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Tutorial-2

Ans 1

```
void fun(int n){
int j=1, i=0
while(i<n){
 i=i+j;
 j++;}}
```

$j=1$ $i=0+1$

$j=2$ $i=0+1+2$

$j=3$ $i=0+1+2+3$

⁞

L loop ends when $i>=n$

$0+1+2+3 - n>n$

$\frac{k(k+1)}{2}>n$

$k^2>n$

$k>\sqrt{n}$

$\Rightarrow O(\sqrt{n})$

Ans-2 Recurrence Rel^n for Fibonacci Series

$T(n)=T(n-1)+T(n-2)$ $T(0)=T(1)=1$

- if $T(n-1) \approx T(n-2)$

$T(n)=2T(n-2)$

Put $n=n-2$

$T(n-2)=2T(n-4)$

$T(n)=2(2T(n-4))=4T(n-4)$

Put $n=n-4$

$T(n-4)=2T(n-6)$

$T(n)=4(2T(n-6))=8T(n-6)$

⁞

$T(n)=2^kT(n-2k)$ $\therefore n-2k=0$, $n=2k$ OR $n-2k=1$, $n=1+2k$, $\frac{n-1}{2}=k$

$T(n)=2^nT(2k-2k)$

$2^nT(0)$

$O(2^n)$

Ans 3

Complexity $n\lg n$, n^3, $\lg(\lg n)$

$O(n\lg n)$

```
for(int i=0; i<n; i++){
  for(int j=1; j<n; j=j*2){
    // O(1)
  }}
```

$O(n^3)$

```
for(i=0 to n; i++)
{
  for(j=0 to n; j++){
    for(k=0 to n; k++){
      // O(1) }}}
```

$O(\lg(\lg n))$:

```
for(i=1 to n  i=i*2){
  for(j=1 to n  j=j*2){
    // O(1) }}
```

Ans 4 $T(n) = T(n/4) + T(n/2) + cn^2$

we know that

$T(\frac{n}{2}) >= T(n/4)$

So $T(n) = 2T(\frac{n}{2}) + cn^2$

applying Master theorem $(T(n) = aT(\frac{n}{b}) + f(n))$

$a = 2,\ b = 2 \quad f(n) = n^2$

$c = \log_b a = \log_2 2 = 1$

$n^c = n$

Compare $f(n) > n^c$ So $T(n) = O(n^2)$

Ans 5

```
int fun( int n){
for(int i=1; i<=n; i++){
for (int j=1; j<n; j+=i) {
//O(1) }}}
```

i=1 — j=1, j=2, j=3, ... — n times

i=2 — j=1, j=3, j=5, j=7, ... Loop ends when j>n

1+3+5+7 > n

k > n/2

~ n terms

i=3 — j=1, j=4, j=7 — 1+4+7 > n

k > n/3

i=4 — k > n/4

...

i=n

So total Complexity

$= O(n^2 + n^2 + n^2 \ldots)$

$= O(n^2)$

Ans 6)

```
for(i=2; i<=n; i=pow(i,k))
{ //O(1) }
```

Complexity Pow (i,k) — $O(\log N) = \log(k)$

i=2
$i=2^k$
$i=2^{k^2}$
...
$i=2^{k^m}$

Loop ends when i>n

$2^{k^m} > n$

$\log(2^{k^m}) > \log n$

$k^m > \log n$

$\log(k^m) > \log(\log n)$

$m \log k > \log(\log n)$

$m > \frac{\log(\log n)}{\log(k)}$

$T(C) = O(\log(\log n))$

حل :

a) $100 < \lg n < \sqrt{n} < n < \lg(\lg(n)) < n\log n < \lg n! < n$

$< n^2 < \lg^{2^n} < 2^n < 3^n < 4^n$

b) $1 < \sqrt{\lg n} < \lg n < 2\lg n < \lg 2N < N < 2N < 4N < \lg($

$< N\lg N < \lg N! < N! < N^2 < 2 \times 2^N$

c) $al < \log_8 N < \log_2 N < n\log_{10} N < n\log_2 N < \lg n! < N! < 5N$

$< 8N^2 < 7N^3 < 8^{2n}$