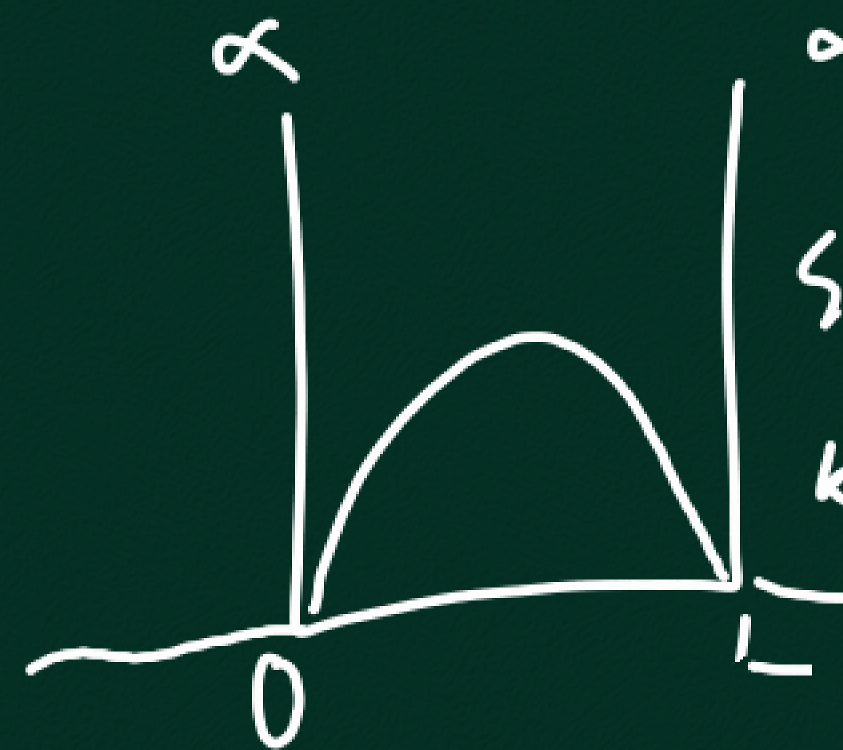
1) Continoust everywhere including boundaries

$E_1 = \alpha$

$E_2 = 4\alpha$

$\vdots \qquad \vdots$



$\sin k_n x$

$k_n = \frac{n\pi}{L}$

$\psi'(x) \neq$ contns

$$E_n = \frac{-13.6\, Z^2}{n^2}$$

$k_B T$

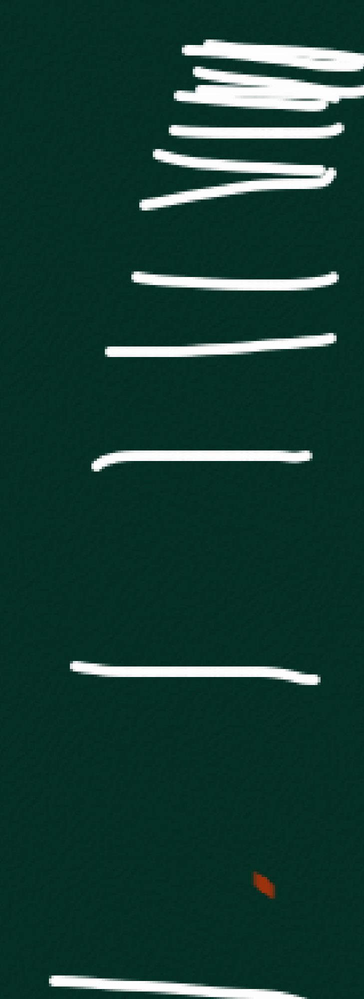
$\Delta E = E_{n+1} - E_n$

$= \alpha(2n+1)$

$\Delta E \uparrow$ with n

gap betwn successv energy levels

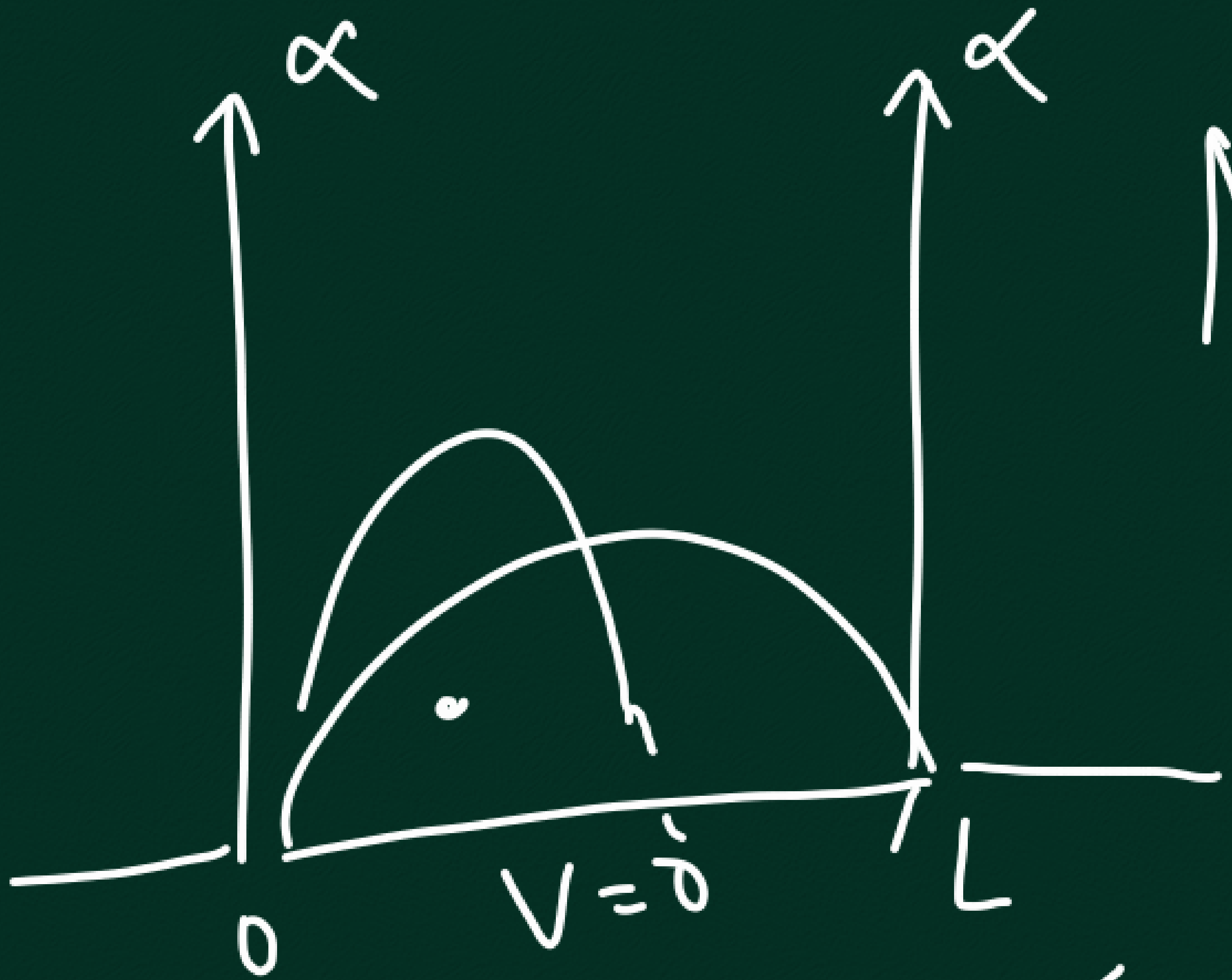
$$|\Delta E| \sim \left|\frac{1}{(n+1)^2} - \frac{1}{n^2}\right| \sim \left|\frac{2n+1}{(n+1)^2 n^2}\right|$$



$$\sim \frac{2n}{n^4} \sim \frac{1}{n^3}$$

$\Delta E \to 0$

as $n \to \infty$



Normalization

$$\psi(x) = C_n \sin k_n x$$

$$\int_0^L |\psi(x)|^2 dx = 1 \quad , \; k_n = \frac{n\pi}{L}$$

$$C_n^2 \underbrace{\int_0^L \sin^2 \frac{n\pi x}{L}\, dx}_{\frac{L}{2}} = 1 \qquad \boxed{\frac{1}{2}\int_0^L \left(1 - \cos \frac{n\pi x}{L}\right)} \quad \Rightarrow C_n = \sqrt{\frac{2}{L}}$$

orthogonality among ψ_n's

$$\int_0^L \underset{E_n}{\underset{\downarrow}{\psi_n(x)}}\, \underset{E_m}{\underset{\downarrow}{\psi_m(x)}}\, dx = \underset{\substack{=0 \text{ if } m \neq n \\ =1 \text{ if } m = n}}{\underset{\uparrow}{\delta_{mn}}}$$

$$\Rightarrow \psi_n(x) = \sqrt{\frac{2}{L}} \sin k_n x$$

Croneker delta

$$\frac{2}{L}\int_0^L \sin\frac{n\pi x}{L}\,\sin\frac{m\pi x}{L}\,dx$$

$$\frac{2}{L}\,\frac{1}{2}\int_0^L\left[\cos\underset{\substack{\uparrow\\ \neq 0}}{(n-m)}\frac{\pi x}{L} - \cos\underset{\substack{\uparrow\\ n\neq 0}}{(n+m)}\frac{\pi x}{L}\right]dx$$

$$\left.\begin{array}{lcl}\text{for } n\neq m & \longrightarrow & 0\\ \text{for } n=m & \longrightarrow & 1\end{array}\right\}\delta_{nm}$$

$$\psi_n \sim \sin\frac{\pi n x}{L}$$

$$p(x) = |\psi_n(x)|^2$$

$$|\psi_n|^2(x)$$

$$\langle O\rangle = \frac{\int \psi_n^*(x)\,\hat{O}\,\psi_n(x)\,dx}{\int \psi_n^*(x)\,\psi_n(x)\,dx}$$

$$\psi_n = \sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi}{L}x\right)$$

$$\langle \hat{p}\rangle = \int_0^L \sin\frac{n\pi x}{L}\left(-i\hbar\frac{d}{dx}\right)\sin\left(\frac{n\pi x}{L}\right)\frac{dx}{2L}$$

$$= -i\hbar\,\frac{n\pi}{L}\int_0^L 2\sin\frac{2n\pi x}{L}\,dx = 0$$

$k = \frac{n\pi}{L}$ from BC

$V=0$ free ptcl

$0 \longrightarrow E > 0$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\psi = Ae^{ikx} + Be^{-ikx}$$

$p = \hbar k$ $\qquad$ $p = -\hbar k$

$$\psi = A e^{ikx}$$

$$= \sqrt{\frac{2}{L}} \sin k_n x$$

$$|\psi|^2 = A^2$$

Not a free ptcl

$$\int |\psi|^2 dx \quad L$$

$$= \lim_{L\to\infty} \frac{2}{L} \int_0^L \sin^2 k_n x \, dx$$

$$= 1$$

(Box normalization)

Ex.

$$z = a + ib$$

$$|z|^2 = z \cdot z^*$$

$$= (a+ib)(a-ib)$$

$$= a^2 + b^2 \quad \because i^2 = -1$$

$$\psi = \sqrt{\frac{1}{L}} e^{ikx}$$ free ptcl

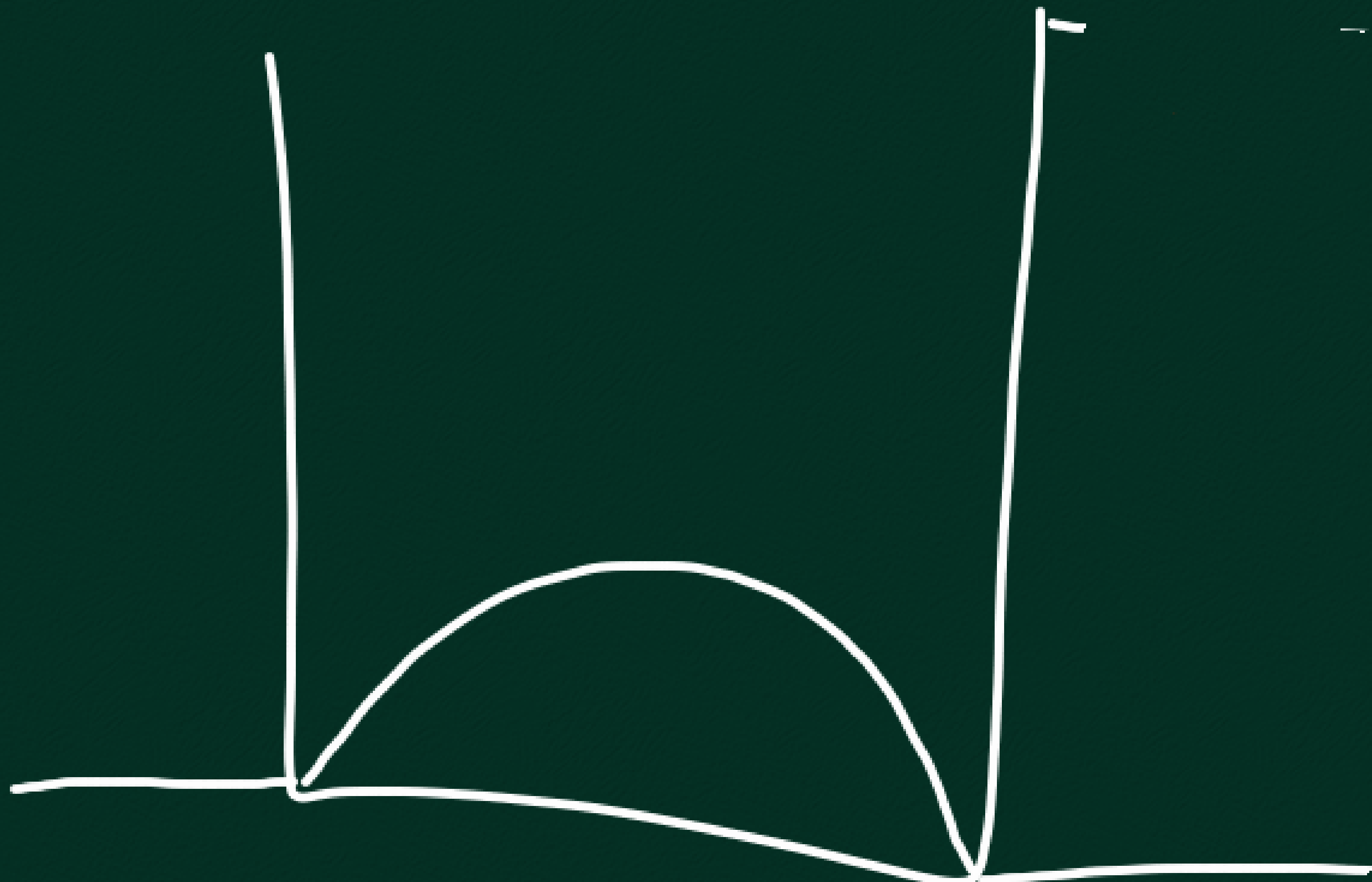
$$|\psi|^2 = \frac{1}{L}$$

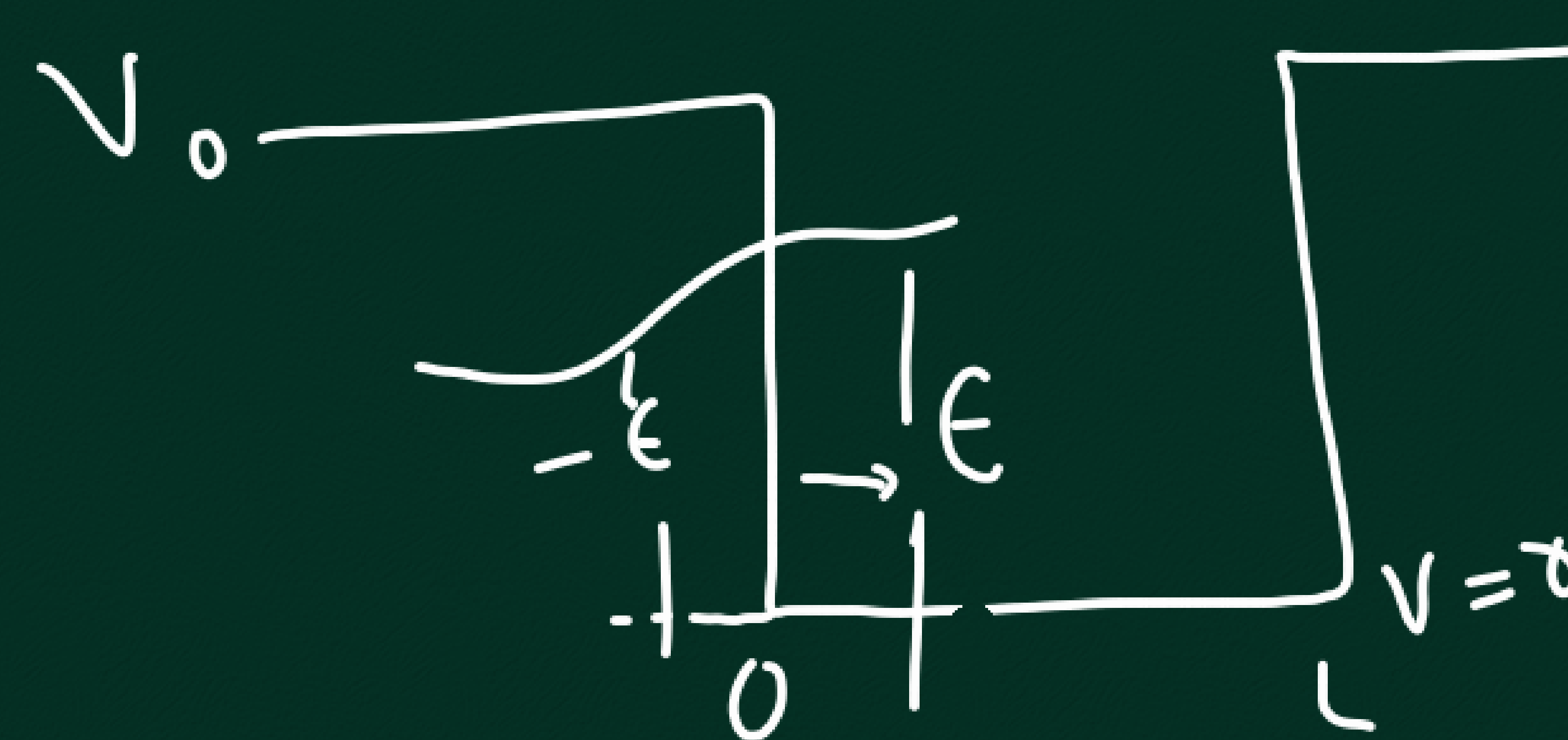
$-L/2$ $L/2$

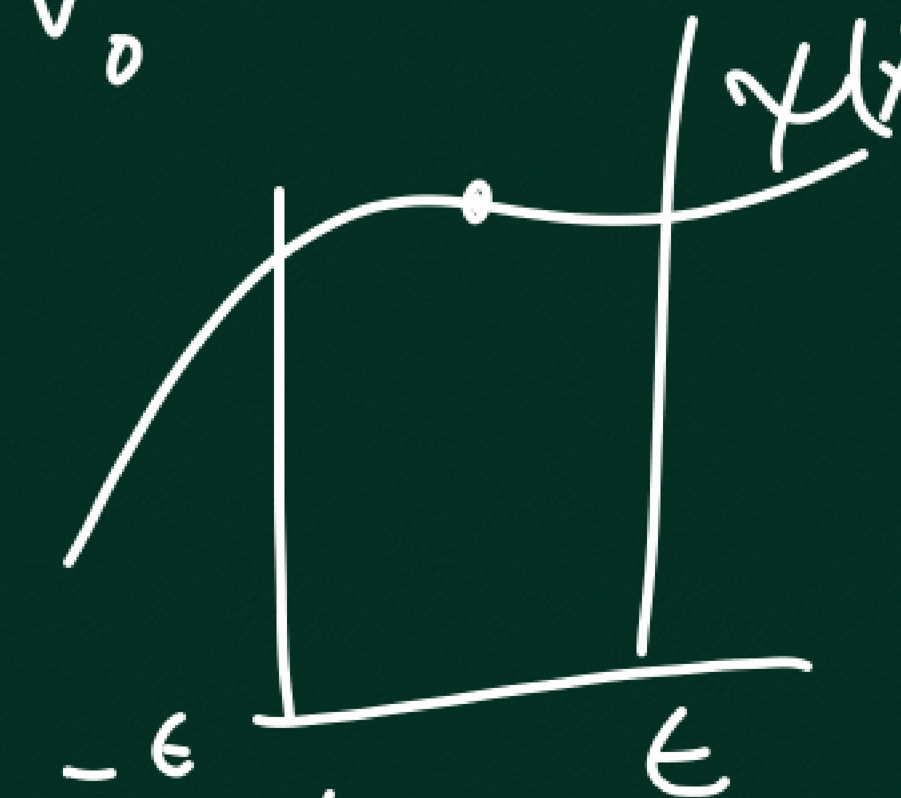
$$\lim_{L\to\infty} \int_0^L |\psi|^2 dx = 1$$

$$\langle \hat{p} \rangle = \frac{\int \psi^* \left(-i\hbar \frac{d}{dx}\right) \psi \, dx}{\int \psi^* \psi \, dx}$$

$$= \frac{1}{L} \int \frac{e^{-ikx} \, (-i\hbar)(ik) \, e^{ikx}}{\frac{1}{L}} = \hbar k$$

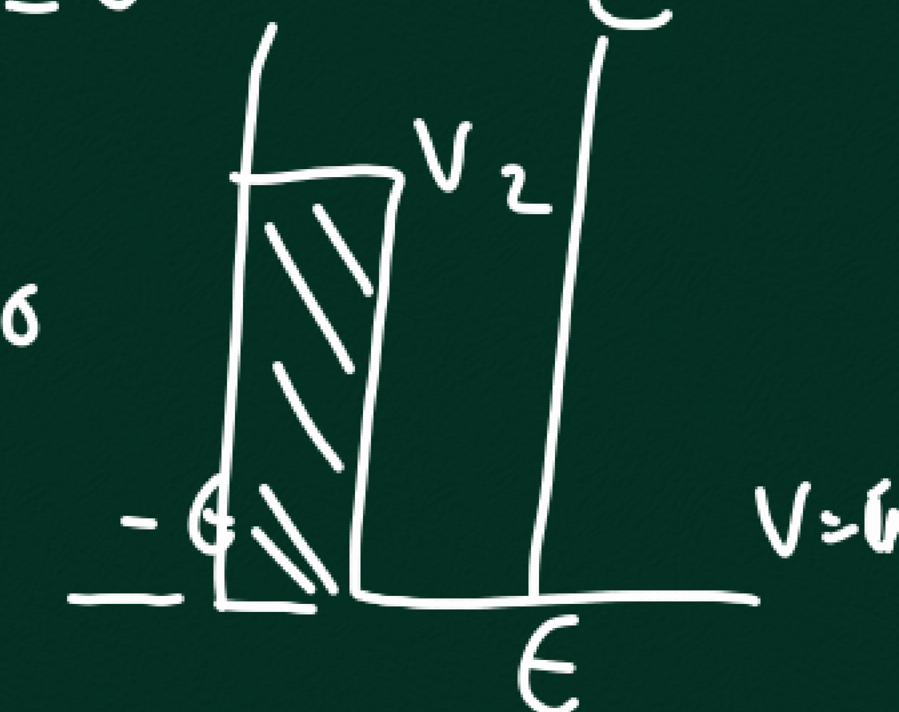






jump in
if V at boundaries is finite then $\psi'(x)$ also has to be contins

$\epsilon \to 0$



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

$$\int_{-\epsilon}^{\epsilon} \to \quad -\frac{\hbar^2}{2m}\left[\frac{d\psi}{dx}\Big|_{x=\epsilon} - \frac{d\psi}{dx}\Big|_{x=-\epsilon}\right] = \int_{-\epsilon}^{\epsilon}\left[E - V\right]\psi(x)\,dx$$

$$= \cancel{2\epsilon\bar{\psi}} - \cancel{\left(V_2 + V_1\right)\epsilon}$$

$\epsilon \to 0$