Q.1) The results shown below were obtained in a small-scale experiment to study the relationship between storage temperature X (in degrees Fahrenheit) and the number of weeks before flavor deterioration of a food product begins to occur (Y).
Using matrix notation for the regression model
Obtain the following:
a. Vector of estimated regression coefficients.
b. Estimated variance-covariance matrix of b
c. Point estimate of E(Y_h) , when $X_h = -6$
d. Estimated variance of Y_h , when $X_h = -6$
e. Interval estimate of E(Y_h) , when$X_h = -6$. Should we call it a confidence or prediction interval? Use $\alpha = 0.05$.

Solution:
a. Vector of estimated regression coefficients.
The matrix is:

Matrix M7

9.940
-0.245

The vector of coefficients are

$\hat{b}_0$= 9.940
$\hat{b}_1$= -0. 245

The regression equation is:
Y=9.940-0.245X

b. Estimated variance-covariance matrix of b:
The variance-covariance matrix of b is:

Matrix M9

0.009866 0.0000000
0.000000 0.0003083

c. Point estimate of E(Y_h) , when $X_h = -6$

QUESTION 1

Prediction for y-variable

Regression Equation

y-variable = 9.9400 - 0.2450 x-variable

Settings

Variable	Setting
x-variable	-6

Prediction

Fit	SE Fit	95% CI	95% PI
11.41	0.144799	(10.9492, 11.8708)	(10.5662, 12.2538)

X is -6 which means the Point estimate of $E(Y_h)$=11.41

d. Estimated variance of $\hat{Y}_h$, when $X_h = -6$

Prediction

Fit	SE Fit	95% CI	95% PI
11.41	0.144799	(10.9492, 11.8708)	(10.5662, 12.2538)

X is -6 which means the estimate variance of $E(Y_h)$= 0.144799

e. Interval estimate of $E(Y_h)$, when$X_h = -6$. Should we call it a confidence or prediction interval? Use $\alpha = 0.05$.

From the above screenshots,

X=-6, This describes that the interval estimate of $E(Y_h)$ is (10.9492,11.8708) when $\alpha = 0.05$

This should be a confidence interval because the X is -6 which is in the range of X

Q.2.15) Byers and Williams (" Viscosities of Binary and Ternary Mixtures of Polynomatic Hydrocarbons, " Journal of Chemical and Engineering Data, 32 , 349 – 354, 1987) studied the impact of temperature on the viscosity of toluene – tetralin blends. The following table gives the data for blends with a 0.4 molar fraction of toluene.

a. Estimate the prediction equation.

b. Perform a complete analysis of the model.

c. Calculate and plot the 95% confidence and prediction bands.

Solution:

a. Estimate the prediction equation.

Regression Equation

Viscosity=1.2815 - 0.008758 Temperature

b. Perform a complete analysis of the model.

HOMEWORK2.15

Regression Analysis: Viscosity versus Temperature

Regression Equation

Viscosity = 1.2815 - 0.008758 Temperature

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	1.2815	0.0469	(1.1668, 1.3962)	27.34	0.000	
Temperature	-0.008758	0.000728	(-0.010540, -0.006976)	-12.02	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
0.0474336	96.02%	95.35%	0.0317038	90.64%	-16.37	-22.13

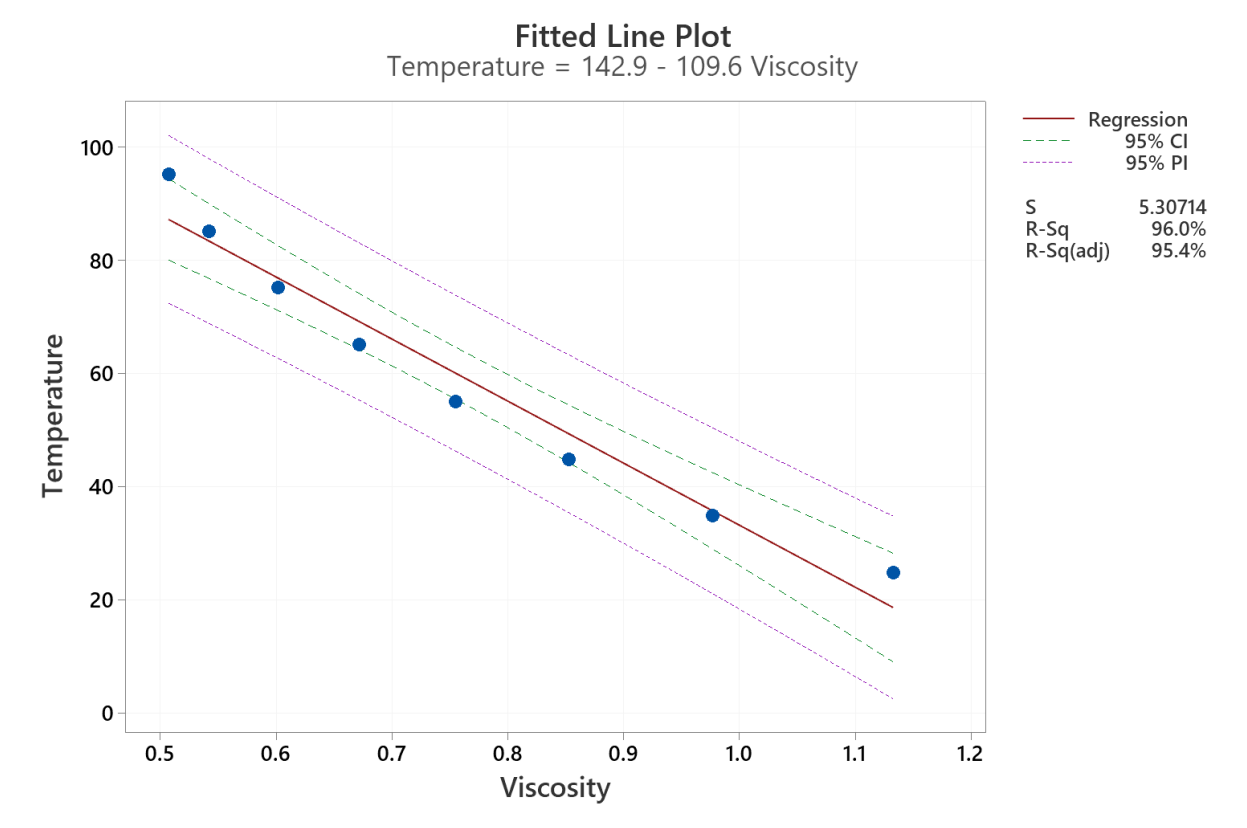
Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Regression	1	0.32529	96.02%	0.32529	0.325292	144.58	0.000
Temperature	1	0.32529	96.02%	0.32529	0.325292	144.58	0.000
Error	6	0.01350	3.98%	0.01350	0.002250		
Total	7	0.33879	100.00%				

Since p-value <0.05, we are rejecting the null Hypothesis. The regression is significant. This indicates that a significant relationship between temperature and viscosity.

c. Calculate and plot the 95% confidence and prediction bands.

The 95% prediction and confidence bands are plotted below.



Q.2.18) On March 1, 1984, the Wall Street Journal published a survey of television advertisements conducted by Video Board Tests, Inc., a New York ad - testing company that interviewed 4000 adults. These people were regular product users who were asked to cite a commercial they had seen for that product category in the past week. In this case, the response is the number of millions of retained impressions per week. The regressor is the amount of money spent by the firm on advertising. The data follow.

a. Fit the simple linear regression model to these data.
b. Is there a significant relationship between the amount a company spends on advertising and retained impressions? Justify your answer statistically.
c. Construct the 95% confidence and prediction bands for these data.
d. Give the 95% confidence and prediction intervals for the number of retained impressions for MCI.

Solution:

a. Fit the simple linear regression model to these data.

Regression Equation

Returned Impressions / week = 21.13 + 0.3678 Amount Spent

b. Is there a significant relationship between the amount a company spends on advertising and retained impressions? Justify your answer statistically.

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value	VIF
Constant	21.13	7.42	(5.47, 36.79)	2.85	0.011	
Amount Spent	0.3678	0.0978	(0.1615, 0.5741)	3.76	0.002	1.00

Model Summary

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)	AICc	BIC
23.3268	45.42%	42.21%	12018.5	29.09%	179.09	180.32

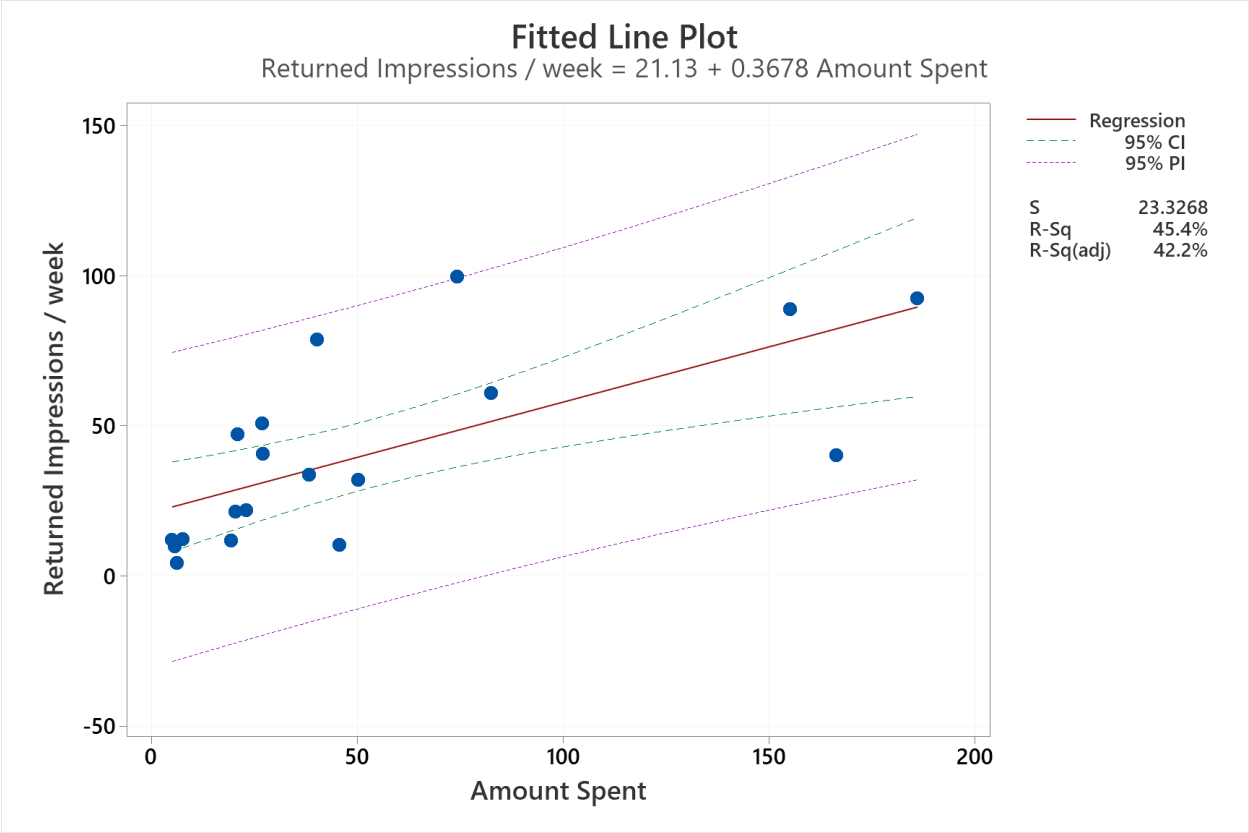
Analysis of Variance

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Regression	1	7698	45.42%	7698	7697.5	14.15	0.002
Amount Spent	1	7698	45.42%	7698	7697.5	14.15	0.002
Error	17	9250	54.58%	9250	544.1		
Total	18	16948	100.00%				

Since p-value <0.05, we are rejecting the null hypothesis. The regression is significant. Therefore we can conclude that there is a significant relationship between the amount a company spends on advertising and retained impressions.

c. Construct the 95% confidence and prediction bands for these data.

The 95% CI and PI bands,



d. Give the 95% confidence and prediction intervals for the number of retained impressions for MCI.

HOMEWORK2.18

Prediction for Returned Impressions / week

Regression Equation

Returned Impressions / week = 21.13 + 0.3678 Amount Spent

Settings

Variable	Setting
Amount Spent	26.9

Prediction

Fit	SE Fit	95% CI	95% PI
31.0214	5.91194	(18.5483, 43.4946)	(-19.7497, 81.7926)

The 95% Confidence Interval and prediction intervals for the number of retained impressions for MCI are (18.5483,43,4946) and (-19.7497,81.7926) respectively.