Inferences on Variances – Homework;

1. The following summary data on bending strength (lb-in/in) of joints is taken from the article Bending Strength of Corner Joints Constructed with Injection Molded Splines." A Scientist would like to know if there are any differences in the variability of bending strength based on coding. Assume that each sample came from a normal distribution. The results from this study are given below:

Type	Sample Size	Sample Mean	Sample Std Dev
Without Coding	**10**	**80.95**	**9.59**
With Coding	**10**	**63.23**	**5.96**

a. List the Null and Alternative Hypotheses
b. Test this with a hypothesis test at =0.05
c. Give the real-world answer
d. Check this hypothesis with a CI. Does the answer change? Explain

Solution:

a. List the Null and Alternative Hypotheses

Assumption
$n = m = 10$, S1= 9.59, S2=5.96
$S_1^2 = (9.59)^2 = 91.96$
$S_2^2 = (5.96)^2 = 35.52$
$\alpha = 0.05$

$$H_a = \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

Work :
The null Hypothesis is :

$$H_0 = \frac{\sigma_1^2}{\sigma_2^2} = 1$$

The Alternate Hypothesis is :

$$H_a = \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

b. Test this with a hypothesis test at =0.05

The formula for F-test statistic is:

$$F_{stat} = \frac{S_1^2}{S_2^2}\frac{\sigma_1^2}{\sigma_2^2}$$

Using the data from the statistic, the test statistic becomes,

$$F_{stat} = \frac{91.96 * 1}{35.52}$$

$$Fstat \ = 2.589$$

For the p-value approach, the p-value is:

$$p - value \ = \ 2 \ * \ min\,(\,P\,(\,F \ \geq F_{stat}\,)\,,\,P(\,F \ \leq F_{stat}\,)\,)$$

$$p - value \ = \ 2 \ * \ min\,(\,1 \ - \ P\,(\,F \ \leq F_{stat}\,)\,,\,P(\,F \ \leq F_{stat}\,)\,)$$

$$p - value \ = \ 2 \ * \ min\,(\,1 \ - \ P\,(\,F \ \leq \ 2.589\,)\,,\,P(\,F \ \leq \ 2.589\,)\,)$$

$$p - value \ = \ 2 \ * \ min\,(\,1 \ - \ 0.91365, \ 0.91365)$$

$$p - value \ = \ 2 \ * \ min\,(0.0863, \ 0.91365)$$

$$p - value \ = \ 2 \ * (0.0863)$$

$$p - value \ = \ 0.1726$$

Decision Rule Based on p-value.
Reject H_0 : p-value $\leq \alpha$
Fail to Reject H_0 : $p - value \ > \ \alpha$

Since 0.1726 > 0.05, we failed to reject the null hypothesis.

c. Give the real-world answer
According to the above calculation, we failed to reject the null hypothesis. There is no evidence to support alternative hypotheses.
I conclude that there is evidence to show that there are any differences in the variability of bending strength based on coding.

d. Check this hypothesis with a CI. Does the answer change? Explain

S1= 9.59, S2=5.96

We will find the interval as:

LB $\leq \frac{\sigma_1^2}{\sigma_2^2} \leq$ UB

Where LB is the lower bound and UB is the upper bound.
The formula for a confidence interval for a ratio of variances is:

$$F_{1-\alpha, v_1, v_2} < \frac{s_1^2}{s_2^2} \frac{\sigma_2^2}{\sigma_1^2} < F_{\alpha/2, v_1, v_2}$$

$$\frac{s_1^2}{s_2^2 * F_{1-(\alpha/2), df_1, df_2}} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{s_1^2}{s_2^2 * F_{(\alpha/2), df_1, df_2}}$$

Now determining the critical values,

$P_L = \alpha/2 = 0.05/2 = 0.025$
$P_L = 0.025$

$P_U = 1-(\alpha/2) = 1-(0.025) = 0.975$

Where, α= 1- confidence level
Confidence level = 1- α

Determining the degree of freedom. For the F-test, we have a degree of freedom for numerator df_1 and denominator df_2.

Now $df_1 = N_1-1=10-1=9$
$df_2 = N_2-1=10-1=9$

Determining the critical value from F-table on F-table tab, The critical value is:
$F_L = F_{0.025,9,9} = 0.2484$
$F_H = F_{0.975,9,9} = 4.026$

Using the data from above, LB becomes,

$$LB = \frac{s_1^2}{s_2^2 * F_H}$$

$$LB = \frac{91.96}{35.52 * 4.026} = 0.64$$

UB becomes,

UB= $\frac{91.96}{35.52 * 0.2484} = 10.4226$

Now combining the LB and UB to get 95% confidence interval:

$$0.64 \leq \frac{\sigma_1^{\ 2}}{\sigma_2^{\ 2}} \leq 10.4226$$

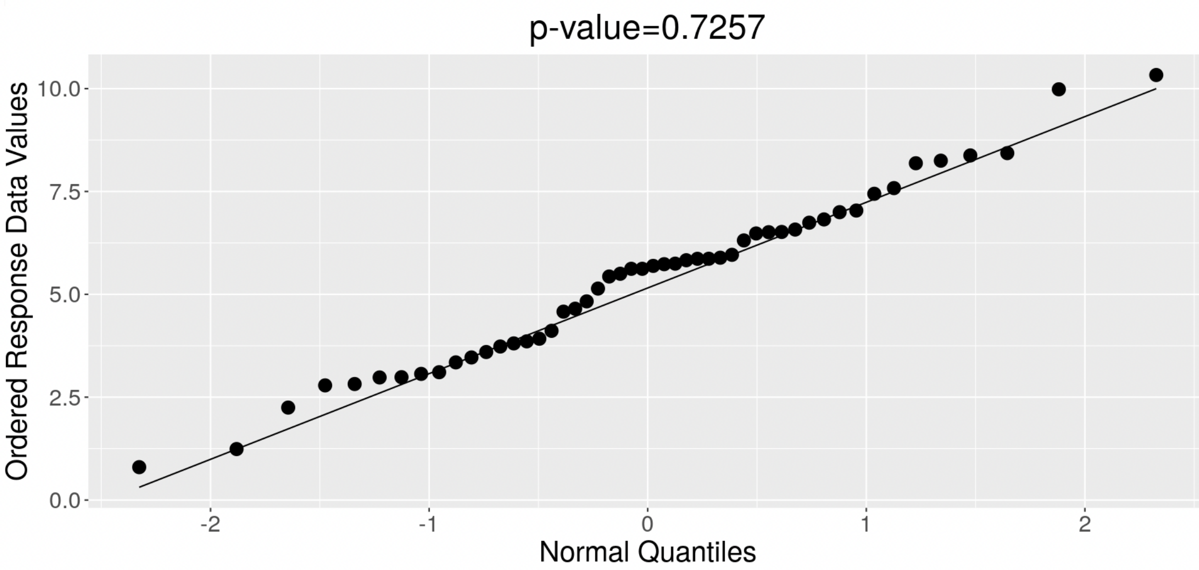
From above calculation, we can conclude that we are 95 % confident that true variance lies in between 0.64 to 10.4226

There is evidence to conclude that there is a difference between the variability of bending strength based on coding.
Answers do not change in part 1c and part 1d, because values lie between 0.0959 and 10.4226 which is greater than 0.05.

2. An engineer is doing a study in a manufacturing setting on the lengths of paperclips. The sample he collected is in the dataset paperclips.xls. The engineer is concerned that the variance is too large. The variance should be no bigger than 5.
a. Check to see if the data comes from the normal distribution
b. List the null and alternative hypotheses
c. Test this with a hypothesis test at =0.05
d. Give the real-world answer
e. Check this hypothesis with a confidence interval. Does the answer change? Explain

Solution:

a. Check to see if the data comes from the normal distribution



This means that we should assume that the given data comes from a normal distribution.
This reflects in the normal probability graph as well.
The plot has points that closely follows line
Since the p-value is greater than 0.05, we conclude that the data comes from a normal distribution.

b. List the null and alternative hypotheses

The null Hypothesis is :

$H_0 = \sigma^2 = 5$

The Alternate Hypothesis is :

$H_a = \sigma^2 \neq 5$

c. Test this with a hypothesis test at =0.05
n=50, variance=4.353

The formula for the chi-square test statistic is :

$$X^2_{stat} = \frac{(n-1)s^2}{\sigma_0^2}$$

$$= \frac{(50-1) * 4.353}{5}$$

$$X^2_{stat} = 42.6594$$

$= \frac{49 * 4.353}{5}$

$X^2_{stat} = 42.6594$

For the p-value approach:

P-value approach: $P(\chi^2 \geq \chi^2_{stat})$

P-value approach: 1 - $P(\chi^2 \leq \chi^2_{stat})$

$p - value = 1 - P(\chi^2 \leq 42.6594)$

$p - value = 1 - 0.27$

$p - value = 0.73$

Decision rule based on p-value:

Reject H_0 : $p - value < \alpha$

Fail to Reject H_0 : $p - value > \alpha$

$\alpha = 0.05;\ p - value = 0.73;\ p - value > \alpha$

As p-value is greater than α

We fail to reject H_0

d. Give the real-world answer

We failed to reject the null hypothesis. There is not enough evidence to support the claim of alternative hypotheses.

Hence, the engineer's concern of variance being too large is true.

e. Check this hypothesis with a confidence interval. Does the answer change? Explain

To calculate CI,

We will find the interval as $LB \leq \sigma^2 \leq \infty$, LB is lower bound

Setting upper value to infinity since there's no limit for upper bound.

The formula for lower bound LB is :

$$LB = \frac{(n-1)s^2}{\chi^2_{1-\alpha,\, df}}$$

Finding critical point,

$p_U = 1 - \alpha = 1 - 0.05 = 0.95$

Now, degree of freedom,

$df = N - 1 = 50 - 1 = 49$

Determining critical value from chi-square table tab.

Critical value is,

$\chi^2_U = \chi^2_{p_U, df} = \chi^2_{0.95,49} = 66.339$

Putting values in below equation,

$$LB = \frac{(n-1)s^2}{\chi^2_{1-\alpha, df}}$$

$$LB = \frac{49 * 4.353}{66.339} = 3.215$$

$3.215 \le \sigma^2 \le \infty$

From the CI calculations, we are 95% confident that paper clip lengths should be at least 3.215cm.

Since the null hypothesis $H_0 = \sigma^2 = 5$ value is greater than 3.215,

we still failed to reject the null hypothesis.

Hence the answer would be the same for both cases.

3. An article reported on an experiment in which 13 computer-proficient medical professionals were timed retrieving images from a library of slides and then retrieving the same image from a computer database. The data can be found in a file called Two-Sample-Tests.

a. Determine if both samples come from normal distributions

1:Slide

Results for the Normality Test

For the Shapiro-Wilk normality test, we assume that:

H_0 : The data comes from a normal distribution

H_A : The data does not comes from a normal distribution

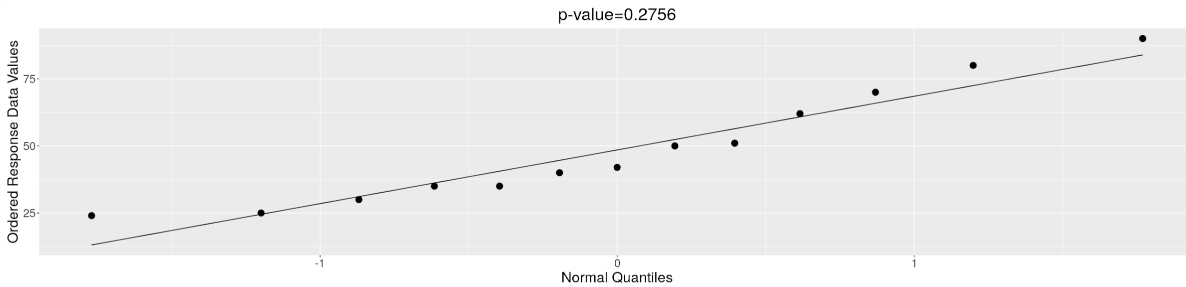
The p-value for this test is 0.2756

Since 0.2756$>$ 0.05 we fail to reject the null hypothesis.

This means that we should assume that the given data comes from a normal distribution.

This is reflected in the normal probability plot below

The plot has points that closely follow the line.



2:Digital slide

Results for the Normality Test

For the Shapiro-Wilk normality test, we assume that:

H_0 : The data comes from a normal distribution

H_A : The data does not comes from a normal distribution

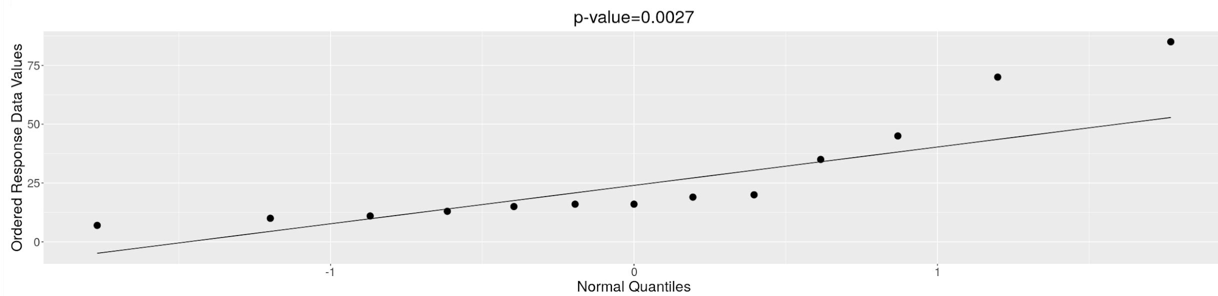
The p-value for this test is 0.0027

Since $0.0027 < 0.05$ we reject the null hypothesis.

This means that we should assume that the given data does not come from a normal distribution.

This is reflected in the normal probability plot below

The plot has points that do not follow the line close enough.



These screenshots were taken from stathelper to understand the probability graph.

b. List the Null and Alternative Hypotheses for testing equality of the variances

The null Hypothesis is :

$$H_0 = \frac{\sigma_1^2}{\sigma_2^2} = 1$$

The Alternate Hypothesis is :

$$H_a = \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

c. Test this with a hypothesis test at =0.05

The null Hypothesis is :

$$H_0 = \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$n = m = 13$$

$$slide - S_1^2 = 445.03$$

$$Digital - S_2^2 = 602.64$$

The Formula for F-test statistic is :

$$F_{stat} = \frac{S_1^2}{S_2^2}\frac{\sigma_1^2}{\sigma_2^2}$$

$$= \frac{445.03 * 1}{602.64}$$

$$= 0.7384$$

$$p - value = 2 * min\,(P\,(F \geq F_{stat}),\, P(F \leq F_{stat}))$$
$$p - value = 2 * min\,(1 - P\,(F \leq F_{stat}),\, P(F \leq F_{stat}))$$
$$p - value = 2 * min\,(1 - P\,(F \leq 0.7384),\, P(F \leq 0.7384))$$
$$p - value = 2 * min\,(0.6962,\ 0.3038)$$
$$p - value = 2 * (0.3038)$$
$$p - value = 0.6077$$

Decision Rule Based on p-value.
Reject H_0 : p-value $\leq \alpha$
Fail to Reject H_0 : $p - value > \alpha$
Since p-value is greater than α=0.05, we fail to reject the null hypothesis.

d. Give the real-world answer

We fail to reject the null hypothesis. There is not enough evidence to support alternate hypotheses. We can say that the image retrieval between slide and digital methods are the same.

e. Check this hypothesis with a CI. Does the answer change? Explain

We will find the interval as:

$$LB \leq \frac{\sigma_1^2}{\sigma_2^2} \leq UB$$

Where LB is the lower bound and UB is the upper bound.
The formula for a confidence interval for a ratio of variances is:

$$F_{1-\alpha, v_1, v_2} < \frac{s_1^2}{s_2^2}\frac{\sigma_2^2}{\sigma_1^2} < F_{\alpha/2, v_1, v_2}$$

$$\frac{s_1^{\ 2}}{s_2^{\ 2} * F_{1-(\alpha/2),\, df_1,\, df_2}} \leq \frac{\sigma_1^{\ 2}}{\sigma_2^{\ 2}} \leq \frac{s_1^{\ 2}}{s_2^{\ 2} * F_{(\alpha/2),\, df_1,\, df_2}}$$

Now determining the critical values,

P_L = α/2= 0.05/2= 0.025
P_L = 0.025

P_U = 1-(α/2)= 1-(0.025) = 0.975

Where, α= 1- confidence level
Confidence level = 1- α

Determining the degree of freedom. For the F-test, we have a degree of freedom for numerator df_1 and denominator df_2.

Now df_1 = N_1-1=13-1=12
df_2 = N_2-1=13-1=12
$p_L = 0.025;\ p_U = 0.975;\ df_1 = 12;\ df_2 = 12$
Determining the critical value from F-table on F-table tab, The critical value is:
F_L= $F_{0.025,12,12} = 0.3051$
F_H= $F_{0.975,12,12} = 3.277$

Using the data from above, LB and UB is,

Lower Bound= $\frac{s_1^{\ 2}}{s_2^{\ 2} * F_H}$

$$Lower\ bound = \frac{445.03}{602.64 * 3.277} = 0.2253$$

Upper Bound= $\frac{s_1^{\ 2}}{s_2^{\ 2} * F_L}$

$$Upper\ bound = \frac{445.03}{602.64 * 0.3051} = 2.4201$$

From the above calculations, we can conclude that we are 95% confident that true variance ratio ($\frac{\sigma_1^{\ 2}}{\sigma_2^{\ 2}}$) lies between 0.2253 and 2.4201.

4. Fusible interlinings are being used with increasing frequency to support outer fabrics and improve the shape and drape of various pieces of clothing. The accompanying data on extensibility (%) at 100g/cm for both high quality (H) and poor quality (P) is given. The data is in a file called Two-Sample-Tests. The experimenter would like to know if the extensibility is better for the high quality fabric than the low quality fabric. Equality of the variances will dictate which 2-sample t-test should be used. Check this at =0.05 and give a recommendation. Make no assumptions about the populations at the start. (Hint: You need to check everything and give the real-world result to get full credit)

Solution:

For the Shapiro-Wilk normality test, we assume that:

H_0 : The data comes from a normal distribution

H_A : The data does not comes from a normal distribution

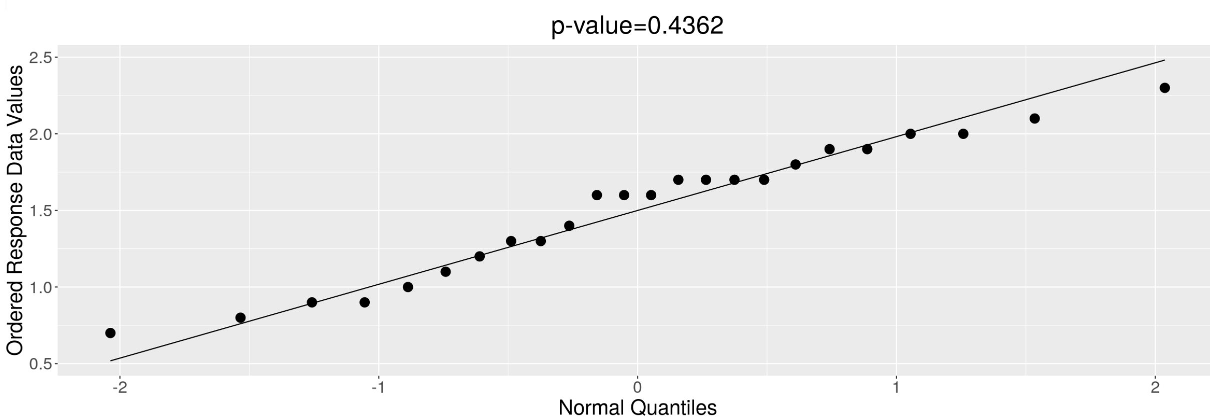
The p-value for this test is 0.4362

Since $0.4362 > 0.05$ we fail to reject the null hypothesis.

This means that we should assume that the given data comes from a normal distribution.

This is reflected in the normal probability plot below

The plot has points that closely follow the line.



Statistic	***Value***
Mean	*1.508*
Median	*1.6*
Sample Size	*24*
Q1	*1.15*
Q3	*1.85*
Min	*0.7*
Max	*2.3*
IQR	*0.7*
Range	*1.6*
Variance	*0.1973*
Standard Deviation	*0.4442*
Lower Fence	*0.1*
Upper Fence	*2.9*

Since p-value 0.4362 > 0.05 we fail to reject the null hypothesis.

The data points follow a normal distribution line.

This means that the data comes from a normal distribution.

For the Shapiro-Wilk normality test, we assume that:

H_0 : The data comes from a normal distribution

H_A : The data does not comes from a normal distribution

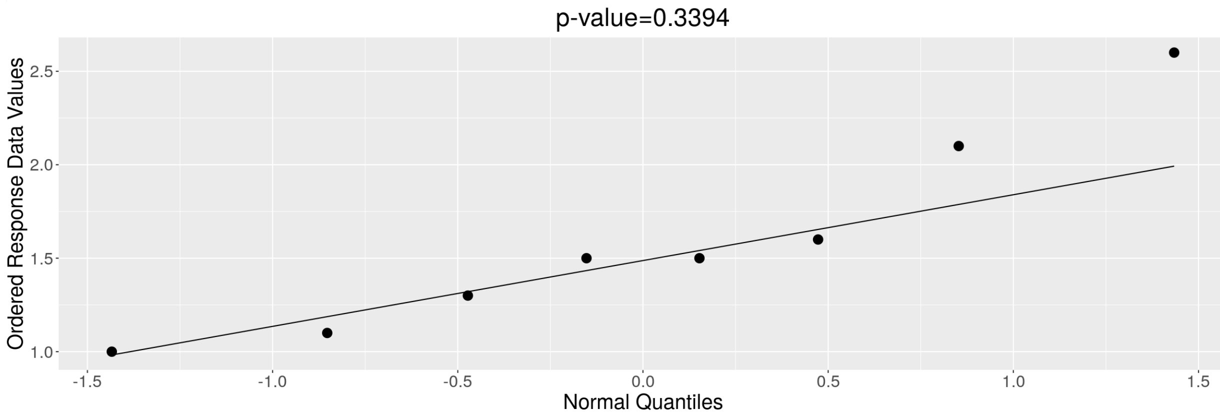
The p-value for this test is 0.3394

Since 0.3394$>$ 0.05 we fail to reject the null hypothesis.

This means that we should assume that the given data comes from a normal distribution.

This is reflected in the normal probability plot below

The plot has points that closely follow the line.



Statistic	*Value*
Mean	*1.588*
Median	*1.5*
Sample Size	*8*
Q1	*1.2*
Q3	*1.85*
Min	*1*
Max	*2.6*
IQR	*0.65*
Range	*1.6*
Variance	*0.2812*
Standard Deviation	*0.5303*
Lower Fence	*0.225*
Upper Fence	*2.825*

Since p-value 0.3394 > 0.05 we fail to reject the null hypothesis.

The data points follow a normal distribution line.
This means that the data comes from a normal distribution.

The null Hypothesis is [Assuming that the variance is same for H and P] :

$$H_0 = \frac{\sigma_1^2}{\sigma_2^2} = 1$$

The Alternate Hypothesis is [Assuming that the variance is not same for H and P] :

$$H_a = \frac{\sigma_1^2}{\sigma_2^2} \neq 1$$

$$n = 24;\ s_1^2 = 0.1973;\ m = 8;\ s_2^2 = 0.2812;\ \alpha = 0.05$$

The formula for F-test statistic is:

$$F_{stat} = \frac{S_1^2}{S_2^2}\frac{\sigma_1^2}{\sigma_2^2}$$

Using the data from the statistic, the test statistic becomes,

$$F_{stat} = \frac{0.1973 * 1}{0.2812} = 0.7015$$

$$F_{stat} = 0.7015$$

For the p-value approach, the p-value is:

$$p - value = 2 * min\,(P\,(F \geq F_{stat}),\ P(F \leq F_{stat}))$$
$$p - value = 2 * min\,(1 - P\,(F \leq F_{stat}),\ P(F \leq F_{stat}))$$
$$p - value = 2 * min\,(1 - P\,(F \leq 0.7015),\ P(F \leq 0.7015))$$
$$p - value = 2 * min\,(0.7569,\ 0.2431)$$
$$p - value = 2 * (0.2431)$$
$$p - value = 0.4862$$

Decision Rule Based on p-value.
Reject H_0 : p-value $\leq \alpha$
Fail to Reject H_0 : $p - value > \alpha$

To conclude,

$Fail\ to\ reject\ H_0 : p - value > \alpha$

Since 0.4862 i.e p-value is greater than 0.05 hence we failed to reject the null hypothesis.

We can say that, from the above calculations, there is not enough evidence to support alternative hypotheses. Hence we can say that there is not enough evidence to conclude that both qualities have differences in quality of fabric.

Confidence Interval calculation :

We will find the interval as:

$$LB \leq \frac{\sigma_1^2}{\sigma_2^2} \leq UB$$

Where LB is the lower bound and UB is the upper bound.
The formula for a confidence interval for a ratio of variances is:

$$F_{1-\alpha, v_1, v_2} < \frac{s_1^2}{s_2^2} \frac{\sigma_2^2}{\sigma_1^2} < F_{\alpha/2, v_1, v_2}$$

$$\frac{s_1^2}{s_2^2 * F_{1-(\alpha/2), df_1, df_2}} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{s_1^2}{s_2^2 * F_{(\alpha/2), df_1, df_2}}$$

Now determining the critical values,

$P_L = \alpha/2 = 0.05/2 = 0.025$
$P_L = 0.025$

$P_U = 1-(\alpha/2) = 1-(0.025) = 0.975$

Where, α= 1- confidence level
Confidence level = 1- α

Determining the degree of freedom. For the F-test, we have a degree of freedom for numerator df_1 and denominator df_2.

Now $df_1 = N_1-1=24-1=23$
$df_2 = N_2-1=8-1=7$

$p_L = 0.025;\ p_U = 0.975;\ df_1 = 23;\ df_2 = 7, s_1^2 = 0.1973;\ s_2^2 = 0.2812$
Determining the critical value from F-table on F-table tab, The critical value is:
$F_L = F_{0.025,23,7} = 0.3445$
$F_H = F_{0.975,23,7} = 4.426$

Using the data from above, LB becomes,

Lower Bound= $\frac{s_1^2}{s_2^2 * F_H}$

$Lower\ bound = \frac{0.1973}{0.2812 * 4.426} = 0.1585$

UB becomes,

Upper Bound= $\frac{s_1^2}{s_2^2 * F_L}$

$Upper\ bound = \frac{0.1973}{0.2812 * 0.3445} = 2.036$

After the confidence interval calculation, we are 95% confident that the true variance ratio ($\frac{\sigma_1^2}{\sigma_2^2}$) is between 0.1585 to 2.036 i.e LB $\le \frac{\sigma_1^2}{\sigma_2^2} \le$ UB

From the above calculation we failed to reject the null hypothesis which we can further conclude that variance would be the same for both high and poor quality of fabric.
Now that we tried to compare 2 different types of fabric quality, I think it is better to use 2 means independent t-test to get a better assumption.