Mrinal Chaudhari- Applied Stat-614

Assignment 3

Homework – Random Variables and Distributions (Binomial and Poisson)

1. (Show Work) Assume that we have a binomial distribution with n=50 and π=0.3. Determine the following probabilities:
a. P(X=3)
b. P(X>1)
c. P(X≤4)

Solution:

n=50, **π**=0.3
a:

P(X=3)
b(x;n,p) = $\binom{n}{x}^{x}\ (1-)^{n-x}$ x=0,1,2….n
P(X=3)= $\binom{50}{3}\ 0.3^{3}\ (1-0.3)^{50-3}$
P(X=3)=0.000028

b:
b(x;n,p) = $\binom{n}{x}^{x}\ (1-)^{n-x}$ x=0,1,2….n
P(X>1)=1-P(X<1))=1-P(X=0)
P(X>1)=1- $\binom{50}{0}\ 0.3^{0}\ (1-0.3)^{50-3}$
P(X>1)=1-0.000000018
P(X>1)=0.999999

c:
b(x;n,p)= $\binom{n}{x}^{x}\ (1-)^{n-x}$ x=0,1,2….n
P(X≤4)= $\binom{n}{i}^{i}\ (1-)^{n-i}$
P(X≤4)= $\binom{50}{4}\ 0.3^{4}\ (1-0.3)^{50-4}$
P(X≤4)=0.0002

2. As the quality engineer, you monitor supplier shipments for quality before accepting the shipment. Historically, this supplier has produced about one defective item out of 100. When a new shipment comes, you take a sample of size twenty items from the shipment, and if there is more than one defective item, you reject the shipment.
a. What is the probability of rejecting the shipment?
b. What is the probability of accepting a shipment?
c. How many defective items would you expect per shipment?

Solution:

n=20, **π**=0.01

a:

b(x;n,p) = $\binom{n}{x}^{x}\ (1-)^{n-x}$ x=0,1,2….n

P(X≥1)=1-P(X <1)=1-P(X=0)

P(X≥1)=1- $\binom{20}{0}0.01^{0}\ (1-0.01)^{20-0}$

P(X≥1)=1-0.8179=0.1821

b:

n=20, **π**=0.01

b(x;n,p = $\binom{n}{x}^{x}\ (1-)^{n-x}$ x=0,1,2….n

P(X≤1)= $\binom{n}{i}^{i}\ (1-)^{n-i}$

P(X≤1)= $\binom{20}{1}0.01^{1}\ (1-0.01)^{20-1}$

P(X≤1)=0.9831

c:

Expected defective items per shipment would be nothing but= n***π**

For binomial distribution , the mean is the expected number of successes and this is calculated by n times pi.

μ=n***π**

20* 0.01=0.2

3. (Show Work) Assume that we have a Poisson distribution with λ=2.8. Determine the following probabilities:

a. P(X=0)

b. P(X>1)

c. P(X≤4)

Solution:

a: P(X=0):

λ=2.8

P(x;**λ**)= $e^{-}\lambda^{x}/x!$

P(X=0):= $e^{-2.8}2.8^{0}/0!$

P(X=0)= 0.0608

b: P(X>1):

λ=2.8
P(x;**λ**)= $e^{-}\lambda^{x}/x!$
P(X>1):= 1-P(X<1)=1-[P(X=0)+P(X=1)]=1- [$e^{-2.8}2.8^{0}/0!$ + $e^{-2.8}2.8^{1}/1!$]
P(X>1):= 1-[0.0608+0.1703]
P(X>1)= 0.7689

c: P(X≤4):
λ=2.8
P(x;**λ**)= $e^{-}\lambda^{x}/x!$
P(X≤4)= P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)
P(X≤4)= $e^{-2.8}2.8^{0}/0!$ + $e^{-2.8}2.8^{1}/1!$ + $e^{-2.8}2.8^{2}/2!$ + $e^{-2.8}2.8^{3}/3!$ + $e^{-2.8}2.8^{4}/4!$
P(X≤4)=0.0608+0.1703+0.2384+0.2225+0.1557
P(X≤4)=0.8477

4. For an automobile painting process an inspector counts the number of minor blemishes found after the painting is complete. Past experience and information tell him that the count of blemishes per automobile is about 5.
a. What is the probability that there are no blemishes on the next automobile to come off the line?
b. What is the probability that there are at least 2 blemishes on the next automobile?
c. Now management is interested in automobiles in sets of 5. Determine the average number of blemishes on a set of five automobiles.
d. Find the probability that no blemishes are found on the next 5 automobiles.

Solution:

a:

The probability that there are no blemishes on the next automobile to come off the line
λ= per automobile is 5=0.2
λ=0.2
P(x;**λ**)= $e^{-}\lambda^{x}/x!$
P(X=0):= $e^{-0.2}2.8^{0}/0!$
P(X=0)=0.8187

b:

The probability that there are at least 2 blemishes on the next automobile
λ=0.2
P(x;**λ**)= $e^{-}\lambda^{x}/x!$
P(X>2)=1-P(X<2)= 1-[P(X=0)+P(X=1)]
P(X>2)=1-[$e^{-0.2}2.8^{0}/0!$ + $e^{-0.2}2.8^{1}/1!$]
P(X>2)=1- [0.8187+0.1637]

P(X>2)=1-0.9824
P(X>2)=0.0176

c:

λ=25
Error or success rate $\lambda = 25$
P(x;**λ**)= $e^{-}\lambda^{x}/x!$
P(X=5)= $e^{-25}2.8^{5}/5!$
P(X=5)= 0.0000011

d:

The probability that no blemishes are found on the next 5 automobiles:
λ=1
Error or success rate $\lambda = 1$
P(x;**λ**)= $e^{-}\lambda^{x}/x!$
P(X=0)= $e^{-1}2.8^{0}/0!$
P(X=0)= 0.3679

5. Repeat #1 using the normal approximation to the binomial and comment on how good the approximation is in this case. Include showing that the approximation holds.

Solution:
Here, n=50 and π=0.3
Perform this by using normal approximation to the binomial.
Here,
S= n*π
= 50*0.3 = 15

F = n (1 - π)
= 50 (1 - 0.3)
= 50 (0.7) = 35

μ=nμ = 15

And σ = nπ(1-π)
= 50 * 0.3 (1 - 0.3)
= 3.24

To calculate:

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a. $P(X=3)$
b. $P(X>1)$
c. $P(X\leq4)$

We calculate $Z= x-\mu/\sigma$
1:$P(X=3)$
$Z= x-\mu/\sigma$
$Z=3-15/3.24$
$Z= -3.70$

$P(X=3)\cong 0$

2:$P(X>1)$
$Z= x-\mu/\sigma$
$Z= 0-15/3.24$
$Z= -4.629$

So, $P(X>1)=1-0\cong 1$

3:$P(X\leq4)$
$Z= x-\mu/\sigma$
$Z=4-15/3.24$
$Z= -3.395$

$P(X\leq4) \cong 0.0003$

So in the first case approximation is not holding the same value but in the second and third case approximation holds value . So we can say that approximation does not hold the correct value every time.