1 Introduction

1.1 THE SUBJECT OF KINEMATICS AND DYNAMICS OF MACHINES

This subject is a continuation of statics and dynamics, which is taken by students in their freshman or sophomore years. In kinematics and dynamics of machines and mechanisms, however, the emphasis shifts from studying general concepts with illustrative examples to developing methods and performing analyses of real designs. This shift in emphasis is important, since it entails dealing with complex objects and utilizing different tools to analyze these objects.

The objective of *kinematics* is to develop *various means of transforming motion* to achieve a specific kind needed in applications. For example, an object is to be moved from point A to point B along some path. The first question in solving this problem is usually: What kind of a mechanism (if any) can be used to perform this function? And the second question is: How does one design such a mechanism?

The objective of *dynamics* is analysis of the behavior of a given machine or mechanism when subjected to dynamic forces. For the above example, when the mechanism is already known, then external forces are applied and its motion is studied. The determination of forces induced in machine components by the motion is part of this analysis.

As a subject, the kinematics and dynamics of machines and mechanisms is disconnected from other subjects (except statics and dynamics) in the Mechanical Engineering curriculum. This absence of links to other subjects may create the false impression that there are no constraints, apart from the kinematic ones, imposed on the design of mechanisms. Look again at the problem of moving an object from A to B. In designing a mechanism, the size, shape, and weight of the object all constitute input into the design process. All of these will affect the size of the mechanism. There are other considerations as well, such as, for example, what the allowable speed of approaching point B should be. The outcome of this inquiry may affect either the configuration or the type of the mechanism. Within the subject of kinematics and dynamics of machines and mechanisms such requirements cannot be justifiably formulated; they can, however, be posed as a learning exercise.

1.2 KINEMATICS AND DYNAMICS AS PART OF THE DESIGN PROCESS

The role of kinematics is to ensure the functionality of the mechanism, while the role of dynamics is to verify the acceptability of induced forces in parts. The functionality and induced forces are subject to various constraints (specifications) imposed on the design. Look at the example of a cam operating a valve (Figure 1.1).

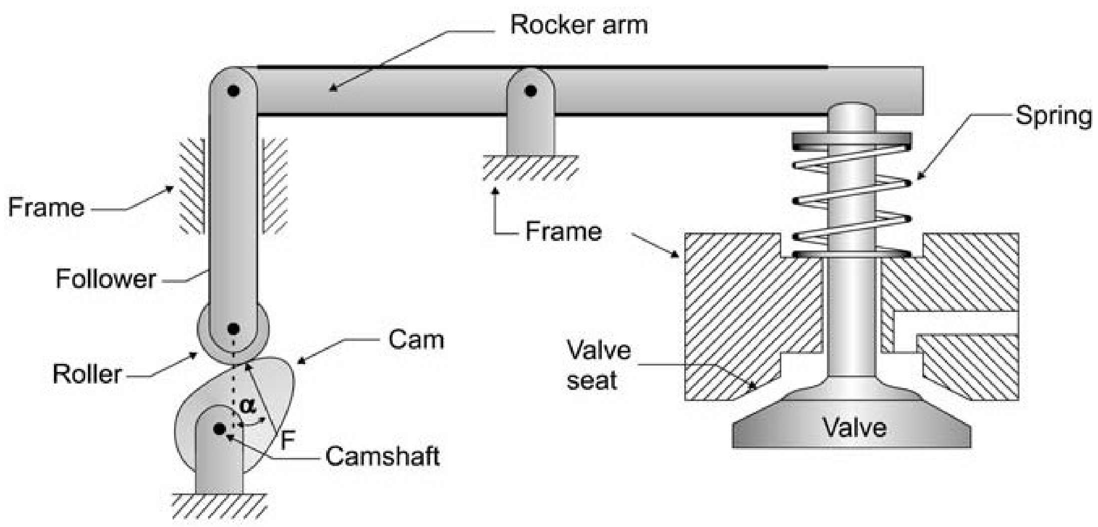

FIGURE 1.1 A schematic diagram of cam operating a valve.

The *design process* starts with meeting the *functional requirements* of the product. The basic one in this case is the proper *opening, dwelling,* and *closing* of the valve as a function of *time*. To achieve this objective, a corresponding cam profile producing the needed follower motion should be found. The rocker arm, being a lever, serves as a displacement amplifier/reducer. The timing of opening, dwelling, and closing is controlled by the speed of the camshaft. The function of the spring is to keep the roller always in contact with the cam. To meet this requirement the inertial forces developed during the follower–valve system motion should be known, since the spring force must be larger than these forces at any time. Thus, it follows that the determination of component accelerations needed to find inertial forces is important for the choice of the proper spring stiffness.

Kinematical analysis allows one to satisfy the functional requirements for valve displacements. Dynamic analysis allows one to find forces in the system as a function of time. These forces are needed to continue the design process. The *design process* continues with meeting the *constraints requirements*, which in this case are:

1. Sizes of all parts;
2. Sealing between the valve and its seat;
3. Lubrication;
4. Selection of materials;
5. Manufacturing and maintenance;
6. Safety;
7. Assembly, etc.

The forces transmitted through the system during cam rotation allow one to determine the proper sizes of components, and thus to find the overall assembly dimension. The spring force affects the reliability of the valve sealing. If any of the requirements cannot be met with the given assembly design, then another set of

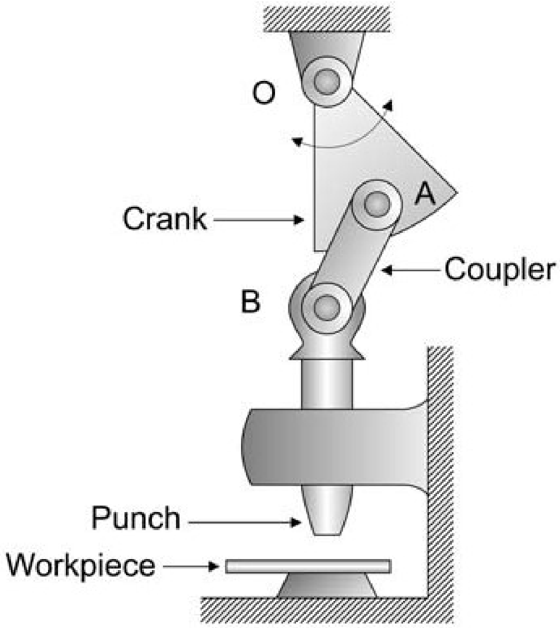

FIGURE 1.2 Punch mechanism.

parameters should be chosen, and the kinematic and dynamic analysis repeated for the new version.

Thus, kinematic and dynamic analysis is an *integral part* of the machine design process, which means it *uses input* from this process and *produces output* for its continuation.

1.3 IS IT A MACHINE, A MECHANISM, OR A STRUCTURE?

The term *machine* is usually applied to a complete product. A *car* is a machine, as is a *tractor*, a *combine*, an *earthmoving machine,* etc. At the same time, each of these machines may have some devices performing specific functions, like a windshield wiper in a car, which are called *mechanisms*. The schematic diagram of the assembly shown in Figure 1.1 is another example of a mechanism. In Figure 1.2 a punch mechanism is shown. In spite of the fact that it shows a complete product, it, nevertheless, is called a mechanism. An internal combustion engine is called neither a machine nor a mechanism. It is clear that there is a historically established terminology and it may not be consistent. What is important, as far as the subject of kinematics and dynamics is concerned, is that the identification of something as a machine or a mechanism has no bearing on the analysis to be done. And thus in the following, the term *machine* or *mechanism* in application to a specific device will be used according to the established custom.

The distinction between the *machine/mechanism* and the *structure* is more fundamental. The former must have moving parts, since it transforms motion, produces work, or transforms energy. The latter does not have moving parts; its function is purely structural, i.e., to maintain its form and shape under given external loads, like a bridge, a building, or an antenna mast. However, an example of a folding

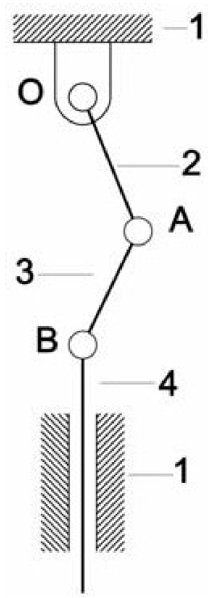

FIGURE 1.3 A skeleton representing the punch mechanism.

chair, or a solar antenna, may be confusing. Before the folding chair can be used as a chair, it must be *unfolded*. The transformation from a folded to an unfolded state is the transformation of motion. Thus, the folding chair meets two definitions: it is a *mechanism* during unfolding and a *structure* when unfolding is completed. Again, the terminology should not affect the understanding of the substance of the matter.

1.4 EXAMPLES OF MECHANISMS; TERMINOLOGY

The punch mechanism shown in Figure 1.2 is a schematic representation of a device to punch holes in a workpiece when the oscillating *crank* through the *coupler* moves the punch up and down. The function of this mechanism is to transform a small force/torque applied to the crank into a large punching force. The specific shape of the crank, the coupler, and the punch does not affect this function. This function depends only on locations of points *O*, *A*, and *B*. If this is the case, then the lines connecting these points can represent this mechanism. Such a representation, shown in Figure 1.3, is called a *skeleton* representation of the mechanism. The power is supplied to crank 2, while punch 4 is performing the needed function.

In Figure 1.3, the lines connecting points *O*, *A*, and *B* are called *links* and they are connected to each other by *joints*. Links are assumed to be rigid. Revolute joints connect link 2 to link 3 and to the frame (at point *O*). A revolute joint is a pin, and it allows rotation in a plane of one link with respect to another. A revolute joint also connects the two links 3 and 4. Link 4 is allowed to slide with respect to the frame, and this connection between the frame and the link is called a *prismatic joint.* The motion is transferred from link 2, which is called the *input link,* to link 4, which is called the *output link*. Sometimes the input link is called a *driver*, and the output link the *follower.*

Another example of a mechanism is the windshield wiper mechanism shown in Figure 1.4. The motion is transferred from the crank driven by a motor through the

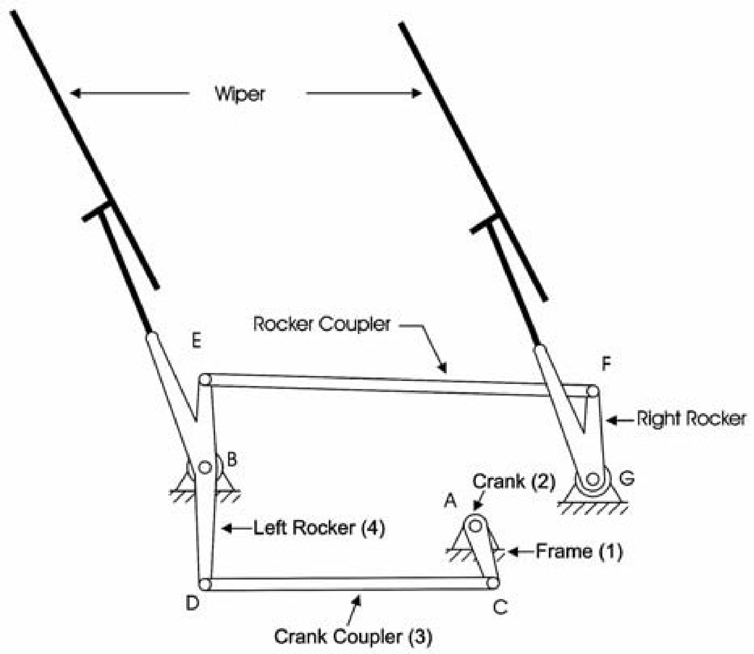

FIGURE 1.4 A windshield wiper mechanism.

coupler to the left rocker. The two rockers, left and right, are connected by the rocker coupler, which synchronizes their motion. The mechanism comprising links 1 (frame), 2 (crank), 3 (coupler), and 4 (rocker) is called a *four-bar mechanism*. In this example, revolute joints connect all links.

A *kinematic chain* is an interconnected system of links in which not a single link is *fixed*. Such a chain becomes a *mechanism* when one of the links in the chain is fixed. The fixed link is called a *frame* or, sometimes, a *base link*. In Figure 1.3 link 1 is a frame. A *planar mechanism* is one in which all points move in parallel planes.

A joint between two links restricts the *relative motion* between these links, thus imposing a *constraining condition* on the mechanism motion. The type of constraining condition determines the number of degrees of freedom (DOF) a mechanism has. If the constraining condition allows only one DOF between the two links, the corresponding joint is called a lower-pair joint. The examples are a revolute joint between links 2 and 3 and a prismatic joint between links 4 and 1 in Figure 1.3. If the constraint allows two DOF between the two links, the corresponding joint is called a high-pair joint. An example of a high-pair joint is a connection between the cam and the roller in Figure 1.1, if, in addition to rolling, sliding between the two links takes place.

A dump truck mechanism is shown in Figure 1.5, and its skeleton diagram in Figure 1.6. This is an example of a *compound mechanism* comprising two simple ones: the first, links 1–2–3, is called the *slider-crank mechanism* and the second, links 1–3–5–6, is called the *four-bar linkage*. The two mechanisms work in sequence (or they are *functionally in series*): the input is the displacement of the piston in the hydraulic cylinder, and the output is the tipping of the dump bed.

All the previous examples involved only links with two connections to other links. Such links are called *binary links*. In the example of Figure 1.6, in addition to binary links, there is link 2, which is connected to three links: 1 (frame), 3, and 5. Such a link is called a *ternary link*. It is possible to have links with more than three connections.

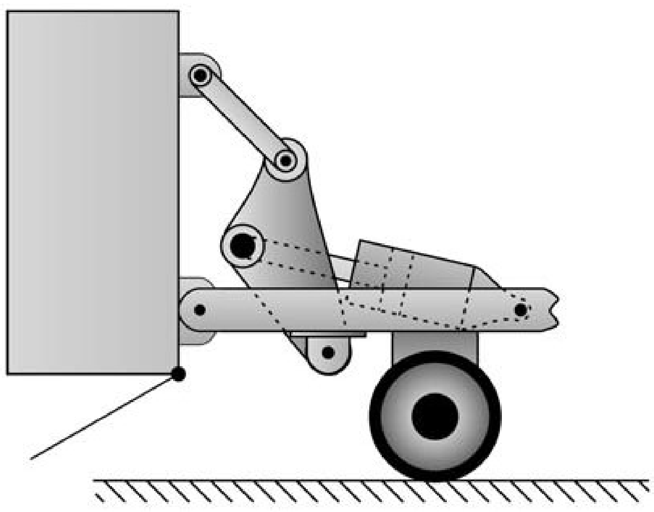

FIGURE 1.5 Dump truck mechanism.

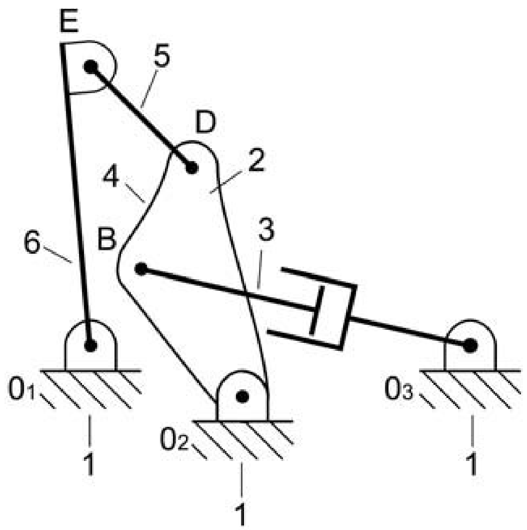

FIGURE 1.6 Skeleton of the dump truck mechanism.

1.5 MOBILITY OF MECHANISMS

The *mobility* of a mechanism is its *number of degrees of freedom*. This translates into a number of independent input motions leading to a single follower motion.

A single unconstrained link (Figure 1.7a) has three DOF in planar motion: two translational and one rotational. Thus, two disconnected links (Figure 1.7b) will have six DOF. If the two links are welded together (Figure 1.7c), they form a single link having three DOF. A revolute joint in place of welding (Figure 1.7d) allows a motion of one link relative to another, which means that this joint introduces an additional (to the case of welded links) DOF. Thus, the two links connected by a revolute joint have four DOF. One can say that by connecting the two previously disconnected links by a revolute joint, two DOF are eliminated. Similar considerations are valid for a prismatic joint.

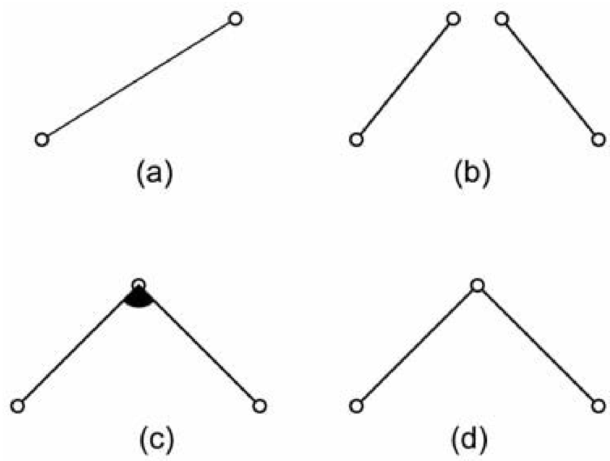

FIGURE 1.7 Various configurations of links with two revolute joints.

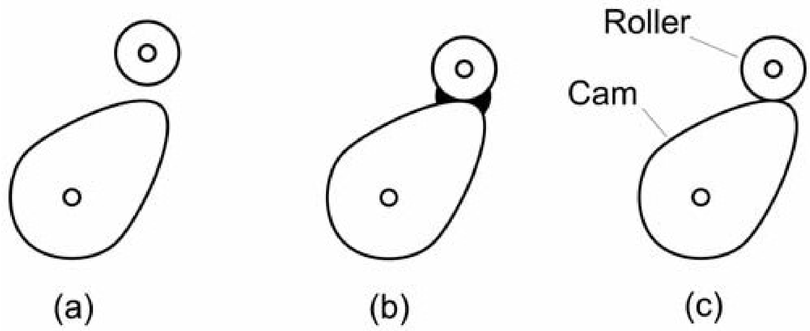

FIGURE 1.8 Various configurations of two links with a high-pair joint.

Since the revolute and prismatic joints make up all low-pair joints in planar mechanisms, the above results can be expressed as a rule: *a low-pair joint reduces the mobility of a mechanism by two DOF.*

For a high-pair joint the situation is different. In Figure 1.8 a roller and a cam are shown in various configurations. If the two are not in contact (Figure 1.8a), the system has six DOF. If the two are welded (Figure 1.8b), the system has three DOF. If the roller is not welded, then two relative motions between the cam and the roller are possible: rolling and sliding. Thus, in addition to the three DOF for a welded system, another two are added if a relative motion becomes possible. In other words, if disconnected, the system will have six DOF; if connected by a high-pair joint, it will have five DOF. This can be stated as a rule: *a high-pair joint reduces the mobility of a mechanism by one DOF.*

These results are generalized in the following formula, which is called *Kutzbach's criterion* of mobility

$$m = 3(n - 1) - 2j_1 - j_2 \tag{1.1}$$

where n is the number of links, j_1 is the number of low-pair joints, and j_2 is the number of high-pair joints. Note that 1 is subtracted from n in the above equation to take into account that the mobility of the frame is zero.

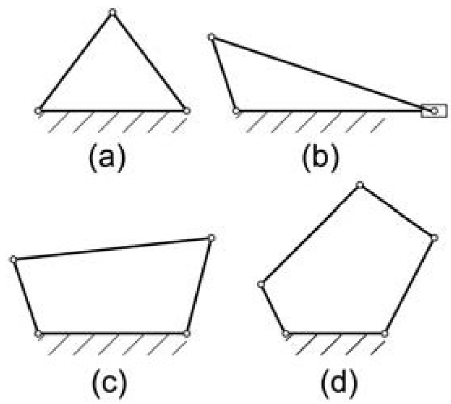

FIGURE 1.9 Mobility of various configurations of connected links: (a) $n = 3, j_1 = 3, j_2 = 0, m = 0$; (b) $n = 4, j_1 = 4, j_2 = 0, m = 1$; (c) $n = 4, j_1 = 4, j_2 = 0, m = 1$; (d) $n = 5, j_1 = 5, j_2 = 0, m = 2$.

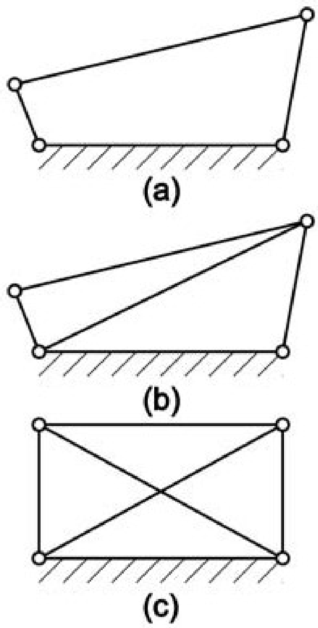

FIGURE 1.10 Effect of additional links on mobility: (a) $m = 1$, (b) $m = 0$, (c) $m = -1$.

In Figure 1.9 the mobility of various configurations of connected links is calculated. All joints are low-pair ones. Note that the mobility of the links in Figure 1.9a is zero, which means that this system of links is not a mechanism, but a structure. At the same time, the system of interconnected links in Figure 1.9d has mobility 2, which means that any two links can be used as input links (drivers) in this mechanism.

Look at the effect of an additional link on the mobility. This is shown in Figure 1.10, where a four-bar mechanism (Figure 1.10a) is transformed into a structure having zero mobility (Figure 1.10b) by adding one link, and then into a structure having negative mobility (Figure 1.10c) by adding one more link. The latter is called an *overconstrained* structure.

In Figure 1.11 two simple mechanisms are shown. Since slippage is the only relative motion between the cam and the follower in Figure 1.11a, then this interface is equivalent to a prismatic low-pair joint, so that this mechanism has mobility 1.

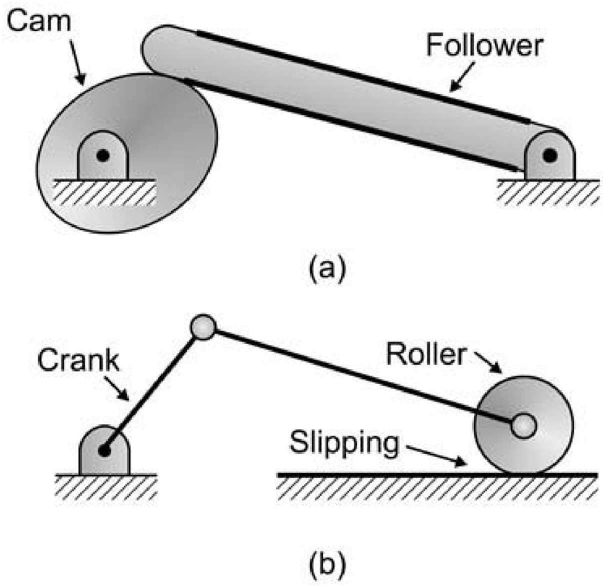

FIGURE 1.11 Mechanisms involving slippage only (a), and slippage and rolling (b).

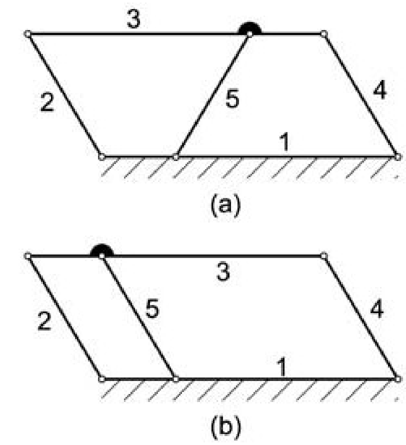

FIGURE 1.12 Example of violation of Kutzbach's criterion: (a) $n = 5, j_1 = 6, j_2 = 0, m = 0$; (b) $n = 5, j_1 = 6, j_2 = 0, m = 0$.

On the other hand, if both slippage and rolling are taking place between the roller and the frame in Figure 1.11b, then this interface is equivalent to a high-pair joint. Then the corresponding mobility of this mechanism is 2.

Kutzbach's formula for mechanism mobility does not take into account the specific geometry of the mechanism, only the connectivity of links and the type of connections (constraints). The following examples show that Kutzbach's criterion can be violated due to the nonuniqueness of geometry for a given connectivity of links (Figure 1.12). If links 2, 5, and 4 are as shown in Figure 1.12a, the mobility is zero. If, however, the above links are parallel, then according to Kutzbach's criterion the mobility is still zero, whereas motion is now possible.

It has been shown that in compound mechanisms (see Figure 1.6) there are links with more than two joints. Kutzbach's criterion is applicable to such mechanisms provided that a proper account of links and joints is made. Consider a simple compound mechanism shown in Figure 1.13, which is a sequence of two four-bar

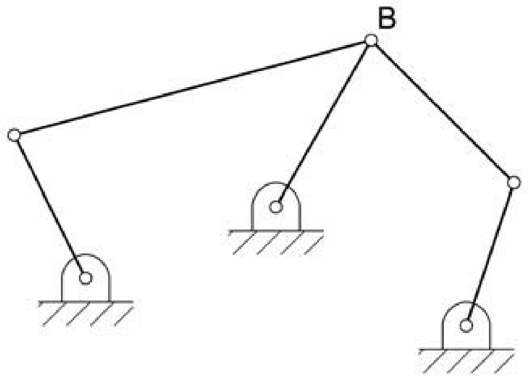

FIGURE 1.13 An example of a compound mechanism with coaxial joints at B.

mechanisms. In this mechanism, joint B represents two connections between three links. A system of three links rigidly coupled at B would have three DOF. If one connection were made revolute, the system would have four DOF. If another one were made revolute, it would have five DOF. Thus, if the system of three disconnected links has nine DOF, their connection by two revolute joints reduces it to five DOF. According to Kutzbach's formula $m = 3 \times 3 - 2 \times 2 = 5$. In other words, it should be taken into account that there are, in fact, *two* revolute joints at B. The axes of these two joints may not necessarily coincide, as in the example of Figure 1.6.

1.6 KINEMATIC INVERSION

Recall that a kinematic chain becomes a mechanism when one of the links in the chain becomes a frame. The process of choosing different links in the chain as frames is known as *kinematic inversion*. In this way, for an n-link chain n different mechanisms can be obtained. An example of a four-link slider-crank chain (Figure 1.14) shows how different mechanisms are obtained by fixing different links functionally. By fixing the cylinder (link 1) and joint A of the crank (link 2), an internal combustion engine is obtained (Figure 1.14a). By fixing link 2 and by pivoting link 1 at point A, a rotary engine used in early aircraft or a quick-return mechanism is obtained (Figure 1.14b). By fixing revolute joint C on the piston (link 4) and joint B of link 2, a steam engine or a crank-shaper mechanism is obtained (Figure 1.14c). By fixing the piston (link 4), a farm hand pump is obtained (Figure 1.14d).

1.7 GRASHOF'S LAW FOR A FOUR-BAR LINKAGE

As is clear, the motion of links in a system must satisfy the constraints imposed by their connections. However, even for the same chain, and thus the same constraints, different motion transformations can be obtained. This is demonstrated in Figure 1.15, where the motions in the inversions of the four-bar linkage are shown. In Figure 1.15, s identifies the smallest link, l is the longest link, and p, q are two other links.

From a practical point of view, it is of interest to know if for a given chain at least one of the links will be able to make a complete revolution. In this case, a motor can drive such a link. The answer to this question is given by Grashof's law, which states

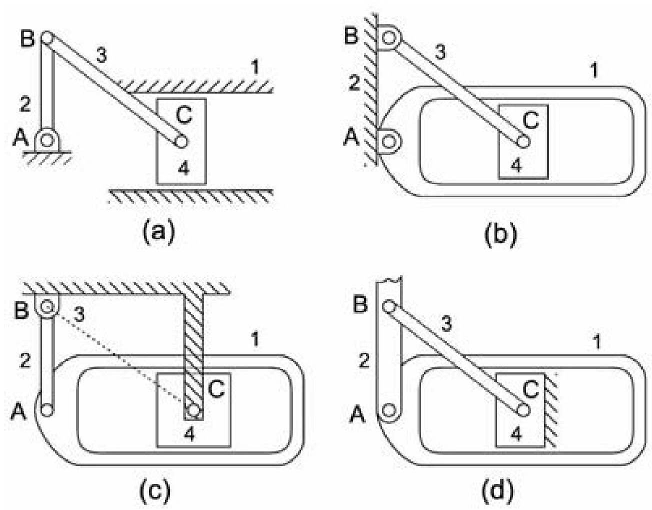

FIGURE 1.14 Four inversions of the slider-crank chain: (a) an internal combustion engine, (b) rotary engine used in early aircraft, quick-return mechanism, (c) steam engine, crank-shaper mechanism, (d) farm hand pump.

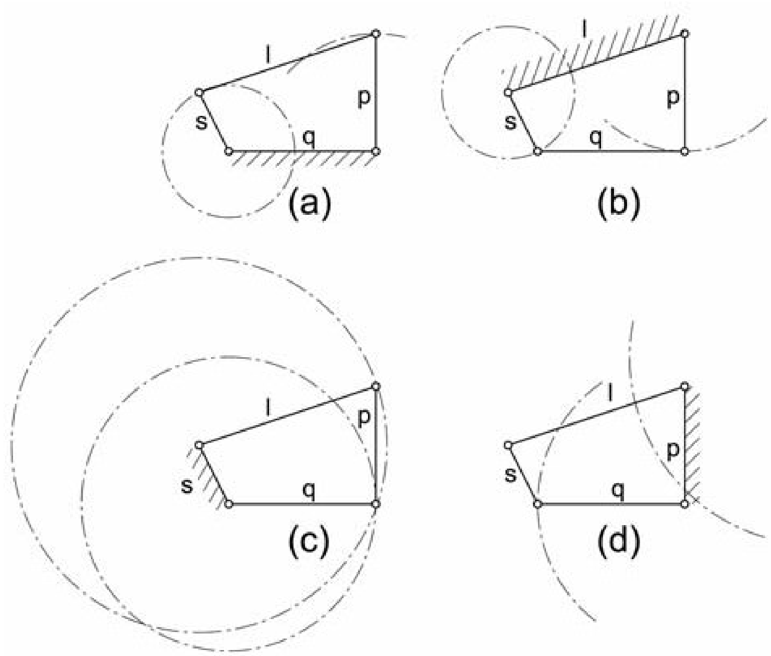

FIGURE 1.15 Inversions of the four-bar linkage: (a) and (b) crank-rocker mechanisms, (c) double-crank mechanism, (d) double-rocker mechanism.

that for a four-bar linkage, if the sum of the shortest and longest links is not greater than the sum of the remaining two links, at least one of the links will be revolving. For the notations in Figure 1.15 Grashof's law (condition) is expressed in the form:

$$s + l \leq p + q \tag{1.2}$$

Since in Figure 1.15 Grashof's law is satisfied, in each of the inversions there is at least one revolving link: in Figure 1.15a and b it is the shortest link s; in Figure 1.15c there are two revolving links, l and q; and in Figure 1.15d the revolving link is again the shortest link s.

PROBLEMS

1. What is the difference between a linkage and a mechanism?
2. What is the difference between a mechanism and a structure?
3. Assume that a linkage has *N* DOF. If one of the links is made a frame, how will it affect the number of DOF of the mechanism?
4. How many DOF would three links connected by revolute joints at point B (Figure P1.1) have? Prove.

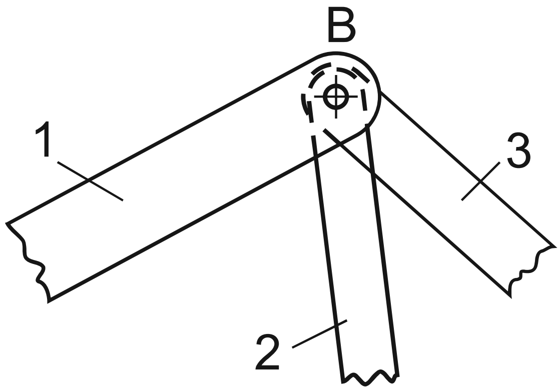

FIGURE P1.1

5. A fork joint connects two links (Figure P1.2). What is the number of DOF of this system? Prove.

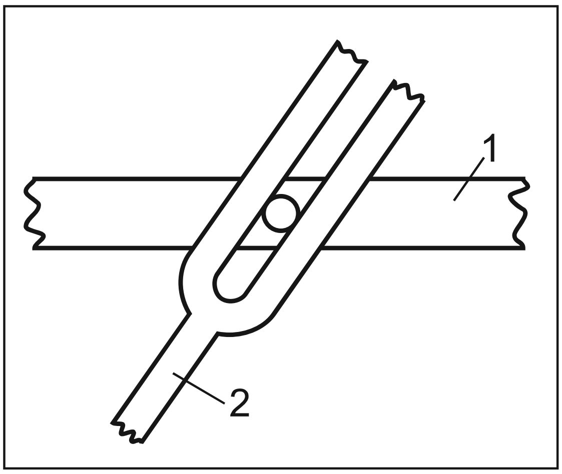

FIGURE P1.2

6. An adjustable slider drive mechanism consists of a crank-slider with an adjustable pivot, which can be moved up and down (see Figure P1.3).
   a. How many bodies (links) can be identified in this mechanism?
   b. Identify the type (and corresponding number) of all kinematic joints.
   c. What is the function of this mechanism and how will it be affected by moving the pivot point up and down?

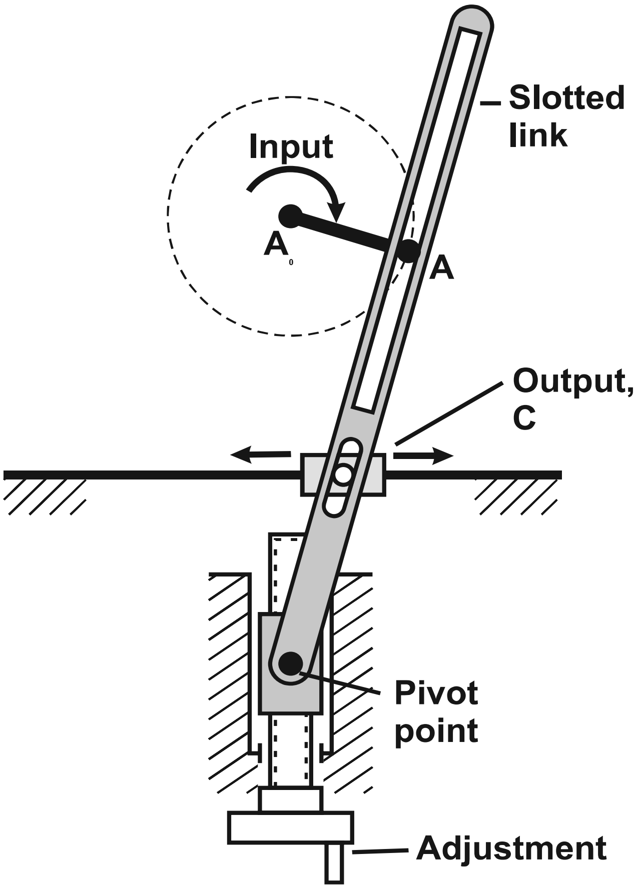

FIGURE P1.3

7. In Figure P.1.4 a pair of locking toggle pliers is shown.
   a. Identify the type of linkage (four-bar, slider-crank, etc.).
   b. What link is used as a driving link?
   c. What is the function of this mechanism?
   d. How does the adjusting screw affect this function?

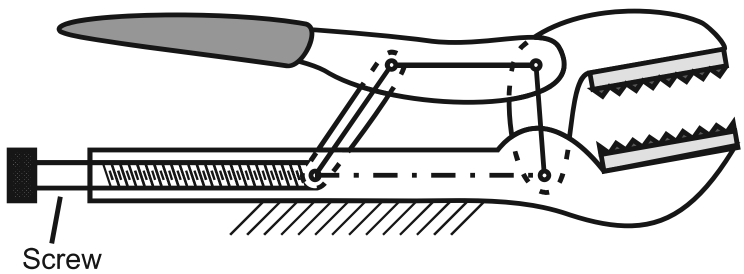

FIGURE P1.4

8. A constant velocity four-bar slider mechanism is shown in Figure P1.5.
   a. How many bodies (links) can be identified in this mechanism?
   b. Identify the type (and corresponding number) of all kinematic joints.
   c. Identify the frame and the number of joints on it.

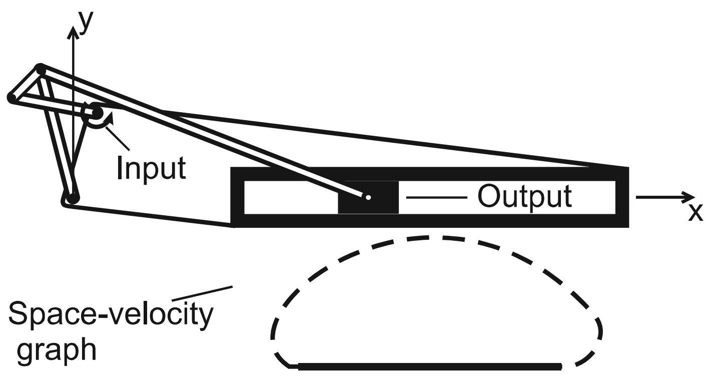

FIGURE P1.5 Constant-velocity mechanism.

9. How many driving links are in the dump truck mechanism shown in Figure P1.6?
10. What are the mobilities of mechanisms shown in Figures P1.3 through P1.5?
11. What are the mobilities of mechanisms shown in Figures P1.6 and P1.7?

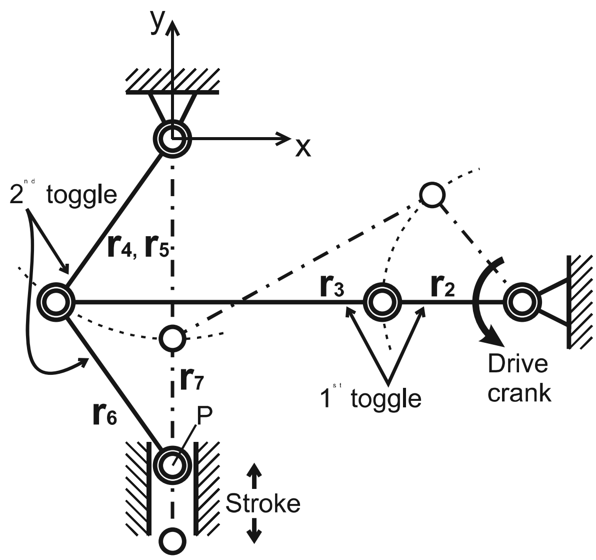

FIGURE P1.6 Double-toggle puncher mechanism.

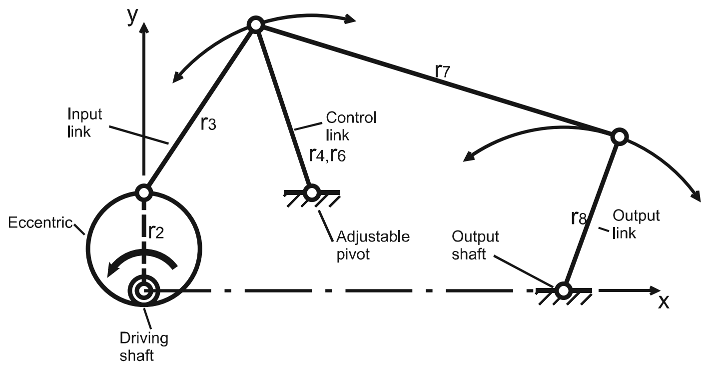

FIGURE P1.7 Variable-stroke drive.

12. Identify the motion transformation taking place in a windshield wiper mechanism (Figure 1.4). What must the relationship between the links in this mechanism be to perform its function?