Errors

- An error is defined as the **difference** between the **actual value** and the **approximate value** obtained from the experimental observation or from numerical computation.
- **Error $=$ Actual value$-$Approximate value**

$$= \boldsymbol{x - \overline{x}} \quad \textbf{Or} \quad \boldsymbol{x - x_a}$$

Types of Errors

- Absolute error
- Relative error
- Percentage error

Absolute Error

- If x is the true value of a quantity and $\bar{x}$ is its approximate value, then Absolute error is denoted by E_a.

$$E_a = |Exact\ value - Approximate\ value|$$

$$E_a = |x - \bar{x}|$$

Relative Error

- The relative error is defined by

$$\mathrm{E_r} = \left|\frac{Exact\ value - Approximate\ value}{Exact\ value}\right|$$

$$\mathrm{E_r} = \left|\frac{x - \bar{x}}{x}\right|$$

Percentage Error

- The percentage error is

$$E_p = E_r \times 100$$

$$E_p = \left|\frac{Exact\ value - Approximate\ value}{Exact\ value}\right| \times 100\%$$

$$E_p = \left|\frac{X - \bar{X}}{X}\right| \times 100\% \qquad \left(\because E_r = \left|\frac{X - \bar{X}}{X}\right|\right)$$

Sources of Errors

- Inherent error
- Truncation error
- Rounding-off error

Inherent Error

- The errors which are **already Present in the statement** of problem before its solution is obtained.

- Such are either due to the given data being approximated or due to limitation of mathematical measurements.

Rounding-off-Error

- ➢ Rounding errors arise from the process of rounding off the numbers during the computation.

- ✓ There are numbers with large number of digits.

i.e., $\frac{22}{7} = 3.142857143.$

This process of dropping unwanted digits is called rounding off.

Truncation Error

- Truncation errors are caused by using approximate results or on replacing an infinite process by a finite one.
- If we are using a decimal computer having a fixed word length of 4 digits, rounding off of 13.658 gives 13.66 whereas truncation gives 13.65
- i.e. if $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = x$ (say) is replaced by $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} = \bar{x}$(say), then the truncation error is $x - \bar{x}$.

Significant Figures

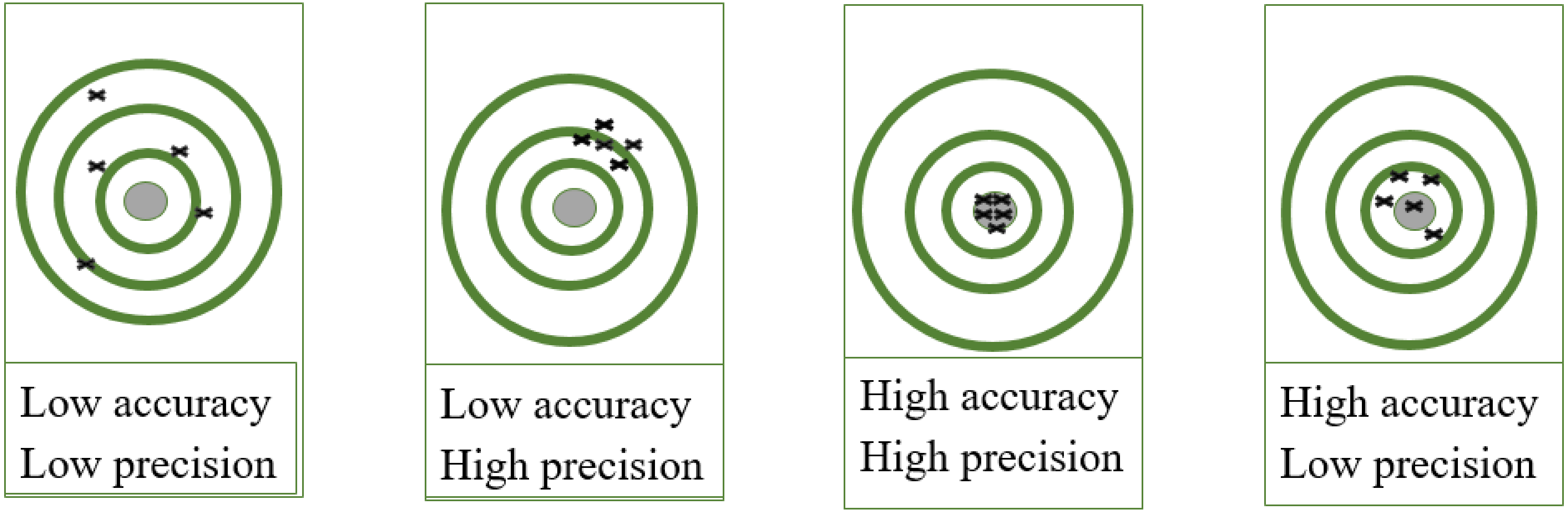
- The digits used to express a number are called significant numbers.
- All nonzero digits are considered as significant, e.g., 9345, 123.9 have four significant figures.
- All zeros between two nonzero digits are significant, e.g., 10011, 120.03 have 5 significant figures.
- Leading zeros are not significant, e.g., 0.0012, 0.13 have two significant figures.

Accuracy and Precision

The concept of accuracy and precision are closely related to significant digits. They are related as follows:

- Accuracy refers to how closely a computed or measured value agree with true value.
- Precision refers to how closely individually computed or measured value agree with each other.

Accuracy and Precision



Example-1

Find error and relative error in the following cases:

$(a)\ x = 3.141592, \bar{x} = 3.14$

Solution:

$(a)\ x = 3.141592, \bar{x} = 3.14$

Absolute error: $E_a = |x - \bar{x}|$

$= |3.141592 - 3.14|$

$= 0.001592$

Relative error: $E_r = \left|\frac{x-\bar{x}}{x}\right|$

$= \left|\frac{3.141592 - 3.14}{3.141592}\right|$

$= 0.0005067$

$= 5.067 * 10^{-4}$

Example-2

Find the relative error is the computation of $x - y$ for $x = 12.05$ and $y = 8.02$ having absolute error $\delta_x = 0.005, \delta_y = 0.001$.

Solution:

$x = 12.05, \qquad \delta_x = 0.005$

$y = 8.02, \qquad \delta_y = 0.001$

Relative error in $x = \frac{absolute\ error\ in\ x}{x}$ $\qquad \left(\because \mathrm{E_r} = \left|\frac{x-\bar{x}}{x}\right|\right)$

$= \frac{\delta_x}{x}$

$= \frac{0.005}{12.05}$

$= 4.15 * 10^{-4}$

Relative error in y= $\frac{absolute\ error\ in\ x}{y}$

$$= \frac{\delta_y}{y}$$

$$= \frac{0.001}{8.02}$$

$$= 1.25 * 10^{-4}$$

Relative error in $x - y = 4.15 * 10^{-4} - 1.25 * 10^{-4}$

$$= 2.903 * 10^{-4}$$

Example-3

Find (i) Absolute error (ii)Relative error (iii)percentage error, If $\frac{2}{3}$ is approximated to four significant digits.

Solution:

Here, $x = \frac{2}{3}, \bar{x} = 0.6666$

Absolute Error $E_a = |x - \bar{x}|$

$= \left|\frac{2}{3} - 0.6666\right|$

$= 6.6666 * 10^{-5}$

Relative error:

$$E_r = \left|\frac{x-\bar{x}}{x}\right|$$

$$= \left|\frac{\frac{2}{3} - 0.6666}{\frac{2}{3}}\right|$$

$$= 0.1 * 10^{-3}$$

Percentage error:

$$E_p = \left|\frac{x-\bar{x}}{x}\right| * 100\%$$

$$= 0.01$$

Example-4

The solution of a problem is given as $\mathbf{3.436}$. It is known that the absolute error in the solution is less than $\mathbf{0.01}$. Find the interval within which the exact value must lie.

Solution:

Here, $\bar{x} = 3.436$

$|x - \bar{x}| < 0.01$
$\therefore |x - 3.436| < 0.01$
$\therefore -0.01 < x - 3.436 < 0.01$
$\therefore -0.01 + 3.436 < x < 0.01 + 3.436$
$\therefore 3.426 < x < 3.446$

Example-5

Given the solution of a problem $x_a = 35.25$ with relative error in the solution at most 2%. Find, to four decimal digits, the range of values within which the exact value of the solution must lie.

Solution:

$\bar{x} = 35.25$

$$\left|\frac{x - \bar{x}}{x}\right| < 0.02$$

$$\therefore \left|\frac{x - 35.25}{x}\right| < 0.02$$

$$\therefore -0.02 < \frac{x - 35.25}{x} < 0.02$$

$$-0.02 < \frac{x - 35.25}{x}$$

$$\therefore -0.02x < x - 35.25$$

$$\therefore 35.25 < x + 0.02x$$

$$\therefore 35.25 < 1.02x$$

$$\therefore \frac{35.25}{1.02} < x$$

$$\therefore 34.5588 < x$$

$$\frac{x - 35.25}{x} < 0.02$$

$$\therefore x - 35.25 < 0.02x$$

$$\therefore x - 0.02x < 35.25$$

$$\therefore 0.98x < 35.25$$

$$\therefore x < \frac{35.25}{0.98}$$

$$\therefore x < 35.9694$$

Solution is,

$$34.5588 < x < 35.9694$$

Mathematical Modelling

- Mathematical Modelling is the method of translating the problems from real life systems into conformable and manageable mathematical expressions whose analytical consideration determines an insight and orientation for solving a problem and provides us with a technique for better development of the system.
- Mathematical models are used in various fields including natural sciences, engineering and social sciences.

Steps of problem solving:

1. Data Analysis:

In this phase problem is analyzed and required data is collected for modelling.

2. Designing of Mathematical model:

In this phase, the structure of the solution like objective of the model, bounds of the system, performance measures, etc. is defined.

3. Computer simulation and post processing or graphic result:

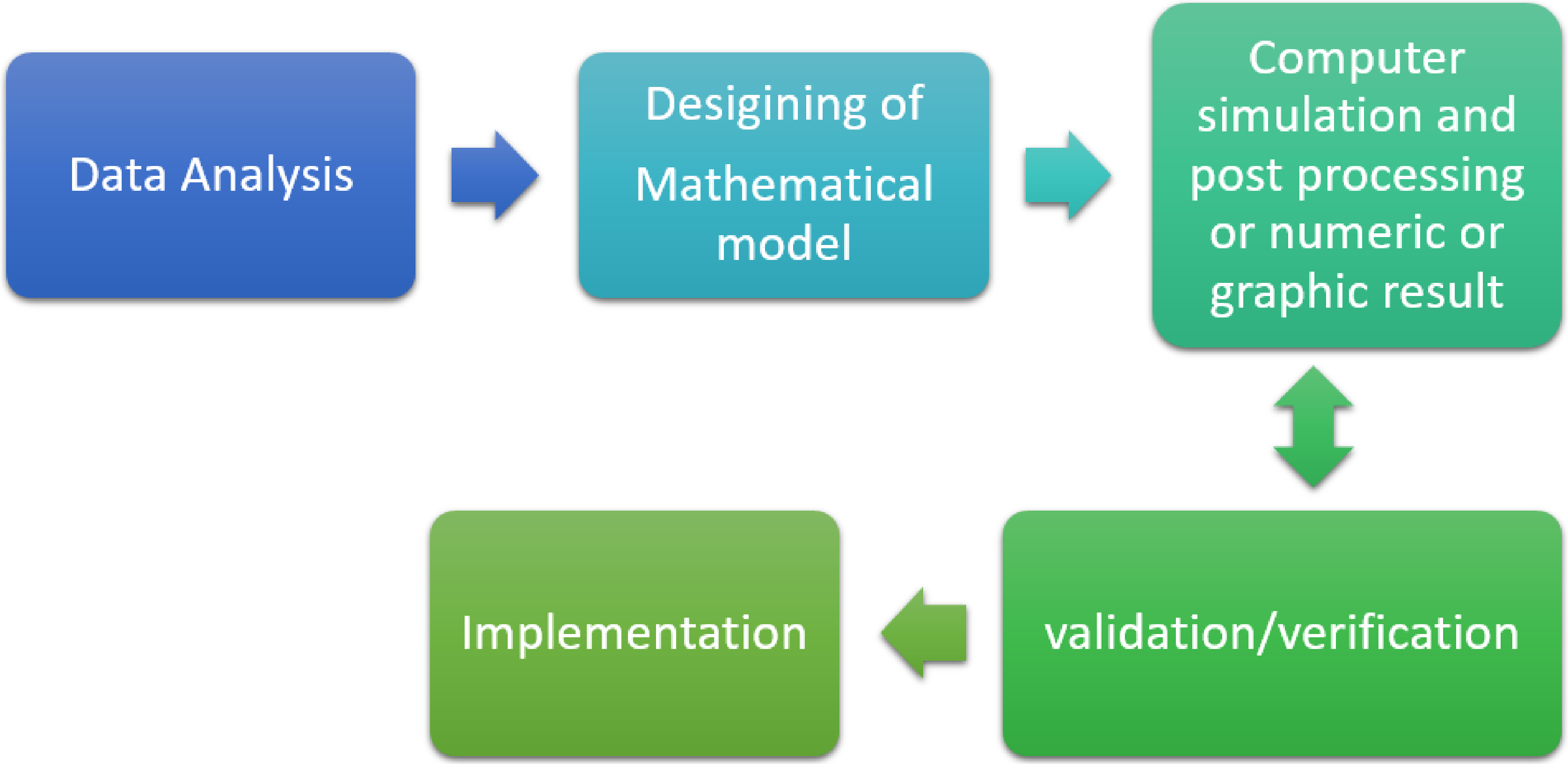
By inputting required data, we get the result in form of data or graph using mathematical model by computer simulation software.

4. Validation/Verification:

During validation phase, Mathematical model's result is verified.

5. Implementation:

We can implement model of problem in real world.



This is the whole view of whole process leading from a problem to its solution by scientific computation.