# 1 Introduction

## 1.1 Background

Macroeconomic modeling aims to describe a complex world of agents interacting with each other and making decisions in a dynamic setting. The models are often very complex, requiring strong underlying assumptions to be made, and use a lot of computational power to solve. One of the most common methods to solve these complex problems is to use a grid search method to solve the model. The endogenous grid method (EGM) developed by Carroll (2006) allows dynamic optimization problems to be solved in a more computationally efficient and faster manner than the previous method of convex optimization using grid search. Many problems that before took hours to solve became much easier to solve and allowed macroeconomists and computational economists to focus on estimation and simulation. However, the engodenous grid method is limited to a few specific classes of problems. Recently, the classes of problems to which EGM can be applied has been expanded[1], but with every new method comes a new set of limitations. This paper introduces a new approach to EGM in a multivariate setting. The method is called Sequential EGM (or EGM$^n$) and introduces a novel way of breaking down complex problems into a sequence of simpler, smaller, and more tractable problems, along with an exploration of new multidimensional interpolation methods that can be used to solve these problems.

## 1.2 Literature Review

The literature review should cite the following:

Carroll (2006) first introduced the endogenous grid method as a way to speed up the solution of dynamic stochastic consumption-savings problems. The method consists of starting with an exogenous grid of post-decision states and using the inverse of the first order condition to find the optimal consumption policy that rationalizes such post-decision states. Given the optimal policy and post-decision states, it is straightforward to calculate the initial pre-decision state that leads to the optimal policy. Although this method is certainly innovative, it only applied to a model with one control variable and one state variable. Barillas and Fernández-Villaverde (2007) further extend this method by including more than one control variable in the form of a labor-leisure choice, as well as a second state variable for stochastic persistence.

What do these do? Hintermaier and Koeniger (2010) and Jørgensen (2013).

Alternative ECM Maliar and Maliar (2013).

Then came White (2015), Iskhakov (2015), Ludwig and Schön (2018), with multidimensions.

For non-concave problems Fella (2014), Iskhakov, Jørgensen, Rust, and Schjerning (2017), Druedahl and Jørgensen (2017).

Clausen and Strub (2020), Druedahl (2021) do nested problems.

[1] Barillas and Fernández-Villaverde (2007); Maliar and Maliar (2013); Fella (2014); White (2015); Iskhakov, Jørgensen, Rust, and Schjerning (2017), among others.

## 1.3 Research Question

The purpose of this paper is to describe a new method for solving dynamic optimization problems efficiently and accurately while avoiding convex optimization and grid search methods with the use of the endogenous grid method and first order conditions. The method is called Sequential EGM (or $\text{EGM}^n$) and introduces a novel way of breaking down complex problems into a sequence of simpler, smaller, and more tractable problems, along with an exploration of new multidimensional interpolation methods that can be used to solve these problems. This paper also illustrates an example of how Sequential EGM can be used to solve a dynamic optimization problem in a multivariate setting.

## 1.4 Methodology

The sequential endogenous grid method consists of 3 major parts: First, the problem to be solved should be broken up into a sequence of smaller problems that themselves don't add any additional state variables or introduce asynchronous dynamics with respect to the uncertainty. If the problem is broken up in such a way that uncertainty can happen in more than one period, then the solution of this sequence of problems might be different from the aggregate problem due to giving the agent additional information about the future by realizing some uncertainty. Second, I evaluate each of the smaller problems to see if they can be solved using the endogenous grid method. This evaluation is of greater scope than the traditional endogenous grid method, as it allows for the resulting exogenous grid to be non-regular. If the sub-problem can not be solved with EGM, then convex optimization is used. Third, if the exogenous grid generated by the EGM is non-regular, then I use a multidimensional interpolation method that takes advantage of machine learning methods to generate an interpolating function. Solving each subproblem in this way, the sequential endogenous grid method is capable of solving complex problems that are not solvable with the traditional endogenous grid method and are difficult and time consuming to solve with convex optimization and grid search methods.

## 1.5 Contributions

The Sequential Endogenous Grid Method is capable of solving multivariate dynamic optimization problems in an efficient and fast manner by avoiding grid search. This should allow researchers and practitioners to solve more complex problems that were previously not easily accessible to them, but more accurately capture the dynamics of the macroeconomy. By using advancements in machine learning techniques such as Gaussian Process Regression, the Sequential Endogenous Grid Method is capable of solving problems that were not previously able to be solved using the traditional endogenous grid method. Additionally, the Sequential Endogenous Grid Method often sheds light onto the problem by breaking it down into a sequence of simpler problems that were not previously apparent. This is because intermediary steps in the solution process generate value and marginal value functions of different pre- and post-decision states that can be used to understand the problem better.

## 1.6 Outline

The first section below presents a basic model that illustrates the sequential endogenous grid method in 1 dimension. Then section 2 introduces a more complex method with 2 state variables to demonstrate the use of machine learning methods to generate an interpolating function. In section 3 I present the unstructured interpolation methos using machine learning in more detail. Section 4 discusses the theoretical requirements to use the Sequential Endogenous Grid Method. Finally, section 5 concludes with some limitations and future work.

# References


Barillas, Francisco, and Jesús Fernández-Villaverde (2007): "A generalization of the endogenous grid method," *Journal of economic dynamics & control*, 31(8), 2698–2712.

Carroll, Christopher D (2006): "The method of endogenous gridpoints for solving dynamic stochastic optimization problems," *Economics letters*, 91(3), 312–320.

Clausen, Andrew, and Carlo Strub (2020): "Reverse calculus and nested optimization," *Journal of economic theory*, 187(105019), 105019.

Druedahl, Jeppe (2021): "A Guide on Solving Non-convex Consumption-Saving Models," *Computational Economics*, 58(3), 747–775.

Druedahl, Jeppe, and Thomas Høgholm Jørgensen (2017): "A general endogenous grid method for multi-dimensional models with non-convexities and constraints," *Journal of economic dynamics & control*, 74, 87–107.

Fella, Giulio (2014): "A generalized endogenous grid method for non-smooth and non-concave problems," *Review of economic dynamics*, 17(2), 329–344.

Hintermaier, Thomas, and Winfried Koeniger (2010): "The method of endogenous gridpoints with occasionally binding constraints among endogenous variables," *Journal of economic dynamics & control*, 34(10), 2074–2088.

Iskhakov, Fedor (2015): "Multidimensional endogenous gridpoint method: Solving triangular dynamic stochastic optimization problems without root-finding operations," *Economics letters*, 135, 72–76.

Iskhakov, Fedor, Thomas H Jørgensen, John Rust, and Bertel Schjerning (2017): "The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks," *Quantitative economics*, 8(2), 317–365.

Jørgensen, Thomas H (2013): "Structural estimation of continuous choice models: Evaluating the EGM and MPEC," *Economics letters*, 119(3), 287–290.

Ludwig, Alexander, and Matthias Schön (2018): "Endogenous Grids in Higher Dimensions: Delaunay Interpolation and Hybrid Methods," *Computational Economics*, 51(3), 463–492.

Maliar, Lilia, and Serguei Maliar (2013): "Envelope condition method versus endogenous grid method for solving dynamic programming problems," *Economics letters*, 120(2), 262–266.

White, Matthew N (2015): "The method of endogenous gridpoints in theory and practice," *Journal of economic dynamics & control*, 60, 26–41.