# Econometrics 2nd Assignment

**Submitted by**
**Fabio Fortis, Nikolas Aschenbrenner, Mridul Jain**

**Berlin 2nd February 2024**

# Table of Contents

# 3 Literature

The book-to-market ratio, the ratio of a firm's book value to its market value, is a significant predictor of stock returns. Empirical evidence indicates that high book-to-market stocks ("value stocks") tend to outperform low book-to-market stocks ("growth stocks") (Fama & French, 1992). This pattern has been observed across various markets and time periods, challenging traditional asset pricing models.

Fama and French (1992) established the book-to-market ratio as a key factor in stock returns, leading to their three-factor model. Their findings were supported by Lakonishok, Shleifer, and Vishny (1994), who argued that the superior performance of value stocks cannot be solely attributed to risk. Similar results have been documented in international markets (Capaul, Rowley, & Sharpe, 1993).

The book-to-market effect contradicts the efficient market hypothesis (EMH), which asserts that asset prices fully reflect all available information (Fama, 1970). The persistence of this return pattern suggests that markets either misprice stocks or fail to incorporate relevant information.

Two primary explanations exist for the anomaly: rational risk-based theories and behavioral biases.

Risk-Based View: Fama and French (1992) argued that value stocks carry higher risk, such as financial distress, justifying their excess returns. However, some researchers question whether risk alone explains the magnitude of the return premium (Lakonishok et al., 1994). Behavioral View: Lakonishok et al. (1994) and De Bondt and Thaler (1985) suggested that investor biases drive mispricing—over-extrapolation and overreaction cause growth stocks to be overvalued and value stocks to be undervalued, leading to their superior subsequent returns.

The book-to-market ratio remains a key factor in asset pricing, yet its persistence challenges market efficiency. While risk-based and behavioral theories both offer explanations, no consensus has been reached. Future research integrating these perspectives may provide a clearer understanding of this anomaly and its implications for financial markets.

# 4 Descriptive statistics

The descriptive statistics table shows that higher book-to-market portfolios (Hi 10) show higher average returns (0.044) and greater volatility (S.D. 1.662) than lower ones (Lo 10: 0.029, S.D. 1.180). Market (MKT) returns are modest (mean 0.031, S.D. 1.079), while size (SMB) and

value (HML) factors show minor average returns but occasional strong outperformance. The factor of the difference portfolio (LMH) has a negative mean (-0.061), indicating overperformance of high book-to-market stocks. Overall, value stocks outperform growth stocks but at the cost of higher return dispersion and risk.

# 5 Analysis of sorted portfolios

## 5.1 CAPM:

$$R_i - R_f = \alpha + \beta(R_m - R_f) + \epsilon \qquad (1)$$

The CAPM results indicate significant differences in performance across book-to-market portfolios. High book-to-market portfolios (value stocks) exhibit higher CAPM alphas than low book-to-market portfolios (growth stocks). The CAPM beta values range between 0.945 and 1.277, suggesting that all portfolios have a similar level of systematic risk compared to the market. However, the $R^2$ values vary from 0.687 for high book-to-market stocks (Hi10) to 0.953 for low book-to-market stocks (Lo10), meaning that CAPM explains a substantial portion of return variation, but not fully.

Notably, the alpha for Hi10 is 1.207, which is highly significant, implying that these stocks generate excess returns beyond what CAPM predicts. In contrast, growth stocks (Lo10) show an alpha of -0.738, suggesting that their excess returns are almost entirely explained by market exposure. This pattern is consistent with the well-documented value premium, where value stocks tend to outperform growth stocks on a risk-adjusted basis.

The CAPM analysis confirms the book-to-market anomaly, as high book-to-market stocks earn significantly higher risk-adjusted returns that CAPM cannot explain. The presence of positive and statistically significant CAPM alphas suggests that market risk alone does not fully determine returns.

## 5.2 3-factormodel:

$$R_i - R_f = \alpha + \beta_m(R_m - R_f) + \beta_s\,\mathrm{SMB} + \beta_h\,\mathrm{HML} + \epsilon \qquad (2)$$

The alphas (unexplained excess returns) now vary between –2.673 and 1.187. Overall, the alphas are now closer to 0, indicating that the Fama&French 3-factor model is able to better explain stock returns and allocate them to the three factors.

The t-statistics for alpha across deciles vary strongly, with some of the alphas not being statistically significant.
The value beta (HML factor) starts negative for Lo 10 (-0.333) and turns positive, increasing to 1.038 for Hi 10. This shift confirms that higher book-to-market portfolios have higher average returns compared to low book-to-market portfolios. This also confirms the anomaly: high book-to-market portfolios (value stocks) generate higher returns, even when controlling market risk, size, and value factors.

The high t-statistics for value beta also validate that the increasing exposure to value stocks is significant.
Market beta remains relatively stable across deciles, ranging from 1.068 (Lo 10) to 1.224 (Hi 10). This suggests that differences in returns are not primarily driven by systematic market risk.
The size beta increases significantly across deciles, from -0.091 (Lo 10) to 0.479 (Hi 10). Higher size beta indicates greater exposure to small-cap stocks in higher book-to-market portfolios. This finding suggests that part of the value premium may be explained by the size effect, as smaller firms are often more volatile and exhibit higher returns.
The increasing value beta is central to the explanation of the anomaly. Higher book-to-market portfolios naturally have higher exposure to the HML factor, which aligns with the essence of the Fama-French model.
The model's explanatory power, as indicated by FF $R^2$, is consistently high for all deciles (above 85%). This suggests that while the Fama-French model explains a large portion of the variation, some unique aspects of the portfolios remain unexplained.

## 5.3 Additional factor(s):

$$R_i - R_f = \alpha + \beta_m(R_m - R_f) + \beta_s \, SMB + \beta_h \, HML + \beta_{mom} \, MOM + \epsilon \qquad (3)$$

The Fama-French 3-Factor Model is expanded with a Momentum factor, analyzing whether past winners (high returns) continue to outperform past losers (low returns). Alpha values vary across deciles, with some (e.g., Dec 2: 1.438, Dec 3: 1.085) showing positive alphas, while others (e.g., Dec 7: -1.571, Dec 8: -0.397) are negative. Overall, alphas remain close to zero, indicating most stock returns are explained by the model. The t-statistics also vary, with some alphas (e.g., Lo 10, Hi 10) being statistically insignificant.

Market Beta remains stable around 1.0, suggesting near-market exposure. Size Beta is mostly negative, implying a preference for large stocks, except in Deciles 8 and 9, where small stocks outperform. Value Beta (HML) increases from Decile 1 to 9, supporting the book-to-market anomaly.

Momentum Beta is mostly around 0, except for Hi 10 (0.852), indicating strong momentum effects. Interestingly, Hi 10 also has a negative Value Beta, deviating from the usual Fama-French pattern. Most Momentum Betas, except Decile 6, are statistically significant.

Overall, the model confirms the book-to-market anomaly, except for Hi 10, where a high Value Beta was expected. R-squared values (0.876 to 0.973) show that the model explains return variation well, outperforming the original Fama-French model and demonstrating the added value of the Momentum factor.

The Momentum factor is added to capture the well-documented tendency of past winners to continue outperforming past losers (Jegadeesh & Titman, 1993), which the original Fama-French model does not explain. Its inclusion improves the model's explanatory power, as shown by higher R-squared values, and accounts for behavioral biases in stock pricing.

# 6 Fama/MacBeth regression

$$\text{First stage: } R_{i,t} - R_{f,t} = \alpha_i + \beta_i F_t + \epsilon_{i,t} \quad (4)$$

$$\text{Second stage: } R_{i,t} = \lambda_{0,t} + \lambda_{1,t}\beta_{i1} + \lambda_{2,t}\beta_{i2} + \cdots + \lambda_{k,t}\beta_{ik} + \epsilon_{i,t} \quad (5)$$

With the Fama-MacBeth regression estimates, we compare the full-sample approach with a rolling window method. The intercept, which represents the average excess return unexplained by the factors, is higher in the full sample (1.267) compared to the rolling window (0.828), with both being statistically significant based on their t-statistics of 4.389 and 4.901, respectively. This result could be explained by the possibility of hindsight bias occurring in the full-sample approach.

The market beta (betasMKT), which measures sensitivity to the market factor, is negative in both cases, with the full sample showing a stronger negative value (-0.551) compared to the rolling window (-0.095). However, its t-statistics suggest that market beta is not statistically significant in the rolling window (-0.428), whereas in the full sample, it is slightly significant (-1.716).

The SMB factor (betasSMB), which captures the size effect, is slightly higher in the rolling window (0.183) than in the full sample (0.142), with the latter having a t-statistic of 1.447,

indicating moderate significance. In contrast, the rolling window's SMB coefficient has a t-statistic of 1.899, suggesting stronger statistical relevance.

The value factor (betasHML), which reflects exposure to high book-to-market stocks, is nearly identical across both methods, with the full sample at 0.362 and the rolling window at 0.364. However, the statistical significance is higher for the full sample (3.377 t-stat) than for the rolling window (3.344 t-stat), though both are clearly significant.

The final factor, betasDIFF, captures the difference portfolio as a factor. The difference portfolio is created by going long in the portfolio with the stocks that have the largest size and highest book-to-market ratio, and by going short in the portfolio with the stocks that have the smallest size and lowest book-to-market ratio. The beta of the difference portfolio is higher in the full sample (0.416) compared to the rolling window (0.311), with the t-statistics indicating that it is strongly significant in both cases, though slightly more so in the full sample (1.570) compared to the rolling window (1.176).

Overall, these results suggest that while the model explains a significant portion of excess returns, the full-sample approach tends to produce stronger coefficients and greater statistical significance compared to the rolling window approach. On the other hand, the unexplained excess return is smaller in the rolling-beta approach, which shows that the rolling-beta approach should be favoured.

We selected this set of test assets to capture key cross-sectional variations in stock returns, focusing on size and value effects, which are central to asset pricing models. These assets allow us to assess how well factors like market beta, SMB, HML, and the difference portfolio explain excess returns. The tested specifications explain returns reasonably well, with statistically significant factors, especially in the full-sample approach. However, some factors, like market beta in the rolling window, show weaker significance, indicating potential instability. The difference portfolio works well as an explanatory factor, showing strong statistical significance in both methods. Its higher coefficient in the full sample suggests a more stable size and value premium over time, reinforcing its relevance in asset pricing models.

# 7 Tables

## 7.1 Descriptive Statistics (Sorted and Difference Portfolios)

Table 1: Descriptive Statistics for Portfolio sorted after Book-to-Market Ratio

| | Min | 10th Quantile | Median | Mean | 90th Quantile | Max | S.D. | No. Observations |
|---|---|---|---|---|---|---|---|---|
| MKT | -17.440 | -1.040 | 0.060 | 0.031 | 1.050 | 15.760 | 1.079 | 25,880 |
| SMB | -11.630 | -0.580 | 0.010 | 0.004 | 0.580 | 8.180 | 0.595 | 25,880 |
| HML | -6.020 | -0.561 | 0.010 | 0.015 | 0.600 | 8.820 | 0.627 | 25,880 |
| LMH | -20.250 | -1.130 | 0 | -0.015 | 1.080 | 10.260 | 1.146 | 25,880 |
| MOM | -10.260 | -1.080 | 0 | 0.015 | 1.130 | 20.250 | 1.146 | 25,880 |
| RF | -0.003 | 0 | 0.010 | 0.012 | 0.028 | 0.061 | 0.012 | 25,880 |
| Lo 10 | -16.737 | -1.196 | 0.059 | 0.029 | 1.185 | 15.938 | 1.180 | 25,880 |
| Dec 2 | -17.947 | -1.090 | 0.059 | 0.033 | 1.117 | 11.976 | 1.104 | 25,880 |
| Dec 3 | -18.397 | -1.045 | 0.049 | 0.032 | 1.087 | 14.718 | 1.075 | 25,880 |
| Dec 4 | -19.267 | -1.053 | 0.058 | 0.029 | 1.060 | 29.268 | 1.140 | 25,880 |
| Dec 5 | -17.687 | -1.040 | 0.056 | 0.033 | 1.073 | 20.708 | 1.116 | 25,880 |
| Dec 6 | -15.747 | -1.030 | 0.059 | 0.035 | 1.059 | 21.888 | 1.162 | 25,880 |
| Dec 7 | -18.357 | -1.106 | 0.050 | 0.031 | 1.120 | 24.318 | 1.230 | 25,880 |
| Dec 8 | -17.917 | -1.140 | 0.059 | 0.041 | 1.174 | 21.658 | 1.300 | 25,880 |
| Dec 9 | -17.537 | -1.260 | 0.060 | 0.046 | 1.309 | 24.308 | 1.410 | 25,880 |
| Hi 10 | -16.586 | -1.481 | 0.058 | 0.044 | 1.539 | 28.468 | 1.662 | 25,880 |

Notes: This table presents the minima, 10th percentiles, means, medians, 90th percentiles, maxima and standard deviations of stock returns of ten sorted portfolios and the difference portfolios. MKT denotes the Market Portfolio, SMB is the Small-Minus-Big factor, HML is the High-Minus-Low factor, LMH is the Low-Minus-High factor, MOM is the Momentum factor and RF denotes the reference portfolio. Lo 10 - Hi 10 describe 10 sorted portfolios which are split up in decentiles regarding the book-to-maket ratio. Observations with the lowest B/M-ratio are in the first decentile (Lo 10) whereas observations with the highest B/M-ratio are in the last decentile (Hi 10).

## 7.2 Descriptive Statistics (25 Sorted Portfolios)

Table 2: Descriptive Statistics for 25 Portfolios sorted after Size and Book-to-Market Ratio

| Size — B/M | Min | 10th Quantile | Median | Mean | 90th Quantile | Max | S.D. | No. Observations |
|---|---|---|---|---|---|---|---|---|
| SMALL LoBM | -49.503 | -10.581 | 0.496 | 0.573 | 10.727 | 147.810 | 11.965 | 1,181 |
| SMALL HiBM | -33.090 | -7.343 | 1.026 | 1.347 | 9.222 | 100.028 | 9.215 | 1,181 |
| ME2 BM1 | -33.230 | -8.109 | 0.902 | 0.644 | 9.063 | 76.456 | 7.964 | 1,181 |
| ME2 BM5 | -37.418 | -7.334 | 1.401 | 1.247 | 8.810 | 89.942 | 8.663 | 1,181 |
| ME3 BM1 | -30.440 | -7.349 | 1.115 | 0.723 | 7.688 | 56.931 | 7.384 | 1,181 |
| ME3 BM5 | -35.852 | -7.103 | 1.133 | 1.109 | 8.613 | 73.775 | 8.367 | 1,181 |
| ME4 BM1 | -29.756 | -6.526 | 1.000 | 0.745 | 7.237 | 35.136 | 6.206 | 1,181 |
| ME4 BM5 | -41.486 | -7.130 | 1.251 | 1.043 | 8.579 | 86.574 | 8.565 | 1,181 |
| BIG LoBM | -29.310 | -5.648 | 0.932 | 0.689 | 6.523 | 32.459 | 5.348 | 1,181 |
| BIG HiBM | -45.586 | -6.964 | 1.116 | 0.956 | 8.038 | 98.009 | 8.491 | 1,181 |
| DIFF | -68.502 | -8.012 | 0.765 | 0.383 | 8.448 | 58.400 | 8.852 | 1,181 |

Notes: This table presents the minima, 10th percentiles, means, medians, 90th percentiles, maxima and standard deviations of stock returns of (theoretically) 25 sorted portfolios and a difference portfolios. For visibility reasons, not all 25 portfolios are included in the table, but only the portfolios split in 5 size groups with the smallest and the largest book-to-market ratio. SMALL LoBM - BIG HiBM describe 25 sorted portfolios which are split up regarding the B/M-ratio and the firm size. Observations with the lowest B/M-ratio and lowest size are in the first portfolio (SMALL LoBM) whereas observations with the highest B/M-ratio are in the last portfolio (BIG HiBM). DIFF denotes the Difference Portfolio which is created by going long in the portfolio with the highest book-to-market ratio and the largest size and by going short in the portfolio with the lowest B/M-ratio and smalles size.

## 7.3 CAPM Results

Table 3: CAPM Results

| | | Lo 10 | Dec 2 | Dec 3 | Dec 4 | Dec 5 | Dec 6 | Dec 7 | Dec 8 | Dec 9 | Hi 10 | LMH |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | CAPM Alpha | -0.738 | 0.68 | 0.682 | -0.398 | 0.802 | 1.139 | -0.247 | 1.932 | 2.796 | 1.207 | -1.922 |
| 2 | t-stat | -1.363 | 1.38 | 1.277 | -0.663 | 1.295 | 1.575 | -0.306 | 2.106 | 2.566 | 0.823 | -1.106 |
| 3 | CAPM Beta | 1.046 | 0.982 | 0.945 | 0.995 | 0.968 | 0.99 | 1.035 | 1.079 | 1.142 | 1.277 | -0.231 |
| 4 | t-stat | 523.461 | 543.695 | 483.444 | 449.22 | 426.744 | 374.198 | 347.841 | 322.808 | 289.037 | 238.208 | -35.751 |
| 5 | CAPM R^2 | 0.914 | 0.92 | 0.9 | 0.886 | 0.876 | 0.844 | 0.824 | 0.801 | 0.763 | 0.687 | 0.047 |

Notes: This table presents the results of a CAPM analysis, where the Market Portfolio is the single regressor. The table includes the CAPM Alpha, which describes excess returns, CAPM Beta and the R-squared of the CAPM regression. Furthermore, this table displays the t-statistics of Alphas and Betas to analyze their statistical significance.

## 7.4 Fama&French 3 Factor Results

Table 4: Fama&Frech 3 Factor Model Results

| | | Lo 10 | Dec 2 | Dec 3 | Dec 4 | Dec 5 | Dec 6 | Dec 7 | Dec 8 | Dec 9 | Hi 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | FF Alpha | 0.412 | 1.187 | 0.9 | -0.766 | 0.187 | 0.155 | -1.723 | -0.363 | -0.106 | -2.673 |
| 2 | t-stat | 0.952 | 2.523 | 1.698 | -1.306 | 0.321 | 0.242 | -2.624 | -0.641 | -0.155 | -2.771 |
| 3 | Market Beta | 1.068 | 0.993 | 0.951 | 0.982 | 0.95 | 0.96 | 0.996 | 1.019 | 1.085 | 1.224 |
| 4 | t-stat | 659.302 | 566.353 | 480.618 | 445.032 | 434.073 | 398.478 | 401.027 | 478.319 | 422.98 | 333.865 |
| 5 | Size Beta | -0.091 | -0.023 | -0.004 | -0.015 | -0.007 | -0.015 | 0.027 | 0.046 | 0.203 | 0.479 |
| 6 | t-stat | -31.222 | -7.261 | -1.099 | -3.776 | -1.768 | -3.522 | 6.048 | 11.994 | 43.96 | 72.61 |
| 7 | Value Beta | -0.333 | -0.154 | -0.07 | 0.131 | 0.207 | 0.333 | 0.48 | 0.732 | 0.841 | 1.038 |
| 8 | t-stat | -120.388 | -51.524 | -20.664 | 34.711 | 55.337 | 80.906 | 113.171 | 200.905 | 191.806 | 165.739 |
| 9 | FF R^2 | 0.946 | 0.927 | 0.902 | 0.892 | 0.889 | 0.876 | 0.882 | 0.922 | 0.904 | 0.859 |

Notes: This table presents the results of a Fama&French 3 Factor analysis, where the Market Portfolio as well as SMB (Small Minus Big) and HML (High minus Low) factors are used as regressors. The table includes the FF Alpha, which describes excess returns, Market Beta, Size Beta, Value Beta and the R-squared of the FF regression. Furthermore, this table displays the t-statistics of Alphas and Betas to analyze their statistical significance.

## 7.5 Fama&French 3 Factor with Momentum Results

Table 5: FamaFrench 3 Factor Model with Additional Factor (Momentum)

| | | Lo 10 | Dec 2 | Dec 3 | Dec 4 | Dec 5 | Dec 6 | Dec 7 | Dec 8 | Dec 9 | Hi 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | FF Alpha | -0.052 | 1.438 | 1.085 | -0.744 | 0.222 | 0.159 | -1.571 | -0.397 | 0.124 | -0.052 |
| 2 | t-stat | -0.13 | 3.102 | 2.059 | -1.269 | 0.38 | 0.247 | -2.398 | -0.701 | 0.182 | -0.13 |
| 3 | Market Beta | 1.091 | 0.981 | 0.942 | 0.981 | 0.948 | 0.96 | 0.988 | 1.021 | 1.074 | 1.091 |
| 4 | t-stat | 700.715 | 551.556 | 465.107 | 431.465 | 420.595 | 386.671 | 387.367 | 465.1 | 408.753 | 700.715 |
| 5 | Size Beta | -0.006 | -0.068 | -0.037 | -0.019 | -0.013 | -0.016 | -0.001 | 0.052 | 0.161 | -0.006 |
| 6 | t-stat | -2.086 | -19.623 | -9.446 | -4.282 | -3.015 | -3.27 | -0.226 | 12.175 | 31.36 | -2.086 |
| 7 | Value Beta | -0.129 | -0.263 | -0.15 | 0.121 | 0.192 | 0.332 | 0.412 | 0.747 | 0.74 | -0.129 |
| 8 | t-stat | -31.002 | -55.193 | -27.679 | 19.867 | 31.696 | 49.793 | 60.268 | 126.751 | 104.981 | -31.002 |
| 9 | Momentum Beta | -0.148 | 0.079 | 0.059 | 0.007 | 0.011 | 0.001 | 0.049 | -0.011 | 0.074 | 0.852 |
| 10 | t-stat | -62.012 | 29.044 | 18.852 | 2.023 | 3.198 | 0.272 | 12.589 | -3.241 | 18.232 | 355.715 |
| 11 | R^2 | 0.953 | 0.929 | 0.903 | 0.892 | 0.889 | 0.876 | 0.883 | 0.922 | 0.905 | 0.976 |

Notes: This table presents the results of a Fama&French 3 Factor analysis with an additional factor, the Momentum factor. The Market Portfolio as well as SMB (Small Minus Big),HML (High minus Low) and MOM (Momentum) factors are used as regressors. The table includes the FF Alpha, which describes excess returns, Market Beta, Size Beta, Value Beta, Momentum Beta and the R-squared of the expanded FF regression. Furthermore, this table displays the t-statistics of Alphas and Betas to analyze their statistical significance.

## 7.6 Fama/MacBeth Results

Table 6: Fama/MacBeth Regression Results

| | | | (Intercept) | betasMKT | betasSMB | betasHML | betasDIFF |
|---|---|---|---|---|---|---|---|
| 1 | Full Sample | Avg. Beta | 1.267 | -0.551 | 0.142 | 0.362 | 0.416 |
| 2 | | t-stat | 4.389 | -1.716 | 1.447 | 3.377 | 1.570 |
| 3 | Rolling Window | Avg. Beta | 0.828 | -0.095 | 0.183 | 0.364 | 0.311 |
| 4 | | t-stat | 4.901 | -0.428 | 1.899 | 3.344 | 1.176 |

Notes: This table presents the results of a Fama/MacBeth Regression. The table displays the averages of the coefficients of the second-stage cross-sectional regression and its t-statistics. The Average Betas in Row 1 are calculated with the use of Full-Sample Betas whereas the Average Betas in Row 3 are calculated with Betas that have been calculated through a 5-year rolling window.

# 8 Sources

**References:**


De Bondt, W. F. M., & Thaler, R. (1985). Does the stock market overreact? *Journal of Finance*, 40(3), 793-805.

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *Journal of Finance*, 25(2), 383-417.

Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. *Journal of Finance*, 47(2), 427-465.

Lakonishok, J., Shleifer, A., & Vishny, R. W. (1994). Contrarian investment, extrapolation, and risk. *Journal of Finance*, 49(5), 1541-1578.

Capaul, C., Rowley, I., & Sharpe, W. F. (1993). International value and growth stock returns. *Financial Analysts Journal*, 49(1), 27-36.


# 9 Packages

1. expss
2. tidyverse
3. xtable
4. readxl
5. stargazer
6. maditr
7. dyplr