Page :
Date : / /

Set -

(Q1) G1 => { S → a, S → AZ, A → a, Z → z }

=> S → a
S → AZ
A → a
Z → z

Step 1) Introducing new non-terminal symbols
G1 :
$S \to a \mid A_1 Z$
$A \to a$
$Z \to z$
$A_1 \to AZ$

Step 2) Eliminating unit productions (none in this case)

Step 3) Eliminating productions with more than two non-terminal on the right-hand-side
G1 :
$S \to a \mid A_1 Z$
$A \to a$
$Z \to z$
$A_1 \to A_2 Z$
$A_2 \to AZ$
CNF

Page :
Date : / /

left most derivation 1 :

$S \rightarrow aaB$ $[S \rightarrow aaB]$

$\rightarrow aab$ $[B \rightarrow b]$

left most derivation 2 :

$S \rightarrow Ab$ $[S \rightarrow Ab]$

$\rightarrow aab$ $[A \rightarrow a, B \rightarrow b]$

2. Consider the string "aaaaB"

left most derivation 1 :

$S \rightarrow aaB$ $[S \rightarrow aaB]$

$\rightarrow aaaS$ $[S \rightarrow Ab, A \rightarrow a]$

$\rightarrow aaaaB$ $[A \rightarrow a]$

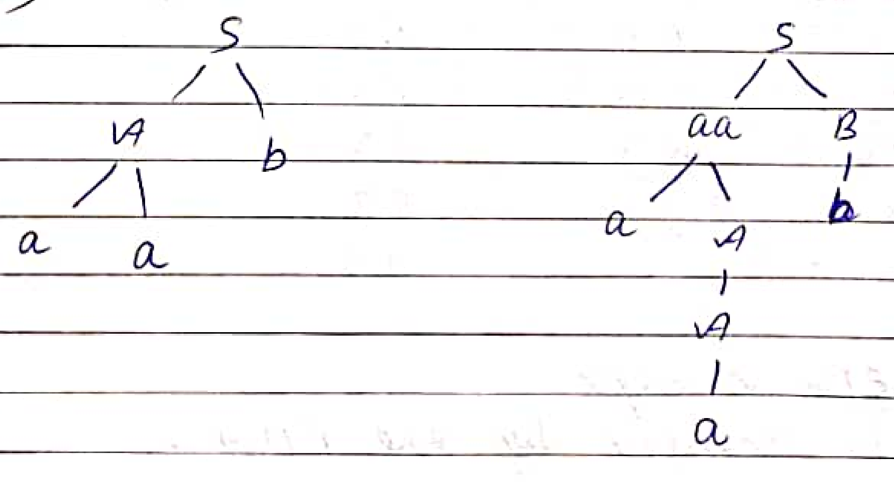
left most derivation 2 :

$S \rightarrow aaB$ $[S \rightarrow aaB]$

$S \rightarrow aaaS$ $[S \rightarrow aaB, A \rightarrow Aa]$

$\rightarrow aaaaB$ $[A \rightarrow a]$

b) derivation tree



Page :
Date : / /

$G_2 = \{ S \to a, S \to aZ, Z \to a \}$

Step 1 :

$S \to a \mid A_1 Z$
$Z \to a$
$A_1 \to a$

Step 2 :

Eliminating unit productions (none in this case)

Step 3 :

Eliminating productions with more than two non-terminals on the right-hand-side :

G_2 =

$S \to a \mid A_1 Z$
$Z \to a$
$A_1 \to a$

~~Note~~ → CNF

Q2) Given the following ambiguous Context free grammar

$S \to Ab \mid aaB$
$A \to a \mid Aa$
$B \to b$

a) $S \to Ab \mid aaB$
$A \to a \mid Aa$
$B \to b$

Page :
Date : / /

Q3) $S \to 0BB$
$B \to 0S \mid 1S \mid 0$

$\Rightarrow$

$A = \{(q), (0,1), (S, B, 0, 1), \delta, q, S, ?\}$

The production rule δ can be :

$R1 : \delta(q, \varepsilon, S) = \{(q, 0BB)\}$
$R2 : \delta(q, \varepsilon, B) = \{(q, 0S) \mid (q, 1S) \mid (q, 0)\}$
$R3 : \delta(q, 0, 0) = \{(q, \varepsilon)\}$
$R4 : \delta(q, 1, 1) = \{(q, \varepsilon)\}$

Testing 010^4
$\delta(q, 0000, S) \vdash \delta(q, 010000, 0BB)$

$\vdash \delta(q, 10000, BB)$ R1
$\vdash \delta(q, 10000, 1SB)$ R3
$\vdash \delta(q, 0000, SB)$ R2
$\vdash \delta(q, 0000, 0BBB)$ R1
$\vdash \delta(q, 000, BBB)$ R3
$\vdash \delta(q, 000, 0BB)$ R2
$\vdash \delta(q, 00, BB)$ R3
$\vdash \delta(q, 00, 0B)$ R2
$\vdash \delta(q, 0, B)$ R3
$\vdash \delta(q, 0, 0)$ R2
$\vdash \delta(q, \varepsilon)$ R3

~~Accept~~ Accept
010^4 is accepted by the PDA.

Page :
Date : / /

Q5) Eliminate null productions

i) $S \rightarrow aSb / aAb / ab / a$
$A \rightarrow \epsilon$

ii) $S \rightarrow aXbX$
$X \rightarrow aY / bY / \epsilon$
$Y \rightarrow X / d$

i) $S \rightarrow aSb \mid aAb \mid ab \mid a$
$A \rightarrow \epsilon$

Step 1) Identify nullable non-terminals
The non terminal A is nullable because it directly produces ϵ.

Step 2) Update productions for nullable non-terminals
We need to generate all possible combination of A being included or excluded from the derivation.
For each production P containing A, create a new production without A, and another production with A removed:

- For $S \rightarrow aAb$, we create $S \rightarrow ab$.
- For $S \rightarrow aSb$, " " $S \rightarrow aSb$, $S \rightarrow ab$, $S \rightarrow aS$, and $S \rightarrow Sb$.
- For a, we keep it as is
- For $A \rightarrow \epsilon$, we don't create any new production.

Page :
Date : / /

$S \rightarrow aSb \mid ab \mid aS \mid Sb \mid a$
$A \rightarrow \varepsilon$

ii) $S \rightarrow aXbX$
$X \rightarrow aY \mid bY \mid \varepsilon$
$Y \rightarrow X \mid d$

Step 1 : Identify nullable non-terminals
The non-terminals X is nullable because it directly produces ε.

Step 2 : Update productions for nullable non-terminals.

• ~~for $S \rightarrow aXbX$, we create~~ X, ~~create a new~~

for $S \rightarrow aXbX$, we create $S \rightarrow ab$ and $S \rightarrow aYb$.
for $\cancel{X} \rightarrow aY$, " " $X \rightarrow aY$, $X \rightarrow a$ and $X \rightarrow Y$.
for $X \rightarrow bY$, " " $X \rightarrow bY$, $X \rightarrow b$ and $X \rightarrow Y$
" $X \rightarrow \varepsilon$, we don't create any new production

$S \rightarrow ab \mid aYb$
$X \rightarrow aY \mid a \mid bY \mid b \mid Y$
$Y \rightarrow X \mid d$