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## 1.Uniformity Test:

| n | ppf(k=10) | Chi-squared (k=10) | Status (k=10) | ppf(k=20) | Chi-squared (k=20) | Status (k=20) |
|---|---|---|---|---|---|---|
| 20 | 14.68 | 7.08 | Not rejected | 27.20 | 17.21 | Not rejected |
| 500 | 14.68 | 2.0 | Not rejected | 27.20 | 7.68 | Not rejected |
| 4000 | 14.68 | 6.95 | Not rejected | 27.20 | 17.34 | not rejected |
| 10000 | 14.68 | 7.08 | Not rejected | 27.20 | 17.21 | Not rejected |

For large n, $\chi^2$ will have an approximate chi-square distribution with $k - 1$ degrees of freedom under the null hypothesis: $H_0$ = $U_i$'s are IID random variables. We reject this hypothesis at level $\alpha$ if $\chi^2 > \chi^2_{(k-1,1-\alpha)}$ where $\chi^2_{(k-1,1-\alpha)}$ is the upper $1 - \alpha$ critical point of the chi-square distribution with $k - 1$ degrees of freedom.

Here for all the test it is “not rejected”.So the uniformity test passes.

## 2.Serial test:

| n | d=2,k=4 (ppf,chi-squared) | d=2,k=4 (status) | d=2,k=8 (ppf,chi-squared) | d=2,k=8 (status) | d=3,k=4 (ppf,chi-squared) | d=3,k=4 (status) | d=3,k=8 (ppf,chi-squared) | d=3,k=8 (status) |
|---|---|---|---|---|---|---|---|---|
| 20 | 22.30, 12.8 | Not rejected | 77.74,38.40 | Not rejected | 77.74,27.20 | Not rejected | 552.37,161.60 | Not rejected |
| 500 | 22.30, 130.04 | rejected | 77.74,152.83 | rejected | 77.74,240.96 | rejected | 552.37,378.944 | Not rejected |
| 4000 | 22.30,1013.45 | rejected | 77.74,1039.26 | rejected | 77.74,1803.45 | rejected | 552.37,1948.52 | rejected |
| 10000 | 22.30,2512.94 | rejected | 77.74,2536.47 | rejected | 77.74,4467.70 | rejected | 552.37,4620.29 | rejected |

The serial test, is really just a generalization of the chi-square test to higher dimensions. If the $U_i$ ’s were really IID U(0, 1) random variates, the nonoverlapping d-tuples should be IID random vectors distributed uniformly on the d-dimensional unit hypercube, $[0, 1]^d$

If the individual $U_i$ ’s are correlated, the distribution of the d-vectors $U_i$ will deviate from d-dimensional uniformity; thus, the serial test provides an indirect check on the assumption that the individual $U_i$ ’s are independent. For example, if adjacent $U_i$ ’s tend to be positively correlated, the pairs ($U_i$ , $U_{i+1}$) will tend to cluster around the southwest-northeast diagonal in the unit square. Finally, it should be apparent that the serial test for d >3 could require a lot of memory to tally the $k^d$ values of $f_{j1j2\ldots jd}$.

Here ,when n is smaller the serial test is “not rejected”.If we increase n the serial test rejects.

## 3.Run test:

| n | ppf | R | status |
|---|---|---|---|
| 20 | 10.64 | 0.9399 | Not rejected |
| 500 | 10.64 | 2.39 | Not rejected |
| 4000 | 10.64 | 7.27 | Not rejected |
| 10000 | 10.64 | 5.77 | Not rejected |

The third empirical test we consider, the runs (or runs-up) test, is a more direct test of the independence assumption. (In fact, it is a test of independence only; i.e., we are not testing for uniformity in particular.) We examine the Ui sequence (or, equivalently, the $Z_i$ sequence) for unbroken subsequences of maximal length within which the $U_i$ ’s increase monotonically; such a subsequence is called a run up.

Here  all the results are “not rejected”.So the run up is distributed independently.

## 4.Corelation test:

| n | J=1 ppf,\|Aj\| | J=1 status | J=3 ppf,\|Aj\| | J=3 status | J=5 ppf,\|Aj\| | J=5 status |
|---|---|---|---|---|---|---|
| 20 | 1.64,1.49 | Not rejected | 1.64,0.18 | Not rejected | 1.64,1.33 | Not rejected |
| 500 | 1.64,0.42 | Not rejected | 1.64,1.46 | Not rejected | 1.64,1.32 | Not rejected |
| 4000 | 1.64,1.10 | Not rejected | 1.64,1.98 | rejected | 1.64,0.53 | Not rejected |
| 10000 | 1.64,0.98 | Not rejected | 1.64,1.32 | Not rejected | 1.64,1.30 | Not rejected |

Under the null hypothesis that $\rho_j = 0$ and assuming that n is large, it can be shown that the test statistic

has an approximate standard normal distribution. This provides a test of zero lag j correlation at level α, by rejecting this hypothesis s if $|A_j| > Z_{1-\alpha/2}$.

The test should probably be carried out for several values of j, since it could be, for instance, that there is no appreciable correlation at lags 1 or 5, but there is dependence between the $U_i$'s at lag 3, due to some anomaly of the generator.