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Time and space Complexity Assignment.

Q1

```
int sum = 0;
for (int i = 1 ; i <= n ; i++)
{
    for (int j = 0 ; j <= n ; j++)
    {
        sum ++;
    }
}
```

time complexity = $O(n^2)$

→ to reduce time complexity we can use other mathematical formula to calculate sum.

Q2

$T(n) = 3T(n-1) + 12n$, $T(0) = 5$

$T(1) = 3T(1-1) + 12(1)$
$= 3T(0) + 12$
$= 3 \times 5 + 12$
$T(1) = 27$

→ $T(2) = 3T(2-1) + 12(2)$
$= 3T(1) + 24$
$= 3(27) + 24$
$T(2) = \boxed{105}$

Ans

$27 \times 3 = 81$, $81 + 24 = 105$

Q3

$$T(n) = T(n-1) + c$$

$$T(n-1) = T(n-2) + c$$

$$T(n) = T(n-2) + 2c \quad -(\text{I})$$

$$T(n-2) = T(n-3) + c$$

$$T(n) = T(n-3) + 3c \quad -(\text{II})$$

On observing.

$$T(n) = T(n-k) + kc$$

$$n-k = 1$$
$$n-1 = k$$

$$T(n) = T(n-n+1) + (n-1)c$$
$$T(n) = \underbrace{T(1)}_{1} + (n-1)c$$

Constant time task.

$$\boxed{T(n) = O(n)}$$

Q4

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$$T(n) = 16T\left(\frac{n}{4}\right) + n^2 \log n.$$

Standard Equation

$$T(n) = aT\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$$

$a = 16$ $k = 2$

$b = 4$ $p = 1$

$b^k = (4)^2 = \underline{16}$

as $a = b^k$

$16 = 16$

$p > -1$

$$T(n) = O(n^{\log_b a} \cdot \log^{p+1} n)$$

$$T(n) = O(n^{\log_4 16} \cdot \log^2 n)$$

$$T(n) = O(2n \cdot \log^2 n)$$

$\frac{\ln 16}{\ln 4}$

$\frac{\log 16}{\log 2}$

$\frac{4 \log 2}{\log 2}$

Q5

$T(n) = 2T(n/2) + n$

$T(n)$

$T(n/2) \quad T(n/2) \qquad n$

$T(n/4) \quad T(n/4) \quad T(n/4) \quad T(n/4) \quad - n$

$T(n/8) \quad T(n/8)$

$\vdots$

k levels

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log n = k \log 2$$

$$\frac{\log n}{\log 2} = k$$

$$\boxed{\log_2 n = k}$$

$$\therefore \quad T(n) = \boxed{n \log n}$$

Q6

$$T(n) = 2T(n/2) + K$$

$$T(n)$$

$$T(n/2) \quad T(n/2) \qquad 2K$$

$$T(n/4) \quad T(n/4) \quad T(n/4) \quad T(n/4) \qquad 4K$$

$$\vdots$$

$$\frac{n}{2^K} = 1$$

$$n = 2^K$$

$$\log n = K \log 2$$

$$\frac{\log n}{\log 2} = K$$

$$\log_2 n = K$$

$$\text{Cost} = K + 2K + 4K \ldots \log_2 n \text{ times}$$

$$\cancel{K \times \log \ldots}$$

$$\text{GP Sum} = \left(\frac{K}{1-2}\right)$$

$$\boxed{T(n) = O(n)}$$