

Design of Experiments project

Team C.O.F.F.E.E.



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## I. Proposal

### A. Problem Description

Acidity is a known problem for avid coffee drinkers. Because coffee is acidic, it can cause heartburn. Because of this, many coffee drinkers are looking into lower acidic types of coffee, as well as different coffee-making methods to lower the chance of discomfort throughout the day-to-day activity.

### B. Response Variable

Our group proposes measuring acidic levels (in pH) of three different types of coffee roast and two different types of coffee making techniques.

### C. Factors

**Factor A:** Type of Coffee machine with 2 Levels –

1. French Press
2. Mr. Coffee coffeemaker

**Factor B:** Type of Coffee with 3 levels –

1. Death Wish, the world's strongest commercially available coffee
2. Urban Alchemy Blonde Roast coffee (which is supposed 'moderately acidic')
3. Java Planet low acid medium dark coffee

### D. Experimental Units

**Cups of coffee** – 2 cups, i.e. replications for each combination below – a combined total of 12.

1. (Death Wish, French Press)
2. (Urban Alchemy Blonde, French Press)
3. (Java Planet, French Press)
4. (Death Wish, Mr. Coffee)
5. (Urban Alchemy Blonde, Mr. Coffee)
6. (Java Planet, Mr. Coffee)

### E. The goal of the study

The goal of this study is to detect whether the coffee brand or the type of coffee machine have any effect on the acidity levels of the coffee. This is to allow coffee drinkers who are sensitive to acidic foods and drinks to be aware of what coffee brands or the brewing process that may be harmful.

### F. Data Collection

#### **Method**

The coffee grinds from each coffee roast will be measured in 1 tablespoon. The acidic levels will be measured with Apera pH pocket tester. There will be two replications for each condition. The pH pocket tester will be recalibrated with the provided calibration solution after each measurement.

**Apparatus and Protocol:**

The team used the following equipment

1. Mr. Coffee Coffeemaker- Electric drip coffeemaker
2. French press
3. Measuring cup
4. Measuring spoon-1 tablespoon
5. Coffee bean Grinder
6. Tap water (City of Arlington)
7. Apparatus: pH meter
8. Death Wish-The World's Strongest coffee beans roast
9. Java Planet- Columbian USDA low acid coffee beans roast
10. Urban Alchemy Blonde Roast coffee beans roast
11. Sandwich bags
12. Coffee Filter
13. Dial Soap
14. Sponge
15. 12 oz coffee cup

Once the equipment was gathered, the team performs the following procedure:

1. To control the coarseness of the coffee grinds; pour  $\frac{1}{2}$  cups of coffee beans from only one of the roasts into the coffee grinder. Grind the coffee for precisely twenty seconds. Once the twenty seconds are over, put the coffee grinds into one of the sandwich bags.
2. Clean the coffee grinder using tap water from the City of Arlington, Dial Soap, and a sponge; and repeat step 1 until you have all three different types of coffee grinds into a sandwich bag.

**Prelude:** Calibrate the pH meter following these instructions:

1. Long press the calibration button to enter calibration mode; short press the power button to exit.
2. Insert the probe into the 7.00 pH calibration solution; stir gently; leave it to stand. Wait for the smiley face to appear on the pH meter screen and stay on the screen. Short press the calibration button to complete the 1st point calibration.
3. Rinse the probe in distilled water. Long press the calibration button to enter calibration mode again. Insert the probe in the pH 4.00 calibration solution, stir gently; leave it to stand; wait for the smiley face to appear and stay on the screen. Then short press the calibration button to complete the second point calibration.

**Procedure for Mr. Coffee and a roast**

1. Use a measuring cup to measure exactly two cups of tap water from the City of Arlington and pour it into the water component of the Mr. Coffee coffeemaker
2. Use a measuring spoon to measure exactly one tablespoon of whichever coffee is assigned to the treatment from the sandwich bag.

3. Put one Coffee Filter on the coffee grind component of the Mr. Coffee coffeemaker and put the coffee grind from the tablespoon on top of the filter.
4. Turn the Mr. Coffeemaker on.
5. Once the coffee is poured, pour the coffee from the coffee pot into the coffee mug and allow the coffee to sit for seven minutes. Turn the coffeemaker off.
6. Take the measurement using a pH meter.
7. Remove the used filter from the coffeemaker and throw it into the trash.
8. Clean the cup with soap and sponge.
9. To clean the Mr. Coffee coffee machine; Pour two cups of tap water into the water component, turn on the Mr. Coffee, and pour the pot into the sink to remove residue.
10. Repeat Steps 1-9 in 1A when the next treatment requires the Mr. Coffee condition.

#### **Procedure for a French press and a roast**

1. Use a measuring cup to measure exactly two cups of tap water from the City of Arlington and pour the water into the water component of the Mr. Coffee
2. Remove the top of the French press
3. Use a measuring spoon to measure exactly one tablespoon of coffee grind that is required for the treatment, and place it at the bottom of the French press
4. Run the Mr. Coffee to get a full pot of hot water (make sure there is no coffee filter in the Mr. Coffee, and make sure the coffeemaker is clean; if not, perform step 9 in part 1A.)
5. Pour the pot of hot water into the French Press.
6. Place the top of the French press back onto the French press
7. Wait for seven minutes.
8. Once seven minutes have passed, *slowly* push down on the handle on the French press
9. Pour the freshly brewed coffee into the coffee mug
10. Take the measurement using the pH meter.

#### **Randomization – Actual Dataset:**

Treatment order before and after randomization. For experimental protocol, the group will follow the randomized order and the treatments in the far-right column. The following abbreviations were used: MCC for Mr. Coffee Coffeemaker, FP for French Press. A for Death Wish Coffee, B for Urban Alchemy Coffee, and C for Java Planet Low Acid coffee.

Initial Order	Treatment	Randomized Order	Treatment
1	(A, MCC)	11	(C, FP)
2	(A, MCC)	7	(A, FP)
3	(B, MCC)	12	(C, FP)
4	(B, MCC)	8	(A, FP)
5	(C, MCC)	4	(B, MCC)
6	(C, MCC)	2	(A, MCC)
7	(A, FP)	9	(B, FP)
8	(A, FP)	1	(A, MCC)
9	(B, FP)	3	(B, MCC)
10	(B, FP)	6	(C, MCC)
11	(C, FP)	10	(B, FP)
12	(C, FP)	5	(C, MCC)

Figure 1: Table - Randomized Dataset

Factors				
Type of Coffee Machine	Type of Coffee Machine	Type of Coffee	TYPE OF COFFEE	PH level
French Press	1	Java Planet Low Acidic	3	6.6
French Press	1	Death Wish	1	6.8
French Press	1	Java Planet Low Acidic	3	6.7
French Press	1	Death Wish	1	6.8
Mr.Coffee Coffee maker	2	Urban Alchemy	2	6.1
Mr.Coffee Coffee maker	2	Death Wish	1	6.9
French Press	1	Urban Alchemy	2	6.1
Mr.Coffee Coffee maker	2	Death Wish	1	6.7
Mr.Coffee Coffee maker	2	Urban Alchemy	2	6.3
Mr.Coffee Coffee maker	2	Java Planet Low Acidic	3	6.7
French Press	1	Urban Alchemy	2	6.1

Figure 2: Table - pH level readings for randomized treatments

## II. Preliminary Analysis of Model Assumptions

### A. Raw Data Plots

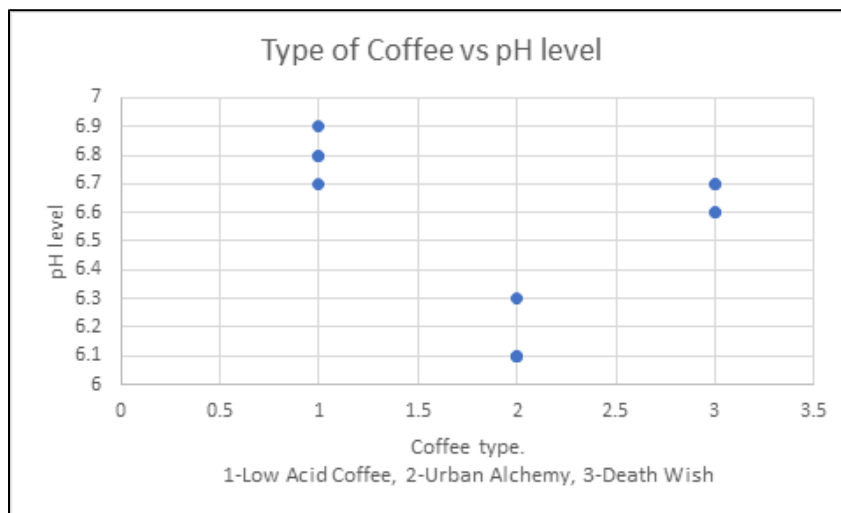


Figure 3: Type of Coffee vs Ph level

Figure 3 shows that the Urban Alchemy brand has a higher pH level than the other two coffee brands. Further tests will be needed to conduct significance. The variances between these treatments seem to be similar with type 3 coffee having slightly lower variance.

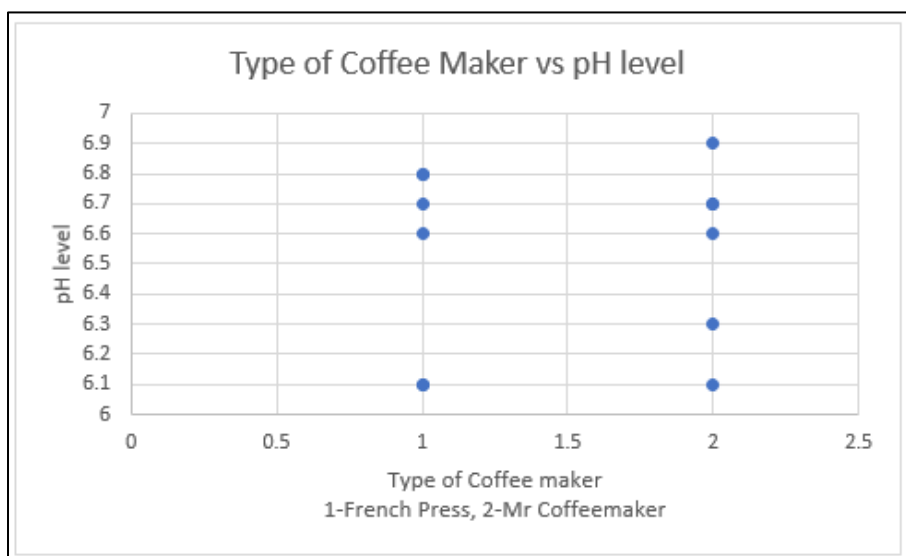


Figure 4: Type of Coffee Maker vs Ph level

Figure 4 shows the pH levels by using different coffee makers. It appears that the Mr. Coffee Coffee maker has slightly lower variance than the French Press. However, a funnel shape looks unlikely. The average pH level of french press is 6.51, and the average for the Mr. Coffeemaker is 6.55. Because of our small dataset, there should be no statistical significance between the two.



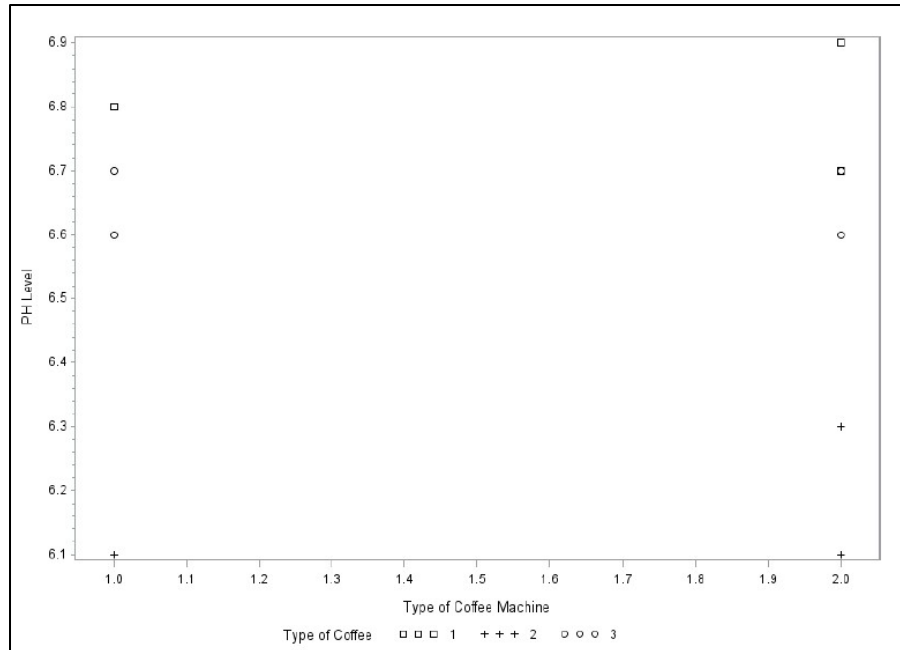


Figure 5: Type of Coffee machine, coffee vs pH level

Figure 5 shows both the type of coffee and the coffee maker type.

## B. Model Form

### Full Interaction Model for two factors:

Our Model contains the full interaction model where

$$\mu_{ij} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ij}$$

Where,

$\mu_{ij}$  = Predicted value of Ph level

$\mu_{..}$  = Overall mean

$\alpha_i$  = Main effect A i.e. Type of Coffee Maker

$\beta_j$  = Main effect B i.e. Type of Coffee

$(\alpha\beta)_{ij}$  = Interaction between Type of Coffee Maker and Type of Coffee.

$\epsilon_{ij}$  = errors iid  $N(0, \sigma^2)$

### C. Model Assumptions

The following are our model's Assumptions

1. Constant error variance

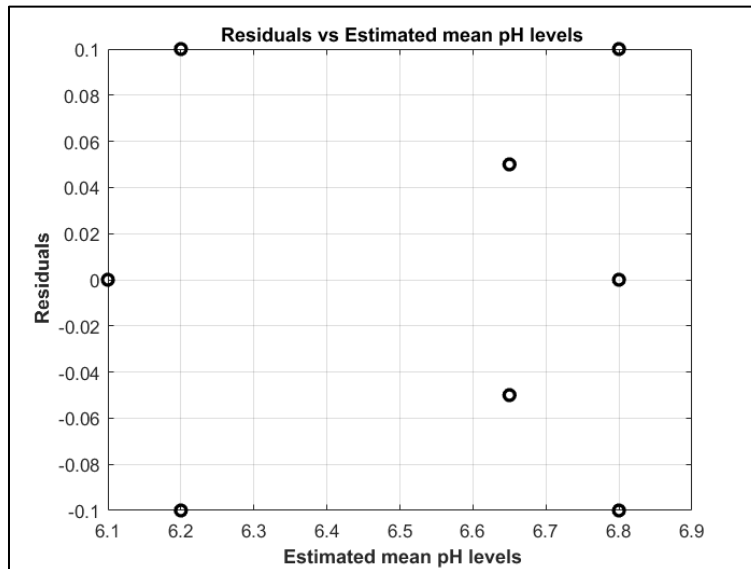


Figure 6: Residuals vs Estimated means plot

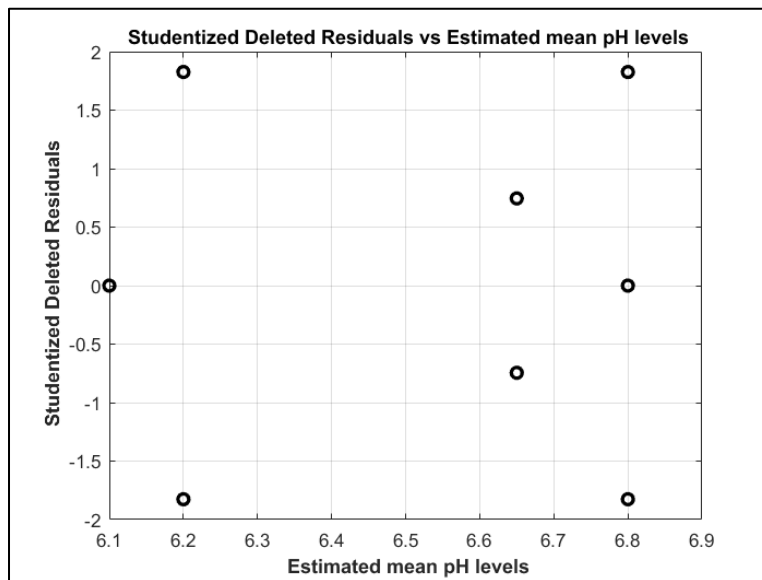


Figure 7: Studentized deleted residuals

The above plots indicate that we have a near constant variance and that there is no funnel shape; therefore, the constant variance assumption is satisfied.

#### Modified Levene

Hypothesis:

H0: Means of the  $d_{it}$  populations are equal

H1: Not all means are equal

Decision Rule: Reject  $H_0$  when  $p \text{ value} < \alpha$

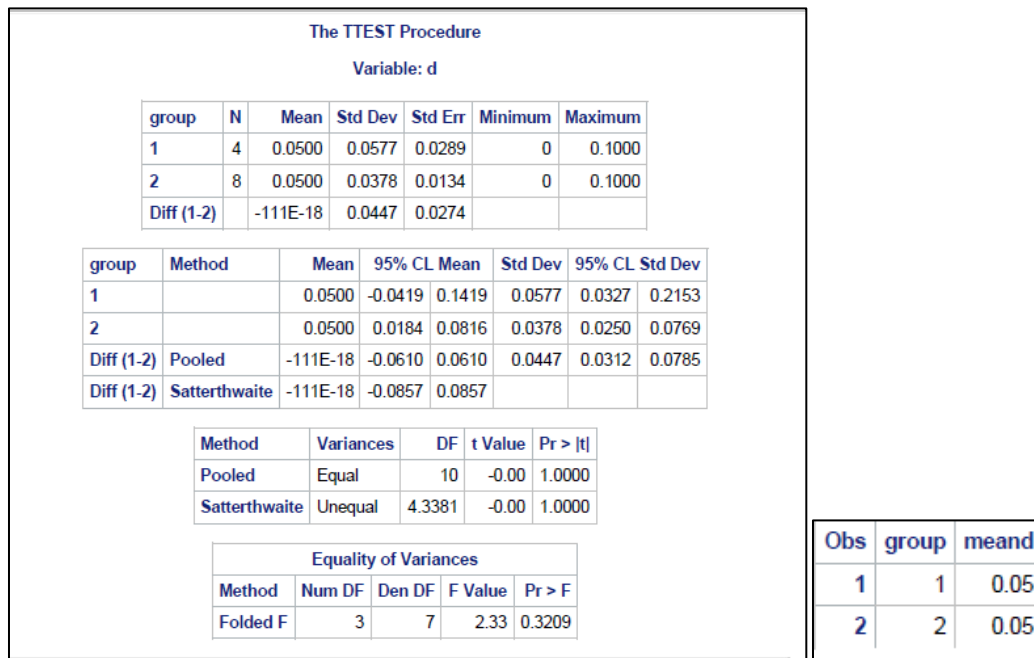


Figure 8: Modified Levene test (SAS)

From the modified Levene test, shown in Figure 8, we can see that  $p\text{-value} = 0.3209$  which is higher than  $\alpha = 0.10$ . Hence we fail to reject  $H_0$  and conclude that means of the dit populations are equal. Thus, the variance is constant for the set of observations.

## 2. Normally distributed errors

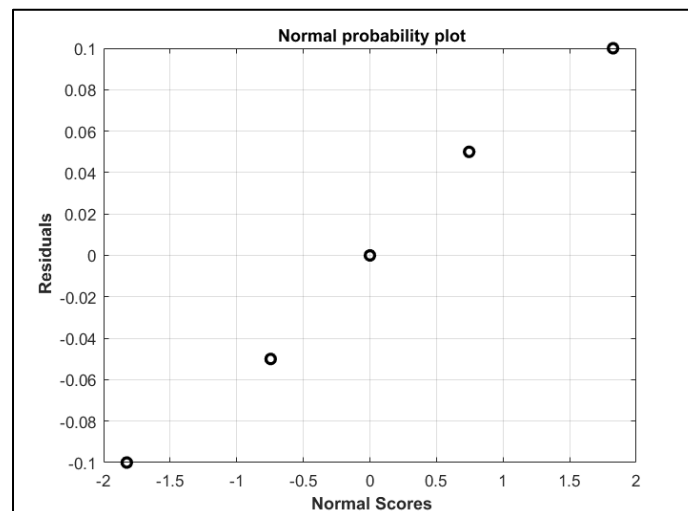


Figure 9: Normal probability plot

Figure 9 shows the normal probability plot. This plot seems pretty straight. Therefore, we can say that residuals are normally distributed. Further, we conducted a normality test to verify our findings.

### Normality test:

Hypothesis:

$H_0$ : Normality is OK

$H_1$ : Normality is violated

Decision Rule: if  $\rho < C(\alpha, n)$ , reject  $H_0$ ,

Where,  $C(\alpha, n) = c(0.10, 12) = 0.942$

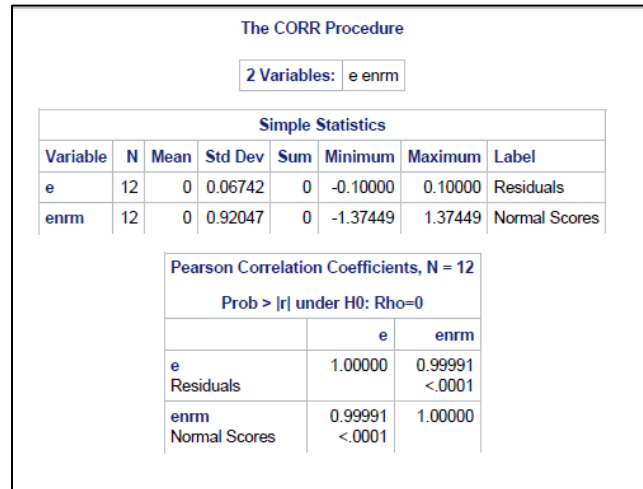


Figure 10: Correlation Procedure

Since  $\rho = 0.999 > C(0.10, 12) = 0.942$ , we Fail to Reject  $H_0$  and conclude that normality is not violated. This result matches with our previous conclusion from the Normal Probability plot.

### 3. Uncorrelated errors

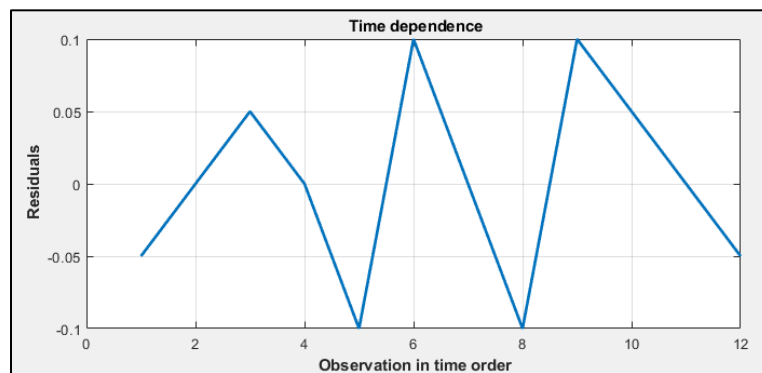


Figure 11: Time Plot

Figure 11 shows the time plot for the observations. Random jaggedness pattern is observed with no trends. We can conclude that there is no time correlation.

#### 4. No outliers

##### **Bonferroni Outlier test:**

We conducted a Bonferroni outlier test to check whether there are any outliers.

Hypothesis:

$H_0$ : No outliers

$H_1$ : At least one outlier

N= number of observations = 12

V= number of treatments = 6

$\alpha = 0.01$

$t(n-v-1, \alpha/2n) = t(12-6-1, 0.000417) = 7.14639$

Decision Rule: if  $|t| < 7.14639$  then conclude  $H_0$ .

Obs	plevel	toem	toce	yhat	e	tres	enrm
1	6.8	1	1	6.80	0.00	0.00000	0.00000
2	6.8	1	1	6.80	0.00	0.00000	0.00000
3	6.1	1	2	6.10	0.00	0.00000	0.00000
4	6.1	1	2	6.10	0.00	0.00000	0.00000
5	6.6	1	3	6.65	-0.05	-0.74536	-0.66391
6	6.7	1	3	6.65	0.05	0.74536	0.66391
7	6.9	2	1	6.80	0.10	1.82574	1.37449
8	6.7	2	1	6.80	-0.10	-1.82574	-1.37449
9	6.1	2	2	6.20	-0.10	-1.82574	-1.37449
10	6.3	2	2	6.20	0.10	1.82574	1.37449
11	6.7	2	3	6.65	0.05	0.74536	0.66391
12	6.6	2	3	6.65	-0.05	-0.74536	-0.66391

Figure 12: Studentized Residuals

From figure 12, we can see that all values are below 7.14639. Therefore, no outliers are present in our dataset.

##### **Validation of model assumptions:**

From the analysis of plots above and respective tests for model assumptions, we can conclude that all model assumptions have been satisfied.

#### D. Transformations

From our observations of Normality and constant variance, we can say that our model assumptions are satisfied. Hence, no transformation is required to conduct further analysis.

### III. Analysis of Variance

#### A. Interaction Plots and analysis

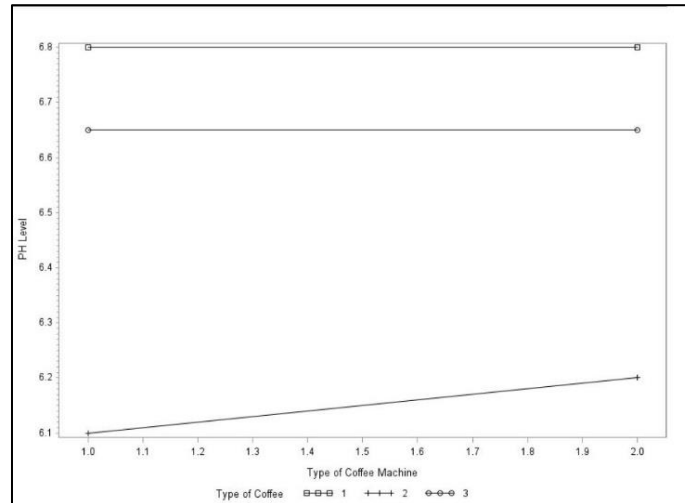


Figure 13: Interaction plot

From the plot, shown in Figure 13, we can see that all lines for the type of coffee 1 and 3 are parallel while the line for coffee type 2 is somewhat non-parallel. We can say that the interaction effect if any would be negligible.

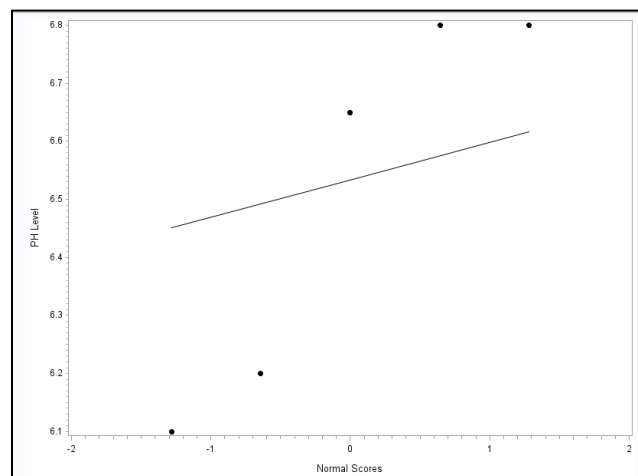


Figure 14: Normal probability plot of effects

Figure 14 shows the normal probability plot of effects. We see that the points are spread out, there might be some interactions. However, the averages are close to each other (6.1-6.8). We would conduct further tests to check for interaction effects in the following sections.

## B. ANOVA Discussion

The SAS System					
The GLM Procedure					
Dependent Variable: plevel PH Level					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	0.93666667	0.18733333	22.48	0.0008
Error	6	0.05000000	0.00833333		
Corrected Total	11	0.98666667			

R-Square	Coeff Var	Root MSE	plevel Mean
0.949324	1.397251	0.091287	6.533333

Source	DF	Type I SS	Mean Square	F Value	Pr > F
to cm	1	0.00333333	0.00333333	0.40	0.5504
to ce	2	0.92666667	0.46333333	55.60	0.0001
to cm*to ce	2	0.00666667	0.00333333	0.40	0.6870

Source	DF	Type III SS	Mean Square	F Value	Pr > F
to cm	1	0.00333333	0.00333333	0.40	0.5504
to ce	2	0.92666667	0.46333333	55.60	0.0001
to cm*to ce	2	0.00666667	0.00333333	0.40	0.6870

Figure 15: GLM Procedure

Figure 14 shows the Anova table and GLM Procedure output from SAS. We can obtain the following ANOVA parameters:

$SSTot = 0.98667$

$SSTr = 0.93667$

$SSA = 0.0033$

$SSB = 0.92667$

$SSAB = 0.00667$

Since we are working with an orthogonal model/design/dataset, our observations for Type I SS and Type III SS are same i.e.  $SSTr = SSTr(adj)$ . Decomposition values of  $SSTr$  into  $SSA$ ,  $SSB$ ,  $SSAB = SSA(adj)$ ,  $SSB(adj)$ ,  $SSAB(adj)$

**Degrees of freedom:** Number of independent quantities

$a = 2$  = levels for factor A

$b = 3$  = levels for factor B

$r = 2$  = number of replications/repeats

$n$  = total number of observations = 12

$v$  = Number of treatment combinations =  $a*b = 2*3 = 6$

$SSTot$  is subject to one constraint:  $abr - 1 = (2*3*2) - 1 = 12 - 1 = 11$

$SSTr$  is subject to one constraint:  $= ab - 1 = v - 1 = 6 - 1 = 5$

$SSE$  is subject to  $ab$  constraints:  $= ab(r-1) = 6*(2-1) = 6$

$SSA$  is subject to one constraint:  $= a - 1 = 2 - 1 = 1$

$SSB$  is subject to one constraint:  $= b - 1 = 3 - 1 = 2$

SSAB is subject to  $a + b - 1$  constraints:  $= (a-1) (b-1) = (2-1) * (3-1) = 1*2 = 2$

### **Mean Squares**

$$MSA = SSA/a-1 = 0.0033$$

$$MSB = SSB/b-1 = 0.4633$$

$$MSAB = SSAB/(a-1) (b-1) = 0.0033$$

$$MSTr = SStr/abr-1 = 0.18733$$

$$MSE = SSE/ab(r-1) = 0.00833$$

### **F-statistics**

$$FA^* = MSA/MSE = 0.40$$

$$FB^* = MSB/MSE = 55.40$$

$$FAB^* = MSAB/MSE = 0.40$$

To understand which significance of factors for our experiment, we perform first perform the test to check interaction effect followed test for main effects A, B - individual factors.

### **Test AB interaction:**

Hypothesis:

$H_0$ : AB interaction is negligible

$H_1$ : AB interaction is not negligible

### **Conclusion:**

Since p-value for interaction effect  $= 0.6870 > \alpha = 0.10$ , we fail to Reject  $H_0$ , and conclude that the interaction is negligible. There is no combined effect of coffee machine and type of coffee on the pH level of coffee. Hence, we can drop the interaction term from our full interaction model. The results of test match with the results we obtained from our interaction model plot. We now test the main effects for both factors to find the additive model.

### **Test main A:**

Hypothesis:

$H_0$ : Main effect A is negligible

$H_1$ : Main effect A is not negligible

### **Conclusion:**

Since P value for factor A, i.e. type of coffee machine is  $0.5504 > \alpha = 0.10$  therefore fail to reject  $H_0$  and conclude that main effect A i.e. type of coffee machine is not significant at  $\alpha = 0.10$  level.

### **Test main B:**

Hypothesis:

$H_0$ : Main effect B is negligible

$H_1$ : Main effect B is not negligible

### **Conclusion:**

Since P value for factor A, i.e. type of coffee is  $0.0001$  which is less than  $\alpha = 0.10$ , we reject  $H_0$  and conclude that main effect B is significant.

From the test for effects, we can conclude that we should use an additive model for our analysis if and when we plan to do the same experiment again.



**Additive model:**

$$\mu_{ij} = \mu_{..} + \beta_j + \epsilon_{ij}$$

Where,

$\mu_{ij}$  = Predicted value of Ph level

$\mu_{..}$  = Overall mean

$\beta_j$  = Main effect B i.e. Type of Coffee

$\epsilon_{ij}$  = errors iid  $N(0, \sigma^2)$

**C. Effects Estimation**

Level of tocm	Level of toce	N	plevel	
			Mean	Std Dev
1	1	2	6.80000000	0.00000000
1	2	2	6.10000000	0.00000000
1	3	2	6.65000000	0.07071068
2	1	2	6.80000000	0.14142136
2	2	2	6.20000000	0.14142136
2	3	2	6.65000000	0.07071068

Level of toce	N	plevel	
		Mean	Std Dev
1	4	6.80000000	0.08164966
2	4	6.15000000	0.10000000
3	4	6.65000000	0.05773503

Level of tocm	N	plevel	
		Mean	Std Dev
1	6	6.51666667	0.33115958
2	6	6.55000000	0.29495762

Figure 16: Means for factors and factor combinations

$$\hat{\mu}_{..} = \bar{Y} \dots = 6.533 \text{ (From SAS output, Figure 15)}$$

$$\hat{\alpha}_1 = \bar{Y}_{1..} - \bar{Y} \dots = 6.51666667 - 6.533333 = -0.01667$$

$$\hat{\alpha}_2 = \bar{Y}_{2..} - \bar{Y} \dots = 6.550 - 6.533333 = 0.01667$$

$$\hat{\beta}_1 = \bar{Y}_{.1.} - \bar{Y} \dots = 6.8 - 6.533333 = 0.266667$$

$$\hat{\beta}_2 = \bar{Y}_{.2.} - \bar{Y} \dots = 6.150 - 6.533333 = -0.383333$$

$$\hat{\beta}_3 = \bar{Y}_{.3.} - \bar{Y} \dots = 6.650 - 6.533333 = 0.116667$$

$$\hat{\alpha\beta}_{11} = \bar{Y}_{11.} - \bar{Y}_{1..} - \bar{Y}_{.1.} + \bar{Y} \dots = 6.8 - 6.51666667 - 6.8 + 6.533333 = 0.01667$$

$$\hat{\alpha\beta}_{12} = \bar{Y}_{12.} - \bar{Y}_{1..} - \bar{Y}_{.2.} + \bar{Y} \dots = 6.1 - 6.51666667 - 6.150 + 6.533333 = -0.03333$$

$$\hat{\alpha\beta}_{13} = \bar{Y}_{13.} - \bar{Y}_{1..} - \bar{Y}_{.3.} + \bar{Y} \dots = 6.65 - 6.51666667 - 6.650 + 6.533333 = 0.016666$$

$$\hat{\alpha\beta}_{21} = \bar{Y}_{21.} - \bar{Y}_{2..} - \bar{Y}_{.1.} + \bar{Y} \dots = 6.8 - 6.550 - 6.8 + 6.533333 = -0.01667$$

$$\hat{\alpha\beta}_{22} = \bar{Y}_{22.} - \bar{Y}_{2..} - \bar{Y}_{.2.} + \bar{Y} \dots = 6.2 - 6.550 - 6.150 + 6.533333 = 0.03333$$

$$\hat{\alpha\beta}_{23} = \bar{Y}_{23.} - \bar{Y}_{2..} - \bar{Y}_{.3.} + \bar{Y} \dots = 6.65 - 6.550 - 6.650 + 6.533333 = -0.016667$$

## IV. Analysis of Effects

### A. Pairwise Comparisons of effects

In our case, the AB interaction is statistically insignificant, and the main effect for B is significant. Hence, we conduct the comparisons on the factor levels  $j$  for B,

*Tukey pairwise comparison:*

$$\text{Hypothesis:} \quad H_0: \mu_i - \mu_s = 0$$

$$H_1: \mu_i - \mu_s \neq 0$$

*Decision Rule:* If the CI contains zero, conclude  $H_0$ , Otherwise  $H_1$ .

$$L^B = \hat{L}^B \pm w_T se(\hat{L}^B)$$

$$w_T = \frac{q_{b, df_{Err}, \alpha}}{\sqrt{2}}$$

$$b = 3$$

$$df_{Err} = 6$$

$$\alpha = 0.05$$

$$w_T = \frac{4.34}{\sqrt{2}} = 3.06884$$

$$msE = 0.00833$$

$$r_i = r_s = 4$$

$$se(\hat{L}^B) = \sqrt{msE \left( \frac{1}{r_i} + \frac{1}{r_s} \right)} = .064537$$

$$w_T \times se(\hat{L}^B) = \mathbf{0.198053}$$

$$\hat{L}_1 = \mu_{.1} - \mu_{.2} = 0.65$$

$$\hat{L}_2 = \mu_{.1} - \mu_{.3} = 0.15$$

$$\hat{L}_3 = \mu_{.2} - \mu_{.3} = -0.50$$

$$L_1 = 0.65 \pm 0.198053 = (0.45194, 0.8480); \quad \textbf{Reject } H_0$$

$$L_2 = 0.15 \pm 0.198053 = (-0.04805, 0.34805); \quad \textbf{Failed to reject } H_0$$

$$L_3 = -0.50 \pm 0.198053 = (-.69805, -.30194); \quad \textbf{Reject } H_0$$

The GLM Procedure Least Squares Means Adjustment for Multiple Comparisons: Tukey			
toce	plevel LSMEAN	LSMEAN Number	
1	6.80000000	1	
2	6.15000000	2	
3	6.65000000	3	

toce	plevel LSMEAN	95% Confidence Limits	
1	6.800000	6.688314	6.911686
2	6.150000	6.038314	6.261686
3	6.650000	6.538314	6.761686

Least Squares Means for effect toce Pr >  t  for H0: LSMean(i)=LSMean(j)			
Dependent Variable: plevel			
i/j	1	2	3
1		0.0001	0.1278
2	0.0001		0.0006
3	0.1278	0.0006	

Least Squares Means for Effect toce				
i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	0.650000	0.451945	0.848055
1	3	0.150000	-0.048055	0.348055
2	3	-0.500000	-0.698055	-0.301945

Figure 17: Tukey Comparisons (SAS)

The analysis from hand calculation and SAS are an exact match for the Tukey analysis which confirms the accuracy of the calculations. From the analysis, the following conclusions are made.

### Conclusion:

The Tukey analysis for Factor B (the type of coffee) shows with 95% simultaneous confidence interval that, there is no significant difference between the mean pH levels for coffee 1 (Death Wish) and coffee 3 (Java Planet Low Acid).

Further, the analysis concludes with 95% simultaneous confidence interval that there is a difference in the mean pH levels between coffee 1 (Death Wish) and coffee 2 (Urban Alchemy), as well as between coffee 2 (Urban Alchemy) and coffee 3 (Java Planet Low Acid).

### B. Line Plots

For the line plot, the treatments are first sorted by their estimated means, as shown in Table 3. Tukey 95% confidence intervals are evaluated for the differences of the adjacent sorted treatments. A line is plotted for the same treatment means, that is, the treatments which have the same means with a 95% simultaneous confidence interval. The plot is shown in Figure 19.

Treatment	$\bar{Y}_i(\text{sorted})$
2(12)	6.1
5(22)	6.2
3(13)	6.65
6(23)	6.65
1(11)	6.8
4(21)	6.8

Figure 18: Sorted Treatment means

Comparison	$\bar{Y}_{i+1} - \bar{Y}_i$	$w_T se\{\hat{L}\}$	CI (95%)	
			Lower Bound	Upper Bound
2--5	0.1	0.36329	<b>-0.2633</b>	<b>0.463296</b>
5--3	0.45	0.36329	0.086704	0.813296
3--6	0	0.36329	<b>-0.3633</b>	<b>0.363296</b>
6--1	0.15	0.36329	<b>-0.2133</b>	<b>0.513296</b>
1--4	0	0.36329	<b>-0.3633</b>	<b>0.363296</b>
2--3	0.55	0.36329	0.186704	0.913296
5--6	0.45	0.36329	0.086704	0.813296
6--4	0.15	0.36329	-0.2133	<b>0.513296</b>
3--1	0.15	0.36329	-0.2133	<b>0.513296</b>
3--4	0.15	0.36329	-0.2133	<b>0.513296</b>

Figure 19: Tukey 95% CI intervals

For example, in the 2-5 treatment comparison, mean differences  $\bar{Y}_5 - \bar{Y}_2$  are calculated to measure the 95% confidence intervals.

*The GLM Procedure*  
*Least Squares Means*  
*Adjustment for Multiple Comparisons: Tukey*

Least Squares Means for Effect tomc*toce				
i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	0.700000	0.336704	1.063296
1	3	0.150000	-0.213296	0.513296
1	4	0	-0.363296	0.363296
1	5	0.600000	0.236704	0.963296
1	6	0.150000	-0.213296	0.513296
2	3	-0.550000	-0.913296	-0.186704
2	4	-0.700000	-1.063296	-0.336704
2	5	-0.100000	-0.463296	0.263296
2	6	-0.550000	-0.913296	-0.186704
3	4	-0.150000	-0.513296	0.213296
3	5	0.450000	0.086704	0.813296
3	6	0	-0.363296	0.363296
4	5	0.600000	0.236704	0.963296
4	6	0.150000	-0.213296	0.513296
5	6	-0.450000	-0.813296	-0.086704

Figure 20: Multiple Tukey Comparison

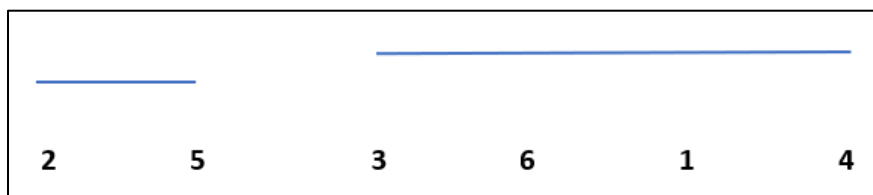


Figure 21: Line Plot

The SAS out show good correlation with the hand calculations, proceeding forth with the analysis. Figure 20 shows that except for the 5-2 treatment all other *adjacent treatments* have similar means with a 95% simultaneous confidence interval. Further, the 95% simultaneous CI show no statistical difference between the treatments 3, 6, 1, 4.

### C. Multiple Comparisons

The following equation represents the simultaneous Bonferroni confidence intervals,

$$\hat{L} \pm w_B se\{\hat{L}\}$$

Where,

$$w_B = t_{n-v, \frac{\alpha}{2m}}$$

$$\alpha = 0.05$$

$$m = 3$$

$$w_B = 3.2874$$

Test form:

$$H_0 = L = 0$$

$$H_1 = L \neq 0$$

Where we reject  $H_0$  if, 0 *does not*  $\in$  CI. With Bonferroni method for preplanned comparisons we get an individual confidence interval of  $100(1 - \alpha/m)\%$ , and a family confidence interval of at least  $100(1 - \alpha)\%$ . Here,  $m$  is the number of comparisons. In the absence of main effect A and interaction AB, we would not see much of difference or contrast in main effect A or AB interaction. Therefore, the following three comparisons were selected for the analysis:

1. Factor B (type of coffee) contrast: mean pH level for coffee 1 and 3 vs coffee 2

$$L_1 = \frac{\mu_1 + \mu_3}{2} - \mu_2 = \frac{\beta_1 + \beta_3}{2} - \beta_2$$

$$C = \left(\frac{1}{2}, \frac{1}{2}, -1\right)$$

$$\hat{L}_B = 0.5749$$

$$se\{\hat{L}\} = \sqrt{msE \sum \frac{c_i^2}{r_i}} = 0.0590058$$

$$w_B se\{\hat{L}\} = (3.2874) \times (0.0590058) = 0.18376$$

$$L_1 = 0.5749 \pm 0.18376 = (0.3911, 0.7587); \quad \textbf{Reject } H_0$$

We are 95% confident that the mean PH level for coffee 1 and 3 differ with the mean pH level for coffee 2, simultaneously with the following two analysis. mean PH level for coffee 1 and 3 is always greater than the mean pH level for coffee 2

2. Factor B (the type of coffee) contrast: mean coffee 1 and 2 vs coffee 3

$$L_2 = \frac{\mu_{.1} + \mu_{.2}}{2} - \mu_{.3} = \frac{\beta_1 + \beta_2}{2} - \beta_3$$

$$C = \left(\frac{1}{2}, \frac{1}{2}, -1\right)$$

$$\hat{L}_B = -0.1749$$

$$se\{\hat{L}\} = 0.0590058$$

$$w_B se\{\hat{L}\} = (3.2874) \times (.0590058) = 0.18376$$

$$L_2 = -0.1749 \pm 0.18376 = (-0.3587, 0.0089); \quad \textbf{Failed to reject } H_0$$

We are 95% confident that the mean pH level for coffee 1 and 2 is similar the mean pH level for coffee 3, simultaneously with the following analysis.

3. Factor B (the type of coffee) contrast: mean coffee 2 and 3 vs coffee 1

$$L_3 = \frac{\mu_{.2} + \mu_{.3}}{2} - \mu_{.1} = \frac{\beta_2 + \beta_3}{2} - \beta_1$$

$$C = \left(\frac{1}{2}, \frac{1}{2}, -1\right)$$

$$\hat{L}_B = -0.4$$

$$se\{\hat{L}\} = 0.0590058$$

$$w_B se\{\hat{L}\} = (3.2874) \times (.0590058) = 0.18376$$

$$L_3 = -0.4 \pm 0.18376 = (-0.5838, -0.2162); \quad \textbf{Reject } H_0$$

We are 95% simultaneously confident that mean pH level for coffee 2 and 3 is different (lower) from the mean pH level for coffee 1.

## V. Final discussion

### A. Summary

In this experiment, we measure the acidity levels of three types of coffee -Death Wish, Urban Alchemy, Java Planet. We perform the experiment to measure the acidity where we control the type of water used to make the coffee, control the temperature of water, amount of water and the amount of coffee being used. We then use an Apera instruments pH meter to measure the acidic levels for each of our 12 treatment combination from two factor a - two type of coffee machines and factor b - 3 types of coffee. From our observed data that we find all the model assumptions for complete factorial model are satisfied without any need for transformations. By analyzing the treatment effects for interactions, main A and main B, we can drop the interaction term and factor A - Type of coffee machine from our initial model. A simple additive model with factor is the correct model to use for analyzing the same experiment if repeated. The pairwise comparison test reveals that there is no significant difference between the mean pH levels for coffee 1 (Death Wish) and coffee 3 (Java Planet Low Acid). The line plot in Figure 19 shows the comparison of means for all the treatments.

### B. Conclusion

From this experiment, we can conclude that the acidic level of coffee does not depend on the type of coffee machine used to make the coffee and any interaction between the type of coffee machine and type of coffee. The acidity of coffee is primarily dependent on the type of coffee beans/ coffee being used to make the coffee under controlled experimental conditions.

### C. Further notes

Upon our first run, we noticed that one set of observation was questionable. In particular, an observation with urban alchemy coffee in a coffee cup. After further investigation, we recalled that some of the acidic pH solutions on the probe contaminated the coffee cup. The coffee cup was discarded, and another measurement was taken in its place.