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Sample Inference on a Mean Difference Homework-week 8

1. An article reported on an experiment in which 13 computer-proficient medical professionals were timed retrieving images from a library of slides and then retrieving the same image from a computer database.
a. Assuming that the differences are normally distributed and that this is a paired situation, test the hypothesis that the mean difference is 14 at =0.10
b. Assume that the data come from two normal distribution, these distributions are independent and the variances are not equal, test the hypothesis that the mean difference is 14 at =0.10
c. Assume that the data come from two normal distribution, these distributions are independent and the variances are equal, test the hypothesis that the mean difference is 14 at =0.10
d. Do the decisions agree? If they don't agree, explain why you think that this is happening and which test is valid.

Solution:

a. Assuming that the differences are normally distributed and that this is a paired situation, test the hypothesis that the mean difference is 14 at =0.10

Statistic	*Value*
Mean	*20.923*
Median	*19*
Sample Size	*13*
Q1	*12*
Q3	*26.5*
Min	*5*
Max	*46*
IQR	*14.5*
Range	*41*
Variance	*136.24*
Standard Deviation	*11.672*
Lower Fence	*-9.75*
Upper Fence	*48.25*

Null and alternative hypothesis,
α=0.10, $n=13$, $D_i=14$, $df=n-1=13-1=12$,

$H_0: \mu_1 - \mu_2 = 14$
$H_a \neq 14$

$T_{stat} = \bar{x} - D_i / (S_D / \sqrt{n})$
$T_{stat} = 20.923-14/(11.672/\sqrt{13})$
$T_{stat} = 20.923-14/3.237$
$T_{stat} = 2.139$
To calculate p-value,
P-value approach= $P(t_{df} \geq t_{stat}) = P(t_{12} \geq 2.139) = 0.0537$
$t_{crit} = t_{0.95,12} = 1.782$
We reject H_0

b. Assume that the data come from two normal distribution, these distributions are independent and the variances are not equal, test the hypothesis that the mean difference is 14 at =0.10

α= 0.10; H0 = $\mu_1-\mu_2$= 14; HA$\neq$14; variance is not equal.

t_{stat}= 0.771; p-value = 0.4481; $t_{0.95,24}$ = 1.711

We fail to reject H0, as $t_{stat} < t_{0.95,24}$ & p-value >α

c. Assume that the data come from two normal distribution, these distributions are independent and the variances are equal, test the hypothesis that the mean difference is 14 at =0.10

α= 0.10; H0 = $\mu_1-\mu_2$= 14; HA$\neq$14; variance is not equal.

t_{stat}= 0.771; p-value = 0.4481; $t_{0.95,24}$ = 1.711

We fail to reject H0, as $t_{stat} < t_{0.95,24}$ & p-value >α

d. Do the decisions agree? If they don't agree, explain why you think that this is happening and which test is valid

The decision does not agree. Because we are perfromning independent t-test on both part. This indicates that the images would be same in both cases hence we can perform 2 means paired T-test in such scenario. Performing 2 means paired test would be correct choice for such examples.

2. Fusible interlinings are being used with increasing frequency to support outer fabrics and improve the shape and drape of various pieces of clothing. The accompanying data on extensibility (%) at 100g/cm for both high quality (H) and poor quality (P) is given.

a. Explain why we should consider these independent distributions

b. Create normal probability plots for both samples to assess normality

c. Test to see if the variances are equal

d. Test to see if the means are equal vs. HA : $\mu_1 \neq \mu_2$

e. Give the real-world answer

f. Create a 95% confidence interval for the mean difference and use it to test the hypothesis from part d

g. Do you get the same answers for parts d and f? Explain why this works.

Solution:

a. Explain why we should consider these independent distributions.

We should have to consider independent distribution because variance of both distributions need to be compared. The data is experienced with two quality of fabric i.e high and poor quality. Therefore, we should consider these as independent distributions.

b. Create normal probability plots for both samples to assess normality.

1: Normal probability plot for high quality(H)

For the Shapiro-Wilk normality test, we assume that:

H_0 : The data comes from a normal distribution

H_A : The data does not comes from a normal distribution

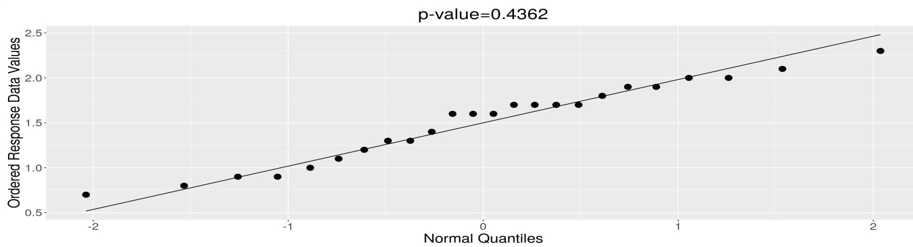
The p-value for this test is 0.4362

Since $0.4362 > 0.05$ we fail to reject the null hypothesis.

This means that we should assume that the given data comes from a normal distribution.

This is reflected in the normal probability plot below

The plot has points that closely follow the line.



2: Normal probability plot for poor quality(P)

For the Shapiro-Wilk normality test, we assume that:

H_0 : The data comes from a normal distribution

H_A : The data does not comes from a normal distribution

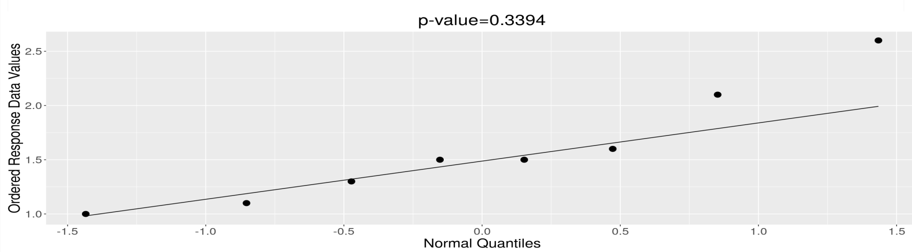
The p-value for this test is 0.3394

Since $0.3394 > 0.05$ we fail to reject the null hypothesis.

This means that we should assume that the given data comes from a normal distribution.

This is reflected in the normal probability plot below

The plot has points that closely follow the line.



From the above figure we can say that data comes from normal distribution. From the plot we can see that lines are quite parallel and the assumption of variance is valid.

c. Test to see if the variances are equal.

Statistic	Value	Statistic	Value
Mean	1.508	Mean	1.588
Median	1.6	Median	1.5
Sample Size	24	Sample Size	8
Q1	1.15	Q1	1.2
Q3	1.85	Q3	1.85
Min	0.7	Min	1
Max	2.3	Max	2.6
IQR	0.7	IQR	0.65
Range	1.6	Range	1.6
Variance	0.1973	Variance	0.2812
Standard Deviation	0.4442	Standard Deviation	0.5303
Lower Fence	0.1	Lower Fence	0.225
Upper Fence	2.9	Upper Fence	2.825

$\bar{x}$=1.508, $\bar{y}$=1.588, variance 1= 0.1973, variance 2=0.2812, m=24,n=8,S1=0.442, S2=0.5303

$H_0 = \sigma 1^2/\sigma 2^2 = 1$ - The null hypothesis, we assume the variance to be the same for both H and P

$H_A = \sigma 1^2/\sigma 2^2 \neq 1$ --The alternate hypothesis, where the variance is not the same for both H and P

Fstat= $S1^2/S2^2 * \sigma 2^2/\sigma 1^2$

p-value approach :- 2 * min (P(F≥Fstat), P(F≤Fstat))

p-value= 2 * min(1 - P(F≥Fstat),P(F≤Fstat))

p-value= 2 * min(1 - P(F≤0.7015), P(F≤0.7015))

p-value= 2 * min(1 - 0.2431, 0.2431)

p-value= 2 * min(0.7569, 0.2431)

p-value= 2 * 0.2431

p-value= 0.4862

p-value= 0.05; p-value = 0.4862; p-value >α

We fail to reject H0

There is not enough evidence to support the claim of the alternate hypothesis.

d. Test to see if the means are equal vs. HA : μ1 ≠ μ2 , assume α=0.05

$\bar{x}$=1.508, $\bar{y}$=1.588, m=24, n=8, S1=0.442, S2=0.5303

Assume equal variances.

Null and alternative hypothesis,

$H0$: $\mu 1 = \mu 2$

$H1$: $\mu 1 \neq \mu 2$

Sp **=** $\sqrt{}(m - 1)S1^2 + (n - 1)S2^2/m + n - 2$

Sp **=** $\sqrt{}23 * (0.442)^2 + 7(0.5303)^2/ 24 + 8 - 2$

Sp **=**$\sqrt{}23 * 0.1953 + 7 * 0.2812/ 30$

Sp **=**$\sqrt{}4.4919 + 1.9684/ 30$

Sp **=**$\sqrt{}4.4919 + 1.9684/ 30$

Sp **=**$\sqrt{}6.4603/ 30$

Sp **=**$\sqrt{}0.2153$

Sp **=**0.4640

Now, we will calculate test statistic,

t=$\bar{x} - \bar{y} - \Delta 0/\sqrt{}Sp^2[1/m + 1/n]$

t=$1.508 - 1.588/\sqrt{}0.4640^2[1/24 + 1/8]$

t=$- 0.08/\sqrt{}0.2152[0.0416 + 0.125]$

t=$- 0.08/\sqrt{}0.2152 * 0.1666$

t=$- 0.08/\sqrt{}0.03585$

t=$- 0.08/0.1893$

t=$- 0.4226$

Now, we will calculate p-value,
p-value=2*P(t>0.4226)
p-value= 0.675504

Critical Value $t_{31,0.025}$ = ±2.1448, Reject Region is anything smaller than -2.1448 or bigger than 2.1448
Since our p-value is 0.675504, We failed to reject null hypothesis.
There is not enough evidence to support alternative hypothesis.

e. Give the real-world answer

At the 5% level of significance, there is not enough evidence to claim a difference between the two population means (although, it is close…).This indicates that the extensibility does not differ significantly when fused with H and P fabric.

f. Create a 95% confidence interval for the mean difference and use it to test the hypothesis from part d

$\bar{x}$=1.508, $\bar{y}$=1.588, m=24, n=8, S1=0.442, S2=0.5303

$\bar{x} - \bar{y} \pm t_{v,\alpha/2}$.**S**$p\sqrt{}$1/m+1/n

1.508-1.588±2.04*0.4640√1/24+1/8
-0.08±0.94656√0.04166+0.125

$-0.08 \pm 0.94656\sqrt{0.166}$
$-0.08 \pm 0.94656 * 0.407$
$-0.08 \le \mu_1 - \mu_2 \le 0.3852$

From stathelper calculation, we get,
Degree of freedom is 30
Critical point=$t_{0.975,30}$=2.042
$\mu 1 - \mu 1 \in (-0.467, 0.307)$

g. Do you get the same answers for parts d and f? Explain why this works.

Yes, since the confidence interval contains zero, we arrive at the same conclusion as the hypothesis test, that we cannot claim the two means are different. These two tests are equivalent.

3. Two different hardening processes, (1) saltwater quenching and (2) oil quenching, are used on samples of a particular type of metal alloy. Assume that different alloy samples were used for each test. Do you believe that the mean hardness level for the saltwater is less than the mean hardness level for the oil? Test this hypothesis at =0.01. If need be, give a range of plausible values for the true mean difference. You need to include all of the steps that we do for a hypothesis test to get full credit.

Statistic	*Value*
Mean	*147.6*
Median	*148*
Sample Size	*10*
Q1	*145*
Q3	*152*
Min	*139*
Max	*154*
IQR	*7*
Range	*15*
Variance	*24.711*
Standard Deviation	*4.971*
Lower Fence	*134.5*
Upper Fence	*162.5*

Statistic	*Value*
Mean	*153.4*
Median	*154.5*
Sample Size	*10*
Q1	*150*
Q3	*156*
Min	*144*
Max	*162*
IQR	*6*
Range	*18*
Variance	*29.822*
Standard Deviation	*5.461*
Lower Fence	*141*
Upper Fence	*165*

$\bar{x}$=147.6, $\bar{y}$=153.4, m=10, n=10, S1=4.971, S2=5.461, α=0.01
Assume equal variances.
Null and alternative hypothesis,
$H0: \mu 1 = \mu 2$

$H1: \mu 1 \neq \mu 2$

S$_p$ **=** $\sqrt{(m-1)S1^2 + (n-1)S2^2/m + n - 2}$

Sp **=** $\sqrt{9 * 4.971^2 + 9 * 5.461^2 / 18}$
Sp **=**$\sqrt{9 * 24.7108 + 9 * 29.822 / 18}$
Sp **=**$\sqrt{222.3972 + 268.398 / 18}$
Sp **=**$\sqrt{490.7952 / 18}$
Sp **=**$\sqrt{27.26}$
Sp **=** 5.2217

Now, we will calculate test statistic,

t=$\bar{x} - \bar{y} - \Delta 0 / \sqrt{Sp^2}[1/m + 1/n]$
t=$147.6 - 153.4 / \sqrt{5.2217^2}[1/10 + 1/10]$
t=$- 5.8 / \sqrt{27.2661}[0.2]$
t=$- 5.8 / \sqrt{5.5432}$
t=$- 5.8 / 2.3544$
t= -2.4634

Now, we will calculate p-value,
p-value=2*P(t>2.4634)
p-value= 0.024067

Critical Value t_{18},0.005 = ±2.1448, Reject Region is anything smaller than -2.1448 or bigger than 2.1448
$t_{0.975,18}$=2.101

Since our p-value is 0.024067 and $t_{stat}<t_{0.975,18}$, We failed to reject null hypothesis.
There is not enough evidence to support alternative hypothesis
Therefore, we can conclude that the mean hardness for the salt water is not less than the mean hardness of oil.

Degree of freedom= 10+10-2= 18

$t_{0.975,18}$=2.101,
We are 95 % confidence that the mean difference value range from,
(μ1−μ2)∈(−10.706,−0.8938)

4. Snee (1981) determined the octane ratings of 32 gasoline blends by two standard methods: motor (method 1) and research (method 2). Each of the 32 blends was tested by each method. Is the difference between the research blend and the motor blend equal to 5? If need be, give a range of plausible values for the true mean difference. You need to include all of the steps that we do for a hypothesis test to get full credit.

Solution:
$H_0: \mu_D = 5$
$H_1: \mu_D \neq 5$

d_i **= research$_i$ - motor$_i$**
T_0 **= D_i-**$\Delta_0/S_D/\sqrt{n}$

Statistic	*Value*
Mean	*93.428*
Median	*92*
Sample Size	*32*
Q1	*89.55*
Q3	*95.4*
Min	*85.4*
Max	*109.5*
IQR	*5.85*
Range	*24.1*
Variance	*30.586*
Standard Deviation	*5.53*
Lower Fence	*80.775*
Upper Fence	*104.17*

Statistic	*Value*
Mean	*96.531*
Median	*97.2*
Sample Size	*32*
Q1	*93.75*
Q3	*99.6*
Min	*83.3*
Max	*106.6*
IQR	*5.85*
Range	*23.3*
Variance	*26.694*
Standard Deviation	*5.167*
Lower Fence	*84.975*
Upper Fence	*108.37*

Assume α=0.01

d_i = research$_i$ - motor$_i$
d_i =96.531-93.428
d_i = 3.103

T_0 **= D_i-**$\Delta_0/S_D/\sqrt{n}$

T_0= 3.1031 − 5 /3.040 $\sqrt{32}$
T_0= -1.8969/(3.040/5.65)
T_0= -1.8969/0.5380
T_0= −3.5298

$p - value$ = 2 · $P(t_{31} > -3.5298)$ = 0.0013
T critical region is= $t_{0.975,31}$=1.696, Since p-value 0.0013 is smaller than 0.01 and $|T_{stat}|>t_{crit}$, **we reject null hypothesis.**
There is strong evidence to support alternative hypothesis.

At the 10% level of significance, there is strong evidence to claim the difference between the two methods does not equal 5.

90 % confidence interval,
$(\mu 1-\mu 2) \in (2.192, 4.014)$
The true mean difference value could be between 2.192 and 4.014.