/SequentialEGM

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[Appendices]Appendices

## Solving the illustrative G2EGM model with EGM

### The problem for a retired household

I designate as the problem of a retired household at time with total resources . The retired household solves a simple consumption-savings problem with no income uncertainty and a certain next period pension of .

(1)

Notice that there is no uncertainty and the household receives a retirement

income every period until death.

### The problem for a worker household

The value function of a worker household is

(2)

where the choice specific problem for a working household that decides to

continue working is

(3)

and the choice specific problem for a working household that decides to retire

is

(4)

### Applying the Sequential EGM

The first step is to define a post-decision value function. Once the household

decides their level of consumption and pension deposits, they are left with

liquid assets they are saving for the future and illiquid assets in their

pension account which they can’t access again until retirement. The

post-decision value function can be defined as

(5)

Then redefine the working agent’s problem as

(6)

Clearly, the structure of the problem remains the same, and this is the problem

that G2EGM solves. We’ve only moved some

of the stochastic mechanics out of the problem. Now, we can apply the

sequential EGM method. Let the agent first decide , the deposit

amount into their retirement; we will call this the deposit problem, or outer loop. Thereafter, the

agent will have net liquid assets

of and pension assets of .

(7)

Now, the agent can move on to picking their consumption and savings; we can call this

the pure consumption problem or inner loop.

(8)

Because we’ve already made the pension decision, the amount of pension assets

does not change in this loop and it just passes through to the post-decision

value function.

### Solving the problem

#### Solving the Inner Consumption Saving Problem

Let’s start with the pure consumption-saving problem, which we can summarize by

substitution as

(9)

The first-order condition is

(10)

We can invert this Euler equation as in standard EGM to obtain the consumption

function.

(11)

Again as before, . To sum up, using an

exogenous

grid of we obtain the trio which

provides an

interpolating function for our optimal consumption decision rule over the

grid. Without loss of generality, assume and define the interpolating

function as

(12)

For completeness, we derive the envelope conditions as well, and as we will

see, these will be useful when solving the next section.

(13)

#### Solving the Outer Pension Deposit Problem

Now, we can move on to solving the deposit problem, which we can also summarize

as

(14)

The first-order condition is

(15)

Rearranging this equation gives

(16)

Assuming that exists and is invertible, we can find

(17)

Using this, we can back out as

(18)

and as

(19)

In sum, given an exogenous grid we obtain the triple

, which

we can use to

create an interpolator for the decision rule .

To close the solution method, the envelope conditions are

(20)

### Is g invertible?

We’ve already seen that is invertible, but is ?

(21)

### The Post-Decision Value and Marginal Value Functions

(22)

and

(23)

and

(24)

### Taste Shocks

From discrete choice theory and from DCEGM paper, we know that

(25)

and

(26)

the first-order conditions are therefore

(27)

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