

Table 1 Microeconomic Model Calibration

Calibrated Parameters			
Description	Parameter	Value	Source
Permanent Income Growth Factor	Γ	1.03	PSID: Carroll (1992)
Interest Factor	R	1.04	Conventional
Time Preference Factor	β	0.96	Conventional
Coefficient of Relative Risk Aversion	ρ	2	Conventional
Probability of Zero Income	\wp	0.005	PSID: Carroll (1992)
Std Dev of Log Permanent Shock	σ_ψ	0.1	PSID: Carroll (1992)
Std Dev of Log Transitory Shock	σ_θ	0.1	PSID: Carroll (1992)

Table 2 Model Characteristics Calculated from Parameters

Description	Symbol and Formula		Approximate Calculated Value
Finite Human Wealth Factor	\mathcal{R}^{-1}	$\equiv \Gamma/R$	0.990
PF Finite Value of Autarky Factor	\beth	$\equiv \beta\Gamma^{1-\rho}$	0.932
Growth Compensated Permanent Shock	$\underline{\psi}$	$\equiv (\mathbb{E}[\psi^{-1}])^{-1}$	0.990
Uncertainty-Adjusted Growth	$\underline{\Gamma}$	$\equiv \Gamma\underline{\psi}$	1.020
Utility Compensated Permanent Shock	$\underline{\underline{\psi}}$	$\equiv (\mathbb{E}[\psi^{1-\rho}])^{1/(1-\rho)}$	0.990
Utility Compensated Growth	$\underline{\underline{\Gamma}}$	$\equiv \Gamma\underline{\underline{\psi}}$	1.020
Absolute Patience Factor	\mathfrak{P}	$\equiv (R\beta)^{1/\rho}$	0.999
Return Patience Factor	\mathfrak{P}_R	$\equiv \mathfrak{P}/R$	0.961
PF Growth Patience Factor	\mathfrak{P}_Γ	$\equiv \mathfrak{P}/\Gamma$	0.970
Growth Patience Factor	$\mathfrak{P}_{\underline{\Gamma}}$	$\equiv \mathfrak{P}/\underline{\Gamma}$	0.980
Finite Value of Autarky Factor	$\underline{\underline{\beth}}$	$\equiv \beta\Gamma^{1-\rho}\underline{\underline{\psi}}^{1-\rho}$	0.941
Weak Impatience Factor	$\wp^{1/\rho}\mathfrak{P}$	$\equiv (\wp\beta R)^{1/\rho}$	0.071

Table 3 Definitions and Comparisons of Conditions

Perfect Foresight Versions	Uncertainty Versions
Finite Human Wealth Condition (FHC)	
$\Gamma/R < 1$ The growth factor for permanent income Γ must be smaller than the discounting factor R for human wealth to be finite.	$\Gamma/R < 1$ The model's risks are mean-preserving spreads, so the PDV of future income is unchanged by their introduction.
Absolute Impatience Condition (AIC)	
$\mathbf{P} < 1$ The unconstrained consumer is sufficiently impatient that the level of consumption will be declining over time: $\mathbf{c}_{t+1} < \mathbf{c}_t$	$\mathbf{P} < 1$ <i>If wealth is large enough, the expectation of consumption next period will be smaller than this period's consumption:</i> $\lim_{m_t \rightarrow \infty} \mathbb{E}_t[\mathbf{c}_{t+1}] < \mathbf{c}_t$
Return Impatience Conditions	
Return Impatience Condition (RIC)	Weak RIC (WRIC)
$\mathbf{P}/R < 1$ The growth factor for consumption \mathbf{P} must be smaller than the discounting factor R , so that the PDV of current and future consumption will be finite: $c'(m) = 1 - \mathbf{P}/R < 1$	$\wp^{1/\rho} \mathbf{P}/R < 1$ If the probability of the zero-income event is $\wp = 1$ then income is always zero and the condition becomes identical to the RIC. Otherwise, weaker. $c'(m) < 1 - \wp^{1/\rho} \mathbf{P}/R < 1$
Growth Impatience Conditions	
PF-GIC	GIC
$\mathbf{P}/\Gamma < 1$ Guarantees that for an unconstrained consumer, the ratio of consumption to permanent income will fall over time. For a constrained consumer, guarantees the constraint will eventually be binding.	$\mathbf{P} \mathbb{E}[\psi^{-1}]/\Gamma < 1$ By Jensen's inequality, stronger than the PF-GIC. Ensures consumers will not expect to accumulate m unboundedly. $\lim_{m_t \rightarrow \infty} \mathbb{E}_t[m_{t+1}/m_t] = \mathbf{P}_{\underline{\Gamma}}$
Finite Value of Autarky Conditions	
PF-FVAC	FVAC
$\beta \Gamma^{1-\rho} < 1$ equivalently $\mathbf{P} < R^{1/\rho} \Gamma^{1-1/\rho}$ The discounted utility of constrained consumers who spend their permanent income each period should be finite.	$\beta \Gamma^{1-\rho} \mathbb{E}[\psi^{1-\rho}] < 1$ By Jensen's inequality, stronger than the PF-FVAC because for $\rho > 1$ and nondegenerate ψ , $\mathbb{E}[\psi^{1-\rho}] > 1$.

Table 4 Sufficient Conditions for Nondegenerate[‡] Solution

Model	Conditions	Comments/Logic
PF Unconstrained Section 2.4.2	RIC, FHWC [°]	RIC $\Rightarrow v(m) < \infty$; FHWC $\Rightarrow 0 < v(m) $ RIC prevents $\bar{c}(m) = 0$ FHWC prevents $\bar{c}(m) = \infty$ RIC+FHWC \Rightarrow PF-FVAC
PF Constrained Section 2.4.3 Appendix A	PF-GIC , RIC	FHWC must hold ($\Gamma < \mathbf{D} < R \Rightarrow \Gamma < R$) Identical to solution to PF Unconstrained for $m > m_{\#}$ for some $0 < m_{\#} < 1$; $c(m) = m$ for $m \leq m_{\#}$ (RIC would yield $m_{\#} = 0$ so degenerate $c(m) = 0$)
	PF-GIC, RIC	$\lim_{m \rightarrow \infty} \dot{c}(m) = \bar{c}(m)$, $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = \underline{\kappa}$ kinks at points where horizon to $b = 0$ changes*
	PF-GIC, RIC	$\lim_{m \rightarrow \infty} \dot{\kappa}(m) = 0$ kinks at points where horizon to $b = 0$ changes*
Buffer Stock Model Section 2.5	FVAC, WRIC	FHWC $\Rightarrow \lim_{m \rightarrow \infty} \dot{c}(m) = \bar{c}(m)$, $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = \underline{\kappa}$ FHWC +RIC $\Rightarrow \lim_{m \rightarrow \infty} \dot{\kappa}(m) = \underline{\kappa}$ FHWC + RIC $\Rightarrow \lim_{m \rightarrow \infty} \dot{\kappa}(m) = 0$ GIC guarantees finite target wealth ratio FVAC is stronger than PF-FVAC WRIC is weaker than RIC

[‡]For feasible m satisfying $0 < m < \infty$, a nondegenerate limiting consumption function defines the unique value of c satisfying $0 < c(m) < \infty$; a nondegenerate limiting value function defines a corresponding unique value of $-\infty < v(m) < 0$. [°]RIC, FHWC are necessary as well as sufficient. *That is, the first kink point in $c(m)$ is $m_{\#}$ s.t. for $m < m_{\#}$ the constraint will bind now, while for $m > m_{\#}$ the constraint will bind one period in the future. The second kink point corresponds to the m where the constraint will bind two periods in the future, etc.

Table 5 Taxonomy of Perfect Foresight Liquidity Constrained Model Outcomes

For constrained \dot{c} and unconstrained \bar{c} consumption functions

Main Condition Subcondition	Math	Outcome, Comments or Results
PF-GIC and RIC and RIC	$1 < \mathbf{D}/\Gamma$ $\mathbf{D}/R < 1$ $1 < \mathbf{D}/R$	Constraint never binds for $m \geq 1$ FHWC holds ($R > \Gamma$); $\dot{c}(m) = \bar{c}(m)$ for $m \geq 1$ $\dot{c}(m)$ is degenerate: $\dot{c}(m) = 0$
PF-GIC and RIC	$\mathbf{D}/\Gamma < 1$ $\mathbf{D}/R < 1$	Constraint binds in finite time for any m FHWC may or may not hold $\lim_{m \uparrow \infty} \bar{c}(m) - \dot{c}(m) = 0$ $\lim_{m \uparrow \infty} \dot{\kappa}(m) = \underline{\kappa}$
and RIC	$1 < \mathbf{D}/R$	FHWC $\lim_{m \uparrow \infty} \dot{\kappa}(m) = 0$

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where ~~PF-GIC~~ and RIC both hold, while the third row indicates that when the PF-GIC and the RIC both fail, the consumption function is degenerate; the next row indicates that whenever the PF-GIC holds, the constraint will bind in finite time.