Topics we are going to do & References

[1]	Brief historical description about the need for quantum theory
[2]	Compton scattering : Xray photon & electron
[3]	Wave-particle duality, de Broglie hypothesis
[4]	Concept of wave packets, group velocity, phase velocity, etc
[5]	Mathematical interlude: discussion of Fourier transform, Gaussian wave packets etc.
[6]	Heisenberg uncertainty principle
[7]	Schrödinger equation and its applications to simple problems such as free particle, particle in a box, potential barriers and wells, and 1D simple harmonic oscillator
[8]	Brief discussion of how to solve problems in 2D & 3D

- **Modern Physics:** R. A.Serway, C. J. Moses, C. A. Moyer, Thomson Learning Inc. 2005

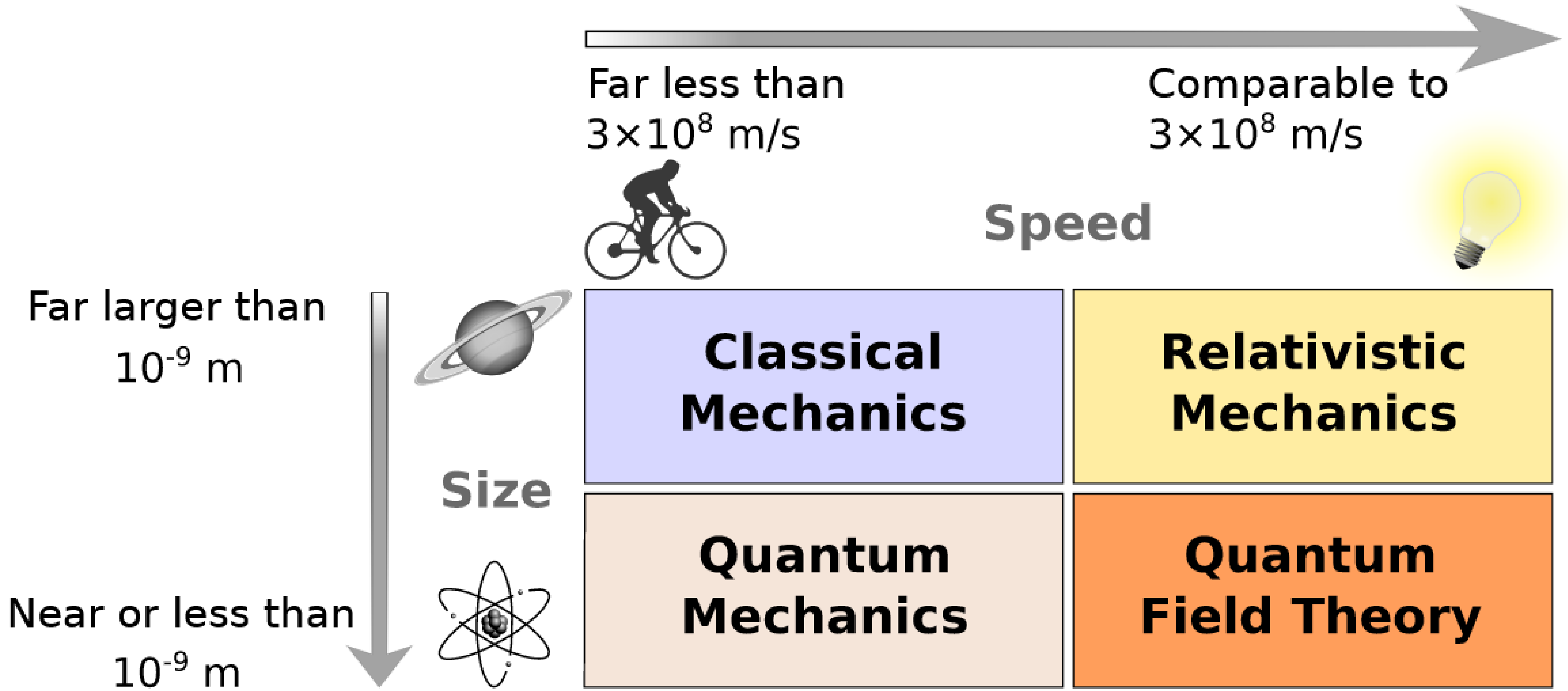
- **Concepts of Modern Physics:** A. Beiser, S. Mahajan, S. Rai Choudhury; McGraw Hill International,1987 4th Ed…

What was understood and what was not : circa ~1900

Successes	Failures
Newtonian Mechanics : Motion of planets etc	Radiation from hot objects (Blackbody radiation)
Electromagnetic basis of light/optics with some phenomenological input	Photoelectric effect : why it has a threshold ?
Gas Laws and fluid mechanics	Specific heats of materials at low temperatures
Thermodynamic principles	Scattering of X-ray by electrons (Compton scattering)
Engineering of ships, trains, engines, motors...which utilised all these.	Stability of the atom...

The atomistic nature of matter was NOT established !
Einstein's "dissertation" is on Brownian motion, which he used to argue for existance of atoms as building blocks.

The new developments in ~ 1900-1930 are still called Modern Physics !!



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Waves and particles

Photoelectric effect , Compton scattering are examples where lightwaves behave like a particle. We treat them using some Energy-momentum conservation rules

There are situations where particles (like electrons) behave like a wave. These are called matter waves.

A critical question: What wave equation can correcty describe the behaviour of waves of matter? We will return to this key question later

Compton scattering
Particle like behaviour of EM wave

What are the expectations and observations ?

Classcial EM can desribe the scattering of light from things like ~1 micron dust particle quite well.

Xray are also EM waves. So think of an electron as a very small particle and repeat the calculation.

One finds that the expectation is NOT met. This is the key problem.

The solution was found by independently by Compton and Debye (~1922-23)

The momentum of light (any EM radiation)

Classical EM theory predicts that electromagnetic field has a certain energy (U) and momentum (P) per unit volume. For a field corresponding to an EM wave U = Pc

Light exerts pressure. If I is the energy density then the radiation pressure is I/c. It is fairly easy to show this !

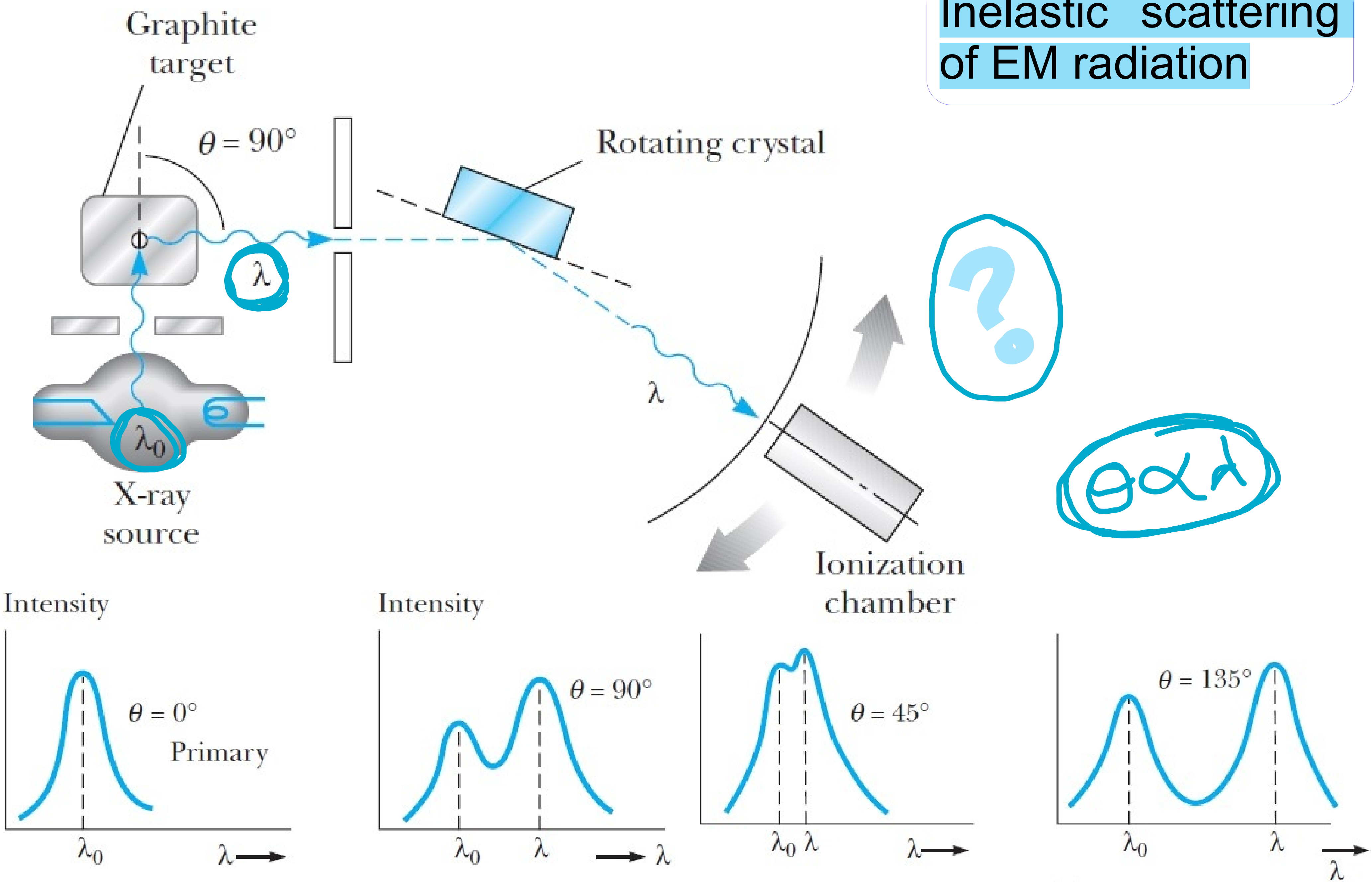
If the EM field is replaced by "quanta" of light, then those must carry energy and Momentum. What experiment demonstrates the momentum of light?

$$U = \frac{1}{2} \int_{\substack{all \\ space}} \left(\epsilon_0 E^2 + \frac{B^2}{\mu_0} \right) d^3 \vec{r}$$

$$\vec{P} = \frac{1}{c^2} \int_{\substack{all \\ space}} \left(\frac{\vec{E} \times \vec{B}}{\mu_0} \right) d^3 \vec{r}$$

What did Compton observe ?

Inelastic scattering of EM radiation



What did Compton observe ?

A few non-trivial details....

Why is the target Graphite ?

The loosely bound electrons are almost free with a work function of ~ 4eV

What is the role of the crystal?

The crystal ~~is like a diffraction grating for X-rays.~~
The lattice of atoms work like opaque-transparent-opaque periodicity of a grating.
Different wavelengths will have different angular deviations.
Notice that here the X-rays are behaving like EM waves!

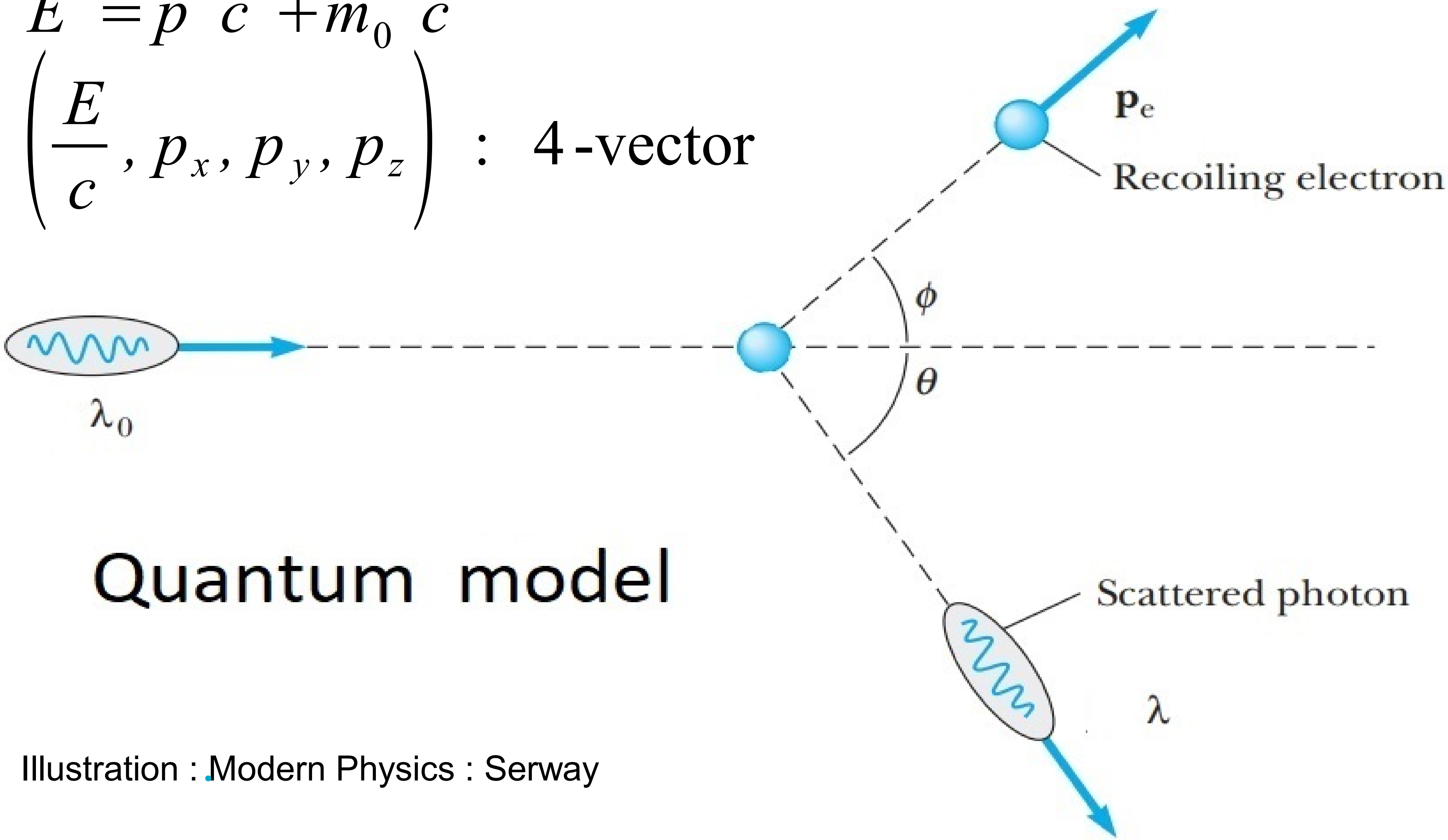
What do the core-electrons of the atoms do ?

The core electrons are very tightly bound. They give characteristic lines. That is not elastic either, but that is not scattering by a free electron at rest

The relativistic quantum view of e^--photon scattering

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$\left(\frac{E}{c}, p_x, p_y, p_z\right) \ : \ \text{4-vector}$$



Quantum model

Illustration : Modern Physics : Serway

Each component of the 4-vector is conserved....

Using the relativistic Energy-momentum relation : $E^2 = p^2 c^2 + m_0^2 c^4$ .....

$$E = h\nu_0 + m_0 c^2 = h\nu + \sqrt{p_e^2 c^2 + m_0^2 c^4}$$

$$P_x = \frac{h\nu_0}{c} = \frac{h\nu}{c}\cos\theta + p_e \cos\phi$$

$$P_y = 0 = \frac{h\nu}{c}\sin\theta - p_e \sin\phi$$

$$p_e \sin\phi = \frac{h\nu}{c}\sin\theta$$

$$p_e \cos\phi = \frac{h\nu_0}{c} - \frac{h\nu}{c}\cos\theta$$

$$\Rightarrow p_e^2 c^2 = (h\nu)^2 + (h\nu_0)^2 - 2h^2 \nu \nu_0 \cos\theta$$

We can calculate the same quantity in another way $\rightarrow$

What does Energy conservation tell us for e^--Xray scattering?

$$p_e^{\,2} c^2 + m_0^{\,2} c^4 = \left[h(\nu_0 - \nu) + m_0 c^2\right]^2$$

$$\Rightarrow \quad p_e^{\,2} c^2 = h^2 (\nu_0 - \nu)^2 + 2h(\nu_0 - \nu) m_0 c^2$$

$$(h\nu)^2 + (h\nu_0)^2 - 2h^2 \nu \nu_0 \cos\theta = h^2(\nu_0 - \nu)^2 + 2h(\nu_0 - \nu) m_0 c^2$$

$$\lambda - \lambda_0 = \frac{h}{m_0 c}(1 - \cos\theta) \quad \begin{cases} \Delta\lambda = 0 & \text{for } \theta = 0 \\ \text{largest} & \text{for } \theta \to \pi \end{cases}$$

$$\frac{h}{m_0 c} = 2.43 \times 10^{-12}\, m \Rightarrow \frac{\Delta\lambda}{\lambda} \approx 0.02 \text{ for } 1\, A^o \text{ X-ray}$$

Does an analogue of Compton effect exist for visible light... ?

Unless the wavelength was already small the fractional change would be too small to detect.

Also a heavier particle (like the nucleus) would give a smaller shift...

Compton scattering of visible light by free electrons would be too small to detect. BUT....could there be other "objects"“ causing ineleastic scattering of visible light ?

If so extra lines would appear in the spectrum of a monochromatic light when scattered by atoms/molecules of a transparent medium.

This indeed happens. This is the Raman Effect.

Can the photon disappear (get absorbed) in the scattering process?

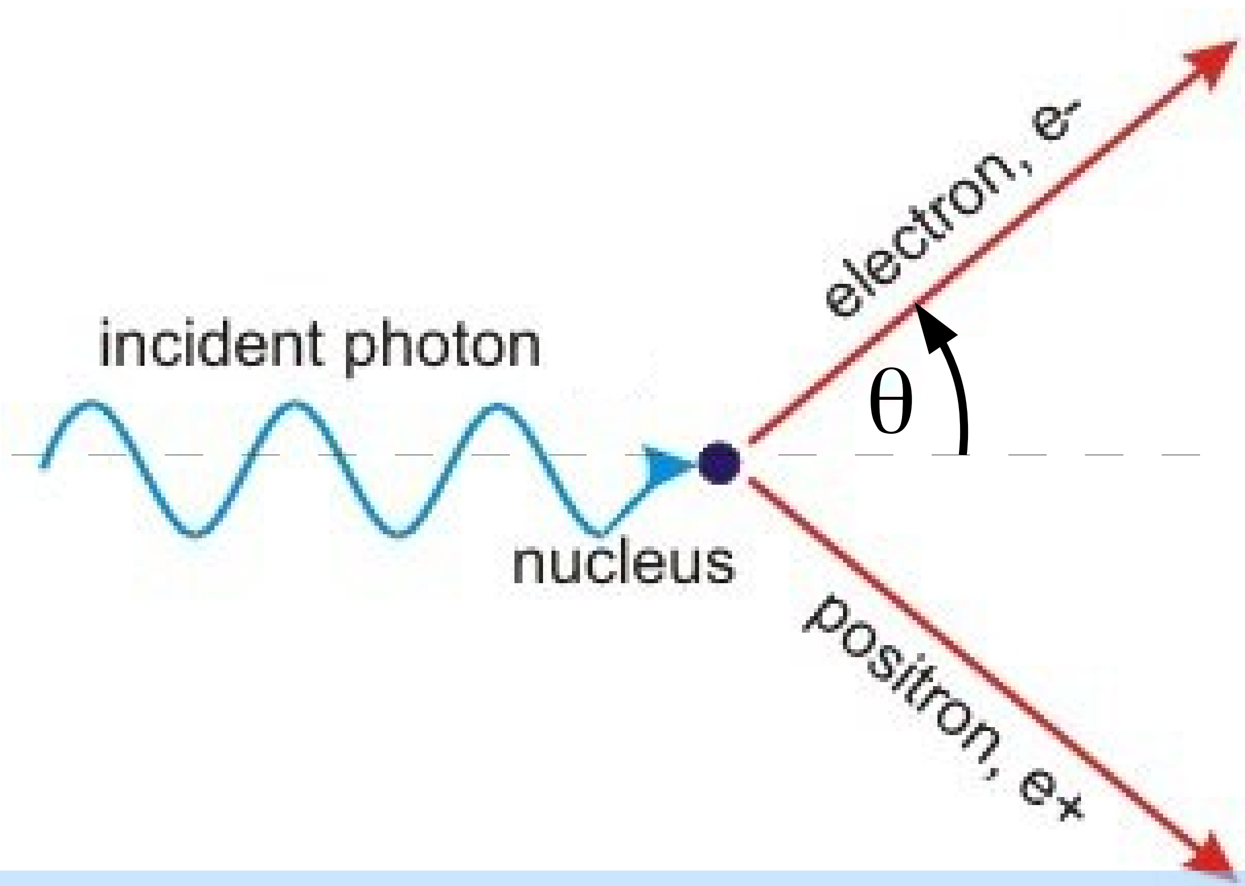
$$\left.\begin{aligned} \frac{h\nu_0}{c} + 0 &= p_e \\ h\nu_0 + m_0 c^2 &= \sqrt{p_e^{\,2} c^2 + m_0^{\,2} c^4} \end{aligned}\right\} \Rightarrow h\nu = 0$$

We assumed the electron was at rest. That is sufficient. WHY? We just go to the rest frame of the electron...the conclusions will not change.

In any "light-matter" interaction, both energy and momentum MUST to be conserved. Then how does an atom go to an excited state ?

The electron needs to be bound. The photon momentum can be absorbed by the much heavier system (atom) with negligible KE. The energy goes to raise the electron to a higher energy state.

When can a photon produce an electron-positron pair ? $h\nu \rightarrow e^{+} + e^{-}$?



Why is the presence of a heavy object (nucleus) needed for this process ?

The E,p equations have no solution unless something can take away the momentum.

The heavy nucleus can take away the momentum but very little Kinetic energy

$$\frac{h\nu_0}{c} = 2\, p_e \cos\theta$$

$$h\nu_0 = 2\sqrt{p_e^{\,2} c^2 + m_0^{\,2} c^4}$$

The various energy regimes of light-matter interaction