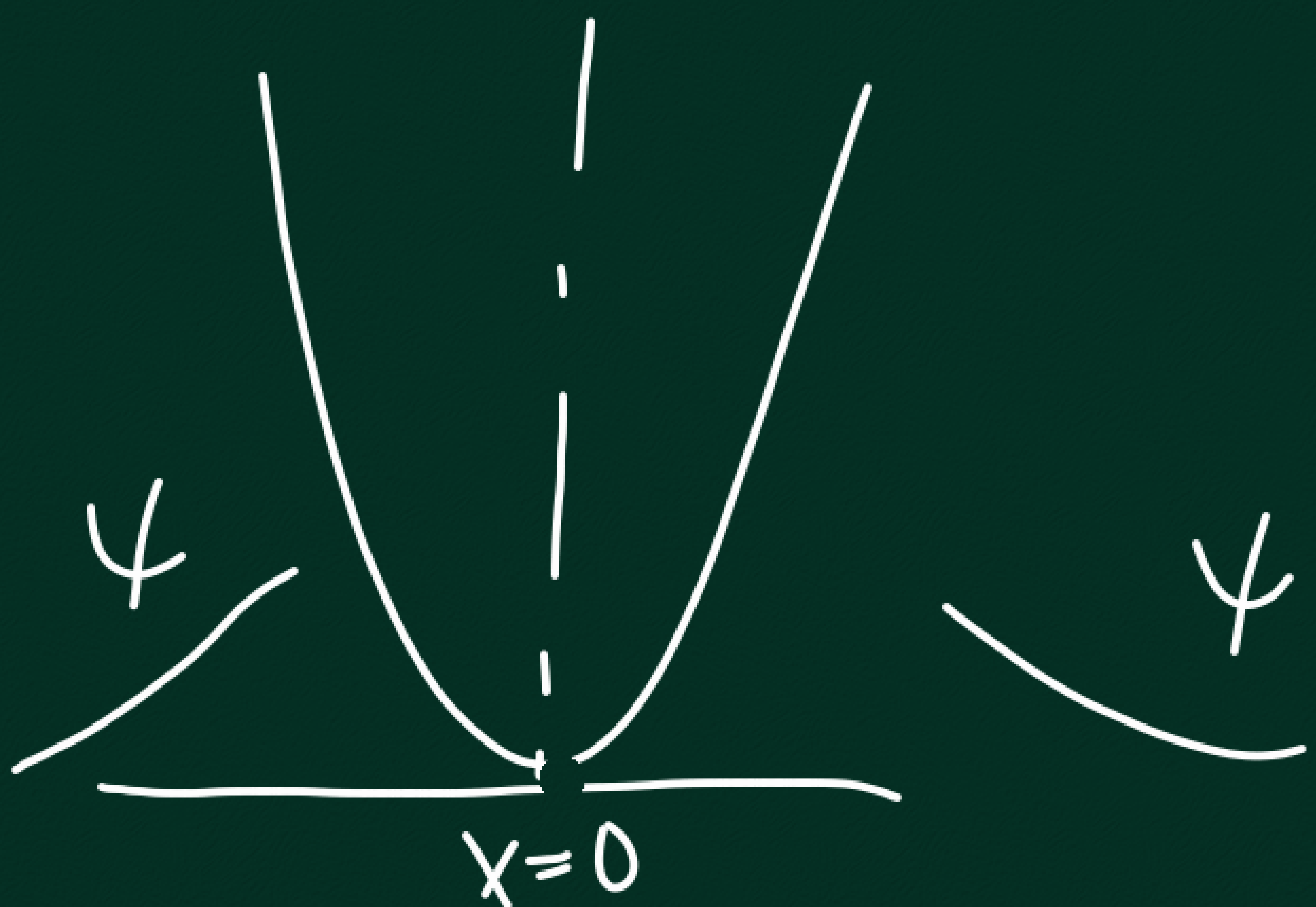
Simple Harmonic Oscillator

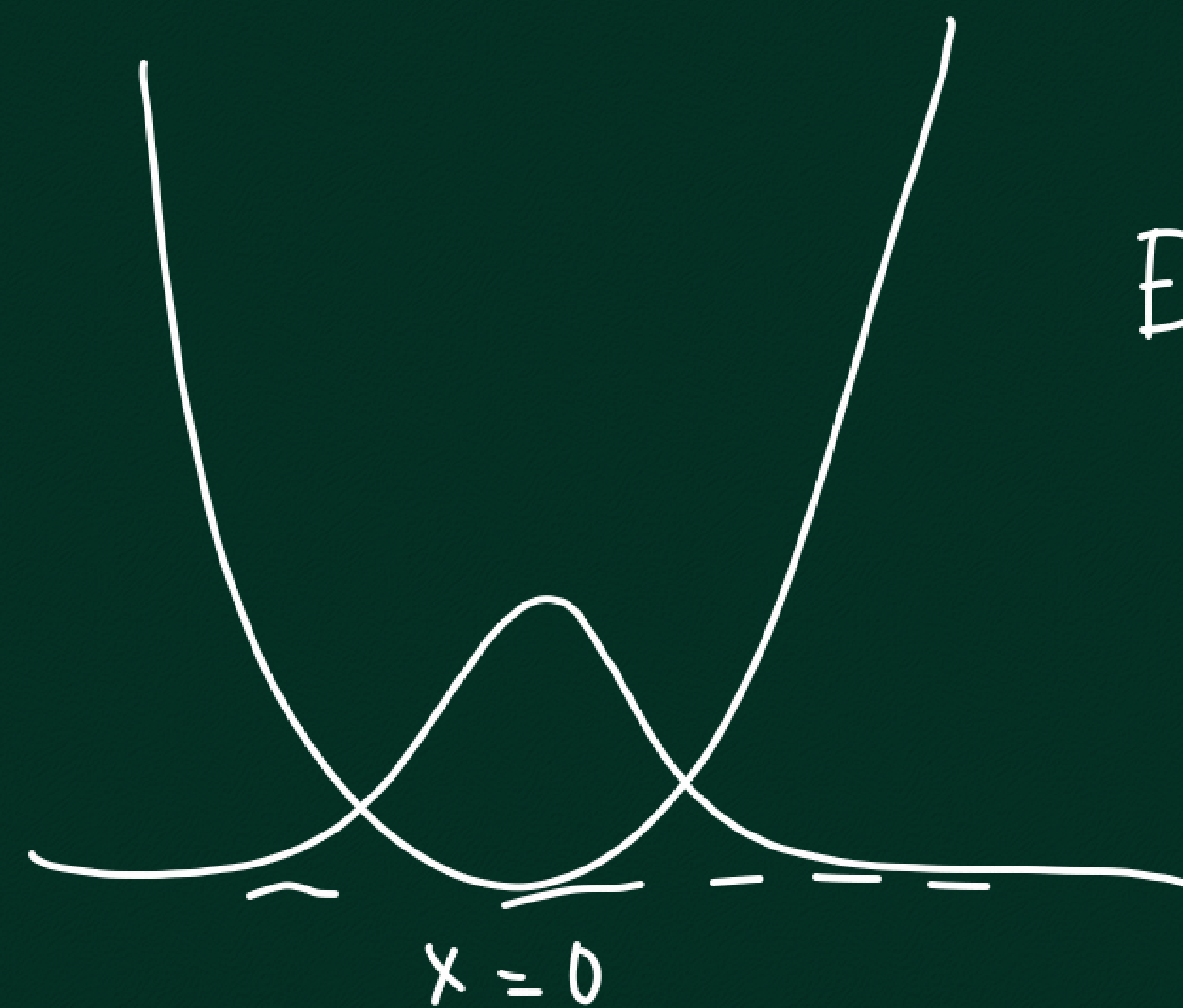
$$V = \frac{1}{2}kx^2$$
$$= \frac{1}{2}m\omega^2 x^2$$



$$\psi'' = -k^2 \psi$$

$$\psi \sim \cos kx$$



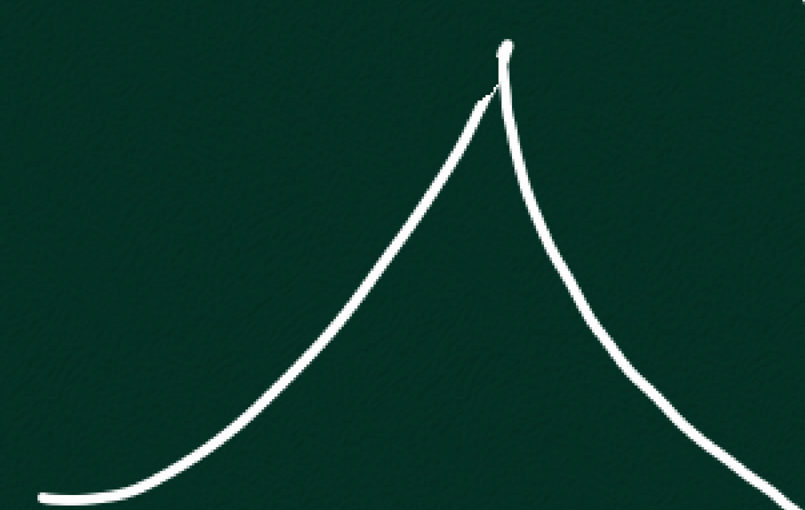


$$E = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$$

$$\Delta x = 0 \Rightarrow \Delta p \to \infty$$

$$x \to 0^+ \,/\, x \to 0^-$$

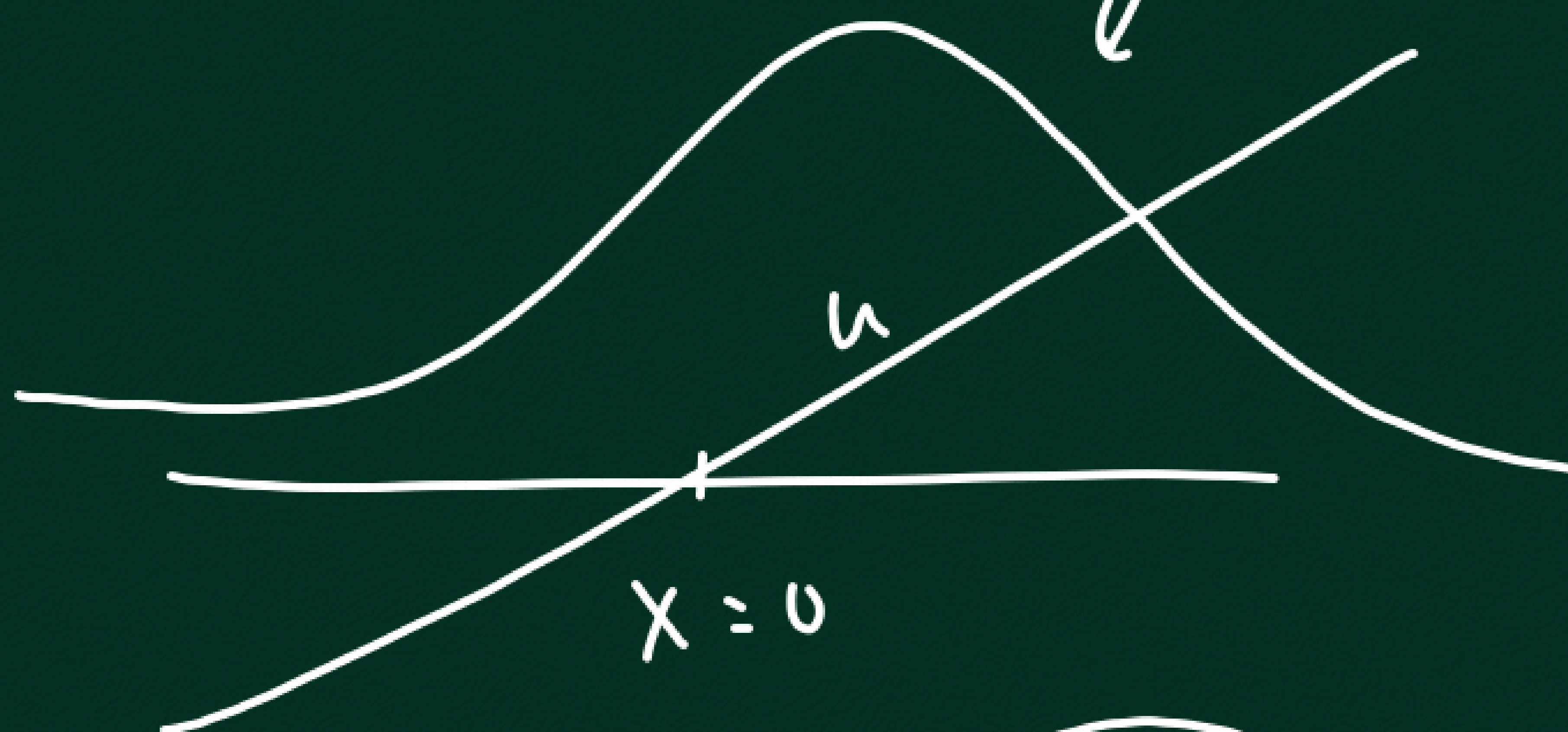
$$e^{-\alpha x}, \quad e^{-\alpha|x|}$$

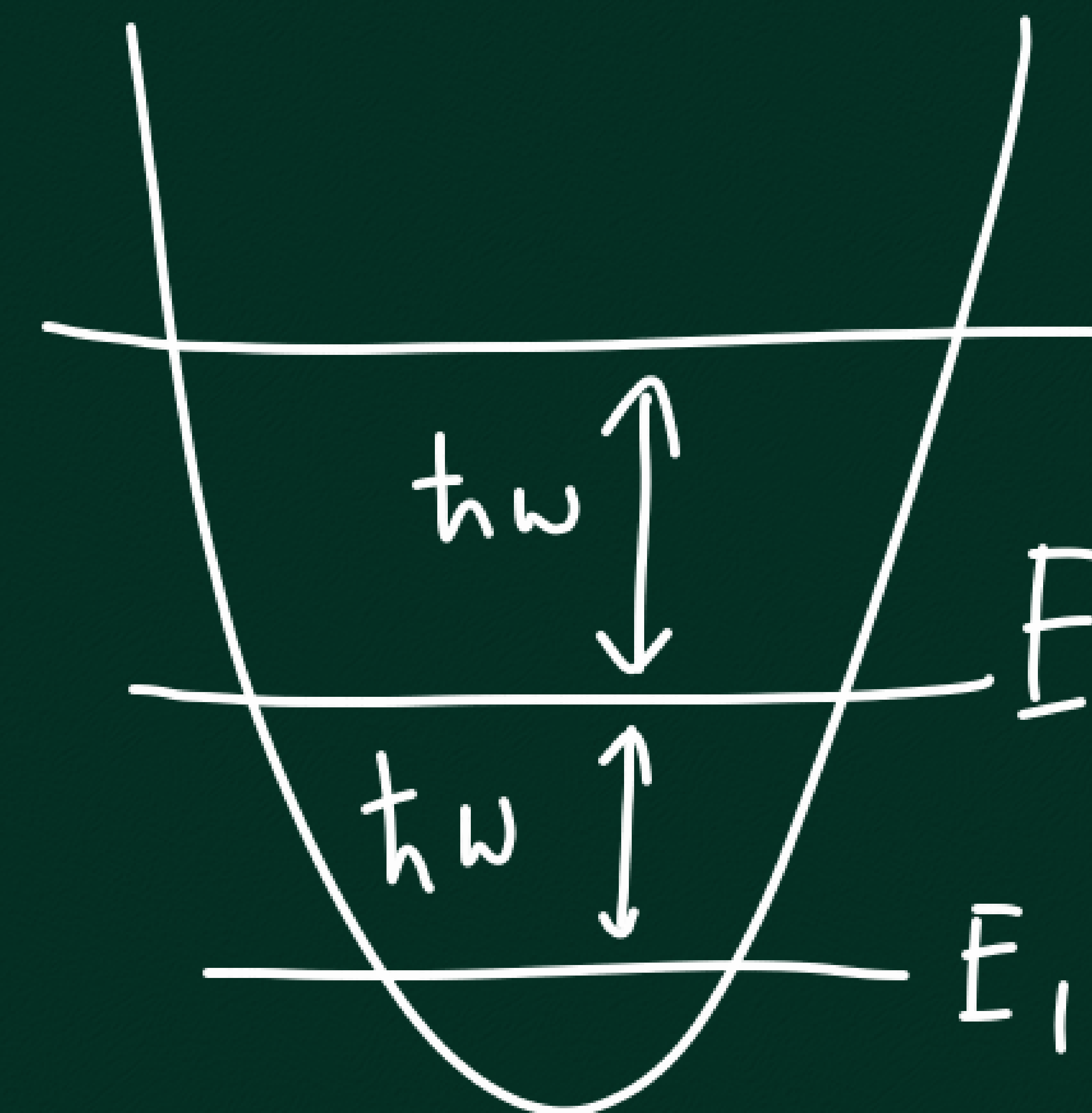


$$, \quad e^{-\alpha x^2}, \quad \frac{1}{x^2+a^2}, \quad e^{-\alpha x^4}$$

$$\psi(x) = f(x)\, e^{-\beta x^2}$$

$$\psi_2(u) = u\, e^{-u^2/2}$$





$$E_3 = 5\frac{\hbar\omega}{2}$$

$$E_2 = 3\frac{\hbar\omega}{2}$$

$$E_1 = \frac{\hbar\omega}{2}$$

$$\psi_3 = \left(u^2 - \frac{1}{2}\right) e^{-\frac{u^2}{2}}$$