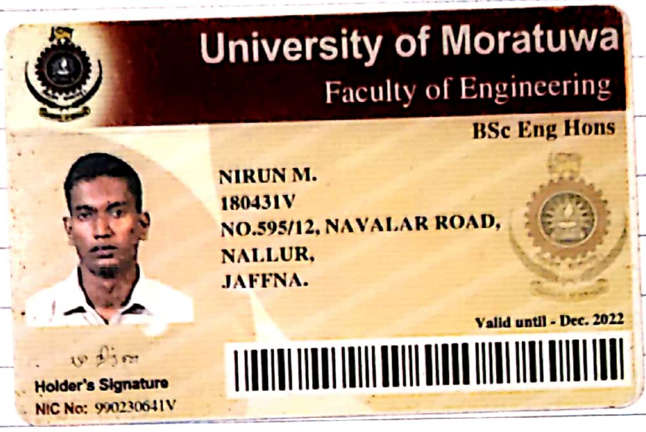

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180431V

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180431V

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1)

1) • U, W are subspace of V

$\therefore \underline{0} \in U$

$\underline{0} \in W$

• $\underline{0} + \underline{0} = \underline{0}$

$\therefore \underline{0} \in U+W$

$\therefore$ $U+W$ is an non empty.

• let $\underline{x_1}, \underline{x_2} \in U+W$

$\Rightarrow \underline{x_1} = \underline{u_1} + \underline{w_1}$ $(\underline{u_1} \in U$ and $\underline{w_1} \in W)$

$\underline{x_2} = \underline{u_2} + \underline{w_2}$ $(\underline{u_2} \in U$ and $\underline{w_2} \in W)$

$$\underline{x_1} + \underline{x_2} = (\underline{u_1} + \underline{w_1}) + (\underline{u_2} + \underline{w_2})$$

$$= (\underline{u_1} + \underline{w_1} + \underline{u_2} + \underline{w_2})$$ association

$$= (\underline{u_1} + \underline{u_2}) + (\underline{w_1} + \underline{w_2})$$ $\left[\begin{array}{l} \text{as} \\ (\underline{u_1}+\underline{u_2}) \in U \\ (\underline{w_1}+\underline{w_2}) \in W \end{array}\right]$

$\therefore \underline{x_1} + \underline{x_2} \in U+W$

↳ satisfies closure under addition

• let a scaler k.

$$k\underline{x_1} = k(\underline{u_1} + \underline{w_1})$$

$$= k\underline{u_1} + k\underline{w_1}$$ distributive law scalar multi

$$= k\underline{u_1} + k\underline{w_1}$$ $\left[\begin{array}{l} \text{as} \\ k\underline{u_1} \in U \\ k\underline{w_1} \in W \end{array}\right]$

$\therefore k\underline{x_1} \in U+W$

$\therefore$ closure under scalar multiplication satisfied

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- $U+W$ is a subset of V $\left[\begin{matrix} U \subseteq V \\ W \subseteq V \end{matrix}\right]$
- $U+W$ is non empty
- $U+W$ satisfy closure under addition
  scalar multiplication

$\therefore$ $U+W$ is a subspace of V //

ii) •

$U+W$ is a subspace of V contains U, W

↳ it must contain linear span of U and W

$\therefore$ $\text{span}(U, W) \subseteq U+W$ —— $*$

• and if $\underline{v} \in U+W$ then.

$\underline{v} = \underline{u} + \underline{w}$.

$= 1 \times \underline{u} + 1 \times \underline{w}$ $\left[\begin{matrix} \underline{u} \in U \\ \underline{w} \in W \end{matrix}\right]$

• v is a linear combination of elements U, V, W
and $v \in \text{span}(U, W)$

$\therefore$ $U+W \subseteq \text{span}(U, W)$ —— $**$

$*, ** \Rightarrow$

$U+W = \text{span}(U, W)$

180431

iii) above proved

$u + w = \text{span}(u, w)$

$u+w = \text{span}\{(1,3,-2,2,3),\ (1,4,-3,4,2),\ (2,3,-1,-2,9),\ (1,3,0,2,1),\ (1,5,-6,6,3),\ (2,5,3,2,1)\}$

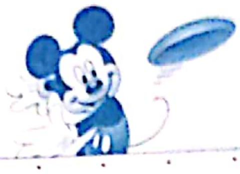
↑ 6 × 5 matrix form change and row reduce.

$$\underset{6\times 5}{\begin{bmatrix} 1 & 3 & -2 & 2 & 3 \\ 1 & 4 & -3 & 4 & 2 \\ 2 & 3 & -1 & -2 & 9 \\ 1 & 3 & 0 & 2 & 1 \\ 1 & 5 & -6 & 6 & 3 \\ 2 & 5 & 3 & 2 & 1 \end{bmatrix}} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & -4 & 7 \\ 0 & 1 & 0 & 2 & -2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis of set $(u+w) = \{(1,0,0,-4,7),\ (0,1,0,2,-2),\ (0,0,1,0,-1)\}$ //

Dimension of $(u+w) = 3$ //

//



18043 IV

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b) $A \to m \times n$ $\quad$ $B \to n \times l$

i) $R \to (1 \times n)$ [$\because$ row vector of n component]

$$\therefore RB = \underset{(1\times n)}{(r_1, r_2, r_3 \dots r_n)} \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1l} \\ b_{21} & & & \\ \vdots & & & \\ b_{n1} & - & - & \dots b_{nl} \end{bmatrix}_{n \times l}$$

$$= (r_1, r_2, \dots r_n) \left\{ \begin{bmatrix} b_{11} & \dots & b_{1l} \\ 0 & 0 \; 0 \; 0 & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \; 0 \dots & 0 \\ b_{21} & \dots & b_{2l} \\ 0 \; 0 & \dots & 0 \\ \vdots & & \vdots \end{bmatrix} + \dots \right]$$

$$= (r_1, r_2 \dots r_n) \begin{bmatrix} b_{11} & \dots & b_{1l} \\ 0 \; 0 & \dots & 0 \\ \vdots & & \\ 0 & \dots & 0 \end{bmatrix} + (r_1, r_2 \dots r_n) \begin{bmatrix} 0 \; 0 \; 0 & \dots & 0 \\ b_{21} & \dots & b_{2l} \\ \vdots & & \\ 0 & \dots & 0 \end{bmatrix}$$

$$+ \dots + (r_1, r_2 \dots r_n) \begin{bmatrix} 0 & \dots & 0 \\ \vdots & & \\ 0 & \dots & 0 \\ b_{n1} & \dots & b_{nl} \end{bmatrix}$$

$$= r_1 [b_{11} \dots b_{1l}] + r_2 [b_{21} \; b_{22} \dots b_{2l}] + \dots$$

$$+ r_n [b_{n1} \dots b_{nl}]$$

$\therefore RB$ is a linear combination of rows of B //

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5

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ii)

$$AB = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & \cdots & b_{1l} \\ \vdots & & \\ b_{n1} & \cdots & b_{nl} \end{bmatrix}$$

$$= \begin{bmatrix} r_{a_1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} b_{11} & \cdots & b_{1l} \\ \vdots & & \\ b_{n1} & \cdots & b_{nl} \end{bmatrix} + \begin{bmatrix} 0 \\ r_{a_2} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} b_{11} & \cdots & b_{1l} \\ \vdots & & \\ b_{n1} & \cdots & b_{nl} \end{bmatrix}$$

$$+ \cdots$$

like that $= r_{a_1}[b_{11} \cdots b_{1l}] + r_{a_2}[b_{21} \cdots b_{2l}] + \cdots$

$+ r_{a_m}[b_{n1} \cdots b_{nl}]$ //

↳ row space AB is spanned by row vectors of B

∴ row space of AB is contained in B's row space //

iii) rank (AB) = dimension of row space of AB

in above result ⇒ row space of AB is contained in B

dimension of row space of AB $\leq$ dimension of row space of B

rank (AB) $\leq$ rank (B) //

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[illegible]

(2) a)

i) If $(\underline{u}_1, \underline{u}_2, \ldots \underline{u}_n)$ linearly ~~in~~dependant

$\Rightarrow k_1 \underline{u}_1 + k_2 \underline{u}_2 + \ldots + k_n \underline{u}_n = \underline{0} \Rightarrow$ {at least one cofficient $\neq 0$}

as T is linear

$$T(k_1 \underline{u}_1 + k_2 \underline{u}_2 + \ldots + k_n \underline{u}_n) = T(\underline{0})$$
[T is linear] //

$$k_1 T.\underline{u}_1 + k_2 T.\underline{u}_2 + \ldots + k_n T.(\underline{u}_n) = T(0.\underline{v}).$$
$$= 0.T.(\underline{v})$$
$$= \underline{0} \text{ //}$$

here atleast one $k_i \neq 0$

$\therefore$

$\{T(\underline{u}_1), T(\underline{u}_2), \ldots\}$ linearly independant in V //

ii) If $\{\underline{u}_1, \underline{u}_2, \ldots \underline{u}_n\}$ linearly independant in V.

$\Rightarrow k_1 \underline{u}_1 + k_2 \underline{u}_2 + \ldots + k_n \underline{u}_n = \underline{0} \Rightarrow$ [all $k_i = 0$]

apply transform of T is linear

$\Rightarrow$

$$k_1 T(\underline{u}_1) + k_2 T(\underline{u}_2) + \ldots + k_n T(\underline{u}_n) = T(0.\underline{v})$$
$$= 0.T(\underline{v})$$
$$= \underline{0} \text{ //}$$

$\Rightarrow k_1 = k_2 = \ldots k_n = 0$

$\therefore \{T(\underline{u}_1), T(\underline{u}_2), \ldots T(\underline{u}_n)\}$ linearly independant in V //

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(2)

b) $T(p) = \begin{bmatrix} p(0) & p(1) \\ p(-1) & p(0) \end{bmatrix}$

•) consider $\left.\begin{array}{l} \underline{p} \in P_2 \\ \underline{q} \in P_2 \end{array}\right\}$

$T(\underline{p}) = \begin{bmatrix} p(0) & p(1) \\ p(-1) & p(0) \end{bmatrix}$

$T(\underline{q}) = \begin{bmatrix} q(0) & q(1) \\ q(-1) & q(0) \end{bmatrix}$

consider $T(\underline{p} + \underline{q}) = \begin{bmatrix} (p+q)(0) & (p+q)(1) \\ (p+q)(-1) & (p+q)(0) \end{bmatrix}$

$= \begin{bmatrix} p(0) & p(1) \\ p(-1) & p(0) \end{bmatrix} + \begin{bmatrix} q(0) & q(1) \\ q(-1) & q(0) \end{bmatrix}$

$= T(p) + T(q)$ — (*)

↳ satisife ~~closure under~~ addition property

• Consider $T(k\underline{p}) = \begin{bmatrix} kp(0) & kp(1) \\ kp(-1) & kp(0) \end{bmatrix}$

$= k^2 \begin{bmatrix} p(0) & p(1) \\ p(-1) & p(0) \end{bmatrix}$

$= k^2 \; T(p) \in P_2$ — (**)

(*), (**) ⇒ ∴ satisfies homogeniety property//

∴ T is a linear transform//

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11) let $P_2 = ax^2 + bx + c$

$$\left.\begin{aligned} P(0) &= c \\ P(1) &= a+b+c \\ P(-1) &= a-b+c \end{aligned}\right\}$$

$$T(P) = \begin{bmatrix} c & a+b+c \\ a-b+c & c \end{bmatrix}$$

$$= c\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + a\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$\therefore$ Range of T is spanned by

$\hookrightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ //

and they are linearly independent //

dim(Range of T) = Rank(T) = 3 //

•> consider

$T(P) = \underline{0}$

$$\begin{bmatrix} c & a+b+c \\ a-b+c & c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ $c = 0$

$$\left.\begin{aligned} a+b &= 0 \\ a-b &= 0 \rightarrow a = b \end{aligned}\quad\begin{aligned} a &= 0 \\ b &= 0 \\ c &= 0 \end{aligned}\right\} //$$

kernal of T spanned by $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ //

$\therefore$ dim(kernal of T) = nullity (T) = 0 //

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iii) $T(1) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$T(1+x) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$T(1+x^2) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$M_{22} = \{ M_1, M_2, M_3, M_4 \}$

$[T(1)]_{B'} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$[T(1+x)]_{B'} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

$[T(1+x^2)]_{B'} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$

$[T]_{B',B} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ //