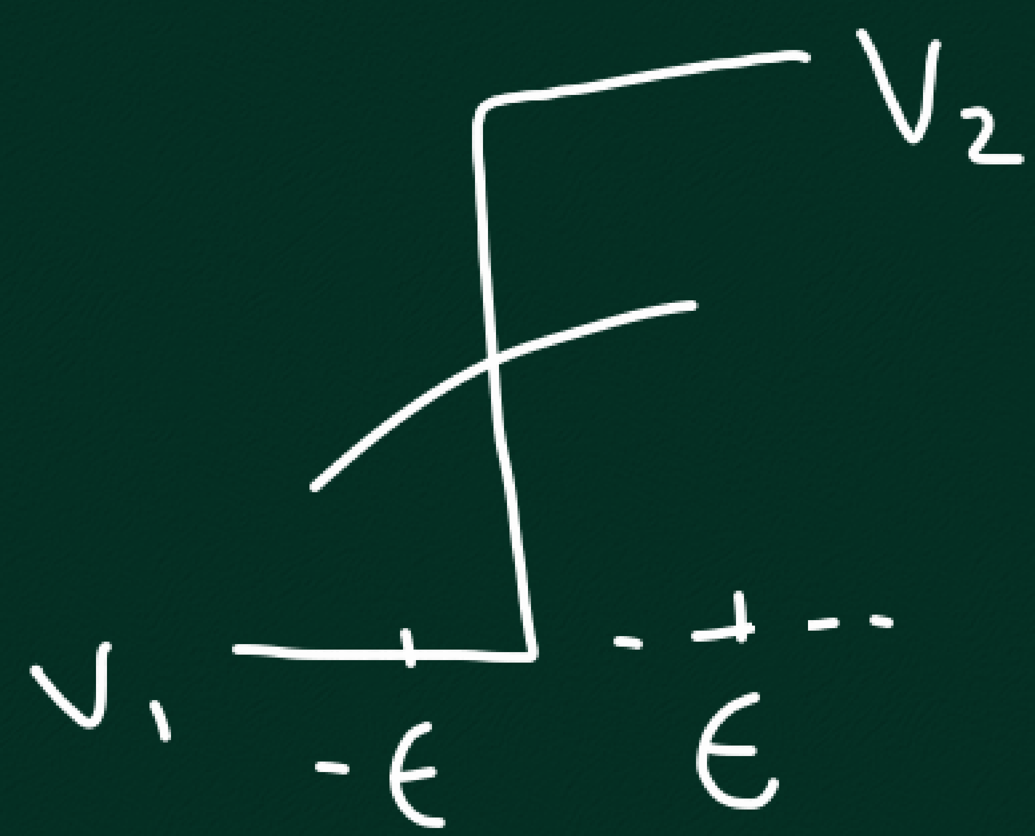
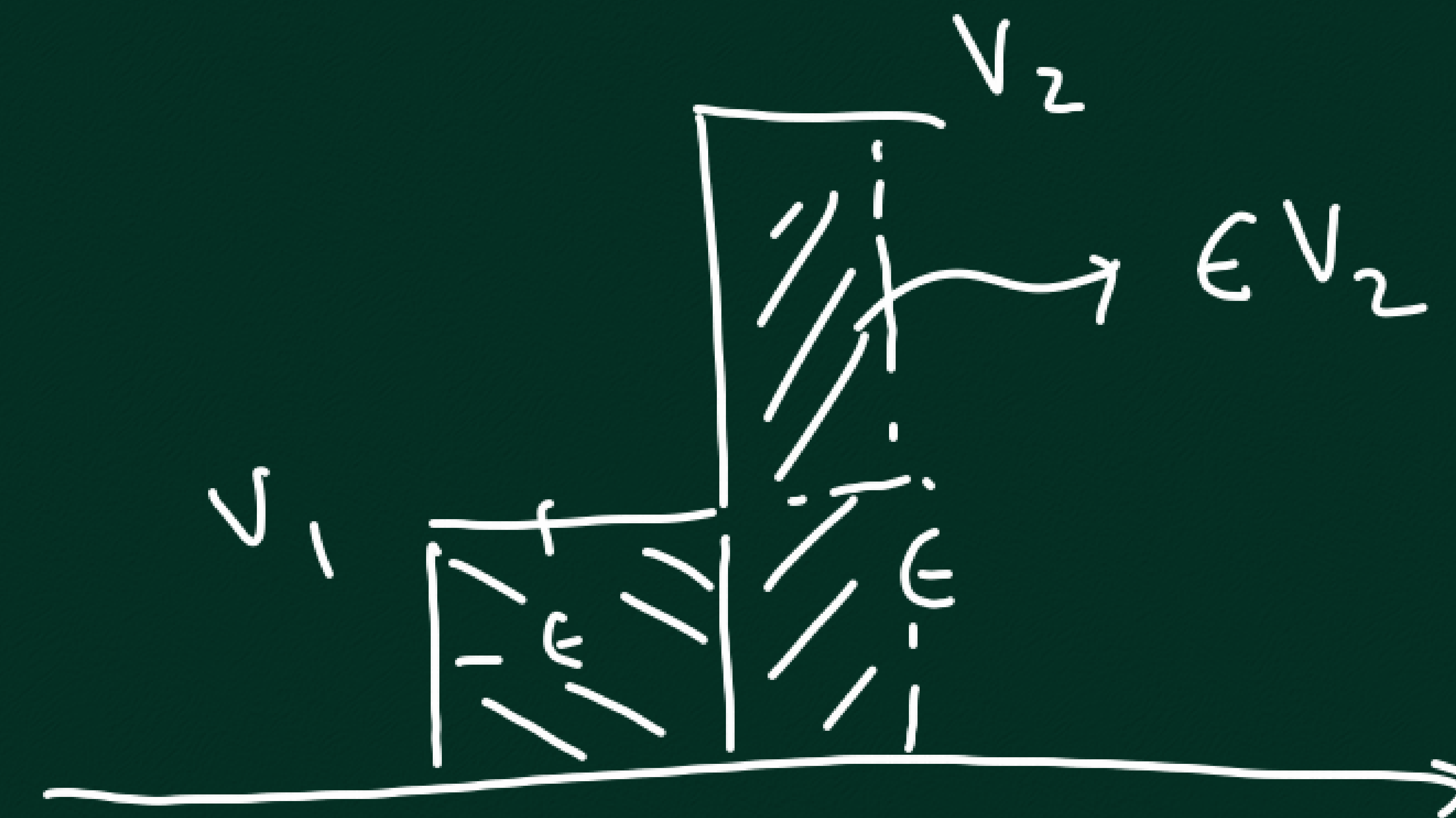
$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) = (E - V)\psi(x)$$



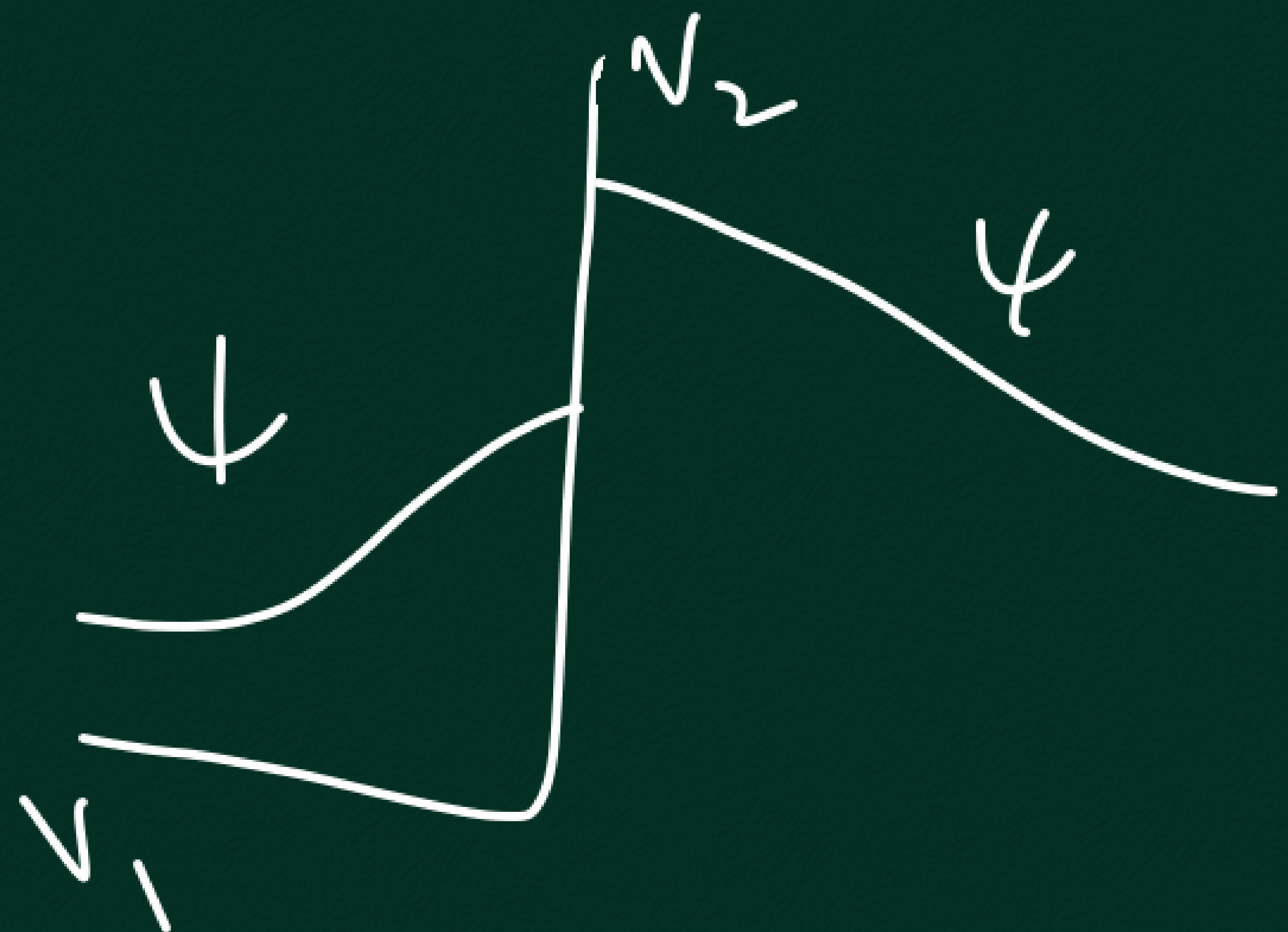


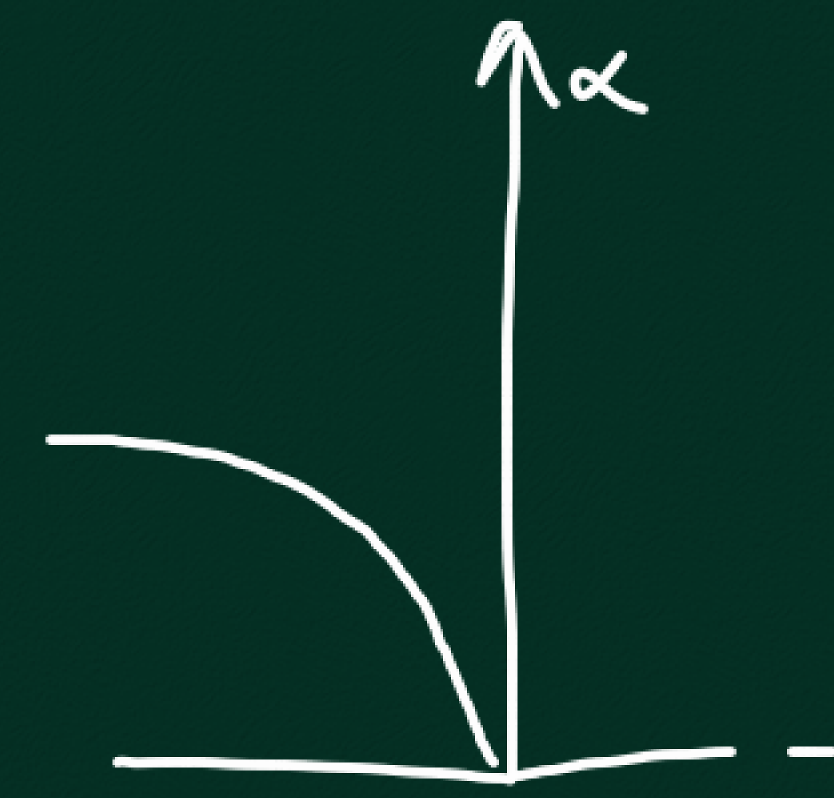
$$-\frac{\hbar^2}{2m}\left(\frac{d\psi}{dx}\Big|_{\epsilon} - \frac{d\psi}{dx}\Big|_{-\epsilon}\right) = E\int_{-\epsilon}^{\epsilon}\psi(x)\,dx - \int_{-\epsilon}^{\epsilon} V\psi(x)\,dx$$

$$\lim_{\epsilon\to 0} E\psi(0)2\epsilon - \psi(0)\int_{-\epsilon}^{\epsilon}V(x)\,dx$$

$$= \epsilon\left[E\psi(0) - \psi(0)(V_1+V_2)\right]$$

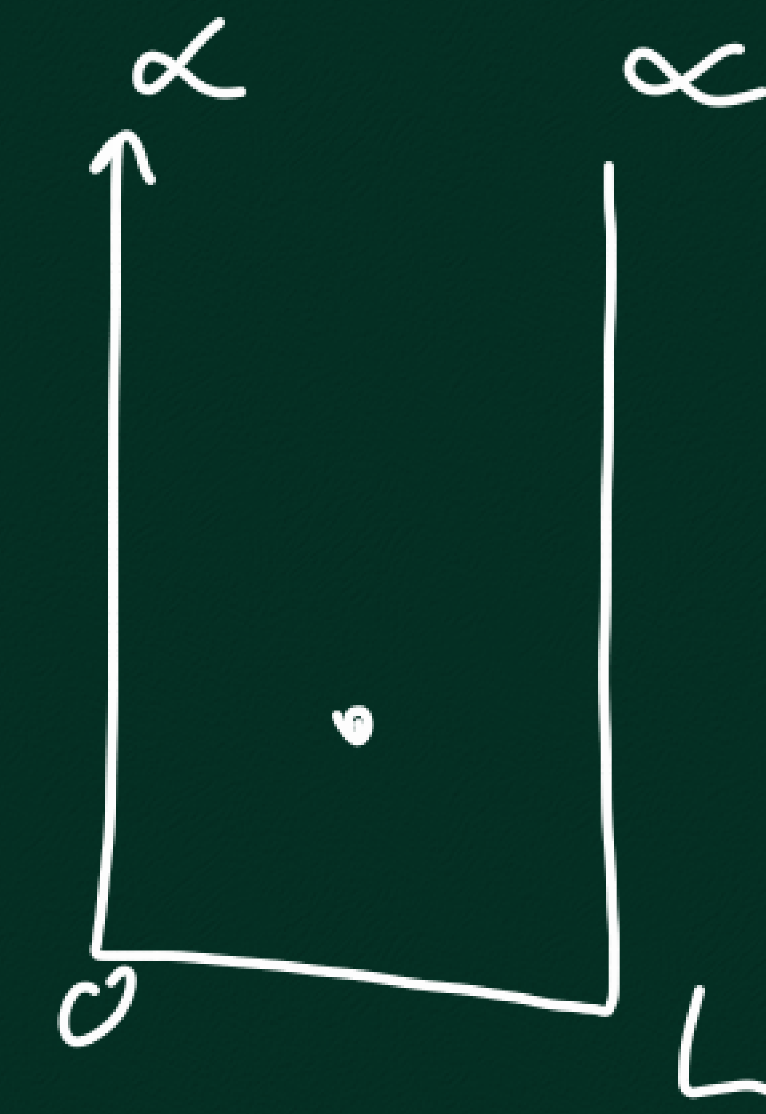
$$\epsilon(V_2+V_1)$$





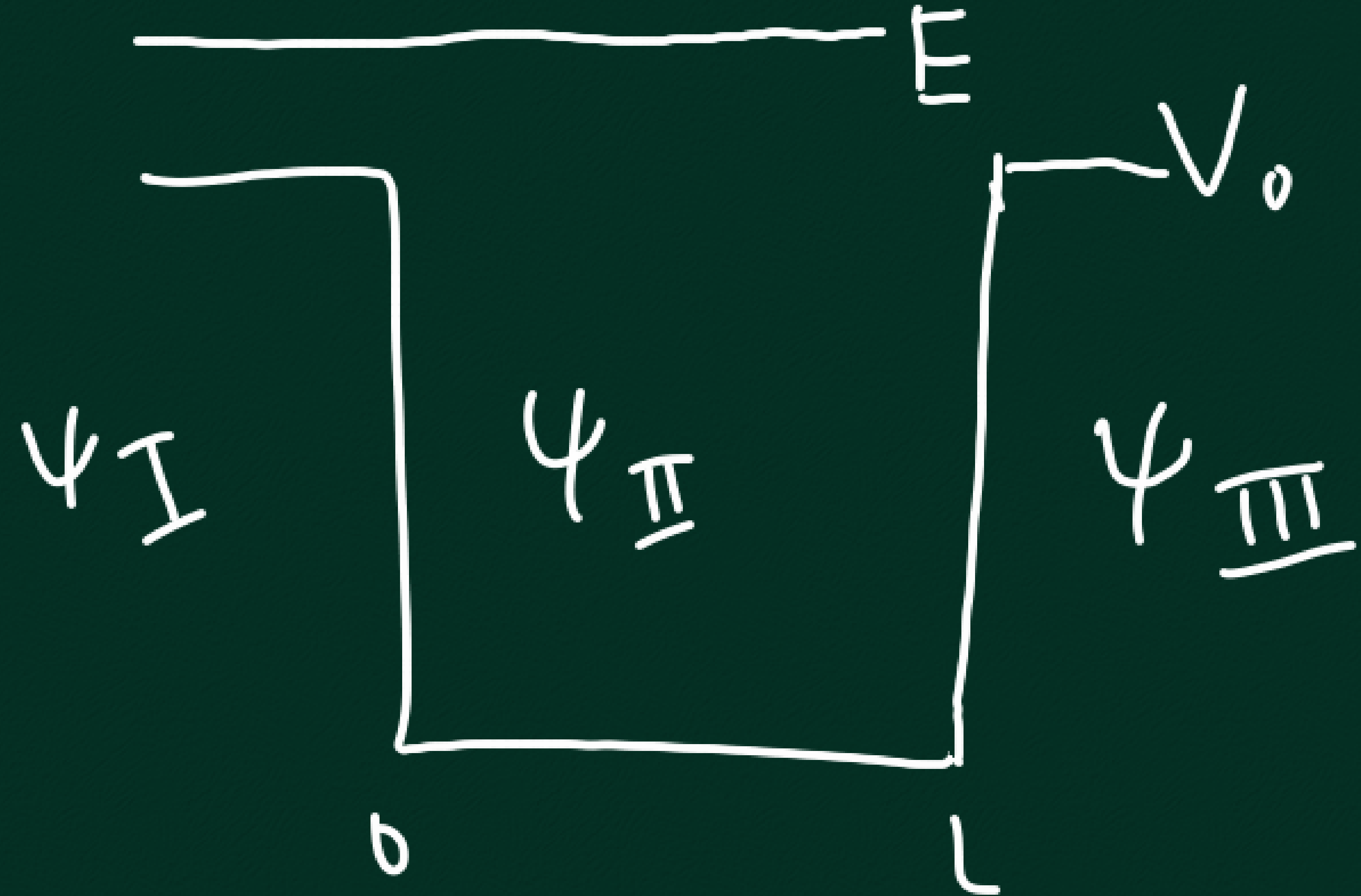
$$\hat{p} = -i\hbar \frac{d}{dx}$$

$$\langle p \rangle = \frac{\int \psi^*(x)\, \hat{p}\, \psi(x)}{\underbrace{\int \psi^* \psi \, dx}_{=1}}$$

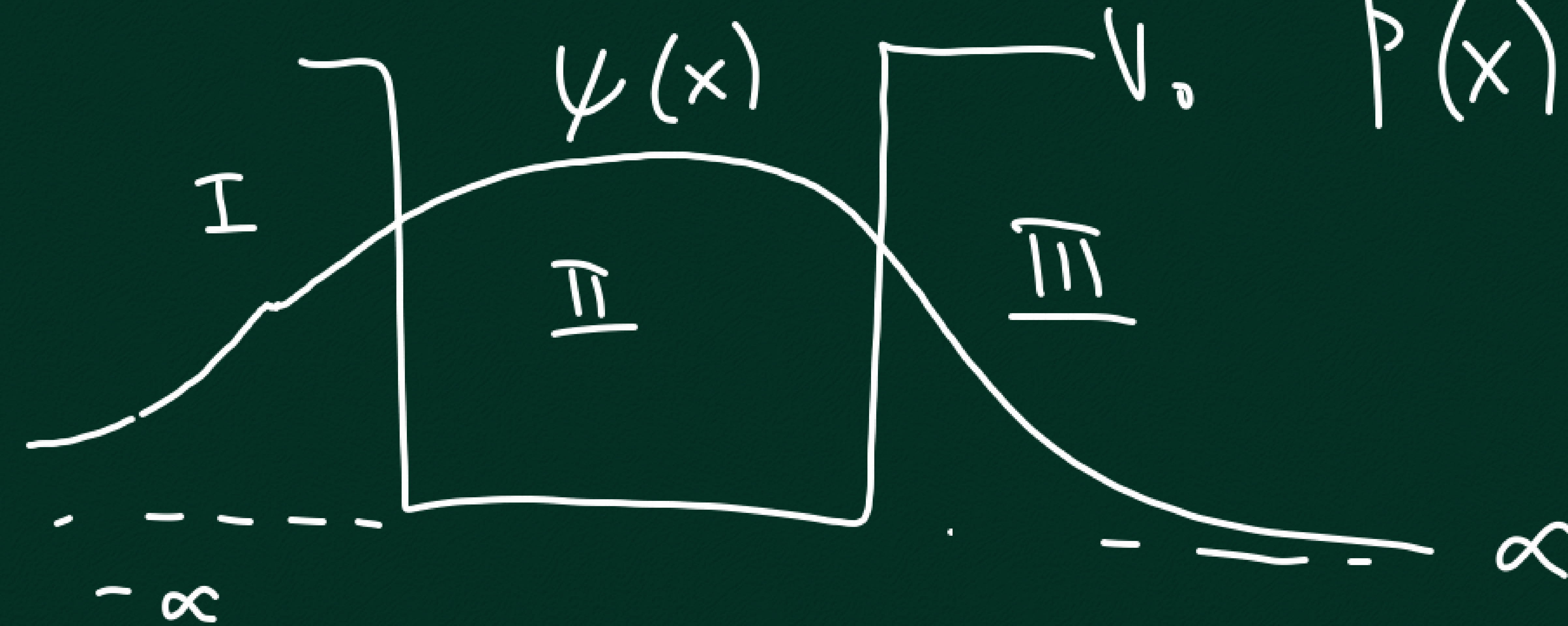


$$E = KE + PE$$

$$\psi(0) = \psi(L) = 0$$



if $E < V_0$



$$P(x) = |\psi(x)|^2$$

$$x = 0, \quad \psi_I(0) = \psi_{II}(0)$$

$$\psi'_I(0) = \psi'_{II}(0)$$

$$\psi(\infty) = \psi(-\infty) \longrightarrow 0$$

$$\underbrace{x = L \quad \psi_{II} \ \& \ \psi_{III}}_{4 \text{ B.C.s}}$$

$$E = V_0 + KE$$

I $\quad -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} = \overbrace{(E-V_0)}^{-ve}\psi$

$$E < V_0$$

$$\frac{d^2}{dx^2} = \alpha^2\psi$$

$$\psi = Ae^{\alpha x} + Be^{-\alpha x}$$

$$\psi(-\infty)\to 0$$

$$\Rightarrow B = 0$$

$$A = ?$$

II $\quad \frac{d^2}{dx^2}\psi = -k^2\psi$

$$\psi_{II}(x) = Ce^{ikx} + De^{-ikx}$$

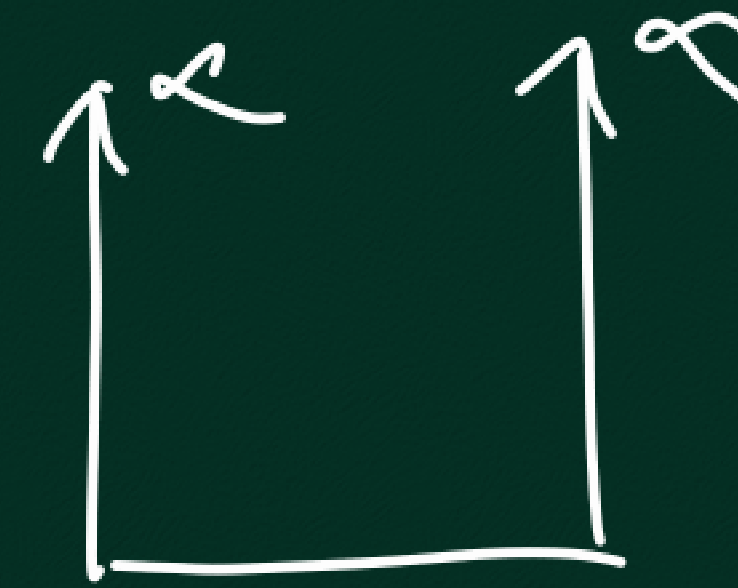
$$= C_1\sin kx + D_1\cos kx.$$

$$= C(\cos kx + i\sin kx) + D(\cos kx - i\sin kx)$$

$$= \cos kx\underbrace{(C+D)}_{D_1} + \sin kx\underbrace{(C-iD)}_{C_1}$$

$$C, D = ?$$

III $\quad \psi_{III} = \overset{\nearrow 0}{G e^{\alpha x}} + H e^{-\alpha x}$

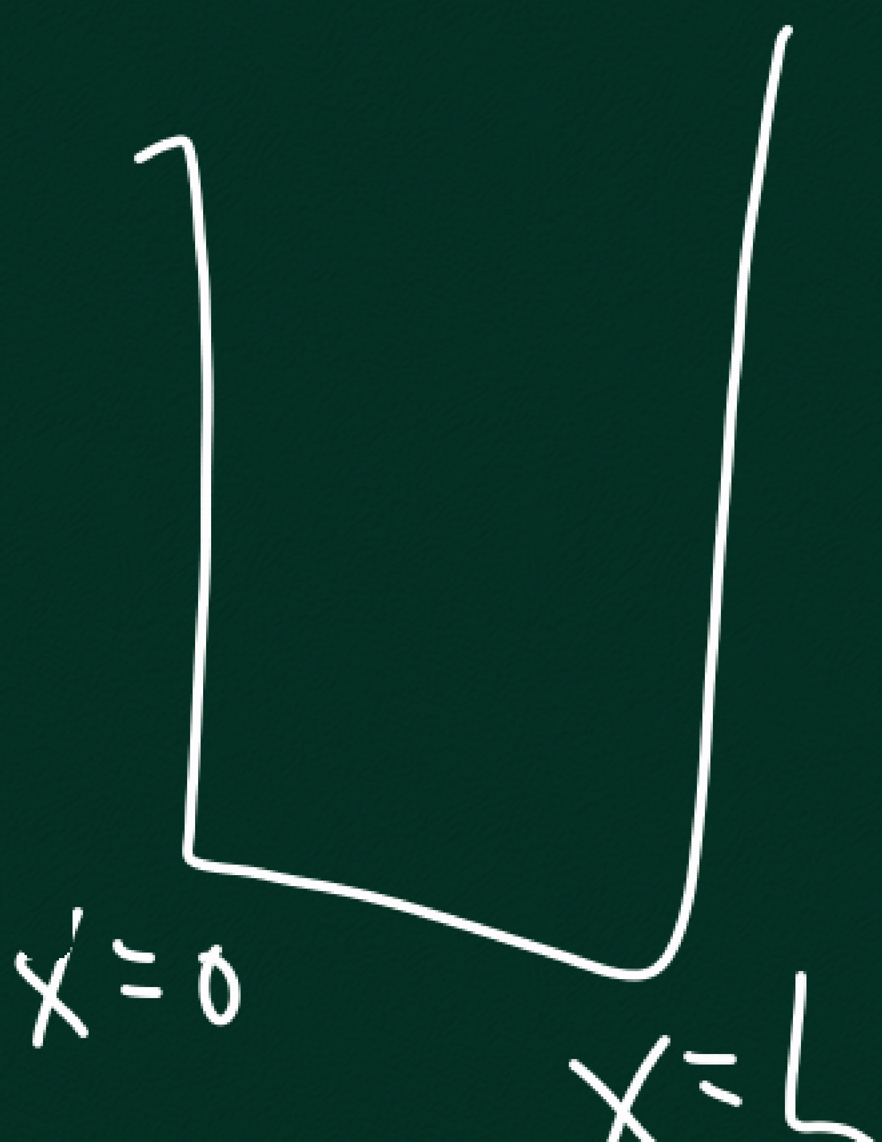


$k_n = ?$

$H = ?$

$A e^{\alpha x}$, $\quad C \sin kx + D \cos kx$, $\quad H e^{-\alpha x}$

I $\qquad$ II $\qquad$ II



$$\alpha^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$k^2 = \frac{2m}{\hbar^2} E$$

$$\frac{\frac{\alpha}{k}\tan kL + 1}{-\alpha + k \tan kL} = \frac{1}{\alpha}$$

$$\frac{\alpha^2}{k}\tan kL + \alpha = -\alpha + k\tan kL$$

$$\tan kL\left(\frac{\alpha^2}{k} - k\right) = -2\alpha$$

$$\tan kL\left(\alpha^2 - k^2\right) = -2\alpha k$$

$$\tan kL = \frac{2\alpha k}{k^2 - \alpha^2}$$

$$\tan kL = \frac{2\alpha k}{k^2 - \alpha^2}$$

$$\downarrow$$

$$\frac{2\tan\frac{kL}{2}}{1 - \tan^2\frac{kL}{2}} = \frac{2}{\frac{k}{\alpha} - \frac{\alpha}{k}}$$

$$\frac{2}{\frac{1}{\tan kL} - \tan kL} = \frac{2}{\frac{k}{\alpha} - \frac{\alpha}{k}}$$

$\tan kL \to y$, $\frac{\alpha}{k} \to x$

$$\frac{1}{y} - y = \frac{1}{x} - x \Rightarrow y = x, -\frac{1}{x}$$

$$\tan\frac{kL}{2} = \frac{\alpha}{k} \quad \cdots \text{①} \quad \to \frac{\alpha^2 + k^2}{k^2}$$

$$\cdots = -\frac{k}{\alpha} \cdots \text{②}$$

$$\alpha^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$k^2 = \frac{2m}{\hbar^2}E$$

$$\alpha^2 + k^2 = \frac{2m}{\hbar^2}V_0 \quad \cdots \text{②}$$

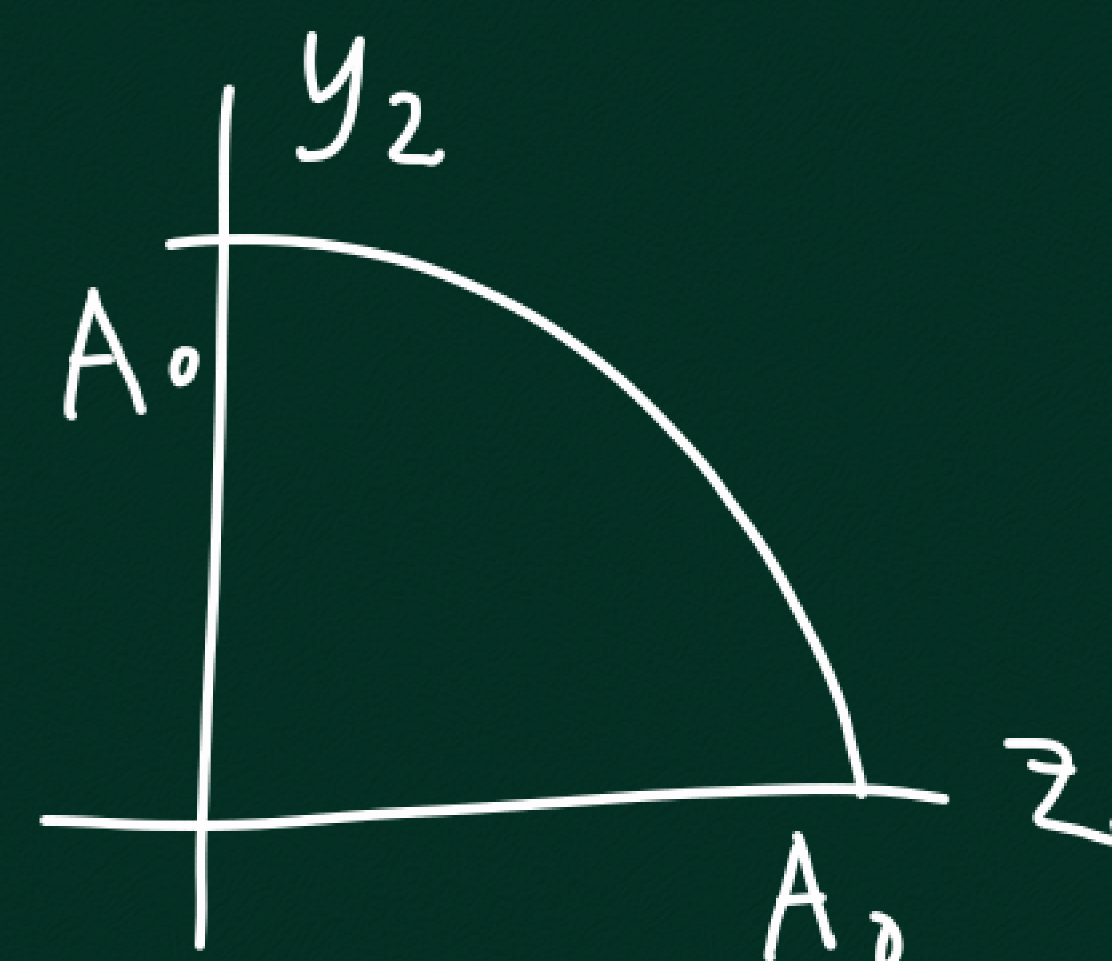
$$\boxed{1 + \tan^2\frac{kL}{2} = \frac{2mV_0}{\hbar^2 k^2}}$$

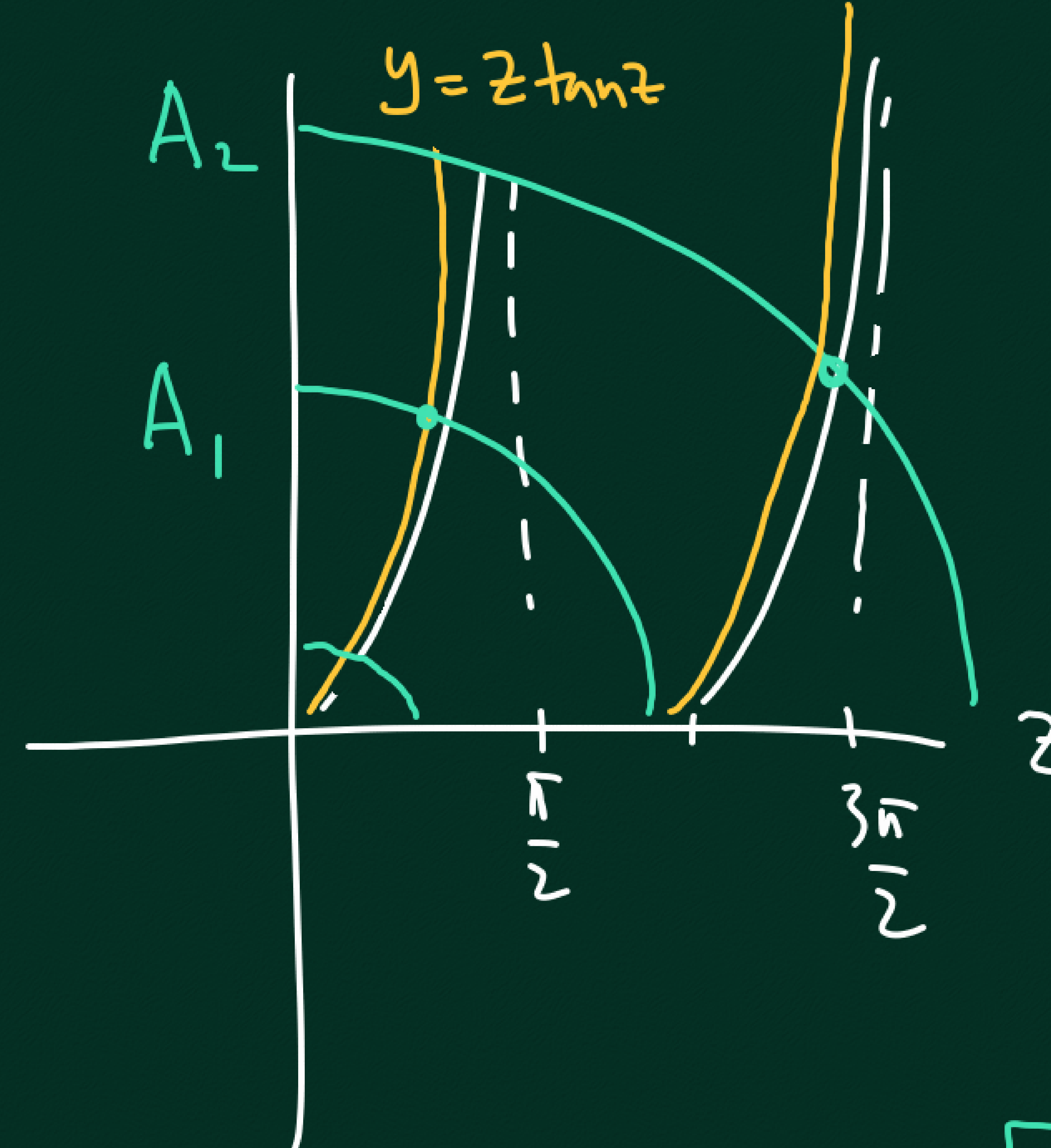
$$\frac{kL}{2}\tan\left(\frac{kL}{2}\right) = \left(\sqrt{\frac{2mV_0}{\hbar^2 k^2} - 1}\right)\frac{kL}{2}$$

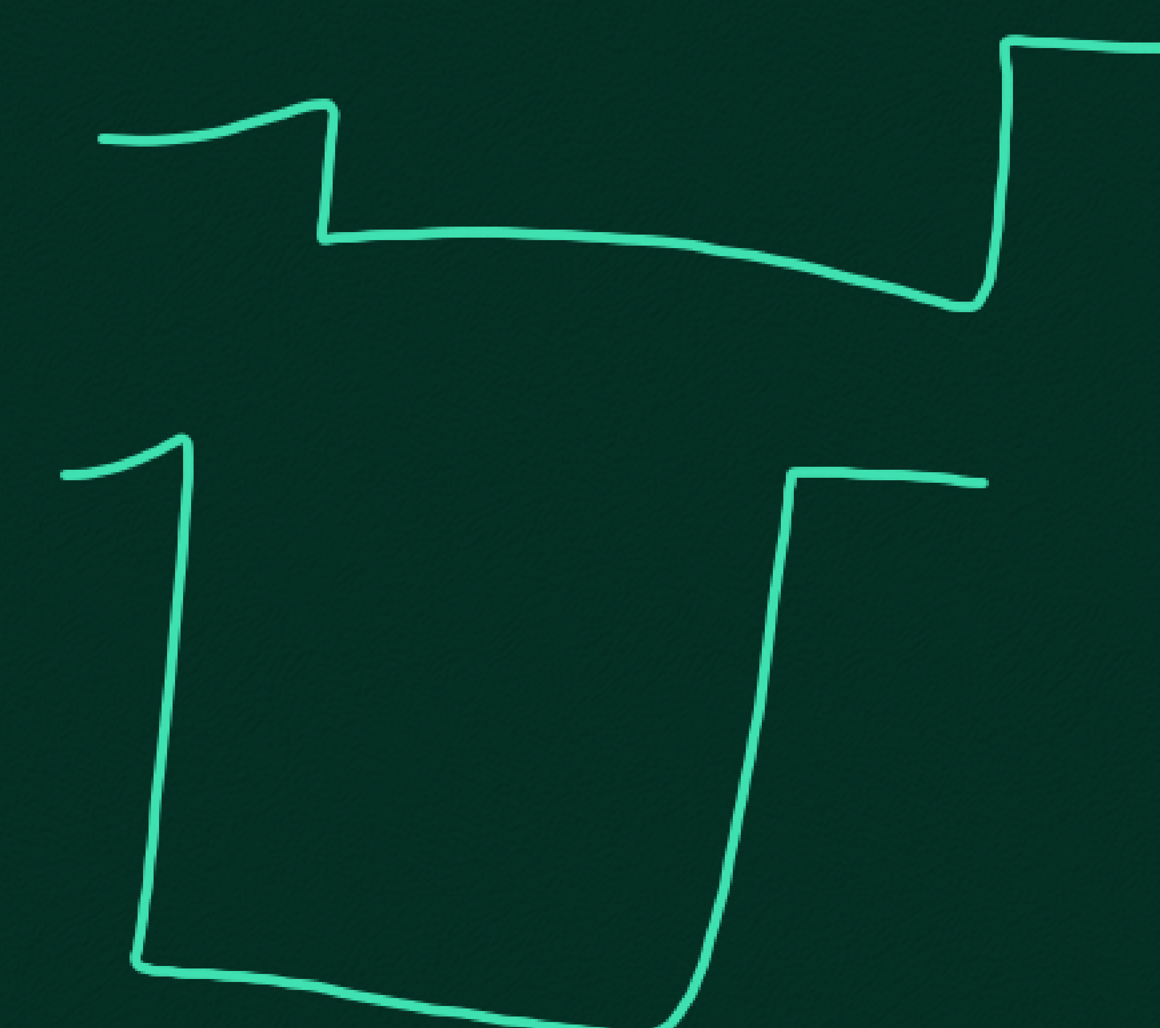
$$\frac{kL}{2}\tan\left(\frac{kL}{2}\right) = \sqrt{\frac{mV_0 L^2}{2\hbar^2} - \left(\frac{kL}{2}\right)^2}$$

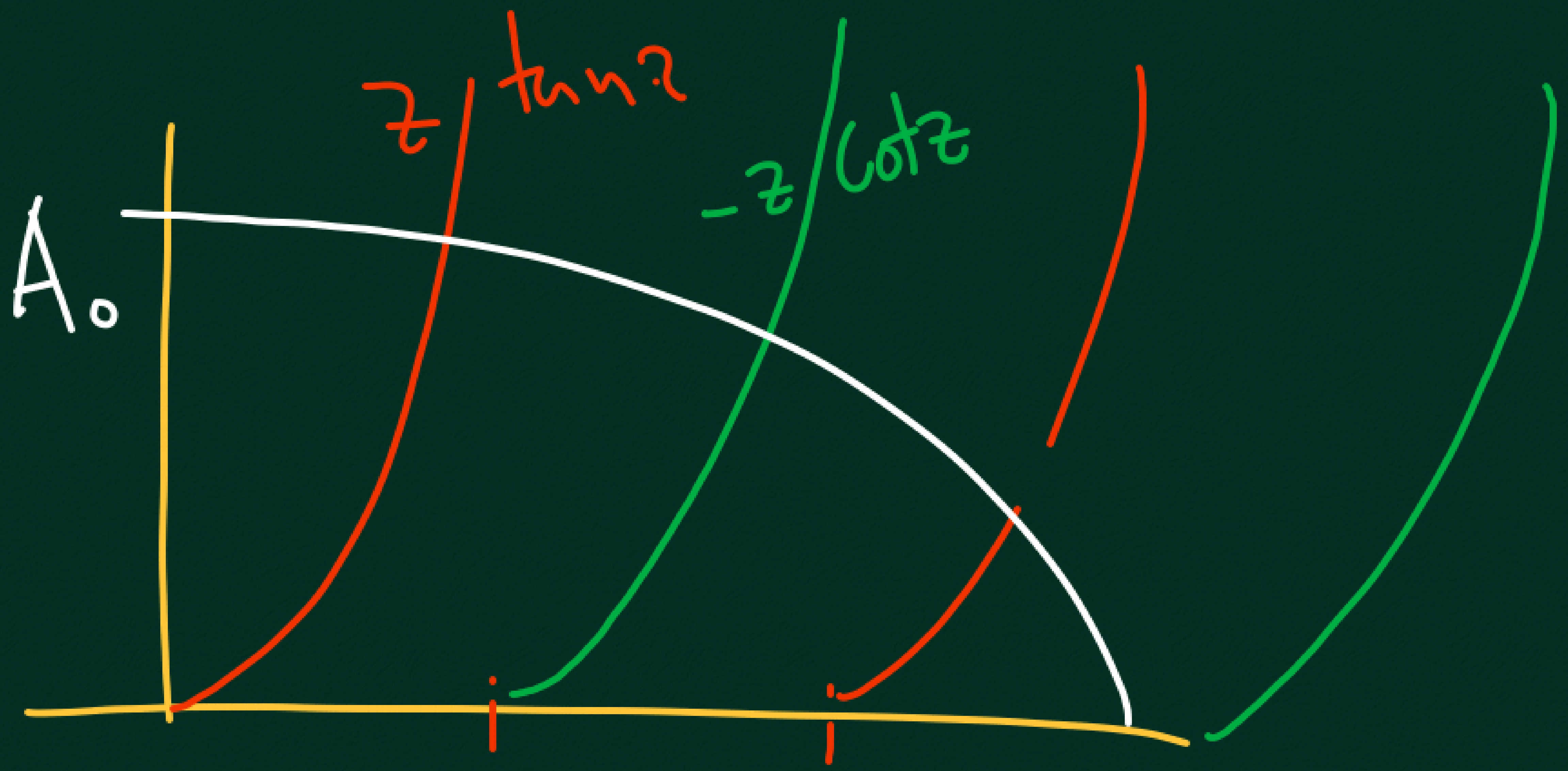
$$z = \frac{kL}{2}$$

$$\underset{\substack{\parallel \\ y_1}}{z\tan z} = \underset{\substack{\parallel \\ y_2}}{\sqrt{A_0^2 - z^2}}$$









$$-z \cot z$$

$$z = \frac{kL}{2} \rightarrow E$$