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Assignment 2

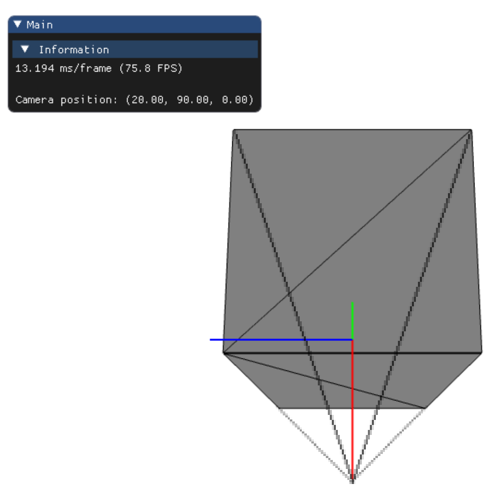
1.

a. (Note: Please press P to see the perspective image of the cuboid)
I started by doing the camera transformation from world coordinate system to camera coordinate system. I did this by forming a new basis u,v,w from e,g,t of the camera.
Gaze vector g = -(camera Position)
Up vector t = (0.0, 0.1,0.0)
Lookat point = (0.0,0.0,0.0)

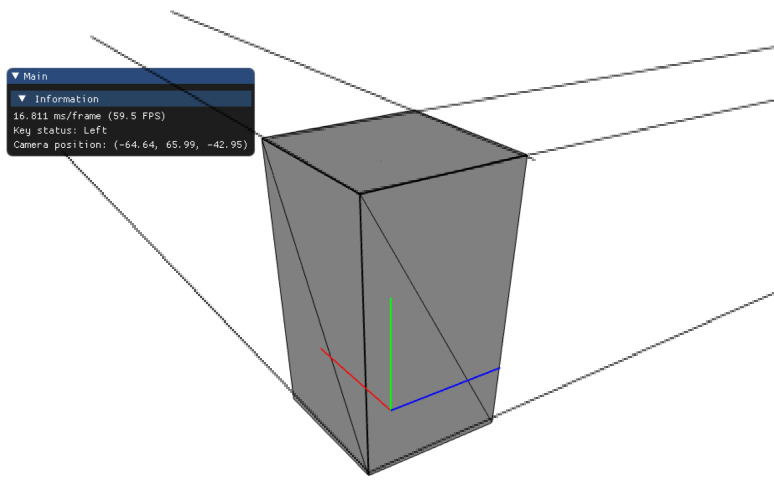
w = -g / ||g||
u= (t x w) / ||t x w||
v=(w x u)

The next step was to perform the translation along axes in the uvw basis.
When right, left arrow keys are pressed the camera position changes accordingly:
camPosition= (camPosition.x-u.x, camPosition.y=u.y, camPosition.z=u.z)
Same steps are followed for the other directions.

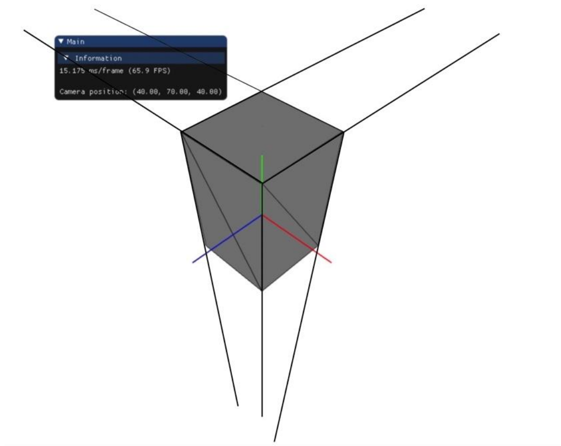
b. (i) 1 point perspective(1 vanishing point)



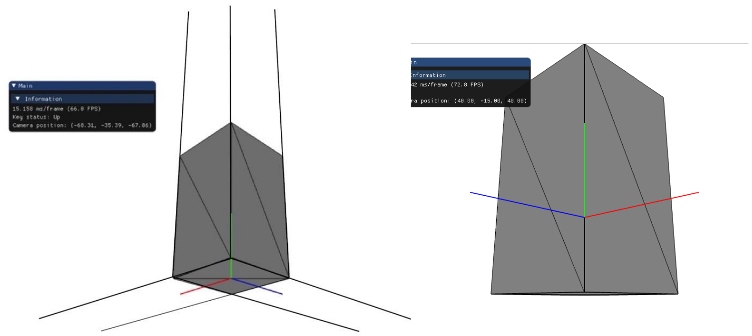
(ii) 2 point perspective(2 vanishing points)



(iii) bird's eye view(3 point perspective)



(iv) rat's eye view(3 point perspective)



2. In the second question I've programed the 'P'/'p' and 'O'/'o' keys to switch between perspective and orthographic positions.
   The coordinates of the orthographic positions are allocated according to the scale factor.

```
if(ImGui::IsKeyPressed('O')){
    projectionT =  glm::ortho(-45.0f, 45.0f, -45.0f, 45.0f, 20.0f, 150.0f);
    //projectT=((GLfloat)screen_width/(GLfloat)screen_height)*glm::scale(glm::mat4(1.0f), projectio
    glUniformMatrix4fv(vProjection_uniform, 1, GL_FALSE, glm::value_ptr(projectionT));
}
else if(ImGui::IsKeyPressed('P')){
    projectionT = glm::perspective(45.0f, (GLfloat)screen_width/(GLfloat)screen_height, 0.1f, 1000.
    glUniformMatrix4fv(vProjection_uniform, 1, GL_FALSE, glm::value_ptr(projectionT));
}
```

For the 2nd part I added an if clause within the if else statements of the arrow keys. The if clause checked for the proximity towards the axes and whether the ctrl key is pressed or not.

```
if (ImGui::IsKeyDown(ImGui::GetKeyIndex(ImGuiKey_LeftArrow))) {
  strcpy(textKeyStatus, "Key status: Left");
  camPosition=glm::vec4(camPosition.x-u.x, camPosition.y-u.y, camPosition.z-u.z, 1.0);

  if(camPosition.x<60.0f && ImGui::IsKeyPressed(GLFW_KEY_LEFT_CONTROL)){
      camPosition=glm::vec4(100.0f, 0.0f, 0.0f, 1.0);
  }

}
```

3.

$$z'(z) = n + f - \frac{nf}{z}$$

Perspective projection $P = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Perspective transformation on x, y, z :

$$P \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} nx \\ ny \\ (n+f)z - fn \\ z \end{bmatrix}$$

$$= z \begin{bmatrix} \frac{nx}{z} \\ \frac{ny}{z} \\ n+f - f\frac{n}{z} \\ 1 \end{bmatrix}$$

Perspective projection

$M_{per} = M_{orth} * P$ $\quad M_{orth} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$M_{per} = \begin{bmatrix} 2/_{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & 2/_{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & 2/_{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$M_{per} = \begin{bmatrix} 2n/(r-l) & 0 & -\frac{(r+l)}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & -\frac{(t+b)}{t-b} & 0 \\ 0 & 0 & -\frac{(f+n)}{f-n} & \frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$z'(z) = n + f - \frac{nf}{z}$$

$$z' = \cancel{(z) \times (-f+n} \; -\frac{f+n}{f-n} \times z + \frac{2fn}{f-n}$$

(after projection)

$$= -\frac{(f+n)}{f-n}\left(n+f-\frac{nf}{z}\right) + \frac{2fn}{f-n}$$

$$= -\frac{(f+n)(n+f)}{f-n} + \frac{nf(f+n)}{z(f-n)} + \frac{2fn}{f-n}$$

$$= -\frac{(f^2+n^2+\cancel{2nf}-\cancel{2fn})}{f-n} + \frac{nf(f+n)}{z(f-n)}$$

$$z' = \cancel{\frac{-z(f^2+n^2)+nf^2+n^2f}{z(f-n)}} \qquad \frac{f^2+n^2}{n-f} + \frac{nf(f+n)}{z(f-n)}$$

~~z~~

divide by w to ~~p~~ map $\boxed{w = -z}$

$$z' = n + f - \frac{nf}{z} \qquad z_1' = n + f - \frac{nf}{z_1}$$

$$z'(n) = n + f - \frac{nf}{n} \qquad z_2' = n + f - \frac{nf}{z_2}$$

$$= n$$

$$z'(f) = n + f - \frac{nf}{f} \qquad \frac{nf}{z} = n + f - z'$$

$$= f \qquad z = \frac{nf}{n + f - z'}$$

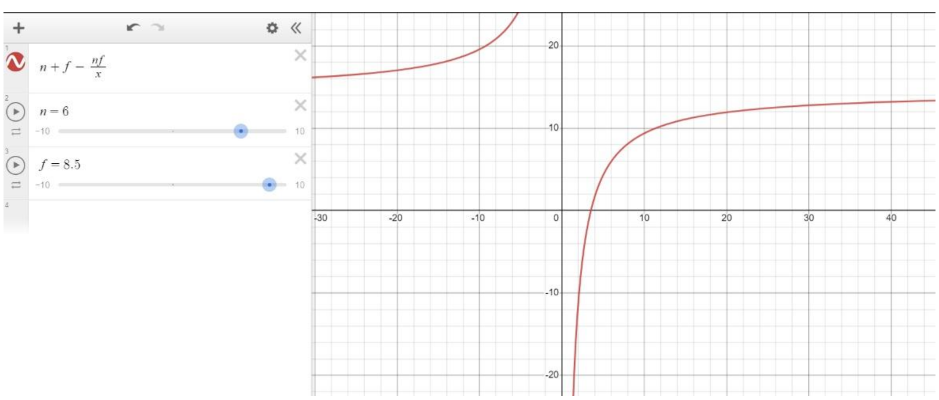
$z_1 < z_2$ (Assumption)

$$\frac{nf}{n+f-z_1'} - \frac{nf}{n+f-z_2'} < 0$$

$$\frac{\cancel{nf}}{n+f-z_1'} < \frac{\cancel{nf}}{n+f-z_2'}$$

$$\cancel{n+f} - z_1' > \cancel{n+f} - z_2'$$

$$z_1' < z_2' \qquad \text{Hyperbolic function}$$



z’ is a Hyperbolic function in terms of z. (The graph is plotted using desmos and n=6, f=8.5)