FME201/FPE 211- Solid & Structural Mechanics I

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Lecture: Mon 11am -1pm (Online)
Tutorial Tue 8am -10am (F-2-F)

Learning outcome

Upon Completion of this lecture, you should be able to;

- Define strain
- Apply small strain analysis in engineering applications
- Determine the stress-strain curve for a material

Outline

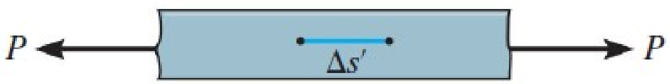
1. Strain
2. Small strain analysis

Strain

Normal Strain

Undeformed body

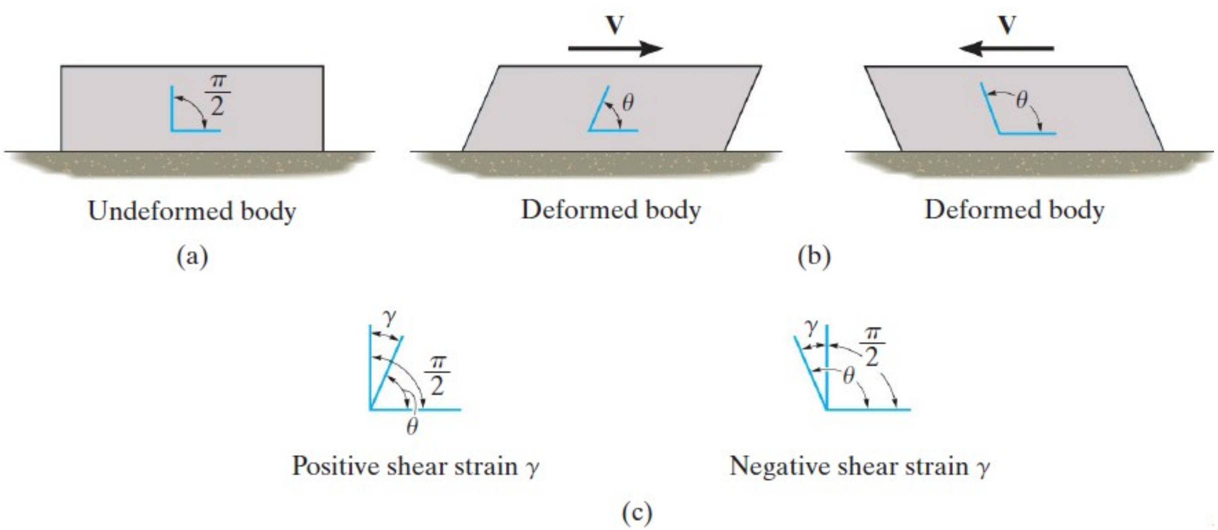
$$\epsilon_{avg} = \frac{L - L_0}{L_0} \qquad (2\text{–}1)$$



Deformed body

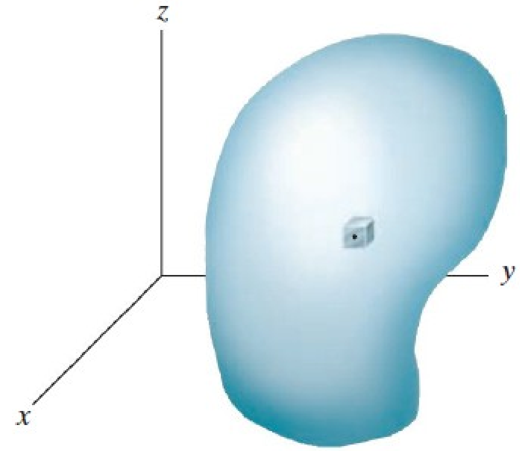
$$\epsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s} \qquad (2\text{–}2)$$

Shear Strain



Shear strain

$$\boxed{\gamma = \frac{\pi}{2} - \theta} \qquad (2\text{–}3)$$



(a)

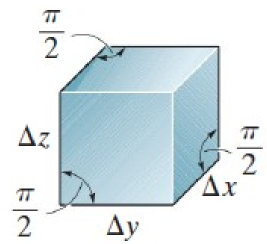
Cartesian Strain Components. We can generalize our definitions of normal and shear strain and consider the undeformed element at a point in a body, Fig. 2–4*a*. Since the element's dimensions are very small, its deformed shape will become a parallelepiped, Fig. 2–4*b*. Here the *normal strains* change the sides of the element to

$$(1 + \epsilon_x)\Delta x \qquad (1 + \epsilon_y)\Delta y \qquad (1 + \epsilon_z)\Delta z$$

which produces a *change in the volume of the element*. And the *shear strain* changes the angles between the sides of the element to

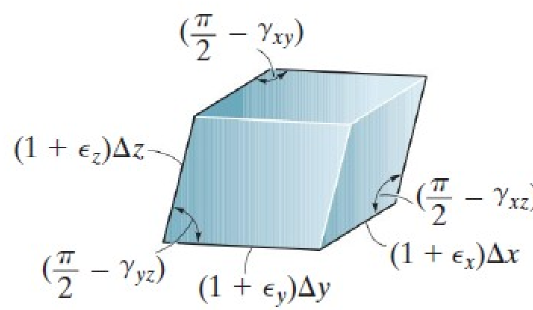
$$\frac{\pi}{2} - \gamma_{xy} \qquad \frac{\pi}{2} - \gamma_{yz} \qquad \frac{\pi}{2} - \gamma_{xz}$$

which produces a *change in the shape of the element*.



Undeformed element

(b)



Deformed element

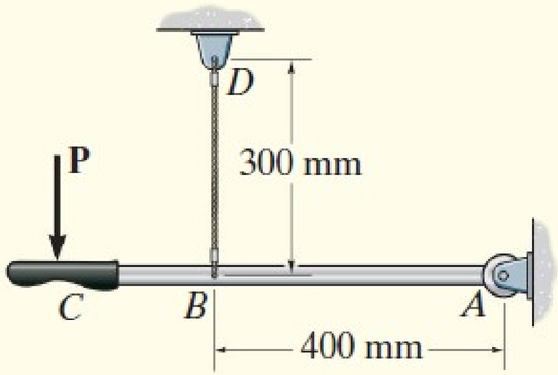
(c)

Small Strain Analysis

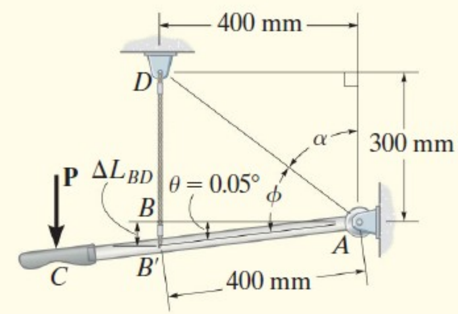
Small Strain Analysis. Most engineering design involves applications for which only *small deformations* are allowed. In this text, therefore, we will assume that the deformations that take place within a body are almost infinitesimal. For example, the *normal strains* occurring within the material are *very small* compared to 1, so that $\epsilon \ll 1$. This assumption has wide practical application in engineering, and it is often referred to as a *small strain analysis*. It can also be used when a change in angle, $\Delta\theta$, is small, so that $\sin \Delta\theta \approx \Delta\theta$, $\cos \Delta\theta \approx 1$, and $\tan \Delta\theta \approx \Delta\theta$.

Example 2.2

When force **P** is applied to the rigid lever arm ABC in Fig. 2–6*a*, the arm rotates counterclockwise about pin A through an angle of 0.05°. Determine the normal strain in wire BD.



(a)



(b)

Fig. 2–6

Geometry. The orientation of the lever arm after it rotates about point A is shown in Fig. 2–6*b*. From the geometry of this figure,

$$\alpha = \tan^{-1}\left(\frac{400\text{ mm}}{300\text{ mm}}\right) = 53.1301^\circ$$

Then

$$\phi = 90^\circ - \alpha + 0.05^\circ = 90^\circ - 53.1301^\circ + 0.05^\circ = 36.92^\circ$$

For triangle ABD the Pythagorean theorem gives

$$L_{AD} = \sqrt{(300\text{ mm})^2 + (400\text{ mm})^2} = 500\text{ mm}$$

Example 2.2

Using this result and applying the law of cosines to triangle $AB'D$,

$$
\begin{aligned}
L_{B'D} &= \sqrt{L_{AD}^2 + L_{AB'}^2 - 2(L_{AD})(L_{AB'})\cos\phi} \\
&= \sqrt{(500\text{ mm})^2 + (400\text{ mm})^2 - 2(500\text{ mm})(400\text{ mm})\cos 36.92^\circ} \\
&= 300.3491\text{ mm}
\end{aligned}
$$

nm

Normal Strain.

$$\epsilon_{BD} = \frac{L_{B'D} - L_{BD}}{L_{BD}}$$

$$= \frac{300.3491\text{ mm} - 300\text{ mm}}{300\text{ mm}} = 0.00116\text{ mm/mm} \qquad \textit{Ans.}$$

SOLUTION II

Since the strain is small, this same result can be obtained by approximating the elongation of wire BD as ΔL_{BD}, shown in Fig. 2–6*b*. Here,

$$\Delta L_{BD} = \theta L_{AB} = \left[\left(\frac{0.05^\circ}{180^\circ}\right)(\pi\text{ rad})\right](400\text{ mm}) = 0.3491\text{ mm}$$

Therefore,

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L_{BD}} = \frac{0.3491\text{ mm}}{300\text{ mm}} = 0.00116\text{ mm/mm} \qquad \textit{Ans.}$$

Mechanical Properties of materials

1. Tension and Compression Test
2. Stress-Strain Diagram
3. Stress-Strain Behavior of Ductile and Brittle Materials
4. Hooke’s Law
5. Strain Energy
6. Poission’s Ratio
7. Shear Stress-Strain Diagram

Introduction

- Show relationship of stress and strain using experimental methods to determine stress-strain diagram of a specific material
- Discuss the behavior described in the diagram for commonly used engineering materials

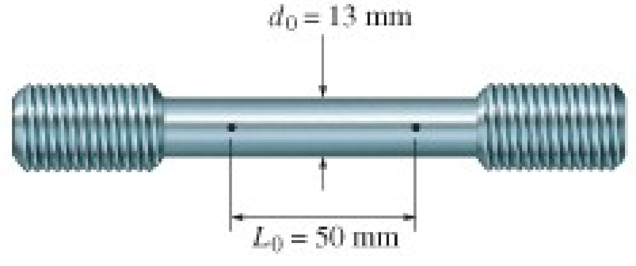
- Discuss the mechanical properties and other test related to the development of mechanics of materials

Tension and compression

- Strength of a material can only be determined by *experiment*
- One test used by engineers is the *tension or compression test*
- This test is used primarily to determine the relationship between the average normal stress and average normal strain in common engineering materials, such as metals, ceramics, polymers and composites

Performing the tension or compression test

- Specimen of material is made into "standard" shape and size
- Before testing, 2 small punch marks identified along specimen's length
- Measurements are taken of both specimen's initial x-sectional area A_0 and gauge-length distance L_0; between the two marks
- Seat the specimen into a testing machine shown below

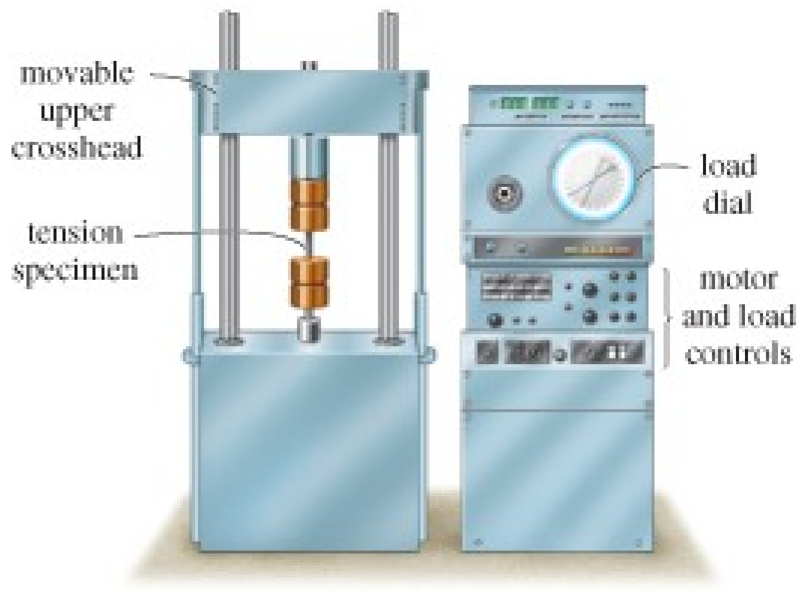


Performing the tension or compression test

- Seat the specimen into a testing machine shown below

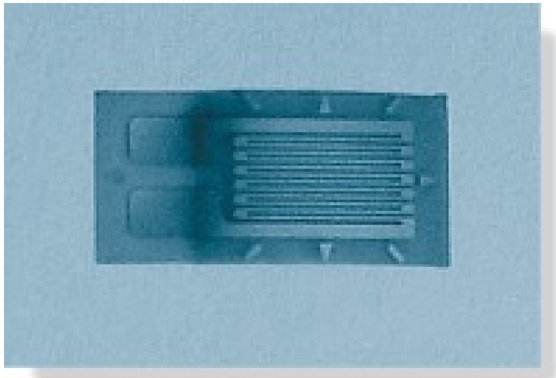
The machine will stretch specimen at slow constant rate until breaking point

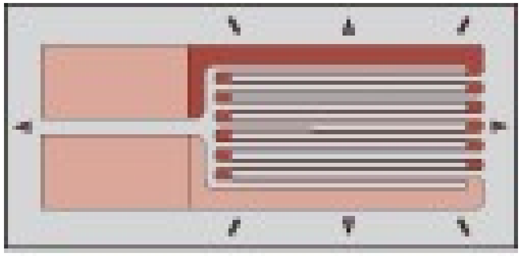
At frequent intervals during test, data is recorded of the applied load P.



Performing the tension or compression test

- Elongation $\delta = L - L_0$ is measured using either a caliper or an extensometer
- δ is used to calculate the normal strain in the specimen
- Sometimes, strain can also be read directly using an *electrical-resistance strain gauge*





Electrical–resistance strain gauge

Stress – strain diagram

- A *stress-strain diagram* is obtained by plotting

the various values of the stress and corresponding strain in the specimen

Conventional stress-strain diagram

- Using recorded data, we can determine nominal or engineering stress by

$$\sigma = \frac{P}{A_0}$$

- Assumption: Stress is constant over the x-section and throughout region between gauge points

Stress-strain diagram cont.

Conventional Stress-Strain Diagram

- Likewise, nominal or engineering strain is found directly from strain gauge reading, or by

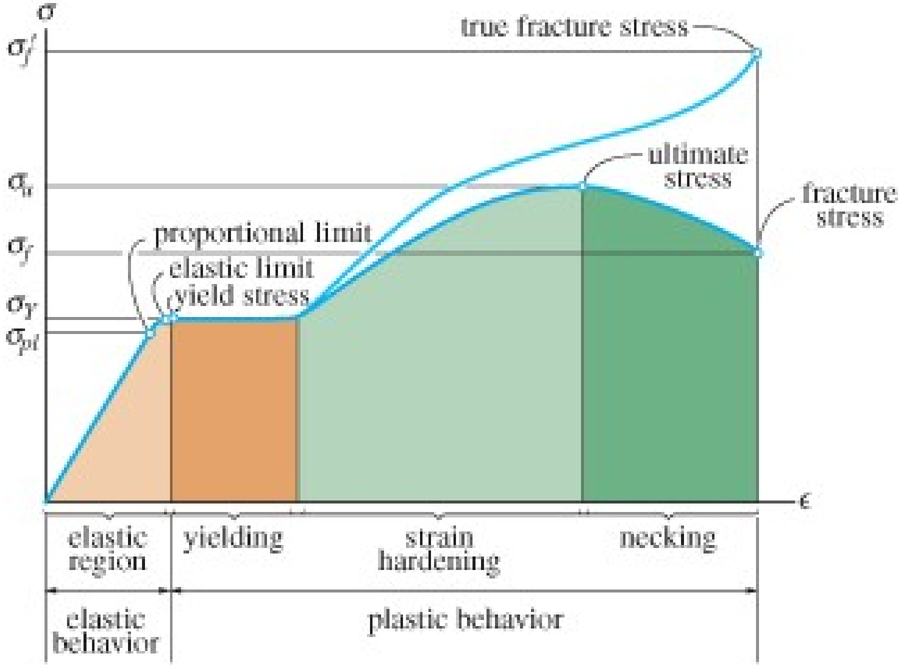
$$= \frac{\delta}{L_0}$$

- Assumption: Strain is constant throughout region between gauge points
- By plotting σ (ordinate) against (abscissa), we get a conventional stress-strain diagram

Conventional stress strain diagram

Conventional stress-strain diagram

- Figure shows the characteristic stress-strain diagram for steel, a commonly used material for structural members and mechanical elements



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

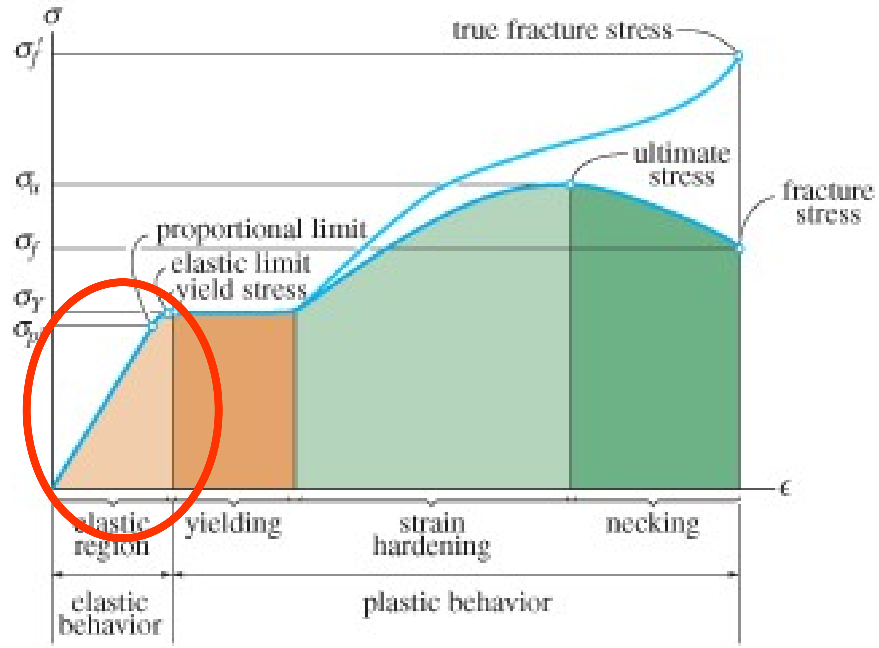
Elastic behaviour

Elastic behavior.

A straight line

Stress is proportional to strain, i.e., linearly elastic Upper stress limit, or *proportional limit*; σ_{pl}

If load is removed upon reaching elastic limit, specimen will return to its original shape



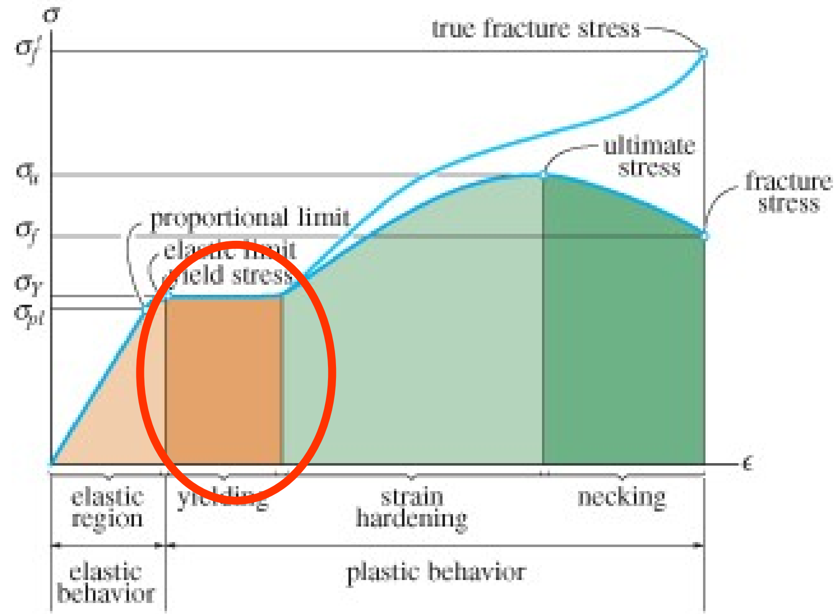
Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Yielding

Yielding.

Material deforms permanently; yielding; plastic deformation

Yield stress, σ_Y



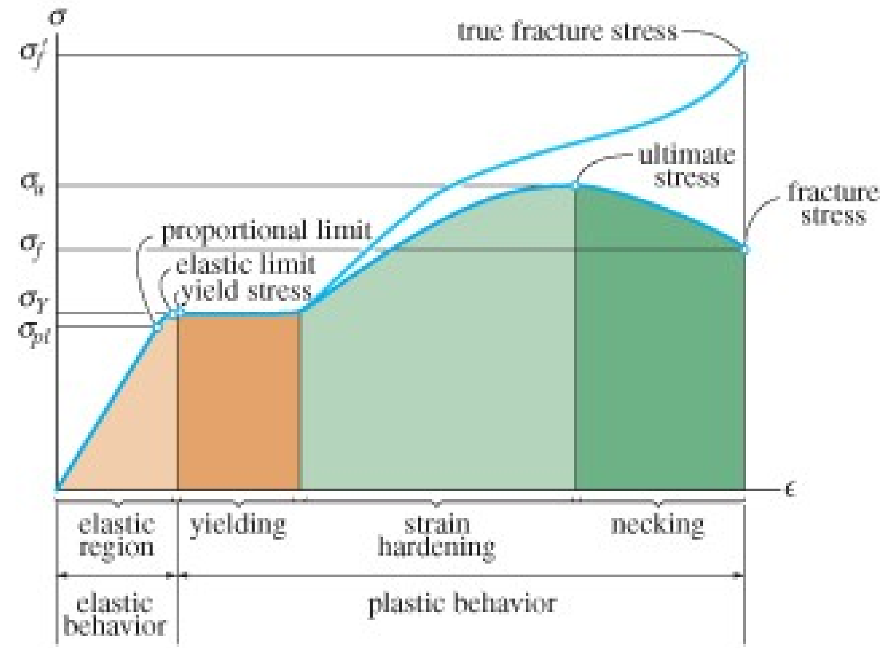
Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Once yield point reached, specimen continues to elongate (strain) *without any increase in load* Note figure not drawn to scale, otherwise induced strains is 10-40 times larger than in elastic limit Material is referred to as being *perfectly plastic*

Conventional stress-strain diagram

Necking.

- At ultimate stress, x-sectional area begins to decrease in a *localized* region
- As a result, a constriction or “neck” tends to form in this region as specimen elongates further



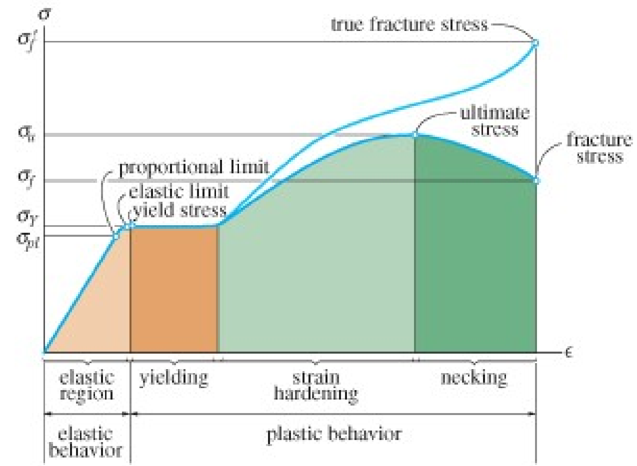
Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

- Specimen finally breaks at fracture stress, σ_f

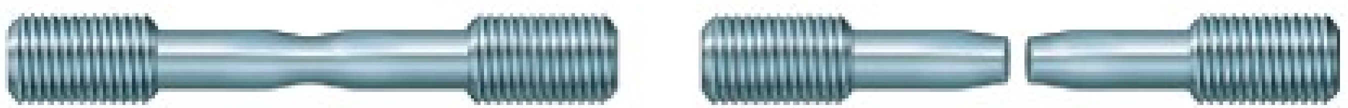
Conventional stress-strain diagram

Necking.

- Specimen finally breaks at fracture stress, σ_f



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)



Necking

(a)

Failure of a ductile material

(b)

True stress-strain diagram

- Instead of using *original* cross-sectional area and length, we can use the actual cross-sectional area and length at the *instant* the load is measured
- Values of stress and strain thus calculated are called *true stress* and *true strain*, and a plot of their values is the *true stress-strain diagram*

True stress-strain diagram

- In strain-hardening range, conventional σ- diagram shows specimen supporting *decreasing load*
- While true σ- diagram shows material to be sustaining *increasing stress*

True stress-strain diagram

- Although both diagrams are different, most engineering design is done within elastic range provided
  1. Material is “stiff,” like most metals
  2. Strain to elastic limit remains small
  3. Error in using engineering values of σ and is very small (0.1 %) compared to true values

Ductile materials

- Defined as any material that can be subjected to large strains before it ruptures, e.g., mild steel
- Such materials are used because it is capable of absorbing shock or energy, and if before becoming overloaded, will exhibit large deformation before failing
- Ductility of material is to report its percent elongation or percent reduction in area at time of fracture

Ductile materials

- **Percent elongation** is the specimen's fracture strain expressed as a percent

$$\text{Percent elongation} = \frac{L_f - L_0}{L_0}(100\%)$$

- **Percent reduction in area** is defined within necking region as

$$\text{Percent reduction in area} = \frac{A_0 - A_f}{A_0}(100\%)$$

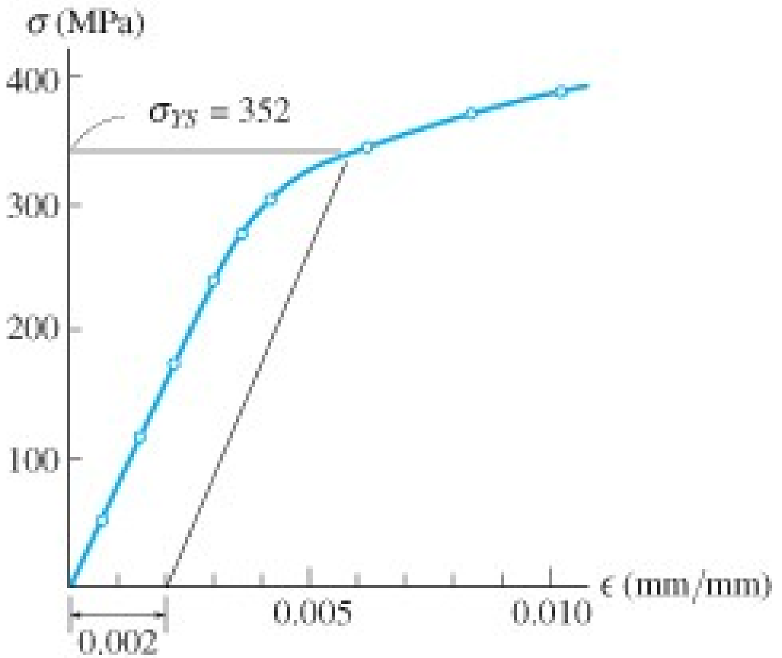
Ductile materials

- Most metals do not exhibit *constant yielding* behavior beyond the elastic range, e.g. aluminum
- It does not have well-defined yield point, thus it is standard practice to define its *yield strength* using a graphical procedure called the offset method

Ductile materials

Offset method to determine yield strength

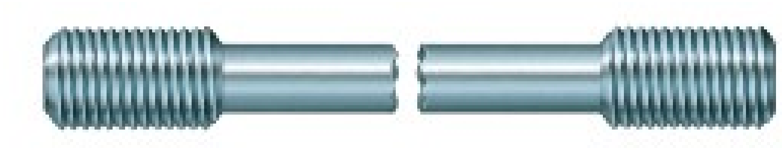
1. Normally, a 0.2 % strain is chosen.
2. From this point on the axis, a line parallel to initial straight-line portion of stress-strain diagram is drawn.
3. The point where this line intersects the curve defines the yield strength.



Yield strength for an aluminum alloy

Brittle Materials

- Material that exhibit little or no yielding before failure are referred to as brittle materials, e.g., gray cast iron
- Brittle materials do not have a well-defined tensile fracture stress, since appearance of initial cracks in a specimen is quite rando



Tension failure of
a brittle material

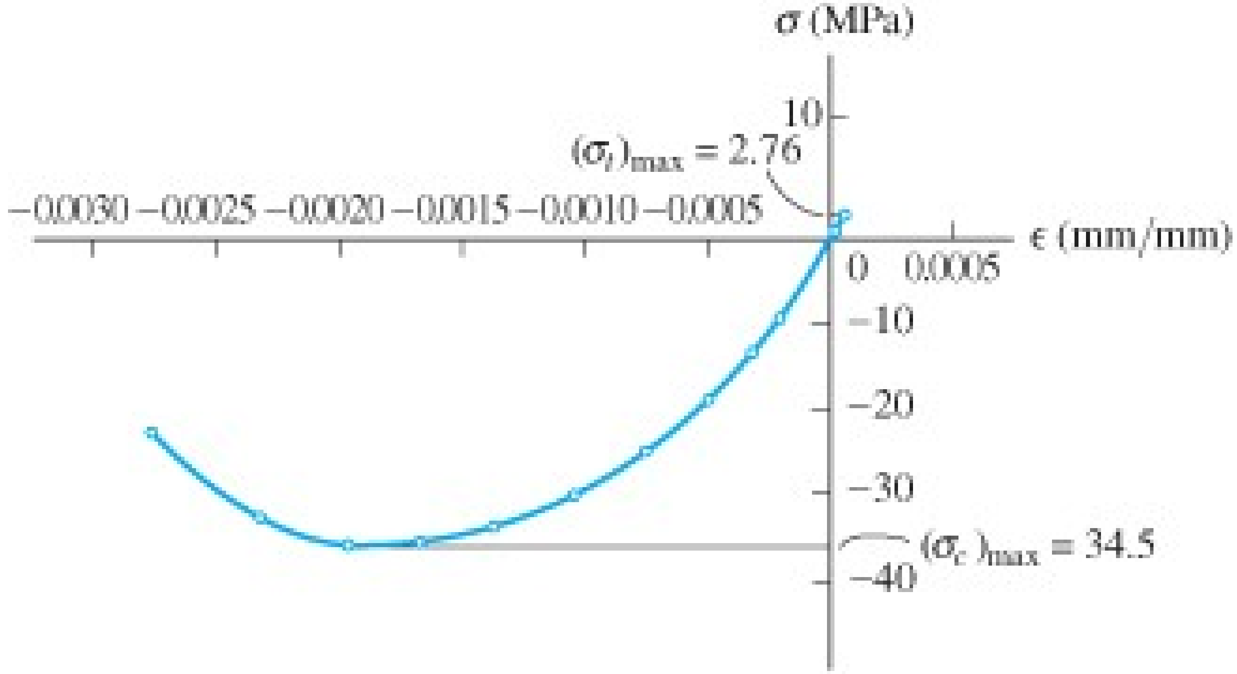
(a)

Compression causes
material to bulge out

(b)

Brittle Materials

- Instead, the *average* fracture stress from a set of observed tests is generally reported



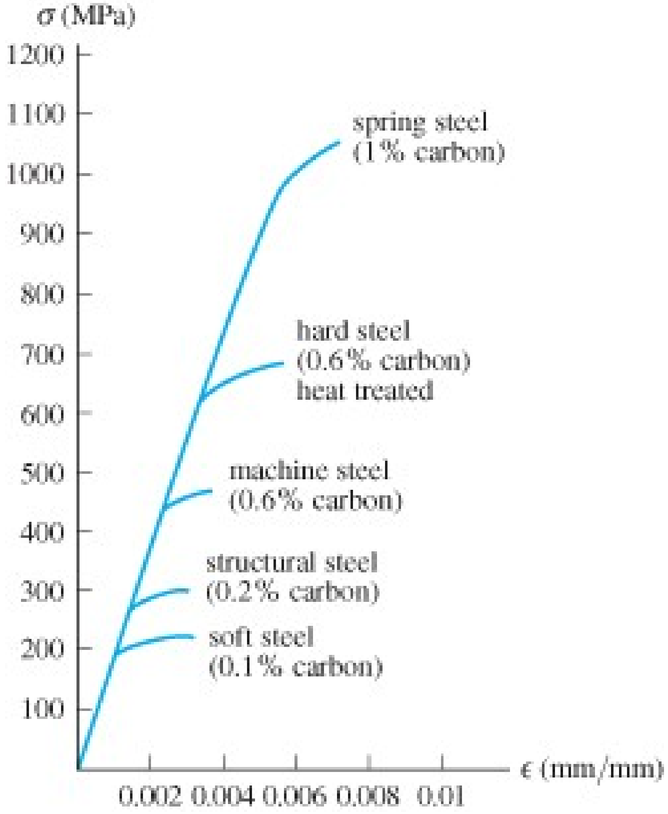
σ–ϵ diagram for typical concrete mix

- Most engineering materials exhibit a *linear relationship* between stress and strain with the elastic region
- Discovered by Robert Hooke in 1676 using springs, known as *Hooke's law*

$$\sigma = E$$

- *E* represents the constant of proportionality, also called the *modulus of elasticity* or *Young's modulus*
- *E* has units of stress, i.e., pascals, MPa or GPa.

- As shown above, most grades of steel have same modulus of elasticity, E_{st} = 200 GPa
- Modulus of elasticity is a mechanical property that indicates the *stiffness* of a material
- Materials that are still have large E values, while spongy materials (vulcanized rubber) have low values



Hookes Law

IMPORTANT

- Modulus of elasticity *E*, can be used only if a material has linear-elastic behavior.
- Also, if stress in material is greater than the proportional limit, the stress-strain diagram ceases to be a straight line and the equation is not valid

Hookes Law

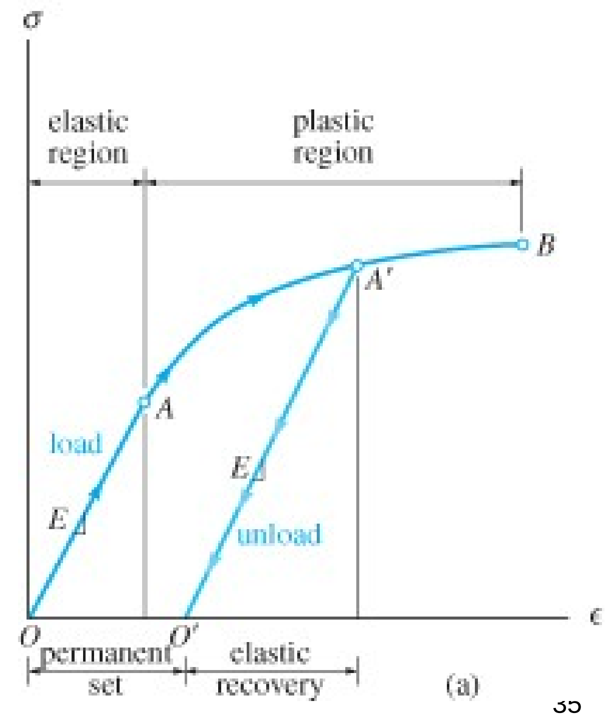
Strain hardening

- If a specimen of ductile material (steel) is loaded into the *plastic region* and then unloaded, *elastic strain* is recovered as material returns to its equilibrium state
- However, *plastic strain* remains, thus material is subjected to a *permanent set*

Strain Hardening

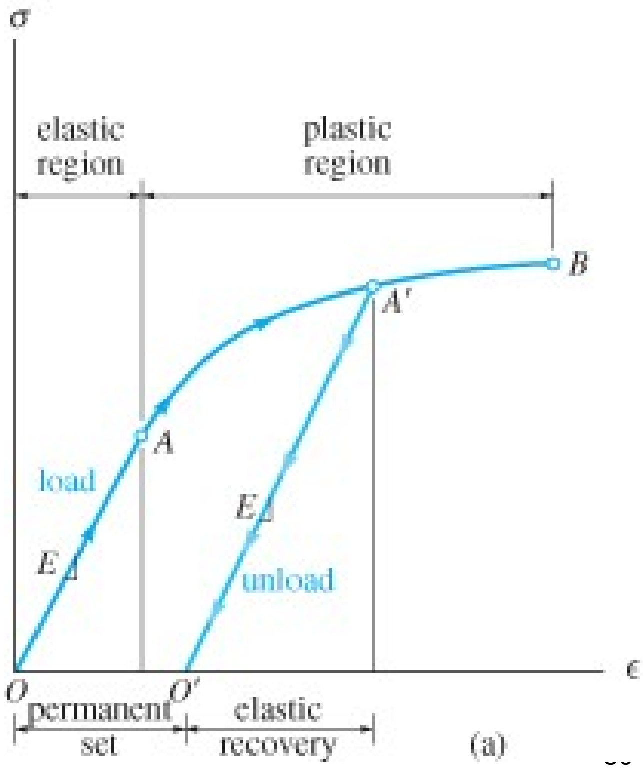
Strain hardening

- Specimen loaded beyond yield point *A* to *A'*
- Inter-atomic forces have to be overcome to elongate specimen *elastically,* these same forces pull atoms back together when load is removed
- Since E is the same, slope of line $O'A'$ is the same as line OA



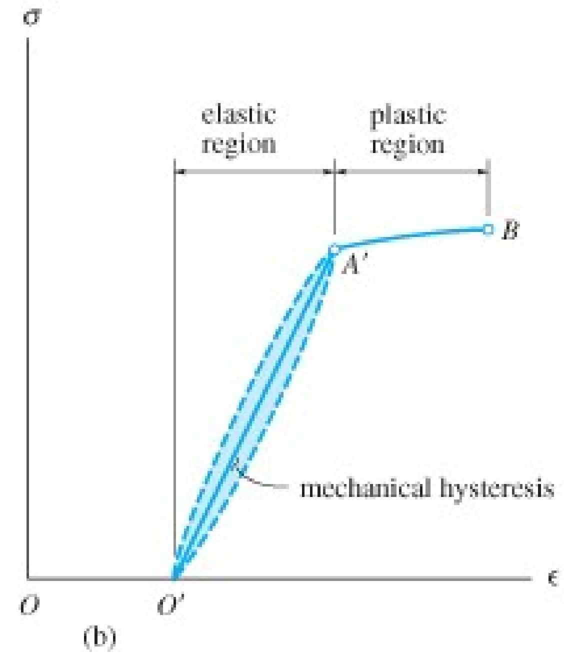
Hookes Law
Strain hardening

- Load reapplied, atoms will be displaced until yielding occurs at or near *A'*, and stress-strain diagram continues along same path as before
- New stress-strain diagram has *higher* yield point (*A'*), a result of strain-hardening
- Specimen has a *greater elastic region* and *less ductility*



Strain hardening

- As specimen is unloaded and loaded, heat or *energy* may be *lost*
- Colored area between the curves represents lost energy and is called *mechanical hysteresis*
- It's an important consideration when selecting materials to serve as dampers for vibrating structures and equipment



Strain

- Strain
  - Definition
  - Units of measurement
  - How to measure

Strain

Definition of **strain**. In engineering this is not a measure of force but is a measure of the deformation produced by the influence of stress. For tensile and compressive loads:

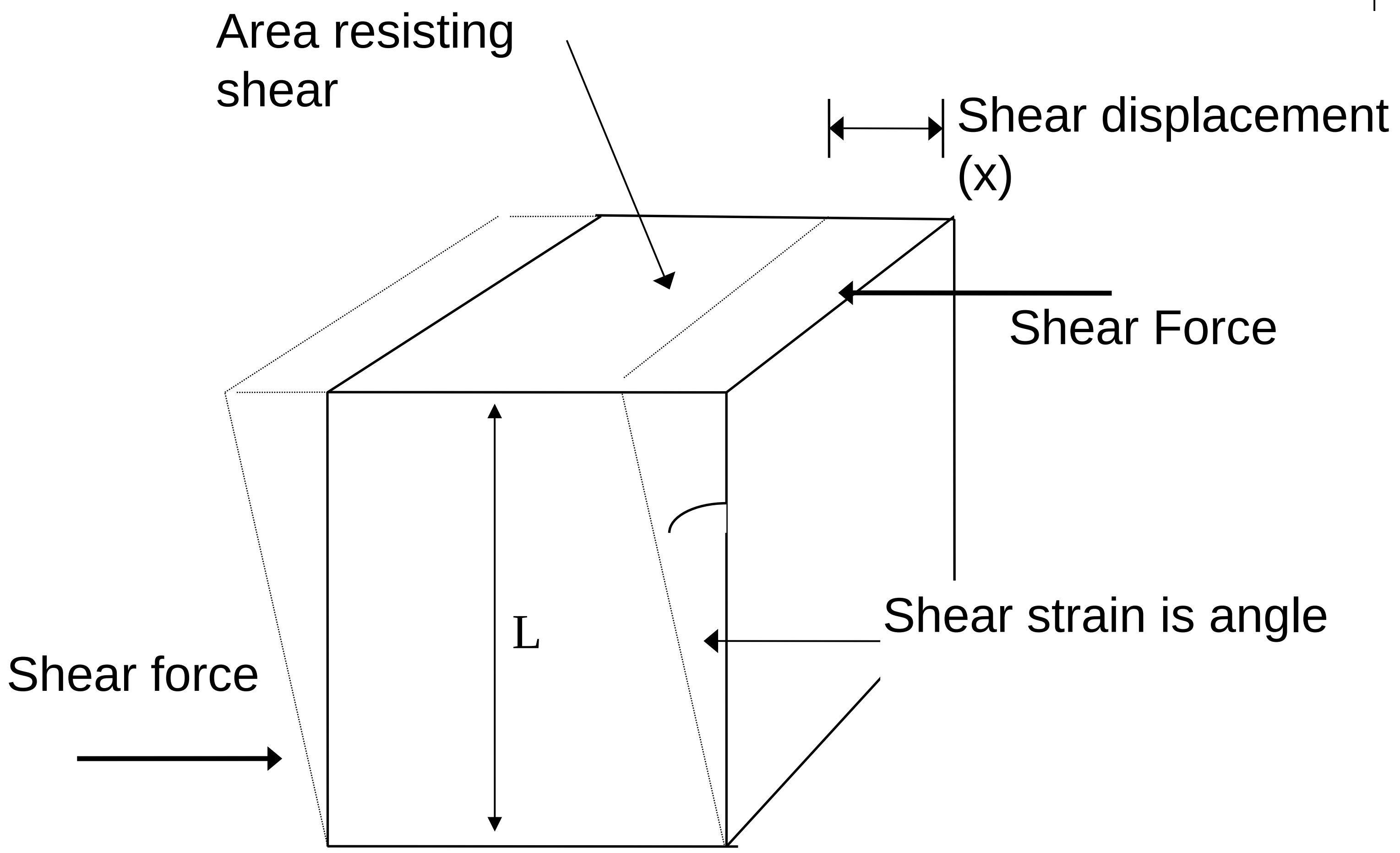
$$strain \quad \varepsilon = \frac{increase\ in\ length\ \ x}{original\ length\ \ L}$$

Strain is dimensionless, i.e. it is not measured in metres, killogrammes etc.

$$shear\ strain \qquad \gamma \approx \frac{shear\ displacement\ \ x}{width\ \ L}$$

For shear loads the strain is defined as the angle This is measured in radians

Shear stress and strain



Units of stress and strain

The basic unit for Force and Load is the Newton (N) which is equivalent to kg m/s^2. One kilogramme (kg) weight is equal to 9.81 N.

In industry the units of stress are normally Newtons per square millimetre (N/mm^2) but this is not a base unit for calculations.

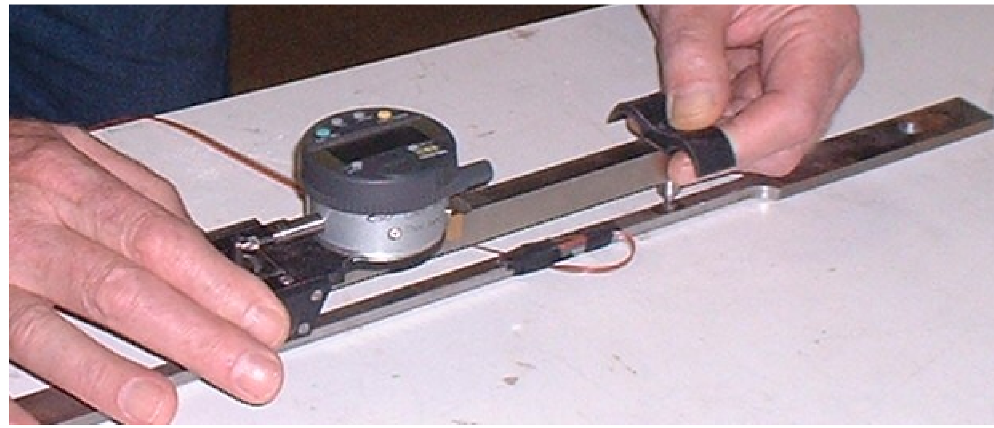
The metric unit for pressure is the Pascal. 1 Pascal = 1 Newton per square metre

Pressure and Stress have the same units 1 MPa = 1 N/mm^2

Strain has no dimensions. It is expressed as a percentage or in microstrain (s).

A strain of 1 s is an extension of one part per million. A strain of 0.2% is equal to 2000 s

Measurement of Strain



Homework

- F2-1
- F2-2
- F2-3
- 3.1
- 3.2
- 3.3
- 3.4
- 3.5

Recommended Texts

- Mechanics of Materials – 2nd Edition, Madhukar Vable – available online **FREE**
- Engineering Mechanics – Statics, R.C. Hibbler,
- Engineering Mechanics – Statics, D.J. McGill & W.W. King
- Mechanics of Materials , J.M. Gere & S.P. Timoshenko
- Mechanics of solids, Abdul Mubeen, Pearson Education Asia