CS : 215
Signal & Data Communication Laboratory

Experiment: V-B

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Problem

Suppose a system is governed by the differential equation,

$$\frac{dy(t)}{dt} + 5y(t) = 3\frac{dx(t)}{dt} - x(t)$$

Find a closed form expression for the complete system response (both zero-state-response [ZSR] and zero-input-response [ZIR]), given that, initial condition $y(0^-) = 3$ and input $x(t) = \sin(t)u(t)$. Also, find both steady-state response and transient response of the system. You may use "dsolve" function of MATLAB. Further, choose time span judiciously.

Plot ZSR, ZIR, complete response, and transient response due to input.

Theoretical Background

Zero Input Response(ZIR) : The zero input part of the response is the response due to initial conditions alone with no input given to the system.

Zero State Response(ZSR) : The zero state part of the response is the response due to the system input alone (with initial conditions set to zero).

Transient Response : The transient response (also called natural response) of a causal, stable LTI differential system is the homogeneous response, i.e., with the input set to zero(initial value).

Steady State Response : The steady-state response (or forced response) is the particular solution corresponding to a constant or periodic input.

Complete Response : The total response generated by a system, considering both initial value and a given input.

$$\text{Complete Response} = \text{ZSR} + \text{ZIR}$$

Methodology

1. Our task is to find the ZSR and ZIR for a given system represented by the given differential equation. To solve the differential equation,'dsolve' function is used wherever required.

2. To find ZSR, we set initial condition $y(0^-) = 0$ and calculate the subsequent expression for ZSR by using 'dsolve' function.

3. Taking a suitable range, we plot values of ZSR against discrete time.

4. Similarly, To find ZIR, we set input $x(t)$ as zero. Thus, RHS of the differential equation becomes zero and the linear differential equation reduces to homogeneous. We can solve the equation manually or using 'dsolve' function.

5. Taking a suitable range, we plot values of ZIR against discrete time.

6. Complete Response(CR) = ZSR + ZIR is calculated and plotted against time using suitable range.

7. Observations are made and the Transient Response and Steady State Response of the system is found using the plot.

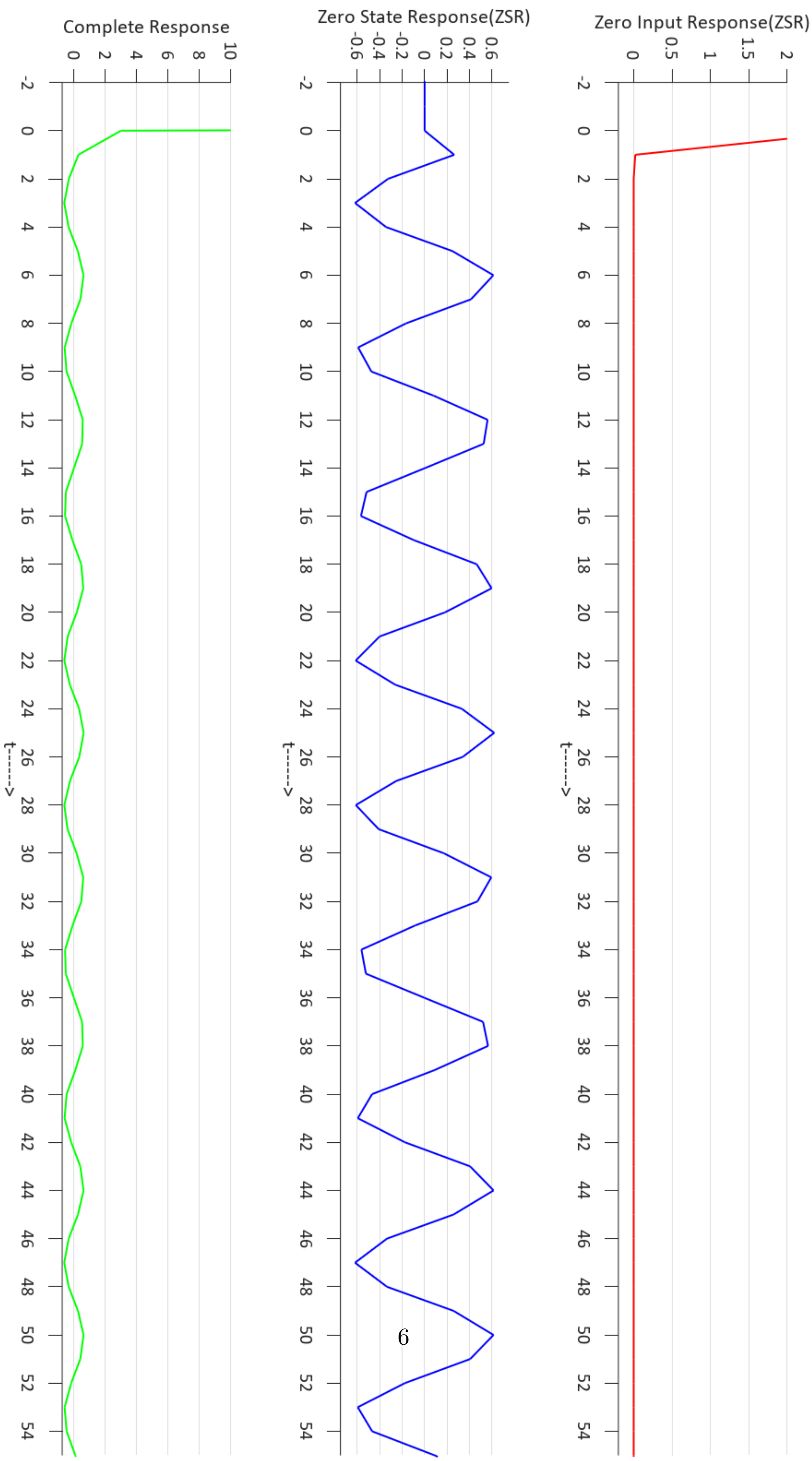
Code

```
clear all;
clc;

syms t x(t) y(t);
x(t) = sin(t) * heaviside(t);
eqn = diff(y, t) + 5*y == 3*diff(x, t) - x;
Sol = dsolve(eqn);

range = [-100:100];

% Zero Input Response Input x(t) = 0
eqn = diff(y, t) + 5*y == 0;
cond = y(0) == 3;
sol1 = dsolve(eqn, cond);
ZIR = function_ handle(rhs(sol1));
VarZIR = double(ZIR(range));
subplot(3,1,1);
plot(range, VarZIR, ’r’, ’linewidth’, 1);

xlabel(’t------>’);
ylabel(’Zero Input Response(ZSR)’);
set(gca, ...
    ’Box’, ’off’, ...
    ’TickDir’, ’out’, ...
    ’YGrid’, ’on’, ...
    ’FontSize’, 20, ...
    ’FontName’, ’Calibri’);
```

```
axis([-255 - 0.22]);
% Zero State Response Initial Condition = 0
x(t) = sin(t) * heaviside(t);
eqn = diff(y, t) + 5*y == 3*diff(x, t) - x;
cond = y(0) == 0;
sol2 = dsolve(eqn, cond);
ZSR = function_ handle(rhs(sol2));
VarZSR = double(ZSR(range));
subplot(3,1,2);
plot(range, VarZSR, 'b', 'linewidth', 1);
xlabel('t------>');
ylabel('Zero State Response(ZSR)');
set(gca, ...
     'Box', 'off', ...
     'TickDir', 'out', ...
     'YGrid', 'on', ...
     'FontSize', 20, ...
     'FontName', 'Calibri'); ([-2 55 -0.75 0.75]);
% Complete Response = ZSR + ZIR
subplot(3,1,3);
plot(range, VarZSR + VarZIR , 'g', 'linewidth', 1);
xlabel('t------>');
ylabel('Complete Response');
set(gca, ...
```

```
    'Box', 'off', ...
    'TickDir', 'out', ...
    'YGrid', 'on', ...
    'FontSize', 20, ...
    'FontName', 'Calibri');
axis([−255 − 0.7510]);

% Total Response Input x(t) = 0 Found to be same as Complete Response
x(t) = sin(t) * heaviside(t);
eqn = diff(y, t) + 5*y == 3*diff(x,t) - x;
cond = y(0) == 3;
finalRes = dsolve(eqn, cond);
```

Graphs And Analysis



Input Data Description

Given Equation,

$$\frac{dy(t)}{dt} + 5y(t) = 3\frac{dx(t)}{dt} - x(t)$$

Input : $x(t) = \sin(t)u(t)$
Initial Condition: $y(0^-) = 3$

Result

By comparing the plots, we observe that the transient response is shown by an exponential decaying curve and approximates to zero as it reaches 1. Thus, transient response runs upto 1.0 approximately on time axis. The graph after t=1.0 is almost constant and oscillatory for the remaining time period. Thus, the Steady State Response for the differential equation is zero.

Discusssion

- We found ZIR = $3e^{-5t}$
- ZIR rapidly decreases and drops down to nearly zero as early a t=1. This is the period for which transient response lasts.
- The rest of the plot is oscillatory and varies between $[-0.6, 0.6]$. Thus, the Steady State Response for the given system is zero.
- It is also to be noted that ZIR, ZSR are not the same as Transient Response, Steady State Response. While the former terms can be obtained on plot using octave, the latter is more conceptual and can only be observed through plot.
- ZIR is independent of system input, while ZSR is independent of initial condition. Thus only one of the two changes when there is a variation in either initial condition or the system input.