# 1 When Is Consumption Growth Declining in $m$?

Henceforth indicating appropriate arguments by the corresponding subscript (e.g. $\mathrm{c}'_{t+1} \equiv \mathrm{c}'(m_{t+1})$), since $\Gamma_{t+1}\mathcal{R}_{t+1} = \mathsf{R}$, the portion of the LHS of equation (58) in brackets can be manipulated to yield

$$\begin{aligned}
\mathrm{c}_t \boldsymbol{\Upsilon}'_{t+1} &= \mathrm{c}'_{t+1}\mathrm{a}'_t\mathsf{R} - \mathrm{c}'_t\Gamma_{t+1}\mathrm{c}_{t+1}/\mathrm{c}_t \\
&= \mathrm{c}'_{t+1}\mathrm{a}'_t\mathsf{R} - \mathrm{c}'_t\boldsymbol{\Upsilon}_{t+1}.
\end{aligned}$$

Now differentiate the Euler equation with respect to $m_t$:

$$\begin{aligned}
1 &= \mathsf{R}\beta\,\mathbb{E}_t[\boldsymbol{\Upsilon}^{-\rho}_{t+1}] \\
0 &= \mathbb{E}_t[\boldsymbol{\Upsilon}^{-\rho-1}_{t+1}\boldsymbol{\Upsilon}'_{t+1}] \\
&= \mathbb{E}_t[\boldsymbol{\Upsilon}^{-\rho-1}_{t+1}]\,\mathbb{E}_t[\boldsymbol{\Upsilon}'_{t+1}] + \mathrm{cov}_t(\boldsymbol{\Upsilon}^{-\rho-1}_{t+1}, \boldsymbol{\Upsilon}'_{t+1}) \\
\mathbb{E}_t[\boldsymbol{\Upsilon}'_{t+1}] &= -\mathrm{cov}_t(\boldsymbol{\Upsilon}^{-\rho-1}_{t+1}, \boldsymbol{\Upsilon}'_{t+1})/\,\mathbb{E}_t[\boldsymbol{\Upsilon}^{-\rho-1}_{t+1}]
\end{aligned}$$

but since $\boldsymbol{\Upsilon}_{t+1} > 0$ we can see from (1) that (58) is equivalent to

$$\mathrm{cov}_t(\boldsymbol{\Upsilon}^{-\rho-1}_{t+1}, \boldsymbol{\Upsilon}'_{t+1}) > 0$$

which, using (1), will be true if

$$\mathrm{cov}_t(\boldsymbol{\Upsilon}^{-\rho-1}_{t+1}, \mathrm{c}'_{t+1}\mathrm{a}'_t\mathsf{R} - \mathrm{c}'_t\boldsymbol{\Upsilon}_{t+1}) > 0$$

which in turn will be true if both

$$\mathrm{cov}_t(\boldsymbol{\Upsilon}^{-\rho-1}_{t+1}, \mathrm{c}'_{t+1}) > 0$$

and

$$\mathrm{cov}_t(\boldsymbol{\Upsilon}^{-\rho-1}_{t+1}, \boldsymbol{\Upsilon}_{t+1}) < 0.$$

The latter proposition is obviously true under our assumption $\rho > 1$. The former will be true if

$$\mathrm{cov}_t\left((\Gamma\psi_{t+1}\mathrm{c}(m_{t+1}))^{-\rho-1}, \mathrm{c}'(m_{t+1})\right) > 0.$$

The two shocks cause two kinds of variation in $m_{t+1}$. Variations due to $\xi_{t+1}$ satisfy the proposition, since a higher draw of $\xi$ both reduces $c^{-\rho-1}_{t+1}$ and reduces the marginal propensity to consume. However, permanent shocks have conflicting effects. On the one hand, a higher draw of $\psi_{t+1}$ will reduce $m_{t+1}$, thus increasing both $c^{-\rho-1}_{t+1}$ and $c'_{t+1}$. On the other hand, the $c^{-\rho-1}_{t+1}$ term is multiplied by $\Gamma\psi_{t+1}$, so the effect of a higher $\psi_{t+1}$ could be to decrease the first term in the covariance, leading to a negative covariance with the second term. (Analogously, a lower permanent shock $\psi_{t+1}$ can also lead a negative correlation.)