University of Rajshahi

Department of Computer Science and Engineering

B. Sc. (Engg) Part-II Even Semester Examination 2020

Course: MATH 2241 (Linear Algebra)

Full Marks: 52.5 Duration: 3 (Three) Hours

Answer 06 (Six) questions taking any 03 (Three) from each section

Section-A

1. a) Define a vector in $\mathbb{R}^n$. What do you mean by linear combination of vectors in $\mathbb{R}^n$? 3

   b) Consider the system of linear equations 3

$$x - 2y = 2$$
$$2x - 4y = -2$$

   Draw row picture and column picture, and explain the solution of the system of the linear equations based on the pictures.

   c) Find a unit vector u in the direction of $v = (3,4)$. Find a unit vector w that is perpendicular to u. How many possibilities for w? 2.75

2. a) Define a *vector space*. Let H be the set of all vectors of the form $(a - 3b, b - a, a, b)$, where a and b are arbitrary scalars. Show that H is a subspace of $\mathbb{R}^4$. 3.75

   b) Define *Null space*. Find a spanning set for the null space of the matrix 3

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}.$$

   c) Let $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$ and $u = (3, -2, -1, 0)$. 2

   Determine if u is in $Nul\ A$. Could u be in $Col\ A$?

3. a) What do you mean by linearly dependent and linearly independent set of vectors? Define basis for a vector subspace H. 2

   b) Consider figure 3(a) and let $H = \text{Span}\{u, v, w\}$ and $w = u + v$. 3.75

   Show that $\text{Span}\{u, v, w\} = \text{Span}\{u, v\}$. Then find a basis for the subspace H.

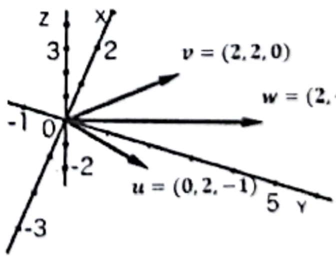

Figure - 3(a)

   c) Let $u = (2,1)$, $v = (-1,1)$, $x = (4,5)$, and $\mathcal{B} = \{u, v\}$. Find the coordinate vector $[x]_{\mathcal{B}}$ of x relative to $\mathcal{B}$. 2

   d) Let $v_1 = (1, -2, 2)$ and $v_2 = (-3, 7, -8)$. Is $\{v_1, v_2\}$ a basis for $\mathbb{R}^2$? 1

4. a) Consider the bases $\{e_1 = (1,0),\ e_2 = (0,1)\}$ and $\{f_1 = (1,3),\ f_2 = (2,5)\}$ of $\mathbb{R}^2$. 2.25+3
(i) Find the transition matrix P from $\{e_i\}$ to $\{f_i\}$.
(ii) Show that $[T]_f = P^{-1}[T]_e P$ for the linear operator T on $\mathbb{R}^2$ defined by $T(x,y) = (2y, 3x - y)$.

b) Let T be the linear operator on $\mathbb{R}^2$ defined by $T(x,y) = (4x - 2y, 2x + y)$. 3.5
Verify that $[T]_f[v]_f = [T(v)]_f$ for any vector $v \in \mathbb{R}^2$.

Section-B

5. a) Consider figure 5(a). Is v an eigenvector of the matrix A ? 1

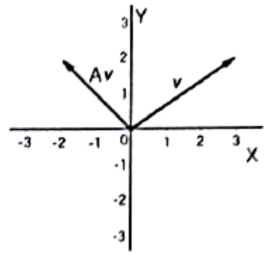

Figure- 5(a)

b) Show that 7 is an eigenvalue of matrix $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, and find the corresponding eigenvectors. 3.75

c) Prove that the set $\{v_1, v_2, \dots\dots, v_r\}$ is linearly independent where $v_1, v_2, \dots\dots, v_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \lambda_2, \dots\dots, \lambda_r$ of an $n \times n$ matrix A. 4

6. a) Define ***similarity*** of two matrices. Prove that two $n \times n$ ***similar*** matrices A and B have the same characteristic polynomial and hence the same eigenvalues. 3

b) Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k, given that $A = PDP^{-1}$, 3.75
where $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$.

c) Suppose A is a 2×2 matrix. One eigenvalue of A is $\lambda_1 = 0.8 - 0.6i$ and its corresponding eigenvector $v_1 = (-2 - 4i, 5)$. Find another eigenvalue and its corresponding eigenvector. 2

7. a) Define inner product of two vectors and length of a vector. 1

b) Let W be the subspace of $\mathbb{R}^2$ spanned by $x = \left(\frac{2}{3}, 1\right)$. Find a unit vector z that is a basis for W. 2

c) The set $S = \{u_1, u_2, u_3\}$ is an orthogonal basis for $\mathbb{R}^3$, where $u_1 = (3,1,1)$, $u_2 = (-1,2,1)$ and $u_3 = \left(-\frac{1}{2}, -2, \frac{7}{2}\right)$. Express the vector $y = (6,1,-8)$ as a linear combination of the vectors in S. 3

d) Let $u_1 = (2,5,-1)$, $u_2 = (-2,1,1)$, and $y = (1,2,3)$. Observe that $\{u_1, u_2\}$ is an orthogonal basis for $W = \text{Span}\,\{u_1, u_2\}$. Write y as the sum of a vector in W and a vector orthogonal to W. 2.75

8. a) Define an inner product space. State and prove Cauchy Schwartz inequality in an inner product space. 4.5

b) Use Gram-Schmidt orthogonalization process to transform the basis $\{v_1 = (1,1,1), v_2 = (0,1,1), v_3 = (0,0,1)\}$ of $\mathbb{R}^3$ into an orthonormal basis $\{u_i\}$. 4.25

University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engg. Part – II, Semester – Even, Examination 2019
Course: **MATH 2241** (Linear Algebra)
Marks: 52.50 Time: 3 Hours
[Answer any Six (06) questions taking at least three (03) from each section.]

Section-A

1.a) Define a vector space. Let X be any non-empty set and K be an arbitrary field. Define addition and scalar multiplication on V, the set of all functions $f: X \to K$, so that V is a vector space. 3

b) Define subspaces of a vector space with example. Show that the intersection of any two subspaces is also a subspace. 3

c) Determine whether or not W is a subspace of $\mathbb{R}^3$if (i) $W = \{(a, b, 0): a, b \in \mathbb{R}\}$, (ii) $\{(a, b, c): a^2 + b^2 + c^2 \leq 1\} = W$. 2.75

2.a) Define pivot position and pivot column in a matrix. 2

b) Let $u = (1, -2, -5)$, $v = (2,5,6)$, and $b = (7,4, -3)$. Determine whether b can be generated as a linear combination of u and v. 3.75

c) Let $u = (1, -2,3)$, $v = (5, -13, -3)$, and $b = (-3,8,1)$. Span$\{u, v\}$ is a plane through the origin in R^3. Is b in that plane? 3

3.a) Given the system 2

$$x_1 + 2x_2 - x_3 = 4$$
$$-5x_2 + 3x_3 = 1.$$

Write the system as a matrix times a vector form, i.e., $Ax = b$ form.

b) Determine if the following homogeneous system has a nontrivial solution. Then describe the solution set (if any). If there is no nontrivial solution set then explain why? 4.75

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 2x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 4x_3 = 0.$$

c) Let $u_1 = (1,2,3)$, $u_2 = (4,5,6)$, and $u_3 = (2,1,0)$. Determine if the set $\{u_1, u_2, u_3\}$ is linearly independent. 2

4.a) Define a linear transformation $T: R^2 \to R^2$ by 2.75

$$T(x) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix}$$

Find the images under T of $u = (4,1)$, $v = (2,3)$, and $u + v = (6,4)$.

b) Define kernel and range of a linear transformation T from a vector space V into a vector space W. 2

c) Define inner product of vector space, Cauchy-Schwarz inequality, Triangle inequality, and Hilbert space. 4

Section-B

5.a) Define vector space. Given u and v in a vertor space W, let $H = \text{Span}\ \{u, v\}$. Show that H is a subspace of W. 3.75

b) Define null space. Let $A = \begin{pmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{pmatrix}$ be a matrix, and let $u = (5,3, -2)$. Determine if 2

u belongs to the null space of A.

c) Let $A = \begin{pmatrix} 2 & 4 & -21 \\ -2 & -5 & 7\,3 \\ 3 & 7 & -86 \end{pmatrix}$ $\begin{pmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{pmatrix}$ 3

i) If the column space of A is a subspace of R^k, what is k?

ii) If the null space of A is a subspace of R^k, what is k?

6.a) Define basis and dimension with an example. Find a basis of the subspace $U = \{(a,b,c,d): b+c+d=0\}$ of $\mathbb{R}^4$. 2.75

b) Define matrix representation of a linear operator. Verify that $[T]_f[V]_f = [T(V)]_f$, where $T(x,y) = (5x+y, 3x-2y)$ and the basis f is given by $\{f_1 = (1,2), f_2 = (2,3)\}$. 3

c) Define row space of a $m \times n$ matrix. Find bases for the row space, the column space, and the null space of the matrix A. 3

$$A = \begin{pmatrix} -2 & -5 & 8 & 0-17 \\ 1 & 3 & -5\,1 & 5 \\ 3 & 11 & -197 & 1 \\ 1 & 7-13 & 5 & -3 \end{pmatrix} \quad \begin{pmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{pmatrix}$$

7.a) Consider two bases $B = \{b_1, b_2\}$ and C$= \{c_1, c_2\}$ for a vector space V, such that $b_1 = 4c_1 + c_2$ and $b_2 = -6c_1 + c_2$. 3

Suppose $x = 3b_1 + b_2$, so we have $[x]_B = (3,1)$. Find $[x]_C$.

b) What do you mean by orthogonal complement of subspace W of R^n? 2.75

Let $y = (7,6)$ and $u = (4,2)$, Find the orthogonal projection of y onto u. Then write y as the sum of two orthogonal vectors, one in Span $\{u\}$ and one orthogonal to u.

c) The set $S = \{u_1, u_2, u_3\}$ is an orthogonal basis for R^3, 3

where $u_1 = (3,1,1)$, $u_2 = (-1,2,1)$, $u_3 = \left(\frac{-1}{2}, -2, \frac{7}{2}\right)$.

Express the vector $y = (6,1,-8)$ as a linear combination of the vectors in S.

8.a) Let $A = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}$. An eigenvalue of A is 2. Find a basis for the corresponding eigenspace. 3

b) For any subset W of a vector space V, define $W^\perp$. Let W be the subspace of $\mathbb{R}^5$ spanned by $u = (1, 2, 3, -1, 2)$ and $v = (2, 4, 7, 2, -1)$. Find basis of the orthogonal complement $W^\perp$. of W. 3

c) Define orthonormal set. Find orthonormal basis $\{u_1, u_2, u_3\}$ from the basis $\{v_1 = (1, 1, 1), v_2 = (0, 1, 1), v_3 = (0, 0, 1)\}$. 2.75

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University of Rajshahi

Department of Computer Science and Engineering

B.Sc. (Engg.) Part-2 Even Semester Examination-2018

Course: MATH2241 (Linear Algebra)

Marks: 52.50 Time: 3 Hours

[N.B. Answer SIX questions taking THREE from each section.]

Section-A

1. (a) Define vector space and its subspace with example. Let $V = \mathbb{R}^3$ and let $W = \{ (x, y, z): x + y + z = 0\}$. Prove that W is a subspace of V. 2.75

   (b) Prove that vector space V is the direct sum of its subspaces U and W, if and only if 3
   $(i)\ V = U + W$ and $(ii)\ U \cap W = \{0\}$.

   (c) Show that $(1,1,1), (0,1,1)$ and $(0,1,-1)$ generate $\mathbb{R}^3$. 3

2. (a) Define linearly independent set. Let V be the vector space of 2×2 matrices over $\mathbb{R}$. Are the matrices $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ linearly dependent? 3

   (b) Define basis and dimension of vector space. Determine whether or not $(1,1,2), (1,2,5)$ and $(5,3,4)$ form a basis for the vector space $\mathbb{R}^3$. 3

   (c) Let U and W be the following subspace of $\mathbb{R}^4$: $U = \{(a, b, c, d): b + c + d = 0\}$, $W = \{(a, b, c, d): a + b = 0,\ c = 2d\}$. Find the dimension and a basis of $(i)\ U$, $(ii) W$. 2.75

3. (a) Define co-ordinate of a vector. Find the co-ordinate of the polynomial $v = 2t^2 - 5t + 6$ where v is the vector in $V = \{ at^2 + bt + 6 : a, b, c \in \mathbb{R}\}$ which generated by the polynomial $e_1 = 1$, $e_2 = t - 1$ and $e_3 = (t - 1)^2$. 3.25

   (b) Define linear mapping with example. Let $F: V \to U$ be a linear mapping. Show that the kernel of F is a subspace of V. 3

   (c) Let $F: \mathbb{R}^3 \to \mathbb{R}$ be defined by $(x, y, z) = 8x - 3y + 5z$. Is the map F linear? 2.5

4. (a) Let $F: \mathbb{R}^4 \to \mathbb{R}^3$ be a linear mapping defined by $F(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$. Find a basis and the dimension of the kernel F. 3

   (b) Define linear operator. Let T be the linear operator on $\mathbb{R}^2$ defined by $T(3,1) = (2, -4)$ and $T(1,1) = (0,2)$. Find $T(a, b)$ and $T(6,5)$. 2.75

   (c) Define a matrix representation of linear operator. Find the matrix represented by the linear operator $\mathbb{R}^2$ defined by $T(x, y) = (3x - 2y, 3x + 5y)$ relative to the basis $\{ e_1 = (1,2), e_2 = (-3,0)\}$. 3

Section-B

5. (a) Let T be the linear operator on $\mathbb{R}^3$ defined by $T(x,y,z) = (2y+z, x-4y, 3x)$ 4

i) Find the matrix of T in the basis $\{ f_1 = (1,2,1), f_2 = (1,2,0), f_3 = (3,0,0)\}$

ii) Verify that $[T]_f[v]_f = [T(v)]_f$, for any $v \in \mathbb{R}^3$.

(b) Define transition matrix. Consider the following bases of 4.75
$\mathbb{R}^3$: $\{ e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1)\}$ and
$\{f_1 = (1,1,1), f_2 = (1,1,0), f_3 = (1,0,0)\}$.

i) Find the transition matrix P from $\{e_i\}$ to $\{f_i\}$.
ii) Find the transition matrix Q from $\{f_i\}$ to $\{e_i\}$.
iii) Verify that $Q = P^{-1}$.

6. (a) Define eigenvalue and eigenvector of a matrix A. Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ be a matrix. Find all eigenvalues and eigenvectors of A. 2.75

3

(b) Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ be a matrix. Find an invertible matrix P such that $P^{-1}AP$ is diagonal. 3

(c) Suppose λ is an eigenvalue of an invertible operator T. Show that λ^{-1} is an eigenvalue of T^{-1}.

7. (a) Define linear functional. Let $V = \{a + bt : a, b \in \mathbb{R}\}$ consider $\phi_1 : V \to \mathbb{R}$ and $\phi_2 : V \to \mathbb{R}$ be defined $\phi_1(f(t)) = \int_0^1 f(t)dt$ and $\phi_2(f(t)) = \int_0^2 f(t)dt$. Find the basis $\{ v_1, v_2\}$ of V which is dual to $\{ \phi_1, \phi_2\}$. 3

2.75

(b) Define dual space and annihilator. Let W be a subset of a vector space V. Show that W^0 is a subspace of V^*. 3

(c) Let W be a subspace of $\mathbb{R}^4$ spanned by $v_1 = (1,2,-3,4)$ and $v_2 = (0,1,4,-1)$. Find a basis of the annihilator of W.

8. (a) Define inner product. Verify that the following in an inner product in $\mathbb{R}^2$ 3
$< u, v > = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$ where $u = (x_1, x_2)$, $v = (y_1, y_2)$.

2.75

(b) Define orthogonal vector. Find a unit vector orthogonal to $v_1 = (1,1,2)$ and $v_2 = (0,1,3)$.

3

(c) Define orthonormal set. Find an orthonormal basis of the subspace W of $\mathbb{C}^3$ spanned by $v_1 = (1, i, 0)$ and $v_2 = (1, 2, 1 - i)$.

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University of Rajshahi

Department of Computer Science and Engineering
B.Sc. (Engg.) Part-2 Even Semester Examination -2017
Course: MATH2241 (Linear Algebra)
Full Marks: 52.5 Time: 3:00 hours

[N.B. Answer any six questions taking THREE from each of the groups]

Section-A

1. a) Define subspace of a vector space. If U and W be subspace of vector space V. Show that $U \cap W$ is a subspace of V. 2.5

   b) Let U and W be the subspaces of$V = \mathbb{R}^3$ defined by $U = \{(a, b, c) | a = b = c\}$ and$W = \{(0, b, c)\}$.Show that $V = U \oplus W$. 3.0

   c) Prove that $\text{L}(S)$ is the intersection of all subspaces of V containing S. 3.25

2. a) Define basis and dimension of a vector space. Determine whether or not the following form a basis for the vector space $\mathbb{R}^3$: $(1,1,1), (1,2,3)$and $(2, -1,1)$. 3.0

   b) Let U and W be the subspaces of $\mathbb{R}^4$ generated by $\{(1,1,0, -1), (1,2,3,0), (2,3,3, -1)\}$ and $\{(1,2,2, -2), (2,3,2, -3), (1,3,4, -3)\}$ respectively. Find dim(U+W). 3

   c) Let $v_1, v_2, \dots \dots, v_m$ be independent vectors and suppose $U = a_1 v_1 + a_1 v_1 + \dots + a_m u_m$ where the a_i are scalars. Show that the above representation of U is unique. 2.75

3. a) Define a linear mapping and kernel. Let f and g be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find $(gof)(4)$ and $(fog)(4)$. Also find formula for fog and gof. 3.25

   b) Define image of a linear mapping. Let $F: V \to U$ be a linear mapping. Show that image of F is a subspace of U. 3

   c) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find a basis and the dimension of *(i)*Image of T and *(ii)* kernel of T. 2.5

4. a) For any operator$S, T \in A(V)$. Show that $[ST]_e = [S]_e[T]_e$ where dim $V = 2$. 3.25

   b) Find the matrix representation of the linear operator T on $\mathbb{R}^3$ relative to the usual basis $\{e_1 = (1,0,0), e_2 = (0,1,0)ande_3 = (0,0,1)\}$, where $T(x, y, z) = (2y + z, z - 4y, 3x)$. 3

   c) Find the trace of T on $\mathbb{R}^3$, where $T(x, y, z) = (x, y + 2z, 3x + 4y - 5z)$.

Section-B

5. a) Define eigenvalue and eigenvectors of a matrix A. Let $A\begin{pmatrix}1 & 4\\2 & 3\end{pmatrix}$ be a matrix. Find all eigenvalues and eigenvectors of A. 3.25

b) Find the minimum polynomial of the matrix $A = \begin{pmatrix}2 & 1 & 0 & 0\\0 & 2 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 5\end{pmatrix}$ 3.00

c) Suppose λ is an eigenvalue of an invertible operator$T$. Show that λ^{-1} is an eigenvalue of T^{-1}. 2.50

6. a) Let $F: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear mapping defined by $F(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$. Find the matrix of F in the following bases of $\mathbb{R}^3$ and $\mathbb{R}^2$: $\{f_1 = (1,1,1), f_2 = (1,1,0), f_3 = (1,0,0)\}$ 3.00

$\{g_1 = (1,3), g_2 = (2,5)\}$

b) Show that similar matrices have the same eigenvalues. 3.00

c) Find all eigenvalues and a basis of each eigenspace of the operator $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (2x + y, y - z, 2y + 4z)$ 2.75

7. a) Define linear functional and dual space. Consider the following basis of $\mathbb{R}^3$: $\{v_1 = (1,-1,3), v_2 = (0,1,-1), v_3 = (0,3,-2)\}$. Find the dual basis $\{\emptyset_1, \emptyset_2, \emptyset_3\}$. 4.75

b) Define annihilator of a set W of a vector space V. Suppose V has a finite dimension and W is a subspace of V, show that $\dim W + \dim W^o = \dim V$. 4

8. a) Define an inner product space. Give an example of it. 2.00

b) Let $\{v_1 = (1,1,1), v_2 = (0,1,1), v_3 = (0,0,1)\}$ be the basis of $\mathbb{R}^3$. Applying Gram-Schmidt orthogonalization process, find the orthogonal basis of $\mathbb{R}^3$. 4.75

c) Prove $|(u, v)| = ||u||\ ||v||$ 2.00

4.25
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University of Rajshahi
Department of Computer Science and Engineering
B.Sc. (Engg.) Part-II (Even Semester) Examination-2016
Course: MATH2241 (Linear Algebra)

Full Marks: 52.5 **Time: 3 Hours**

[Answer any **Six (06)** questions taking **Three (03)** questions from each part]

Part-A

1. (a) Define a subspace of a vector space. Let $V = \mathbb{R}^3$, and let $W = \{(x, y, z): x + y + z = 0\}$. Prove that W is a subspace of V. 2.5

   (b) Define a vector space. Prove that vector space V is the direct sum of its subspace U and W, if and only if (i) $V = U + W$ and (ii) $U \cap W = \{0\}$. 3.25

   (c) Write the matrix $E = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}$ as a linear combination of $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$, and $C = \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}$. 3

2. (a) Let u, v, and w be linearly independent vectors. Prove that $u + v$, $u - v$, and $u - 2v + w$ are also linearly independent. 2

   (b) Suppose U and W are finite dimensional subspaces of a vector space V. Prove that $dim(U + W) = dim(U) + dim(W) - dim(U \cap W)$. 3.75

   (c) Find the coordinate vectors of $v = (3,1,-4)$ relative to the basis 2.75

   $f_1 = (1,1,1)$, $f_2 = (0,1,1)$, and $f_3 = (0,0,1)$.

3. (a) Define: a linear mapping; Kernel and Image of a linear mapping. Suppose $F: V \to U$ is a linear mapping. Prove that (i) the image of F is a subspace of U, and (ii) the kernel of F is a subspace of V. 3

   (b) Find $T(a, b, c)$ where $T: \mathbb{R}^3 - \mathbb{R}$ is defined by $T(1,1,1) = 3$, $T(0,1,-2) = 1$, and $T(0,0,1) = -2$. 3

   (c) Show that the mapping $F: \mathbb{R}^2 - \mathbb{R}^2$ defined by $F(x, y) = (x + y, x)$ is linear. 2.75

4. (a) Consider the bases $\{e_1 = (1,0), e_2 = (0,1)\}$ and $\{f_1 = (1,3), f_2 = (2,5)\}$ of $\mathbb{R}^2$.

   i. Find the transition matrix P from $\{e_i\}$ to $\{f_i\}$. 2.25

   ii. Show that $[T]_f = P^{-1}[T]_e P$ for the linear operator on $\mathbb{R}^2$ defined by $T(x, y) = (2y, 3x - y)$. 3

   (b) Let T be the linear operator on $\mathbb{R}^2$ defined by $T(x, y) = (4x - 2y, 2x + y)$. Verify that $[T]_f[v]_f = [T(v)]_f$ for any vector $v \in \mathbb{R}^2$. 3.5

Part-B

5. (a) Define characteristic equation. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 6 & 16 \\ -1 & -4 \end{bmatrix}$. 3

(b) If $v_1, v_2, \dots \dots, v_n$ be nonzero eigenvectors of an operator T belonging to distinct eigenvalues $\lambda_1, \lambda_2, \dots \dots, \lambda_n$, then prove that $v_1, v_2, \dots \dots, v_n$ are linearly independent. 3

(c) If A and B are n-square matrices, then show that AB and BA have same eigenvalues. 2.75

6. (a) Define minimal polynomial. Find the minimal polynomial of $A = \begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{bmatrix}$. 3

(b) Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be eigenvectors belonging to distinct eigenvalues λ_1 and λ_2 of a symmetric matrix A. Prove that $\boldsymbol{x}$ and $\boldsymbol{y}$ are orthogonal. 2.75

(c) State Cayley-Hamilton theorem. Using Cayley-Hamilton theorem, find the inverse of the matrix $A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$. 3

7. (a) Find a basis of the annihilators W^0 of the subspace W of $\mathbb{R}^4$ spanned by $v_1 = (1, 2, -3, 4)$ and $v_2 = (0, 1, 4, -1)$. 3

(b) Consider the basis $\{v_1 = (2, 1), v_2 = (3, 1)\}$ of $\mathbb{R}^2$. Find the dual basis $\{\phi_1, \phi_2\}$. 3

(c) Suppose V has finite dimension. Show that if $v \in V, v \neq 0$, then there exists a $\varphi \in V^*$ such that $\varphi(v) \neq 0$. 2.75

8. (a) Define an inner product space. State and prove Cauchy Schwartz inequality in an inner product space. 4.5

(b) Use Gram-Schmidt orthogonalization process to transform the basis $\{v_1 = (1,1,1), v_2 = (0,1,1), v_3 = (0,0,1)\}$ of $\mathbb{R}^3$ into an orthonormal basis $\{u_i\}$. 4.25

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University of Rajshahi

Department of Computer Science and Engineering

B.Sc. (Engg.) Part-II (Even Semester) Examination-2015

MATH2241 (Linear Algebra)

Full Marks: 52.5 **Time:** 3 Hours

[Answer any **six (06)** questions taking **three** questions from each part]

Part-A

1. (a) Let **U** and **W** be the subspaces of $\mathbb{R}^3$ defined by $U = \{(a,b,c)|a = b = c\}$ and $W = \{(0,b,c)\}$. Show that $\mathbb{R}^3 = \mathbf{U} \oplus \mathbf{W}$. 2.75
   (b) Let $V = \{f|f: \mathbb{R} \to \mathbb{R}\}$ be the vector space. Show that **W** is a subspace of **V** where W consists of all even functions. 3
   (c) Show that (1,1,1), (0,1,1) and (0,1,−1) generate $\mathbb{R}^3$. 3

2. (a) Let $V = \{at^2 + bt + c|a,b,c \in \mathbb{R}\}$ be the vector space. The polynomials $e_1 = 1$, $e_2 = t - 1$ and $e_3 = (t-1)^2$ form a basis for **V**. Let $v = 2t^2 - 5t + 6$. Find $[v]_e$. 3
   (b) Let $\{e_i\}$ and $\{f_i\}, i = 1,2$ be the bases of a vector space **V**. Suppose $f_1 = a_1e_1 + a_2e_2$ and $f_2 = b_1e_1 + b_2e_2$. Let **P** be the matrix whose rows are the coordinate vectors of f_1 and f_2 respectively relative to the basis $\{e_i\}$: $P = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}$. Show that for any vector $v \in \mathbf{V}$, $[v]_f\mathbf{P} = [v]_e$. 3
   (c) Let $\mathbf{W} = \{(a,b,c,d)|a = d, b = 2c\}$ be subspace of $\mathbb{R}^4$. Find a basis and dimension of **W**. 2.75

3. (a) Suppose $\mathrm{F}: \mathrm{V} \to \mathrm{U}$ and $\mathrm{G}: \mathrm{V} \to \mathrm{U}$ are linear mappings over a field K. The mappings $\mathrm{F} + \mathrm{G}: \mathrm{V} \to \mathrm{U}$ and $\mathrm{KF}: \mathrm{V} \to \mathrm{U}$ be defined by $(\mathrm{F}+\mathrm{G})(\mathrm{u}) = \mathrm{F}(\mathrm{v}) + \mathrm{G}(\mathrm{v})$ and $(\mathrm{kF})(\mathrm{v}) = \mathrm{kF}(\mathrm{v})$, $\mathrm{k} \in \mathrm{K}$. Show that $(\mathrm{F}+\mathrm{G})$ and (kF) are linear. 3
   (b) Let $\mathrm{T}: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\mathrm{T}(3,1) = (2,-4)$ and $\mathrm{T}(1,1) = (0,2)$. Find $\mathrm{T}(7,4)$. 3
   (c) Let $\mathrm{F}: \mathbb{R}^3 \to \mathbb{R}^2$, $\mathrm{G}: \mathbb{R}^3 \to \mathbb{R}^2$ and $\mathrm{H}: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $\mathrm{F}(x,y,z) = (x+y+z, x+y)$, $\mathrm{G}(x,y,z) = (2x+z, x+y)$ and $\mathrm{H}(x,y,z) = (2y, x)$. Show that $\mathrm{F}, \mathrm{G}, \mathrm{H} \in (\mathbb{R}^3, \mathbb{R}^2)$ are linearly independent. 2.75

4. (a) For any operator $\mathrm{S}, \mathrm{T} \in \mathrm{A}(\mathrm{V})$ show that $[\mathrm{ST}]_e = [\mathrm{S}]_e[\mathrm{T}]_e$ where $dim\ \mathrm{V} = 2$. 2.75
   (b) Find the matrix representation of the linear operator T on $\mathbb{R}^3$ relative to the usual basis $\{e_1 = (1,0,0),\ e_2 = (0,1,0)$ and $e_3 = (0,0,1)\}$ where $\mathrm{T}(x,y,z) = (2y + z, z - 4y, 3x)$. 3
   (c) Show that all the matrices similar to an invertible matrix are invertible. 3

Part-B

5. (a) Define eigenvalue and eigenvectors of a matrix **A**. Let $\mathbf{A} = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ be a matrix. Find all eigenvalues and eigenvectors of **A**. 3
   (b) Let $\mathbf{A} = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ be a matrix. Find an invertible matrix **P** such that $\mathbf{P^{-1}AP}$ is diagonal. 2.75

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(c) Find all eigenvalues and a basis of each eigenspace of the operator $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (2x + y, y - z, 2y + 4z)$. 3

6. (a) State Cayley-Hamilton Theorem. Verify Cayley-Hamilton Theorem for the matrix $A = \begin{pmatrix} 1 & 4 & -3 \\ 0 & 3 & 1 \\ 0 & 2 & -1 \end{pmatrix}$. 3

(b) Show that a matrix $\mathbf{A}$ and its transpose $\mathbf{A}'$ have the same characteristics polynomial. 2.75

(c) Find the minimum polynomial $m(t)$ of $A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}$. 3

7. (a) Find a polynomial for which the matrix $\mathbf{A} = \begin{pmatrix} 2 & 3 & -2 \\ 0 & 5 & 4 \\ 1 & 0 & -1 \end{pmatrix}$ is a root and verify. 3

(b) Suppose λ is an eigenvalue of an invertible operator T. Show that λ^{-1} is an eigenvalue of T^{-1}. 2.75

(c) Let $\mathbf{A} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$. Show that $\mathbf{A}$ and $\mathbf{B}$ have different characteristic polynomial. 3

8. (a) Define $\mathbf{W}^{+}$. Show that $\mathbf{W}^{+}$is a subspace of a vector space $\mathbf{V}$. 3

(b) Let $\{v_1 = (1, 1, 1), v_2 = (0, 1, 1), v_3 = (0, 0, 1)\}$ be the basis of $\mathbb{R}^3$. Find the orthogonal basis of $\mathbb{R}^3$. 3

(c) Find an orthogonal transformation of co-ordinates which diagonalizes the quadratic form $q(x, y) = 2x^2 - 4xy + 4y^2$. 2.75

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University of Rajshahi

Department of Computer Science & Engineering

2nd Year 2nd Semester Examination 2014

Course: MATH-2241 (Linear Algebra)

Full Marks: 52.5 Duration: 3(Three) Hours

Answer 06(Six) questions taking any 03(Three) questions from each part

Part-A

1. a) Define a vector space. Let V be the set of ordered pairs of real numbers. Show that V is not a vector space over $\mathbb{R}$ with addition in V and scalar multiplication on V defined by $(a,b) + (c,d) = (a+c, b+d)$ and $k(a,b) = (a,b)$ 2.75
   b) Define linear combination of vectors. Write u as a linear combination of polynomials $v = 2t^2 + 3t - 4$ and $w = t^2 - 2t - 3$, where $u = 4t^2 - 6t - 1$ 3
   c) Define linear span. Show that the xy-plane in $\mathbb{R}^3$ is generated by $u = (2,-1,0)$ and $v = (1,3,0)$. 3

2. a) Define basis and dimension of a vector space. Determine whether or not the following form a basis for the vector space $\mathbb{R}^3$: (1,1,2), (1,2,5) and (5,3,4). 3
   b) Define linear independent vectors of a vector space. Determine whether the vectors (1, -2,4,1), (2,1,0, -3), (3, -6,1,4) in $\mathbb{R}^4$ are linearly dependent or independent. 3
   c) Let V be the vector space of 2×2 symmetric matrices over K. Show that dim$V$=3. 2.75

3. a) Define *kerF* and *ImF* of a linear mapping $F: V \to U$. Show that ImF is a subspace of U. 3
   b) Determine whether or not the mapping F is linear, where $F: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $F(x,y,z) = (x+1, y+z)$. 3
   c) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by $T(x,y,z) = (x+2y-z, y+z, x+y-2z)$. Find a basis of the dimension of the kernel of T. 2.75

4. a) For any operators $S, T \in A(V)$, show that $[T+S]_e = [T]_e + [S]_e$, where dim$V$=3. 3
   b) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x,y) = (2x-3y, x+4y)$. Find $[T]_f^g$ and $[T]_g^f$, where $\{g_1 = (1,0), g_2 = (0,1)\}$ and $\{f_1 = (1,3), f_2 = (2,5)\}$ are the bases of $\mathbb{R}^2$. 3
   c) Define matrix representation of T relative to the bases $\{f_i\}$ and $\{g_i\}$. Find the transition matrix from the bases $\{e_1 = (1,0), e_2 = (0,1)\}$ and $\{f_1 = (1,1), f_2 = (-1,0)\}$. 2.75

Part-B

5. a) Define eigenvalues and eigenvectors of a matrix A. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ be a matrix. Show that A is diagonalizable matrix. 3
   b) State Cayley-Hamilton theorem. Verify the Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & 5 \\ 1 & -3 \end{pmatrix}$. 3
   c) Find the minimum polynomial $m(t)$ of $A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}$. 2.75

6. a) Define linear functional on a vector space V. Further, define dual space and dual basis of V. 3
   b) Let $\{v_1 = (2,1), v_2 = (3,1)\}$ be the basis of $\mathbb{R}^2$. Find the dual basis $\{\phi_1, \phi_2\}$ of $\mathbb{R}^2$. 2.75
   c) Define an annihilator. Let W be a subset of a vector space V. Let V^* be the dual space of V and W^0 be the annihilator of W. Prove that W^0 is a subspace of V^*. 3

7. a) Let W be the subspace of $\mathbb{R}^4$ spanned by $v_1 = (1,2,-3,4)$ and $v_2 = (0,1,4,-1)$. Find the basis of the annihilator of W. 2.75
   b) Define inner product space and normed space. Establish the relation $\|u\| = \sqrt{\langle u, u \rangle}$. 3
   c) For any vectors $u, v \in V$, prove that $|\langle u, v \rangle| \le \|u\| \|v\|$. 3

8. a) Define orthogonal vectors and orthogonal complement of a subset of a vector space. Let W be a subset of a vector space V. Define $W^{\perp} = \{v \in V: \langle v, w \rangle = 0, \forall w \in W\}$. Then show that $W^{\perp}$ is a subspace of V.

b) Define orthonormal set of vectors. Let $\{e_1 = (1,0,0),\ e_2 = (0,1,0),\ e_3 = (0,0,1)\}$ be the usual basis of $\mathbb{R}^3$. Show that $\{e_1, e_2, e_3\}$ is an orthonormal basis of $\mathbb{R}^3$. 2.75

c) Let $\{v_1 = (1,1,1),\ v_2 = (0,1,1),\ v_3 = (0,0,1)\}$ be the basis of $\mathbb{R}^3$. Applying Gran-Schmidt orthogonalization process, find the orthonormal basis of $\mathbb{R}^3$. 3

Time:

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University of Rajshahi
Department of Computer Science and Engineering
B. Sc. (Engg.) Part-II Even Semester Examination 2013
Course: ~~CSE~~ MATH 2241 (Linear Algebra) MATH2241
Full Marks: 52.5 **Time:** 3 Hours
[Answer any six questions taking any 3 from each Group]

Group A

1. a) Define a Subspace of a Vector space. — 3.75
Let $V = \mathbb{R}^3$. Show that $W = \{(a,b,c); a+b+c=0\}$ is a subspace of V.
b) Let $V = \mathbb{R}^3$. Show that $W = \{(a,b,c); a^2+b^2+c^2 \leq 1\}$ is not a subspace of V. — 2.5
c) Let U and W be subspaces of a vector space V. Prove that $U \cap W$ is a subspace of V. — 2.5

2. a) Define sum of two Subspaces of a Vector space. Let U and W be subspaces of a vector space V. Prove that U+W is a subspace of V. — 3
b) Prove that the vector space V is the direct sum of its subspaces U and W if and only if — 3
(i) $V = U+W$ and (ii) $U \cap W = \{0\}$.
c) Let V be the vector space of n-square matrices over the field IR. Let U and W be the Subspaces of Symmetric and Antisymmetric matrices respectively. Show that $V = U \oplus W$. — 2.75

3. a) Find a basis and dimension of the subspace W of $\mathbb{R}^3$, where — 5
(i) $W = \{(a, b, c) : a+b+c = 0\}$
(ii) $W = \{(a, b, c) : a=b=c\}$
b) Define direct sums. Show that the vector space $V=\mathbb{R}^3$ is not the direct sum of the subspaces $U = \{(a, b, 0) : a, b \in \mathbb{R}\}$ and $W = \{(0, b, c) : b, c \in \mathbb{R}\}$ — 3.75

4. a) Define linear mapping. Show that every linear mapping takes the zero vector into the zero vector. — 2.75
b) Determine whether the following are linear mapping or not: — 3
(i) $F(x, y) = (x+y, x)$
(ii) $F(x, y) = (x+1, y)$
Where $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$
c) Let $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear mapping for $F(1,0) = (2,3)$ and $F(0,1) = (1,4)$. Find a formula for F; that is, find F(a,b). — 3

Group B

5. a) Let V be of finite dimension, and let $F:V \rightarrow U$ be a linear mapping with image U' and kernel W. Prove that: — 3
$$\dim U' + \dim W = \dim V$$
b) Let V be a vector space of 2x2 matrices over IR and let $M = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$. — 2.75

Let $F: V \to V$ be the linear map defined by $F(A) = AM - MA$. Find a basis and dimension of the Kernel W of F.

c) Let $\{e_1, e_2, \ldots, e_n\}$ be a basis of V. 3

For any operators $S, T \in A(V)$, prove that $[ST]_e = [S]_e [T]_e$

6. a) Define matrix representation of a Linear Operator. Let $F: IR^2 \to IR^2$ be the linear operator defined by $F(x,y) = (2x+3y, 4x-5y)$. 3.75

Find the matrix representation of F relative to the basis

$$S = \{u_1, u_2\} = \{(1,2), (2,5)\}$$

b) Consider the following basis of IR^2 5

$$E = \{e_1, e_2\} = \{(1,0), (0,1)\}$$
$$S = \{u_1, u_2\} = \{(1,3), (1,4)\}$$

(i) Find the change-of-basis matrix P from the basis E to S.

(ii) Find the change-of-basis matrix Q from the basis S to E.

(iii) Find the co-ordinate of $V=(5, -3)$ relative to S.

7.

a) Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$. Find all eigenvalues of A and the corresponding eigenvectors. 3

Find an invertible matrix P such that $P^{-1}AP$ is diagonal.

b) Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. Find an orthogonal matrix P such that $P^{-1}AP$ is diagonal. 3

c) Show that similar matrices have the same eigenvalues. 2.75

8. a) Define an inner product space. Let A be a real nxn positive definite matrix. 4

Prove that the function $\langle u,v \rangle = u^t Av$ is an inner product on iR^n.

b) Prove that the symmetric matrix $A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$ is positive definite if and only if $a>0$, 2.75

$d>0$ and $|A| = ad - b^2 > 0$. 2

c) Let $u = (x_1, x_2)$ and $v = (y_1, y_2)$ belong to IR^2. Verify that the function

$$f(u,v) = x_1y_1 - 2x_1y_2 - 2x_2y_1 + 5x_2y_2$$

defines an inner product on IR^2.

University of Rajshahi
Department of Computer Science and Engineering
B.Sc(Engg) 2^{nd} year 2^{nd} semister, 2012

S

Course- MATH2241 (Linear Algebra)

Time: 4 Hours **Marks :52.5**

(Answer **SIX** questions taking any **THREE** from each group.) $8\frac{3}{4}\times 6 = 52\frac{1}{2}$

Part-A

1. (a) Define subspace of a vector space V. Let V be a vector space over a field K. Then prove that a subset W of V is a subspace of V if and only if $\alpha u + \beta v \in W$, for all u, v $\in$ V, α, $\beta \in$ K. $4\frac{3}{4}$

   (b) What do you mean by a linear combination of vectors? Express (1, -2, 5) as a linear combination of vectors (1, 1, 1), (1, 2, 3) and (-1, -5, 1) in $\Re^3$. 4

2. (a) Define linearly dependent set of vectors. Show that the set of vectors (1, -2, 1), (2, 1, -1) and (7, -4, 1) in $\Re^3$ are linearly dependent. 3

   (b) Define basis of a vector space. Show that the set of vectors {(1, 1, 2), (1, 2, 5), (5, 3, 4)} is not a basis for the vector space $\Re^3$. 3

   (c) Let $v_1, v_2, \ldots\ldots, v_m$ be independent vectors and suppose $u = a_1v_1 + a_2v_2 + \text{------} + a_mv_m$, where the a_i are scalars. Show that the above representation of u is unique. $2\frac{3}{4}$

3. (a) Define linear mapping. Show that the mapping $F:\Re^2 \to \Re^2$ defined by F(x,y) = (x+y, x) is linear. $2\frac{3}{4}$

   (b) Define kernel of a linear mapping. Let $T:\Re^3 \to \Re^3$ be the linear mapping defined by T(x,y,z) = (x+2y-z, y+z, x+y-2z). Find a basis and the dimension of kernel of T. 3

   (c) Define image of a linear mapping. Let F: V$\to$ U be a linear mapping, then show that the image of F is a subspace of U. 3

4. (a) Define a linear operator and then define its matrix representation. Let { $e_1, e_2, \ldots\ldots, e_n$ } be a basis of V and let T be any operator on V. Then prove that $[T]_e[v]_e = [T(v)]_e$ for any vector $v \,\varepsilon$ V. $4\frac{3}{4}$

   (b) Let T be the linear operator on $\Re^3$ defined by $T(x,y,z) = (2y+z, x-4y, 3x)$. Find the matrix representation of T in the basis { $f_1 = (1,1,1)$, $f_2 = (1,1,0)$, $f_3 = (1,0,0)$ } . 4

Marks: 52

Part-B

5. (a) Define a transition matrix. Consider the following basis of $\Re^2$: $\{e_1 = (1,0), e_2 = (0,1)\}$ and $\{f_1 = (1,2), f_2 = (2,3)\}$. Find the transition matrix P from $\{e_i\}$ to $\{f_i\}$ and then verify that $[v]_e = P[v]_f$ for any vector $v \; \varepsilon \; \Re^2$. 4

(b) Define the followings:
Similar matrices, Trace of a matrix A and Diagonalizable operator T. $1\frac{3}{4}$

(c) Let V be a vector space of 2×2 matrices over $\Re$ and let $M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Let T be linear operator on V defined by T(A)=MA, where A ε V. Find the trace of T. 3

6. (a) Show that every matrix is a zero of its characteristic polynomial. 4

(b) Find the minimum polynomial m(t) of $4\frac{3}{4}$

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}$$

7. (a) Define a linear functional on a vector space V and then define the dual space of V. Also define the dual basis of V^*. Consider the following basis of $\Re^3$: $\{ v_1 = (1, -1, 3), v_2 = (0, 1, -1), v_3 = (0, 3, -2) \}$. Find the dual basis $\{\phi_1, \phi_2, \phi_3\}$. 4

(b) Let V be the vector space of polynomials over $\Re$ of degree ≤ 2. Let ϕ_1, ϕ_2 and ϕ_3 be the linear functionals on V defined by $\phi_1(f(t)) = \int_0^1 f(t)dt$, $\phi_2(f(t)) = f'(1)$, $\phi_3(f(t)) = f(0)$ where $f(t) = a + bt + ct^2 \; \varepsilon$ V and $f'(t)$ denotes the derivative of $f(t)$. Find the basis $\{ f_1(t), f_2(t), f_3(t)\}$ of V which is dual to $\{\phi_1, \phi_2, \phi_3\}$. $2\frac{3}{4}$

(c) Define an annihilator of a subset W of a vector space V. Let W^0 be the set of all annihilators of W. Then prove that W^0 is a subspace of V^*. 2

8. (a) Define an inner product. Space V over K. Verify that the following is an inner product in $\Re^2$: $\langle u,v\rangle = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$, where $u = (x_1, x_2)$, $v = (y_1, y_2)$. 4

(b) Define orthogonal vectors and orthogonal complements. Let W be a subset of a vector space V. Show that the orthogonal complements, denoted by $W^{\perp}$, is a subspace of V. $2\frac{3}{4}$

(c) Express the norm of a vector u in terms of its inner product. Find the norm of $v = (3,4) \; \varepsilon \; \Re^2$ with respect to : 2

(i) The usual inner product;
(ii) The inner product in $v = (3,4)$ by the relation $\langle u,v\rangle = x_1y_1 - x_2y_1 - x_1y_2 + 3x_2y_2$ as $u = (x_1, x_2)$, $v = (y_1, y_2)$.