E213 : Analysis of Decays of heavy vector boson Z^0

A lab report written by
Group P20: Mrunmoy Jena and Ajay Shanmuga Sakthivasan

Supervisor: Martin Angelsmark

Universität Bonn
May 18, 2022

Contents

Introduction **1**

1 Theoretical Background **2**

1.1 The Standard Model- A Brief Overview . . . 2
1.2 The Unified Theory of Electromagnetic and Weak Interactions . . . 2
1.3 Physics Related to the Z^0 Resonance . . . 3
1.3.1 e^+e^- Interactions . . . 3
1.3.2 Forward-Backward Asymmetry . . . 3
1.3.3 Background Processes: Radiative Corrections . . . 3
1.3.4 Important Parameters of a Resonance Particle . . . 3
1.4 Physics of High Energy Colliders . . . 3

2 Pre-Lab Exercises **4**

2.1 Calculation of partial decay widths for $Z^0 \to f\bar{f}$ . . . 4
2.2 Calculation of hadronic, leptonic and total decay widths and cross section . . . 5
2.3 Effect of additional generation on width of Z^0 resonance curve . . . 6
2.4 Angular distributions of differential cross sections . . . 6
2.5 Calculation of forward-backward asymmetry . . . 7

Introduction

vcnfjdvfdnvk

Theoretical Background

1.1 The Standard Model- A Brief Overview

The Standard Model of particle physics which has been one of the most successful and well-tested theories so far, provides the most fundamental description of nature by incorporating the elementary particles and their interactions. These elementary particles are categorized into two families: fermions (having half integer spins) which form matter as we know it, and bosons (with integer spins) serving as mediators of the three fundamental forces. While electromagnetic interactions are mediated by the photon (γ); strong and weak interactions are mediated by gluons (g) and by $W^{\pm}, Z^0$ bosons respectively. A fundamental particle that mediates gravitation has been only postulated theoretically, and is left out of the Standard Model, since the effects of gravity are too weak to play any important role in the realm of particle physics. The fermions consists of three generations of quarks and leptons. The quarks have six flavours: up (u), down (d), charm (c), strange (s), top (t) and bottom (b). Similarly, the leptons consist of the electron (e), muon (μ) and tau (τ), each having its own associated charge-less and almost massless neutrino (ν_e, ν_μ and ν_τ). Furthermore, each particle in the standard model has its own antiparticle. The quarks are able to form composite particles in either three quark combinations, called baryons ($qqq/\bar{q}\bar{q}\bar{q}$) or a quark-antiquark pair, called a meson ($q\bar{q}$). Mathematically, the elementary particles are described as elements of representations of certain symmetry groups. The gauge fields that couple to these particles (i.e. mediate the interactions) arise naturally as a consequence of invariance of their corresponding Lagrangian under local group transformations [1]. As such, the gauge symmetry that governs the Standard Model is given by:

$$SU(3)_{\text{Colour}} \times SU(2)_{\text{Left chiral}} \times U(1)_{\text{Y(Weak hypercharge)}}$$

1.2 The Unified Theory of Electromagnetic and Weak Interactions

Since the experiment deals with verifying some of the properties of the Z^0 bosons, it is of interest to touch upon the theory of electroweak unification.

While electromagnetism and the theory of weak interactions were formulated separately, it was later on postulated that at higher energies ($\sim$ 246 GeV [2]), both these interactions would be unified into a single force. As such, the GSW(Glashow-Salam-Weinberg) electroweak model was developed in the 1960s to describe this unified force.

One finds that imposing the principle of local gauge invariance on the $SU(2)_L$ symmetry group leads to the introduction of three gauge fields: $W^{(1)}$, $W^{(2)}$ and $W^{(3)}$ (or W^0 in some references) [1]. The physical W^+ and W^- bosons (that mediate the weak charged current interaction) can be seen as the linear combinations:

$$W^{\pm} = \frac{1}{\sqrt{2}} \left(W^{(1)} \mp W^{(2)} \right) \tag{1.1}$$

However, the $W^{(3)}$ field has no physical interpretation of its own. Therefore an additional symmetry, the $U(1)_Y$ group is introduced. The field B (or equivalently Y^0) arising as a consequence of this new symmetry, similarly does not have a physical meaning on its own. Rather, it was seen that linear combinations of the $W^{(3)}$ (W^0) and B (Y^0) fields gives rise to the photon and the Z^0 boson:

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B \\ W^{(3)} \end{pmatrix} \tag{1.2}$$

where θ_W is the weak mixing/Weinberg angle

In addition to this, it is to be noted that the gauge fields $W^{(1),(2),(3)}$ and B have to be massless, in order to respect gauge invariance under local $SU(2)_L \times U(1)_Y$ gauge transformation. However, the physical gauge bosons $W^{\pm}$ and Z^0 are predicted to be massive, whereas the photon should remain massless. To explain this, the concept of electroweak spontaneous symmetry breaking was introduced. A massive scalar field (the Higgs field) is introduced, to which these bosons ($W^{\pm}$, Z^0) must couple to, in order to get their physical masses, while the photon does not interact with it [3]. The intricacies of the Higgs mechanism are not of immediate interest here, and can be understood from standard references [1, 4].

1.3 Physics Related to the Z^0 Resonance

1.3.1 e^+e^- Interactions

1.3.2 Forward-Backward Asymmetry

1.3.3 Background Processes: Radiative Corrections

1.3.4 Important Parameters of a Resonance Particle

1.4 Physics of High Energy Colliders

Pre-Lab Exercises

In this chapter, we have provided our solutions to some theoretical questions that were needed to be solved before conducting the experiment. All the equations and standard values of parameters used to calculate the numerical results for these exercises are taken from [5], unless mentioned otherwise.

2.1 Calculation of partial decay widths for $Z^0 \to f\bar{f}$

In this exercise, we are required to calculate the partial decay widths of $Z^0 \to f\bar{f}$; where $f\bar{f}$ represent the following fermion-antifermion pairs : (i) e^+e^- (ii) $\mu^+\mu^-$ (iii) $\tau^+\tau^-$ (iv) $q\bar{q}$, where q represents all the flavours of quarks (except for t quark, because it is too heavy ($M_t \approx 172.76$ GeV [6]) to be produced from Z^0 decays). The partial decay widths have been calculated with the following formula:

$$\Gamma_f = \frac{N_c^f \sqrt{2}}{12\pi} G_F M_Z^3 \left(\left(g_V^f\right)^2 + \left(g_A^f\right)^2 \right) \tag{2.1}$$

where:

N_c^f: colour factor, (1 for leptons, 3 for quarks)

$G_F = 1.16637 \times 10^{-5} \text{GeV}^{-2}$, Fermi's constant

$M_Z = 91.182$ GeV, mass of Z^0 boson

$g_V^f = I_3^f - 2Q_f \sin^2\theta_W$, vector coupling strength of Z^0 to fermions

$g_A^f = I_3^f$, axial-vector coupling strength of Z^0 to fermions

Q_f: electric charge of fermion f

I_3: third component of weak isospin

$\sin^2\theta_W = 0.2312$, θ_W is the Weinberg (weak-mixing) angle

Fermion	$\mathbf{Q_f}$	$\mathbf{I_3^f}$	$\mathbf{g_V^f}$	$\mathbf{g_A^f}$	$\mathbf{N_c^f}$	$\mathbf{\Gamma_f^{(calc)}}$ **/MeV**	$\mathbf{\Gamma_f^{(ref)}}$ **/MeV**
e^-, μ^-, τ^-	-1	-0.5	-0.0376	-0.5	1	83.89	83.8
u, c	2/3	0.5	0.1917	0.5	3	285.34	299
d, b, s	-1/3	-0.5	-0.3459	-0.5	3	367.84	378
ν_e, ν_μ, ν_τ	0	0.5	0.5	0.5	1	165.85	167.6

Table 2.1: Parameters $Q_f, I_3^f, g_V^f, g_A^f, N_c^f$ for various fermion pairs and their partial decay widths

The calculated partial decay widths for the required fermion pairs have been listed under $\Gamma_f^{(calc)}$ in Table 2.1. Further, the reference [5] values of partial decay widths for the same fermion pairs are listed under $\Gamma_f^{(ref)}$ for comparison. We have also included the partial decay widths of the three neutrinos since they would be used in the solution to the next exercise.

One finds that calculated values of partial decay width for the lepton pairs are in close agreement with the literature value, deviating by about 0.1% to 1%. The slight deviation could be caused because the

$\gamma \to f\bar{f}$ term and interference terms have been neglected. In case of the quarks, the deviations from the reference values are higher ($\sim$ 2.7% to 4.6%). This may be due to the fact that additionally, the effect of strong interactions have not been accounted for in our calculations.

2.2 Calculation of hadronic, leptonic and total decay widths and cross section

Hadronic decay width: The decay widths for the hadronic mode is given by the sum of the partial decay widths of the u,d,c,s and b quarks:

$$\Gamma_{had} = \Gamma_u + \Gamma_c + \Gamma_d + \Gamma_s + \Gamma_b = 2 \cdot \Gamma_{u,c} + 3 \cdot \Gamma_{d,s,b} = 1674.20 \text{ MeV}$$

'Charged' decay width: The charged leptons, e, μ, τ will contribute to this decay width:

$$\Gamma_{charged\ leptonic} = \Gamma_e + \Gamma_\mu + \Gamma_\tau = 3 \cdot \Gamma_{e,\mu,\tau} = 250.17 \text{ MeV}$$

'Neutral' (invisible) decay width: The uncharged leptons ($\nu_e,\ \nu_\mu,\ \nu_\tau$) will contribute to this:

$$\Gamma_{neutral\ leptonic} = \Gamma_{\nu_e} + \Gamma_{\nu_\mu} + \Gamma_{\nu_\tau} = 3 \cdot \Gamma_{\nu_e} = 497.55 \text{ MeV}$$

Total Z^0 decay width: The total decay width for Z^0 will just be the sum of the hadronic, charged leptonic and neutral leptonic decay widths:

$$\Gamma_{total} = \Gamma_{hadronic} + \Gamma_{charged\ leptonic} + \Gamma_{neutral\ leptonic} = 2421.92 \text{ MeV}$$

Partial cross sections at maximum of resonance: At resonance, the formula for calculating partial cross section for $Z^0 \to f\bar{f}$ becomes:

$$\sigma_f^{peak} = \frac{12\pi\Gamma_e\Gamma_f}{M_Z^2\Gamma_Z^2} \tag{2.2}$$

The calculated partial cross sections for the different decay channels along with the respective decay widths are tabulated in Table 2.2.

Decay channel	**Decay width / MeV**	**Partial cross section / 10^{-11} MeV^{-2}**
Hadronic (u,d,c,s,b)	1674.20	10.79
Charged leptonic (e, μ, τ)	250.17	1.61
Neutral leptonic ($\nu_e,\ \nu_\mu, \nu_\tau$)	497.55	3.21
Total	2491.92	15.61

Table 2.2: Calculated decay widths and partial cross sections for different Z^0 decay channels

2.3 Effect of additional generation on width of Z^0 resonance curve

In case it is possible for the Z^0 to decay into an extra generation of light fermions (u,d,e,ν), the total decay width will increase, and the new total decay width will be:

$$\Gamma_{total}^{(new)} = \Gamma_{total} + \Gamma_e + \Gamma_\nu + \Gamma_u + \Gamma_d = 3324.34 \text{ MeV}$$

The percentage increase in the width of the Z^0 resonance curve will be:

$$\frac{\Gamma_{total}^{(new)} - \Gamma_{total}}{\Gamma_{total}} = \frac{902.42}{2421.92} \times 100\% = 37.26\%$$

2.4 Angular distributions of differential cross sections

On studying the dependence of differential cross sections of e^+e^- processes on the azimuthal angle θ, it is found that the s-channel has a symmetrical dependence on θ (with an additional small asymmetric term, in the case of Z^0 mediated process; which will be calculated later):

$$\left(\frac{d\sigma}{d\Omega}\right)_s \propto \left(1 + \cos^2\theta\right) \tag{2.3}$$

For the case of a t-channel process, it is found that the differential cross section diverges quickly at small θ values [5]:

$$\left(\frac{d\sigma}{d\Omega}\right)_t \propto (1 - \cos\theta)^{-2} \tag{2.4}$$

While the process $e^+e^- \to \mu^-\mu^+$ can only take place through the s-channel; the $e^+e^- \to e^+e^-$ process can occur through both s and t channels. The angular distributions of these two processes are shown in Figure 2.1. For the $e^+e^- \to e^+e^-$ process, the contributions from the s and t channels are shown separately as well as their combined contribution to the differential cross section.

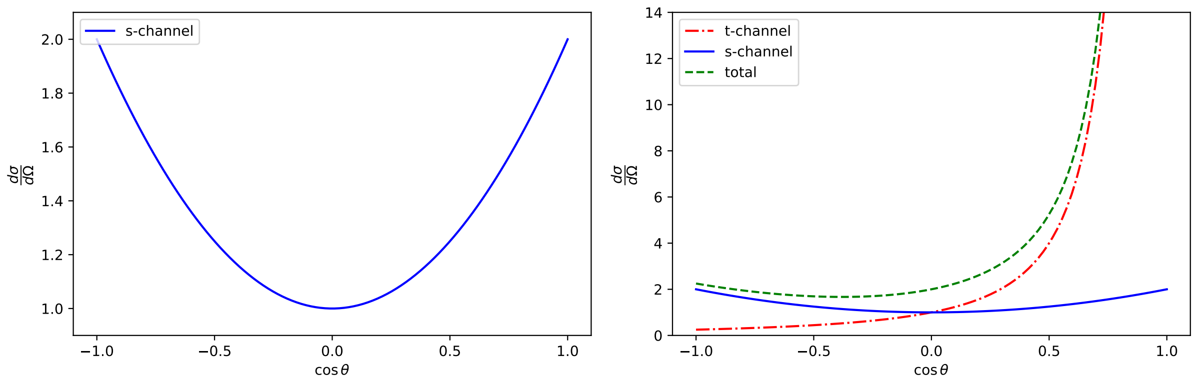

Figure 2.1: Angular distributions of $e^+e^- \to \mu^+\mu^-$ (left) and $e^+e^- \to \mu^+\mu^-$ (right)

2.5 Calculation of forward-backward asymmetry

We are required to calculate the forward-backward asymmetry factor, A_{FB} in the process $e^+e^- \to \mu^+\mu^-$. In order to do this, the following formula is used:

$$A_{FB} = \frac{3F_2}{4F_1} \tag{2.5}$$

where the parameters are given by:

$$F_1(s) = Q_f^2 - 2v_e v_f Q_f \mathfrak{Re}(\chi) + (v_e^2 + a_e^2)(v_f^2 + a_f^2)|\chi|^2$$

$$F_1(s) = -2a_e a_f Q_f \mathfrak{Re}(\chi) + 4v_e a_e v_f a_f |\chi|^2$$

$$v_f = \frac{g_V^f}{2\sin\theta_W \cos\theta_W}$$

$$a_f = \frac{g_A}{2\sin\theta_W \cos\theta_W}$$

$$\chi(s) = \frac{s}{\left((s - M_Z^2) + is\dfrac{\Gamma_Z}{M_Z}\right)}, \text{ Propagator}$$

s: Square of centre-of-mass energy

$\sin^2\theta_W$ \ $\sqrt{s}$ / GeV	89.225	91.225	93.225
0.21	-0.0937	0.0762	0.2317
0.23	-0.1639	0.0228	0.1965
0.25	-0.1948	0.0042	0.1906

Table 2.3: Forward backward asymmetry factors for different Weinberg angles (θ_W) and centre of mass energies ($\sqrt{s}$)

Bibliography

[1] M. Thomson, *Modern particle physics* (Cambridge University Press, 2013).
[2] J. Erler and A. Freitas, *Electroweak Model and Constraints on New Physics*, English, Mar. 2018.
[3] S. K. Dooling, "Differential Cross Section Measurement of Drell-Yan Production and associated Jets with the CMS Experiment at the LHC", OA; Universität Hamburg, Diss., 2015, Dr. (Universität Hamburg, 2015), p. 242.
[4] D. J. Griffiths, *Introduction to elementary particles; 2nd rev. version*, Physics textbook (Wiley, New York, NY, 2008).
[5] Universität Bonn, *Instructions for E213: analysis of Z^0 decay.*
[6] P. Zyla et al. (Particle Data Group), "Review of Particle Physics", PTEP **2020**, and 2021 update, 083C01 (2020).