Ans 5.1. a) $Z_0 \to e^+e^-$

b) " $\mu^+\mu^-$

c) $\tau^+\tau^-$

d) q^+q^-

Lab Ques (E213: Analysis of Decays of Z^0 boson)

$$\Gamma_f = \frac{N_c^f}{12\pi}\sqrt{2}\, G_F\, M_Z^3 \left((g_V^f)^2 + (g_A^f)^2\right)$$

where $g_V^f = I_3^f - 2Q_f \sin^2\theta_W$

$g_A^f = I_3^f$

	Fermions	Q_f	I_3^f	g_V^f	g_A^f	N_c^f	$\Gamma_{f(calc)}$ (MeV)	$\Gamma_{f(obs)}$ (MeV)
a)	e	-1	-1/2	-0.0376	-1/2	1	~~0.823~~	
b)	μ	-1	-1/2	"	-1/2	1	– 83.39	– 83.8
c)	τ	-1	-1/2	"	-1/2	1		
d)	u	+2/3	1/2	0.1917	1/2	3	285.34	299
	d	-1/3	-1/2	-0.3459	-1/2	3	367.84	378
	ν_e	0	1/2	1/2	1/2	1	165.85	167.6

$s = E_{cm}^2$

Ans 5.2 b) $\Gamma_{had} = \underbrace{2\Gamma_u}_{(u+c)} + \underbrace{3\Gamma_d}_{(d,s,b)} = 1674.2 \text{ MeV}$

u c ~~t~~
d s b

c) $\Gamma_{\text{charged lepton}} = 3\Gamma_e = 250.17 \text{ MeV}$

d) $\Gamma_{\text{neutral invisible}} = \Gamma_{\nu_e} + \Gamma_{\nu_\mu} + \Gamma_{\nu_\tau} = 3\Gamma_{\nu_e} = 3 \times 165.85$

$= 497.55 \text{ MeV}$

a) $\Gamma_{tot} = \Gamma_{had} + \Gamma_{\text{charged lepton}} + \Gamma_{\text{neutral invisible}}$

$= 2421.92 \text{ MeV}$

e) $\sigma_f^{peak} = \dfrac{12\pi}{M_Z^2} \dfrac{\Gamma_e \cdot \Gamma_f}{\Gamma_Z^2}$

$\Rightarrow \sigma_{f}^{peak} = \cancel{6.44625 \times 10^{-8} \times \Gamma_f}\ 6.44625 \times 10^{-4} \times \Gamma_f$

$\sigma_{had}^{peak} = 1.079 \times 10^{-10} = 10.79 \times 10^{-11}$

$\sigma_{\text{charged lepton}}^{peak} = 1.612 \times 10^{-11}$

$\sigma_{\text{neutral invisible}}^{peak} = 3.207 \times 10^{-11}$

$\sigma_{tot}^{peak} = 15.612 \times 10^{-11}$

Ans 5.3

If there's another generation :

$\Gamma_{new} = \cancel{3\Gamma_u + 3\Gamma}\ \Gamma_{old} + \Gamma_e + \Gamma_\nu + \Gamma_u + \Gamma_d$

$\Rightarrow \Gamma_{new} - \Gamma_{old} = \Gamma_e + \Gamma_\nu + \Gamma_u + \Gamma_d$

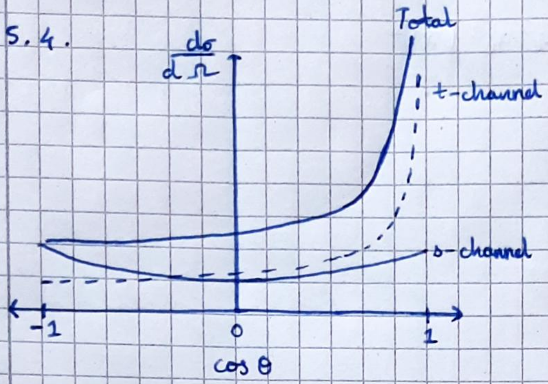
$\% \text{ change} = \dfrac{\Gamma_{new} - \Gamma_{old}}{\Gamma_{old}} \times 100\%$

$= \dfrac{\Gamma_e + \Gamma_\nu + \Gamma_u + \Gamma_d}{\Gamma_{old}} \times 100\%$

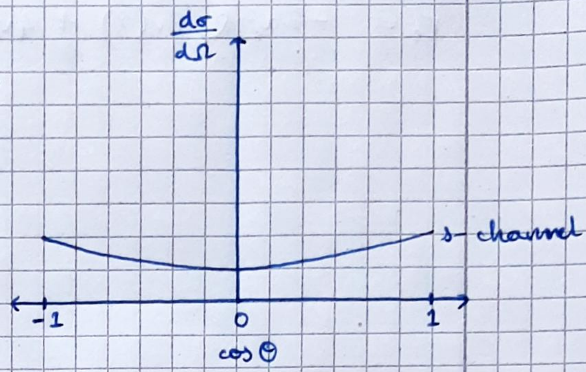
$= \dfrac{902.42}{2421.92} \times 100\%$

$\simeq 37.26\%$

Ans 5.4.



$e^+e^- \to e^+e^-$



$e^+e^- \to \mu^+\mu^-$

For $e^+e^- \to e^+e^-$: both s-channel & t-channel contribute.

$$\left.\frac{d\sigma}{d\Omega}\right|_{s\text{-channel}} \propto (1+\cos^2\theta)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{t\text{-channel}} \propto (1-\cos\theta)^{-2}$$

← Here only s-channel contributes.

Ans 5.5.

GeV / Angle	0.21	0.23	0.25
89.225	-0.0937	-0.1639	-0.1948
91.225	0.0762	0.0228	0.0042
93.225	0.2317	0.1965	0.1906

$$\chi(s) = \frac{s}{(s-M_Z^2)+is\Gamma_Z/M_Z} = \frac{s\{(s-M_Z^2)-is\Gamma_Z/M_Z\}}{(s-M_Z^2)^2+s^2\Gamma_Z^2/M_Z^2}$$

$$\mathrm{Re}\,\chi(s) = \frac{s(s-M_Z^2)}{(s-M_Z^2)^2+s^2\Gamma_Z^2/M_Z^2}$$

$$|\chi(s)|^2 = \chi\chi^* = \frac{s^2\cancel{\{(s-M_Z^2)^2+s^2\Gamma_Z^2/M_Z^2\}}}{\{(s-M_Z^2)^2+s^2\Gamma_Z^2/M_Z^2\}^2}$$

$$F_1 = Q_f^2 - 2v_e v_f Q_f\,\mathrm{Re}(\chi) + (v_e^2+a_e^2)\cdot(v_f^2+a_f^2)\,|\chi|^2$$
$$= Q_e^2 - 2v_e^2 Q_e\,\mathrm{Re}(\chi) + (v_e^2+a_e^2)^2\,|\chi|^2$$

$$F_2 = -2a_e^2 Q_e \,\mathrm{Re}(\chi) + 4v_e^2 a_e^2 |\chi|^2$$