1 Formulae Derived from ?

?, pp. 303–304, shows that the signal-extracted estimate of permanent income is

$$\widetilde{P}_t = v_1 Y_t + v_2 Y_{t-1} + v_3 Y_{t-2} + \ldots \tag{1}$$

for a sequence of v's given by

$$v_k = (1-\lambda_1)\lambda_1^{k-1} \tag{2}$$

for $k = 1, 2, 3, \ldots$. So:

$$\begin{aligned}
\widetilde{P}_t &= (1-\lambda_1)(\qquad\qquad Y_t + \lambda_1 Y_{t-1} + \lambda_1^2 Y_{t-2} \ldots) && (3)\\
\widetilde{P}_{t+1} &= (1-\lambda_1)(Y_{t+1} + \lambda_1 Y_t + \lambda_1^2 Y_{t-1} + \lambda_1^3 Y_{t-2} \ldots) && (4)\\
&= (1-\lambda_1)\ Y_{t+1} + \lambda_1 \underbrace{(1-\lambda_1)(Y_t + \lambda_1^2 Y_{t-1} + \lambda_1^3 Y_{t-2} \ldots)}_{\widetilde{P}_t} && (5)\\
&= (1-\lambda_1) Y_{t+1} + \lambda_1 \widetilde{P}_t && (6)
\end{aligned}$$

This compares with (??) in the main text

$$\widetilde{P}_{t+1} = \Pi Y_{t+1} + (1-\Pi)\widetilde{P}_t \tag{7}$$

so the relationship between our Π and Muth's λ_1 is:

$$\lambda_1 = 1 - \Pi \tag{8}$$

Defining the signal-to-noise ratio $\varphi = \sigma_{\boldsymbol{\psi}}/\sigma_{\boldsymbol{\theta}}$, starting with equation (3.10) in ? we have

$$\begin{aligned}
\lambda_1 &= 1 + (1/2)\varphi^2 - \varphi\sqrt{1+\varphi^2/4}\\
(1-\Pi) &= 1 + (1/2)\varphi^2 - \varphi\sqrt{1+\varphi^2/4}\\
-\Pi &= (1/2)\varphi^2 - \varphi\sqrt{1+\varphi^2/4} && (9)
\end{aligned}$$

yielding equation (10) in the main text.

$$\Pi = \varphi\sqrt{1+\varphi^2/4} - (1/2)\varphi^2 \tag{10}$$