# Normalized Cuts and Image Segmentation

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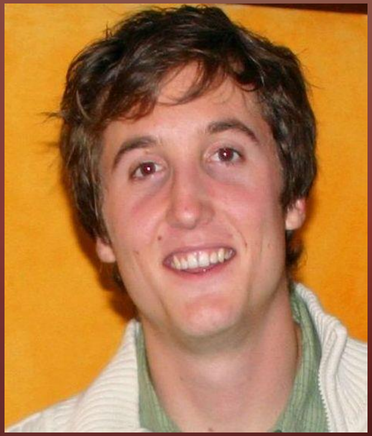


# Presentation Flow

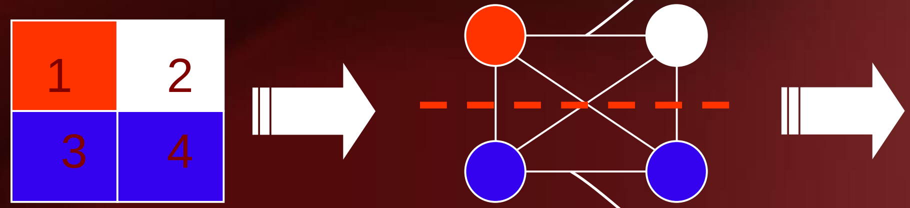
- Motivate image segmentation
- Image segmentation as a graph problem
- Min-Cut
- Normalized cut
- Optimal partition – computing the norm-cut
- Segmentation affinity measures
- Norm-cut algorithm
- Segmented Examples using Ncut
- Conclusion

# Motivation

- **Image segmentation is the problem of associating pixels to an object**
- **Images have a large search space can have a large number of pixels**
- **Knowing which pixels relate to which objects can help for recognition purposes**
- **One method to do image segmentation is to view it as a graph problem**

# Image Segmentation as a Graph Partition Problem

- The image segmentation problem will be transformed into a connected undirected weighted graph



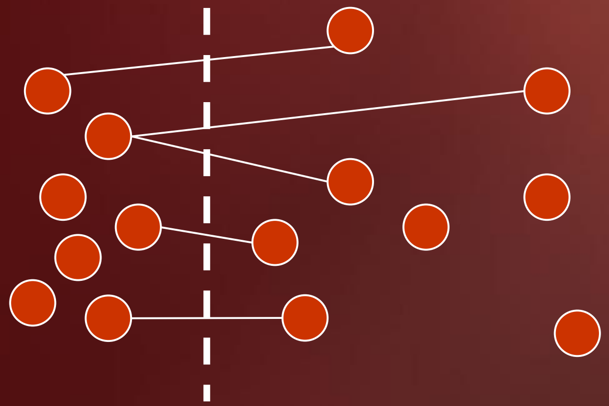

$$\begin{bmatrix} w(1,1) & w(1,2) & w(1,3) & w(1,4) \\ w(2,1) & w(2,2) & w(2,3) & w(2,4) \\ w(3,1) & w(3,2) & w(3,3) & w(3,4) \\ w(4,1) & w(4,2) & w(4,3) & w(4,4) \end{bmatrix}$$

- The weights are called affinity measures
- Affinity measures are a function of similarity between the nodes (e.g. Colour, distance, etc…)
- What is the precise criterion for a good partition?
- How can such a partition be computed efficiently?

# Graph Cut

$$cut(A,B) = \sum_{u \in A, v \in B} w(u,v)$$

- **Where A,B $\in$ V are two disjoint sets of nodes ($A \cap B = \emptyset$)**
- **A graph cut is the sum of the weights for all the edges crossing the cut**

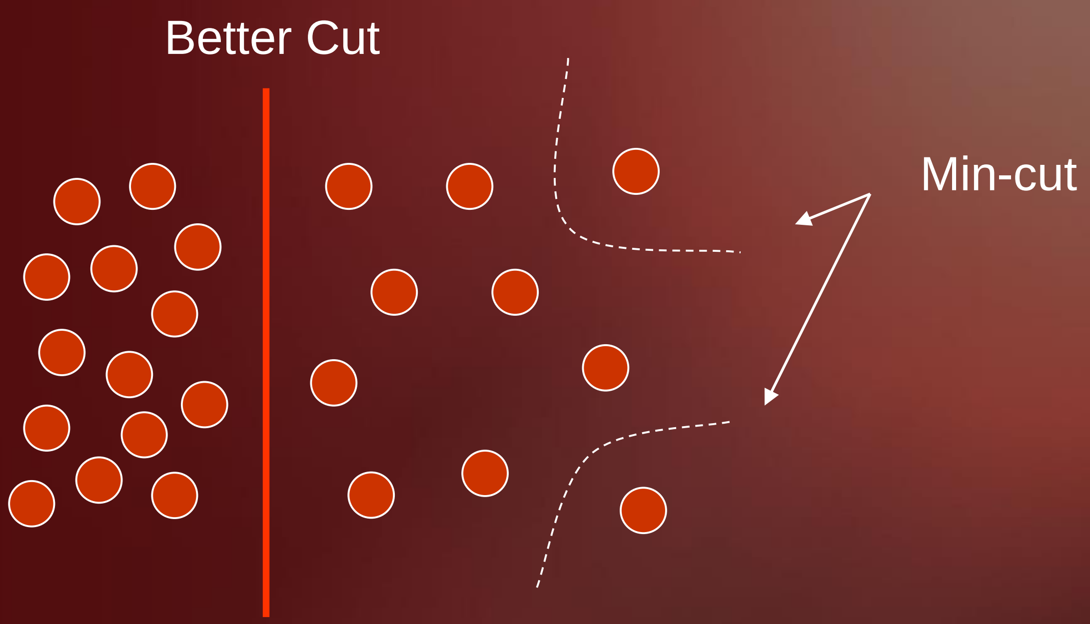


# Minimum Cut

- **The optimal partition is one which minimizes the cut, a.k.a, *Minimum Cut***
- **In terms of image segmentation, the *min-cut* represents pixels which have the least in common (since weights represents similarity measures)**
- **The *min-cut* is a measure of disassociation between two sets**
- ***Wu and Leahy, in 1993* proposed a clustering method based on the *min-cut***
- **The *min-cut* does produces good segmentation results, but not always…**

# Minimum Cut

- *Min-cut* favors individual nodes
- Using a distance affinity
  - Large distances have small weights
  - Small distances have greater weight



# Normalized Cuts

- *Shi* and *Malik* in 2000 proposed a new measure of disassociation:

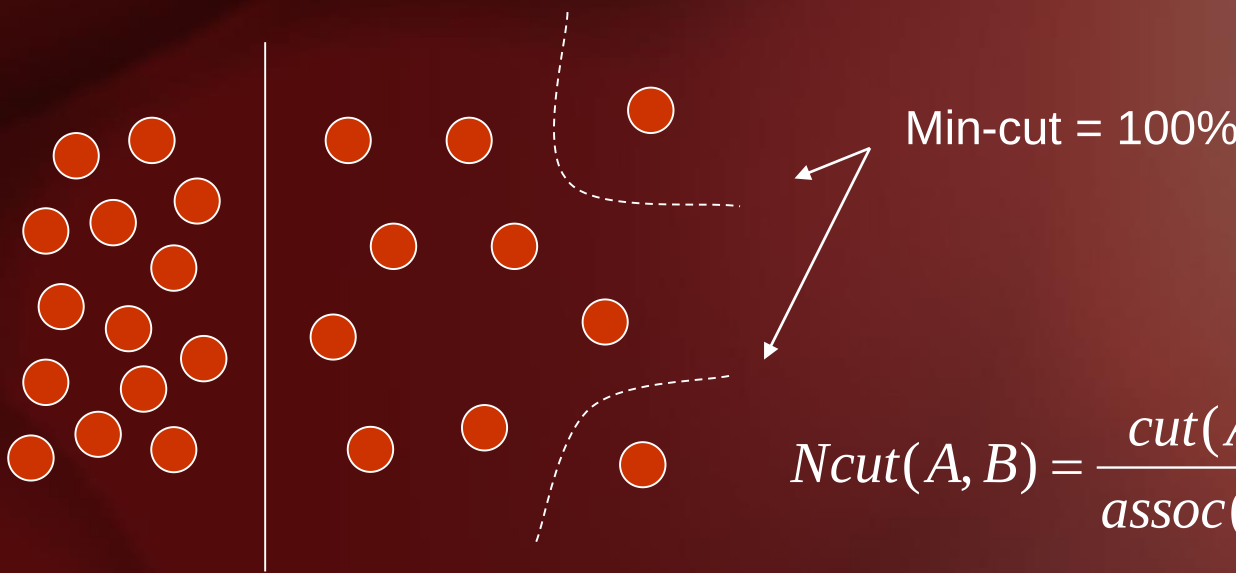
$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

- Where, $assoc(A,V) = \sum_{u \in A, t \in V} w(u,t)$

- The *norm-cut* is the cost of the cut over the total edge connection

# Normalized Cut

- ***Norm-Cut*** **does not favor single nodes**
- **Remember, we need to minimize the cut**




$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

$$assoc(A,V) = \sum_{u \in A, t \in V} w(u,t)$$

# Normalized Cut

- We can also define a Normalized Association measure

$$Nassoc(A,B) = \frac{assoc(A,A)}{assoc(A,V)} + \frac{assoc(B,B)}{assoc(B,V)}$$

- This measure is representative of an average of how tightly connected the nodes are in a set
- We want to maximize the association and minimize the disassociation
- By minimizing the Norm-Cut actually maximizes the association since

$$Ncut(A,B) = 2 - Nassoc(A,B)$$

# Disadvantage of Norm-Cut

- *Papadimitrou* in 97 proved that finding the *norm-cut* is NP-Complete
- Fortunately, in practice, an approximation to the *norm-cut* can be found efficiently
- How do we compute the optimal *norm-cut* partition?

# Optimal Partition

- Minimizing the norm-cut can be transformed into an eigenvalue problem:

$$\min(NCut(A,B)) = \min_y \frac{y^T(D-W)y}{y^T Dy}$$

- D is a Diagonal matrix of the total connection of a node in the graph
- W is the weight matrix
- y is a vector of with two discrete values {1, -b}
- This is an integer programming problem, and we must find y that will satisfy the minimization
- Problem isn't any easier unless…

# Optimal Partition

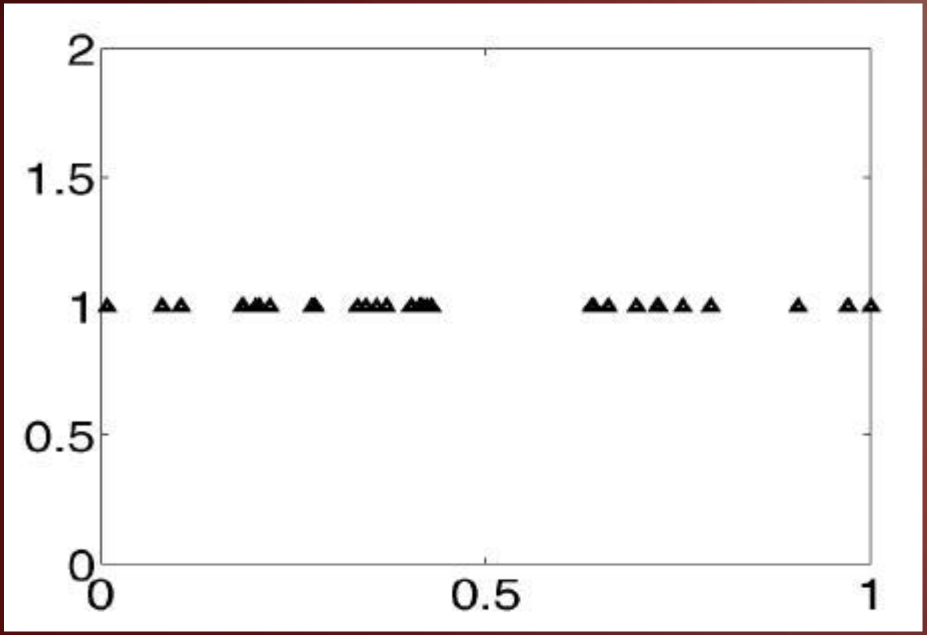
- But relaxing y to take real values we are solving this problem:

$$(D-W)y = \lambda Dy$$

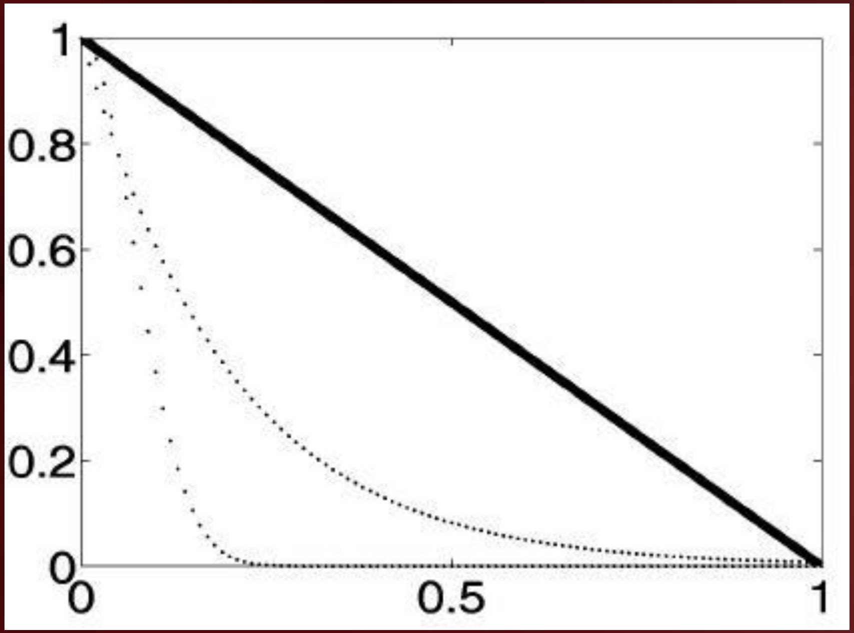
- Now we have an approximation to the solution and can set some threshold value
- Since the values in y are not discrete, we have to find a splitting point
- Which eigenvector do we use to represent the cut?
- The smallest eigenvalue will always be 0
- We choose the second smallest

# Optimal Partition Example

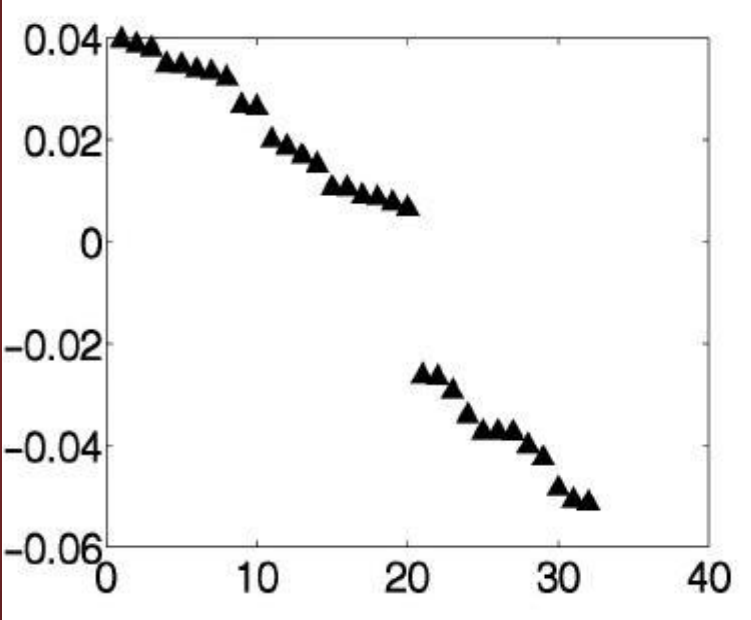
- **1-dimensional segmentation problem**



# Optimal Partition Example

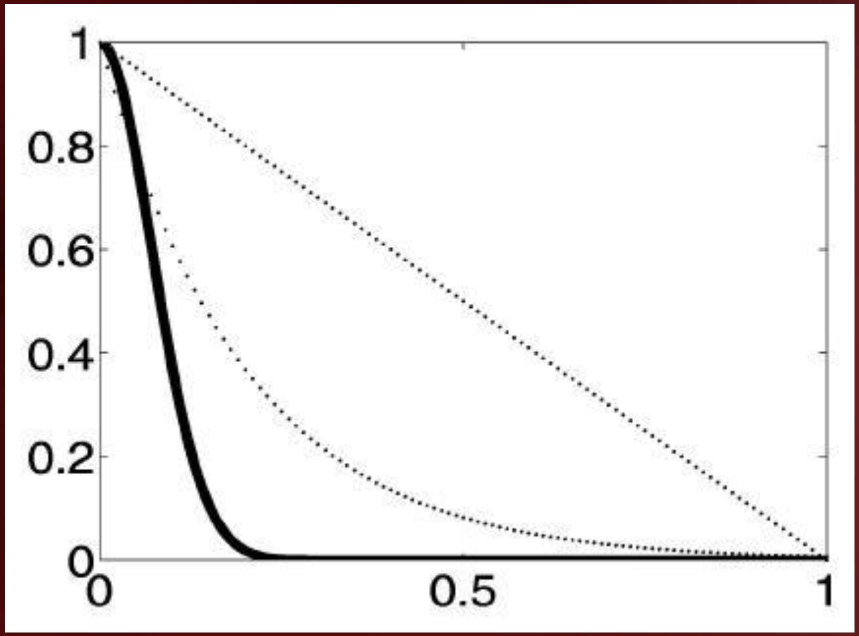



**Affinity measure (distance)**

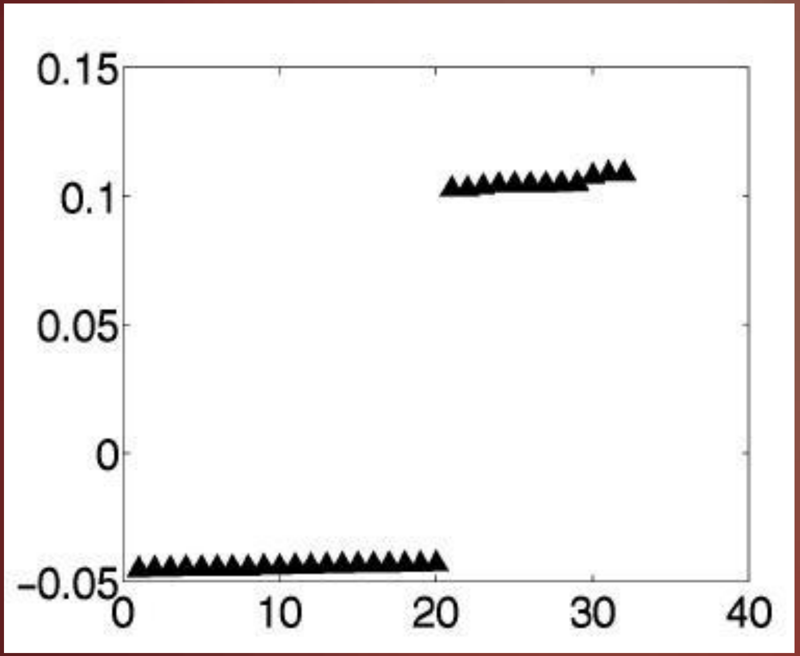



**Second smallest eigenvector**

# Optimal Partition Example




Affinity measure (distance)




Second smallest eigenvector

# Wow, that was a lot!

- Image segmentation can be transformed as a graph partitioning problem
- *Min-cut* was not the optimal partition measure
- *Norm-cut* is a better measure of disassociation since it does not favor any particular nodes
- Minimizing the *norm-cut* also maximizes the internal association within clusters
- Computing the *norm-cut* is NP-Complete but can be approximated to an eigenvalue problem
- Affinity measure is important to your problem

# Ncut applied to image segmentation

- Affinity measures
  - Distance
    - Closer pixels should have greater weight
    - Find a balance between neighbourhood size
  - Intensity
    - Large differences in intensity should have small affinity
  - Color
    - Similar colors should have greater affinity
  - Texture
    - Similar textures should have greater affinity

# Image Segmentation Algorithm Using Norm-Cut

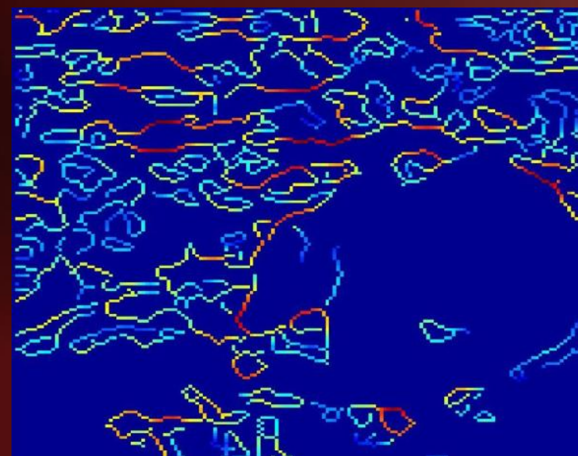
- The image segmentation algorithm is described in 4 steps:

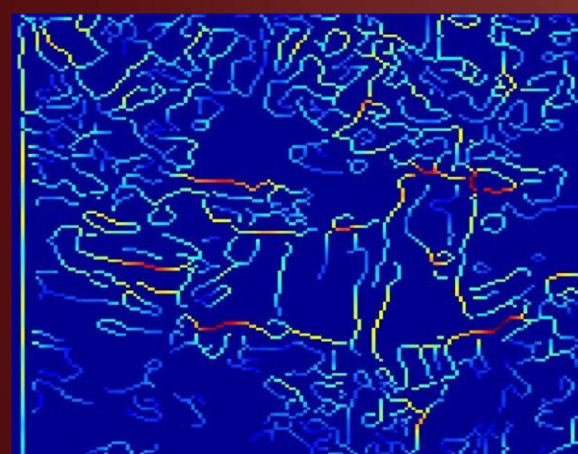
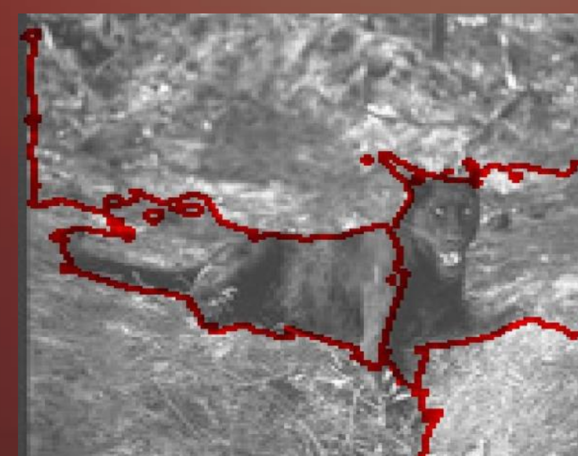
1. Create the weighted graph according to the affinities used
2. Solve the eigenvalue problem: $(D-W)y = \lambda Dy$
3. Use the second smallest eigenvector to bipartition the graph
4. Decide if the current partition should be subdivided and recursively repartition the segmented parts if necessary

# Segmentation Examples using Norm-Cut

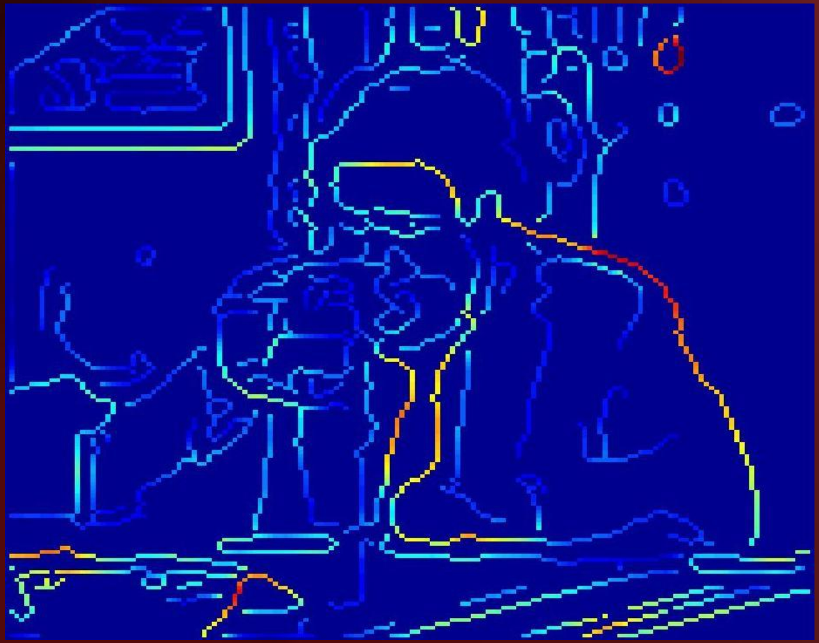
- The following images have been segmented following the norm-cut approach using Intervening contours

# Segmentation Examples using Norm-Cut

# NORM!!



# Conclusion

- Introduce and motivate the problem of image segmentation
- Approach to this problem as a graph cut problem
- Introduce the *norm-cut* approach
- Show some examples

# Thank you for listening ☺

## Any Questions?

# References


- Jianbo Shi and Jitendra Malik, "*Normalized Cuts and Image Segmentation*," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 22, no 8, Aug. 2000
- David A. Forsyth and Jean Ponce, "*Computer Vision: A Modern Approach*", Chapter 14, Prentice Hall, NJ, Reprint 2003
- Z. Wu and R. Leahy, "*An Optimal Graph Theoretic Approach to Data Clustering: Theory and Its Application to Image Segmentation*," IEEE Trans. Pattern Analysis and Machine Intelligence, vol. 15, no 11, pp. 1,101-1,113, Nov. 1993