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1 Intro

This document details the calibration of the quarterly version of the idiosyncratic model using estimates obtained from annual idiosyncratic data. The derivations are verified in the associated Stata program MicroShkQtrVsAnnStata.do.

2 Variable Definitions

Stata	LaTeX	Description
`tShkQtr`	θ_t	transitory shock, $\log\theta \sim \mathcal{N}(-\sigma_\theta^2/2, \sigma_\theta^2) \Rightarrow E_t[\theta_{t+n}] = 1\ \forall\ n > 0$
`pShkQtr`	ψ_t	permanent shock, $\log\psi \sim \mathcal{N}(-\sigma_\psi^2/2, \sigma_\psi^2) \Rightarrow E_t[\psi_{t+n}] = 1\ \forall\ n > 0$
`ltShkQtr`	$l\theta_t$	An `l` in front of a variable indicates the log
`lpShkQtr`	$l\psi_t$	
`tDevQtr`	$\theta_t^\nabla = \theta_t - 1$	additive form of transitory income shock in quarter t
`pDevQtr`	$\psi_t^\nabla = \psi_t - 1$	additive form of permanent income shock in quarter t
`pLevQtr`	$\mathbf{p}_t$	permanent income at a quarterly rate
`tLevQtr`	$\mathbf{t}_t$	transitory income at a quarterly rate (=θ)
`yLevQtr`	$\mathbf{p}_t\theta_t = \mathbf{p}_t(1+\theta_t^\nabla)$	actual income in quarter t
`pLevAnn`	$\vec{\mathbf{p}}_t$	permanent income for four quarters beginning at t eqn (2)
`tLevAnn`	$\vec{\theta}_t$	transitory income for four quarters beginning at t eqn (6)
`tDevAnn`	$\vec{\theta}_t - 1$	transitory shock for four quarters beginning at t eqn (6)
`yLevAnn`	$\vec{\mathbf{y}}_t$	income for four quarters beginning at t
`tShkAnn`	$\vec{\theta}_t$	transitory shock to income over year beginning at t
`pShkAnn`	$\vec{\psi}_t$	permanent shock to income over year beginning at t
`stdltShkAnn`	$\sigma_{\vec{\theta}}$	standard deviation of transitory component of annual income
`stdlpShkAnn`	$\sigma_{\vec{\psi}}$	standard deviation of permanent component of annual income
`stdltShkQtr`	σ_θ	standard deviation of transitory component of quarterly income
`stdlpShkQtr`	σ_ψ	standard deviation of permanent component of quarterly income

3 Relationship Between Annual and Quarterly Transitory Shocks

Start by assuming that the shocks to permanent income occur only once every four quarters, so that, e.g.,

$$\mathbf{p}_t \quad = \quad \mathbf{p}_{t+1} = \mathbf{p}_{t+2} = \mathbf{p}_{t+3} \tag{1}$$

which implies

$$
\begin{aligned}
\texttt{pLevAnn} = \vec{\mathbf{p}}_t &= 4\mathbf{p}_t && (2)\\
\texttt{yLevAnn} = \vec{\mathbf{y}}_t &= \mathbf{p}_t(1+\theta_t^{\nabla}) + \mathbf{p}_{t+1}(1+\theta_{t+1}^{\nabla}) + \mathbf{p}_{t+2}(1+\theta_{t+2}^{\nabla}) + \mathbf{p}_{t+3}(1+\theta_{t+3}^{\nabla}) && (3)\\
&= \mathbf{p}_t\left(4+\theta_t^{\nabla}+\theta_{t+1}^{\nabla}+\theta_{t+2}^{\nabla}+\theta_{t+3}^{\nabla}\right) && (4)\\
&= \vec{\mathbf{p}}_t\left(1+(\theta_t^{\nabla}+\theta_{t+1}^{\nabla}+\theta_{t+2}^{\nabla}+\theta_{t+3}^{\nabla})/4\right) && (5)\\
\vec{\theta}_t &= \left(1+(\theta_t^{\nabla}+\theta_{t+1}^{\nabla}+\theta_{t+2}^{\nabla}+\theta_{t+3}^{\nabla})/4\right) && (6)
\end{aligned}
$$

$$
\begin{aligned}
\log\vec{\theta}_t &\approx (\theta_t^{\nabla}+\theta_{t+1}^{\nabla}+\theta_{t+2}^{\nabla}+\theta_{t+3}^{\nabla})/4 && (7)\\
\mathrm{var}(\log\vec{\theta}) &\approx \mathrm{var}\left((\theta_t^{\nabla}+\theta_{t+1}^{\nabla}+\theta_{t+2}^{\nabla}+\theta_{t+3}^{\nabla})/4\right) && (8)
\end{aligned}
$$

but since $\mathrm{var}(\theta^{\nabla}) \approx \sigma_{\theta}^2$, this implies

$$
\begin{aligned}
\sigma_{\vec{\theta}}^2 &= 4\sigma_{\theta}^2/16 && (9)\\
\hat{\sigma}_{\theta}^2 &= 4\sigma_{\vec{\theta}}^2 && (10)
\end{aligned}
$$

4 Permanent Quarterly and Annual Shocks

Now let's relax the assumption that permanent shocks arrive only once a year, but (to keep the algebra simple), suppose we are looking at semiannual rather than quarterly data.

$$
\begin{aligned}
\vec{\mathbf{p}}_t &= \mathbf{p}_t + \mathbf{p}_t\psi_{t+1} && (11)\\
&= \mathbf{p}_t(1+\psi_{t+1}) && (12)\\
&= \mathbf{p}_t(1+(1+l\psi_{t+1})) && (13)\\
&= 2\mathbf{p}_t(1+l\psi_{t+1}/2) && (14)\\
\vec{\mathbf{p}}_{t+2} &= \mathbf{p}_t\psi_{t+1}\psi_{t+2}(1+\psi_{t+3}) && (15)\\
&= 2\mathbf{p}_t(1+l\psi_{t+1})(1+l\psi_{t+2})(1+l\psi_{t+3}/2) && (16)\\
l\vec{\mathbf{p}}_{t+2} &\approx \log 2\mathbf{p}_t + l\psi_{t+1} + l\psi_{t+2} + l\psi_{t+3}/2 && (17)\\
l\vec{\mathbf{p}}_{t+2} - l\vec{\mathbf{p}}_t &\approx l\psi_{t+1} + l\psi_{t+2} + l\psi_{t+3}/2 - l\psi_{t+1}/2 && (18)\\
&= l\psi_{t+1}/2 + l\psi_{t+2} + l\psi_{t+3}/2 && (19)\\
\mathrm{var}(l\vec{\mathbf{p}}_{t+2} - l\vec{\mathbf{p}}_t) &\approx (1+1/2)\,\mathrm{var}(l\psi) && (20)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
l\vec{\mathbf{p}}_{t+4} &\approx \log 2\mathbf{p}_t + l\psi_{t+1} + l\psi_{t+2} + l\psi_{t+3} + l\psi_{t+4} + l\psi_{t+5}/2 && (21)\\
l\vec{\mathbf{p}}_{t+4} - l\vec{\mathbf{p}}_t &\approx l\psi_{t+1}/2 + l\psi_{t+2} + l\psi_{t+3} + l\psi_{t+4} + l\psi_{t+5}/2 && (22)\\
\mathrm{var}(l\vec{\mathbf{p}}_{t+4} - l\vec{\mathbf{p}}_t) &\approx (3+1/2)\,\mathrm{var}(l\psi) && (23)
\end{aligned}
$$

so

$$
\mathrm{var}(l\vec{\mathbf{p}}_{t+4} - l\vec{\mathbf{p}}_t) - \mathrm{var}(l\vec{\mathbf{p}}_{t+2} - l\vec{\mathbf{p}}_t) = 2\,\mathrm{var}(l\psi) \qquad (24)
$$

The corresponding logic for quarterly (as opposed to semiannual) data leads to the conclusion that

$$
\mathrm{var}(l\vec{\mathbf{p}}_{t+8} - l\vec{\mathbf{p}}_t) - \mathrm{var}(l\vec{\mathbf{p}}_{t+4} - l\vec{\mathbf{p}}_t) = 4\,\mathrm{var}(l\psi) = \mathrm{var}(l\vec{\psi}) = \sigma_{\vec{\psi}}^2 \qquad (25)
$$

Thus, our estimate of the variance of quarterly shocks to permanent income is

$$
\hat{\sigma}_{\psi}^2 = \sigma_{\vec{\psi}}^2 \qquad (26)
$$

5 Calibration

The key parameters are taken from Carroll and Samwick (1997), Table 1, first row: The estimated annual variances of transitory and permanent components of income. For purposes of this paper, those figures are rounded off, as below. All other statistics are derived from these two.

Source	$\sigma^2_{\vec{\psi}}$	$\sigma^2_{\vec{\theta}}$	$\sigma_{\vec{\psi}}$	$\sigma_{\vec{\theta}}$
Annual				
C-S Table 1	0.0217	0.0440	0.1473	0.1473
This Paper	0.0160	0.0300	0.1265	0.1732
Quarterly				
C-S Table 1	0.0054	0.1760	0.0737	0.0737
This Paper	0.0040	0.1200	0.0632	0.3464

References

Carroll, Christopher D., and Andrew A. Samwick (1997): "The Nature of Precautionary Wealth," *Journal of Monetary Economics*, 40(1), 41–71.