E213 : Analysis of Decays of heavy vector boson Z^0

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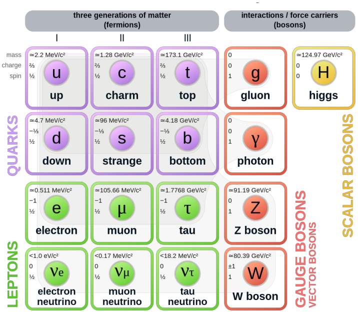
Outline

Introduction

- Goal: to understand how data from a particle accelerator is analysed and to deduce different properties of the Z^0 boson
- Important physical quantities: Z^0 mass and decay width
- Data collected from the OPAL (Omni-Purpose Apparatus at LEP) detector
- Part I: Carried out event display analysis on smaller datasets to understand how to separate out different Z^0 decay channels
- Part II: Cuts (constraints) imposed on the data are refined and statistical analysis done on larger real world data $\rightarrow$ deduce physical quantities

Standard Model

- Standard Model: provides the most fundamental description of nature by incorporating the elementary particles and their interactions
- Two families: fermions (half integer spins), and bosons (integer spins)
  - EM interactions → photon (γ)
  - Strong force → gluons (g)
  - Weak force → $W^{\pm}, Z^0$
  - Gravity → graviton (hypothesized; not included in SM)



Standard Model [1]

[1] *Standard Model*. https://en.wikipedia.org/wiki/Standard_Model.

Standard Model

- Fermions: three generations of quarks and leptons
  - Six flavours of quarks: up (u), down (d), charm (c), strange (s), top (t) and bottom (b)
  - Six flavours of leptons: electron (e), muon (μ) and tau (τ), and associated neutrinos (ν_e, ν_μ and ν_τ)
- Composite particles: three quark combinations, called baryons ($qqq/\bar{q}\bar{q}\bar{q}$) or quark-antiquark pairs, called mesons ($q\bar{q}$)
- Mathematically, elementary particles $\rightarrow$ elements of representations of certain symmetry groups
- Gauge fields coupling to these particles $\rightarrow$ consequence of invariance of corresponding Lagrangian under local phase transformations [2]
- Gauge symmetry that governs the Standard Model is given by:

$$SU(3)_{\mathrm{Colour}} \times SU(2)_{\mathrm{Left\ chiral}} \times U(1)_{\mathrm{Y(Weak\ hypercharge)}}$$

[2] Mark Thomson. *Modern Particle Physics*. Cambridge University Press, 2013. DOI: 10.1017/CBO9781139525367.

Electroweak Theory

- Initially, EM and the theory of weak interactions formulated separately
- At higher energies ($\sim$ 246 GeV [3]), unified into single force $\rightarrow$ GSW electroweak model - 1960s
- Impose local gauge invariance on $SU(2)_L$ symmetry group $\rightarrow$ three gauge fields: $W^{(1)}$, $W^{(2)}$ and $W^{(3)}$
- Physical W^+ and W^- bosons found to be linear combinations:

$$W_\mu^{\pm} = \frac{1}{\sqrt{2}}\left(W_\mu^{(1)} \mp \mathrm{i}\mathrm{W}_\mu^{(2)}\right) \tag{1}$$

[3] J. Erler and A. Freitas. *Electroweak Model and Constraints on New Physics*. English. Mar. 2018. URL: https://pdg.lbl.gov/2019/reviews/rpp2019-rev-standard-model.pdf.

Electroweak Theory

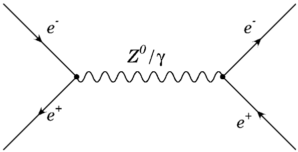
- $W_\mu^{(3)}$ field (no physical interpretation ?)
- Additional symmetry, the $U(1)_Y$ group is introduced
- B_μ field arising from $U(1)_Y$ symmetry (no physical meaning ?)
- Linear combinations of $W_\mu^{(3)}$ and B_μ fields $\rightarrow$ photon and the Z^0 boson:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^{(3)} \end{pmatrix} \tag{2}$$

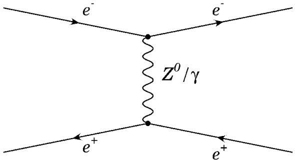
θ_W : weak mixing/Weinberg angle

Angular Dependence of γ/Z^0 Mediated Processes

- $e^+e^- \to e^+e^-$: t-channel as well as s-channel component
- $e^+e^- \to f\bar{f}$ (f other than e^-): only s-channel



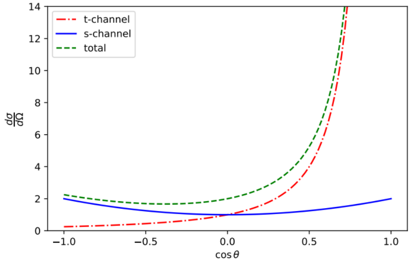
s-channel Bhaba scattering



t-channel Bhaba scattering

Angular Dependence of γ/Z^0 Mediated Processes

- s channel angular dependence: $\left(\frac{d\sigma}{d\Omega}\right)_s \propto (1+\cos^2\theta)$ Cross section has a major contribution at large angles (or small values of $\cos\theta$)
- t channel angular dependence: $\left(\frac{d\sigma}{d\Omega}\right)_t \propto (1-\cos\theta)^{-2}$ Cross section increases asymptotically at small angles (or large values of $\cos\theta$)[4]
- Essential step ! : Remove t-channel contribution while finding inherent forward backward asymmetry in $e^+e^- \rightarrow e^+e^-$ process



s and t-channel angular distribution

[4] Universität Bonn. *Instructions for E213: Analysis of Z^0 decay.*

Forward-Backward Asymmetry

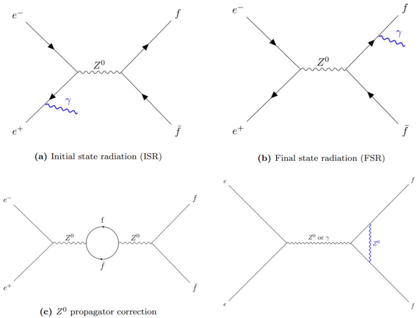
- Consider Z^0 mediated s-channel process $e^+e^- \to f\bar{f}$
- Angular dependence: $\left(\frac{d\sigma}{d\Omega}\right)_{s\ (Z^0)} \propto a(1+\cos^2\theta) + 2b\cos\theta$
- No. of fermions in forward dir., $(\theta > \pi/2) \neq$ no. of fermions in backward dir., $(\theta < \pi/2)$
- Asymmetry term b: Due to unequal coupling of Z^0 to right handed and left handed fermions $b = \left[(g_L^e)^2 - (g_R^e)^2\right]\left[\left(g_L^f\right)^2 - \left(g_R^f\right)^2\right]$
  - g_L^f: coupling of Z^0 to left handed fermions
  - g_R^f: coupling of Z^0 to right handed fermions

Forward-Backward Asymmetry

- FB asymmetry factor given as the ratio: $\mathcal{A}_{fb} = \dfrac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \dfrac{3b}{4a}$
  - σ_F, σ_B: cross sections in forward and backward directions respectively
- At Z^0 resonance, $\mathcal{A}_{fb}$ simplifies to [5]: $\mathcal{A}^f_{fb} \approx 3\left(\dfrac{g^f_V}{g^f_A}\right) = 1 - 4\sin^2\theta_W$
- From this, ratio of g^f_V to g^f_A can be found
- In turn gives us the Weinberg (weak mixing) angle θ_W

[5] Thomson, *Modern Particle Physics*.

Background Processes: Radiative Corrections



(a) Initial state radiation (ISR)

(b) Final state radiation (FSR)

(c) Z^0 propagator correction

(d) Z^0 vertex correction

Some radiative corrections

To test out predictions of the Standard Model at high precision, need to account for higher order processes [6]:

- **ISR:** radiation of photons in the initial state $\rightarrow$ decreases $\sqrt{s}$ and affects Z^0 peak parameters
- **FSR:** radiation of photons or gluons in final state $\rightarrow$ partial widths increase
- **Electroweak corrections:** Virtual processes like loops in Z^0 propagator and vertex corrections

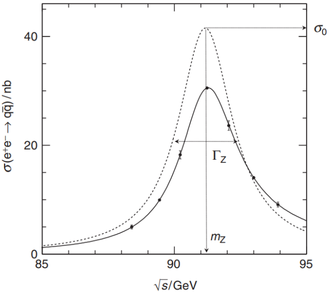
[6] G. Abbiendi et al. and "The OPAL" Collaboration. "Precise determination of the Z resonance parameters at LEP: "Zedometry"". In: *The European Physical Journal C - Particles and Fields* 19.4 (Mar. 2001), pp. 587–651. ISSN: 1434-6052. DOI: 10.1007/s100520100627. URL: https://doi.org/10.1007/s100520100627.

Breit Wigner Distribution

- Contribution of Z^0 boson exchange propagator to the matrix element:
$$\mathcal{M}_{Z^0} \propto \frac{g_{Z^0}^2}{q^2 - m_{Z^0}^2} = \frac{g_{Z^0}^2}{s - m_{Z^0}^2}$$
- Around the Z^0 resonance ($\sqrt{s} \sim m_{Z^0}$), propagator diverges
- Correction $\rightarrow$ modify propagator for a decaying state
- For unstable particle having decay rate Γ, wavefunction modified to:
$\psi \propto e^{-imt} \rightarrow e^{-imt} e^{-\Gamma t/2}$
- Equivalent to introducing an additional imaginary term in the mass:
$m \rightarrow m - i\frac{\Gamma}{2}$
- Z^0 propagator then changes to: $\dfrac{1}{s - m_{Z^0}^2} \rightarrow \dfrac{1}{s - (m_{Z^0} - i\Gamma_{Z^0}/2)^2}$

Breit Wigner Distribution

- Complete form of cross section in process $e^+e^- \rightarrow Z^0 \rightarrow f\bar{f}$ is: $\sigma_f(s) = \frac{12\pi}{M_Z^2} \frac{s\Gamma_e\Gamma_f}{(s-M_Z^2)^2+\left(\frac{s\Gamma_Z}{M_Z}\right)^2}(\hbar^2c^2)$
- **Breit-Wigner distribution:** Probability distribution that characterizes this dependence of cross section on centre of mass energy
- Various physical parameters can be extracted by fitting this theoretical distribution to the observed data



Breit Wigner distribution of the cross section for $e^+e^- \rightarrow q\bar{q}$ process [7]

[7] Thomson, *Modern Particle Physics*.

The LEP Experiment

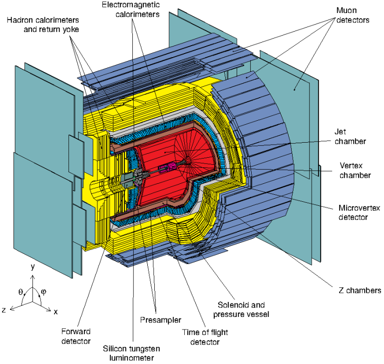
- LEP built at CERN; started operating in 1989
- One of the major goals: make high precision measurements of Z^0 boson properties
- Produced e^+e^- collisions at $\sqrt{s}$ close to Z^0 resonance
- Recorded about 17 million $e^+e^- \to Z^0$ events (1989-95) [8]
- Collisions at four different points in the circular collider $\rightarrow$ four detectors:
    - ALEPH (Apparatus for LEP PHysics)
    - DELPHI (DEtector with Lepton, Photon and Hadron Identification)
    - L3 (Third LEP experiment)
    - OPAL (Omni-Purpose Apparatus for LEP)

[8] Thomson, *Modern Particle Physics*.

OPAL Detector and its Components

Starting from origin, as e^+e^- collision products fly outwards, various detector components are encountered. These are discussed here in brief:

- **Vertex detector:**
  - Surrounds the central beam pipe
  - Key role: locating vertices of short-lived decay products
  - Improves momentum resolution
- **Jet chamber:**
  - Has good spatial and track resolution
  - Records events associated with jets
  - Also determines dE/dx of charged particles



Cross sectional view of the OPAL detector [9]

[9] *Das OPAL Experiment*. https://www-static.etp.physik.uni-muenchen.de/fp-versuch/node7.html.

OPAL Detector and its Components

- **z chambers:**
  - Locates z coordinates of decay particles
  - Helpful in improving the resolutions of the polar angle
- **Solenoid:**
  - Surrounds central detector
  - Generates a uniform magnetic field of 0.435 T along beam dirn.
  - $\vec{B}$ field $\rightarrow$ charged particles have helical path $\rightarrow$ momenta can be measured
- **ECAL:**
  - $\gamma,\ e^-$ and $e^+ \xrightarrow[\text{Pair production}]{\text{Bremsstrahlung}}$ deposit all energy in ECAL
  - Hadrons $\rightarrow$ loose some energy (don't stop completely)
  - Lead glass used (high Z; entire EM shower contained in small region of space)

OPAL Detector and its Components

- **HCAL:**
  - Just like ECAL; detects mesons and baryons
  - Hadronic energy loss : EM shower + strong interaction with nuclei
  - Numerous decay products
  - HCAL occupies more detector volume (significant distance between two consecutive nuclear interactions)
  - Decays more complicated $\rightarrow$ large uncertainty in determining energy loss $\rightarrow$ worse energy resolution compared to ECAL
- **Muon detector:**
  - Only remaining particles: muons
  - Four layers of muon detectors outside HCAL
  - Provides a coverage of almost the entire solid angle of 4π
  - Has a barrel region and two endcap regions

Analysis

- Analysis is divided into two parts - Analysis of event displays and Statistical analysis of Z^0 decays.
- The main objective of the first part is to understand how different channels affect different observed quantities.
- We use this knowledge to come up with cut criteria - a filter to separate out different channels.
- The main object of the second part is to use the knowledge from first part and use it to analyse large sets of real world data.

Analysis of Event Displays

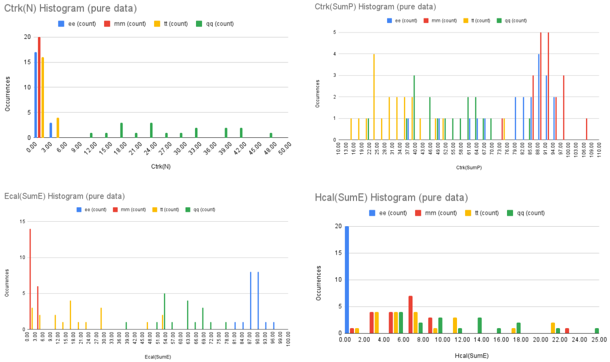
- We analyse event displays of pure events - events of specific channel.
- Every event display contains different parameters - number of charged tracks, momentum of all charged tracks, total energy in the EM calorimeter and the total energy in the hadronic calorimeter.
- We then plot histograms of different parameteres try to come up with a cut criteria.
- Since we already know the behaviour of our detectors to different kinds of particles, we have a rough idea of what to expect.

- $e^- e^+ -$channel: We expect two charged tracks, SumP will have the value of centre of mass energy and all the energy is expected to be deposited in the EM calorimeter and no signal is expected in the hadronic calorimeter.
- $\mu^- \mu^+$-channel: Again, two charged tracks, SumP will have the value of centre of mass energy, but no energy depsoited in the calorimeters. Instead, signals in the muon chamber.
- $\tau^- \tau^+$-channel: Short life of τ results in interesting outcomes. We expect to see a large number of charged tracks due to the decay of τ. The dominant decay products are μ and π, which can then be detected. This channel can be classified by the number of charge tracks, called prongs.
- $q\bar{q}$-channel: Since quarks instantly hadronise, we again have interesting outcomes. The produced hadrons decay further and can be detected. This results in the so-called *jets*, which are much bigger clusters of charged tracks. SumP value is expected to be less than the centre of mass energy, since neutrinos are produced which then carry away some of the energy.

- We plot histograms of all the quantities to develop cuts.
- The bin sizes or chosen so that we are able to see the best possible separation of the different channels.
- These cuts are tested on the mixed samples. The mixed samples were classified and checked visually if it matched the channel as determined by the cuts.

Channel	Ctrk(N)	Ctrk(Sump)	Ecal(SumE)	Hcal(SumE)
$e^- e^+$	(0,6)		$>=80$	<1
$\mu^- \mu^+$	<6	$>=75$	$<=10$	
$\tau^- \tau^+$	<6	$<=75$	$<=60$	
$q\bar{q}$	$>=8$		[36,79]	

Table: Cuts determined from event display analysis



Histograms of *Ctrk(N)*, *Ctrk(Sump)*, *Ecal(SumE)* and *Hcal(SumE)* for the four channels, generated from *pure* samples.

Statistical Analysis of Z^0 Decays

- A simple analysis of event displays is not possible when we have a large number of events. We resort to statistical analysis methods. We use the software, *ROOT* for this purpose.
- *ROOT* works with *.root* files, which contain all the information in a tree-like structure, called *ntuple*.
- The *ntuple* contains various parameters - run number, event number, number of charged tracks, total scalar sum of track momenta, total energy in the EM calorimeter, total energy in the hadronic calorimeter, LEP beam energy, polar angle beween beam axis and thrust axis, polar angle between incoming positron and outgoing positive particle.
- We first use Monte Carlo events to refine our cuts. Then we work with the actual data to calculate different physical quantities.

Refining the cuts

- For the $e^- e^+$-channel, we would like to exclude t-channel events. This is because t-channel is possible only in this mode and we omit this for the sake of consistency.
- This is achieved by limiting ourselves to smaller angles, since t-channel dominates at large angles. Therefore we take cos$\theta < 0.5$.
- But this means that we are excluding some of the s-channel events in this region. The correction factor to account for this, 1.5829 in our case, is taken from theory.
- For the $\mu^- \mu^+$-channel, we observed a lot of events with scalar momenta exactly equal to zero. These are not physical and we ignore them by modifying our cuts.
- For the above two channels, we also modified our cuts to ignore angles very close to the beam axis, since the resolution of our detector is far from perfect in that region.
- The other two channels did not require any additional cut, but we did need cosθ to take only the values between -1 and 1 to exclude unphysical events.

Efficiency matrix

- We now begin our ultimate analysis. We used *data6.root* in our analysis.
- Our cuts are not perfect. Ideally, the cuts should completely separate out every channel perfectly, but this is not the case.
- To account for this, we use efficiency matrix. Efficiency matrix, as the name suggests, is a measure of how efficient our cuts are.
- It is given by,

$$\epsilon_{ij} = \frac{N_j^{i,cut}}{N_j^{j,all}}. \tag{3}$$

- This translates to i-th cut applied to j-th channel divided by the total number of j-th channel events. This then gives the actual counts as,

$$N_{obs} = \epsilon N_{actual} \implies N_{actual} = \epsilon^{-1} N_{obs}. \tag{4}$$

- To summarise, we start with the observed counts with our cuts, understand that this is not the true counts and we use the efficiency matrix to calculate the true counts.
- The efficiency matrix can be found in the report. The true counts are used for all our calculations.

Cross Sections

- With the actual counts, we can calculate the cross section values.
- This is given by,

$$\sigma = \frac{N_{actual}}{\int \mathcal{L} dt} + \text{cf(radiation)}. \quad (5)$$

- The term $\int \mathcal{L} dt$ is the integrated luminosity and cf(radiation) is the radiative correction factor, the values for which are taken from [10].
- We get the following cross section values,

$\sqrt{s}$ [GeV]	σ_{ee} [nb]	σ_{mm} [nb]	σ_{tt} [nb]	σ_{qq} [nb]
88.47	0.39 ± 0.03	0.30 ± 0.02	0.47 ± 0.03	7.19 ± 0.10
89.46	0.82 ± 0.04	0.65 ± 0.03	0.72 ± 0.03	14.20 ± 0.14
90.22	1.26 ± 0.04	1.16 ± 0.03	1.12 ± 0.03	25.70 ± 0.21
91.22	1.69 ± 0.02	1.82 ± 0.02	1.75 ± 0.02	40.75 ± 0.22
91.97	1.12 ± 0.05	1.32 ± 0.04	1.21 ± 0.04	28.96 ± 0.27
92.96	0.45 ± 0.04	0.55 ± 0.03	0.66 ± 0.04	13.67 ± 0.20
93.71	0.28 ± 0.03	0.34 ± 0.02	0.40 ± 0.03	8.20 ± 0.13

Table: Calculated cross section values for different $\sqrt{s}$ values.

[10] Universität Bonn, *Instructions for E213: Analysis of Z^0 decay.*

Forward Backward Asymmetry and Weak Mixing Angle

- As discussed in the theory section, Z^0 couples differently to left- and right-handed fermions, which leads to an asymmetry in the angular distribution of the final state particles.
- This asymmetry depends on the weak mixing angle. Therefore, by calculating this simple asymmetry we can calculate the weak mixing angle.
- We perform this calculation for the $\mu^-\mu^+$ channel, since we do not have the issue of t-channel and we also get two clear tracks.
- The forward backward asymmetry is given by,

$$A_{fb} = \frac{N_+ - N_-}{N_+ + N_-} + \text{cf}(A_{fb}). \tag{6}$$

- cf(A_{fb}) are the correction factors corresponding to forward bacward asymmetry, the values for which are taken from [11].

[11] Universität Bonn, *Instructions for E213: Analysis of Z^0 decay.*

- A_{fb} is then used to calculate the weak mixing angle. We get $sin^2\theta_W = 0.2369 \pm 0.0085$ for the data event at resonance energy and $sin^2\theta_W = 0.2352 \pm 0.0098$ for the MC event.
- The literature value is $sin^2\theta_W = 0.2352 \pm 0.0098$ [12], which is withing one standard deviation of both our results.

[12] M. Tanabashi et al. "Review of Particle Physics". In: *Physical Review D* 98.3 (Aug. 2018). DOI: 10.1103/physrevd.98.030001. URL: https://doi.org/10.1103/physrevd.98.030001.

Lepton Universality

- Since the masses of leptons are much smaller compared to that of the Z^0, the cross sections of all the three leptonic decay modes must be the same at resonance.
- We see that, at resonance, the cross sections are given by,

$$\begin{aligned} \sigma_e &= 1.69 \pm 0.02\text{nb} \\ \sigma_\mu &= 1.82 \pm 0.02\text{nb} \\ \sigma_\tau &= 1.75 \pm 0.02\text{nb} \end{aligned} \tag{7}$$

- The theoretical value is $\sigma_f = 1.99$nb. The values we have calculated are several standard deviations away, unfortunately. But they are roughly the same and match up to the order.
- We postulate that this is because we don't have the best possible cut criteria. We are undercounting the event numbers to some extent and since this is a very sensity quatity, it gets affected significantly.
- We suggest that we increase the number of events significantly, to improve statistical accuracy.

- The ratios of hadronic to the leptonic modes follow a similar discrepancy,

$$\begin{aligned}\frac{\sigma_{had}}{\sigma_e} &= 24.18 \pm 0.30 \\ \frac{\sigma_{had}}{\sigma_\mu} &= 22.43 \pm 0.24 \\ \frac{\sigma_{had}}{\sigma_\tau} &= 23.28 \pm 0.26\end{aligned} \tag{8}$$

- The literature value is 20.804 ± 0.050, 20.785 ± 0.033 and 20.764 ± 0.045 respectively [13].

[13] al., "Review of Particle Physics".

References

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