

## Time and space complexity.

### Assignment.

Q1

```

int sum = 0;
for (int i = 1 ; i <= n ; i++)
{
    for (int j = 0 ; j <= n ; j++)
    {
        sum++;
    }
}

```

Time complexity =  $O(n^2)$

→ to reduce time complexity we can use other mathematical formula to calculate sum.

Q2

$$T(n) = 3T(n-1) + 12n, \quad T(0) = 5$$

$$\begin{aligned}
 T(1) &= 3T(1-1) + 12(1) \\
 &= 3T(0) + 12 \\
 &= 3 \times 5 + 12 \\
 T(1) &= 27
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow T(2) &= 3T(2-1) + 12(2) \\
 &= 3T(1) + 24 \\
 &= 3(27) + 24
 \end{aligned}$$

$$T(2) = \boxed{105}$$

Ans

$$\begin{array}{r}
 27 \\
 27 \\
 \times 3 \\
 \hline
 81 \\
 27 \times 3 \\
 \hline
 105
 \end{array}$$

Q3

$$T(n) = T(n-1) + C$$

$$T(n-1) = T(n-2) + C$$

$$T(n) = T(n-2) + 2C \quad \text{--- (I)}$$

$$T(n-2) = T(n-3) + C$$

$$T(n) = T(n-3) + 3C \quad \text{--- (II)}$$

On observing,

$$T(n) = T(n-k) + kC$$

$$n-k = 1$$

$$n-1 = k$$

$$T(n) = T(n-n+1) + (n-1)C$$

$$T(n) = \frac{T(1)}{1} + (n-1)C$$

Constant time task.

$$T(n) = O(1)$$



Q4

$$T(n) = 16T\left(\frac{n}{4}\right) + \Theta(n^2 \log n)$$

Standard Equation

$$T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^k \log^p n)$$

$$a = 16$$

$$k = 2$$

$$b = 4$$

$$p = 1$$

$$b^k = (4)^2 = 16$$

as  $a = b^k$   
 $16 = 16$

$$p > -1$$

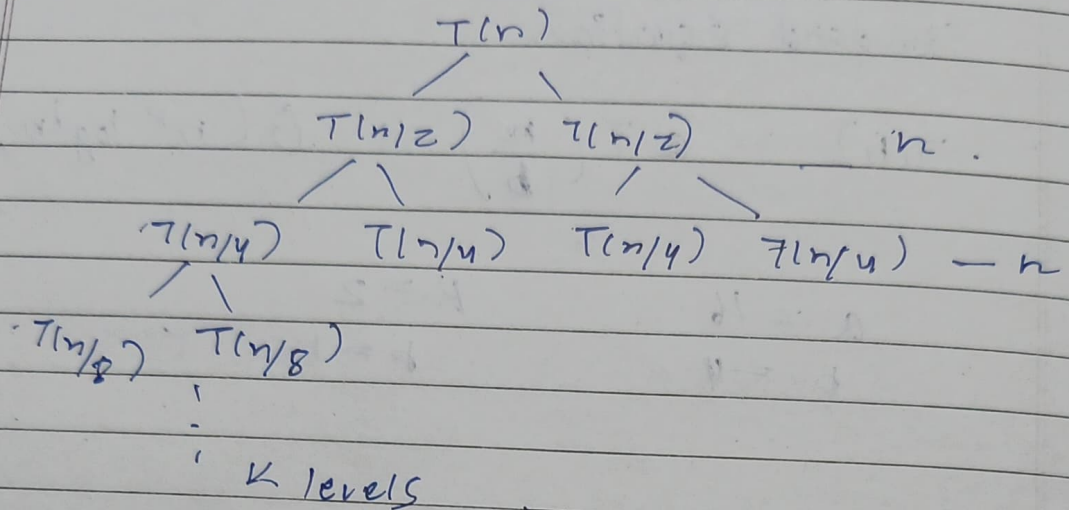
$$T(n) = O(n^{\log_b a} \cdot \log^{p+1} n)$$

$$T(n) = O(n^{\log_4 16} \cdot \log^2 n)$$

$$T(n) = O(2n \cdot \log^2 n)$$

Q5

$$T(n) = 2T(n/2) + n$$



$$\frac{n}{2^K} = 1$$

$$n = 2^K$$

$$\log n = K \log 2$$

$$\frac{\log n}{\log 2} = K$$

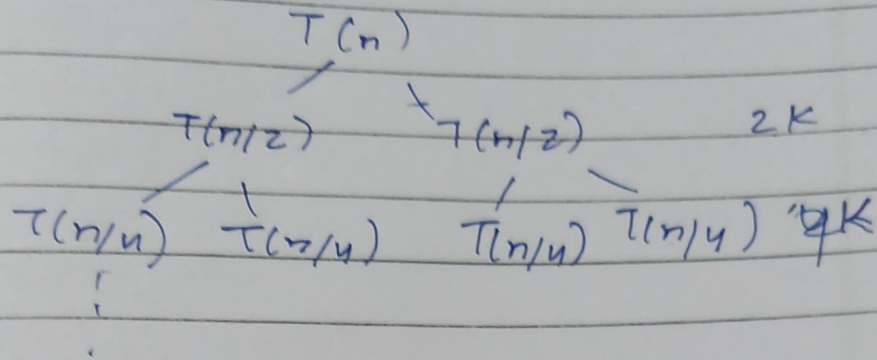
$$\boxed{\log_2 n = K}$$

$$\therefore T(n) = \boxed{n \log n}$$



06

$$T(n) = 2T(n/2) + K$$



$$\frac{n}{2^K} = 1$$

$$n = 2^K$$

$$\log n = K \log 2$$

$$\frac{\log n}{\log 2} = K$$

$$\log_2 n = K$$

~~Rec~~ Cost =  $K + 2K + 4K + \dots + \log_2 n$  times

~~$K \log_2 n$~~

$$\text{GP Sum} = \left( \frac{K}{1-2} \right)$$

$$\boxed{T(n) = O(n)}$$