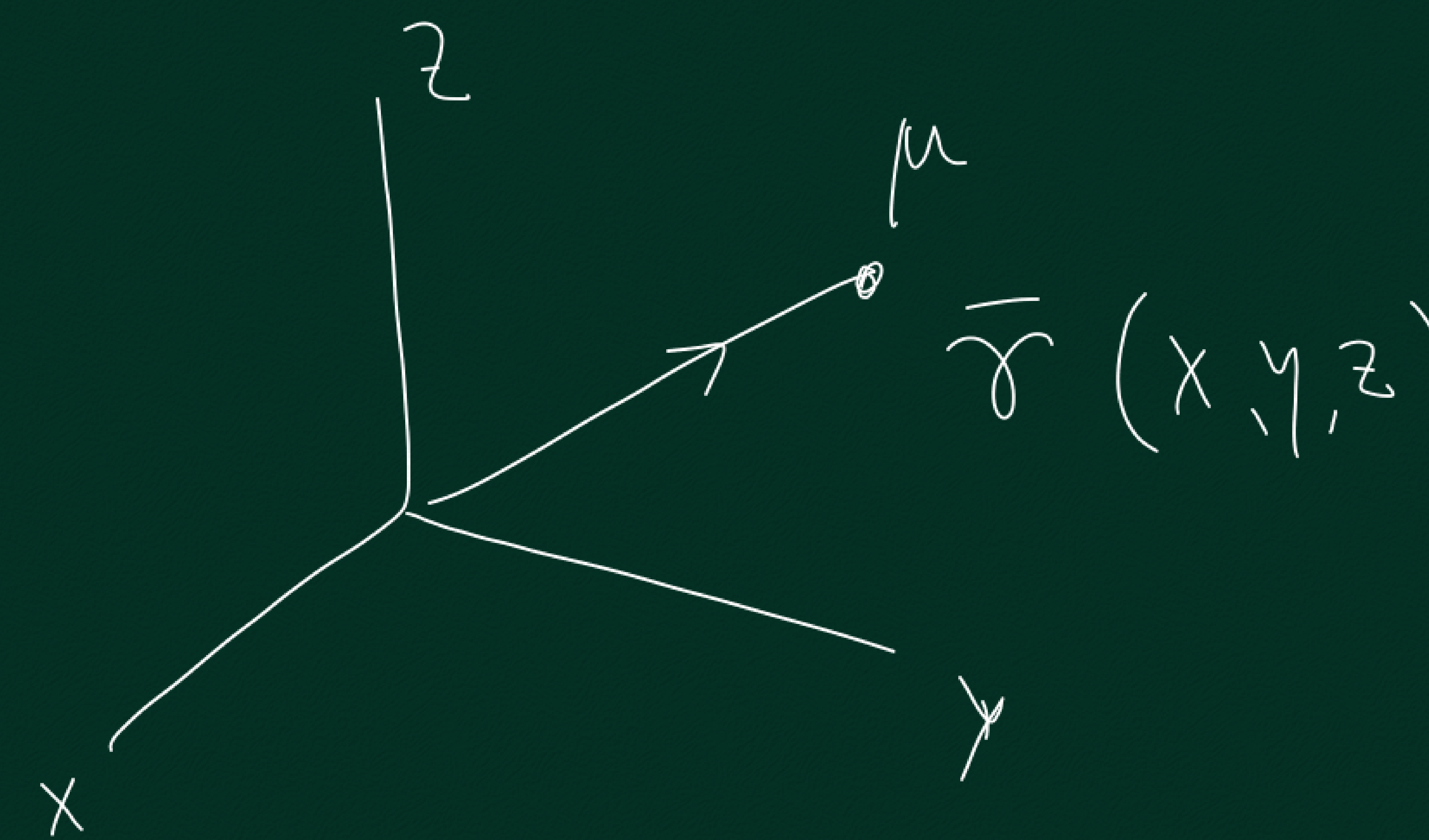
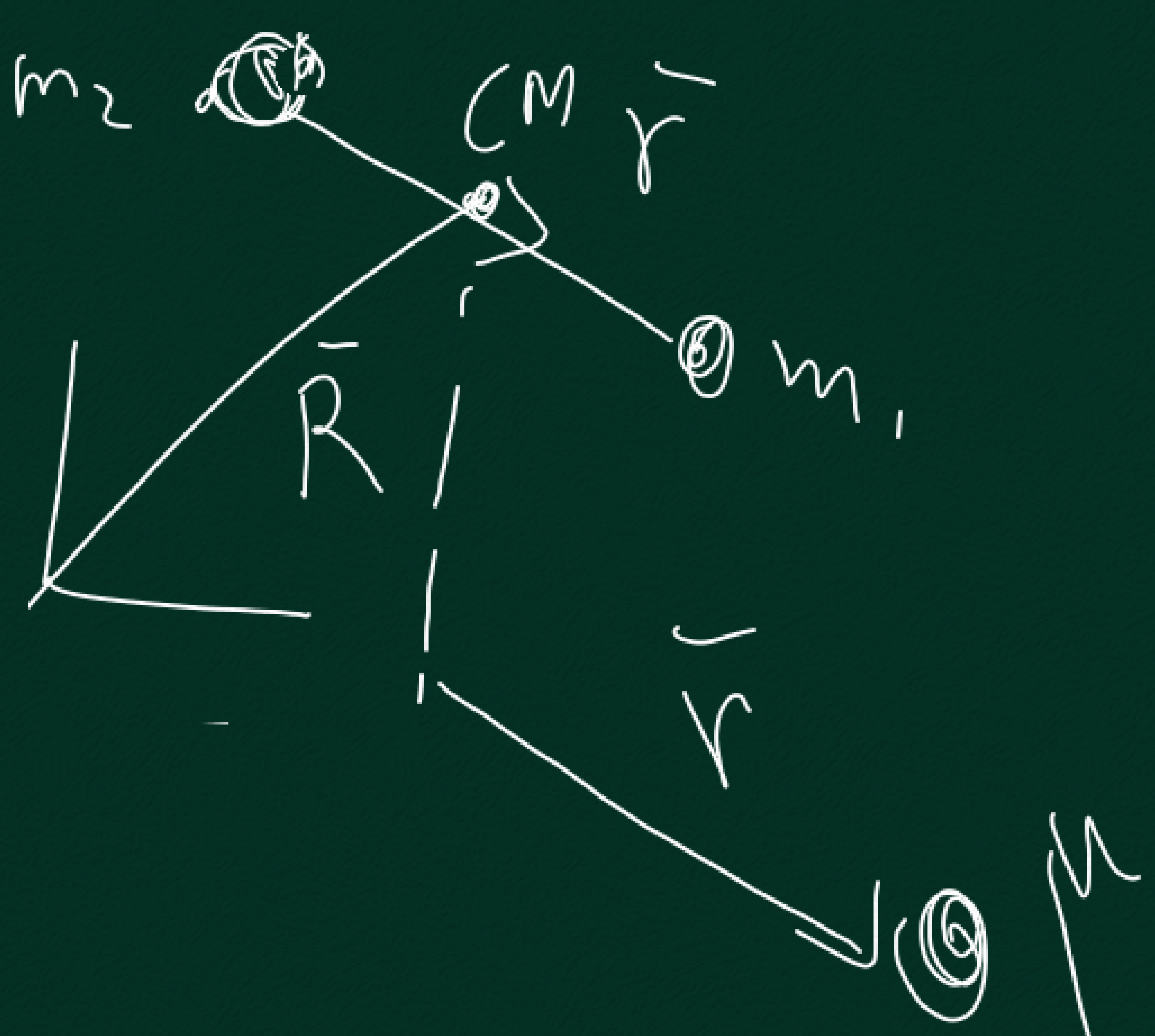
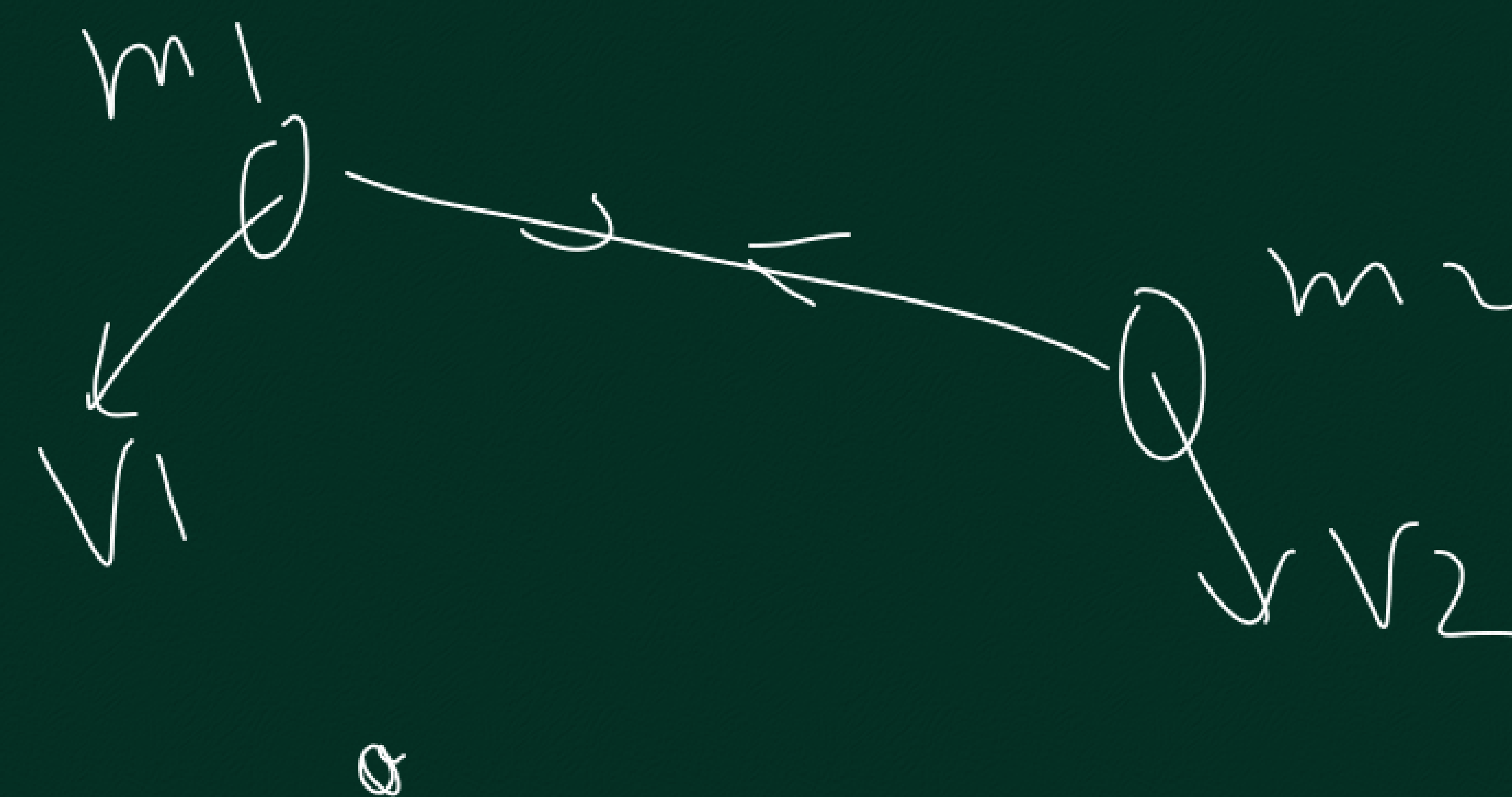
$$\mu \ddot{\bar{r}} = f(r)\,\hat{r}$$

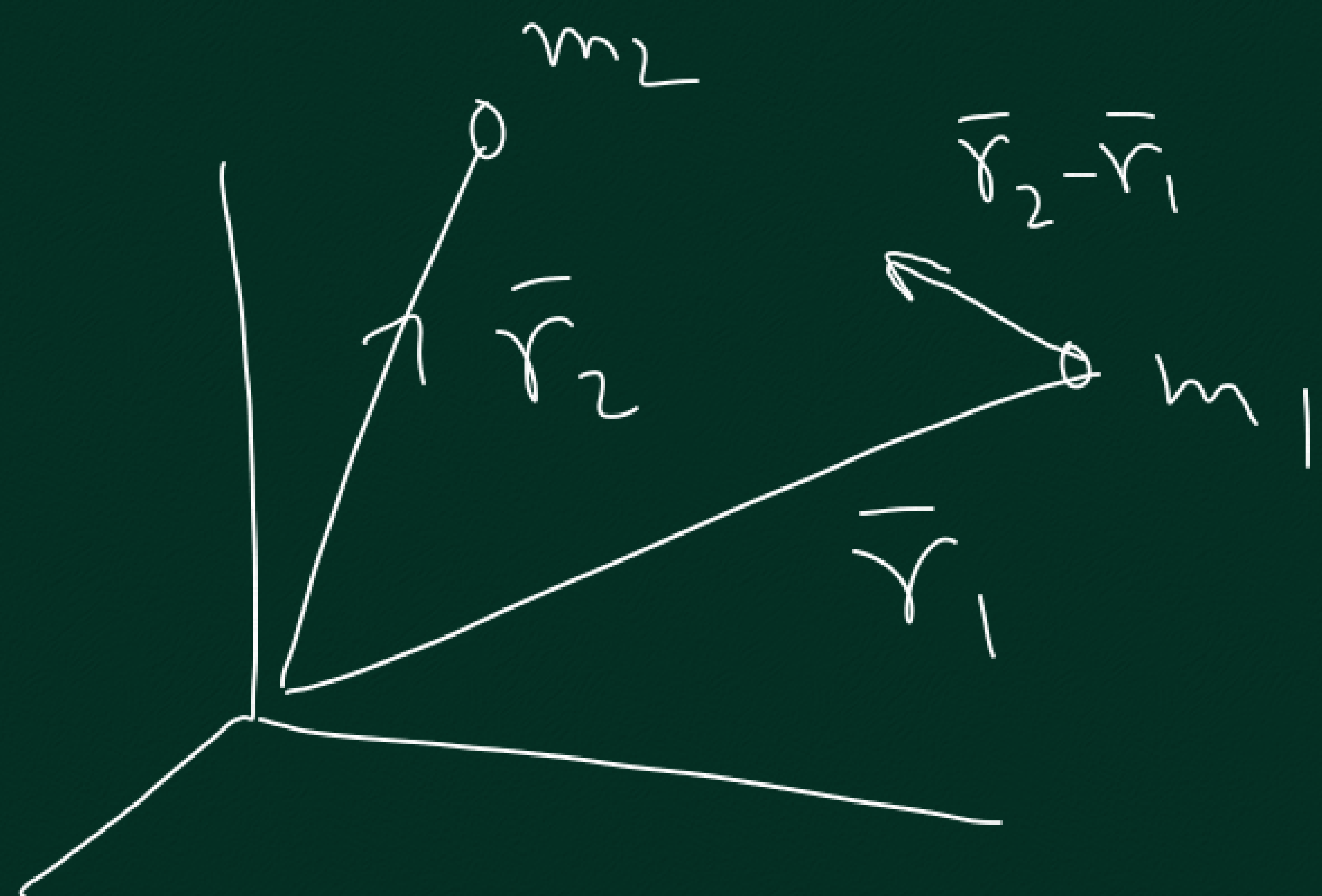
$$\ddot{R} = 0 \qquad \bar{R} = \bar{V}_0 t + \bar{R}_0$$

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$









$$\bar{L} = m_1 \bar{r}_1 \times \dot{\bar{r}}_1 + m_2 \bar{r}_2 \times \dot{\bar{r}}_2$$

$$\frac{d}{dt} L = m_1 \left(\cancel{\dot{\bar{r}}_1 \times \dot{\bar{r}}_1} + \bar{r}_1 \times \ddot{\bar{r}}_1 \right)$$
$$+ m_2 \left(\cancel{\dot{\bar{r}}_2 \times \dot{\bar{r}}_2} + \bar{r}_2 \times \ddot{\bar{r}}_2 \right)$$

$$m_1 \ddot{\bar{r}}_1 = f(r) \hat{r}_{12}$$
$$m_2 \ddot{\bar{r}}_2 = -f(r) \hat{r}_{12}$$

$f(r)$ is -ve

$$\hat{r}_{12} = \frac{\bar{r}_1 - \bar{r}_2}{|\bar{r}_1 - \bar{r}_2|}$$

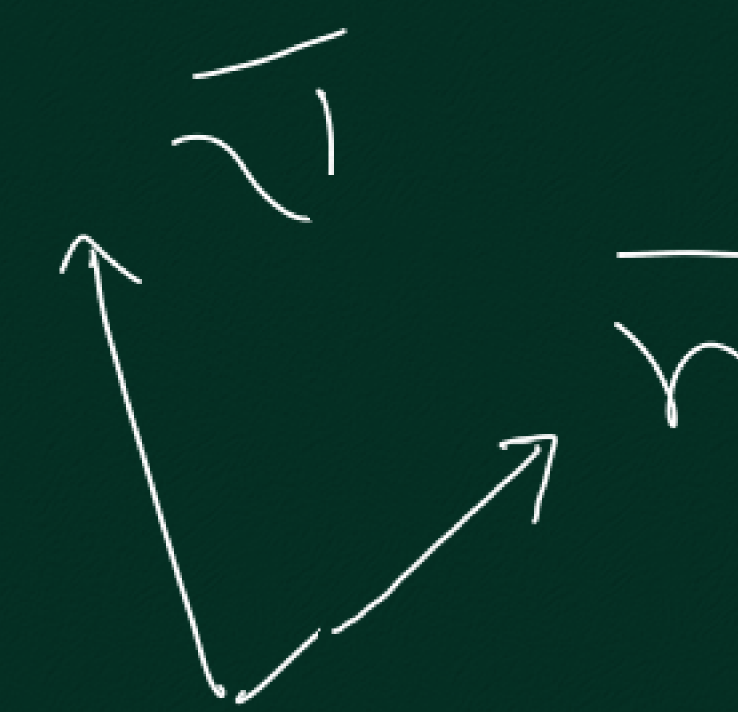
$$= \bar{r}_1 \times f(r) \frac{\bar{r}_1 - \bar{r}_2}{r}$$
$$- \bar{r}_2 \times f(r) \frac{\bar{r}_1 - \bar{r}_2}{r}$$
$$= \frac{f(r)}{r} \left[\bar{r}_1 \times (r_1 - \bar{r}_2) - \bar{r}_2 \times (r_1 - r_2) \right]$$
$$\underbrace{-\bar{r}_1 \times \bar{r}_2 - \bar{r}_2 \times \bar{r}_1}$$

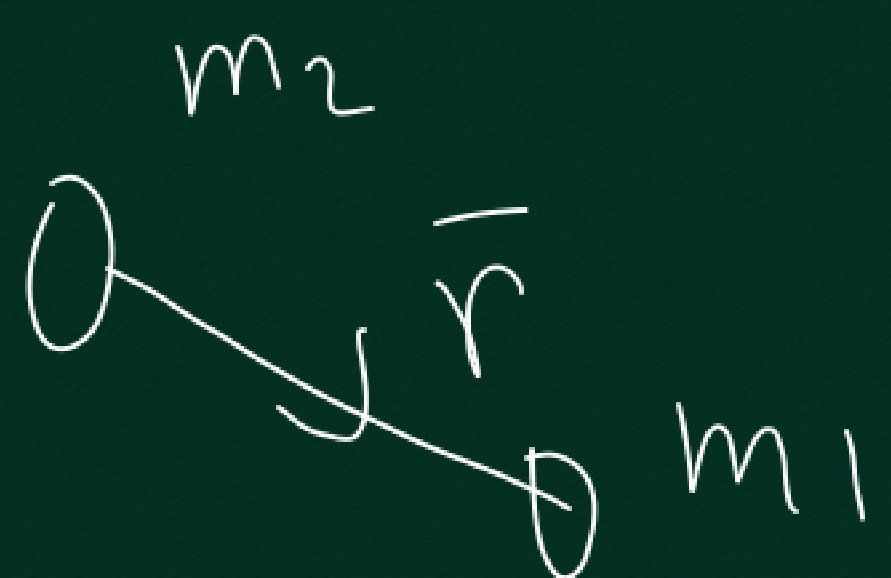
$$\bar{L}' = \bar{r} \times \mu\dot{\bar{r}} = m\bar{r} \times \bar{v}$$

$$\mu\ddot{\bar{r}} = f(r)\hat{r}$$

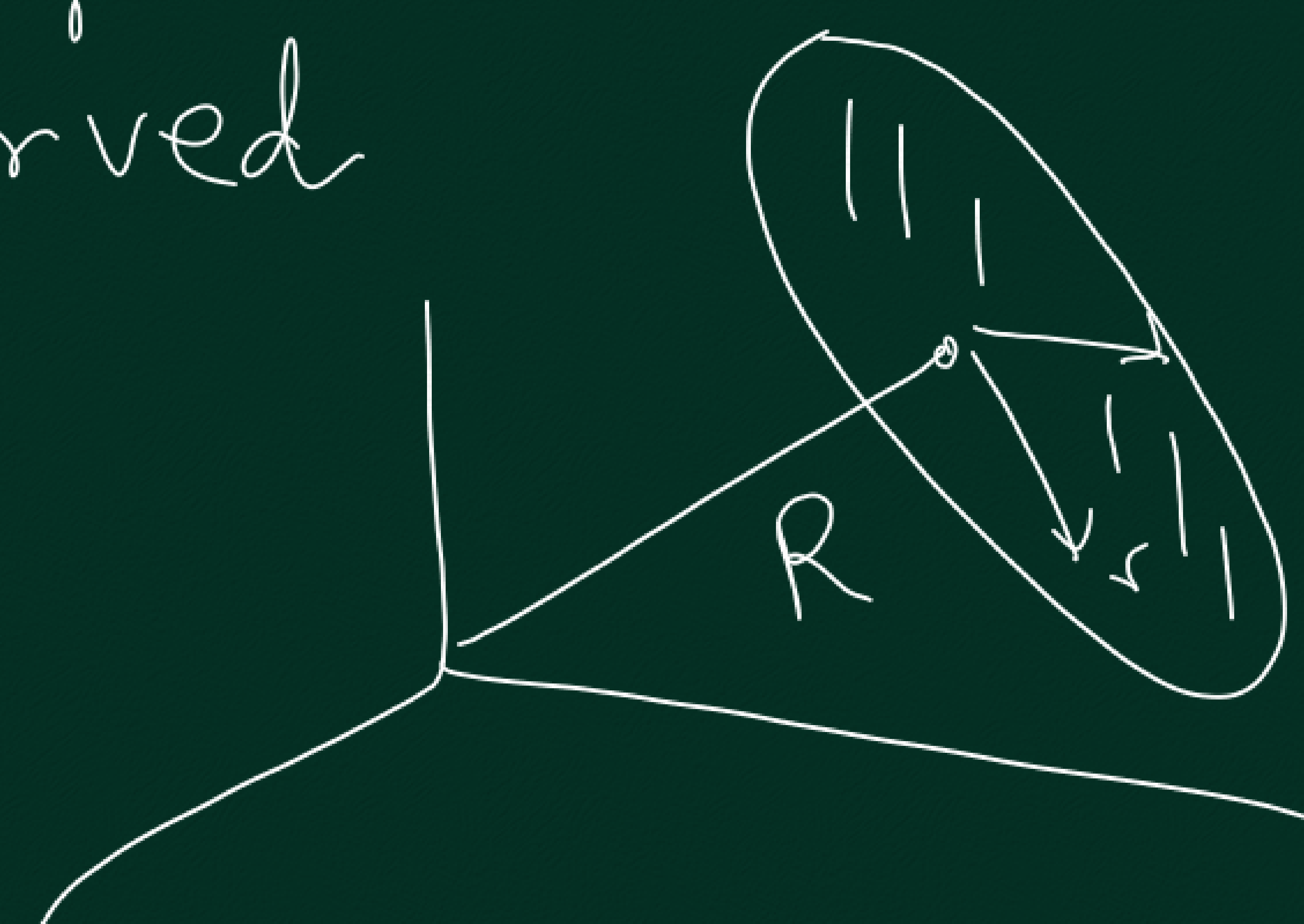
μ

$$\frac{dL'}{dt} = \mu\left[\cancel{\dot{\bar{r}} \times \dot{\bar{r}}} + \bar{r} \times \underbrace{\ddot{\bar{r}}}_{\hat{r}}\right] = 0$$





L' is conserved



$$\mu \ddot{\bar{r}} = f(r)\,\hat{r}$$

$\bar{r}$: $\hat{r} \times \hat{\theta} = \hat{z}$

$$\mu \bar{a} = \left[(\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}\right]\mu = f(r)\,\hat{r}$$

(1) $\ddot{r} - r\dot{\theta}^2 = f(r)$

(2) $2\dot{r}\dot{\theta} + r\ddot{\theta} = 0$

$r\times$

$$(2r\dot{r}\dot{\theta} + r^2\ddot{\theta}) = 0$$

$$\frac{d}{dt}\underbrace{(r^2\dot{\theta})}_{} = 0$$

$\mu r^2\dot{\theta}$ = const of motion

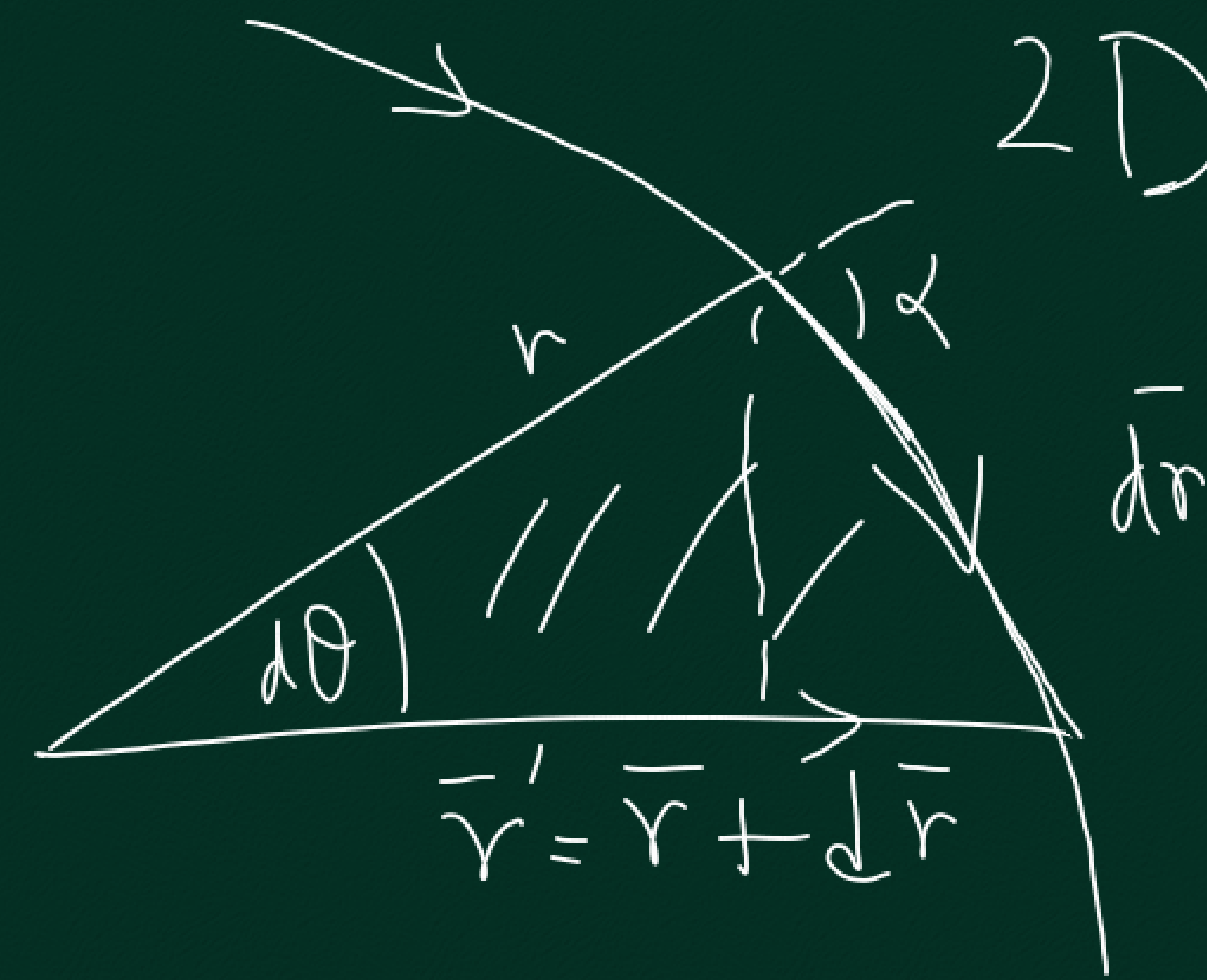
= ang mom in relative frame

$$\bar{r} = r\hat{r}$$

$$\dot{\bar{r}} = \bar{V} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\bar{r} \times \bar{V} = r^2\dot{\theta}\,\hat{r} \times \hat{\theta} = r^2\dot{\theta}\,\hat{z}$$

$$\underbrace{\mu r^2}_{I}\ \underbrace{\dot{\theta}}_{\omega}$$



2D pln (r,θ)

$$\frac{dA}{dt} = \text{const.}$$

$$d\bar{A} = \frac{1}{2}\,\bar{r} \times \bar{r}'$$

$$= \frac{1}{2}\, r\, \underset{\approx}{r'} \sin d\theta$$

$$\simeq \frac{1}{2}\, r\, r\, d\theta$$

$$= \frac{1}{2}\, r\left(r + \underset{\text{order}}{O}\left(\frac{dr}{r}\right)\right) d\theta$$

$$\bar{r}' = \bar{r} + d\bar{r}$$

$$r' = \sqrt{r^2 + dr^2 + 2\,\bar{r}\cdot d\bar{r}}$$

$$= r\left(1 + \left(\frac{dr}{r}\right)^2 + 2\frac{dr}{r}\cos\alpha\right)^{1/2}$$

$$r' \simeq r\left(1 + \frac{dr}{r}\cos\alpha\right)$$

$$\boxed{\frac{dA}{dt} \simeq \frac{1}{2} r^2 \frac{d\theta}{dt} \qquad = \frac{1}{2} r^2 \dot{\theta}} + * \frac{d\theta\, dr}{dt}$$

$$= \frac{L}{2\mu} \quad (\text{const})$$

Conservative or not

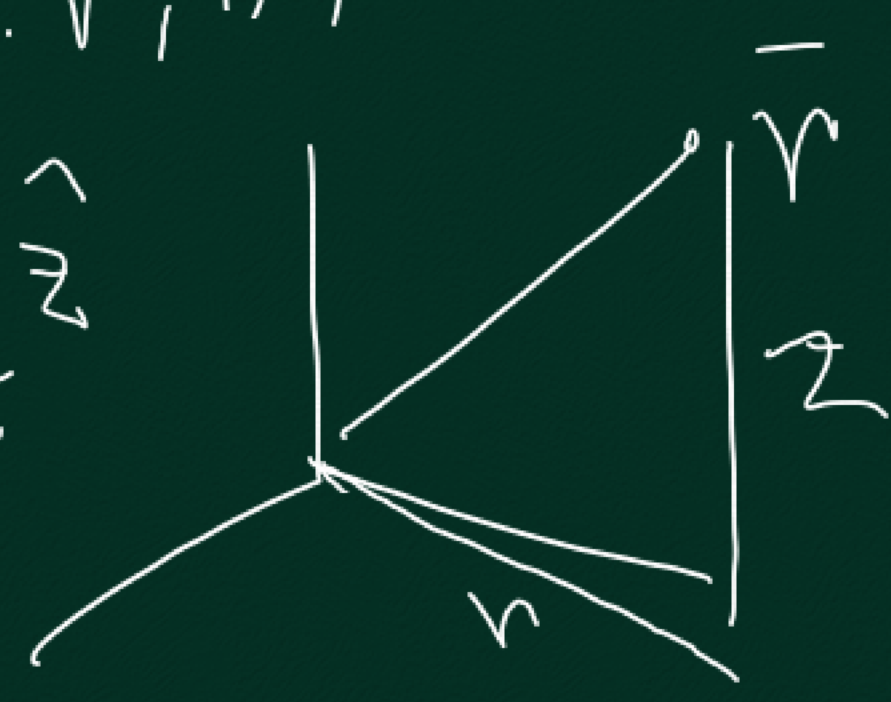
$$f(r) = -\frac{K}{r^2}, \quad K = G m_1 m_2$$

$$\bar{F} = f(r)\hat{r} = -\bar{\nabla} V(\bar{r})$$

$$\text{curl}\left(f(r)\,\hat{r}\right) = 0$$

<u>HW</u> cyl polr: r, θ, z

$$\bar{F} = A_r \hat{r} + A_z \hat{z} + A_\theta \hat{\theta}$$



$$\dot{\bar{r}} \cdot \dot{\bar{r}}$$

$$E = \frac{\mu}{2} \dot{\bar{r}}^2 + V(r)$$

$$\mu \ddot{\bar{r}} = -\bar{\nabla} V(r)$$

$$\frac{dE}{dt} = 0$$

$$\frac{dE}{dt} = \mu \dot{\bar{r}} \cdot \ddot{\bar{r}} + \frac{d}{dt} V(r)$$

$$= \mu \dot{\bar{r}} \cdot \ddot{\bar{r}} + \bar{\nabla} V \cdot \dot{\bar{r}}$$

$$= \dot{\bar{r}} \cdot \Big(\underbrace{\mu \ddot{\bar{r}} + \bar{\nabla} V}_{=0 \text{ from EOM}} \Big)$$

$$= 0$$

$$r = \sqrt{x^2 + y^2}$$

$$\frac{d}{dt} V(r) = \frac{\partial V}{\partial x} \dot{x} + \frac{\partial V}{\partial y} \dot{y}$$

$$= \bar{\nabla} V \cdot \dot{\bar{r}}$$

$$\mu r^2 \dot{\theta} = L$$

$$\dot{\theta} = \frac{L}{\mu r^2}$$

$$\mu r^2 \dot{\theta}^2 = \mu r^2 \frac{L^2}{\mu^2 r^4}$$

$$= \frac{L^2}{\mu r^2}$$

$$E = \frac{\mu}{2}\dot{r}^2 + \underbrace{\frac{L^2}{2\mu r^2} + V(r)}_{V_{eff}(r)}$$

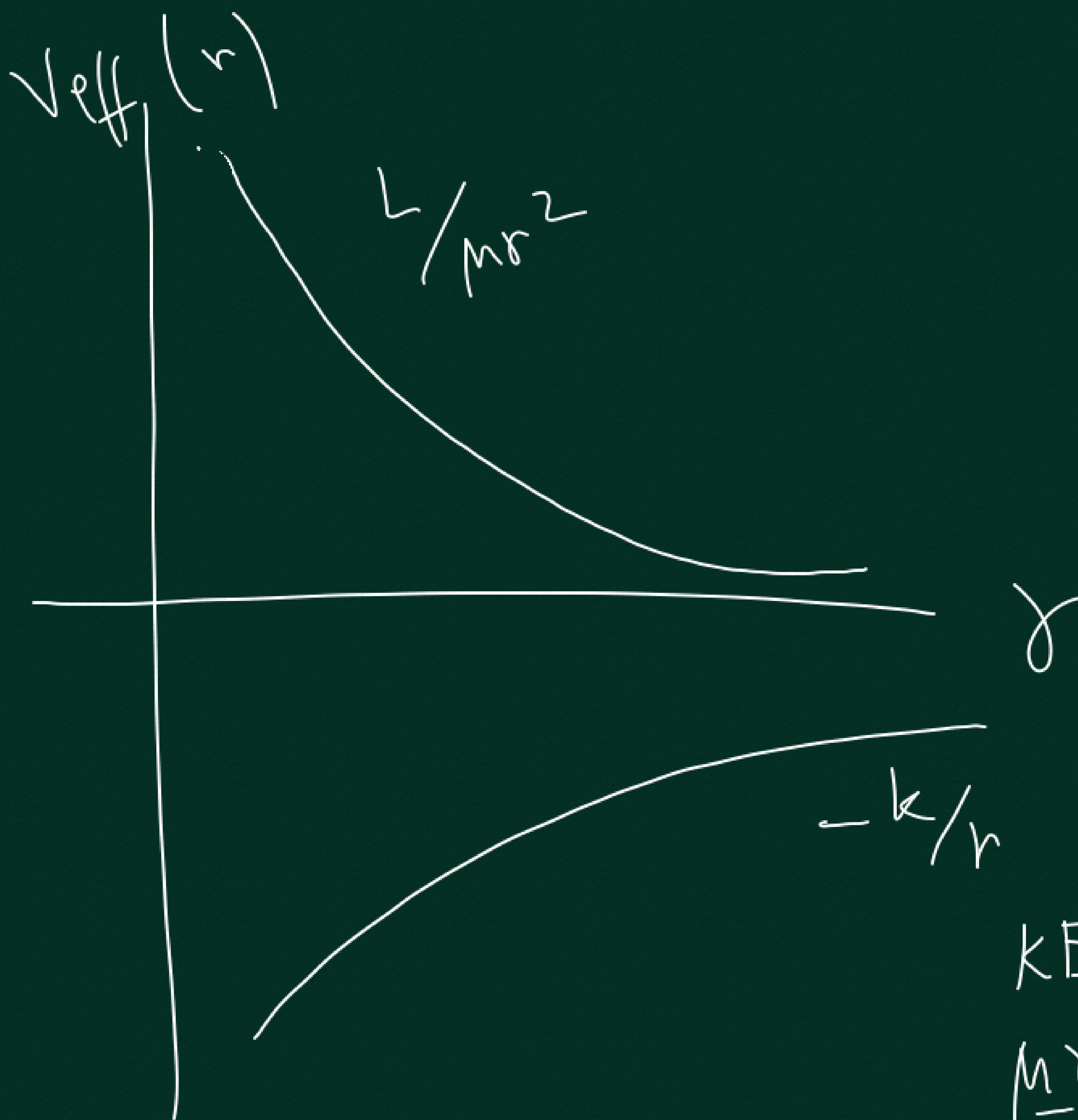
$$r(t)$$

$$E = \frac{\mu}{2}\dot{r}^2 + V_{eff}(r)$$

$$\downarrow$$

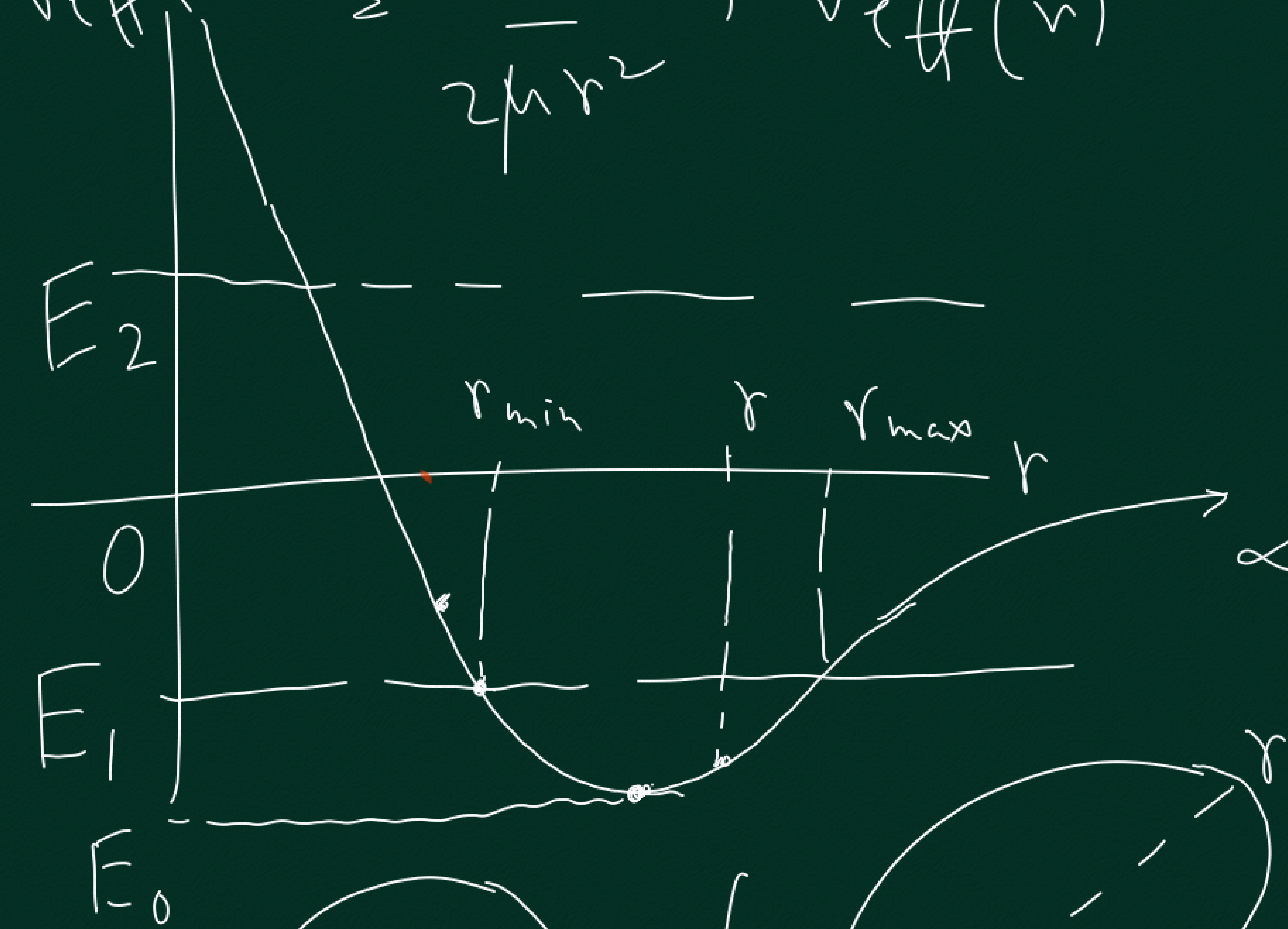
$$\dot{r} = g\left(E, \mu, V_{eff}(r)\right)$$

$$\boxed{\frac{d\theta}{dt} = \frac{L}{\mu r^2}}$$



$\Rightarrow$

$$V_{eff}(r) = \frac{L^2}{2\mu r^2} + V_{eff}(r)$$



$KE \propto V_{eff} =$

$\frac{\mu \dot{r}^2}{2}$