Homework 5– One Sample Hypothesis Tests

1. The null hypothesis for a test is H_A : μ =5 . The population variance is 2.
The sample mean in this case is 5.4 and the sample size is 36. Test the null hypothesis using the p-value approach:
a. H_A : μ >5, with =0.05
b. H_A : μ >5, with =0.01
c. H_A : μ <5, with =0.01
d. H_A : μ ≠5, with =0.05
e. H_A : μ ≠5, with =0.10

Solution:

We have x = 5.4
σ2 = 2
n = 36
SD=$\sqrt{variance}$=$\sqrt{2}$= 1.41

a) H_A : μ >5 , with α = 0.05.
We have σ = 1.414
α=0.05
H_0 : μ = 5
We calculate $Z_{stat} = \bar{x} - \mu 0 \ / \ \sigma/\sqrt{n}$
Z_{stat} = 5.4 − 5/1.414/6
Z_{stat} = 1.697
P-value = P(Z > 1.697) = 0.0446
As this value is less than the α , we reject the hypothesis.

b. H_A : μ >5, with =0.01
We have σ = 1.414
α=0.05
H_0 : μ = 5
We calculate $Z_{stat} = \bar{x} - \mu 0 \ / \ \sigma/\sqrt{n}$
Z_{stat} = 5.4 − 5/1.414/6
Z_{stat} = 1.697
P-value = P(Z > 1.697) = 0.0446

As this value is more than the α , we will have to fail to reject the
Hypothesis.

c. H_A : μ <5, with =0.01

We have σ = 1.414

α=0.01
H_0 : μ = 5
We calculate $Z_{stat} = \bar{x} - \mu 0 / \sigma/\sqrt{n}$
Z_{stat} = 5.4 − 5/1.414/6
Z_{stat} = 1.697
P-value = P(Z < 1.697) = 0.9554
As this value is more than the α , we will have to fail to reject the hypothesis.

d. H_A : $\mu \neq 5$, with =0.05
We have σ = 1.414
α=0.05
H_0 : μ = 5
We calculate $Z_{stat} = \bar{x} - \mu 0 / \sigma/\sqrt{n}$
Z_{stat} = 5.4 − 5/1.414/6
Z_{stat} = 1.697
P-value =2[P(Z > 1.697) = 0.0891

As this value is more than the α , we will have to fail to reject the hypothesis.

e. H_A : $\mu \neq 5$, with =0.10

We have σ = 1.414
α=0.10
H_0 : μ = 5
We calculate $Z_{stat} = \bar{x} - \mu 0 / \sigma/\sqrt{n}$
Z_{stat} = 5.4 − 5/1.414/6
Z_{stat} = 1.697
P-value = 2[P(Z > 1.697)] = 0.0891
As this value is less than α , we reject the hypothesis.

2. Now redo parts a-e using the rejection region method. Do the 2 methods agree for each part?

Solution:
a. H_A : μ >5, with =0.05
Here, we can take values for mean, standard deviation and sample size as follows
H_0: $\mu 0$=5 , $\bar{x}$=5.4, n=36, α=0.05
SD=$\sqrt{variance}$=$\sqrt{2}$= 1.41

$Z_{stat} = \bar{x} - \mu 0 / \sigma/\sqrt{n}$

$Z_{stat} = 5.4 - 5/1.41/\sqrt{36}$
$Z_{stat} = 5.4 - 5/1.41/6$
$Z_{stat} = 0.4/0.235$
$Z_{stat} = 1.7$

Rejection region : $Z_{stat} > Z_{\alpha}$
$Z_{stat} > Z_{\alpha}$ I.e $Z_{stat} > Z_{0.05} = 1.7 > 1.645$
Since Z_{stat} is greater than Z_{α} i.e 1.7>1.645, we are rejecting this hypothesis.

b. H_A : μ >5, with =0.01

H_0: $\mu 0=5$, $\bar{x}=5.4$, n=36, α=0.01
SD=$\sqrt{variance}$=$\sqrt{2}$= 1.41

$Z_{stat} = \bar{x} - \mu 0 / \sigma/\sqrt{n}$

$Z_{stat} = 5.4 - 5/1.41/\sqrt{36}$
$Z_{stat} = 5.4 - 5/1.41/6$
$Z_{stat} = 0.4/0.235$
$Z_{stat} = 1.7$

$Z_{stat} > Z_{\alpha}$ I.e $Z_{stat} > Z_{0.01}$ but $1.7 < 2.326$
Since Z_{stat} is less than Z_{α} i.e 1.7 < 2.326, we failed to reject this hypothesis.

c. H_A : μ <5, with =0.01

H_0: $\mu 0=5$, $\bar{x}=5.4$, n=36, α=0.01
SD=$\sqrt{variance}$=$\sqrt{2}$= 1.41

$Z_{stat} = \bar{x} - \mu 0 / \sigma/\sqrt{n}$

$Z_{stat} = 5.4 - 5/1.41/\sqrt{36}$
$Z_{stat} = 5.4 - 5/1.41/6$
$Z_{stat} = 0.4/0.235$
$Z_{stat} = 1.7$

$Z_{stat} < Z_{\alpha-1}$ I.e $Z_{stat} < Z_{-0.99}$ but 1.7 > -2.326
Since Z_{stat} is greater than $Z_{\alpha-1}$ we failed to reject the hypothesis.

d. H_A : μ ≠5, with =0.05
H_0: μ0=5 , $\bar{x}$=5.4, n=36, α=0.05
SD=$\sqrt{variance}$=√2= 1.41

$Z_{stat} = \bar{x} - \mu 0 / \sigma/\sqrt{n}$

$Z_{stat} = 5.4 - 5/1.41/\sqrt{36}$
$Z_{stat} = 5.4 - 5/1.41/6$
Z_{stat} =0.4/0.235
Z_{stat} =1.7

$|Z_{stat}| > Z_{\alpha/2}$ I.e $Z_{stat} > Z_{0.025}$ but 1.7<1.96
Since Z_{stat} is less than $Z_{\alpha/2}$ we failed to reject the hypothesis.

e. H_A : μ ≠5, with =0.10

H_0: μ0=5 , $\bar{x}$=5.4, n=36, α=0.10
SD=$\sqrt{variance}$=√2= 1.41

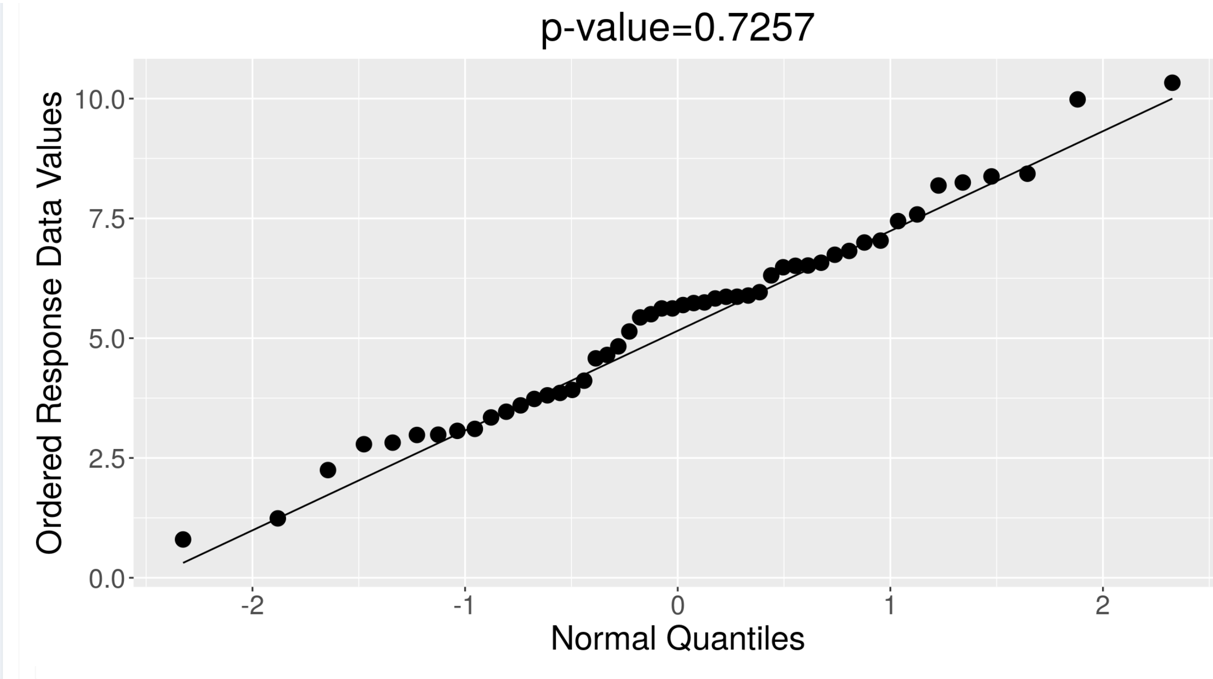
$Z_{stat} = \bar{x} - \mu 0 / \sigma/\sqrt{n}$
$Z_{stat} = 5.4 - 5/1.41/\sqrt{36}$
$Z_{stat} = 5.4 - 5/1.41/6$
Z_{stat} =0.4/0.235
Z_{stat} =1.7

$|Z_{stat}| > Z_{\alpha/2}$ I.e $Z_{stat} > Z_{0.05}$ = 1.7>1.645
Since Z_{stat} is greater than $Z_{\alpha/2}$ we reject this hypothesis.

3. An engineer is doing a study in a manufacturing setting on the lengths of paperclips. The sample he collected is in the dataset paperclips.xlsx. Assume that the target value for the paperclip length is 6 cm. He would like to see if this assumption still holds.
a. Check to see if the data seems to come from a normal distribution
b. Give the null and alternative hypotheses for this situation
c. Determine the test statistic in this case (show work)
d. Using α=0.05, test the hypothesis and draw conclusions
e. Give the real-world answer

Solution:

a. Check to see if the data seems to come from a normal distribution



This means that we should assume that the given data comes from a normal distribution.
This reflects in the normal probability graph as well.
The plot has points that closely follows line
Since the p-value is greater than 0.05, we conclude that the data comes from a normal distribution.

b. Give the null and alternative hypotheses for this situation

From stathelper calculation, we get
Mean=$\overline{x}$=5.368, n=50, σ=2.086

Null hypothesis becomes,
H_0: μ= 6

The alternative hypothesis is typically the opposite of the null hypothesis or the statement we wish to prove true.
Alternative hypothesis will be,
H_A: $\mu \neq 6$

c. Determine the test statistic in this case (show work)

Statistic	*Value*
Mean	*5.368*
Median	*5.656*
Sample Size	*50*
Q1	*3.732*
Q3	*6.575*
Min	*0.7993*
Max	*10.331*
IQR	*2.843*
Range	*9.532*
Variance	*4.353*
Standard Deviation	*2.086*
Lower Fence	*-0.5322*
Upper Fence	*10.839*

So, for test statistic,

Mean=$\bar{x}$=5.368, n=50, σ=2.086

$t_{stat} = \bar{x} - \mu 0 / \sigma / \sqrt{n}$

$t_{stat} = 5.368 - 6/2.086/\sqrt{50}$

$t_{stat} = 5.368 - 6/2.086/7.071$

t_{stat} = -0.632/0.295

t_{stat} = -2.142

d. Using α=0.05, test the hypothesis and draw conclusions.

Mean=$\bar{x}$=5.368, n=50, σ=2.086, α=0.05

H0 : μ = 6

We calculate $Z_{stat} = \bar{x} - \mu 0 / \sigma / \sqrt{n}$

t_{stat} = 5.368 – 6/2.086/$\sqrt{50}$

$t_{stat} = 5.368 - 6/2.086/7.071$

t_{stat} = -0.632/0.295

t_{stat}= -2.142

For the p-value approach, the p-value is,

p-value= $2P(t \geq |t_{stat}|)$

p-value= 2(1-P(t≤|2.142|))

p-value= 2(1-0.0324)

p-value=2*0.9676

p-value=1.9352

Interpretation for a 1-sample t-test
Decision rule based on p-value:
Reject H_0 : p-value $\leq \alpha$
Fail to reject H_0 : p-value > α

P-value for this test is 1.9352
α=0.05

For p-value approach:
Since 1.9252>0.05, we failed to reject the null hypothesis.
There is not enough evidence to support the claim of the alternative hypothesis.

e. Give the real-world answer
There is not enough evidence to support the claim of the alternative hypothesis.
We conclude that the target value for the paperclip length is 6 cm.

4. Suppose that an analyst believes that the number of parking tickets for parking on the street given out on average in a month is 512. Suppose that we sample 256 weeks and the average number of parking tickets for parking in the street per week was found to be 130. Is the assumption valid? Test this at α=0.05 (include justification for the test that you chose and the real-world answer).

Solution:

If μ is the average number of successes occurring in a given time interval or region in the Poisson distribution, then the mean and the variance of the Poisson distribution are both equal to μ.
Hence $variance$ and $\bar{x}$=130
Average per month is 512 i.e 512/4=128
Mean=$\bar{x}$=130, n=256, α=0.05
H_0: $\mu 0$=128
σ =SD=$\sqrt{variance}$=$\sqrt{130}$= 11.40
Z_{stat} =$\bar{x} - \mu 0 / (\sigma/\sqrt{n})$

Z_{stat} =$130 - 128/(11.40/\sqrt{256})$
Z_{stat} =$130 - 128/(11.40/16)$
Z_{stat} =$2/0.7125$

Z_{stat} =2.80

For the p-value approach, the p-value is,
p-value= $P(Z>Z_{stat})$
p-value=0.0025

Interpretation for a 1-sample t-test
Decision rule based on p-value:
Reject H_0 : p-value ≤α
Fail to reject H_0 : p-value > α

P-value for this test is 0.0025
α=0.05

For p-value approach:
Since 0.0025 ≤ 0.05, we reject the null hypothesis.
There is evidence to support the claim of the alternative hypothesis.

Decision rule based on rejection region:
Reject H_0 : $Z_{stat} > z$
Fail to reject H_0 : $Z_{stat} \leq Z$

Since 2.8 > 1.645 , we reject the null hypothesis
There is evidence to support the claim of the alternative hypothesis.

Statistical answer: reject the hypothesis

Real-world answer:
We cannot conclude that the number of parking tickets for parking on the street given out on average in a month is 512.