



3D Localization in Autoware.Auto Using Normal Distribution Transform

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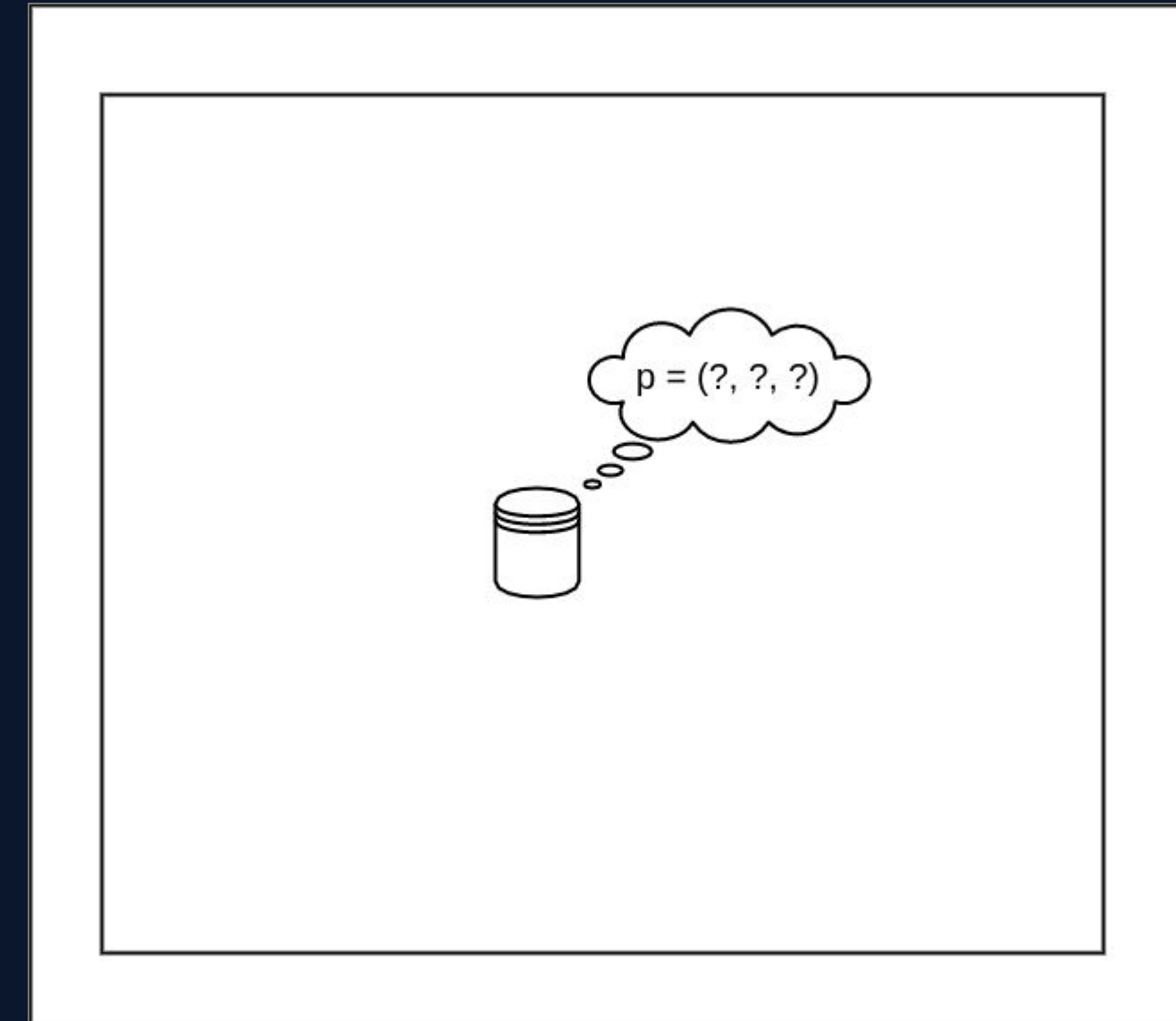
01 Localization Using External Sensors

Relative Localization: Recap

In order to navigate safely, a mobile robot needs to be informed on where it is located with respect to a static reference point.

This reference point could be:

- A landmark with a known location
- Origin of a map
- Center of the world



How to localize?  By sensing and measuring!

Internal sensors:

- IMU
- Wheel encoder

External sensors:

- GNSS
- Camera
- Lidar

Lidar Based Relative Localization

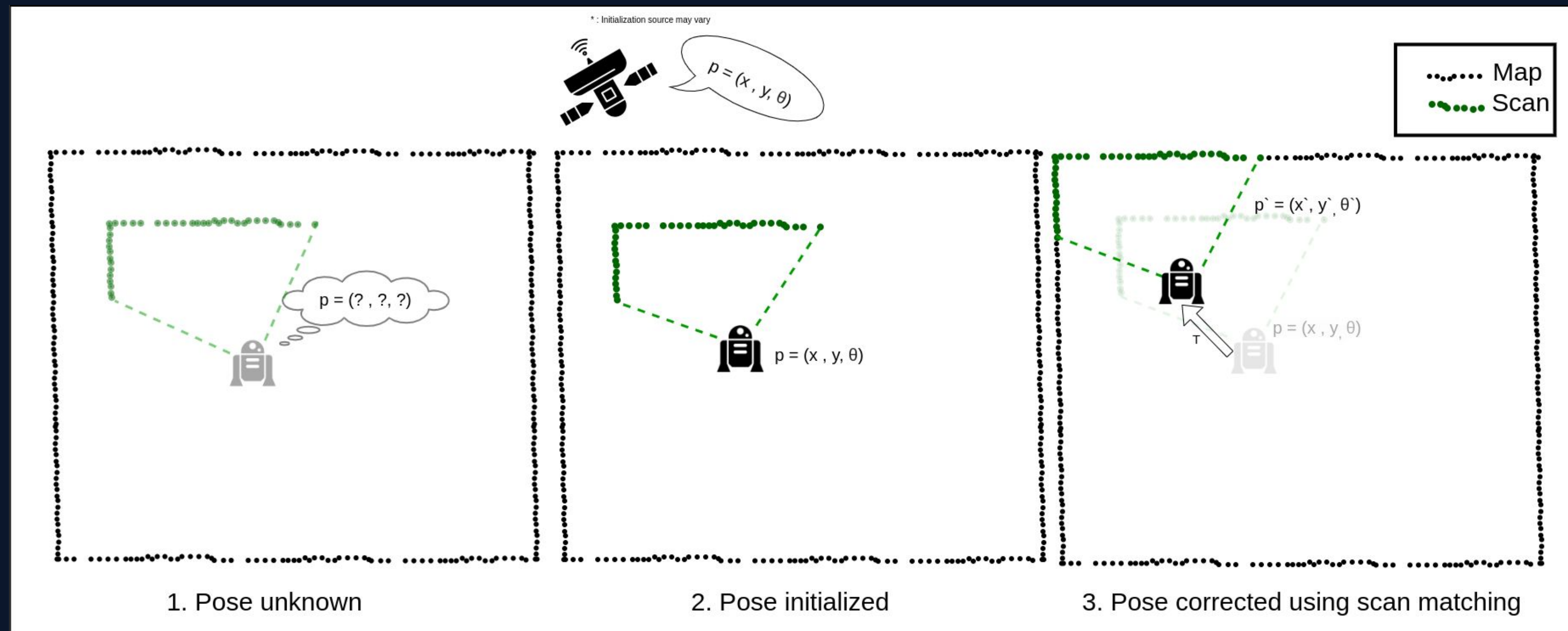
Why have lidar based localization?

- GNSS is subject to limitations:
 - Subject to blockage: Not applicable indoors.
 - Subject to attacks: interference/spoofing.
- Lidar sensors are more precise and robust in sensing the 3D environment compared to other environmental sensors like camera and radar.
 - Robust scale estimation is a challenging problem for camera based techniques.
- Lidar scan matching techniques are proven to be quite precise tools for localizing against high resolution point cloud maps.

Limitations:

- Relies on the existence of an accurate map.
- Processing 3d data can be expensive depending on the algorithm.
- Local algorithms relies on the assumption of a somewhat accurate initial guess.

Lidar Based Relative Localization



Given:

- Map \mathcal{M}
- Initial pose estimate p
 - Only for local algorithms
- scan S

Find:

- Best alignment: T
- Corrected pose estimate $p' = T p$

How?

- ICP [\[Besl & McKay, 1992\]](#)
- NDT [\[Biber & Straßer, 2003\]](#)
- Others

Lidar Scan Matching

- Treat the alignment as a minimization problem
- Minimize alignment error E.

What is E?

Plain `Iterative closest points` (ICP) algorithm:

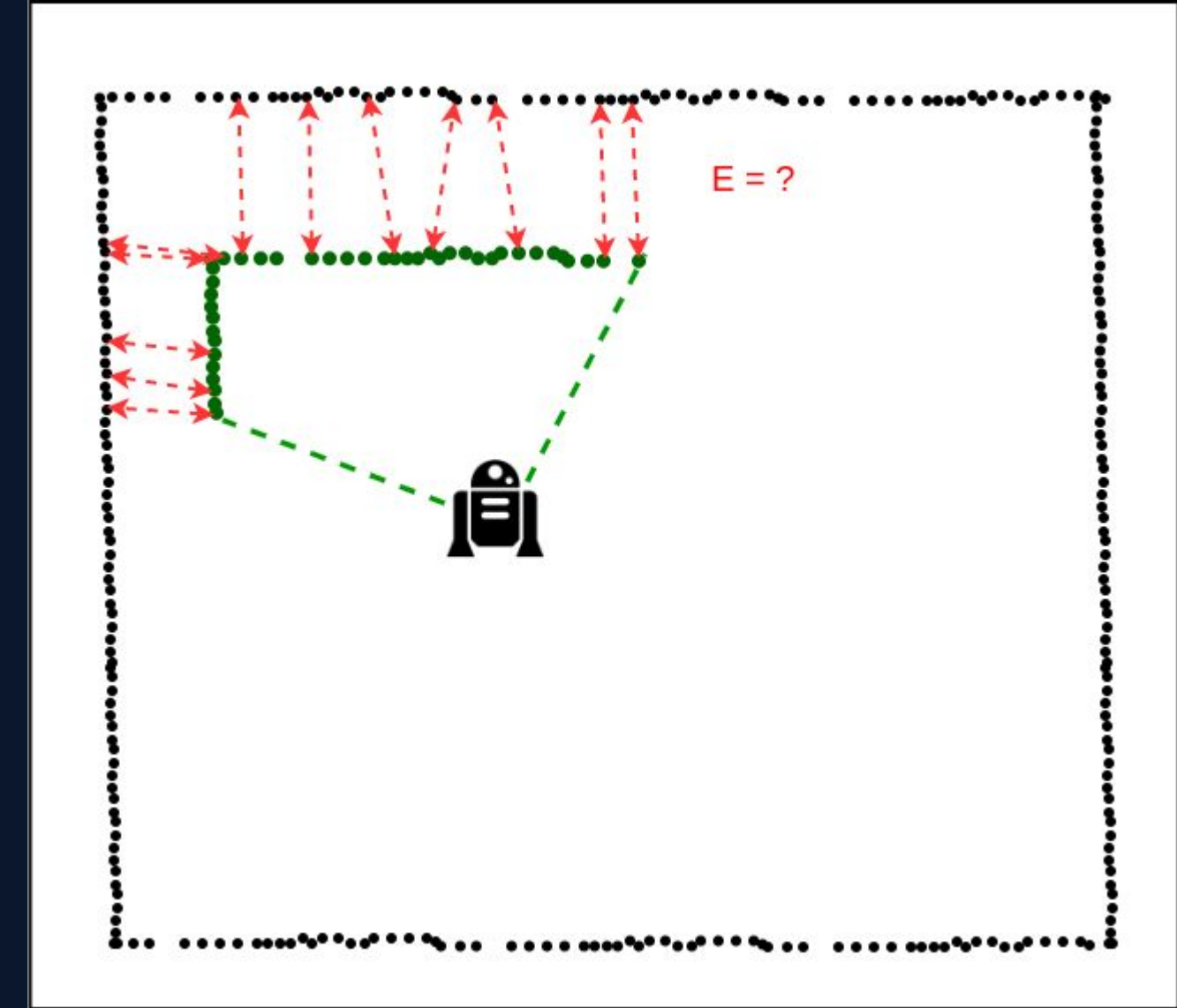
- Given the points in the scan S and the map M:

$$x_s \in S \wedge x_m \in M$$

- Define the Error term E:
 - Closest point-to-point distance:

$$E = \sum_{i=1}^{N_p} (Tx_{s_i} - x_{m_i})^2$$

- Find the T that minimizes E



Cons*:

- More sensitive to initial estimate and the level of alignment than NDT ^{[1] [2]}
 - Bad initial alignment often leads to getting stuck at a local optimum. ^[2]
- Benchmarked to be less accurate than the base(P2D) NDT method in varying scenarios. ^{[1] [3]}

*: depends on the variant

[1]: [\[Magnusson et al., 2015\]](#)

[2]: [\[He et al., 2017\]](#)

[3]: [\[Magnusson et al., 2009\]](#)



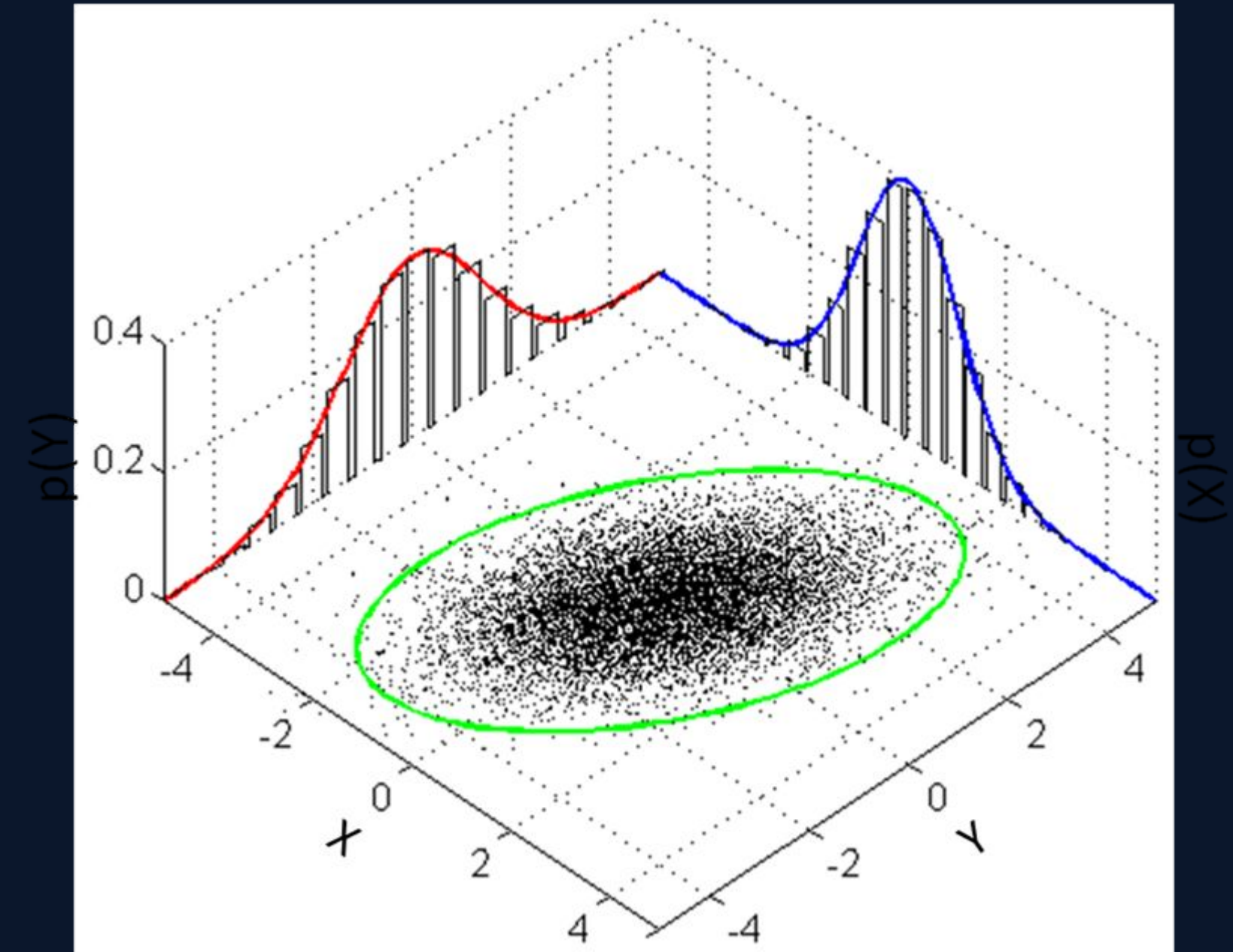
02 Normal Distribution Transform

Multivariate Normal Distribution

Probability density function:

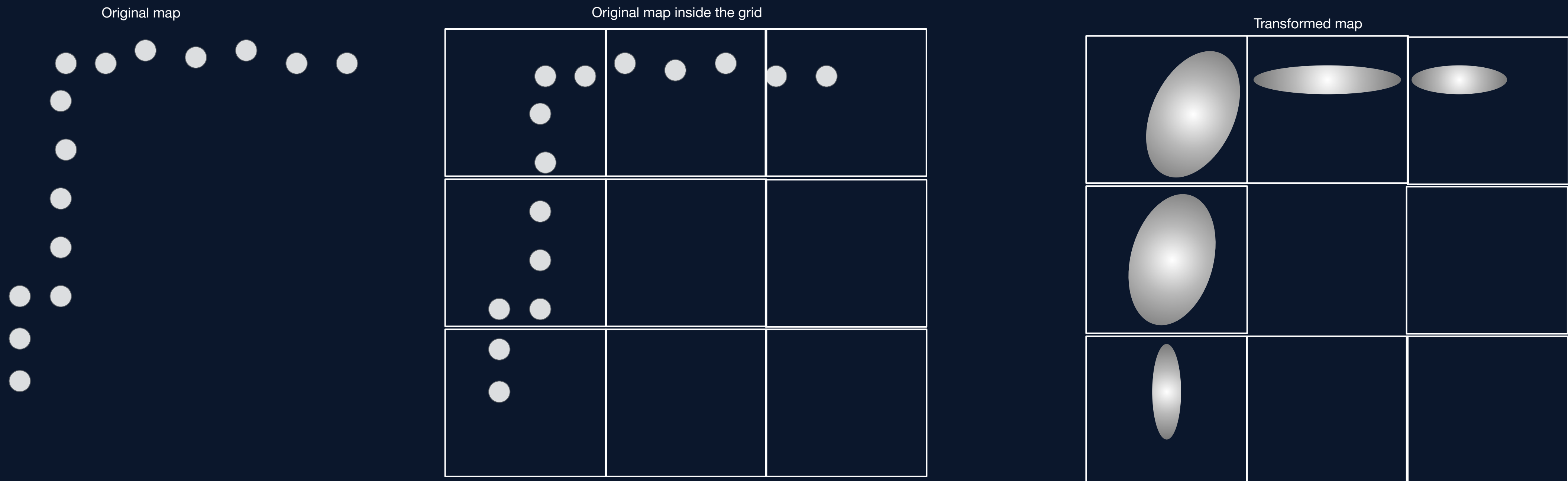
$$f(x) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} e^{-\frac{1}{2}(x-\mu)\Sigma^{-1}(x-\mu)^T}$$

Where μ is the mean of distribution and Σ is the covariance of the distribution.



[1] Sample 2D gaussian distribution

2D Normal Distribution Transform [Bibeni & Straßer, 2003]



- Divide the map into a grid.
- Each cell represents a normal distribution of points.
- Mean and covariance is computed using each point x in a cell with N number of points:

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\Sigma = \frac{1}{N-1} \sum_{i=1}^N (x_i - \mu)(x_i - \mu)^T$$

NDT Alignment Problem

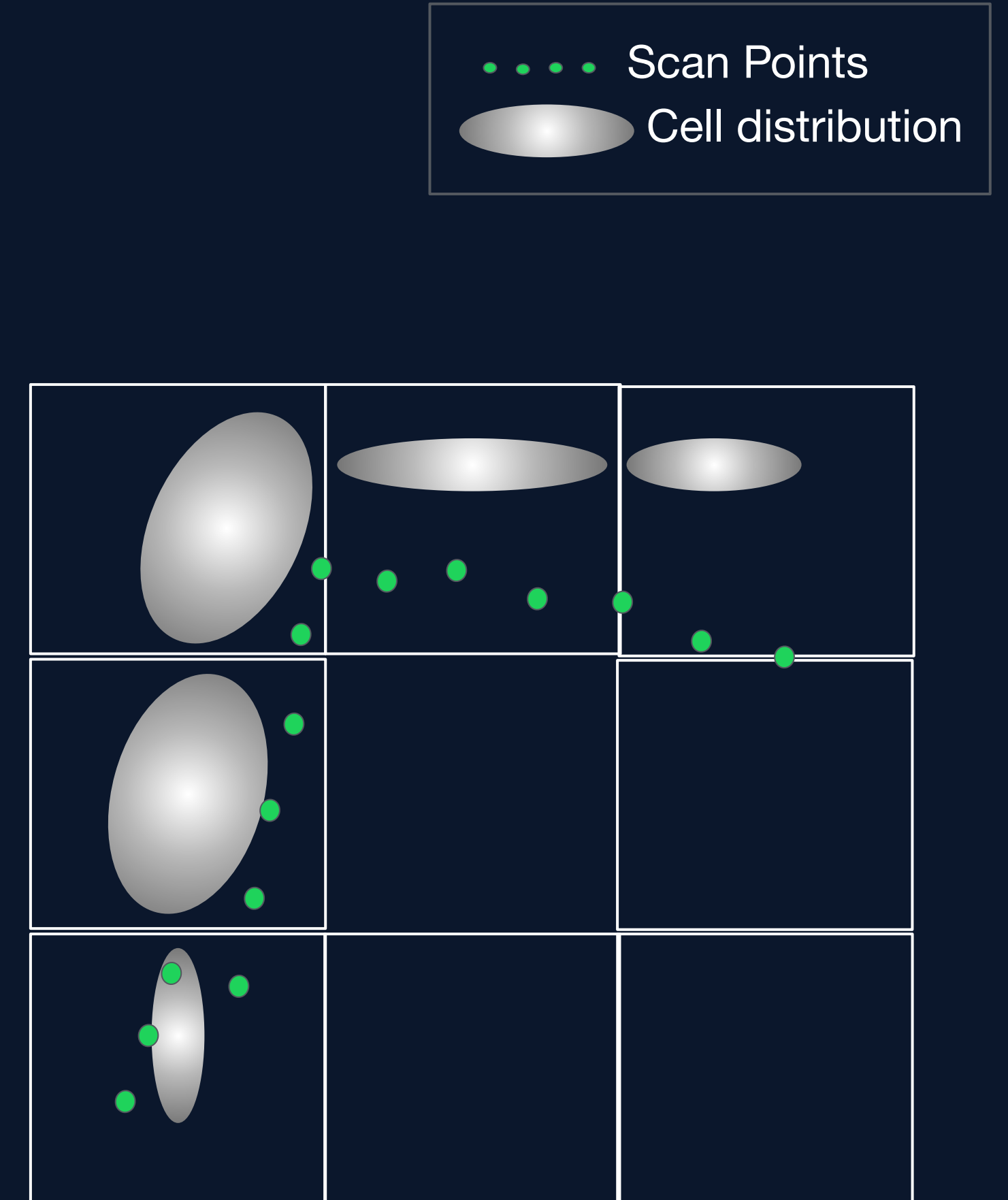
$$p(x) = \frac{1}{c} e^{\frac{-1}{2}(x-\mu)\Sigma^{-1}(x-\mu)^T}$$

❖ Probability density function $p(x)$ of a cell C for a point x is correlated with the probability that the scan point belongs to the cell C .

- If a high number of points in a scan has a high probability of being in an occupied cell in the map grid, we can assume a good alignment.
- Hence the following sum must be maximized for an optimal alignment:

$$\sum_{i=1}^{N_s} e^{\frac{-1}{2}(T(x_{s_i}, p) - \mu_i)\Sigma_i^{-1}(T(x_{s_i}, p) - \mu_i)^T}$$

Where $T(x_{s_i}, p)$ is the transformation function that transforms a point x_s from sensor frame to the map frame using the pose p . μ_i and Σ_i are mean and covariance of the cell the transformed point.



NDT Alignment Optimization

$$p_{2d} = (x, y, \theta) \quad p_{3D} = (x, y, z, \phi, \theta, \psi)$$

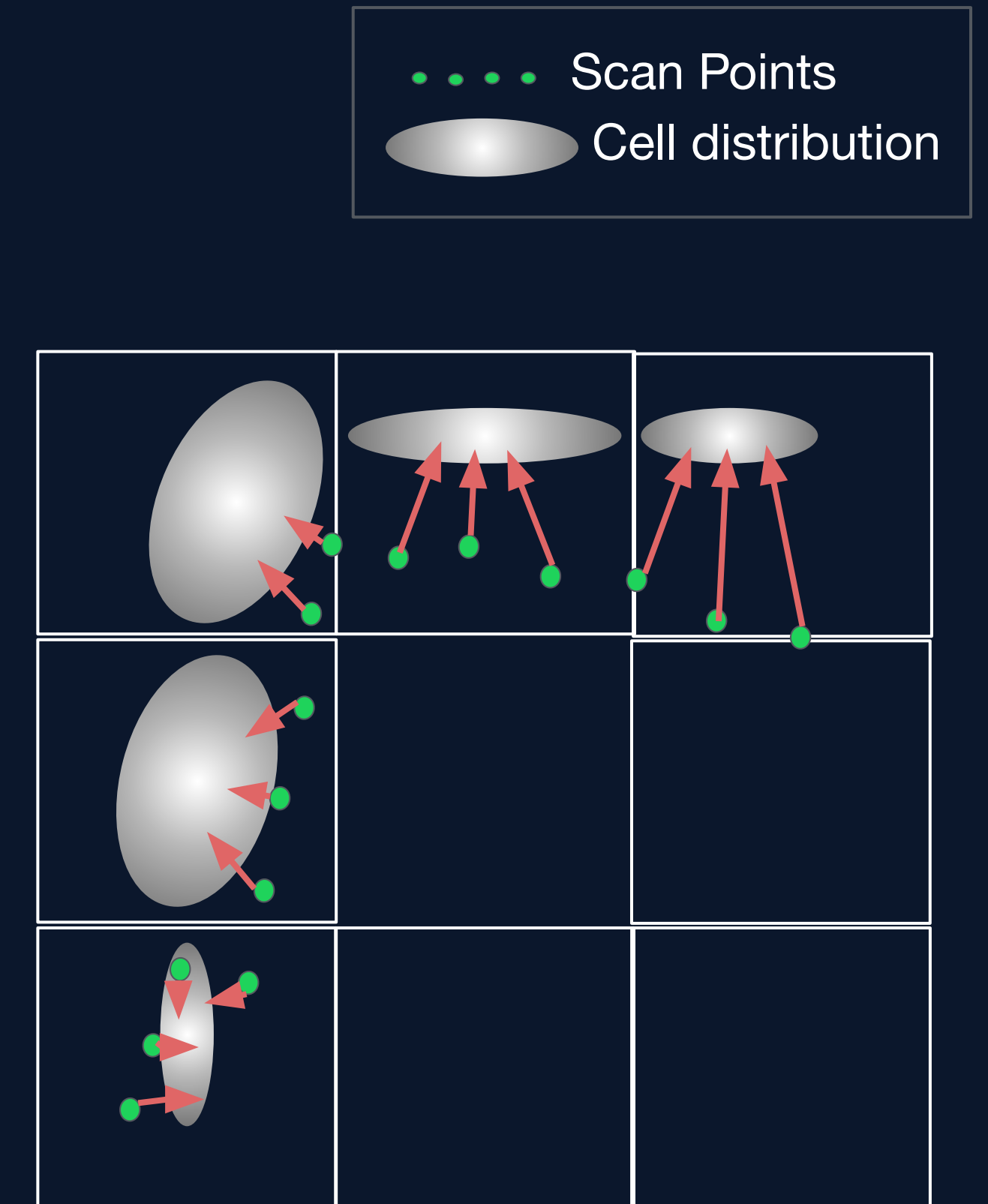
$$\text{score}(p) = - \sum_{i=1}^{N_s} e^{\frac{-1}{2} (T(x_{s_i}, p) - \mu_i)^T \Sigma_i^{-1} (T(x_{s_i}, p) - \mu_i)}$$

Now that we have a minimization problem, we can select an appropriate optimization technique!

Example: Newton's method: $\Delta p = H^{-1} g$

Where H is the hessian and g is the gradient of $\text{score}(p)$

- Either directly use direction Δp using fixed step length.
- Or compute a step length for Δp using a line search algorithm.
 - In autoware.auto, an algorithm proposed in [\[More & thunte, 1994\]](#) is used.



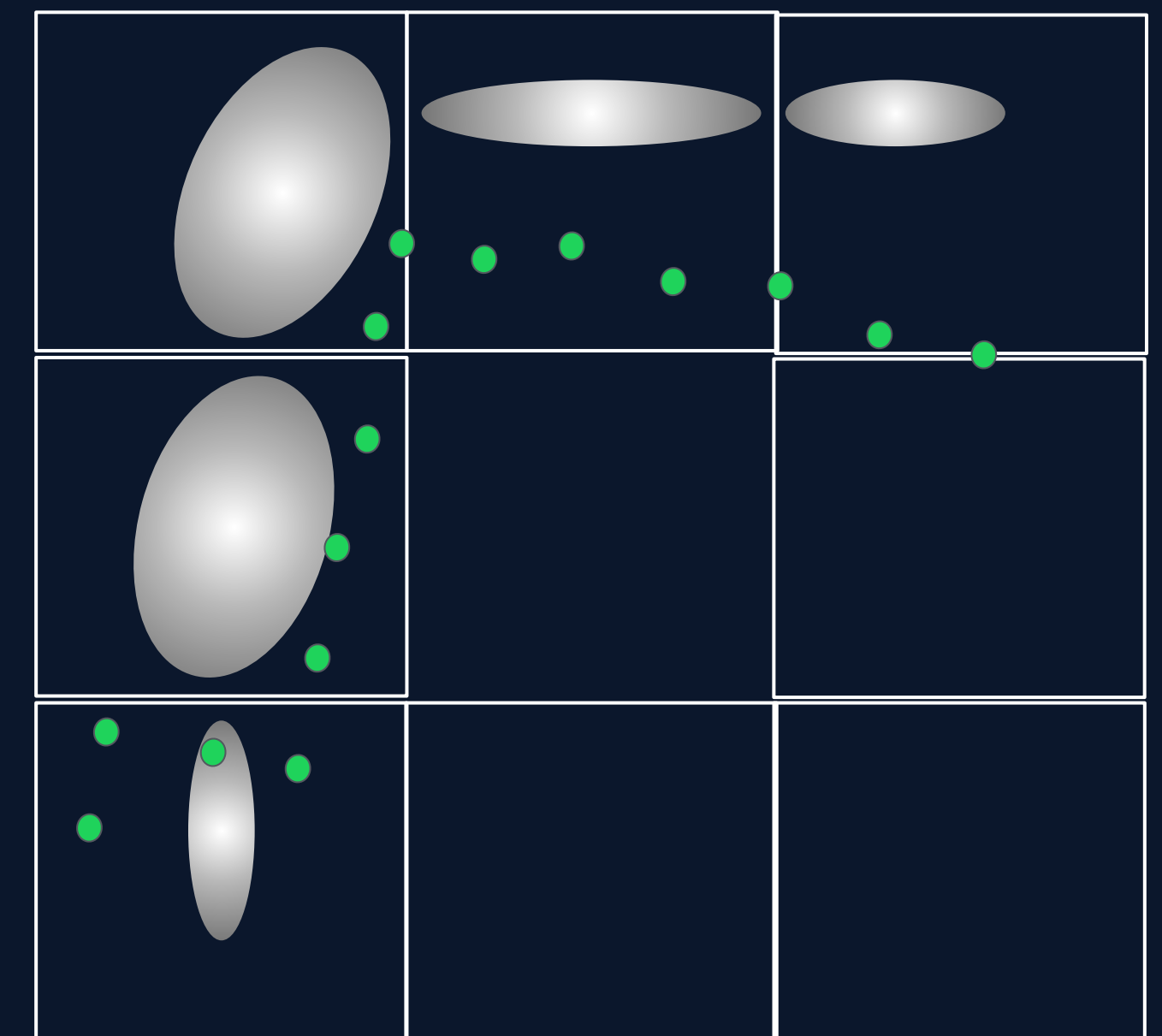
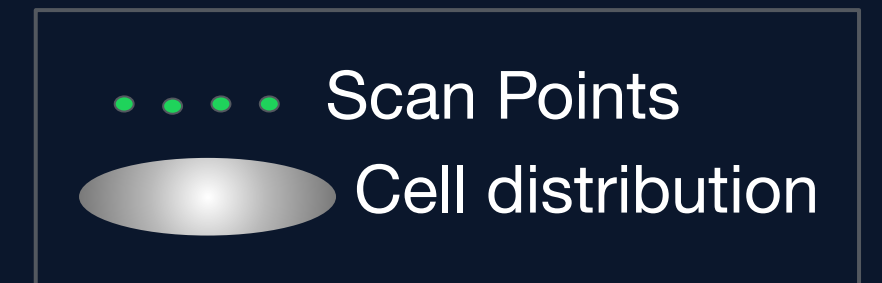
NDT Algorithm [Magnusson, 2009]

Transform the dense point cloud map into an NDT grid

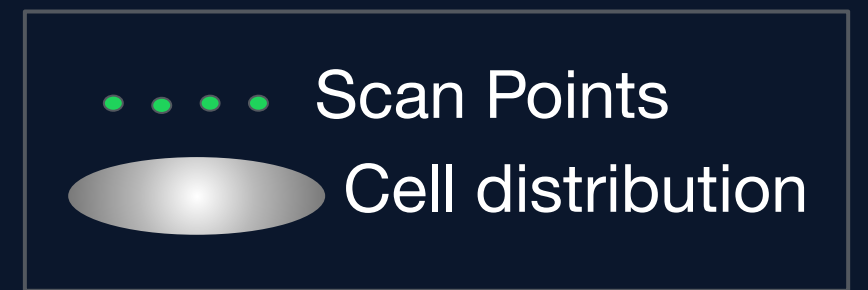
1. Discretize the map into cells.
2. Compute mean and covariance for each cell.

For each received scan:

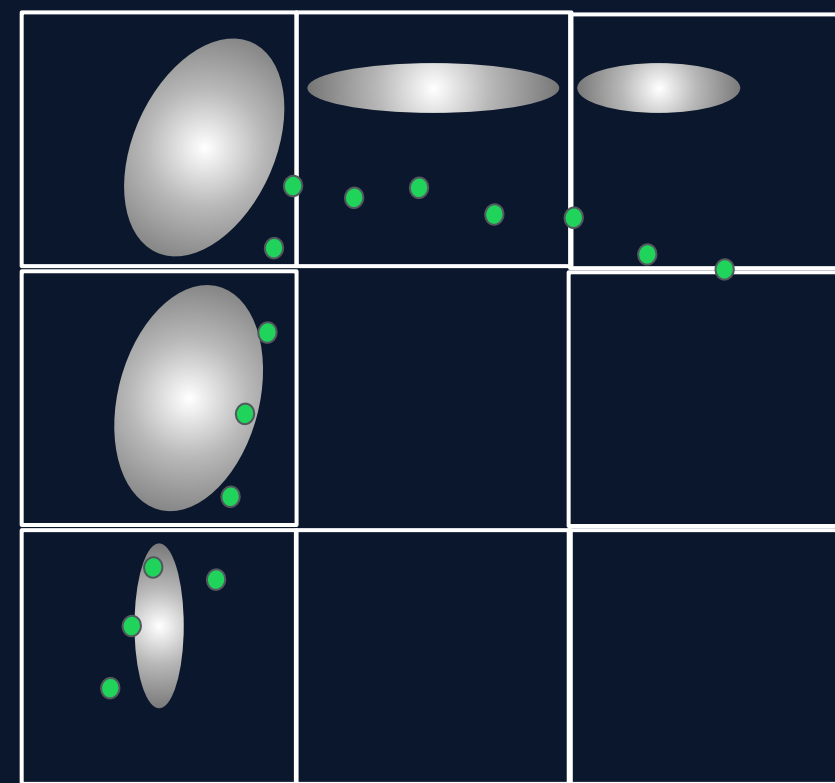
1. Generate an initial pose estimate p to start the optimization.
2. Start optimization:
 - a. Compute $\text{score}(p)$, H and g for p .
 - b. compute Δp using Newton's method.
 - c. Estimate a step length α using line search.
 - d. Update p : $p = p + \alpha \Delta p$
 - e. Repeat until converged or iteration limit is reached
3. Publish the final value of p .



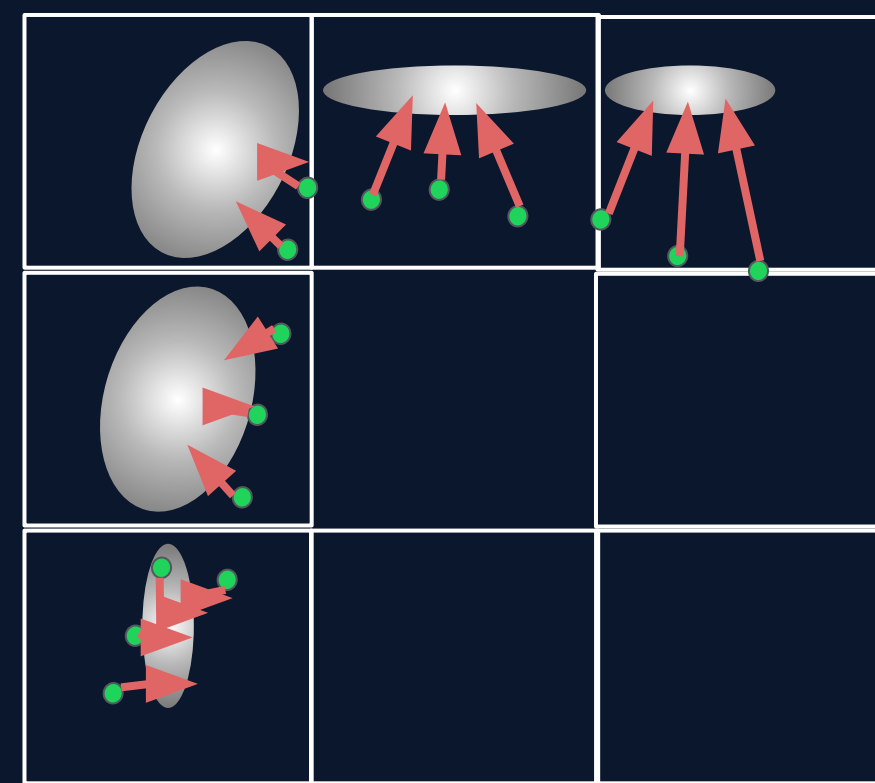
NDT Algorithm



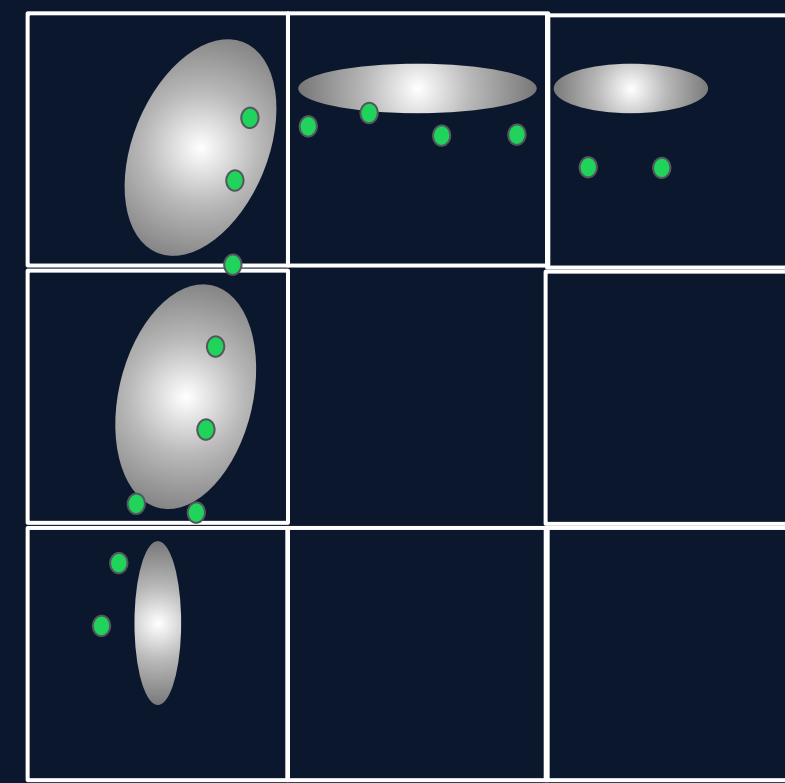
Initial alignment



1. Iteration

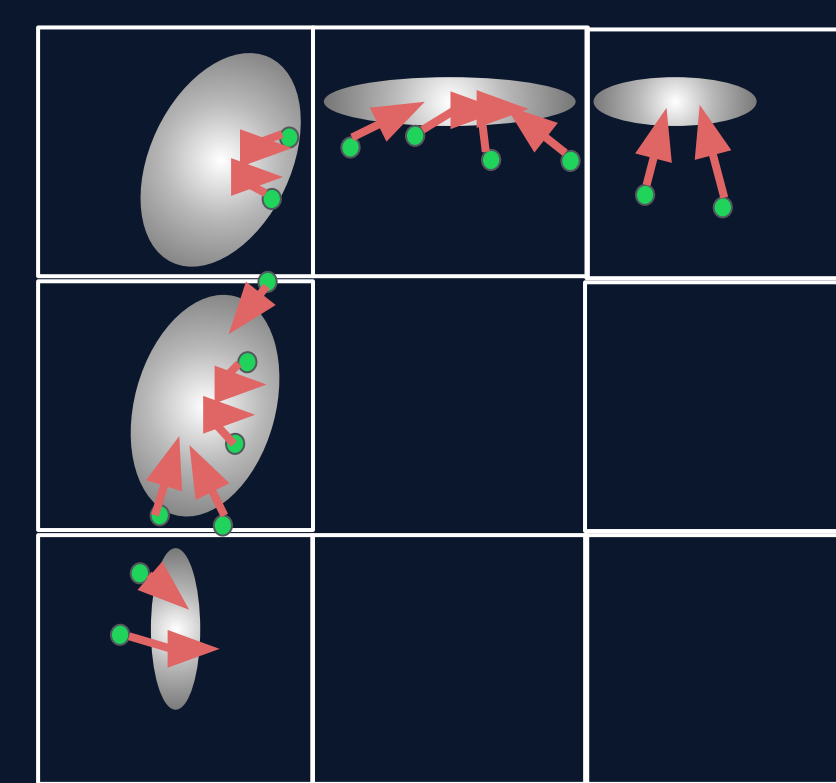


Compute score

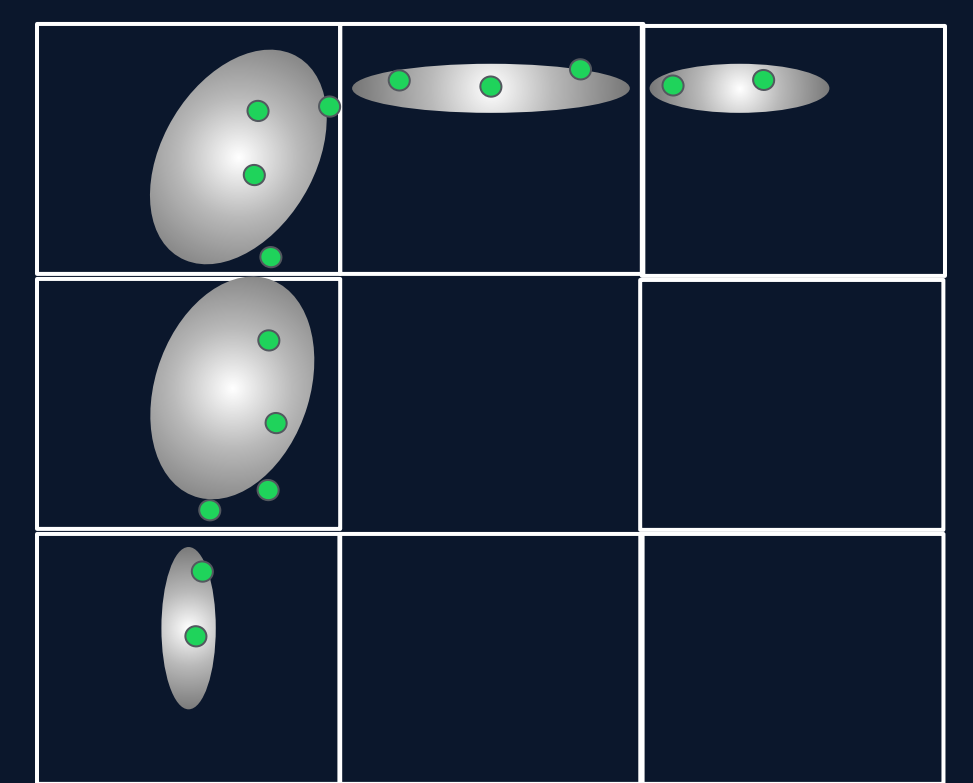


Update alignment

2. Iteration



Compute score



Update alignment

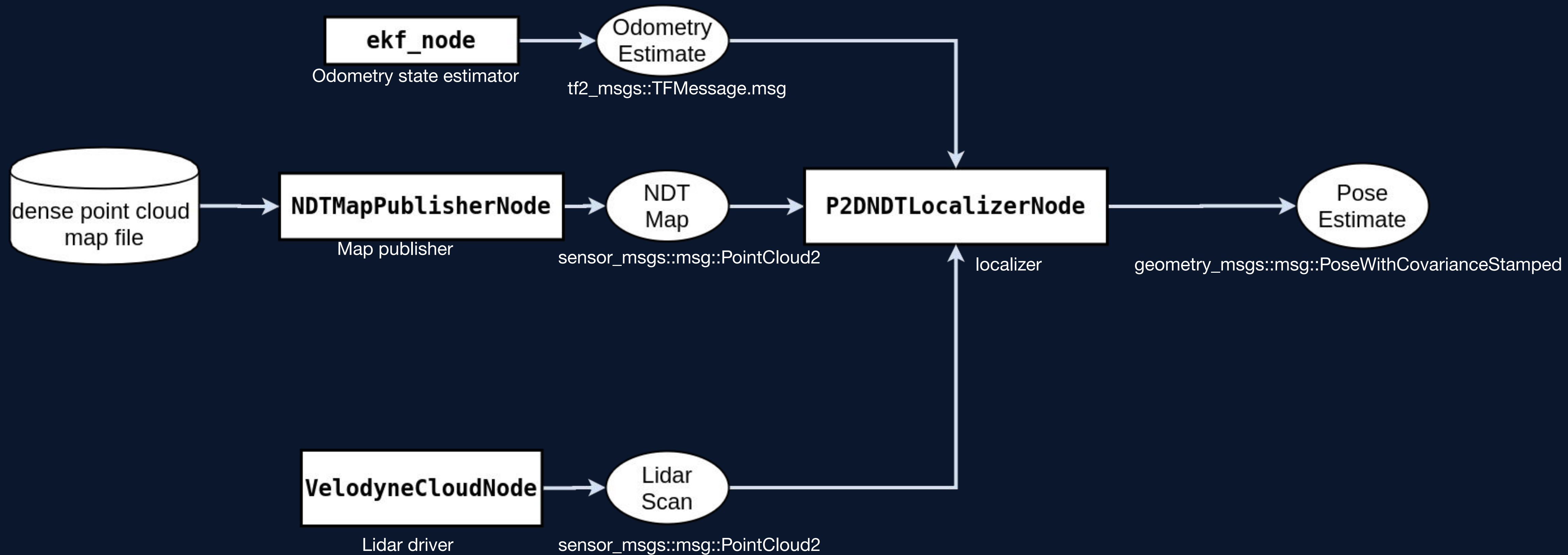
Strengths of NDT localization

- Map has a sparse representation:
 - Lower memory cost
 - Lower computational cost
- No need to engineer a point-correspondence criterion for alignment
- Normal distribution transform addresses the uncertainty in data.
 - More robust to noisy data.
- Variants allow faster computation and/or better accuracy:
 - D2D NDT: [Stoyanov et al., 2012]
 - NDT with constraints on pose: [Andreasson et al., 2017]
 - See more: <https://autowarefoundation.gitlab.io/autoware.auto/AutowareAuto/ndt-literature-review.html>

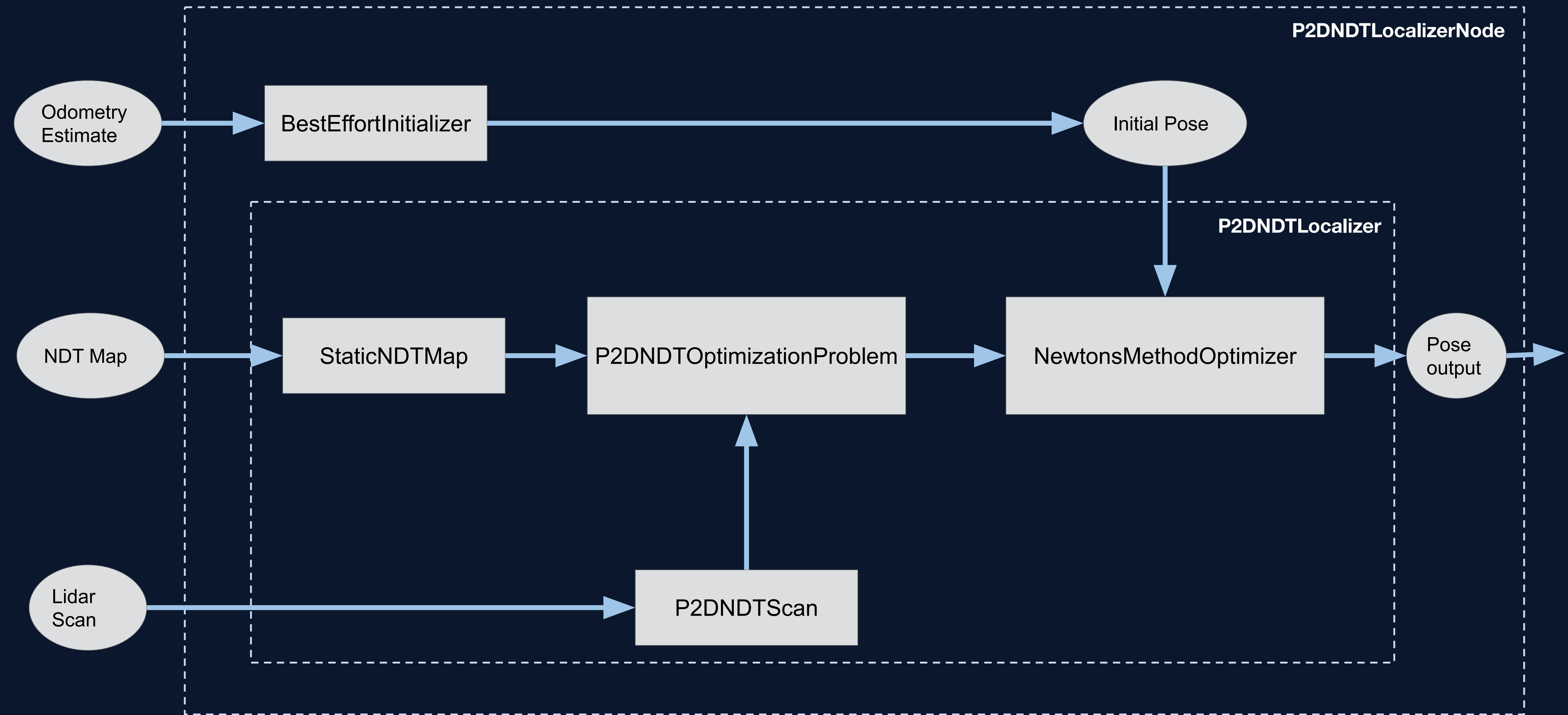


03 Autoware.Auto Implementation

Autoware Auto NDT Localization Architecture



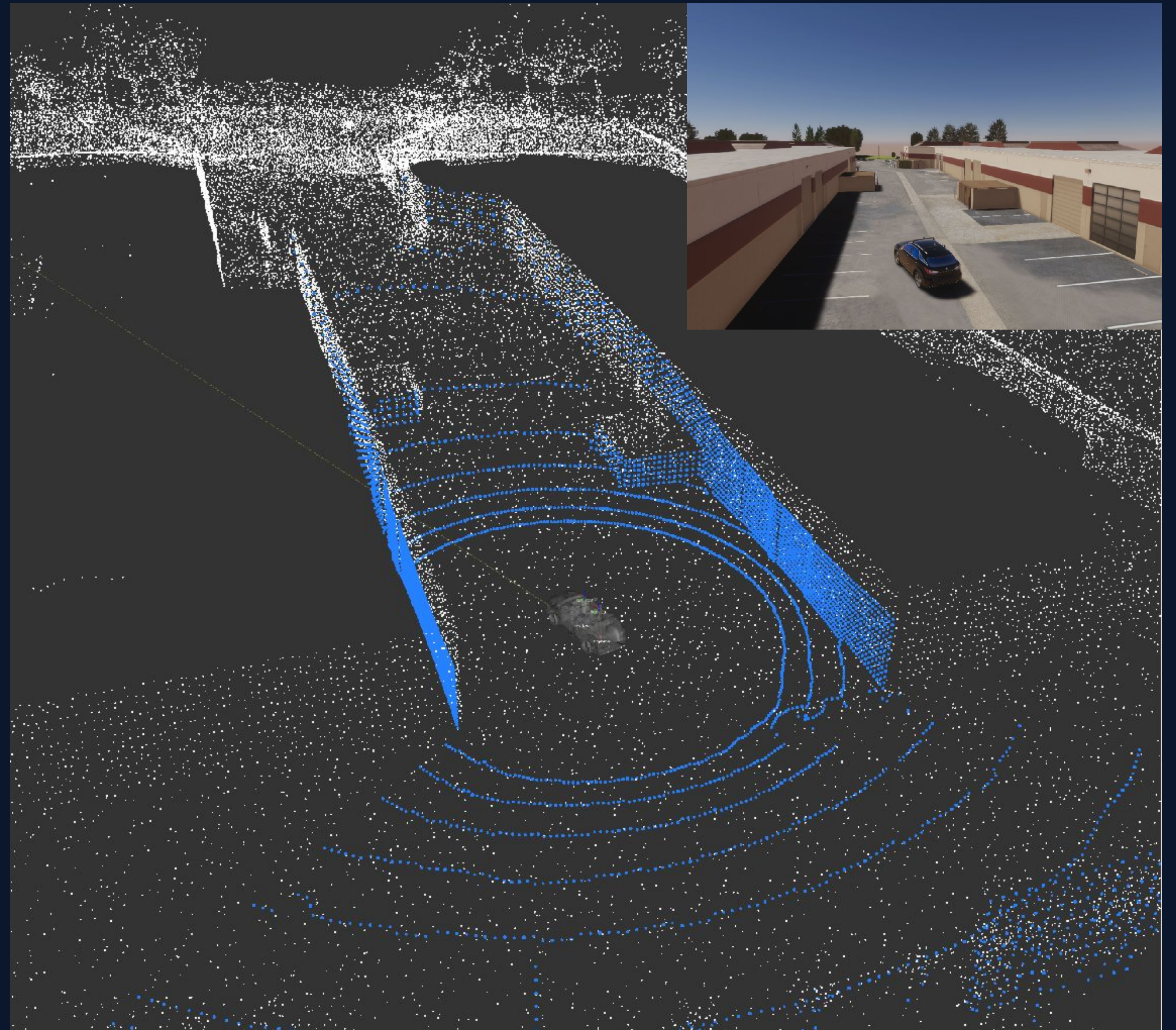
A closer look into the P2DNDTLocalizerNode



NDT Implementation Details

Experimental findings:

- Fixed step-length optimization was not really good at converging in hard alignment problems so `More-thuente` line search algorithm is utilized to improve.
- Cells with co-linear or co-planar points result in singular covariance matrices.
 - Stabilizing covariance matrices before computing the score improves the numerical stability.
 - This stabilization step helped with the convergence characteristics in low information environments with large planar surfaces.



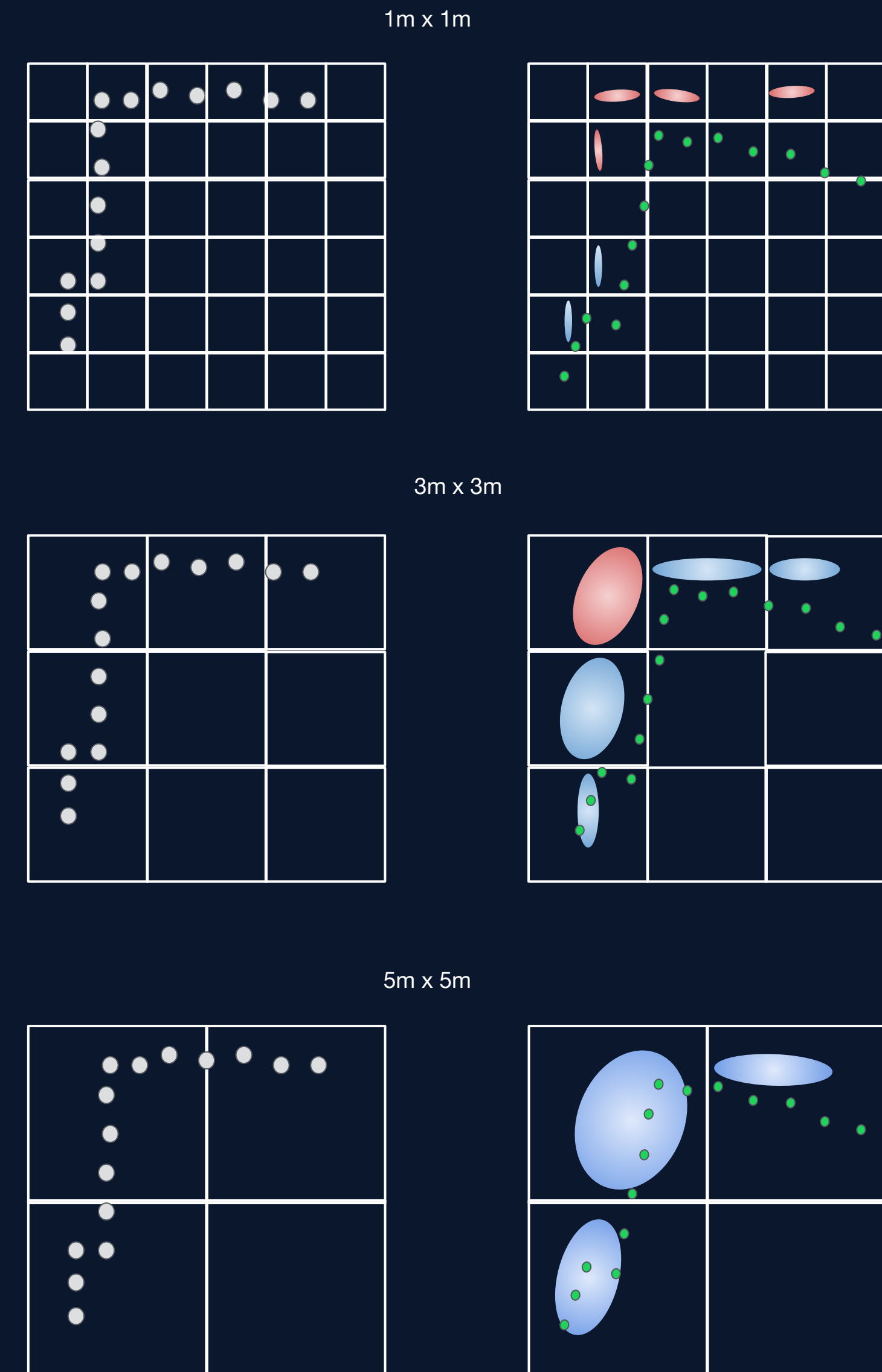
NDT Implementation Details

Effect of parameterization:

- High resolution requires a smaller voxel size.
 - Small voxel size makes the convergence more sensitive on the initial alignment.
 - In higher velocities, the gap between the predicted location (initial guess) and the actual lidar scan may not be close enough to correspond to unoccupied voxels.
- Larger voxels are less sensitive to initial alignment.
 - Lower resolution: less features to match.
- It's hard to find a voxel size that is fit for all scenarios.

Solution:

- Multi-resolution maps
- Octree maps



Live Demo

