Theoretical Foundations of Buffer Stock Saving

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Drawbacks of Numerical Solutions

A Black Box

- Can Construct Solution to Model Without Really Understanding It
- Hard Even To Be Sure Your Numerical Solution Is *Right*
- Little Intuition for How Results Might Change With
  - Calibration
  - Structure
- *Very* Hard To Teach!

I Am A *Big* Fan Of Numerical Methods

- Have Done A Good Deal Of Work With Them Myself
- But As A Result, Have Felt All These Drawbacks Keenly

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Foundations For Microeconomic Household's Problem With

- Uncertain Labor Income
- No Liquidity Constraints
- CRRA Utility
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Key Result

Restrictions On Parameter Values Such That

- Problem Defines A Contraction Mapping
  - $\Rightarrow$ $\exists$ A Unique Consumption Function $\mathrm{c}(m)$
- There Is A 'Target' Ratio Of Assets to Permanent Income
  - Requires A Key 'Impatience' Condition To Hold
  - Good News
    - Condition Is Weaker (Easier To Satisfy) Than Previous Papers Imposed

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Limit as horizon T goes to infinity of

$$\begin{aligned} \mathbf{a}_t &= \mathbf{m}_t - \mathbf{c}_t \\ \mathbf{b}_{t+1} &= \mathbf{a}_t \mathsf{R} \\ \mathbf{p}_{t+1} &= \mathbf{p}_t \underbrace{\Gamma\psi_{t+1}}_{\equiv \Gamma_{t+1}} \\ \mathbf{m}_{t+1} &= \mathbf{b}_{t+1} + \mathbf{p}_{t+1}\xi_{t+1}, \end{aligned} \tag{1}$$

$$\xi_{t+n} = \begin{cases} 0 & \text{with probability } \wp > 0 \\ \theta_{t+n}/\cancel{\wp} & \text{with probability } \cancel{\wp} \end{cases} \tag{2}$$

- $u(\bullet) = \bullet^{1-\rho}/(1-\rho)$; $\mathbb{E}_t[\psi_{t+n}] = \mathbb{E}_t[\xi_{t+n}] = 1 \ \forall \ n > 0$; $\beta < 1, \rho > 1$

Surely This Problem Has Been Solved?

No.

- Can't Use Stokey et. al. theorems because CRRA utility
- Lit thru Matkowski and Nowak (2011) Can't Handle Permanent Shocks
- Must Use Boyd's 'Weighted' Contraction Mapping Theorem
- Surprisingly Subtle

Fortunately, the Conclusions Are Simple!

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Benchmark: Perfect Foresight Model

Definitions:

Absolute Patience Factor	$Þ$	$=$	$(R\beta)^{1/\rho}$
Return Patience Factor	$Þ_{R}$	$=$	$Þ/R$
Perfect Foresight Growth Patience Factor	$Þ_{\Gamma}$	$=$	$Þ/\Gamma$

Name	Condition	Implication
(AIC) Absolute Impatience Condition	$Þ < 1$	$c \downarrow$ over time
(RIC) Return Impatience Condition	$Þ_{R} < 1$	$c/a \downarrow$ over time
(PF-GIC) Growth Impatience Condition	$Þ_{\Gamma} < 1$	$c/\boldsymbol{p} \downarrow$ over time

When Does A Useful Limiting Solution Exist?

Finite Human Wealth (FHWC) condition:

$$\Gamma < \mathsf{R} \tag{3}$$

Return Impatience Condition:

$$\text{Þ}_{\mathsf{R}} < \mathsf{R} \tag{4}$$

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What If There Are Liquidity Constraints?

- FHWC is *not* necessary for solution to exist
- Other Key Condition For Useful Solution is
  'Perfect Foresight Finite Value of Autarky Condition (PF-FVAC)':

$$\beta\Gamma^{1-\rho} < 1 \tag{5}$$

- Without RIC, Constraints Are Irrelevant
  - Because Wealth Always Wants To Rise, So Constraint Never Binds

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Liquidity Constraints and Uncertainty

- Introduce permanent shocks to income
- Finite Value of Autarky Condition Becomes

$$\begin{aligned} \overbrace{\beta \underline{\underline{\Gamma}}^{1-\rho}}^{\equiv \underline{\underline{\beth}}} &< 1 \\ \beta &< \underline{\underline{\Gamma}}^{\rho-1} \end{aligned} \tag{6}$$

Contraction Mapping Requirements

Finite Value of Autarky Condition: Same As In Liq Constr Problem!

$$\begin{aligned} \overbrace{\beta \underline{\underline{\Gamma}}^{1-\rho}}^{\equiv \beth} &< 1 \\ \beta &< \underline{\underline{\Gamma}}^{\rho-1} \end{aligned} \tag{7}$$

'Weak Return Impatience Condition' (WRIC)

$$0 \leq \wp^{1/\rho} \text{Þ}_{\mathsf{R}} < 1 \tag{8}$$

Requirement For Existence Of A Target

Definitions: 'Uncertainty-Adjusted' Growth:

$$\underline{\Gamma} = \Gamma \underline{\psi} \tag{9}$$

Adjusted Growth Patience Factor:

$$\text{Þ}_{\acute{\Gamma}} = \text{Þ}/\underline{\Gamma} \tag{10}$$

Growth Impatience Condition:

$$\text{Þ}_{\acute{\Gamma}} < 1 \tag{11}$$

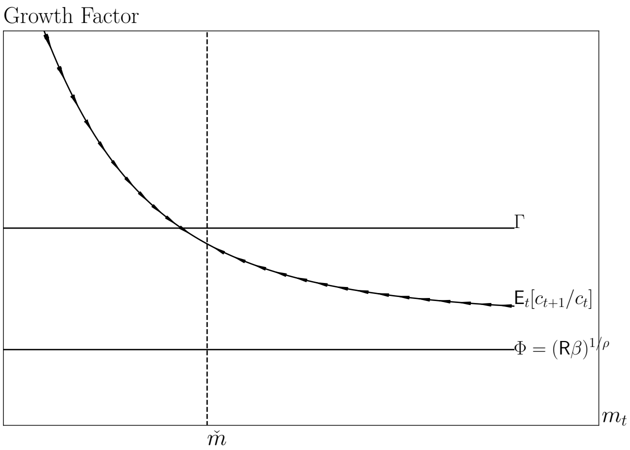
Why? Because it can be shown that

$$\lim_{m_t \to \infty} \mathbb{E}_t \left[\frac{m_{t+1}}{m_t} \right] = \text{Þ}_{\acute{\Gamma}} \tag{12}$$

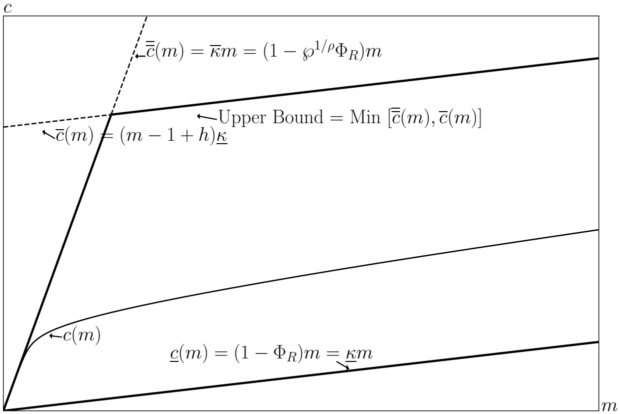
Five Propositions

1. $\lim_{m_t \to \infty} \mathbb{E}_t[c_{t+1}/c_t] = \text{Þ}$
2. $\lim_{m_t \to 0} \mathbb{E}_t[c_{t+1}/c_t] = \infty$
3. $\exists$ a unique target value of m, called $\check{m}$
4. $\mathbb{E}_t[c_{t+1}/c_t | m_t = \check{m}] = \Gamma - \epsilon$
5. $\left(\frac{d\mathbb{E}_t[c_{t+1}/c_t]}{dm_t}\right) < 0$

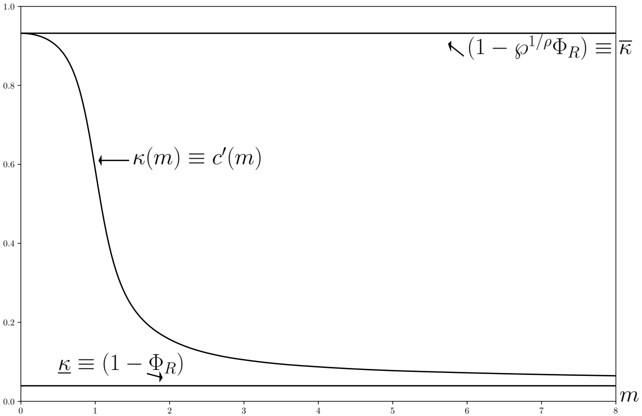
The Target Saving Figure



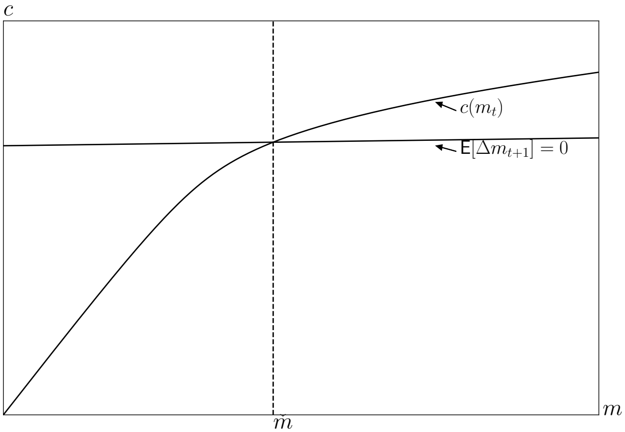
Bounds On the Consumption Function



The Marginal Propensity to Consume

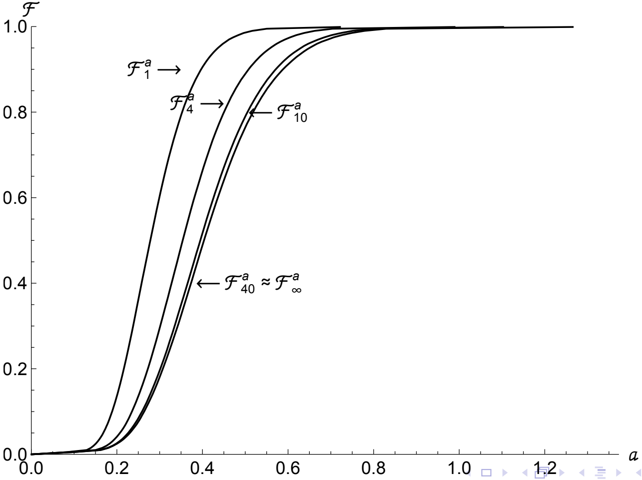


The Consumption Function and Target Wealth



Convergence To The Invariant Distribution

Szeidl (2012) Proves Existence of an Invariant Distribution of m, c, a, etc.



Balanced Growth Equilibrium

Achieved When Cross Section Distribution Reaches Invariance

$$Y_{t+1}/Y_t = C_{t+1}/C_t \quad = \quad \Gamma \tag{13}$$

Fisherian Separation Fails, Even Without Liquidity Constraints!

Insight:

- Precautionary Saving $\approx$ Liquidity Constraints
- If $\grave{\mathrm{c}}(m)$ is solution for constrained consumer,

$$\lim_{\wp \downarrow 0} \mathrm{c}(m;\wp) = \grave{\mathrm{c}}(m) \tag{14}$$

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The MPC Out Of Permanent Shocks

`http://econ.jhu.edu/people/ccarroll/papers/MPCPerm.pdf`

Lots of Recent Papers Trying to Measure the MPCP

Paper Proves:

- MPCP < 1
- But not a lot less:
  - 0.75 to 0.95 (annual rate) for wide range of parameter values

- Defined Conditions Under Which Widely Used Problem Has Solution
    - Finite Value of Autarky Condition Guarantees Contraction (with WRIC)
    - Growth Impatience Condition Prevents $m \to \infty$
- Economy Of Buffer Stock Consumers Exhibits Balanced Growth
    - Even In Absence of General Equilibrium Adj of Interest Rate

Matkowski, Janusz, and Andrzej S. Nowak (2011): "On Discounted Dynamic Programming With Unbounded Returns," *Economic Theory*, 46, 455–474.

Szeidl, Adam (2012): "Stable Invariant Distribution in Buffer-Stock Saving and Stochastic Growth Models," *Manuscript, Central European University*.