Tutorial - 4

$T(n) = 3T(n/2) + n^2$ $\quad f(n = n^2)$

$a = 3 \quad b = 2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing $n^{\log_2 3}$ & n^2

$n^{\log_2 3} < n^2$ (case 3)

$\therefore$ according to master theorem

$T(n) = \theta(n^2)$.

$T(n) = 4T(n/2) + n^2$

$a = 4,\ b = 2$

$n^{\log_b 4} = n^{\log_2 4} = n^2 = f(n)$ (case 2)

$\therefore$ according to master theorem $T(n) = \theta(n^2 \log n)$

$T(n) = T(n/2) + 2^n$

$a = 1,\ b = 2 \quad n^{\log_2 1} = n^0 = 1 \quad 1 < 2^n$ (case 3)

$\therefore$ according to master theorem $T(n) = \theta(2^n)$

(4) $T(n) = 2^n T(n/2) + n^n$

= Not applicable as a is a funⁿ of n

(5) $T(n) = 16T(n/4) + n$

$a = 16,\ b = 4$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n > f(n)$ (case 1)

$T(n = \theta(n^2)$

(6) $T(n) = 2T(n/2) + n \log n$

$a = 2,\ b = 2$

$n^{\log_b a} = n^{\log_2 2} = n$

Now $f(n) > n$ $\therefore$ According to master's Th $= T(n) = \theta(n \log n)$

(7) $T(n) = 2T(n/2) + n/\log n$

$a=2,\ b=2 \quad f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

$\therefore$ According to master T/m $T(n) = O(n)$

(8) $T(n) = 2T(n/4) + n^{0.51}$

$a=2,\ b=4 \quad f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

According to master T/m $T(n) = \Theta(n^{0.51})$

(9) $T(n) = 0.5T(n/2) + \frac{1}{n}$

Not applicable as $a < 1$

(10) $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4 \quad , f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

$\therefore$ According master $T(n) = \Theta(n!)$

(11) $T(n) = 4T(n/2) + \log n$

$a = 4,\ b = 2 \qquad f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

$\therefore$ According to master, $T(n) = O(n^2)$

$\Rightarrow = 3T(n/3) + n/2$

$a=3, b=3 \quad f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\Theta(n) = \Theta(n/2)$

According to master thm. $T(n) = \Theta(n \log n)$

$T(n) = 6T(n/3) + n^2 \log n$

$a=6, b=3, f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63} < n^2 \log n$

$\therefore$ According to master thm $T(n) = \Theta(n^2 \log n)$

(19) $T(n) = 4T(n/2) + n/\log n$

$a=4, b=2, f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2 > n/\log n$

$\therefore$ According to master's thm $T(n) = \Theta(n^2)$

(20) $T(n) = 64T(n/8) - n^2 \log n$

Not applicable as $f(n)$ is not increasing func.

(21) $T(n) = 7T(n/3) + n^2$

$a=7, b=3 \quad f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

$\therefore$ According to master, $T(n) = \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

Not applicable since regularity condition is violated in case 3.

(12) $T(n) = \text{sqrt}(n)T(n/2) + \log n$

$\therefore$ Not applicable as a is not constant

(13) $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2 \qquad f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

According to master's T/m, $T(n) = \Theta(n^{\log_2 3})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3,\ b = 3,\ f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore$ According to master's t/m $T(n) = \Theta(n)$

(15) $T(n) = 4T(n/2) + cn$

$a = 4,\ b = 2,\ f(n) = c * n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c * n$

$\therefore$ Accordg to Master Tm, $T(n) = \Theta(n^2)$

(16) $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

$\therefore$ According to master's tm, $T(n) = \Theta(n\log n)$