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TIME VALUE OF MONEY

CONCEPT

The value of money changes with time. That is, the worth or benefit that can be derived from ₹ 1,000 today will not be the same as ₹ 1,000 after one year.

This is because of few reasons :

(a) The purchasing power of money declines over time (in an inflationary economy) so the amount of goods and services that can be purchased with ₹ 1,000 today will be more than the amount of goods and services that can be purchased with ₹ 1,000 today will be more than the amount of goods and services that can be bought for the same ₹ 1,000 after one year. And the more the time passes, the lesser will be the purchasing power of the same ₹ 1,000. This is why any rational individual (with consumption needs) will prefer ₹ 1,000 today as compared to ₹ 1,000 in future.

(b) Money can be invested to earn returns, even risk-free returns like in fixed deposits or government securities. Here the investor is only parting with his money for some defined interval of time, without any risk of loss, and as a reward for this, he gets some amount more after the period is completed. For example, ₹ 1,000 invested in a risk-free bank deposit for a year at 8% p.a. will become ₹ 1,080 after one year. So, an individual, who is rational, and has investment need, will prefer ₹ 1,000 today as compared to ₹ 1,000 in future because he has a potential of earning a return on that money during the intervening time period.

(c) The third reason for the preference of rational individuals towards money now, rather than the same money in future is 'Risk' or 'Uncertainty'. No one knows what is going to happen in future. So, any rational individual would prefer what is available now to what may be available in future.

So, having looked at the reasons why future value is not equal to present value, we need to look at a concept which would make it possible to compare the future and present values.

This is usually done using an 'Interest Rate' or 'Rate of Return' which would make it possible to compare the future and present values.

$$\text{Present Value} + \text{Interest} = \text{Future Value}$$

$$\text{Future Value} - \text{Interest} = \text{Present Value}$$

Therefore, if we want to know the actual 'value' of any amount in the future, then we need to add an interest rate to it, while we want to know the 'value' of any future amount as of now, then we need to subtract the interest from it.

The process of calculating the future value from a given present sum is called "Compounding" (as it is based on compound interest). And the process of finding the Present Value of a future sum of money is called "Discounting".

The question is why do we need to know the FV or the PV of an FV?

The reason is that the cashflows in our life do not happen at one point in time. They happen throughout our lives. So, in order to compare the flows, we need to bring them to a common point in-time-in order to understand our gain or loss. For example, we invest money today, but we get the payback after 10 years or we take a loan today, but we pay it back after 5 years. It is not possible to simply subtract the end value from the start value to get the gain/loss. The sums of money have to be either compounded or discounted before a conclusion made. For example :

(a) An investment opportunity which promises to double your investment in 10 years, cannot be evaluated at face value till the present value of the end amount is calculated and compared to the Internal value.

(b) Another example can be if a machine is purchased for ₹ 1 Lakh and it gives return of ₹ 35,000 for three years, then we cannot simply subtract 1 lakh from 1 lakh 5 thousand (35,000 × 3) to get ₹ 5,000 as gained amount, because all the four cashflows are happening at different point of time.

COMPOUNDING

- To add compound interest to the principal amount is called Compounding.
- A man deposits ₹ 10,000 in a bank for 3 years @10% p.a. compounded annually. What does this statement mean? It means that ₹ 10,000 will earn interest @10% for one year (*i.e.* 1000), then at the end of one year, the interest will be added to the original principal amount to get the new principal amount (*i.e.* 10,000 + 1,000 = 11,000). For the second year, interest will be calculated on ₹ 11,000 @10% (*i.e.* ₹ 1100) which will be added to ₹ 11,000 to be ₹ 12,100. Now ₹ 12,100 in the principal for the third year and will earn interest @10%, to get ₹ 1,210. So, the final amount becomes ₹ 13,310.
- In mathematical terms :

  Final Amount = Principal $(1 + i)^n$

  where i= rate of interest in decimals

  n = number of years.

Future Value of Single System

Final amount in financial language is called Future Value and the Original principal is called Present Value. So,

$$FV = PV(1 + i)^n$$

Illustration 1. Calculate the total amount accumulated of ₹ 1,00,000 is invested in the bank @ 8%, compounded annually, for 3 years?

Solution : PV-1,00,000, i = 8% = 0.08, n = 3 years

$$\begin{aligned} FV = PV(1+i)^n &= 1{,}00{,}000(1+0.08)^3 \\ &= 1{,}00{,}000 \times 1.259712 \\ &= 125971.20 \end{aligned}$$

Illustration 2. How much will be ₹ 50,000 invested in a bank FD accumulate to if the rate of interest is 7.5% p.a. and duration is 5 years?

Solution :

$$FV = PV(1 + i)^n$$

$$FV = 50{,}000(1 + 0.075)^5$$
$$= 50{,}000 \times 1.435629$$
$$= 71781.45$$

The above examples were related to yearly compounding. However, in real life, compounding can be done multiple times in a year. This means that the interest can be added to the initial principal to form new principal as often as desired. In such cases, the formula for compounding is adopted as under :

$$FV = PV\left(1 + \frac{i}{m}\right)^{n \times m}$$

Where m = number of times compounding is done in one year.

Illustration 3. What will be ₹ 10,000 amount to after 2 years @12% in a bank deposit if compounding is done :

(a) Annually
(b) Half yearly
(c) Three times a year
(d) Quarterly
(e) Monthly

(a) $m=1$

$$FV = 10{,}000\left(1 + \frac{0.12}{1}\right)^{2 \times 1}$$
$$= 12544$$

(b) $m=2$

$$FV = 10{,}000\left(1 + \frac{0.12}{2}\right)^{2 \times 2}$$
$$= 12624.77$$

(c) $m=3$

$$FV = \left(1 + \frac{0.12}{3}\right)^{2 \times 3}$$
$$= 12653.19$$

(d) $m=4$

$$FV = \left(1 + \frac{0.12}{4}\right)^{2 \times 4}$$
$$= 12667.68$$

(e) $m=12$

$$FV = \left(1 + \frac{0.12}{2}\right)^{2 \times 12}$$
$$= 12697.35$$

(A) Through the above illustration, we can conclude that the more the number of times compounding is done during one year, the higher will be the amount accumulated in the end.

(B) Use of financial tables : Since the FV can be calculated by multiplying the PV with $(1 + i)^n$, therefore, to make calculation easy, these are published tables (called PV tables on financial tables) where the value of $(1 + i)^n$ has been calculated for all possible values of i and n, as under :

Compound Value of ₹1 or FV Factor (Table A)

Ni	1%	2%	3%.........	40%
1	1.010	1.030	1.040	
2.....	1.020	1.061	1.082	
30	1.030	1.093		
			1.125	

So, when we want to know the value of $(1 + i)^n$ for 4%for 3 years, we look at the CVF of FVF for the 4%column and 3 years row and it is 1.093. Now we can directly multiply the PV by 1.25 to get the FV.

$$FV = PV\ (FVF\ i\%;\ n\ \text{years})$$

Illustration 4. The fixed deposit scheme of HDFC Bank offers the following interest rates :

58 days to 187 days	6%
188 days to 1 year	6.5%
1 year and above	7%

Calculate the amount accumulated if ₹ 1,00,000 are invested in HDFC Bank FD for 3 years, assume compounding is done semi-annually.

Solution : PV=1,00,000; i=7%; n=3 years; m=2

$$FV = PV\left(1 + \frac{.07}{2}\right)^{3 \times 2}$$

$$= PV(FVF_{3.5\%},\ 6)$$

Looking at the compound Value Factor for 6 years 3% is 1.194 and 6 years 4% is 1.265, so for 3.5%, the average of 1.194 and 1.265 will be used.

$$FVF(3.5\%,\ 6) = 1.194 + \frac{1.265}{2} = 1.2295$$

$$FV = 1{,}00{,}000 \times 1.2295 = 1{,}22{,}950$$

(C) **Future Value of multiple cashflows** : Suppose instead of depositing only once, an investor deposits money every year (or multiple times) into the same investment then how will be the total accumulated amount be calculated?

Illustration 5. A man invests ₹ 10,000 today, ₹ 12,000 after one year, ₹ 15,000 after two years, and ₹ 18,000 after three years in a deposit account earning 10% interest p.a. What will be the total amount accumulated at the end of 4 years from now? Assume compounding is done annually.

Solution : First a timeline is made to visualize the cashflows :

0	1	2	3
10,000 10% 4 years FV = 10,000 (FVF = 10%, 4)	12,000 10% 3 years FV = 10,000 (FVF = 10%, 3)	15,000 10% 2 years FV = 10,000 (FVF = 10%, 2)	18,000 10% 1 years FV = 10,000 (FVF = 10%, 1)

So, as shown in the table, FV of multiple cash sum, of money is calculated separately, depending on the time for which the amount has been invested.

$$\text{FV}(10{,}000) = 10{,}000(\text{FVF } 10\%, 4\text{yrs}) = 10{,}000 \times 1.464 = 14{,}640$$
$$\text{FV}(12{,}000) = 10{,}000(\text{FVF } 10\% \ 3\text{yrs}) = 10{,}000 \times 1.331 = 15{,}972$$
$$\text{FV}(15{,}000) = 10{,}000(\text{FVF } 10\% \ 2\text{yrs}) = 10{,}000 \times 1.210 = 18{,}150$$
$$\text{FV}(18{,}000) = 10{,}000(\text{FVF } 10\% \ 1\text{yr}) = 10{,}000 \times 1.100 = 19{,}800$$
$$\text{Total FV} = 68{,}562$$

Therefore to determine the FV of multiple cashflows, we need to find the individual FV of each cashflow depending on the time for which it has been invested then add up all the FV's to get the total FV.

Future Value of Annuity (Multiple equal cashfflows) : If there are a series of cashflows, which are equal in amount and occur at equal intervals from zero time to nth time, then such a series of cashflows are called Annuities. These are two kinds of annuities :

(a) *Annuity* : which occurs at the end of each year or time period.

0	1	2	5	4	5
0	5,000	5,000	5,000	5,000	5,000

Examples of a simple annuity of ₹ 5,000 which occurs at the end of every year from year 1 to year 5.

(b) *Annuity Due* : which occurs at the beginning of each year or time periods.

0	1	2	3	4	5
5,000	5,000	5,000	5,000	5,000	

Examples of an annuity due of ₹ 5,000 which occurs at the beginning of each year from year 1 to 5.

$$\text{FV of Annuity} = \text{Annuity (FVAF } i\%, n \text{ years)}$$

This is the second table called future value factor for an annuity of ₹ 1 called Table 2 or Table B.

$$\text{FV of Annuity Due} = \text{Annuity due (FVAF } i\%, n \text{ years)} (1 + i)$$

Illustration 6. A man deposits ₹ 1,00,000 in an annual recurring deposit in a bank paying 9% p.a., every year for 10 years. Calculate the amount accumulated at the end of 10 years if :

(a) the 1,00,000 p.a. is deposited at the end of every year.

(b) the 1,00,000 p.a. is deposited at the beginning of every year.

Solution :

(a) This cashflow is a simple Annuity

$$\text{FV} = \text{Annuity}(\text{FVAF}_{i\%}, n \text{ yrs})$$
$$= 1{,}00{,}000(\text{FVAF}_{9\%}, 10 \text{ yrs})$$
$$= 15{,}19{,}300$$

(b) This cashflow is an annuity due

$$FV = \text{Annuity due } (FVAF_{i\%}, n)(1 + i)$$
$$= 1{,}00{,}000(FVAF_{9\%}, 10 \text{ yrs})\ (1 + 0.09)$$
$$= 1{,}00{,}000 \times 15.193 \times 1.09$$
$$= 16{,}56{,}037$$

From the above illustration, it is clear that the amount accumulated in case of an annuity due will always be more than the amount accumulated in case of a simple annuity. This is because the amount instead gets more time to grow (or earn interest) in case of an annuity due.

However, it must be remembered that these formulae of Annuity FV and table 2 factors can be used only in case of an annuity cashflow. If the cashflow are unequal or unevenly spaced, then the FV of each individual cashflow has to be separately calculated and then the total FV will be found by adding the separate FV's.

Calculation of Rate and Time using Table 1 and 2

In addition to calculation of FV, the tables 1 and 2 can also be used to calculate the rate being earned and time taken.

Illustration 7. Calculate the time taken to grow ₹ 10,000 to ₹ 20,000, if the rate of interest is 20%

Solution : $FV = PV(FVF_{i\%}, n)$

$$(FVF_{8\%}, n) = \frac{FV}{PV}$$
$$= \frac{20000}{10000} = 2$$

Now, in table 1, in the 8% column, we search the factor closest to 2.000 and check which year it corresponds to. We find that the factor of 9 years is 1.999, which is almost equal to 2.000. Therefore, we can say that it will take approximately 9 years to grow ₹ 10,000 to ₹ 20,000 @ 10%.

Hence, the answer is 9 years.

Illustration 8. An investment promises to triple the invested amount in 9 years. What is the rate of return?

Solution : Assume PV= 10000 and FV= 30000

$$FVF_{(i\%,9)} = \frac{FV}{PV}$$
$$= \frac{30000}{10000} = 3.000$$

Now, in table 1 in the 9 years row, we search that factor corresponding to 3.000. We find that the factor of 13% is3.004 which is nearest to 3.000.

Therefore we can say that the rate of return that will triple our investment of years is 13% approx.

Illustration 9. A man deposit ₹ 10,000 at the end of every year in a bank deposit paying 8%. After how many years will amount accumulate to ₹ 2,00,000?

Solution : This is the case of an annuity so table 2

$$FV = \text{Annuity}(FVA\ F_{8\%}, n)$$

$$2{,}00{,}000 = 1{,}00{,}00(FVA\ F_{8\%}, n)$$

$$FVA\ F_{8\%} = \frac{2{,}00{,}000}{10{,}000} = 20$$

Now in table 2 we search the factor closest to 20.000 in the 8% column. We find that the factor for 12 years is 18.977 and for 13 years is 21.495. Our required factor 20.000 is between these two. Therefore we can say that it will take 12-13 years to grow on annuity of ₹ 10,000 to ₹ 2,00,000 @8%.

Note : The exact time can also be found b/w 12y and 13y using unitary method)

Discounting

- Discounting is the opposite of compounding.
- It means that the present value of the future sum of money is to be calculated. This is done by removing the accumulated compound interest from the FV to arrive at the PV.

$$PV = FV\frac{1}{(1+1)n} = FV(PVF_{i\%}, n) \quad \text{.......Table 3 Present value of ₹ 1}$$

In case of compounding in case of discounting also tables have been published which have free calculated values of $\frac{1}{(1+i)}$ n for various values of i and n such that it becomes easy to calculate PV by simply multiplying the given FV by corresponding $PVF_{(i\%,\ n)}$ obtained from table 3.

Illustration 10. what is the present value of ₹ 1 lakh which will be received after 5 years if the rate interest is 10% per annum?

Solution :

$$PV = FV(PVF_{i\%}, n)$$

$$PV = 1{,}00{,}000\ (PVF_{10\%}, 5\text{ years})$$

$$= 1{,}00{,}000 \times 0.621 = 62{,}100$$

This means that, in terms of current was the 1 lakh to be received after 5 years is now worth ₹ 62,100 only and during 5 years a compound interest on ₹ 37,900 (*i.e* 1,00,000 – 62,100) will be added to it.

Illustration 11. If I want to accumulate ₹ 10,00,000, at the end of 10 years then how much would I have to invest today in a bank FD giving 8% to get my target?

Solution :

$$PV = (PVF_{i\%}, n)$$

$$= 10{,}00{,}000\ (PVF_{8\%}, 10)$$

$$= 10{,}00{,}000\ (0.463)$$

$$= 4{,}63{,}000$$

Discounting of uneven multiple cashflows

If we did not have a single FV, but instead multiple unequal FVs and we want to know the PV of all these future values, then, this is called Discounting of Multiple cash flows. In this situation, the PV of all these future values then this is called Discounting of multiple cash flows. In this situation, the PV of each individual cashflows is calculated separately and then all the PVs are added to get the total PV.

Illustration 12. My target is to accumulate 5 lacs for my daughters graduation which is scheduled 15 years from now and 10 lacs for the post-graduation which is scheduled 18 years from now how much should I invest today in a bank deposit earning 10% pa to achieve both goals?

Solution :

$$PV\ (5\ \text{lacs}) = FV(PVF_{10\%}\ 15yrs)$$
$$= 500000 \times 0.239 = 1,19,500$$
$$PV(10\ \text{Lacs}) = FV(PVF_{10\%}, 18)$$
$$= 10,00,000 \times 0.180 = 1,80,000$$

Total $PV = 1,19,500 + 1,80,000 = 2,99,500$

This means that if I deposit ₹ 2,99,500 @10% today then I will be able to withdraw 5 lacs after 15 years and 10 lacs after 18 years from this investment.

If however, the futures cashflows are in the form of the annuity or annuity due, then the calculation become more simple because then we will use table 4 which is the present value factor of the the annuity.

Discounting an Annuity

In case of annuity (equal annual cashflows) at the end of each year and annuity due (equal annual cashflows) at the beginning of each year the PV is calculated using Table 4 :

$$\text{PV of annuity} = \text{Annuity}\ (PVFA_{i\%}, n)$$
$$\text{PV of annuity Due} = \text{Annuity due}\ (PVFA_{i\%}, n)\ (1 + i)$$

Illustration 13. A pensioner wishes to deposit a lumpsum amount in an Annual Pension Scheme today which pays a certain amount of ₹ 1,20,000 every year for 10 years? How much should he deposit today to get ₹ 1,20,000 every year.

(a) at the end of every year, (b)at the beginning of every year?

Solution :

(a) This is the case of an annuity

$PV = \text{Annuity}\ (PVAF_{i\%}, \text{litalicn})$

$= 120000\ (PVAF_{10\%}, 10)$

$= 1,20,000 \times 6.145$

$= 737400$

This means that if he deposits ₹ 7,37,400 now @ 10% then he will get an annuity of ₹ 1,20,000 every year end for 10 years from his investment as annual pension

(b) This is an annuity due

$PV = \text{Annuity due}\ (PVAF_{10\%}, 10) \times (1 + 0.10)$ c=10/=0.10

$= 1,20,000 \times (6.145) \times (1.10)$

$= 8,11,140$

This means that if the pensioner wants ₹ 1,20,000 at the beginning of each year for 10 years then he should deposit ₹ 8,11,140 today @10 p.a.

Illustration 14. An equal yearly payment end of each year of ₹ 1,50,000 will completely pay off a loan taken for 5 years @11%. What is the loan amount?

Solution :

This is an annuity payment. The loan is the PV

$$\begin{aligned}PV &= \text{Annuity } (PVAF_{i\%}, n)\\ &= 1,50,000\ (PVAF_{11\%}, 5)\\ &= 1,50,000 \times (3.696)\\ &= 554400\end{aligned}$$

The principal amount of the loan is ₹ 5,54,400.

Illustration 15. Calculate the equal yearly investment that will completely pay off a loan of ₹ 20,00,000 taken today @12% p.a. for 15 years. Assume end of year payment.

Solution :

$$PV = \text{Annuity } (PVAF_{i\%}, n)$$

$$PV = 20 \text{ lacs}$$

$$\text{Annuity} = \frac{PV}{(PVAF_{12\%}, 15y)}$$

$$= \frac{20,00,000}{6.811} = 2,93,642.64$$

This means that an equal yearly end of year payment of ₹ 2,93,642.64 will completely pay off the loan principal + interest amount in 15 years.

Applications of Time Value of Money : (In Banking)

From the preceding discussion, it is clear that, the time value of money affects every transaction in the financial world. In fact, it is impossible to make an investment or take loans without exercising time value of money.

As the banking industries main function is to accept deposits from the public in the form of savings, current, recurring and fixed deposits and to give loans to retails individuals and to corporates as well as the govt. the banking industry is totally based on the time value concept. Whether they want to calculate the expected amount from an F. D. or whether they want to calculate the EMI for loans, the banking sector cannot function without TVM.