PH110: QM : Tutorial Sheet 4

Notes:

1. * marked problems will be solved in the Wednesday tutorial class.
2. Please make sure that you do the assignment by yourself. You can consult your classmates and seniors and ensure you understand the concept. However, do not copy assignments from others.

Free particle

1. *Show that
$$\psi(x) = A\,\sin(kx) + B\,\cos(kx)$$
and
$$\psi(x) = Ce^{ikx} + De^{-ikx}$$
are equivalent solutions of TISE of a free particle. A, B, C and D can be complex numbers.

2. Show that
$$\psi(x,t) = A\sin(kx - \omega t) + B\cos(kx - \omega t)$$
does not obey the time-dependant Schroedinger's equation for a free particle.

3. The wave function for a particle is given by,
$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$
where A and B are constants. Calculate the quantity $\frac{\hbar}{m}\Im\left(\psi^* \frac{d\psi}{dx}\right)$, where $\Im$ denotes the imaginary part of a complex number. What is its physical significance ?

4. * A free proton has a wave function given by
$$\psi(x,t) = Ae^{i(5.02*10^{11}x - 8{:}00*10^{15}t)}$$
The coefficient of x is inverse meters, and the coeffcient of t is inverse seconds. Find its momentum and energy.

5. A particle moving in one dimension is in a stationary state whose wave function,
$$\psi(x) = \begin{cases} 0, & x < -a \\ A\left(1 + \cos\frac{\pi x}{a}\right), & -a \le x \le a \\ 0, & x > a \end{cases}$$
where A and a are real constants.
(a) Is this a physically acceptable wave function? Explain.
(b) Find the magnitude of A so that $\psi(x)$ is normalized.
(c) Evaluate Δx and Δp. Verify that $\Delta x \Delta p \ge \hbar/2$.
(d) Find the classically allowed region.

Particle in a Box:

1. * For a particle in a 1-D box of side L, show that the probability of finding the particle between $x = a$ and $x = a + b$ approaches the classical value b/L, if the energy of the particle is very high.

2. Consider a particle confined to a 1-D box. Find the probability that the particle in its ground state will be in the central one-third region of the box.

3. Consider a particle of mass m moving freely between $x = 0$ and $x = a$ inside an infinite square well potential.
(a) Calculate the expectation values $\langle \hat{X} \rangle_n, \langle \hat{P} \rangle_n, \left\langle \hat{X}^2 \right\rangle_n$, and $\left\langle \hat{P}^2 \right\rangle_n$, and compare them with their classical counterparts.
(b) Calculate the uncertainties product $\Delta x_n \Delta p_n$.
(c) Use the result of (b) to estimate the ground state energy.

4. Consider a one dimensional infinite square well potential of length L. A particle is in $n = 3$ state of this potential well. Find the probability that this particle will be observed between $x = 0$ and $x = L/6$. Can you guess the answer without solving the integral? Explain how.

5. * Consider a one-dimensional particle which is confined within the region $0 \leq x \leq a$ and whose wave function is $\psi(x,t) = \sin(\pi x/a)\exp(-i\omega t)$.
(a) Find the potential $V(x)$.
(b) Calculate the probability of finding the particle in the interval $a/4 \leq x \leq 3a/4$.

6. An electron is moving freely inside a one-dimensional infinite potential box with walls at $x = 0$ and $x = a$. If the electron is initially in the ground state $(n = 1)$ of the box and if we suddenly quadruple the size of the box (i.e., the right-hand side wall is moved instantaneously from $x = a$ to $x = 4a$), calculate the probability of finding the electron in:
(a) the ground state of the new box and
(b) the first excited state of the new box.

7. Solve the time independent Schrodinger equation for a particle in a 1-D box, taking the origin at the centre of the box and the ends at $\pm L/2$, where L is the length of the box.

8. * Consider a particle of mass m in an infinite potential well extending from $x = 0$ to $x = L$. Wave function of the particle is given by

$$\psi(x) = A\left[\sin\left(\frac{\pi x}{L}\right) + \sin\left(\frac{2\pi x}{L}\right)\right]$$

where A is the normalization constant

(a) Calculate A
(b) Calculate the expectation values of x and x^2 and hence the uncertainty Δx.
(c) Calculate the expectation values of p and p^2 and hence the uncertaintyΔp.
(d) What is the probability of finding the particle in the first excited state, if an energy measurement is made?

(given, $\int_0^L x\cos\left(\frac{n\pi x}{L}\right)dx = 0$, $\int_0^L x^2\cos\left(\frac{n\pi x}{L}\right)dx = 0$, for all n)