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# Review of Multi-Agent Algorithms for Collective Behavior: a Structural Taxonomy

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**Abstract:** In this paper, we review multi-agent collective behavior algorithms in the literature and classify them according to their underlying mathematical structure. For each mathematical technique, we identify the multi-agent coordination tasks it can be applied to, and we analyze its scalability, bandwidth use, and demonstrated maturity. We highlight how versatile techniques such as artificial potential functions can be used for applications ranging from low-level position control to high-level coordination and task allocation, we discuss possible reasons for the slow adoption of complex distributed coordination algorithms in the field, and we highlight areas for further research and development.



*Keywords:* Autonomous mobile robots, Agents, Distributed Control, Decentralized Control

## 1. INTRODUCTION

Multi-agent robotic systems hold promise to enable new classes of missions in aerospace, terrestrial, and maritime applications, delivering higher resilience and adaptability at lower cost compared to existing monolythic systems. In particular, in the aerospace domain, multi-agent systems hold great promise for applications including multi-UAV patrolling, satellite formations for astronomy and Earth observation, and multi-robot planetary exploration. A number of algorithms have been proposed to control the collective behavior of such systems, ranging from low-level position control to high-level motion planning and task allocation algorithms.

Many excellent surveys of algorithms for collective behavior exist in the literature; however, such papers generally focus either on single applications (e.g., formation control (Oh et al., 2015) or coverage (Schwager et al., 2009)) or on specific control techniques (e.g., consensus (Garin and Schenato, 2010; Cao et al., 2013)). Several works study the fundamental limitations of performance of multi-agent systems: e.g., Martinez et al. (2007) and Rossi and Pavone (2014) explore time and communication complexity in synchronous and asynchronous systems respectively, and Gupta et al. (2006) studies robustness to agent failures. However, these works only survey the performance of a limited number of applications and algorithms. In contrast, in this paper, we survey the *general* family of collective behavior algorithms for multi-agent systems and classify them according to their underlying mathematical *structure*, without restricting our focus to specific tasks or individual classes of algorithms. In doing so, we aim to capture fundamental mathematical properties of algorithms (e.g. scalability with respect to the number of agents and bandwidth use) and to show how the same algorithm or family of algorithms can be applied to multiple tasks and missions. In particular, the goal of this paper is threefold:

- to act as a guide to practitioners in the selection of control algorithms for a given task or application;
- to highlight how mathematically similar algorithms can be used for a variety of tasks, ranging from low-level control to high-level coordination;
- to explore the state-of-the-art in the field of control of multi-agent systems and identify areas for future research.

*Tasks in multi-agent systems* can be broadly categorized into the following classes (Brambilla et al., 2013):
(1) **Spatially-organizing behaviors**, where agents coordinate to achieve a given spatial configuration and have negligible interactions with the environment. These tasks can be further classified into: (a) *Aggregation:* converging to one location. (b) *Pattern Formation:* achieving a desired formation. (c) *Coverage:* covering an area.
(2) **Collective explorations**, where agents interact with the environment but have minimal interaction among themselves. These tasks can be classified into: (a) *Area Exploration:* exploring the environment for mapping or surveillance. (b) *Goal Searching:* searching for targets.
(3) **Cooperative decision making**, where agents both coordinate among themselves and interact with the environment to accomplish complex tasks. These tasks can be further classified into: (a) *Task Allocation:* distributing tasks among agents. (b) *Collective Transport:* coordinating to transport large objects. (c) *Motion Planning:* finding paths in cluttered environments. (d) *Distributed Estimation:* estimating the state of one or multiple targets.
These simple tasks are the fundamental building blocks of many complex multi-agent applications.

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| | Aggregation | Pattern Formation | Coverage | Area Exploration | Goal Searching | Task Allocation | Collective Transport | Motion Planning | Distributed Estimation | High Scalability | Low Bandwidth Use | Maturity |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Consensus** | ✓ | ✓ | ✓ | | | | | | ✓ | ✓ | ✓ | H |
| **Artificial Potential Functions (APF)** | ✓ | ✓ | ✓ | ✓ | | ✓ | ✓ | ✓ | | ✓ | ✓ | F |
| **Distributed Feedback Control** | ✓ | ✓ | | | | | | | ✓ | ✓ | ✓ | F |
| **Geometric Algorithms** | | | | | | | | | | | | |
| Voronoi-based Algorithms | ✓ | | ✓ | ✓ | | | | ✓ | | ✓ | ✓ | H |
| Circumcenter Algorithms | ✓ | ✓ | | | | | | | | ✓ | ✓ | S |
| Bearing-only Algorithms | ✓ | ✓ | | | | | | | | ✓ | ✓ | H |
| Maze Searching Algorithms | | | | | | | | ✓ | | ✓ | ✓ | S |
| Leader-Follower (LF) Algorithms | | ✓ | | | | | | | | ✓ | ✓ | S |
| Velocity Obstacle (VO) based Algorithms | | | | | | | | ✓ | | ✓ | ✓ | F |
| **State Machines and Behavior Composition** | | | | | | | | | | | | |
| Automata-based Algorithms | | | | | | ✓ | | | ✓ | ✓ | ✓ | S |
| Behavior Composition | | | | | | ✓ | ✓ | | | | | H |
| Petri Networks | | | | | | ✓ | | | | | - | H |
| Game Theory based Algorithms | | | | | | ✓ | | | | | - | S |
| Resource Allocation Systems | | | | | | | | ✓ | | ✓ | - | S |
| **Bio-Inspired Algorithms** | | | | | | | | | | | | |
| Kilobot Self-Assembly Algorithm | | ✓ | | | | | | | | ✓ | ✓ | H |
| Optimotaxis Source-Searching Algorithm | | | | | ✓ | | | | | ✓ | ✓ | S |
| Beeclust Foraging Algorithm | | | | ✓ | | | | | | ✓ | ✓ | S |
| Shepherding Algorithm | ✓ | | | | | | | | | ✓ | ✓ | S |
| Termite-Inspired Collective Construction Algorithm | | | | | | ✓ | ✓ | | | ✓ | ✓ | H |
| Fish-inspired Goal Searching Algorithms | | ✓ | | | ✓ | | | | | ✓ | ✓ | H |
| Gillespie Self-Assembly Algorithm | | ✓ | | | | | | | | ✓ | ✓ | H |
| Mergeable Modular Robots | | ✓ | | | | | | | | ✓ | ✓ | H |
| **Density based Control** | | | | | | | | | | | | |
| Markov Chain-based Algorithms | | ✓ | ✓ | | | ✓ | | | | ✓ | ✓ | H |
| Smoothed Particle Hydrodynamics (SPH) | | ✓ | ✓ | | | | | | | ✓ | ✓ | H |
| Optimal Transport based Algorithm | | ✓ | | | | | | ✓ | | ✓ | ✓ | S |
| **Distributed Optimization Algorithms** | | | | | | | | | | | | |
| Distributed Linear Programming | | ✓ | | | | ✓ | | | | ✓ | ✓ | S |
| Distributed Convex Optimization | | ✓ | | | | ✓ | | | ✓ | ✓ | ✓ | S |
| Distributed Dynamic Programming | | | | | | ✓ | | ✓ | | | | H |
| Sequential Convex Programming | | | | | | | | ✓ | | ✓ | ✓ | H |
| Distributed Auction | | | | | | ✓ | | | | ✓ | ✓ | H |
| **Local Optimization Algorithms for Global Behavior** | | | | | | | | | | | | |
| Decentralized Model Predictive Control (DMPC) | | ✓ | | | | | | ✓ | | ✓ | | H |
| Formal Methods | | | | | | | | ✓ | | ✓ | ✓ | S |
| Sampling-based Motion-Planning Algorithms | | | | | | | | ✓ | | | | H |
| **Centralized Optimization Algorithms** | | | | | | | | | | | | |
| MILPs and MINLPs | | ✓ | | | | ✓ | | ✓ | ✓ | | - | H |
| Linear and Convex Optimization | | | | | | ✓ | | ✓ | ✓ | ✓ | - | S |
| Markov Decision Processes (MDP) | | | | | | ✓ | | ✓ | | | - | H |
| Multi-Agent Traveling Salesman Problems | | | ✓ | ✓ | ✓ | ✓ | | | | | - | H |
| Multi-Armed Bandits | | | | ✓ | ✓ | ✓ | | | ✓ | ✓ | - | S |
| Direct Methods for Optimal Control | | | ✓ | ✓ | | | | ✓ | | | - | F |
| Multiagent Reinforcement Learning | | | | ✓ | | ✓ | | | | | - | H |
| Frontier Techniques | | | | ✓ | ✓ | | | | | | - | F |
| Network Flow Algorithms | | | | | | ✓ | | ✓ | | ✓ | - | S |
| Combinatorial Motion Planning | | | | | | | | ✓ | | ✓ | - | S |

Table 1. Categorization of collective behavior algorithms according to their mathematical structure and applicability of each algorithm to common multi-agent tasks. The scalability, bandwidth use, and level of demonstrated maturity of each algorithm (formally defined in Section 1) are also reported.

*Communication structure* In **centralized** algorithms, all agents share their information with a central node, which computes and issues a joint set of control actions. In **distributed** algorithms, agents can only explicitly share information with their neighbors. Centralized algorithms can be implemented in a distributed fashion with a **shared-world** approach, discussed in Section 2.10.

*Methodology* We performed a thorough review of papers on multi-agent systems in major controls and robotics journals and conferences. It is not feasible to cite all existing works on control of multi-agent systems; accordingly, in this paper, we focus on identifying and classifying the key mathematical *structures* and *techniques* that drive coordination algorithms, as opposed to individual contributions. We refer the reader to the extended version of this paper (Rossi et al., 2018a) for a more thorough literature review and a detailed discussion of the mathematical formulation and properties of each technique.

We classify mathematical techniques according to their: (1) **Scalability:** Highly scalable algorithms have been demonstrated on systems with more than 50 agents (in simulations or hardware). (2) **Bandwidth use:** In low bandwidth algorithms, agents only communicate with their physical neighbors and do not exchange large messages. (3) **Maturity:** The three classes of algorithms are: (i) only demonstrated in 'simulation' (S) (ii) demonstrated in 'hardware' (H) either in the lab or in technology demonstration missions (iii) demonstrated in 'field' (F) deployments (excluding technology demonstrator missions). A formal description and discussion of these metrics is provided in the extended version (Rossi et al., 2018a).

*Organization* Our key contribution is Table 1, which reports the proposed taxonomy of mathematical techniques for collective behavior, highlights the tasks that each mathematical technique can achieve, and lists relevant performance metrics. In Sections 2.1–2.10 we provide a synthetic description of the classification and relevant references. Finally, in Section 3 we draw conclusions and suggest directions for future research.

## 2. A STRUCTURAL TAXONOMY OF MULTI-AGENT COLLECTIVE BEHAVIOR ALGORITHMS

### 2.1 Consensus algorithms

**Consensus** is among the oldest and most widely used distributed algorithms. Each agent shares and averages its state with its neighbors (Tsitsiklis et al., 1986; Ren et al., 2007). Applications include synchronization (Li and Rus, 2006), flocking (Tanner et al., 2007; Olfati-Saber, 2006), formation flying (Chung et al., 2013), and distributed estimation (Rabbat and Nowak, 2004). In *gossip algorithms* (Boyd et al., 2006), each agent communicates with a single randomly-selected neighbor at each step. In *cyclic pursuit algorithms* (Marshall et al., 2004), the consensus algorithm is executed on a directed ring communication topology.

### 2.2 Artificial Potential Functions (APF)

**APF** algorithms synthesize agents' control inputs using the gradient of a suitably-defined potential function (Khatib, 1986). These algorithms are very popular due to their simplicity, scalability, and ability to adapt to a number of tasks. Applications include pattern formation (Sepulchre et al., 2007) , flocking (Zavlanos et al., 2007), path planning (Koditschek and Rimon, 1990), and task allocation (Weigel et al., 2002).

### 2.3 Distributed Feedback Control

Each agent is endowed with a feedback controller that uses the agent's and its neighbors' states as the input (Bamieh et al., 2002; Feddema et al., 2002). In particular, tools for synthesis of **distributed LQG control** are available that can adapt to noisy communication links (Sahai and Mitter, 2006), and packet losses (Liu and Goldsmith, 2004), with applications to formation flying (Ogren et al., 2002) and distributed estimation.

### 2.4 Geometric Algorithms

In geometric algorithms, agents leverage their neighbors' location and speed information to perform spatially organizing tasks and path planning. **Voronoi algorithms** compute Voronoi partitions for coverage (Cortés et al., 2004), path planning (Zhou et al., 2017), and task allocation problems (Pavone et al., 2011). Other geometric algorithms include **circumcenter algorithms** for rendezvous (Cortés et al., 2006), **bearing-only algorithms** for formation control (Fredslund and Mataric, 2002) and rendezvous (Yu et al., 2008), **maze searching algorithms** for path planning (Lumelsky and Harinarayan, 1997), **leader-follower algorithms** for formation flying (Mesbahi and Hadaegh, 1999), and **velocity obstacles** for collision avoidance (van den Berg et al., 2008).

### 2.5 State Machines and Behavior Composition

**Automata-based algorithms** leverage complex state machines and message-passing among agents to establish communication graphs and elect leaders for task allocation (Lynch, 1997; Rossi and Pavone, 2014). **Behavior composition** algorithms rely on composition of elementary behaviors for collective transport (Rus et al., 1995). **Petri networks** (King et al., 2003) and **game theory** (Arslan et al., 2007) algorithms are used for centralized task allocation. **Resource allocation systems** are used for multi-agent motion planning (Reveliotis and Roszkowska, 2011).

### 2.6 Bio-Inspired Algorithms

Bio-inspired algorithms mimic the behavior of swarms of animals such as insects and fish. We present a non-exhaustive list: the **Kilobot algorithm** achieves complex two-dimensional shapes and was demonstrated on a thousand-agent testbed (Rubenstein et al., 2014); the **Optimotaxis source-searching algorithm** is inspired by the run and tumble behaviors of bacteria (Mesquita et al., 2008); the **Beeclust foraging algorithm** is inspired by the behavior of honey bees (Hereford, 2011); **Shepherding algorithms** enable control of large numbers of uncontrolled agents with few controlled agents (Strömbom et al., 2014); a **Termite-inspired algorithm** generates low-level rules for construction of complex structures (Werfel et al., 2014); a **Fish-inspired goal-searching algorithm** switches between individual and collective behavior based on confidence level (Wu and Zhang, 2012); the **Gillespie self-assembly algorithm** leverages chemical kinetics; **Mergeable modular robots** connect to form larger bodies or split into separate bodies, with self-healing properties (Mathews et al., 2017).

### 2.7 Density based Control

As opposed to the agent-based *Lagrangian* framework, density-based algorithms adopt an *Eulerian* framework by treating agents as a continuum and controlling their density. **Markov chain** based algorithms partition the workspace into disjoint cells and control the transition probabilities between cells for pattern formation and goal searching applications (Açıkmeşe and Bayard, 2015; Bandyopadhyay et al., 2017b). **Smoothed particle hydrodynamics (SPH)** (Zhao et al., 2011) and **optimal transport** (Bandyopadhyay et al., 2014) based algorithms are also used for swarm formation control.

### *2.8 Distributed Optimization Algorithms*

Distributed optimization algorithms allow agents to jointly solve optimization problems through information exchange and local computations. **Distributed linear programming** (Bürger et al., 2012) is used for pattern formation and task allocation; **distributed convex optimization** can encode richer convex constraints (Boyd et al., 2011). **Distributed dynamic programming** (Bertsekas, 1982) is used for task allocation and motion planning. **Sequential Convex Programming** can solve non-convex motion planning problems through local convexification and iteration (Morgan et al., 2016). The above algorithms can also be used in a **distributed model-predictive control** framework (Scattolini, 2009). Market-based protocols like **distributed auction** (Gerkey and Mataric, 2002), mechanism design (Dias, 2004), and coalition formation (Shehory and Kraus, 1998) are widely used for task allocation.

### *2.9 Local optimization algorithms for global behavior*

In local optimization algorithms, each agent solves an optimization problem; while the resulting behavior is not generally optimal for the entire system, favorable global properties such as collision avoidance can be guaranteed. In **decentralized model predictive control (DMPC)** each agent employs a local model-predictive control algorithms; inter-agent communication is used to coordinate the agents' plans (Richards and How, 2007). Distributed MPC has been used for flocking and motion planning (Dunbar and Murray, 2002; Schouwenaars et al., 2006). **Formal methods** are used in concert with low-level control primitives for multi-agent motion planning with guaranteed collision avoidance (Kress-Gazit et al., 2008). Decentralized multi-agent **sampling-based motion planning algorithms** have enjoyed significant practical success because of their simplicity, ability to handle higher-dimensional spaces, and probabilistic completeness (Bandyopadhyay et al., 2017a; Desaraju and How, 2012).

### *2.10 Centralized optimization algorithms*

**Mixed-integer linear programs (MILPs)** and mixed-integer convex programs (MICPs), can solve simultaneous task allocation and path planning (Bellingham et al., 2003), tracking (Xu et al., 2013), formation flying (Richards et al., 2002), and defend-the-flag problems (Earl and D'Andrea, 2002). **Linear and convex optimization** problems can also be used to solve task allocation problems (Bertsekas, 1998; Turpin et al., 2014) with collision avoidance constraints (Açıkmeşe et al., 2006), and for distributed estimation and target tracking (Aslam et al., 2003). **Markov decision processes (MDPs)** and partially observable MDPs capture the stochastic nature of the environment and model the agents' *coordination mechanism* (Boutilier, 1999). POMDPs have been used for multi-agent path planning (Omidshafiei et al., 2015) and task allocation. Several approximation algorithms are available to solve the **m-vehicle traveling salesman problem (TSP)** and the team orienteering problem, building blocks for spatial task allocation, persistent monitoring, and information-gathering problems (Yu et al., 2014). **Multi-agent multi-armed bandit** problems (Gittins, 1979) capture the trade-off between exploration and exploitation: they have been employed for task allocation (Le Ny et al., 2008) , goal searching, and tracking applications (Landgren et al., 2016). **Direct methods for trajectory optimization** (Von Stryk and Bulirsch, 1992) are used for area coverage, goal searching, and motion planning (Leonard et al., 2010). **Multi-agent reinforcement learning (MARL)** has been used for exploration (Chalkiadakis and Boutilier, 2003) and task allocation (Liu and Nejat, 2016). **Frontier techniques** (Burgard et al., 2000) are used for urban search-and-rescue, reconnaissance (Olson et al., 2012) and sample collection (Eich et al., 2014). **Network flow** formulations have been proposed for Air Traffic Control (Menon et al., 2004) and for control of autonomous vehicles offering on-demand transportation (Pavone et al., 2012; Rossi et al., 2018b). Several **cooperative combinatorial motion planning** algorithms have been proposed for multi-agent systems: we refer the reader to (Sharon et al., 2015) for a thorough review. Centralized optimization algorithms can be implemented in a distributed fashion with a **shared-world** approach, where agents exchange their state and observations so that every robot has full knowledge of the entire system's state. However, shared-world algorithms have very onerous communication requirements (due to large messages and all-to-all communication) and high computation complexity, since each agent must solve the full centralized optimization problem.

## 3. CONCLUSION

The proposed taxonomy and the properties shown in Table 1 highlight some surprising characteristics of collective behavior algorithms. The majority of existing mathematical techniques is tailored to either low-level spatially organizing tasks (e.g., bio-inspired algorithms and density-based control) or high-level coordination applications (e.g., state machines and optimization-based algorithms). Only a small number of mathematical techniques (in particular, Artificial Potential Functions) can be adapted to a wide variety of tasks that include both low-level and high-level application. This prompts further research into non-APF algorithms for multi-agent systems that share APF's key properties of simplicity, scalability, and high expressivity.

Very few algorithms are mature and field-tested. Such algorithms exchange very simple information (e.g. the agents' locations) or rely on centralized implementations: this may be justified by the difficulty of characterizing and certifying the behavior of an entire multi-agent system when distributed algorithms are used. To overcome this, (i) research in formal methods and adoption of tools from the distributed algorithms literature to provide stronger guarantees for distributed systems and (ii) creation of standardized software and hardware test-beds to characterize the end-to-end behavior of such systems are needed.

Several avenues for future research are of interest. In particular, we hope to evaluate the performance of collective behavior algorithms according to additional metrics including 1) bandwidth use in broadcast and in point-to-point networks, 2) computational complexity, 3) availability of formal guarantees, 4) resilience to disruptions in communication network and to *adversarial* failures, and 5) availability of a reference implementation. We also wish to explore other possible taxonomies for coordination algorithms based, e.g., on the content of messages exchanged by the agent (which vary from simple "beacon" messages reporting the agent's location to complex messages carrying intentions and bids), and the communication topology induced by the algorithm (single-hop vs. multi-hop) Finally, we plan to further explore high-level multi-agent tasks, including adversarial "swarm vs. swarm" problems, and to assess the applicability and performance of collective behavior algorithms with respect to such tasks.

## REFERENCES


Açıkmeşe, B. and Bayard, D.S. (2015). Markov chain approach to probabilistic guidance for swarms of autonomous agents. *Asian Journal of Control*, 17(4), 1105–1124.

Açıkmeşe, B., Scharf, D.P., Murray, E.A., and Hadaegh, F.Y. (2006). A convex guidance algorithm for formation reconfiguration. In *AIAA Guidance, Navigation, and Control Conference*.

Arslan, G., Marden, J.R., and Shamma, J.S. (2007). Autonomous vehicle-target assignment: A game theoretical formulation. *ASME J. Dyn. Syst. Meas. Control*, 129(5), 584–596.

Aslam, J., Butler, Z., Constantin, F., Crespi, V., Cybenko, G., and Rus, D. (2003). Tracking a moving object with a binary sensor network. In *Proceedings of the 1st international conference on Embedded networked sensor systems*.

Bamieh, B., Paganini, F., and Dahleh, M.A. (2002). Distributed control of spatially invariant systems. *IEEE Trans. Autom. Control*, 47(7), 1091–1107.

Bandyopadhyay, S., Baldini, F., Foust, R., Rahmani, A., de la Croix, J.P., Chung, S.J., and Hadaegh, F.Y. (2017a). Distributed fast motion planning for spacecraft swarms in cluttered environments using spherical expansions and sequence of convex optimization problems. In *9th International Workshop on Satellite Constellations and Formation Flying*.

Bandyopadhyay, S., Chung, S.J., and Hadaegh, F.Y. (2014). Probabilistic swarm guidance using optimal transport. In *Proc. IEEE Conf. Control Applicat.*

Bandyopadhyay, S., Chung, S.J., and Hadaegh, F.Y. (2017b). Probabilistic and distributed control of a large-scale swarm of autonomous agents. *IEEE Trans. Robotics*, 33(3).

Bellingham, J., Tillerson, M., Richards, A., and How, J.P. (2003). Multi-task allocation and path planning for cooperating uavs. In *Cooperative Control: Models, Applications and Algorithms*. Springer.

Bertsekas, D. (1982). Distributed dynamic programming. *IEEE Trans. Autom. Control*, 27(3), 610–616.

Bertsekas, D.P. (1998). *Network optimization: continuous and discrete models*. Athena Scientific.

Boutilier, C. (1999). Sequential optimality and coordination in multiagent systems. In *IJCAI*, volume 99.

Boyd, S., Ghosh, A., Prabhakar, B., and Shah, D. (2006). Randomized gossip algorithms. *IEEE Trans. Inf. Theory*, 52, 2508 – 2530.

Boyd, S., Parikh, N., Chu, E., Peleato, B., and Eckstein, J. (2011). Distributed optimization and statistical learning via the alternating direction method of multipliers. *Foundations and Trends in Machine Learning*, 3(1), 1–122.

Brambilla, M., Ferrante, E., Birattari, M., and Dorigo, M. (2013). Swarm robotics: a review from the swarm engineering perspective. *Swarm Intelligence*, 7(1), 1–41.

Burgard, W., Moors, M., Fox, D., Simmons, R., and Thrun, S. (2000). Collaborative multi-robot exploration. In *IEEE Int. Conf. on Robotics and Automation*.

Bürger, M., Notarstefano, G., Bullo, F., and Allgöwer, F. (2012). A distributed simplex algorithm for degenerate linear programs and multi-agent assignments. *Automatica*, 48, 2298–2304.

Cao, Y., Yu, W., Ren, W., and Chen, G. (2013). An overview of recent progress in the study of distributed multi-agent coordination. *IEEE Transactions on Industrial Informatics*, 9(1), 427–438.

Chalkiadakis, G. and Boutilier, C. (2003). Coordination in multiagent reinforcement learning: A bayesian approach. In *Proceedings of the second international joint conference on Autonomous agents and multiagent systems*.

Chung, S.J., Bandyopadhyay, S., Chang, I., and Hadaegh, F.Y. (2013). Phase synchronization control of complex networks of Lagrangian systems on adaptive digraphs. *Automatica*, 49(5), 1148–1161.

Cortés, J., Martinez, S., and Bullo, F. (2006). Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions. *IEEE Trans. Autom. Control*, 51(8), 1289–1298.

Cortés, J., Martinez, S., Karatas, T., and Bullo, F. (2004). Coverage control for mobile sensing networks. *IEEE Trans. Robotics and Automation*, 20(2).

Desaraju, V.R. and How, J.P. (2012). Decentralized path planning for multi-agent teams with complex constraints. *Autonomous Robots*, 32(4), 385–403.

Dias, M.B. (2004). Traderbots: A new paradigm for robust and efficient multirobot coordination in dynamic environments. *Robotics Institute*, 153.

Dunbar, W.B. and Murray, R.M. (2002). Model predictive control of coordinated multi-vehicle formations. In *IEEE Conf. Decision Control*.

Earl, M.G. and D'Andrea, R. (2002). Modeling and control of a multi-agent system using mixed integer linear programming. In *IEEE Conf. Decision Control*.

Eich, M., Hartanto, R., Kasperski, S., Natarajan, S., and Wollenberg, J. (2014). Towards coordinated multirobot missions for lunar sample collection in an unknown environment. *Journal of Field Robotics*, 31(1).

Feddema, J.T., Lewis, C., and Schoenwald, D.A. (2002). Decentralized control of cooperative robotic vehicles: theory and application. *IEEE Trans. Robotics and Automation*, 18(5), 852–864.

Fredslund, J. and Mataric, M.J. (2002). A general algorithm for robot formations using local sensing and minimal communication. *IEEE Trans. Robotics and Automation*, 18(5), 837–846.

Garin, F. and Schenato, L. (2010). A survey on distributed estimation and control applications using linear consensus algorithms. In *Networked Control Systems*, 75–107. Springer.

Gerkey, B.P. and Mataric, M.J. (2002). Sold!: auction methods for multirobot coordination. *IEEE Trans. Robotics and Automation*, 18(5), 758–768.

Gittins, J.C. (1979). Bandit processes and dynamic allocation indices. *Journal of the Royal Statistical Society. Series B (Methodological)*, 148–177.

Gupta, V., Langbort, C., and Murray, R.M. (2006). On the robustness of distributed algorithms. In *IEEE Conf. Decision Control*.

Hereford, J.M. (2011). Analysis of BEECLUST swarm algorithm. In *IEEE Symp. Swarm Intell.*

Khatib, O. (1986). Real-time obstacle avoidance for manipulators and mobile robots. *Int. J. Robotics Research*, 5(1), 90–98.

King, J., Pretty, R.K., and Gosine, R.G. (2003). Coordinated execution of tasks in a multiagent environment. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, 33(5), 615–619.

Koditschek, D.E. and Rimon, E. (1990). Robot navigation functions on manifolds with boundary. *Advances in applied mathematics*, 11(4), 412–442.

Kress-Gazit, H., Conner, D.C., Choset, H., Rizzi, A.A., and Pappas, G.J. (2008). Courteous cars. *IEEE Robotics Automation Magazine*, 15(1), 30–38.

Landgren, P., Srivastava, V., and Leonard, N.E. (2016). On distributed cooperative decision-making in multiarmed bandits. In *IEEE European Control Conf.*

Le Ny, J., Dahleh, M., and Feron, E. (2008). Multi-UAV dynamic routing with partial observations using restless bandit allocation indices. In *IEEE American Control Conf.*

Leonard, N.E., Paley, D.A., Davis, R.E., Fratantoni, D.M., Lekien, F., and Zhang, F. (2010). Coordinated control of an underwater glider fleet in an adaptive ocean sampling field experiment in monterey bay. *Journal of Field Robotics*, 27(6), 718–740.

Li, Q. and Rus, D. (2006). Global clock synchronization in sensor networks. *IEEE Transactions on computers*, 55(2), 214–226.

Liu, X. and Goldsmith, A. (2004). Kalman filtering with partial observation losses. In *IEEE Conf. Decision Control*.

Liu, Y. and Nejat, G. (2016). Multirobot cooperative learning for semiautonomous control in urban search and rescue applications. *Journal of Field Robotics*, 33(4), 512–536.

Lumelsky, V. and Harinarayan, K. (1997). Decentralized motion planning for multiple mobile robots: The cocktail party model. *Autonomous Robots*, 4(1), 121–135.

Lynch, N.A. (1997). *Distributed Algorithms*. Morgan Kaufmann.

Marshall, J.A., Broucke, M.E., and Francis, B.A. (2004). Formations of vehicles in cyclic pursuit. *IEEE Trans. Autom. Control*, 49(11), 1963–1974.

Martinez, S., Bullo, F., Cortes, J., and Frazzoli, E. (2007). On synchronous robotic networks; part I: Models, tasks, and complexity. *IEEE Trans. Autom. Control*, 52(12), 2199 –2213.

Mathews, N., Christensen, A.L., O'Grady, R., Mondada, F., and Dorigo, M. (2017). Mergeable nervous systems for robots. *Nature Communications*, 8(1), 439.

Menon, P.K., Sweriduk, G.D., and Bilimoria, K.D. (2004). New approach for modeling, analysis, and control of air traffic flow. *J. Guid. Control Dyn.*, 27(5).

Mesbahi, M. and Hadaegh, F.Y. (1999). Formation flying control of multiple spacecraft via graphs, matrix inequalities, and switching. In *IEEE Int. Conf. on Control Applications*, volume 2.

Mesquita, A.R., Hespanha, J.P., and Åström, K. (2008). Optimotaxis: A stochastic multi-agent optimization procedure with point measurements. In *Hybrid Systems: Computation and Control*.

Morgan, D., Subramanian, G.P., Chung, S.J., and Hadaegh, F.Y. (2016). Swarm assignment and trajectory optimization using variable-swarm, distributed auction assignment and sequential convex programming. *Int. J. Robotics Research*, 35, 1261–1285.

Ogren, P., Egerstedt, M., and Hu, X. (2002). A control lyapunov function approach to multiagent coordination. *IEEE Trans. Robotics and Automation*, 18(5), 847–851.

Oh, K.K., Park, M.C., and Ahn, H.S. (2015). A survey of multi-agent formation control. *Automatica*, 53, 424–440.

Olfati-Saber, R. (2006). Flocking for multi-agent dynamic systems: algorithms and theory. *IEEE Trans. Autom. Control*, 51(3), 401–420.

Olson, E., Strom, J., Morton, R., Richardson, A., Ranganathan, P., Goeddel, R., Bulic, M., Crossman, J., and Marinier, B. (2012). Progress toward multi-robot reconnaissance and the MAGIC 2010 competition. *Journal of Field Robotics*, 29(5), 762–792.

Omidshafiei, S., Agha-Mohammadi, A.A., Amato, C., and How, J.P. (2015). Decentralized control of partially observable markov decision processes using belief space macro-actions. In *IEEE Int. Conf. on Robotics and Automation*.

Pavone, M., Arsie, A., Frazzoli, E., and Bullo, F. (2011). Distributed algorithms for environment partitioning in mobile robotic networks. *IEEE Trans. Autom. Control*, 56(8), 1834–1848.

Pavone, M., Smith, S.L., Frazzoli, E., and Rus, D. (2012). Robotic load balancing for mobility-on-demand systems. *Int. J. Robotics Research*, 31(7), 839–854.

Rabbat, M. and Nowak, R. (2004). Distributed optimization in sensor networks. In *Proceedings of the 3rd international symposium on Information processing in sensor networks*.

Ren, W., Beard, R.W., and Atkins, E.M. (2007). Information consensus in multivehicle cooperative control. *IEEE Control Syst. Mag.*, 27(2), 71–82.

Reveliotis, S.A. and Roszkowska, E. (2011). Conflict resolution in free-ranging multivehicle systems: A resource allocation paradigm. *IEEE Trans. Robotics*, 27(2), 283–296.

Richards, A., Schouwenaars, T., How, J., and Feron, E. (2002). Spacecraft trajectory planning with avoidance constraints using mixed-integer linear programming. *J. Guid. Control Dyn.*, 25(4).

Richards, A. and How, J.P. (2007). Robust distributed model predictive control. *International Journal of Control*, 80(9), 1517–1531.

Rossi, F., Bandyopadhyay, S., Wolf, M., and Pavone, M. (2018a). Review of multi-agent algorithms for collective behavior: a structural taxonomy (extended version). URL `https://arxiv.org/abs/1803.05464`.

Rossi, F. and Pavone, M. (2014). On the fundamental limitations of performance for distributed decision-making in robotic networks. In *IEEE Conf. Decision Control*.

Rossi, F., Zhang, R., Hindy, Y., and Pavone, M. (2018b). Routing autonomous vehicles in congested transportation networks: structural properties and coordination algorithms. *Autonomous Robots*. In press.

Rubenstein, M., Cornejo, A., and Nagpal, R. (2014). Programmable self-assembly in a thousand-robot swarm. *Science*, 345(6198), 795–799.

Rus, D., Donald, B., and Jennings, J. (1995). Moving furniture with teams of autonomous robots. In *IEEE/RSJ Int. Conf. on Intelligent Robots & Systems*.

Sahai, A. and Mitter, S. (2006). The necessity and sufficiency of anytime capacity for stabilization of a linear system over a noisy communication link - part i: Scalar systems. *IEEE Trans. on Information Theory*, 52(8), 3369–3395.

Scattolini, R. (2009). Architectures for distributed and hierarchical model predictive control - a review. *Journal of Process Control*, 19(5), 723 – 731.

Schouwenaars, T., Stubbs, A., Paduano, J., and Feron, E. (2006). Multivehicle path planning for nonline-of-sight communication. *Journal of Field Robotics*, 23(3-4), 269–290.

Schwager, M., McLurkin, J., Slotine, J.J., and Rus, D. (2009). From theory to practice: Distributed coverage control experiments with groups of robots. In *Experimental Robotics*.

Sepulchre, R., Paley, D.A., and Leonard, N.E. (2007). Stabilization of planar collective motion: All-to-all communication. *IEEE Trans. Autom. Control*, 52(5), 811–824.

Sharon, G., Stern, R., Felner, A., and Sturtevant, N.R. (2015). Conflict-based search for optimal multi-agent pathfinding. *Artificial Intelligence*, 219(Supplement C), 40 – 66.

Shehory, O. and Kraus, S. (1998). Methods for task allocation via agent coalition formation. *Artificial Intelligence*, 101(1), 165–200.

Strömbom, D., Mann, R.P., Wilson, A.M., Hailes, S., Morton, A.J., Sumpter, D.J., and King, A.J. (2014). Solving the shepherding problem: heuristics for herding autonomous, interacting agents. *Journal of the Royal Society Interface*, 11(100), 20140719.

Tanner, H.G., Jadbabaie, A., and Pappas, G.J. (2007). Flocking in fixed and switching networks. *IEEE Trans. Autom. Control*, 52(5), 863–868.

Tsitsiklis, J.N., Bertsekas, D.P., and Athans, M. (1986). Distributed asynchronous deterministic and stochastic gradient optimization algorithms. *IEEE Trans. Autom. Control*, 31(9), 803 – 812.

Turpin, M., Michael, N., and Kumar, V. (2014). CAPT: Concurrent assignment and planning of trajectories for multiple robots. *Int. J. Robotics Research*, 33(1), 98–112.

van den Berg, J., Lin, M., and Manocha, D. (2008). Reciprocal velocity obstacles for real-time multi-agent navigation. In *IEEE Int. Conf. on Robotics and Automation*.

Von Stryk, O. and Bulirsch, R. (1992). Direct and indirect methods for trajectory optimization. *Annals of operations research*, 37(1), 357–373.

Weigel, T., Gutmann, J.S., Dietl, M., Kleiner, A., and Nebel, B. (2002). CS Freiburg: coordinating robots for successful soccer playing. *IEEE Trans. Robotics and Automation*, 18(5), 685–699.

Werfel, J., Petersen, K., and Nagpal, R. (2014). Designing collective behavior in a termite-inspired robot construction team. *Science*, 343(6172), 754–758.

Wu, W. and Zhang, F. (2012). Robust cooperative exploration with a switching strategy. *IEEE Trans. Robotics*, 28(4), 828–839.

Xu, Z., Fitch, R., Underwood, J., and Sukkarieh, S. (2013). Decentralized coordinated tracking with mixed discrete-continuous decisions. *Journal of Field Robotics*, 30(5), 717–740.

Yu, J., LaValle, S.M., and Liberzon, D. (2008). Rendezvous without coordinates. In *IEEE Conf. Decision Control*.

Yu, J., Schwager, M., and Rus, D. (2014). Correlated orienteering problem and its application to informative path planning for persistent monitoring tasks. In *IEEE/RSJ Int. Conf. on Intelligent Robots & Systems*.

Zavlanos, M.M., Jadbabaie, A., and Pappas, G.J. (2007). Flocking while preserving network connectivity. In *IEEE Conf. Decision Control*.

Zhao, S., Ramakrishnan, S., and Kumar, M. (2011). Density-based control of multiple robots. In *IEEE American Control Conf.*

Zhou, D., Wang, Z., Bandyopadhyay, S., and Schwager, M. (2017). Fast, on-line collision avoidance for dynamic vehicles using buffered Voronoi cells. *IEEE Robotics and Automation Letters*, 2(2), 1047–1054.