PH110: Tutorial Sheet 1

1. Consider two vectors $\mathbf{A} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$. Find a third vector $\mathbf{C}$ (say), which is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.

2. Again, consider the vectors $\mathbf{A}$ and $\mathbf{B}$ of the previous problem. Find the angle between them.

3. Find a unit vector, which lies in the xy plane, and which is perpendicular to $\mathbf{A}$ of previous problems. Similarly, find a unit vector which is perpendicular to $\mathbf{B}$, and lies in the xz plane.

4. Calculate $\mathbf{A}\cdot(\mathbf{B} \times \mathbf{A})$, for the vectors of the previous problem. Does the result obtained hold only for the given $\mathbf{A}$ and $\mathbf{B}$ vectors, or will it hold for any general vectors $\mathbf{A}$ and $\mathbf{B}$.

5. Consider two distinct general vectors $\mathbf{A}$ and $\mathbf{B}$. Show that $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$ implies that $\mathbf{A}$ and $\mathbf{B}$ are perpendicular.

6. Position of a particle in the xy plane is given by

$$\mathbf{r}(t) = A\left(e^{\alpha t}\hat{\mathbf{i}} + e^{-\alpha t}\hat{\mathbf{j}}\right),$$

where α and A are constants. Calculate the velocity and acceleration of the particle, as functions of time t. Plot the vectors corresponding to $\mathbf{r}(0)$ and $\mathbf{v}(0)$.

7. Acceleration of a particle in the xy plane is given by $\mathbf{a}(t) = -\omega^2\mathbf{r}(t)$, where $\mathbf{r}(t)$ denotes its position, and ω is a constant. If $\mathbf{r}(0) = a\hat{\mathbf{j}}$, and $\mathbf{v}(0) = a\omega\hat{\mathbf{i}}$ ($\mathbf{v}$ is the velocity), integrate the equation of motion to obtain the expression for $\mathbf{r}(t)$, in Cartesian coordinates.

8. Rate of change of acceleration of a particle is called "jerk" ($\mathbf{j}(t)$) in physics. If the jerk of a particle is given by

$$\mathbf{j}(t) = a\hat{\mathbf{i}} + bt\hat{\mathbf{j}} + ct^2\hat{\mathbf{k}},$$

where a, b, and c are constants. Assuming that at time $t = 0$, particle was located at the origin, and its velocity and acceleration were zero, obtain its position $\mathbf{r}(t)$, as a function of time, in Cartesian coordinates.

9. A rocket of mass M accelerates in free space by expelling hot gas in the backward direction. The speed of the exhaust gas depends on the energy released in the combustion process and can be taken to be a constant, say $|u|$ w.r.t to the rocket. Assume that in time Δt the rocket loses an amount of mass $\Delta m = -\frac{dM}{dt}\Delta t$, where $\frac{dM}{dt}$ denotes the rate of change of the mass of the rocket. Answer the following questions.

   (a) If the instantaneous velocity of the rocket is v, in an inertial frame, what is the velocity of the exhaust in that frame? Write down the total momentum of the system as seen from that inertial frame at t and $t + \Delta t$.

(b) From this obtain a differential equation connecting the changes in mass and velocity.

(c) Show that the final velocity increases only as the *logarithm* of the amount of fuel.

(d) What is the significance of such a dependence ?

10. Consider a parallelopiped with sides of length a, b, and c. The angle between the side with length a and length b is γ, the angle between sides of length a and c is β and the one between sides of length b and c is α. Calculate the volume of the paralleopiped.

11. A ship sails from the southernmost point of India (6.75^oN, 93.84^oE) to the southernmost point of Africa (34.5^oS, 20.00^oE) following the shortest possible path.

    (a) Given that the radius of the earth is 6400 km, what is the distance it has covered?

    (b) If instead of sailing, one had travelled in an aeroplane - by what percentage would the shortest possible distance change?