CSE300 Assignment1
Introduction to LaTeX
Enumeration via Euler Characteristic Integrals

Madhusudan Basak
Mahmudur Rahman Hera
Student ID: 1505071

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Department of Computer Science and Engineering
Bangladesh University of Ensigneering and Technology
(BUET)
Dhaka 1000
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Abstract

We solve the problem of counting the total number of observable targets (e.g., persons, vehicles, landmarks) in a region using local counts performed by a network of sensors, each of which measures the number of targets nearby but neither their identities nor any positional information. We formulate and solve several such problems based on the types of sensors and mobility of the targets. The main contribution of this paper is the adaptation of a topological sheaf integration theory integration with respect to Euler characteristic to yield complete solutions to these problems.

Key words: Euler characteristic, sensor network, enumeration, integration

1 Topological enumeration.

1.1 Sensors.

Sensor networks are poised to impact society in fundamental ways analogous to the impact of the personal computer and the internet. The rapid development of small-scale sensor devices coupled with wireless ad hoc networking capability is giving birth to a wide variety of sensor networks for applications to agriculture, defense, environmental monitoring, and more.

1.2 A simple example: Geometric enumeration.

The following simple example of target enumeration has a trivial geometric solution that motivates our topological techniques. Consider a field of (infinitesimally small) sensors, one at each point x in a planar domain $\mathbb{R}$. Assume there is a finite set of points acting as observables or targets $O_\alpha \subset \mathbb{R}$, and that the sensor at x $\in \mathbb{R}$ returns a quantized count h(x)$\in N$.

How can sensors merge their local counts into a global count of the number of targets? If the targets are sufficiently separated, then this becomes a simple problem of counting the number of connected on clusters in the network . If we want to allow for more complex target interactions, the following critical (if not entirely realistic) assumption makes the problem very simple.Then the total number of targets may be computed as an integral:

$$O_\alpha = \tfrac{1}{M} \int_{\mathbf{R}^2} h(x)dx \qquad\qquad (1)$$

Where

$$M = \pi\mathbf{R}^2$$

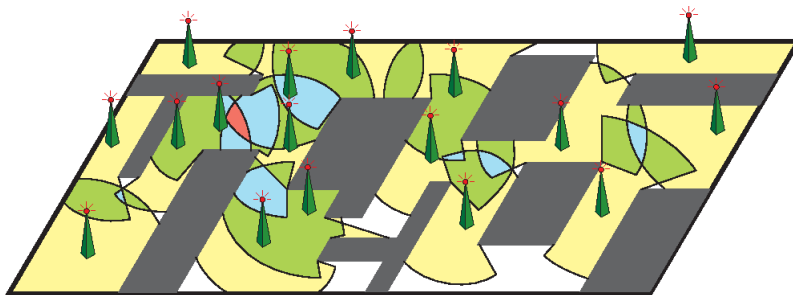$$h = \sum_\alpha \mathbb{1}U_\alpha$$

Fig.1 . A sensor field counts the number of in-range visible beacons based on line-of-sight. Target supports (the region in the sensor space from which the target is sensed) may vary greatly in size and shape, depending on beacon strength and target space obstacles; however, target supports are star-convex with respect to the target and thus all topologically trivial.

1.3   Other problem statements.

There are numerous interesting and challenging problems that involve determining a global count based on a local count. Some variants we address in this paper include the followin

1. Each sensor sweeps a finite length beam or cone over all bearings, recording the number of targets sensed as a function of bearing and locatio

2. Each sensor increments an internal counter whenever a moving target comes within rang.

3. Each sensor increments an internal counter whenever a moving wave front. initiated by a target event passes within rang

2   Integration with respect to Euler characteristic.

Our results follow from the classical and elegant theory of integration with respect to Euler characteristic. We restrict attention to subsets of $\mathbf{R}^2$.

2.1   The simplicial approach.

For simplicity, we begin our treatment of topological integration with simplicial complexes (triangulated piecewise-linear sets) in Rn. For $\kappa \geq 0$, a $\kappa - simplex, \sigma$, is the convex hull of a set of $\kappa + 1$ affinely independent vertices V_{i0}^k in

$$\mathbf{R}^n : \sigma = \{\sum_i t_i v_i : t_i \in (0,1), \sum_i t_i = 1\}.$$

The geometric Euler characteristic of a simplicial complex X is

$$\chi\chi = \sum_\sigma (-1)^{dim\sigma} = \sum_\kappa^\infty (-1)^\kappa \qquad (2)$$
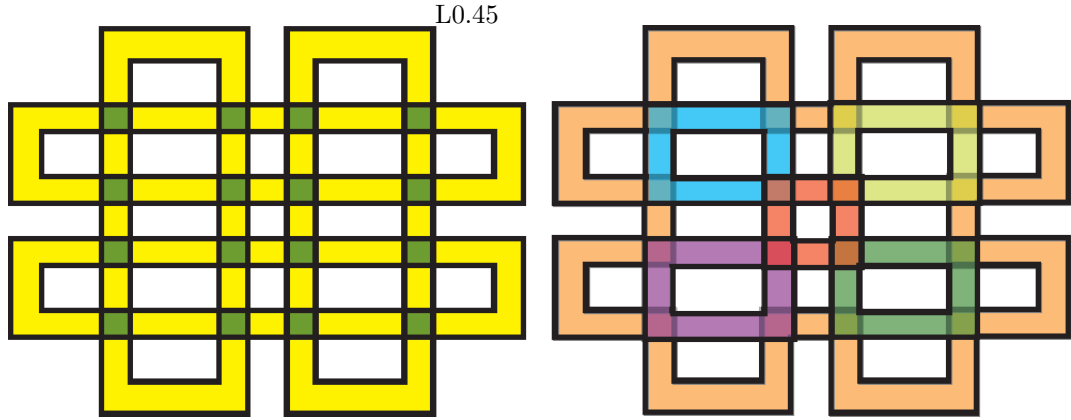
Here κ simplices in χ.

L0.45

fig.2 .$h = \sum_\alpha \mathbb{1} U_\alpha$ with $\chi(U_\alpha) = 0$,one has $\sum h dx = 0$. It is not possible in general to determine α. (Left) The height function of a collection of annuli. Four? or (right) six? Any even number between two and twelve is possible with embedded annuli

2.2   The definable approah.

We extend the definitions to a wide class of spaces which are not obviously simplicial. The reader who is in a hurry may skip ahead to section 3 and remain in the class of simplicial complexes. Not all sets are measurable in $d\chi$: Cantor sets, Hawaiian earrings, and other wild sets do not possess a well-defined Euler characteristic.

2.3   The sheaf approach.

. There is, as intimated earlier, a deeper treatment in the literature on sheaves. There are significant and deep perspectives from sheaf theory that can be easily interpreted in the setting of Euler integration. A sheaf is, roughly speaking, a means of assigning to open sets of a topological space some algebraic object (e.g., a group) in a manner that respects the operations of (1) restriction and (2) gluing. The canonical example of a sheaf on a space X is C(X), the continuous functions $X \to R$.
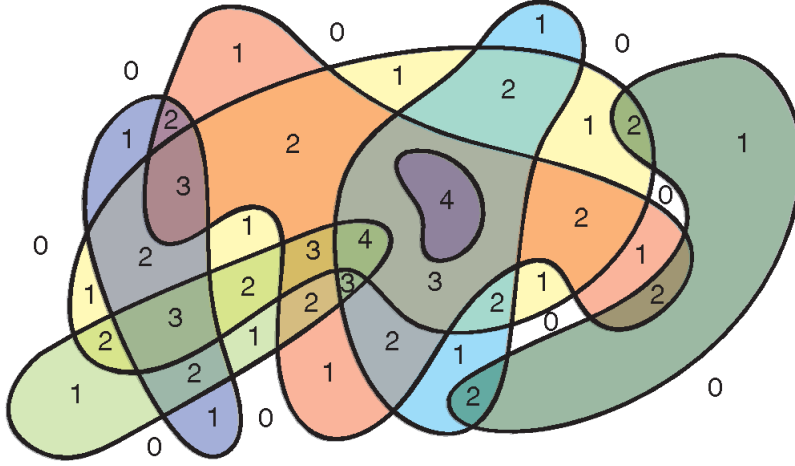

Fig.3 .A collection of contractible patches U_α in
$\mathbf{R}^2$ **corresponding to the supports or visibility regions of seven targets.**

3   From fields to networks.

The mathematical tools formulated here for enumeration problems depend on having a sensor field with counting data at all points in a continuum of the sensor space. Any realistic implementation must occur over a discrete collection of sensors: a network, where nodes N (typically within the target space $N \subset W) record values$.

Trying to parameterize the sensor space X as a discrete set based on the nodes N is doomed to failure, as the target supports will be likewise discrete and of unknown and nonuniform Euler characteristic. Likewise, if the communication links between nodes give the sensor network the structure of a graph, then the network graph is an equally bad candidate for the sensor space X: cycles in the graph can make the Euler characteristics of the target supports vary greatly. In fact, the denser the network in W, the more negative the Euler characteristics of the target supports become (e.g., in Figures 5, 6, and 7, the target supports intersected with the network are graphs with negative Euler characteristic).

3.1 Numerical analysis and Euler integrals.

A more sensible parametrization is to model the sensor space X as a simplicial approximation to the target space W, using the nodes N as vertices. Assume that enough structure is known about N to give a simplicial structure (a triangulation) that has N as the vertex set. In analogy with the problem of computing a numerically approximate Riemann integral of an integrand h based on a discrete sampling, one guesses that the integral of the piecewise-linear (PL) interpolation h_{PL} of h based on the sampled values at N is a good approximation.
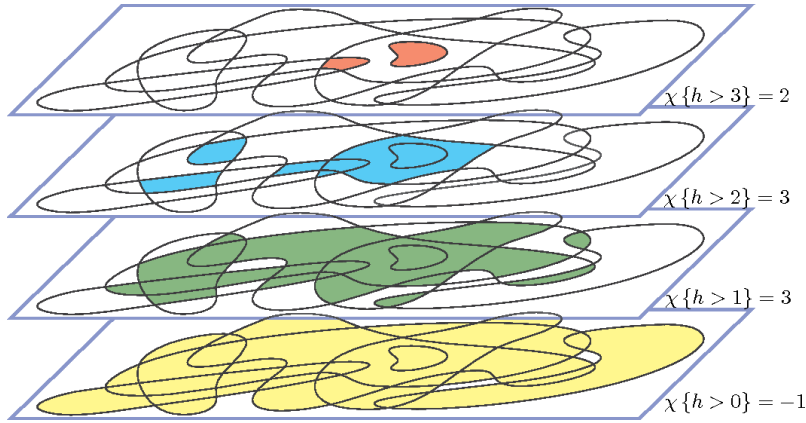

Fig.4 Decomposing h into upper excursion sets and computing yields the integral $\int h d\kappa$.

3.2 Ad hoc planar networks.

We note that the strategy of converting the sampling of the true impact function h over N to a PL interpolation h_{PL} does not necessarily require knowing the coordinates of the nodes. Indeed, the evaluation of

$$\int_\kappa d\kappa$$

is conspicuous in its freedom from coordinate geometry: it is a topological integral. If one is given a triangulation, the extension of the counting function h on vertices over the domain is automatic. However, if no geometry associated with N is known, it may not be possible to determine a canonical extension h_{PL} over the domain. Such a situation is not uncommon in sensor networks based on ad hoc wireless communications, an increasingly common protocol for distributed sensor networks and robotics.