## How does one measure length ? Why does time get involved in this ?

Measure the co-ordinates of two ends of a stick at the same instant of time.

Is "at the same instant" (simultaneous) an unambiguous notion?

In an inertial frame, yes . But NOT across different frames.

Two clocks may be synchronised in frame S. To S' they will not be so.

A rod of length $L_0$ is at rest in S' : between $x_2$ and $x_1\,'$

S' measures the rest frame length $L_0 = x_2\,' - x_1\,'$

S tries to measure the length

he must measure the co-incience of the endpoints at the same instant.

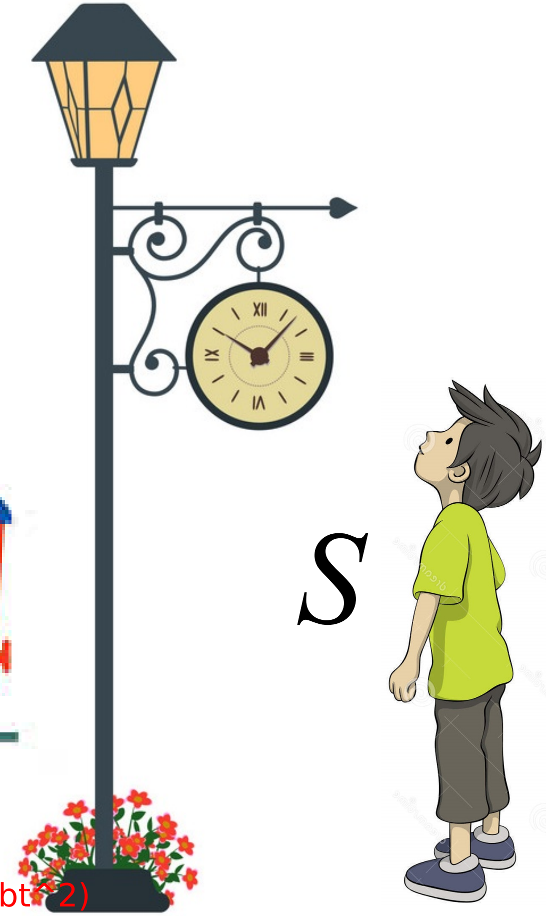
$$\left.\begin{aligned} x_1\,' &= \frac{x_1 - vt}{\sqrt{1-\beta^2}} \\ x_2\,' &= \frac{x_2 - vt}{\sqrt{1-\beta^2}} \end{aligned}\right\} \quad L = (x_2\,' - x_1\,')\sqrt{1-\beta^2} = L_0\sqrt{1-\beta^2}$$

Let's do it another way....

A train moving with a speed $v$ crosses a lamp-post in time $t$.

Q: What is its length ?

A: classically it is $vt$


In the boy's frame S, which is moving right wrt to S' at speed v, the 2 events are occuring at the post (same point), So he sees the Del t shortened. S' sees it dialated. So S sees, L=v.delt=v. delt' sqrt(1-beta^2) and delt'=Lo/v. So L=Lo*sqrt(1-bt^2)

In $S\,'$ tip & rear ends are at $x\,'=0$ and $x\,'=-L_0$ always

Q: At what time in $S$ does $x\,'=-L_0$ give $x=0$?

A: $-L_0=\dfrac{0-vt}{\sqrt{1-\beta^2}}$ $\Rightarrow$ for $S$ train's length : $vt=L_0\sqrt{1-\beta^2}$

# How does one compare measurement of time intervals ?

Consider a clock at rest in the S' frame at the origin of S' (so x'=0 always)

At t'=0 we have t=0 : What happens when t' =1 (say)

$$t = \frac{t' + vx'/c^2}{\sqrt{1-\beta^2}}$$

$$x' = 0 \quad \text{and} \quad t' = 1$$

$$t = \frac{1}{\sqrt{1-\beta^2}}$$

A clock at rest in S' will appear to run slow when timed by clocks in S.

If the "clock" is a particle with a 1 second lifetime it will appear to live longer when seen by S

This is why many fast moving particles in cosmic rays manage to reach the earth's surface.

The universe is a clock.. ☑
The universe has a clock ☒

"Length contraction" and "time dilation" are consequences of the two postulates of relativity.

# Transformation of velocities : velocity addition rule

Problem: We need to relate $u_x = \dfrac{dx}{dt}$ with $u_x{}' = \dfrac{dx'}{dt'}$

$$\delta x' = \frac{\delta x - v\,\delta t}{\sqrt{1-\beta^2}}$$

$$\delta t' = \frac{\delta t - (v/c^2)\,\delta x}{\sqrt{1-\beta^2}}$$

$$\frac{\delta x'}{\delta t'} = \frac{\delta x - v\,\delta t}{\delta t - (v/c^2)\,\delta x}$$

$$u_x{}' = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u_x = \frac{u_x{}' + v}{1 + u_x{}' v/c^2}$$

what happens if $u_x{}' = \dfrac{c}{n}$ ? (Fizeau)

The acceleration seen by two inertial observers is not the same.
Derive the relation by calculating the 2nd derivatives.

How would you define Force ?