Mrinal Chaudhari **Applied Statistics**

Homework 4 – Confidence Intervals on a Mean

1. A CI is desired for the true average stray-load loss (watts) for a certain type of induction motor when the line current is held at 10 amps for a speed of 1500 rpm. Assume that stray-load loss is normally distributed with sample variance = 3.0.
a. Compute a 95% CI for μ when n = 25 and $\bar{x}$ = 60
b. Compute a 95% CI for μ when n =100 and $\bar{x}$ =60
c. Compute a 99% CI for μ when n =100 and $\bar{x}$ = 60
d. Compute an 90% CI for μ when n =100 and $\bar{x}$ = 60
e. What does this imply about sample size and confidence level as pertaining to the length of the intervals, justify your answer?

Solution:

a. Compute a 95% CI for μ when n = 25 and $\bar{x}$ = 60

$\sigma=3$
$\mu \in \bar{x} \pm Z_{\alpha/2}\, \sigma/\sqrt{n}$

If confidence level is 95 %, then Z value is 1.96

We put values in equation,

$\mu \in 60 \pm 1.96 * 3/\sqrt{25}$
$\mu \in 60 \pm 1.96 * 0.6$
$\mu \in 60 \pm 1.176$
$\mu \in (61.176, 58.824)$

μ values will be between 58.824 watts and 61.176 watts.

b. Compute a 95% CI for μ when n =100 and $\bar{x}$ =60

$\sigma=3$
$\mu \in \bar{x} \pm Z_{\alpha/2}\, \sigma/\sqrt{n}$

If confidence level is 95 %, then Z value is 1.96

We put values in equation,

$\mu \in 60 \pm 1.96 * 3/\sqrt{100}$
$\mu \in 60 \pm 1.96 * 0.3$
$\mu \in 60 \pm 0.588$
$\mu \in (60.588, 59.412)$

μ values will be between 59.412 watts and 60.588 watts.

c: Compute a 99% CI for μ when n =100 and $\bar{x}$ = 60

$\sigma=3$
$\mu \in \bar{x} \pm Z_{\alpha/2}\, \sigma/\sqrt{n}$

If confidence level is 99 %, then Z value is 2.575

We put values in equation,

$\mu \in 60 \pm 2.575 * 3/\sqrt{100}$
$\mu \in 60 \pm 2.575 * 0.3$
$\mu \in 60 \pm 0.7725$
$\mu \in (60.7725, 59.2275)$

μ values will be between 59.2275 watts and 60.7725 watts.

d. Compute an 90% CI for μ when n =100 and $\bar{x}$ = 60

$\mu \in \bar{x} \pm Z_{\alpha/2}\, \sigma/\sqrt{n}$

If confidence level is 90 %, then Z value is 1.645

We put values in equation,

$\mu \in 60 \pm 1.645 * 3/\sqrt{100}$
$\mu \in 60 \pm 1.645 * 0.3$
$\mu \in 60 \pm 0.4935$
$\mu \in (60.4935, 59.5065)$

μ values will be between 59.5065 watts and 60.4935 watts.

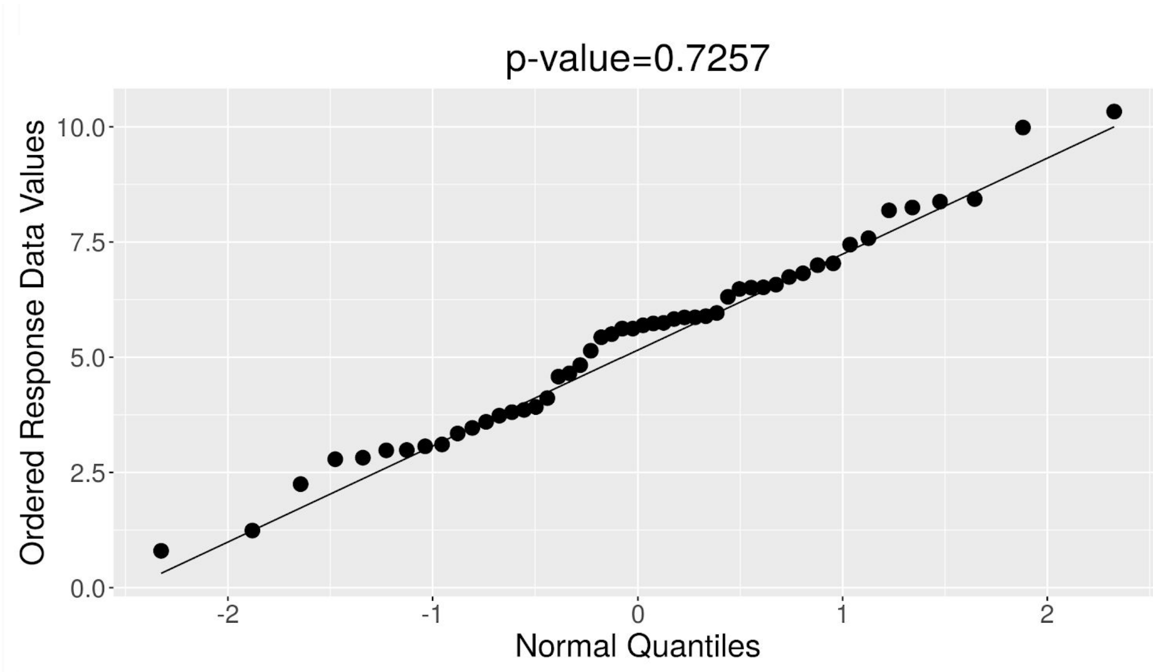
e. What does this imply about sample size and confidence level as pertaining to the length of the intervals, justify your answer?

The relationship between the sample size and length confidence intervals is inversely proportional to each other. If we increase the sample size of the data will decrease which inturn results in the estimates of the confidence intervals to be accurate and interval length to be shorter.

2. An engineer is doing a study in a manufacturing setting on the lengths of paperclips. The sample he collected is in the dataset paperclips.xls.
a. Create a normal probability plot to see if the data is normal
b. Find a two-sided 90% confidence interval for the mean and interpret
c. Now create an 95% upper bound only and interpret
d. Explain why the critical point does not change between parts b and c

Solution:

a. Create a normal probability plot to see if the data is normal



From the above graph, we can say that the graph follows normal distribution as all points lie in the same line and all data points follow the 45 degrees of the normal distribution line.

b. Find a two-sided 90% confidence interval for the mean and interpret

$\mu \in \bar{x} \pm Z_{\alpha/2}\, \sigma/\sqrt{n}$
If confidence level is 90 %, then Z value is 1.645

Mean= 5.368
Variance= 4.353
n=50

We put values in equation,

$\mu \in 5.368 \pm 1.645 * 4.353 / \sqrt{50}$

$\mu \in 5.368 \pm 1.645 * 4.353/7.071$
$\mu \in 5.368 \pm 1.645 * 0.6156$
$\mu \in 5.368 \pm 1.012$
$\mu \in (6.38, 4.356)$

c. Now create an 95% upper bound only and interpret

$\mu < \bar{x} + Z_\alpha * \sigma/\sqrt{n}$

$\alpha = 0.05$
Mean= 5.368
Variance= 4.353
n=50
$Z_{0.05} = 1.64$

We put values in equation,

$\mu < \bar{x} + Z_\alpha * \sigma/\sqrt{n}$
$\mu < 5.368 + Z_{0.05} * 4.353/\sqrt{50}$
$\mu < 5.368 + 1.64 * 4.353/7.071$
$\mu < 5.368 + 1.64 * 0.6156$
$\mu < 5.368 + 1.009584$
$\mu < 6.37758$

d. Explain why the critical point does not change between parts b and c

The Z value in part b is 1.645 and we have calculated a 2 sided 90% confidence interval, which means that we should have to leave out around 5 % of data on either side of the distribution. In the Z the value 0.94950 is the nearest possible value for 95% and it's value is 1.645 .
In part c, Z value for 95% is also 1.64 and we are calculating only the upper bound by which we are not leaving out any data on the lower bound of distribution. Hence we can say that the critical point doesn't change from part b to c.

3. A product has a target length of 5 inches. Assume that we know from historical data that the population variance is 2. A sample was taken. The sample mean in This case is 5.4 and the sample size is 36. Assume that the distribution of the data is normal. The supervisor wants to know if the target is being met. Use confidence intervals to determine this. What would you tell the supervisor? (To get full credit, justify why you chose the confidence interval that you did.)

Solution:

Mean $\bar{x}$ = 5.4
$\sigma 2$=2
σ= 1.414

n=36

Data is assumed to be normally distributed
And the target value is 5 inches.

$\mu \in \bar{x} \pm Z_{\alpha/2}\, \sigma / \sqrt{n}$
To calculate 95% confidence interval.
5.4 ± 1.96(1.414 / √36) = 5.4 ± 0.4619 = (4.9381 to 5.8619)
Since the range is away from the target, let's try CI at 90%.

To calculate 90% confidence interval.
5.4 ± 1.645(1.414 / √36) = 5.4 ± 0.3876 = (5.0124 to 5.7876)

After calculating the confidence interval score at 95% and 90% , We can say that the target is being met in terms of product length and is at least 5.0124 inches with 90% confidence and the interval varies 0.7752 which indicates that the product length is varying by 0.7752 inches for the whole sample size.
To conclude, I am 90% confident that the product is meeting its target length which is at least 5.01 inches.