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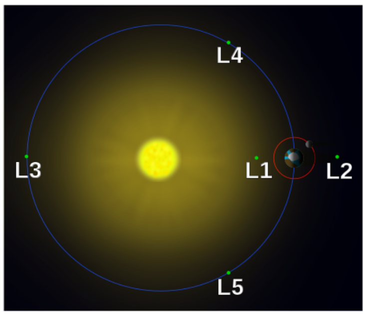
Lagrange point

In celestial mechanics, the **Lagrange points** (/ləˈɡrɑːndʒ/; also **Lagrangian points** or **libration points**) are points of equilibrium for small-mass objects under the gravitational influence of two massive orbiting bodies. Mathematically, this involves the solution of the restricted three-body problem.[1]



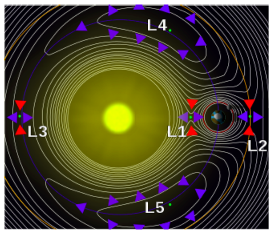
Lagrange points in the Sun–Earth system (not to scale). Earth's orbit here is counterclockwise.

Normally, the two massive bodies exert an unbalanced gravitational force at a point, altering the orbit of whatever is at that point. At the Lagrange points, the gravitational forces of the two large bodies and the centrifugal force balance each other.[2] This can make Lagrange points an excellent location for satellites, as orbit corrections, and hence fuel requirements, needed to maintain the desired orbit are kept at a minimum.

For any combination of two orbital bodies there are five Lagrange points, L_1 to L_5, all in the orbital plane of the two large bodies. There are five Lagrange points for the Sun–Earth system, and five *different* Lagrange points for the Earth–Moon system. L_1, L_2, and L_3 are on the line through the centers of the two large bodies, while L_4 and L_5 each act as the third vertex of an equilateral triangle formed with the centers of the two large bodies.

When the mass ratio of the two bodies is large enough, the L_4 and L_5 points are stable points meaning that objects can orbit them, and that they have a tendency to pull objects into them. Several planets have trojan asteroids near their L_4 and L_5 points with respect to the Sun; Jupiter has more than one million of these trojans.

Some Lagrange points are being used for space exploration. Two important Lagrange points in the Sun-Earth system are L_1, between the Sun and Earth, and L_2, on the same line at the opposite side of the Earth; both are well outside the Moon's orbit. Currently, an artificial satellite called the Deep Space Climate Observatory (DSCOVR) is located at L_1 to study solar wind coming toward Earth from the Sun and to monitor Earth's climate, by taking images and sending them back.[3] The James Webb Space Telescope, a powerful infrared space observatory, is located at L_2.[4] This allows the satellite's large sunshield to protect the telescope from the light and heat of the Sun, Earth and Moon.



A contour plot of the effective potential due to gravity and the centrifugal force of a two-body system in a rotating frame of reference. The arrows indicate the downhill gradients of the potential around the five Lagrange points, toward them (red) and away from them (blue). Counterintuitively, the L_4 and L_5 points are the high points of the potential. At the points themselves these forces are balanced.

The European Space Agency's earlier Gaia telescope, and its newly launched Euclid, also occupy orbits around L_2. Gaia keeps a tighter Lissajous orbit around L_2, while Euclid follows a halo orbit similar to JWST. Each of the space observatories benefit from being far enough from Earth's shadow to utilize solar panels for power, from not needing much power or propellant for station-keeping, from not being subjected to the Earth's magnetospheric effects, and from having direct line-of-sight to Earth for data transfer.

History

The three collinear Lagrange points (L_1, L_2, L_3) were discovered by the Swiss mathematician Leonhard Euler around 1750, a decade before the Italian-born Joseph-Louis Lagrange discovered the remaining two.[5][6]

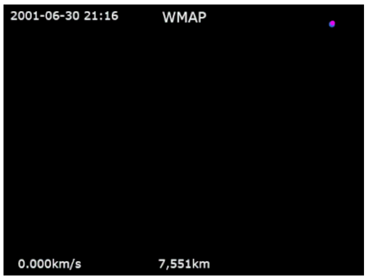
In 1772, Lagrange published an "Essay on the three-body problem". In the first chapter he considered the general three-body problem. From that, in the second chapter, he demonstrated two special constant-pattern solutions, the collinear and the equilateral, for any three masses, with circular orbits.[7]

Lagrange points

The five Lagrange points are labelled and defined as follows:

L_1 point

The L_1 point lies on the line defined between the two large masses M_1 and M_2. It is the point where the gravitational attraction of M_2 and that of M_1 combine to produce an equilibrium. An object that orbits the Sun more closely than Earth would typically have a shorter orbital period than Earth, but that ignores the effect of Earth's gravitational pull. If the object is directly between Earth and the Sun, then Earth's gravity counteracts some of the Sun's pull on the object, increasing the object's orbital period. The closer to Earth the object is, the greater this effect is. At the L_1 point, the object's orbital period becomes exactly equal to Earth's orbital period. L_1 is about 1.5 million kilometers, or 0.01 au, from Earth in the direction of the Sun.[1]



An example of a spacecraft at Sun-Earth L2
WMAP · Earth

L_2 point

The L_2 point lies on the line through the two large masses beyond the smaller of the two. Here, the combined gravitational forces of the two large masses balance the centrifugal force on a body at L_2. On the opposite side of Earth from the Sun, the orbital period of an object would normally be greater than Earth's. The extra pull of Earth's gravity decreases the object's orbital period, and at the L_2 point, that orbital period becomes equal to Earth's. Like L_1, L_2 is about 1.5 million kilometers or 0.01 au from Earth (away from the sun). An example of a spacecraft at L_2 is the James Webb Space Telescope, designed to operate near the Earth–Sun L_2.[8] Earlier examples include the Wilkinson Microwave Anisotropy Probe and its successor, *Planck*.

L_3 point

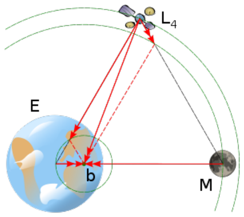
The L_3 point lies on the line defined by the two large masses, beyond the larger of the two. Within the Sun–Earth system, the L_3 point exists on the opposite side of the Sun, a little outside Earth's orbit and slightly farther from the center of the Sun than Earth is. This placement occurs because the Sun is also affected by Earth's gravity and so orbits around the two bodies' barycenter, which is well inside the body of the Sun. An object at Earth's distance from the Sun would have an orbital period of one year if only the Sun's gravity is considered. But an object on the opposite side of the Sun from Earth and directly in line with both "feels" Earth's gravity adding slightly to the Sun's and therefore must orbit a little farther from the barycenter of Earth and Sun in order to have the same 1-year period. It is at the L_3 point that the combined pull of Earth and Sun causes the object to orbit with the same period as Earth, in effect orbiting an Earth+Sun mass with the Earth-Sun barycenter at one focus of its orbit.

L_4 and L_5 points

The L_4 and L_5 points lie at the third vertices of the two equilateral triangles in the plane of orbit whose common base is the line between the centers of the two masses, such that the point lies 60° ahead of (L_4) or behind (L_5) the smaller mass with regard to its orbit around the larger mass.

Stability

The triangular points (L_4 and L_5) are stable equilibria, provided that the ratio of $\frac{M_1}{M_2}$ is greater than 24.96.[note 1] This is the case for the Sun–Earth system, the Sun–Jupiter system, and, by a smaller margin, the Earth–Moon system. When a body at these points is perturbed, it moves away from the point, but the factor opposite of that which is increased or decreased by the perturbation (either gravity or angular momentum-induced speed) will also increase or decrease, bending the object's path into a stable, kidney bean-shaped orbit around the point (as seen in the corotating frame of reference).[9]



Gravitational accelerations at L_4

The points L_1, L_2, and L_3 are positions of unstable equilibrium. Any object orbiting at L_1, L_2, or L_3 will tend to fall out of orbit; it is therefore rare to find natural objects there, and spacecraft inhabiting these areas must employ a small but critical amount of station keeping in order to maintain their position.

Natural objects at Lagrange points

Due to the natural stability of L_4 and L_5, it is common for natural objects to be found orbiting in those Lagrange points of planetary systems. Objects that inhabit those points are generically referred to as 'trojans' or 'trojan asteroids'. The name derives from the names that were given to asteroids discovered orbiting at the Sun–Jupiter L_4 and L_5 points, which were taken from mythological characters appearing in Homer's *Iliad*, an epic poem set during the Trojan War. Asteroids at the L_4 point, ahead of Jupiter, are named after Greek characters in the *Iliad* and referred to as the "Greek camp". Those at the L_5 point are named after Trojan characters and referred to as the "Trojan camp". Both camps are considered to be types of trojan bodies.

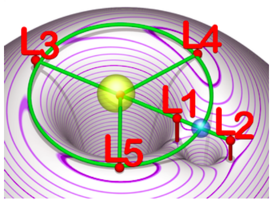
As the Sun and Jupiter are the two most massive objects in the Solar System, there are more known Sun–Jupiter trojans than for any other pair of bodies. However, smaller numbers of objects are known at the Lagrange points of other orbital systems:

- The Sun–Earth L_4 and L_5 points contain interplanetary dust and at least two asteroids, 2010 TK_7 and 2020 XL_5.[10][11][12]
- The Earth–Moon L_4 and L_5 points contain concentrations of interplanetary dust, known as Kordylewski clouds.[13][14] Stability at these specific points is greatly complicated by solar gravitational influence.[15]
- The Sun–Neptune L_4 and L_5 points contain several dozen known objects, the Neptune trojans.[16]
- Mars has four accepted Mars trojans: 5261 Eureka, 1999 UJ_7, 1998 VF_{31}, and 2007 NS_2.
- Saturn's moon Tethys has two smaller moons of Saturn in its L_4 and L_5 points, Telesto and Calypso. Another Saturn moon, Dione also has two Lagrange co-orbitals, Helene at its L_4 point and Polydeuces at L_5. The moons wander azimuthally about the Lagrange points, with Polydeuces describing the largest deviations, moving up to 32° away from the Saturn–Dione L_5 point.
- One version of the giant impact hypothesis postulates that an object named Theia formed at the Sun–Earth L_4 or L_5 point and crashed into Earth after its orbit destabilized, forming the Moon.[17]
- In binary stars, the Roche lobe has its apex located at L_1; if one of the stars expands past its Roche lobe, then it will lose matter to its companion star, known as Roche lobe overflow.[18]

Objects which are on horseshoe orbits are sometimes erroneously described as trojans, but do not occupy Lagrange points. Known objects on horseshoe orbits include 3753 Cruithne with Earth, and Saturn's moons Epimetheus and Janus.

Physical and mathematical details

Lagrange points are the constant-pattern solutions of the restricted three-body problem. For example, given two massive bodies in orbits around their common barycenter, there are five positions in space where a third body, of comparatively negligible mass, could be placed so as to maintain its position relative to the two massive bodies. This occurs because the combined gravitational forces of the two massive bodies provide the exact centripetal force required to maintain the circular motion that matches their orbital motion.



Visualisation of the relationship between the Lagrange points (red) of a planet (blue) orbiting a star (yellow) counterclockwise, and the effective potential in the plane containing the orbit (grey rubber-sheet model with purple contours of equal potential).[19] Click for animation.

Alternatively, when seen in a rotating reference frame that matches the angular velocity of the two co-orbiting bodies, at the Lagrange points the combined gravitational fields of two massive bodies balance the centrifugal pseudo-force, allowing the smaller third body to remain stationary (in this frame) with respect to the first two.

L_1

The location of L_1 is the solution to the following equation, gravitation providing the centripetal force:

$$\frac{M_1}{(R-r)^2} - \frac{M_2}{r^2} = \left(\frac{M_1}{M_1+M_2}R - r\right)\frac{M_1+M_2}{R^3}$$

where r is the distance of the L_1 point from the smaller object, R is the distance between the two main objects, and M_1 and M_2 are the masses of the large and small object, respectively. The quantity in parentheses on the right is the distance of L_1 from the center of mass. The solution for r is the only real root of the following quintic function

$$x^5 + (\mu-3)x^4 + (3-2\mu)x^3 - (\mu)x^2 + (2\mu)x - \mu = 0$$

where

$$\mu = \frac{M_2}{M_1+M_2}$$

and

$$x = \frac{r}{R}$$

However, if the mass of the smaller object (M_2) is much smaller than the mass of the larger object (M_1) then L_1 and L_2 are at approximately equal distances r from the smaller object, equal to the radius of the Hill sphere, given by:

$$r \approx R\sqrt[3]{\frac{\mu}{3}}$$

We may also write this as:

$$\frac{M_2}{r^3} \approx 3\frac{M_1}{R^3}$$

Since the tidal effect of a body is proportional to its mass divided by the distance cubed, this means that the tidal effect of the smaller body at the L_1 or at the L_2 point is about three times of that body. We may also write:

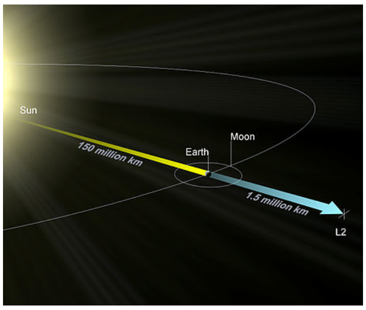
$$\rho_2 (d_2/r)^3 \approx 3\rho_1 (d_1/R)^3$$

where ρ_1 and ρ_2 are the average densities of the two bodies and d_1 and d_2 are their diameters. The ratio of diameter to distance gives the angle subtended by the body, showing that viewed from these two Lagrange points, the apparent sizes of the two bodies will be similar, especially if the density of the smaller one is about thrice that of the larger, as in the case of the earth and the sun.

This distance can be described as being such that the orbital period, corresponding to a circular orbit with this distance as radius around M_2 in the absence of M_1, is that of M_2 around M_1, divided by $\sqrt{3} \approx 1.73$:

$$T_{s,M_2}(r) = \frac{T_{M_2,M_1}(R)}{\sqrt{3}}.$$

L_2



The Lagrangian L_2 point for the Sun–Earth system

The location of L_2 is the solution to the following equation, gravitation providing the centripetal force:

$$\frac{M_1}{(R+r)^2} + \frac{M_2}{r^2} = \left(\frac{M_1}{M_1+M_2}R + r\right)\frac{M_1+M_2}{R^3}$$

with parameters defined as for the L_1 case. The corresponding quintic equation is

$$x^5 + x^4(3-\mu) + x^3(3-2\mu) - x^2(\mu) - x(2\mu) - \mu = 0$$

Again, if the mass of the smaller object (M_2) is much smaller than the mass of the larger object (M_1) then L_2 is at approximately the radius of the Hill sphere, given by:

$$r \approx R\sqrt[3]{\frac{\mu}{3}}$$

The same remarks about tidal influence and apparent size apply as for the L_1 point. For example, the angular radius of the sun as viewed from L_2 is $\arcsin(695.5 \times 10^3/151.1 \times 10^6) \approx 0.264°$, whereas that of the earth is $\arcsin(6371/1.5 \times 10^6) \approx 0.242°$. Looking toward the sun from L_2 one sees an annular eclipse. It is necessary for a spacecraft, like Gaia, to follow a Lissajous orbit or a halo orbit around L_2 in order for its solar panels to get full sun.

L_3

The location of L_3 is the solution to the following equation, gravitation providing the centripetal force:

$$\frac{M_1}{(R-r)^2} + \frac{M_2}{(2R-r)^2} = \left(\frac{M_2}{M_1+M_2}R + R - r\right)\frac{M_1+M_2}{R^3}$$

with parameters M_1, M_2, and R defined as for the L_1 and L_2 cases, and r being defined such that the distance of L_3 from the centre of the larger object is $R-r$. If the mass of the smaller object (M_2) is much smaller than the mass of the larger object (M_1), then:[20]

$$r \approx R\frac{7}{12}\mu.$$

Thus the distance from L_3 to the larger object is less than the separation of the two objects (although the distance between L_3 and the barycentre is greater than the distance between the smaller object and the barycentre).

L_4 and L_5

The reason these points are in balance is that at L_4 and L_5 the distances to the two masses are equal. Accordingly, the gravitational forces from the two massive bodies are in the same ratio as the masses of the two bodies, and so the resultant force acts through the barycenter of the system; additionally, the geometry of the triangle ensures that the resultant acceleration is to the distance from the barycenter in the same ratio as for the two massive bodies. The barycenter being both the center of mass and center of rotation of the three-body system, this resultant force is exactly that required to keep the smaller body at the Lagrange point in orbital equilibrium with the other two larger bodies of the system (indeed, the third body needs to have negligible mass). The general triangular configuration was discovered by Lagrange working on the three-body problem.

Radial acceleration

The radial acceleration a of an object in orbit at a point along the line passing through both bodies is given by:

$$a = -\frac{GM_1}{r^2}\operatorname{sgn}(r) + \frac{GM_2}{(R-r)^2}\operatorname{sgn}(R-r) + \frac{G\big((M_1+M_2)r - M_2R\big)}{R^3}$$

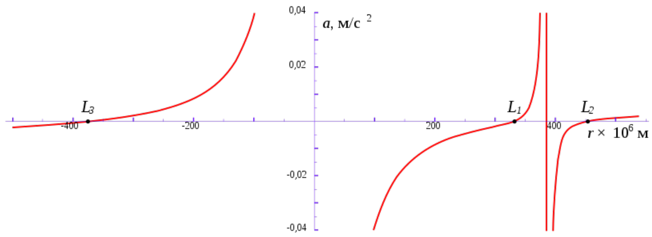
where r is the distance from the large body M_1, R is the distance between the two main objects, and sgn(x) is the sign function of x. The terms in this function represent respectively: force from M_1; force from M_2; and centripetal force. The points L_3, L_1, L_2 occur where the acceleration is zero — see chart at right. Positive acceleration is acceleration towards the right of the chart and negative acceleration is towards the left; that is why acceleration has opposite signs on opposite sides of the gravity wells.

Stability

Although the L_1, L_2, and L_3 points are nominally unstable, there are quasi-stable periodic orbits called *halo orbits* around these points in a three-body system. A full n-body dynamical system such as the Solar System does not contain these periodic orbits, but does contain quasi-periodic (i.e. bounded but not precisely repeating) orbits following

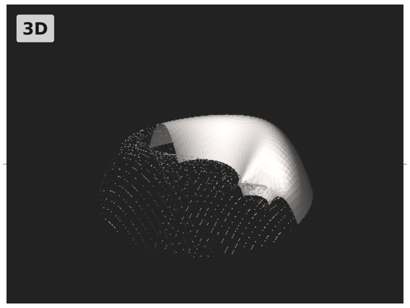
Lissajous-curve trajectories. These quasi-periodic Lissajous orbits are what most of Lagrangian-point space missions have used until now. Although they are not perfectly stable, a modest effort of station keeping keeps a spacecraft in a desired Lissajous orbit for a long time.

For Sun–Earth-L_1 missions, it is preferable for the spacecraft to be in a large-amplitude (100,000–200,000 km or 62,000–124,000 mi) Lissajous orbit around L_1 than to stay at L_1, because the line between Sun and Earth has increased solar interference on Earth–spacecraft communications. Similarly, a large-amplitude Lissajous orbit around L_2 keeps a probe out of Earth's shadow and therefore ensures continuous illumination of its solar panels.



Net radial acceleration of a point orbiting along the Earth–Moon line

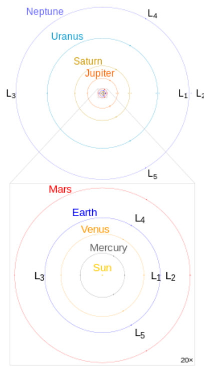
The L_4 and L_5 points are stable provided that the mass of the primary body (e.g. the Earth) is at least 25[note 1] times the mass of the secondary body (e.g. the Moon),[21][22] and the mass of the secondary is at least 10 times that of the tertiary (e.g. the satellite). The Earth is over 81 times the mass of the Moon (the Moon is 1.23% of the mass of the Earth[23]). Although the L_4 and L_5 points are found at the top of a "hill", as in the effective potential contour plot above, they are nonetheless stable. The reason for the stability is a second-order effect: as a body moves away from the exact Lagrange position, Coriolis acceleration (which depends on the velocity of an orbiting object and cannot be modeled as a contour map)[22] curves the trajectory into a path around (rather than away from) the point.[22][24] Because the source of stability is the Coriolis force, the resulting orbits can be stable, but generally are not planar, but "three-dimensional": they lie on a warped surface intersecting the ecliptic plane. The kidney-shaped orbits typically shown nested around L_4 and L_5 are the projections of the orbits on a plane (e.g. the ecliptic) and not the full 3-D orbits.



STL 3D model of the Roche potential of two orbiting bodies, rendered half as a surface and half as a mesh

Solar System values

This table lists sample values of L_1, L_2, and L_3 within the Solar System. Calculations assume the two bodies orbit in a perfect circle with separation equal to the semimajor axis and no other bodies are nearby. Distances are measured from the larger body's center of mass (but see barycenter especially in the case of Moon and Jupiter) with L_3 showing a negative direction. The percentage columns show the distance from the orbit compared to the semimajor axis. E.g. for the Moon, L_1 is 326 400 km from Earth's center, which is 84.9% of the Earth–Moon distance or 15.1% "in front of" (Earthwards from) the Moon; L_2 is located 448 900 km from Earth's center, which is 116.8% of the Earth–Moon distance or 16.8% beyond the Moon; and L_3 is located −381 700 km from Earth's center, which is 99.3% of the Earth–Moon distance or 0.7084% inside (Earthward) of the Moon's 'negative' position.



Sun–planet Lagrange points to scale (Click for clearer points.)

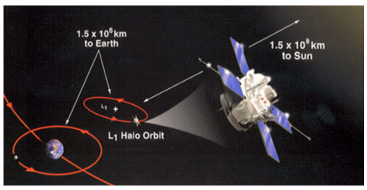
Lagrangian points in Solar System

Body pair	Semimajor axis, SMA ($\times 10^9$ m)	L_1 ($\times 10^9$ m)	$1 - L_1/SMA$ (%)	L_2 ($\times 10^9$ m)	$L_2/SMA - 1$ (%)	L_3 ($\times 10^9$ m)	$1 + L_3/SMA$ (%)
Earth–Moon	0.3844	0.326 39	15.09	0.4489	16.78	−0.381 68	0.7084
Sun–Mercury	57.909	57.689	0.3806	58.13	0.3815	−57.909	0.000 009 683
Sun–Venus	108.21	107.2	0.9315	109.22	0.9373	−108.21	0.000 1428
Sun–Earth	149.598	148.11	0.997	151.1	1.004	−149.6	0.000 1752
Sun–Mars	227.94	226.86	0.4748	229.03	0.4763	−227.94	0.000 018 82
Sun–Jupiter	778.34	726.45	6.667	832.65	6.978	−777.91	0.055 63
Sun–Saturn	1 426.7	1 362.5	4.496	1 492.8	4.635	−1 426.4	0.016 67
Sun–Uranus	2 870.7	2 801.1	2.421	2 941.3	2.461	−2 870.6	0.002 546
Sun–Neptune	4 498.4	4 383.4	2.557	4 615.4	2.602	−4 498.3	0.003 004

Spaceflight applications

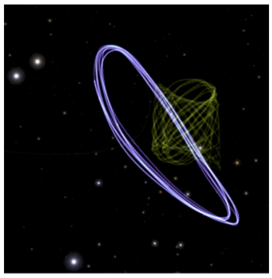
Sun–Earth

Sun–Earth L_1 is suited for making observations of the Sun–Earth system. Objects here are never shadowed by Earth or the Moon and, if observing Earth, always view the sunlit hemisphere. The first mission of this type was the 1978 International Sun Earth Explorer 3 (ISEE-3) mission used as an interplanetary early warning storm monitor for solar disturbances.[25] Since June 2015, DSCOVR has orbited the L_1 point. Conversely, it is also useful for space-based solar telescopes, because it provides an uninterrupted view of the Sun and any space weather (including the solar wind and coronal mass ejections) reaches L_1 up to an hour before Earth. Solar and heliospheric missions currently located around L_1 include the Solar and Heliospheric Observatory, Wind, and the Advanced Composition Explorer. Planned missions include the Interstellar Mapping and Acceleration Probe (IMAP) and the NEO Surveyor.



The satellite ACE in an orbit around Sun–Earth L_1

Sun–Earth L_2 is a good spot for space-based observatories. Because an object around L_2 will maintain the same relative position with respect to the Sun and Earth, shielding and calibration are much simpler. It is, however, slightly beyond the reach of Earth's umbra,[26] so solar radiation is not completely blocked at L_2. Spacecraft generally orbit around L_2, avoiding partial eclipses of the Sun to maintain a constant temperature. From locations near L_2, the Sun, Earth and Moon are relatively close together in the sky; this means that a large sunshade with the telescope on the dark-side can allow the telescope to cool passively to around 50 K – this is especially helpful for infrared astronomy and observations of the cosmic microwave background. The James Webb Space Telescope was positioned in a halo orbit about L_2 on January 24, 2022.


The Gaia (yellow) and James Webb Space Telescope (blue) orbits around Sun–Earth L_2

Sun–Earth L_1 and L_2 are saddle points and exponentially unstable with time constant of roughly 23 days. Satellites at these points will wander off in a few months unless course corrections are made.[9]

Sun–Earth L_3 was a popular place to put a "Counter-Earth" in pulp science fiction and comic books, despite the fact that the existence of a planetary body in this location had been understood as an impossibility once orbital mechanics and the perturbations of planets upon each other's orbits came to be understood, long before the Space Age; the influence of an Earth-sized body on other planets would not have gone undetected, nor would the fact that the foci of Earth's orbital ellipse would not have been in their expected places, due to the mass of the counter-Earth. The Sun–Earth L_3, however, is a weak saddle point and exponentially unstable with time constant of roughly 150 years.[9] Moreover, it could not contain a natural object, large or small, for very long because the gravitational forces of the other planets are stronger than that of Earth (for example, Venus comes within 0.3 AU of this L_3 every 20 months).

A spacecraft orbiting near Sun–Earth L_3 would be able to closely monitor the evolution of active sunspot regions before they rotate into a geoeffective position, so that a seven-day early warning could be issued by the NOAA Space Weather Prediction Center. Moreover, a satellite near Sun–Earth L_3 would provide very important observations not only for Earth forecasts, but also for deep space support (Mars predictions and for crewed missions to near-Earth asteroids). In 2010, spacecraft transfer trajectories to Sun–Earth L_3 were studied and several designs were considered.[27]

Earth–Moon

Earth–Moon L_1 allows comparatively easy access to Lunar and Earth orbits with minimal change in velocity and this has as an advantage to position a habitable space station intended to help transport cargo and personnel to the Moon and back. The SMART-1 Mission [28] passed through the L_1 Lagrangian Point on 11 November 2004 and passed into the area dominated by the Moon's gravitational influence

Earth–Moon L_2 has been used for a communications satellite covering the Moon's far side, for example, Queqiao, launched in 2018,[29] and would be "an ideal location" for a propellant depot as part of the proposed depot-based space transportation architecture.[30]

Earth–Moon L_4 and L_5 are the locations for the Kordylewski dust clouds.[31] The L5 Society's name comes from the L4 and L5 Lagrangian points in the Earth–Moon system proposed as locations for their huge rotating space habitats. Both positions are also proposed for communication satellites covering the Moon alike communication satellites in geosynchronous orbit cover the Earth.[32][33]

Sun–Venus

Scientists at the B612 Foundation were[34] planning to use Venus's L_3 point to position their planned Sentinel telescope, which aimed to look back towards Earth's orbit and compile a catalogue of near-Earth asteroids.[35]

Sun–Mars

In 2017, the idea of positioning a magnetic dipole shield at the Sun–Mars L_1 point for use as an artificial magnetosphere for Mars was discussed at a NASA conference.[36] The idea is that this would protect the planet's atmosphere from the Sun's radiation and solar winds.

See also

Astronomy portal
Spaceflight portal

- Co-orbital configuration
- Euler's three-body problem
- Gegenschein
- Interplanetary Transport Network
- Klemperer rosette
- L_5 Society
- Lagrange point colonization
- Lagrangian mechanics
- List of objects at Lagrange points
- Lunar space elevator
- Oberth effect

Explanatory notes

1. Actually $(25 + 3\sqrt{69})/2 \approx 24.959\,935\,7944$ (sequence A230242 in the OEIS)

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- What are Lagrange points? (https://www.esa.int/Enabling_Support/Operations/What_are_Lagrange_points)—European Space Agency page, with good animations
- Explanation of Lagrange points (https://web.archive.org/web/20030605101737/http://www.physics.montana.edu/faculty/cornish/lagrange.html)—Neil J. Cornish
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- See the Lagrange Points and Halo Orbits subsection under the section on Geosynchronous Transfer Orbit in *NASA: Basics of Space Flight*, Chapter 5 (https://solarsystem.nasa.gov/basics/chapter5-1#critical)