Hidden Markov Models

Forward Algorithm:

1. Initialization:

α1 (j)=a0j bj (o1 ) 1≤j≤N

1. Recursion (recall that states 0 and qF are non-emitting):

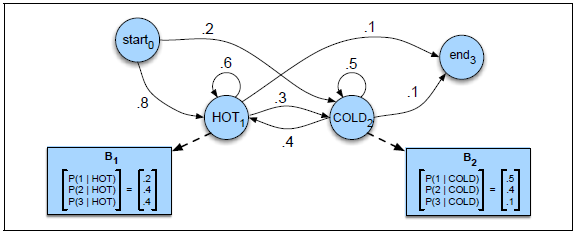
αt (j)=

1≤j≤N, 1<t≤T

1. Termination:

P(O|λ) =

Örnek 1: Dondurma yeme gizli Markov otomatı için ilk 3 gün boyunca yenen dondurma sayısının 3 1 3 olması olasılığı kaçtır?



İleri algoritması bu sorunun cevabını verir.

P(3 1 3) = ?

T(1 2 3)

1. Initialization:

α1 (H)=a0H bH (3) = 0.8x0.4 = 0.32

α1 (C)=a0C bC (3) = 0.2x0.1 = 0.02

1. Recursion (recall that states 0 and qF are non-emitting):

T=2

α2 (H)= =+ =(0.32x0.6+ 0.02x0.4)x0.2= 0.04

α2 (C)= =+ =(0.32x0.3+0.02x0.5)X0.5=0.053

T=3

α3 (H)= =+

=(0.04x0.6 +0.053x0.4)x0.4=0.018

α3 (C)= =+

=(0.04x0.3 +0.053x0.5)x0.1=0.0038

1. Termination:

P(313|λ) = = = (0.018+0.0038)x0.1=0.0022

Bu değerlerin bitim olasılıklarıyla ağırlıklandırılmış toplamı ileri algoritmasının sonucudur.

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| --- | --- | --- | --- |
| HIDDEN STATE | FORWARD ALGORITHM TRELLIS | | |
| HOT | 0.32 | 0.04 | 0.018 |
| COLD | 0.02 | 0.053 | 0.004 |
| OBSERVATION | 3 | 1 | 3 |

Viterbi Algorithm:

1. Initialization:

v1 (j)=a0j bj (o1 ) 1≤j≤N

ψ1(j)=0 1≤j≤N

1. Recursion (recall that states 0 and qF are non-emitting):

vt (j)= 1≤j≤N, 1<t≤T

ψt(j)= 1≤j≤N, 1<t≤T

1. Termination:

The best score:

P\* = =

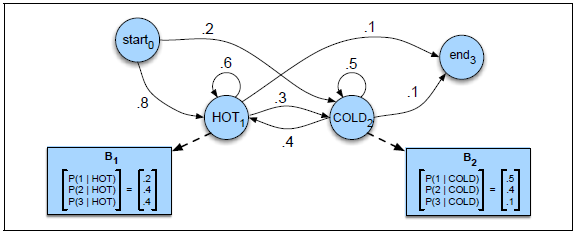
The start of backtrace:

qT \* = ψT (qF) =

1. Backtracing:

qt\* = ψt+1 (qt+1\* ) t = T-1, T-2, …, 1

Örnek 2: Dondurma yeme gizli Markov otomatı için ilk 3 gün boyunca yenen dondurma sayısının 3 1 3 olması olasılığını en iyileyen gizli durum dizisi nedir?



Viterbi algoritması bu sorunun cevabını verir.

Dys: 3 1 3

Zaman: 1 2 3

1. Initialization:

v1 (H)=a0H bH (3) = 0.8x0.4 = 0.32

v1 (C)=a0C bC (3) = 0.2x0.1 = 0.02

ψ1(H)=0 ψ1(C)=0

1. Recursion (recall that states 0 and qF are non-emitting):

T=2

v2 (H)= max(, )

=max(0.32x0.6 x0.2, 0.02x0.4 x0.2) = max(0.038,0.0016) = 0.038

v2 (C)= max(, )=

=max(0.32x0.3 x0.5, 0.02x0.5 x0.5) = 0.048

Ψ2(H)=H Ψ2(C)=H

T=3

v3 (H)= max(, )

=max(0.038x0.6x0.4, 0.048x0.4x0.4) = max(0.0091, 0.0077)=0.0091

v3 (C)= max(, )=

= max(0.038x0.3x0.1, 0.048x0.5x0.1) = max(0.0011, 0.0024)=0.0024

Ψ3(H)=H Ψ3(C)=C

1. Termination:

The best score:

P\* = = = max()=max(0.00091,0.00024)= 0.00091

The start of backtrace:

q3 \* = ψ3 (qF) = = H,

* qF is not considered in the next time step 4 but considered in time step 3

1. Backtracing:

q3 \*=H

q2\* = ψ3 (q3\*)= ψ3 (H)=H

q1\* = ψ2 (q2\*)= ψ2 (H)=H

P(3 1 3)🡪most likely sequence(decoding)= HHH

|  |  |  |  |
| --- | --- | --- | --- |
| HIDDEN STATE | VITERBI ALGORITHM TRELLIS | | |
| HOT | 0.32 | 0.038 | 0.0091 |
| COLD | 0.02 | 0.048 | 0.0024 |
| OBSERVATION | 3 | 1 | 3 |