ICS51: Intro to Computer Organization
Boolean Algebra Practice Problems

Important Notes:

- These practice problems will provide you a sense of the kinds of questions you may encounter on the quizzes and examinations.
- The real quizzes and exams might include questions on material not covered by these practice problems.
- The questions on the real quizzes and exams might not have the same formats as these practice problems.

1. Expressions F through L are defined in the following way:

   $F = B + AC'$

   $G = AB'C' + BC' + BC + AB$

   $H = A + A'BC + BC'$

   $I = A'B + BC + AC'$

   $J = A'C' + A'B'C$

   $K = A'B' + A'C'$

   $L = A'B'C' + A'C' + B'C$

   Which of the expressions are equivalent (produce the same output for all possible input values)?

2. Use DeMorgan's Law to write an expression for F', the negation of the expression.

   a. $F(x, y, z) = x(y' + z)$

   b. $F(x, y, z) = xy + x'z + yz'$

   c. $F(w, x, y, z) = xyz'(y'z + x)' + (w'yz + x')$

3. Using the postulates of Boolean algebra, prove the following formulas:

   a. $A(x, y) = x'y' + x'y + xy = x' + y$

   b. $B(x, y, z) = y + x'z + xy' = x + y + z$

   c. $C(x, y, z) = x'y + y'z' + xy + y'z = 1$

   d. $D(x, y, z) = x'y'z' + x'y'z + x'yz + xy'z + xyz = x'y' + z$

   e. $E(w, x, y, z) = wxy + w'xy + x'(zw + zy') + z(x'w' + y'x) = xy + z$

   f. $F(x, y, z) = x'y + xyz' + xyz = y$

   g. $G(w, x, y, z) = (xy' + w'z)(wx' + yz') = 0$
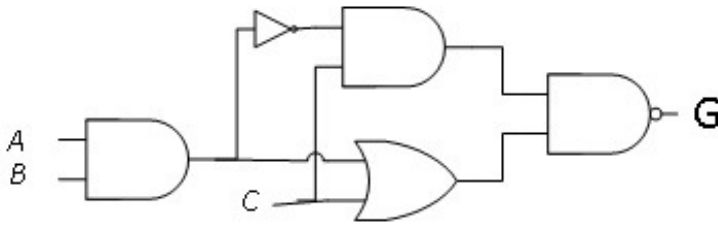
   h. $H(x, y) = (x + y)(x' + y')' = xy$

   i. $I(A, B, C, D) = ABC' + A'C'D + AB'C' + BC'D + A'D = AC' + A'D$

   j. $J(x, y, z) = x'y'z' + x'y'z + x'yz + xy'z + xyz = x'y' + z$

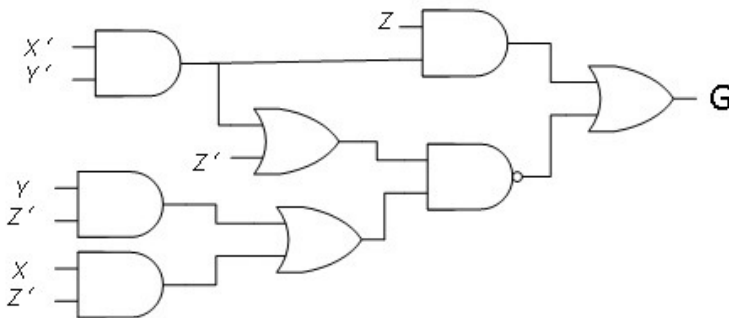   k. $K(X, Y, Z) = X'Y' + Y'Z + XZ + XY + YZ' = X'Y' + XZ + YZ'$

   l. $L(A, B, C, D) = ((A' + B')' + A'B')(C'D' + CD) + (AC)' = A' + C' + BD$

4. Given the following gate network:

    a. Write the expression for G(A,B,C) as shown in the diagram.

    b. Use boolean algebra and De Morgan's law, to simplify the expression.

    c. Redraw the gate network for the simplified expression in part (b) with AND and OR gates.

5. Given the following gate network:

    a. Write the expression for G(X,Y,Z) as shown in the diagram.

    b. Use boolean algebra and De Morgan's law, to simplify the expression.

    c. Redraw the gate network for the simplified expression in part (b) with AND and OR gates.

6. Minimize the following boolean expression, X, using the Boolean Identities. (Hint: final expression has 3 terms and 5 literals)

    a. $X(a, b, c) = a'bc + ((a'b' + b'c)'(a' + b'))'$

    b. Draw the 2-level AND-OR gate network for the simplified expression of Y.

7. Minimize the following boolean expression, Y, using the Boolean Identities. (Hint: final expression has 1 terms and 3 literals)

    a. $Y(x, y, z) = ((x'z')(y' + z'))' + xyz' + x'y'z'$

    b. Draw the 2-level AND-OR gate network for the simplified expression of F.

8. Minimize the following boolean expression, Z, using the Boolean Identities. (Hint: final expression has 2 terms and 3 literals)

    a. $Z(x, y, z) = y'z + x'(zy)' + (x' + y + z)'$

    b. Draw the 2-level AND-OR gate network for the simplified expression of F.

The following questions are from past quizzes. Solutions will not be provided, but you are welcome to discuss them together

9. For each of the following functions:

$$J = ((A' + B)' + C')' + DC' + AB'$$

$$K = x(y + w'z) + (w' + x' + z')'$$

a. Give the boolean expressions for J and K in canonical sum-of-products form (eg. unsimplify the expression until each term contains a literal for each input varaible).

b. Simplify K using boolean logic. Hint: the minimal expression has 2 terms and 4 literals.

c. Write the Boolean expressions for the complement of K (eg. K') in product of sums form.

10. Using the Boolean identities, simplify each of the following formulas

a. $F(x, y, z) = x'yz + x'yz' + xyz + zyz'$ (HINT: 2 terms, 4 literals)

b. $G(w, x, y, z) = w'x'y'z' + w'xy'z' + xx'y'z'$ (HINT: 1 terms, 3 literals)

c. $H(x, y, z) = x'(y'z + yz) + xyz$ (HINT: 2 terms, 4 literals)