

Sol - 1

(Q1)  $G_1 = \{ S \rightarrow a, S \rightarrow AZ, A \rightarrow a, Z \rightarrow z \}$

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$$\begin{aligned} S &\rightarrow a \\ S &\rightarrow AZ \\ A &\rightarrow a \\ Z &\rightarrow z \end{aligned}$$

Step 1) Introducing new non-terminal symbols

$G_1$ :

$$\begin{aligned} S &\rightarrow a \mid A_1 Z \\ A &\rightarrow a \\ Z &\rightarrow z \\ A_1 &\rightarrow AZ \end{aligned}$$

Step 2) Eliminating unit productions (none in this case)

Step 3) Eliminating productions with more than two non-terminal on the right-hand-side

$G_1$ :

$$\begin{aligned} S &\rightarrow a \mid A_1 Z \\ A &\rightarrow a \\ Z &\rightarrow z \\ A_1 &\rightarrow A_2 Z \\ A_2 &\rightarrow AZ \end{aligned}$$

CNF

left most derivation 1:

$$\begin{aligned} S &\rightarrow aaB & [S \rightarrow aaB] \\ &\rightarrow aab & [B \rightarrow b] \end{aligned}$$

left most derivation 2:

$$\begin{aligned} S &\rightarrow Ab & [S \rightarrow Ab] \\ &\rightarrow aab & [A \rightarrow a, B \rightarrow b] \end{aligned}$$

2. Consider the string "aaaaB"

left most derivation 1:

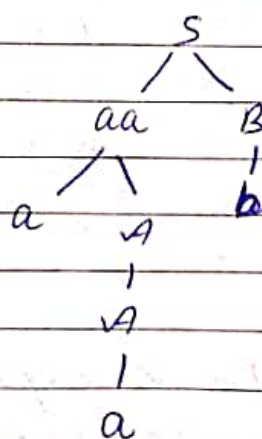
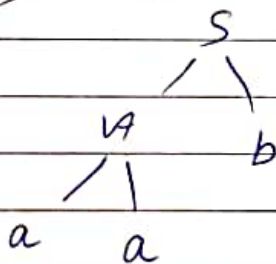
$$\begin{aligned} S &\rightarrow aaB & [S \rightarrow aaB] \\ &\rightarrow aaaS & [S \rightarrow Ab, A \rightarrow a] \\ &\rightarrow aaaaB & [A \rightarrow a] \end{aligned}$$

or

left most derivation 2:

$$\begin{aligned} S &\rightarrow aaB & [S \rightarrow aaB] \\ S &\rightarrow aaaS & [S \rightarrow aaB, A \rightarrow aa] \\ &\rightarrow aaaaB & [A \rightarrow a] \end{aligned}$$

b) derivation tree



Gr 2 :  $\{ S \rightarrow a, S \rightarrow aZ, Z \rightarrow a \}$

Step 1 :

$S \rightarrow a \mid A, Z$

$Z \rightarrow a$

$A, \rightarrow a$

Step 2 :

Eliminating unit productions (none in this case)

Step 3 :

Eliminating productions with more than two non-terminals on the right-hand-side :

Gr 2 =

$S \rightarrow a \mid A, Z$

$Z \rightarrow a$

$A, \rightarrow a$

~~Not~~

CNF

Q2) Given the following ambiguous Context free grammar

$S \rightarrow Ab \mid aaB$

$A \rightarrow a \mid Aa$

$B \rightarrow b$

a)  $S \rightarrow Ab \mid aaB$

$A \rightarrow a \mid Aa$

$B \rightarrow b$



$$Q3) \quad S \rightarrow OBB \\ B \rightarrow OS \mid IS \mid O$$

⇒

$$A = \{ (q), (0, 1), (S, B, 0, 1), \delta, q, S, ? )$$

The production rule  $\delta$  can be :

$$R1 : \delta(q, \epsilon, S) = \{ (q, OBB) \}$$

$$R2 : \delta(q, \epsilon, B) = \{ (q, OS) \mid (q, IS) \mid (q, O) \}$$

$$R3 : \delta(q, 0, 0) = \{ (q, \epsilon) \}$$

$$R4 : \delta(q, 1, 1) = \{ (q, \epsilon) \}$$

Testing  $010^4$

$$\delta(q, 0000, S) \vdash \delta(q, 010000, OBB)$$

$$\vdash \delta(q, 10000, BB) \quad R1$$

$$\vdash \delta(q, 10000, ISB) \quad R3$$

$$\vdash \delta(q, 0000, SB) \quad R2$$

$$\vdash \delta(q, 0000, OBBB) \quad R1$$

$$\vdash \delta(q, 000, BBB) \quad R3$$

$$\vdash \delta(q, 000, OBB) \quad R2$$

$$\vdash \delta(q, 00, BB) \quad R3$$

$$\vdash \delta(q, 00, OB) \quad R2$$

$$\vdash \delta(q, 0, B) \quad R3$$

$$\vdash \delta(q, 0, O) \quad R2$$

$$\vdash \delta(q, \epsilon) \quad R3$$

~~Accept~~ Accept

$010^4$  is accepted by the PDA.

Q5) Eliminate null productions

$$P) S \rightarrow a s b / a A b / a b / a$$

$$A \rightarrow \epsilon$$

$$ii) S \rightarrow a x b x$$

$$x \rightarrow a y / b y / \epsilon$$

$$y \rightarrow x / d$$

$$P) S \rightarrow a s b / a A b / a b / a$$

$$A \rightarrow \epsilon$$

Step 1) Identify nullable non-terminals  
The non-terminal  $A$  is nullable because it directly produces  $\epsilon$ .

Step 2) Update productions for nullable non-terminals

We need to generate all possible combinations of  $A$  being included or excluded from the derivation.

for each production  $P$  containing  $A$ , create a new production without  $A$ , and another production with  $A$  removed:

- for  $S \rightarrow a A b$ , we create  $S \rightarrow a b$ .

- for  $S \rightarrow a s b$ , " " "  $S \rightarrow a s b$ ,  $S \rightarrow a b$ ,

$S \rightarrow a s$ , and  $S \rightarrow s b$ .

- for  $a$ , we keep it as is

- for  $A \rightarrow \epsilon$ , we don't create any new production.



$$S \rightarrow a s b \mid a b \mid a s \mid s b \mid a$$

$$A \rightarrow \epsilon$$

$$\text{ii) } S \rightarrow a x b x$$

$$x \rightarrow a y \mid b y \mid \epsilon$$

$$y \rightarrow x \mid d$$

Step 1: Identify nullable non-terminals  
The non-terminals  $x$  is nullable because  
it directly produces  $\epsilon$ .

Step 2: Update productions for nullable non-terminals.

• ~~for~~  $S \rightarrow a x b x$ , we ~~create~~  $x$ , ~~create a new~~

for  $S \rightarrow a x b x$ , we create  $S \rightarrow ab$  and  $S \rightarrow a y b$ .

for  $x \rightarrow a y$ , " " "  $x \rightarrow a y$ ,  $x \rightarrow a$  and  $x \rightarrow y$ .

for  $x \rightarrow b y$ , " " "  $x \rightarrow b y$ ,  $x \rightarrow b$  and  $x \rightarrow y$

"  $x \rightarrow \epsilon$ , we don't create any new production

$$S \rightarrow ab \mid a y b$$

$$x \rightarrow a y \mid a \mid b y \mid b \mid y$$

$$y \rightarrow x \mid d$$