FME 202/FPE 211 Solid & Structural Mechanics II

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Tutorial: Mon 11am -1pm
Lecture: Tues 3 - 4 pm

Learning outcomes

Upon completion of this lecture, the student should be able to;

- Determine shear flow in built up members

Lecture outline

This lecture is based on Chapter 8 of R C Hibbeler’s Book – Mechanics of Materials -12th Edition

- 8.2 State of Stress – Combined Loading

8.2 Combined Loading

In the previous chapters we showed how to determine the stress in a member subjected to either an internal axial force, a shear force, a bending moment, or a torsional moment. Most often, however, the cross section of a member will be subjected to several of these loadings simultaneously, and when this occurs, then the method of superposition should be used to determine the resultant stress. The following procedure for analysis provides a method for doing this.

This chimney is subjected to the combined internal loading caused by the wind and the chimney's weight.

Conditions for analysis

- Material is homogenous
- Material is loaded in the linear- elastic manner
- St Vernant’s principal to apply – stress to be determined in sections that are far removed from the point of load applications and from discontinuities.

Procedure

Internal Loading.

- Section the member perpendicular to its axis at the point where the stress is to be determined; and use the equations of equilibrium to obtain the resultant internal normal and shear force components, and the bending and torsional moment components.
- The force components should act through the *centroid* of the cross section, and the moment components should be calculated about *centroidal axes*, which represent the principal axes of inertia for the cross section.

Procedure

Normal Force.

- The normal force is related to a uniform normal-stress distribution determined from $\sigma = N/A$.

Shear Force.

- The shear force is related to a shear-stress distribution determined from the shear formula, $\tau = VQ/It$.

Bending Moment.

- For *straight members* the bending moment is related to a normal-stress distribution that varies linearly from zero at the neutral axis to a maximum at the outer boundary of the member. This stress distribution is determined from the flexure formula, $\sigma = -My/I$. If the member is *curved*, the

Procedure

Torsional Moment.

- For circular shafts and tubes the torsional moment is related to a shear-stress distribution that varies linearly from zero at the center of the shaft to a maximum at the shaft's outer boundary. This stress distribution is determined from the torsion formula, $\tau = T\rho/J$.

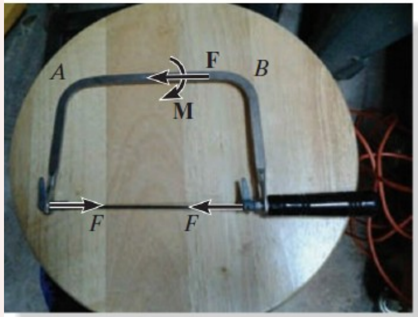
Thin-Walled Pressure Vessels.

- If the vessel is a thin-walled cylinder, the internal pressure p will cause a biaxial state of stress in the material such that the hoop or circumferential stress component is $\sigma_1 = pr/t$, and the longitudinal stress component is $\sigma_2 = pr/2t$. If the vessel is a thin-walled sphere, then the biaxial state of stress is represented by two equivalent components, each having a magnitude of $\sigma_2 = pr/2t$.

Procedure

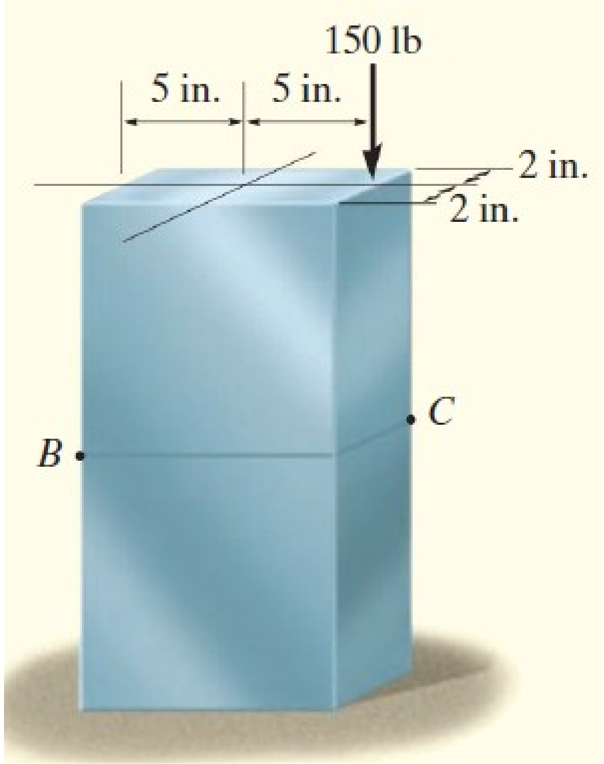
Superposition.

- Once the normal and shear stress components for each loading have been calculated, use the principle of superposition and determine the resultant normal and shear stress components.
- Represent the results on an element of material located at a point, or show the results as a distribution of stress acting over the member's cross-sectional area.



When a pretension force F is developed in the blade of this coping saw, it will produce both a compressive force F and bending moment M at the section AB of the frame. The material must therefore resist the normal stress produced by both of these loadings.

Example 8.2



A force of 150 lb is applied to the edge of the member shown in Fig. 8–3*a*. Neglect the weight of the member and determine the state of stress at points *B* and *C*.

Example 8.2

SOLUTION

Internal Loadings. The member is sectioned through B and C, Fig. 8–3b. For equilibrium at the section there must be an axial force of 150 lb acting through the *centroid* and a bending moment of 750 lb · in. about the centroidal principal axis, Fig. 8–3b.

Stress Components.

Normal Force. The uniform normal-stress distribution due to the normal force is shown in Fig. 8–3c. Here

$$\sigma = \frac{N}{A} = \frac{150 \text{ lb}}{(10 \text{ in.})(4 \text{ in})} = 3.75 \text{ psi}$$

Bending Moment. The normal-stress distribution due to the bending moment is shown in Fig. 8–3d. The maximum stress is

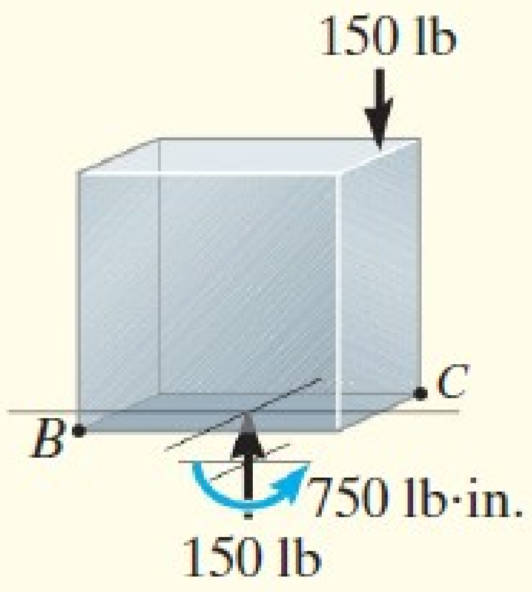
$$\sigma_{\max} = \frac{Mc}{I} = \frac{750 \text{ lb} \cdot \text{in.}\ (5 \text{ in.})}{\frac{1}{12}\,(4 \text{ in.})\,(10 \text{ in.})^3} = 11.25 \text{ psi}$$

Example 8.2

Superposition. Algebraically adding the stresses at B and C, we get

$$\sigma_B = -\frac{N}{A} + \frac{Mc}{I} = -3.75 \text{ psi} + 11.25 \text{ psi} = 7.5 \text{ psi} \quad \text{(tension)} \qquad \textit{Ans.}$$

$$\sigma_C = -\frac{N}{A} - \frac{Mc}{I} = -3.75 \text{ psi} - 11.25 \text{ psi} = -15 \text{ psi (compression)} \qquad \textit{Ans.}$$

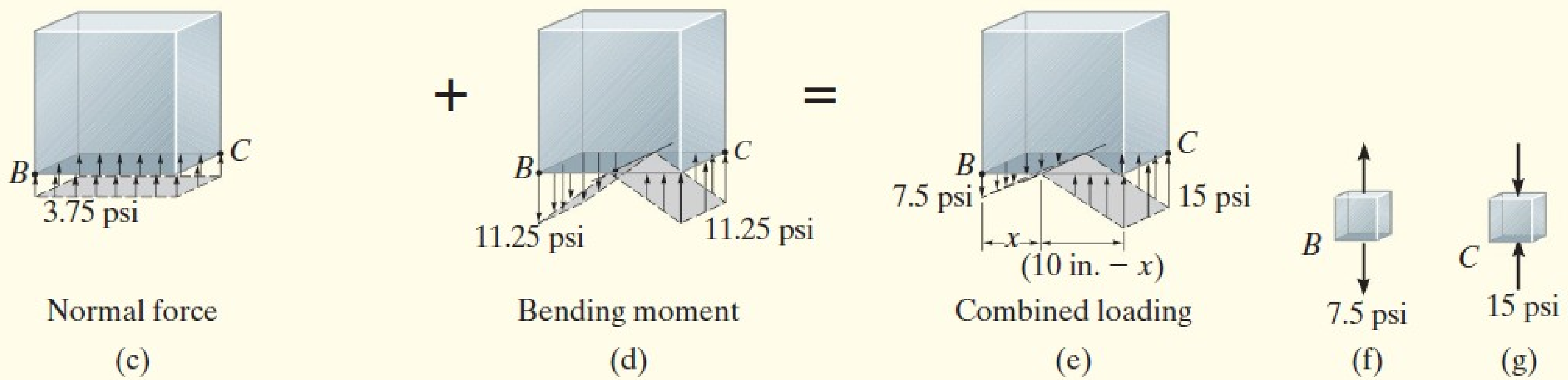


(b)

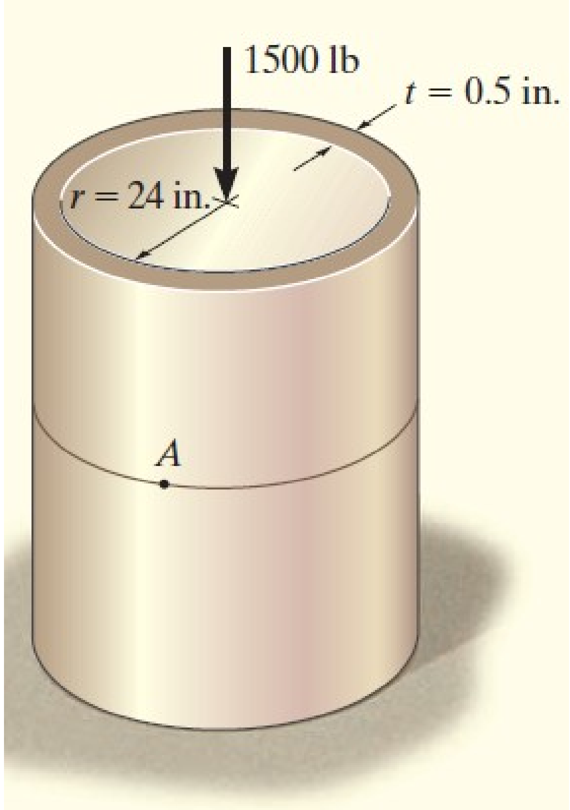
Example 8.2

NOTE: The resultant stress distribution over the cross section is shown in Fig. 8–3*e*, where the location of the line of zero stress can be determined by proportional triangles; i.e.,

$$\frac{7.5 \text{ psi}}{x} = \frac{15 \text{ psi}}{(10 \text{ in.} - x)}; \quad x = 3.33 \text{ in.}$$



Example 8.3



(a)

The gas tank in Fig. 8–4*a* has an inner radius of 24 in. and a thickness of 0.5 in. If it supports the 1500-lb load at its top, and the gas pressure within it is $2\ \text{lb/in}^2$, determine the state of stress at point A.

Example 8.3

Internal Loadings. The free-body diagram of the section of the tank above point A is shown in Fig. 8–4b.

Stress Components.

Circumferential Stress. Since $r/t = 24\text{ in.}/0.5\text{ in.} = 48 > 10$, the tank is a thin-walled vessel. Applying Eq. 8–1, using the inner radius $r = 24$ in., we have

$$\sigma_1 = \frac{pr}{t} = \frac{2\text{ lb/in}^2\,(24\text{ in.})}{0.5\text{ in.}} = 96\text{ psi} \qquad \textit{Ans.}$$

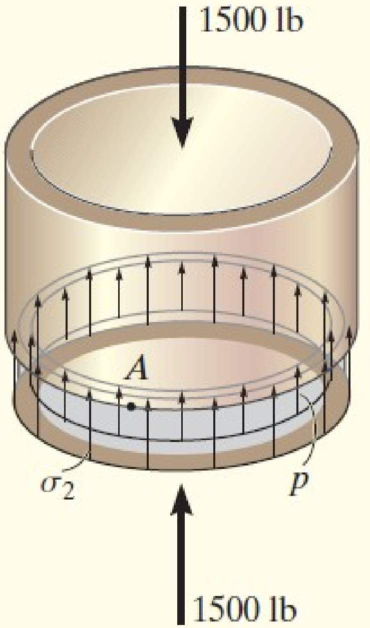
Longitudinal Stress. Here the wall of the tank uniformly supports the load of 1500 lb (compression) and the pressure stress (tensile). Thus, we have

$$\sigma_2 = -\frac{N}{A} + \frac{pr}{2t} = -\frac{1500\text{ lb}}{\pi[(24.5\text{ in.})^2 - (24\text{ in.})^2]} + \frac{2\text{ lb/in}^2\,(24\text{ in.})}{2\,(0.5\text{ in.})}$$

$$= 28.3\text{ psi} \qquad \textit{Ans.}$$

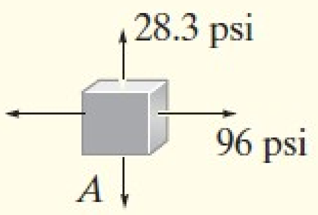
Example 8.3

Point A is therefore subjected to the biaxial stress shown in Fig. 8–4c.



(b)

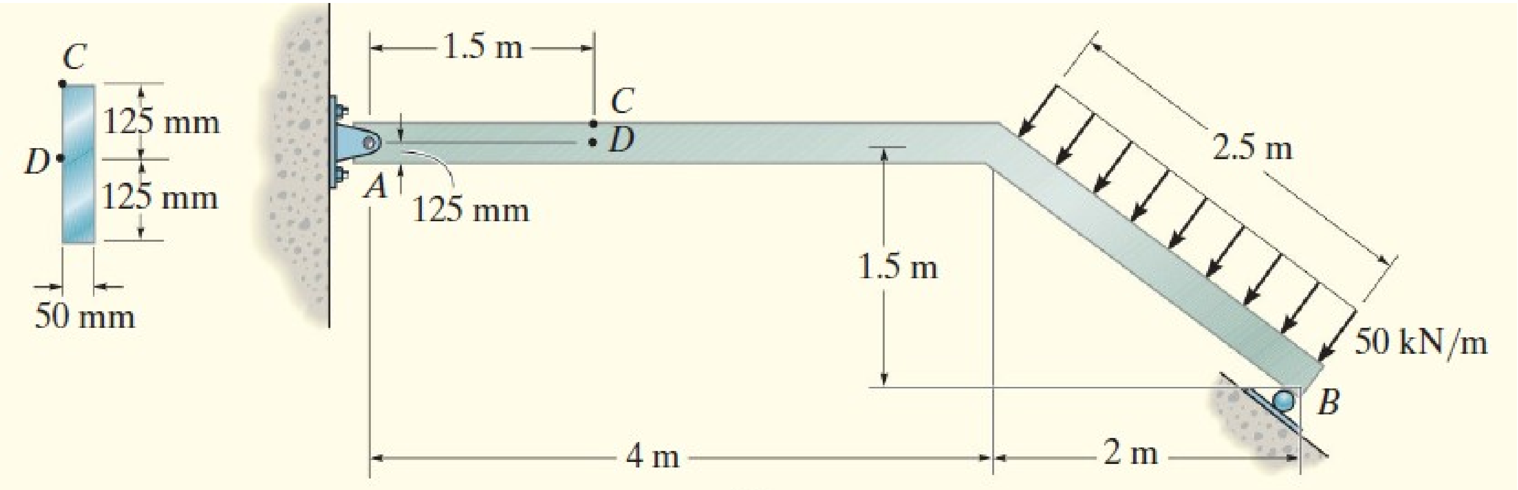
Fig. 8–4



(c)

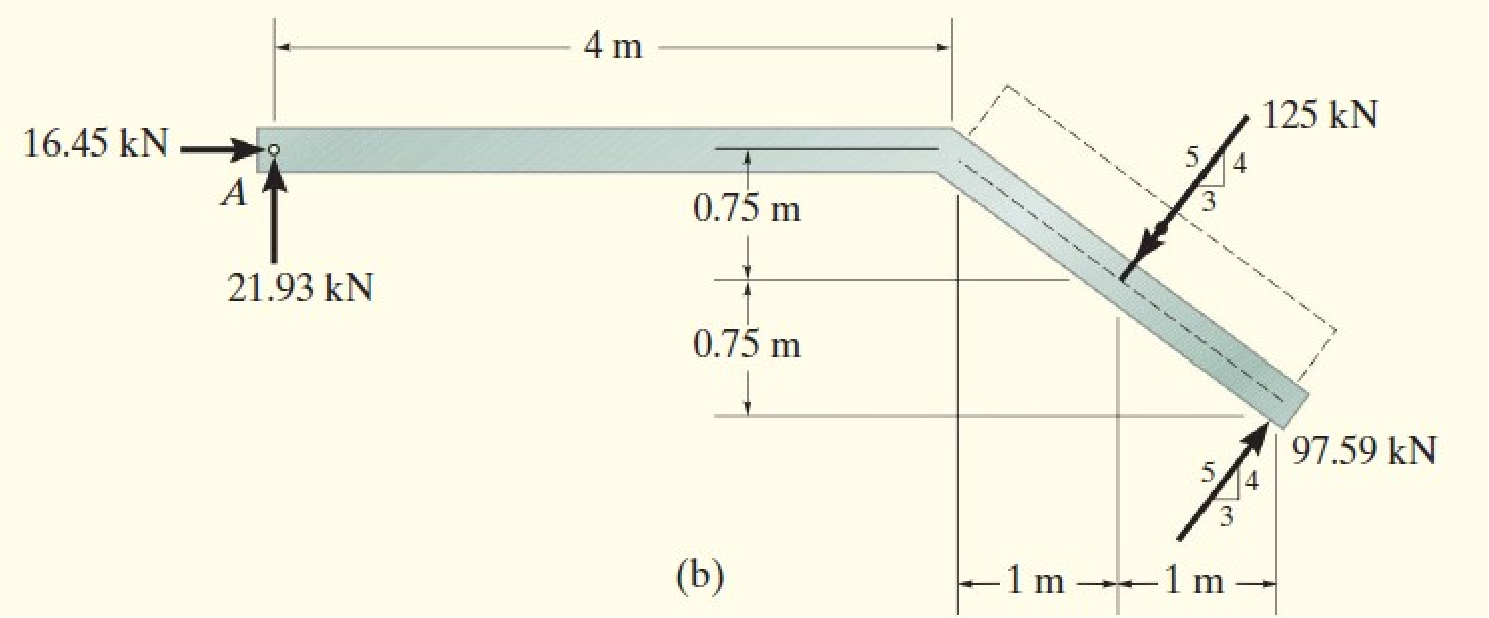
Example 8.4

The member shown in Fig. 8–5*a* has a rectangular cross section. Determine the state of stress that the loading produces at point *C* and point *D*.



(a)

Example 8.4



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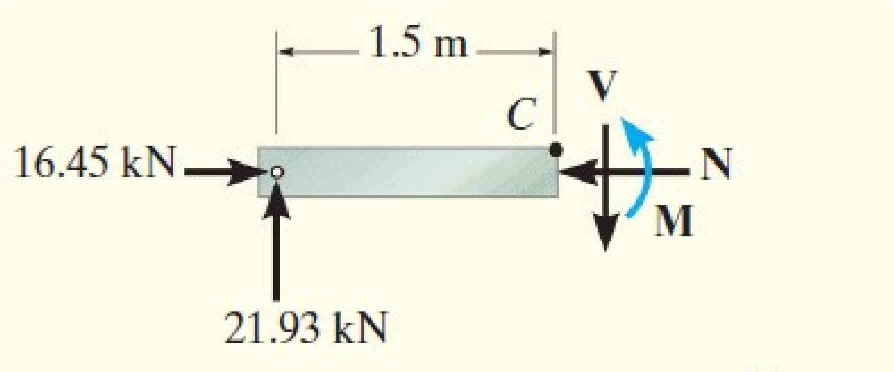


(c)

Example 8.4

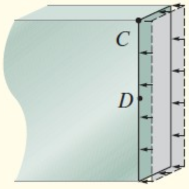
Internal Loadings. The support reactions on the member have been determined and are shown in Fig. 8–5*b*. (As a review of statics, apply $\Sigma M_A = 0$ to show $F_B = 97.59$ kN.) If the left segment *AC* of the member is considered, Fig. 8–5*c*, then the resultant internal loadings at the section consist of a normal force, a shear force, and a bending moment. They are

$$N = 16.45\text{ kN} \qquad V = 21.93\text{ kN} \qquad M = 32.89\text{ kN}\cdot\text{m}$$

Stress Components at C.

Normal Force. The uniform normal-stress distribution acting over the cross section is produced by the normal force, Fig. 8–5*d*. At point *C*,

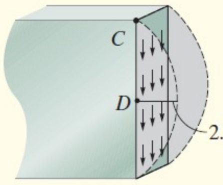
$$\sigma_C = \frac{N}{A} = \frac{16.45(10^3)\text{ N}}{(0.050\text{ m})(0.250\text{ m})} = 1.32\text{ MPa}$$



Normal force

Shear Force. Here the area $A' = 0$, since point *C* is located at the top of the member. Thus $Q = \bar{y}'A' = 0$, Fig. 8–5*e*. The shear stress is therefore

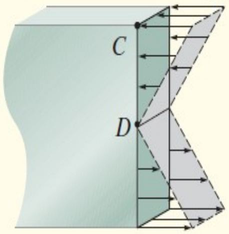
$$\tau_C = 0$$



Shear force

Bending Moment. Point *C* is located at $y = c = 0.125$ m from the neutral axis, so the bending stress at *C*, Fig. 8–5*f*, is

$$\sigma_C = \frac{Mc}{I} = \frac{(32.89(10^3)\text{ N}\cdot\text{m})(0.125\text{ m})}{\left[\frac{1}{12}(0.050\text{ m})(0.250\text{ m})^3\right]} = 63.16\text{ MPa}$$



Bending moment

(f)

Example 8.4

Superposition. There is no shear-stress component. Adding the normal stresses gives a compressive stress at C having a value of

$$\sigma_C = 1.32 \text{ MPa} + 63.16 \text{ MPa} = 64.5 \text{ MPa} \qquad \textit{Ans.}$$

(g)

This result, acting on an element at C, is shown in Fig. 8–5*g*.

Stress Components at *D*.

Normal Force. This is the same as at C, $\sigma_D = 1.32$ MPa, Fig. 8–5*d*.

Shear Force. Since D is at the neutral axis, and the cross section is rectangular, we can use the special form of the shear formula, Fig. 8–5*e*.

$$\tau_D = 1.5\frac{V}{A} = 1.5\frac{21.93(10^3) \text{ N}}{(0.25 \text{ m})(0.05 \text{ m})} = 2.63 \text{ MPa} \qquad \textit{Ans.}$$

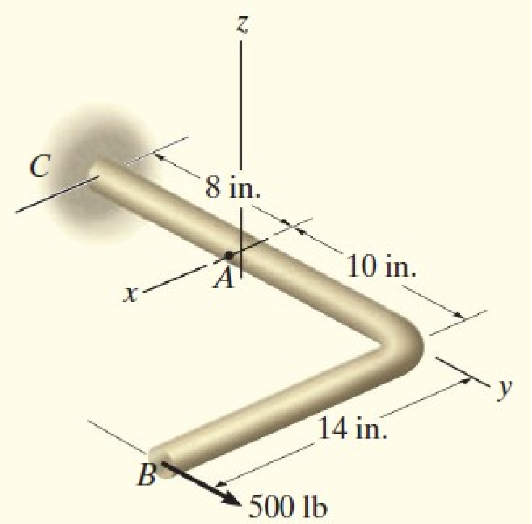
Bending Moment. Here D is on the neutral axis and so $\sigma_D = 0$.

Superposition. The resultant stress on the element is shown in Fig. 8–5*h*.

(h)

Example 8.5

The solid rod shown in Fig. 8–6*a* has a radius of 0.75 in. If it is subjected to the force of 500 lb, determine the state of stress at point *A*.



(a)

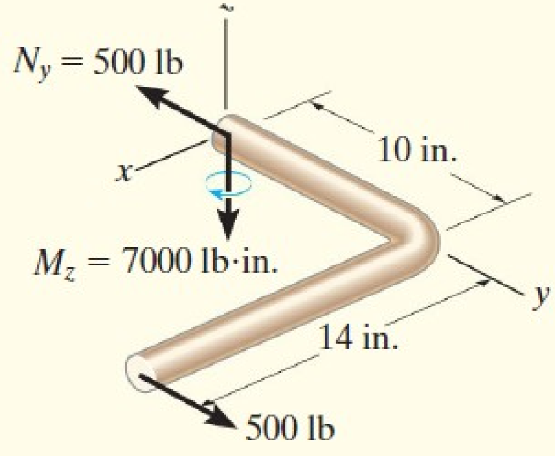
Example 8.5

Internal Loadings. The rod is sectioned through point A. Using the free-body diagram of segment AB, Fig. 8–6b, the resultant internal loadings are determined from the equations of equilibrium.

$$\Sigma F_y = 0; \quad 500\text{ lb} - N_y = 0; \quad N_y = 500\text{ lb}$$

$$\Sigma M_z = 0; \quad 500\text{ lb}(14\text{ in.}) - M_z = 0; \qquad M_z = 7000\text{ lb}\cdot\text{in.}$$

In order to better "visualize" the stress distributions due to these loadings, we can consider the *equal but opposite resultants* acting on segment AC, Fig. 8–6c.



(b)

Example 8.5

Stress Components.

Normal Force. The normal-stress distribution is shown in Fig. 8–6*d*. For point A, we have

$$(\sigma_A)_y = \frac{N}{A} = \frac{500 \text{ lb}}{\pi(0.75 \text{ in.})^2} = 283 \text{ psi} = 0.283 \text{ ksi}$$

Bending Moment. For the moment, $c = 0.75$ in., so the bending stress at point A, Fig. 8–6*e*, is

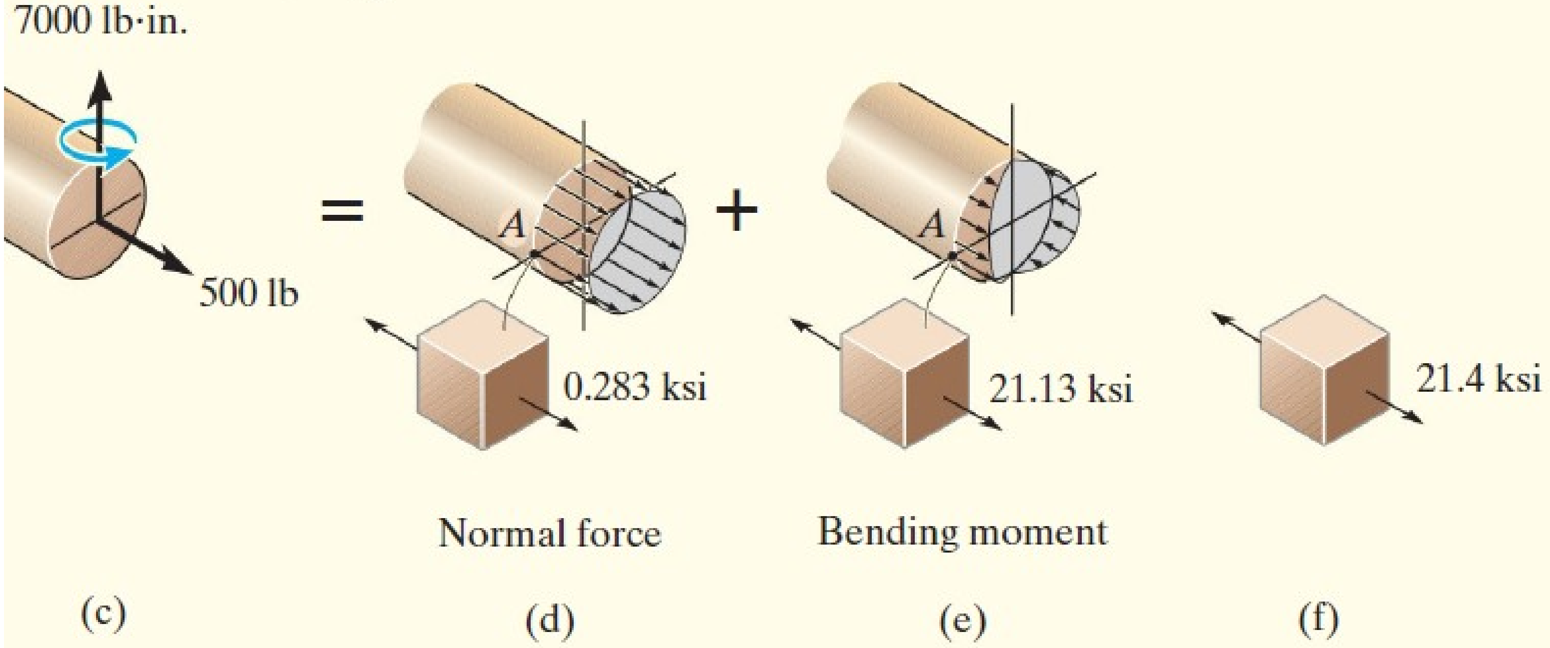
$$(\sigma_A)_y = \frac{Mc}{I} = \frac{7000 \text{ lb} \cdot \text{in.}(0.75 \text{ in.})}{\left[\frac{1}{4}\pi(0.75 \text{ in.})^4\right]}$$

$$= 21\,126 \text{ psi} = 21.13 \text{ ksi}$$

Example 8.5

Superposition. When the above results are superimposed, it is seen that an element at A, Fig. 8–6f, is subjected to the normal stress

$$(\sigma_A)_y = 0.283\ \text{ksi} + 21.13\ \text{ksi} = 21.4\ \text{ksi} \qquad \textit{Ans.}$$



Homework

- P8.1 a
- P8.1 b
- F8-3
- F8-6
- F8-7

Recommended Texts

- **Mechanics of materials – R C Hibbeler 12th Edition**
- Introductory Mechanics of Materials – 2nd Edition, Madhukar Vable – available online **FREE**
- Mechanics of Materials , J.M. Gere & S.P. Timoshenko