* Sample space (örnek uzay):
* Sample point (örnek nokta), atomic event: ,
* Probability space or probability model (Olasılık uzayı veya olasılık modeli):

for , a new sample space that includes is determined such that

and

* Event (olay):

A is an arbitrary subset of such that

is complementary probability of A such that

Example: For the rolling a dice problem

* Sample space:
* Sample point:
* Probability model:

P()= P()= P()= P()= P()= P()=

* An event A and its complement :

A = = P()+ P()+ P()=

= = P()+ P()+ P() ) =

Random variable (Rastgele değişken):

* a function that maps sample points to real numbers or Boolean values.

1. Boolean random variable:

Odd() = True | False = odd|odd

1. Discrete (ayrık) random variable:

Weather() = sunny | rainy | cloudy | snow

1. Continuous (sürekli) random variable:

Temperature() = 21.6 or Temperature() 22.0

Propositions:

* ꓦ: or, ꓥ: and, : not, : if, : if and only if
* Simple propositions: Weather = sunny, cavity
* Complex propositions: Weather = sunny cavity

Taking the probability for a random variable X results in the probability distribution:

Exp: P(Weather) = <sunny, rainy, cloudy, snowy> = <0.72, 0.1, 0.08, 0.1>

*normalized to sum to 1.*

P()

Exp: P(Odd=true)=P(1)+P(3)+P(5)=

Prior probability or Unconditional probability (öncül olasılık veya koşulsuz olasılık):

* Also called inconditional probability
* Gives the probability of a proposition before a new information is learnt
* Exp: P(cavity) = 0.1, P(Weather=sunny)=0.72

Joint probability (birleşik olasılık):

* Gives the probability of more than one propositions
* Exp: P(cavity, Weather=sunny) = 0.14

P(cavity, Weather=sunny) = P(cavity).P(Weather=sunny), if P(cavity) P(Weather=sunny)

independence

Joint probability distribution (birleşik olasılık dağılımı):

* Gives the probabilities for all the sample points of a random variable set
* Events are sum of sample points. So, joint distributions can answer all related questions.
* Exp: P(Cavity, Weather) = a 4x2 matrix

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Weather | sunny | rainy | cloudy | snowy | TOTAL |
| Cavity = True | 0.144 | 0.02 | 0.016 | 0.02 | 0.2 |
| Cavity = False | 0.576 | 0.08 | 0.064 | 0.08 | 0.8 |
| TOTAL | 0.72 | 0.1 | 0.08 | 0. 1 | 1 |

P(cavity)=1- P(cavity)

P(Weather=sunny) = 1- P(sunny) = 1- ( P(rainy)+P(cloudy)+ P(snowy) )

Posterior probability or Conditional probability (ardıl olasılık veya koşullu olasılık):

* Gives the probability of propositions depending on the change in available information.
* Exp:

P(Cavity=true| Toothache=true) = P(cavity| toothache) = 0.8

If my tooth aches, then I have 80% cavity.

conditional

More information changes probability if relevant:

P(cavity|toothache, cavity) = 1

If my tooth aches and I have cavity, then I have 100% cavity.

More information does not change probability if irrelevant:

P(cavity|toothache, FenerbahçeWin) = 0.8

If my tooth aches and Fenerbahçe wins or loses, then I have 80% cavity.

Mathematical definition of conditional probability:

Product rule (derived from above two equations):

Chain rule: Application of multiplication rule repetitively

P(X1, X2, …, Xn) = P(Xn|X1, …, Xn-1) P(X1, …, Xn-1)

= P(Xn|X1, …, Xn-1) P(Xn-1|X1, …, Xn-2) P(X1, …, Xn-2) = ….

= P(Xn|X1, …, Xn-1) P(Xn-1|X1, …, Xn-2) …P(X2|X1).P(X1)

= Xi| X1, …, Xi-1 )

Inference by enumeration:

* the posterior joint distribution of the query variables Y
* given set of random variables X
* given specific values e for the evidence variables E
* Then the hidden variables are H = X – Y – E
* the required summation is done by summing out the hidden variables:

Independence (bağımsızlık):

* If two random variables are independent , then

AB ⇒ P(A|B) = P(A)

P(B|A) = P(B)

P(A,B) = P(A).P(B)

* Exp: When Weather is an independent random variable from other random variables.

Weather = {sunny, rainy, cloudy, snowy}

Toothache = {true, false}

Catch = {true, false}

Cavity = {true, false}

P(Toothache, Catch, Cavity, Weather) = P(Toothache, Catch, Cavity).P(Weather)

Conditional Independence (koşullu bağımsızlık):

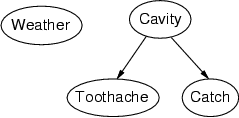
* If a known variable is the cause of other variables that do not have direct influence on each other, then other variables are conditionally independent.

Toothache and Catch are not independent.

But they get independent when Cavity is known.

Toothache depends on the state of tooth nerves.

Catch depends on the ability of the dentist to use the probe.



Cause

Effect 2

Effect 1

Therefore:

P(Toothache, Catch | Cavity) = P(Toothache| Cavity).P(Catch| Cavity)

P(Toothache| Catch, Cavity) = P(Toothache| Cavity)

Bayes’ Rule (Bayes kuralı):

* A common method used for probabilistic inference
* Mathematical definition:

P(X,Y) = P(X|Y). P(Y) (Bayes’ rule)

normalization factor

⇒

P(X,Y) = P(Y|X). P(X)

* Exp:

P(Cavity| toothache, catch) = P(toothache, catch| Cavity). P(Cavity)

Considering independence of toothache and catch given cavity

P(Cavity| toothache, catch) = P(toothache| Cavity). P(catch| Cavity). P(Cavity)

Naïve Bayes’ (Toy Bayes):

* Assumes independence of all effects given the cause to simplify the inference problem
* Drops the number of probabilities that needs to be known to O(n).
* Can provide good inference results even for the cases that are not conditionally independent
* Mathematical definition:

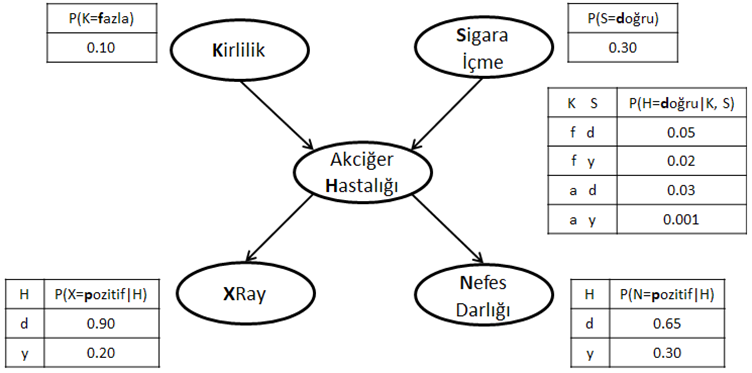
P(Cause, Effect1, Effect2, …, Effectn) = P(Cause).(Effecti| Cause)

Bayesian Networks (Bayes ağları):

* A graphical notation for conditional independence assertions in inference problems
* Nodes represent random variables,
* Links indicate that nodes have direct influences on others,
* Conditional probability tables (CPT) give the distribution over a random variable *Xi* for each combination of parent values
* A CPT for Boolean *Xi* with k Boolean parents has 2k rows
* Each row requires one number p for *Xi* = true (the number for *Xi* = false is 1-p)
* The full joint distribution is the product of the local conditional probabilities

Örnek:

Aşağıda verilen Bayes ağında değişken olasılıkları ve koşullu olasılıklar tanımlanmış olup değişken isimleri ve bunların alabilecekleri değerler kısaca kalın yazı tipiyle vurgulanmış harfler ile tanımlanmıştır. Bu Bayes ağına göre mesela akciğer hastalığı direkt olarak kirlilik ve sigara içme değişkenlerine; xray veya nefes darlığı değişkenleri ise direkt akciğer hastalığı değişkenine bağlıdır.



a) P(X=p, N=n, H=d, S=d, K=a) birleşik olasılığını hesaplayınız.

P(X=true| N=false,H=true,S=true,K=false)\*P(N=false|H=true,S=true, K=false)

\*P(H=true|S=true, K=false)\*P(S=true| K=false)\*P(K=false)

= P(X=true|H=true)\* P(N=false|H=true) \*P(H=true|S=true, K=false)\* P(S=true)\* P(K=false)

=0.9 \*(1-0.65)\*0.03\*0.3\*(1-0.1)

b) Herhangi bir kanıt olmadığında Xray’de hastalığa dair pozitif bulgu olması olasılığı yani P(X=p) nedir? İpucu: Önce P(H=d) olasılığını hesaplayıp P(X=p) = P(X=p|H=d) P(H=d)+P(X=p|H=y) P(H=y) açılımı üzerinden gidiniz.

P(X=p) = P(X=p|H=d).P(H=d)+P(X=p|H=y).P(H=y) = 0,9x0,0116 + 0,2x0,988 =0,208

P(H=d) = P(H=d|K=f,S=d).P(K=f).P(S=d)+ P(H=d|K=f,S=y).P(K=f).P(S=y)+ P(H=d|K=a,S=d).P(K=a).P(S=d)+ P(H=d|K=a,S=y).P(K=a).P(S=y)

= 0,05x 0,1x0,3 + 0,02 x0,1x0,7+ 0,03x0,9x0,3+ 0,001x0,9x0,7 = 0,0116

P(H=y) = 1- P(H=d) = 1-0,0116 = 0,988

c) Nefes darlığının pozitif olması durumunda akciğer hastalığının doğru olması olasılığı yani P(H=d | N=p) nedir? İpucu: Bayes’ kuralını uygulayınız.

P(H=d | N=p) = P(N=p | H=d) . P(H=d) / P(N=p) = 0,65 x 0,0116 / 0,304 = 0,0248

P(N=p)= P(N=p|H=d).P(H=d)+P(N=p|H=y).P(H=y) = 0,65x0,0116+ 0,3 x 0,988 = 0,304

d) Kişinin sigara içmesi durumunda akciğer hastası olması olasılığı yani P(H=d | S=d) nedir?

P(H=d | S=d) = P(H=d | S=d, K=a) P(K=a)+ P(H=d | S=d, K=f) P(K=f) = 0,03 x0,9 + 0,05x 0,1=0,032