

# 1 Unique And Stable Target and Steady State Points

The two theorems and lemma to be proven in this appendix are:

**Theorem 2.** *For the nondegenerate solution to the problem defined in section 1.1 when FVAC, WRIC, and GIC-Nrm all hold, there exists a unique cash-on-hand-to-permanent-income ratio  $\tilde{m} > 0$  such that*

$$\mathbb{E}_t[m_{t+1}/m_t] = 1 \text{ if } m_t = \tilde{m}. \quad (1)$$

Moreover,  $\tilde{m}$  is a point of ‘wealth stability’ in the sense that

$$\begin{aligned} \forall m_t \in (0, \tilde{m}), \quad \mathbb{E}_t[m_{t+1}] &> m_t \\ \forall m_t \in (\tilde{m}, \infty), \quad \mathbb{E}_t[m_{t+1}] &< m_t. \end{aligned} \quad (2)$$

**Theorem 3.** *For the nondegenerate solution to the problem defined in section 1.1 when FVAC, WRIC, and GIC all hold, there exists a unique pseudo-steady-state cash-on-hand-to-income ratio  $\hat{m} > 0$  such that*

$$\mathbb{E}_t[\psi_{t+1}m_{t+1}/m_t] = 1 \text{ if } m_t = \hat{m}. \quad (3)$$

Moreover,  $\hat{m}$  is a point of stability in the sense that

$$\begin{aligned} \forall m_t \in (0, \hat{m}), \quad \mathbb{E}_t[\mathbf{m}_{t+1}]/\mathbf{m}_t &> \Gamma \\ \forall m_t \in (\hat{m}, \infty), \quad \mathbb{E}_t[\mathbf{m}_{t+1}]/\mathbf{m}_t &< \Gamma. \end{aligned} \quad (4)$$

**Lemma 1.** *If both  $\tilde{m}$  and  $\hat{m}$  exist, then  $\hat{m} < \tilde{m}$ .*

## 1.1 Proof of Theorem 2

The elements of the proof of theorem 2 are:

- Existence and continuity of  $\mathbb{E}_t[m_{t+1}/m_t]$
- Existence of a point where  $\mathbb{E}_t[m_{t+1}/m_t] = 1$
- $\mathbb{E}_t[m_{t+1}] - m_t$  is monotonically decreasing

### 1.1.1 Existence and Continuity of $\mathbb{E}_t[m_{t+1}/m_t]$ .

The consumption function exists because we have imposed the sufficient conditions (the WRIC and FVAC; theorem 1). (Indeed, Appendix C shows that  $c(m)$  is not just continuous, but twice continuously differentiable.)

Section 1.7 shows that for all  $t$ ,  $a_{t-1} = m_{t-1} - c_{t-1} > 0$ . Since  $m_t = a_{t-1}\mathcal{R}_t + \xi_t$ , even if  $\xi_t$  takes on its minimum value of 0,  $a_{t-1}\mathcal{R}_t > 0$ , since both  $a_{t-1}$  and  $\mathcal{R}_t$  are strictly positive. With  $m_t$  and  $m_{t+1}$  both strictly positive, the ratio  $\mathbb{E}_t[m_{t+1}/m_t]$  inherits continuity (and, for that matter, continuous differentiability) from the consumption function.

### 1.1.2 Existence of a point where $\mathbb{E}_t[m_{t+1}/m_t] = 1$ .

Existence of a point where  $\mathbb{E}_t[m_{t+1}/m_t] = 1$  follows from:

1. Existence and continuity of  $\mathbb{E}_t[m_{t+1}/m_t]$  (just proven)
2. Existence a point where  $\mathbb{E}_t[m_{t+1}/m_t] < 1$
3. Existence a point where  $\mathbb{E}_t[m_{t+1}/m_t] > 1$
4. The Intermediate Value Theorem

*Existence of a point where  $\mathbb{E}_t[m_{t+1}/m_t] < 1$ .*

**If RIC holds.** Logic exactly parallel to that of section 2.1 leading to equation (49), but dropping the  $\Gamma_{t+1}$  from the RHS, establishes that

$$\begin{aligned} \lim_{m_t \uparrow \infty} \mathbb{E}_t[m_{t+1}/m_t] &= \lim_{m_t \uparrow \infty} \mathbb{E}_t \left[ \frac{\mathcal{R}_{t+1}(m_t - c(m_t)) + \xi_{t+1}}{m_t} \right] \\ &= \mathbb{E}_t[(\mathcal{R}/\Gamma_{t+1})\mathbf{P}_R] \\ &= \mathbb{E}_t[\mathbf{P}/\Gamma_{t+1}] \\ &< 1 \end{aligned} \tag{5}$$

where the inequality reflects imposition of the GIC-Nrm (36).

**If RIC fails.** When the RIC fails, the fact that  $\lim_{m \uparrow \infty} c'(m) = 0$  (see equation (40)) means that the limit of the RHS of (5) as  $m \uparrow \infty$  is  $\bar{\mathcal{R}} = \mathbb{E}_t[\mathcal{R}_{t+1}]$ . In the next step of this proof, we will prove that the combination GIC-Nrm and ~~RIC~~ implies  $\bar{\mathcal{R}} < 1$ .

So we have  $\lim_{m_t \uparrow \infty} \mathbb{E}_t[m_{t+1}/m_t] < 1$  whether the RIC holds or fails.

*Existence of a point where  $\mathbb{E}_t[m_{t+1}/m_t] > 1$ .*

Paralleling the logic for  $c$  in section 2.2: the ratio of  $\mathbb{E}_t[m_{t+1}]$  to  $m_t$  is unbounded above as  $m_t \downarrow 0$  because  $\lim_{m_t \downarrow 0} \mathbb{E}_t[m_{t+1}] > 0$ .

*Intermediate Value Theorem.* If  $\mathbb{E}_t[m_{t+1}/m_t]$  is continuous, and takes on values above and below 1, there must be at least one point at which it is equal to one.

### 1.1.3 $\mathbb{E}_t[m_{t+1}] - m_t$ is monotonically decreasing.

Now define  $\zeta(m_t) \equiv \mathbb{E}_t[m_{t+1}] - m_t$  and note that

$$\begin{aligned} \zeta(m_t) < 0 &\leftrightarrow \mathbb{E}_t[m_{t+1}/m_t] < 1 \\ \zeta(m_t) = 0 &\leftrightarrow \mathbb{E}_t[m_{t+1}/m_t] = 1 \\ \zeta(m_t) > 0 &\leftrightarrow \mathbb{E}_t[m_{t+1}/m_t] > 1, \end{aligned} \tag{6}$$

so that  $\zeta(\check{m}) = 0$ . Our goal is to prove that  $\zeta(\bullet)$  is strictly decreasing on  $(0, \infty)$  using the fact that

$$\begin{aligned} \zeta'(m_t) &\equiv \left( \frac{d}{dm_t} \right) \zeta(m_t) = \mathbb{E}_t \left[ \left( \frac{d}{dm_t} \right) (\mathcal{R}_{t+1}(m_t - c(m_t)) + \xi_{t+1} - m_t) \right] \\ &= \bar{\mathcal{R}} (1 - c'(m_t)) - 1. \end{aligned} \tag{7}$$

Now, we show that (given our other assumptions)  $\zeta'(m)$  is decreasing (but for different reasons) whether the RIC holds or fails.

**If RIC holds.** Equation (22) indicates that if the RIC holds, then  $\underline{\kappa} > 0$ . We show at the bottom of Section 1.8.1 that if the RIC holds then  $0 < \underline{\kappa} < c'(m_t) < 1$  so that

$$\begin{aligned} \bar{\mathcal{R}}(1 - c'(m_t)) - 1 &< \bar{\mathcal{R}}(1 - \underbrace{(1 - \mathbf{P}_R)}_{\underline{\kappa}}) - 1 \\ &= \bar{\mathcal{R}}\mathbf{P}_R - 1 \\ &= \mathbb{E}_t \left[ \frac{\mathbf{R}}{\Gamma\psi} \frac{\mathbf{P}}{\mathbf{R}} \right] - 1 \\ &= \underbrace{\mathbb{E}_t \left[ \frac{\mathbf{P}}{\Gamma\psi} \right]}_{=\mathbf{P}_\Gamma} - 1 \end{aligned}$$

which is negative because the GIC-Nrm says  $\mathbf{P}_\Gamma < 1$ .

**If RIC fails.** Under RIC, recall that  $\lim_{m \uparrow \infty} c'(m) = 0$ . Concavity of the consumption function means that  $c'$  is a decreasing function, so everywhere

$$\bar{\mathcal{R}}(1 - c'(m_t)) < \bar{\mathcal{R}}$$

which means that  $\zeta'(m_t)$  from (11) is guaranteed to be negative if

$$\bar{\mathcal{R}} \equiv \mathbb{E}_t \left[ \frac{\mathbf{R}}{\Gamma\psi} \right] < 1. \quad (8)$$

But the combination of the GIC-Nrm holding and the RIC failing can be written:

$$\underbrace{\mathbb{E}_t \left[ \frac{\mathbf{P}}{\Gamma\psi} \right]}_{\mathbf{P}_\Gamma} < 1 < \underbrace{\frac{\mathbf{P}_R}{\mathbf{R}}}_{\mathbf{P}_\Gamma},$$

and multiplying all three elements by  $\mathbf{R}/\mathbf{P}$  gives

$$\mathbb{E}_t \left[ \frac{\mathbf{R}}{\Gamma\psi} \right] < \mathbf{R}/\mathbf{P} < 1$$

which satisfies our requirement in (8).

## 1.2 Proof of Theorem 3

The elements of the proof are:

- Existence and continuity of  $\mathbb{E}_t[\psi_{t+1}m_{t+1}/m_t]$
- Existence of a point where  $\mathbb{E}_t[\psi_{t+1}m_{t+1}/m_t] = 1$
- $\mathbb{E}_t[\psi_{t+1}m_{t+1} - m_t]$  is monotonically decreasing

### 1.2.1 Existence and Continuity of The Ratio

Since by assumption  $0 < \underline{\psi} \leq \psi_{t+1} \leq \bar{\psi} < \infty$ , our proof in 1.1.1 that demonstrated existence and continuity of  $\mathbb{E}_t[\bar{m}_{t+1}/m_t]$  implies existence and continuity of  $\mathbb{E}_t[\psi_{t+1}m_{t+1}/m_t]$ .

### 1.2.2 Existence of a stable point

Since by assumption  $0 < \underline{\psi} \leq \psi_{t+1} \leq \bar{\psi} < \infty$ , our proof in subsection 1.1.1 that the ratio of  $\mathbb{E}_t[m_{t+1}]$  to  $m_t$  is unbounded as  $m_t \downarrow 0$  implies that the ratio  $\mathbb{E}_t[\psi_{t+1}m_{t+1}]$  to  $m_t$  is unbounded as  $m_t \downarrow 0$ .

The limit of the expected ratio as  $m_t$  goes to infinity is most easily calculated by modifying the steps for the prior theorem explicitly:

$$\begin{aligned}
\lim_{m_t \uparrow \infty} \mathbb{E}_t[\psi_{t+1}m_{t+1}/m_t] &= \lim_{m_t \uparrow \infty} \mathbb{E}_t \left[ \frac{\Gamma_{t+1} ((R/\Gamma_{t+1})a(m_t) + \xi_{t+1}) / \Gamma}{m_t} \right] \\
&= \lim_{m_t \uparrow \infty} \mathbb{E}_t \left[ \frac{(R/\Gamma)a(m_t) + \psi_{t+1}\xi_{t+1}}{m_t} \right] \\
&= \lim_{m_t \uparrow \infty} \left[ \frac{(R/\Gamma)a(m_t) + 1}{m_t} \right] \\
&= (R/\Gamma)\mathbf{P}_R \\
&= \mathbf{P}_\Gamma \\
&< 1
\end{aligned} \tag{9}$$

where the last two lines are merely a restatement of the GIC (30).

The Intermediate Value Theorem says that if  $\mathbb{E}_t[\psi_{t+1}m_{t+1}/m_t]$  is continuous, and takes on values above and below 1, there must be at least one point at which it is equal to one.

### 1.2.3 $\mathbb{E}_t[\psi_{t+1}m_{t+1}] - m_t$ is monotonically decreasing.

Define  $\zeta(m_t) \equiv \mathbb{E}_t[\psi_{t+1}m_{t+1}] - m_t$  and note that

$$\begin{aligned}
\zeta(m_t) < 0 &\leftrightarrow \mathbb{E}_t[\psi_{t+1}m_{t+1}/m_t] < 1 \\
\zeta(m_t) = 0 &\leftrightarrow \mathbb{E}_t[\psi_{t+1}m_{t+1}/m_t] = 1 \\
\zeta(m_t) > 0 &\leftrightarrow \mathbb{E}_t[\psi_{t+1}m_{t+1}/m_t] > 1,
\end{aligned} \tag{10}$$

so that  $\zeta(\check{m}) = 0$ . Our goal is to prove that  $\zeta(\bullet)$  is strictly decreasing on  $(0, \infty)$  using the fact that

$$\begin{aligned}
\zeta'(m_t) &\equiv \left( \frac{d}{dm_t} \right) \zeta(m_t) = \mathbb{E}_t \left[ \left( \frac{d}{dm_t} \right) (\mathcal{R}(m_t - c(m_t)) + \psi_{t+1}\xi_{t+1} - m_t) \right] \\
&= (R/\Gamma) (1 - c'(m_t)) - 1.
\end{aligned} \tag{11}$$

Now, we show that (given our other assumptions)  $\zeta'(m)$  is decreasing (but for different reasons) whether the RIC holds or fails ( $\mathbf{RIC}$ ).

**If RIC holds.** Equation (22) indicates that if the RIC holds, then  $\underline{\kappa} > 0$ . We show at the bottom of Section 1.8.1 that if the RIC holds then  $0 < \underline{\kappa} < c'(m_t) < 1$  so that

$$\begin{aligned} \mathcal{R}(1 - c'(m_t)) - 1 &< \mathcal{R}(1 - \underbrace{(1 - \mathbf{P}_R)}_{\underline{\kappa}}) - 1 \\ &= (\mathbf{R}/\Gamma)\mathbf{P}_R - 1 \end{aligned}$$

which is negative because the GIC says  $\mathbf{P}_R < 1$ .

**If RIC fails.** Under ~~RIC~~, recall that  $\lim_{m \uparrow \infty} c'(m) = 0$ . Concavity of the consumption function means that  $c'$  is a decreasing function, so everywhere

$$\mathcal{R}(1 - c'(m_t)) < \mathcal{R}$$

which means that  $\zeta'(m_t)$  from (11) is guaranteed to be negative if

$$\mathcal{R} \equiv (\mathbf{R}/\Gamma) < 1. \tag{12}$$

But we showed in section 1.5 that the only circumstances under which the problem has a nondegenerate solution while the RIC fails were ones where the FHC also fails (that is, (12) holds).

### 1.3 Proof of Lemma

## 2 The ‘Individual Steady-State’

$$\begin{aligned}
\overbrace{\mathbb{E}_t[\mathbf{m}_{t+1}]}^{\equiv \nu} / \mathbf{m}_t &= \mathbb{E}_t[m_{t+1} \Gamma \psi_{t+1} \mathbf{p}_t] / (m_t \mathbf{p}_t) \\
&= \Gamma \mathbb{E}_t[m_{t+1} \psi_{t+1}] / m_t \\
&= \Gamma \mathbb{E}_t[(m_t - c(m_t))(R/\Gamma) + \xi_{t+1}] \psi_{t+1} / m_t \\
&= \Gamma \mathbb{E}_t[(m_t - c(m_t))(R/\psi_{t+1} \Gamma) + \xi_{t+1}] \psi_{t+1} / m_t \\
&= \Gamma \mathbb{E}_t[(m_t - c(m_t))(R/\Gamma) + \psi_{t+1} \xi_{t+1}] / m_t \\
&= ((\tilde{m} - c(\tilde{m}))R + \Gamma) / \tilde{m} \\
&= ((\tilde{m} - c(\tilde{m}))R + \Gamma) / \tilde{m} \\
&= ((\tilde{m} - c(\tilde{m}))R + \Gamma) / \tilde{m}
\end{aligned}$$

Turn off shocks:

$$\begin{aligned}
\mathbf{c}_{t+1} / \mathbf{c}_t &= c_{t+1} \Gamma \mathbf{p}_t / (c_t \mathbf{p}_t) \\
&= \Gamma c(m_{t+1}) / c_t \\
((\tilde{m} - c(\tilde{m}))R + \Gamma) / \tilde{m} &= \Gamma c((\tilde{m} - c(\tilde{m}))(R/\Gamma) + 1) / c(\tilde{m}) \\
((\tilde{m} - c(\tilde{m}))(R/\Gamma) + 1) / \tilde{m} &= c((\tilde{m} - c(\tilde{m}))(R/\Gamma) + 1) / c(\tilde{m})
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}_t[\mathbf{c}_{t+1}] / \mathbf{c}_t &= \mathbb{E}_t[c_{t+1} \Gamma \psi_{t+1} \mathbf{p}_t] / (c_t \mathbf{p}_t) \\
&= \Gamma \mathbb{E}_t[c(m_{t+1}) \psi_{t+1}] / c_t \\
&= \Gamma \mathbb{E}_t[\psi_{t+1} c((m_t - c_t)(R/\psi_{t+1} \Gamma) + \xi_{t+1})] / c_t \\
&\approx \Gamma \mathbb{E}_t[(c(\tilde{m}) + (m_{t+1} - \tilde{m})c' + (1/2)(m_{t+1} - \tilde{m})^2 c'') \psi_{t+1}] / c_t \\
&= (\Gamma c(\tilde{m}) + \mathbb{E}_t[(\Gamma m_{t+1} \psi_{t+1} - \tilde{m} \psi_{t+1})c' + (1/2)(m_{t+1} - \tilde{m})^2 c'']) \psi_{t+1} / c_t \\
&= (\Gamma c(\tilde{m}) + \tilde{m}(\nu - 1)c' + (1/2)\Gamma \mathbb{E}_t[(m_{t+1}^2 - 2m_{t+1}\tilde{m} + \tilde{m}^2)c'']) \psi_{t+1} / c_t \\
&= (\Gamma c(\tilde{m}) + \tilde{m}(\nu - 1)c' + (1/2)\Gamma \mathbb{E}_t[(m_{t+1}^2 - 2m_{t+1}\tilde{m} + \tilde{m}^2)c'']) \psi_{t+1} / c_t
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}_t[\mathbf{a}_{t+1}] / \mathbf{a}_t &= \mathbb{E}_t[a_{t+1} \Gamma \psi_{t+1} \mathbf{p}_t] / (a_t \mathbf{p}_t) \\
&= \Gamma \mathbb{E}_t[a(m_{t+1}) \psi_{t+1}] / a_t \\
&= \Gamma \mathbb{E}_t[\psi_{t+1} a((m_t - c_t)(R/\psi_{t+1} \Gamma) + \xi_{t+1})] / a_t \\
&\approx \Gamma \mathbb{E}_t[(a(\tilde{m}) + (m_{t+1} - \tilde{m})a' + (1/2)(m_{t+1} - \tilde{m})^2 a'') \psi_{t+1}] / a_t \\
&= (\Gamma a(\tilde{m}) + \mathbb{E}_t[(\Gamma m_{t+1} \psi_{t+1} - \tilde{m} \psi_{t+1})a' + (1/2)(m_{t+1} - \tilde{m})^2 a'']) \psi_{t+1} / a_t \\
&= (\Gamma a(\tilde{m}) + \tilde{m}(\nu - 1)a' + (1/2)\Gamma \mathbb{E}_t[(m_{t+1}^2 - 2m_{t+1}\tilde{m} + \tilde{m}^2)a'']) \psi_{t+1} / a_t \\
&= (\Gamma a(\tilde{m}) + \tilde{m}(\nu - 1)a' + (1/2)\Gamma \mathbb{E}_t[(m_{t+1}^2 - 2m_{t+1}\tilde{m} + \tilde{m}^2)a'']) \psi_{t+1} / a_t
\end{aligned}$$

$$\begin{aligned}
\overbrace{\mathbb{E}_t[\mathbf{m}_{t+1}]}^{\equiv \nu} / \mathbf{m}_t &= \mathbb{E}_t[\mathbf{c}_{t+1}] / \mathbf{c}_t \\
\mathbb{E}_t[m_{t+1} \Gamma \psi_{t+1} \mathbf{p}_t] / (m_t \mathbf{p}_t) &= \mathbb{E}_t[c_{t+1} \Gamma \psi_{t+1} \mathbf{p}_t] / (c_t \mathbf{p}_t) \\
\mathbb{E}_t[m_{t+1} \psi_{t+1}] / m_t &= \mathbb{E}_t[c(m_{t+1}) \psi_{t+1}] / c_t \\
\mathbb{E}_t[(m_t - c(m_t))(R/\Gamma) + \xi_{t+1} \psi_{t+1}] / m_t &= \mathbb{E}_t[c((m_t - c_t)(R/\psi_{t+1} \Gamma) + \xi_{t+1}) \psi_{t+1}] / c_t \\
\mathbb{E}_t[(m_t - c(m_t))(R/\psi_{t+1} \Gamma) + \xi_{t+1} \psi_{t+1}] / m_t &\approx \mathbb{E}_t[(c(\tilde{m}) + (m_{t+1} - \tilde{m})c' + (1/2)(m_{t+1} - \tilde{m})^2 c'') \psi_{t+1}] / c_t \\
\mathbb{E}_t[(m_t - c(m_t))(R/\Gamma) + \psi_{t+1} \xi_{t+1}] / m_t &= (c(\tilde{m}) + (m_{t+1} - \tilde{m})c' + (1/2)(m_{t+1} - \tilde{m})^2 c'') \psi_{t+1} / c_t \\
((\tilde{m} - c(\tilde{m}))(R/\Gamma) + 1) / \tilde{m} &= \mathbb{E}_t[(c(\tilde{m}) + (m_{t+1} - \tilde{m})c' + (1/2)(m_{t+1} - \tilde{m})^2 c'') \psi_{t+1}] / c_t \\
((\tilde{m} - c(\tilde{m}))(R/\Gamma) + 1) / \tilde{m} &= (c(\tilde{m}) + (\nu/\Gamma - \tilde{m})c' + (1/2)(m_{t+1}^2 - 2(\nu/\Gamma)\tilde{m}c' + \tilde{m}^2 c'')) / c_t
\end{aligned}$$

Define  $\mu_t = m_t / \tilde{m}$ , and

$$\chi(\mu) = c(m\mu) \quad (13)$$

$$\chi(\mu) \approx \chi(1) + (\mu - 1)\chi'(1) + (1/2)(\mu - 1)^2 \chi''(1) \quad (14)$$

$$\begin{aligned}
((\tilde{m} - c(\tilde{m}))(R/\Gamma) + 1) / \tilde{m} &\approx (\chi(1) + \mathbb{E}_t[(\mu_{t+1} - 1)\chi'(1) + (1/2)(\mu_{t+1} - 1)^2 \chi''(1)\psi_{t+1}]) / c_t \\
&= (\chi(1) + (\nu/\Gamma - 1)\chi'(1) + \mathbb{E}_t[(1/2)(\mu_{t+1}^2 - 2\mu_{t+1} + 1)\chi''(1)\psi_{t+1}]) / c_t \\
&= (\chi(1) + (\nu/\Gamma - 1)\chi'(1) + (1/2)(\mu_{t+1}^2 - 2\nu/\Gamma + 1)\chi''(1)\psi_{t+1}) / c_t
\end{aligned}$$

Define  $\mu_t = m_t / \tilde{m}$ , and

$$\chi(\mu) = c(m\mu) \quad (15)$$

$$\chi(\mu) \approx \chi(1) + (\mu - 1)\chi'(1) + (1/2)(\mu - 1)^2 \chi''(1) \quad (16)$$

If some value  $m_t = \hat{m}$  exists for which equation (53) holds, for reasons articulated below we will call that point the ‘individual pseudo-steady-state’ (alternatively the ‘**m**-balanced-growth’ point):

$$\begin{aligned}
\mathbb{E}_t \left[ (\hat{m} - \mathring{c}(\hat{m})) \overbrace{\hat{R}/\Gamma}^{\mathcal{R}} + \psi_{t+1} \xi_{t+1} \right] &= \hat{m} \\
(\hat{m} - \mathring{c}(\hat{m}))\mathcal{R} + 1 &= \hat{m}.
\end{aligned} \quad (17)$$

$$\mathbb{E}_t[\mathbf{m}_{t+1}] = \mathbf{m}_t \quad (18)$$

$$\mathbb{E}_t[m_{t+1} \mathbf{p}_{t+1}] = m_t \mathbf{p}_t \quad (19)$$

$$\mathbb{E}_t[m_{t+1} \mathbf{p}_t \psi_{t+1}] = m_t \mathbf{p}_t \quad (20)$$

$$\mathbb{E}_t[m_{t+1} \psi_{t+1}] = m_t \quad (21)$$

$$\mathbb{E}_t[(m_t - c_t)(R/\Gamma) + \psi_{t+1} \xi_{t+1}] = m_t \quad (22)$$

$$(m_t - c(m_t))\mathcal{R} + 1 = m_t \quad (23)$$

$$(\tilde{m} - c(\tilde{m}))\mathcal{R} + 1 = \tilde{m} \quad (24)$$

$$c(\tilde{m})\mathcal{R} = \left( \frac{1 - \tilde{m}(1 - \mathcal{R})}{1} \right) \quad (25)$$

$$c(\tilde{m}) = \left( \frac{1 + (\mathcal{R} - 1)\tilde{m}}{\mathcal{R}} \right) \quad (26)$$

If some value  $m_t = \hat{m}$  exists for which equation (53) holds, for reasons articulated below we will call that point the ‘individual pseudo-steady-state’ (alternatively the ‘**m**-balanced-growth’ point):

$$\begin{aligned} \mathbb{E}_t \left[ (\hat{m} - \mathring{c}(\hat{m})) \overbrace{\mathbf{R}/\Gamma}^{\mathcal{R}} + \psi_{t+1} \xi_{t+1} \right] &= \hat{m} \\ (\hat{m} - \mathring{c}(\hat{m}))\mathcal{R} + 1 &= \hat{m}. \end{aligned} \quad (27)$$