

E213 : Analysis of Decays of heavy vector boson Z^0

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1 Introduction

2 Prerequisite Knowledge

- Standard Model
- Electroweak Theory
- Physics Related to the Z^0 Resonance
 - Angular Dependence of γ/Z^0 Mediated Processes
 - Forward-Backward Asymmetry
 - Background Processes: Radiative Corrections
 - Breit Wigner Distribution
 - LEP Experiment and OPAL Detector

3 Analysis

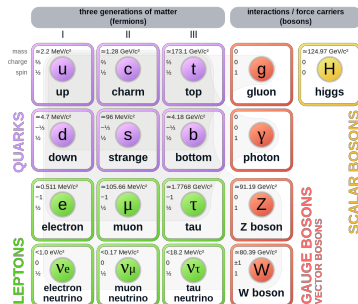
- Analysis of Event Displays
- Statistical Analysis of Z^0 Decays

Introduction

- Goal: to understand how data from a particle accelerator is analysed and to deduce different properties of the Z^0 boson
- Important physical quantities: Z^0 mass and decay width
- Data collected from the OPAL (Omni-Purpose Apparatus at LEP) detector
- Part I: Carried out event display analysis on smaller datasets to understand how to separate out different Z^0 decay channels
- Part II: Cuts (constraints) imposed on the data are refined and statistical analysis done on larger real world data → deduce physical quantities

Standard Model

- Standard Model: provides the most fundamental description of nature by incorporating the elementary particles and their interactions
- Two families: fermions (half integer spins), and bosons (integer spins)
 - EM interactions \rightarrow photon (γ)
 - Strong force \rightarrow gluons (g)
 - Weak force $\rightarrow W^{\pm}, Z^0$
 - Gravity \rightarrow graviton (hypothesized; not included in SM)



Standard Model ¹

¹Standard Model. https://en.wikipedia.org/wiki/Standard_Model.

Standard Model

- Fermions: three generations of quarks and leptons
 - Six flavours of quarks: up (u), down (d), charm (c), strange (s), top (t) and bottom (b)
 - Six flavours of leptons: electron (e), muon (μ) and tau (τ), and associated neutrinos (ν_e , ν_μ and ν_τ)
- Composite particles: three quark combinations, called baryons ($qqq/\bar{q}\bar{q}\bar{q}$) or quark-antiquark pairs, called mesons ($q\bar{q}$)
- Mathematically, elementary particles \rightarrow elements of representations of certain symmetry groups
- Gauge fields coupling to these particles \rightarrow consequence of invariance of corresponding Lagrangian under local phase transformations ²
- Gauge symmetry that governs the Standard Model is given by:

$$SU(3)_{\text{Colour}} \times SU(2)_{\text{Left chiral}} \times U(1)_{\text{Y(Weak hypercharge)}}$$

²Mark Thomson. *Modern Particle Physics*. Cambridge University Press, 2013. DOI: 10.1017/CB09781139525367.

Electroweak Theory

- Initially, EM and the theory of weak interactions formulated separately
- At higher energies ($\sim 246 \text{ GeV}$ ³), unified into single force \rightarrow GSW electroweak model - 1960s
- Impose local gauge invariance on $SU(2)_L$ symmetry group \rightarrow three gauge fields: $W^{(1)}$, $W^{(2)}$ and $W^{(3)}$
- Physical W^+ and W^- bosons found to be linear combinations:

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} \left(W_{\mu}^{(1)} \mp i W_{\mu}^{(2)} \right) \quad (1)$$

³J. Erler and A. Freitas. *Electroweak Model and Constraints on New Physics*. English. Mar. 2018. URL: <https://pdg.lbl.gov/2019/reviews/rpp2019-rev-standard-model.pdf>.

Electroweak Theory

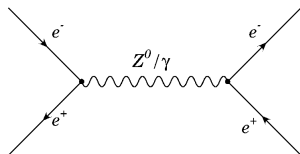
- $W_\mu^{(3)}$ field (no physical interpretation ?)
- Additional symmetry, the $U(1)_Y$ group is introduced
- B_μ field arising from $U(1)_Y$ symmetry (no physical meaning ?)
- Linear combinations of $W_\mu^{(3)}$ and B_μ fields \rightarrow photon and the Z^0 boson:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^{(3)} \end{pmatrix} \quad (2)$$

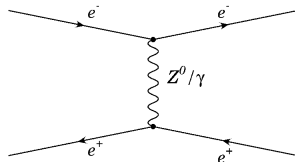
θ_W : weak mixing/Weinberg angle

Angular Dependence of γ/Z^0 Mediated Processes

- $e^+e^- \rightarrow e^+e^-$: t-channel as well as s-channel component
- $e^+e^- \rightarrow f\bar{f}$ (f other than e^-): only s-channel



s-channel Bhabha scattering



t-channel Bhabha scattering

Angular Dependence of γ/Z^0 Mediated Processes

- s channel angular dependence:

$$\left(\frac{d\sigma}{d\Omega}\right)_s \propto (1 + \cos^2 \theta)$$

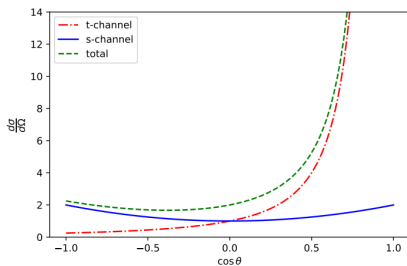
Cross section has a major contribution at large angles (or small values of $\cos \theta$)

- t channel angular dependence:

$$\left(\frac{d\sigma}{d\Omega}\right)_t \propto (1 - \cos \theta)^{-2}$$

Cross section increases asymptotically at small angles (or large values of $\cos \theta$)⁴

- Essential step ! : Remove t -channel contribution while finding inherent forward backward asymmetry in $e^+e^- \rightarrow e^+e^-$ process



s and t -channel angular distribution

⁴Universität Bonn. Instructions for E213: Analysis of Z^0 decay.

Forward-Backward Asymmetry

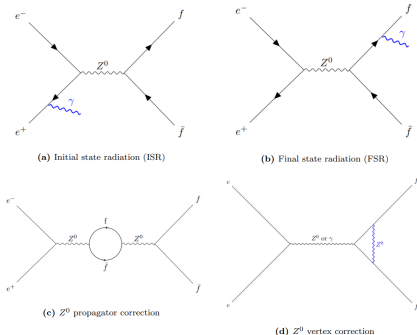
- Consider Z^0 mediated s -channel process $e^+e^- \rightarrow f\bar{f}$
- Angular dependence: $\left(\frac{d\sigma}{d\Omega}\right)_{s(Z^0)} \propto a(1 + \cos^2 \theta) + 2b \cos \theta$
- No. of fermions in forward dir., $(\theta > \pi/2) \neq$ no. of fermions in backward dir., $(\theta < \pi/2)$
- Asymmetry term b : Due to unequal coupling of Z^0 to right handed and left handed fermions $b = \left[(g_L^e)^2 - (g_R^e)^2\right] \left[(g_L^f)^2 - (g_R^f)^2\right]$
 - g_L^f : coupling of Z^0 to left handed fermions
 - g_R^f : coupling of Z^0 to right handed fermions

Forward-Backward Asymmetry

- FB asymmetry factor given as the ratio: $\mathcal{A}_{fb} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3b}{4a}$
 - σ_F, σ_B : cross sections in forward and backward directions respectively
- At Z^0 resonance, \mathcal{A}_{fb} simplifies to ⁵: $\mathcal{A}_{fb}^f \approx 3 \left(\frac{g_V^f}{g_A^f} \right) = 1 - 4 \sin^2 \theta_W$
- From this, ratio of g_V^f to g_A^f can be found
- In turn gives us the Weinberg (weak mixing) angle θ_W

⁵Thomson, *Modern Particle Physics*.

Background Processes: Radiative Corrections



Some radiative corrections

To test out predictions of the Standard Model at high precision, need to account for higher order processes ⁶:

- **ISR:** radiation of photons in the initial state \rightarrow decreases \sqrt{s} and affects Z^0 peak parameters
- **FSR:** radiation of photons or gluons in final state \rightarrow partial widths increase
- **Electroweak corrections:** Virtual processes like loops in Z^0 propagator and vertex corrections

⁶G. Abbiendi et al. and "The OPAL" Collaboration. "Precise determination of the Z resonance parameters at LEP: "Zedometry"". In: *The European Physical Journal C - Particles and Fields* 19.4 (Mar. 2001), pp. 587–651. ISSN: 1434-6052. DOI: 10.1007/s100520100627. URL: <https://doi.org/10.1007/s100520100627>.

Breit Wigner Distribution

- Contribution of Z^0 boson exchange propagator to the matrix element:

$$\mathcal{M}_{Z^0} \propto \frac{g_{Z^0}^2}{q^2 - m_{Z^0}^2} = \frac{g_{Z^0}^2}{s - m_{Z^0}^2}$$

- Around the Z^0 resonance ($\sqrt{s} \sim m_{Z^0}$), propagator diverges
- Correction \rightarrow modify propagator for a decaying state
- For unstable particle having decay rate Γ , wavefunction modified to:
 $\psi \propto e^{-imt} \rightarrow e^{-imt} e^{-\Gamma t/2}$

- Equivalent to introducing an additional imaginary term in the mass:

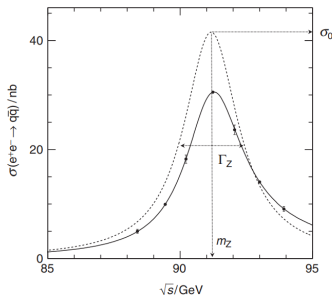
$$m \rightarrow m - i\frac{\Gamma}{2}$$

- Z^0 propagator then changes to: $\frac{1}{s - m_{Z^0}^2} \rightarrow \frac{1}{s - (m_{Z^0} - i\Gamma_{Z^0}/2)^2}$

Breit Wigner Distribution

- Complete form of cross section in process $e^+e^- \rightarrow Z^0 \rightarrow f\bar{f}$ is:

$$\sigma_f(s) = \frac{12\pi}{M_Z^2} \frac{s\Gamma_e\Gamma_f}{(s-M_Z^2)^2 + \left(\frac{s\Gamma_Z}{M_Z}\right)^2} (\hbar^2 c^2)$$
- Breit-Wigner distribution:**
 Probability distribution that characterizes this dependence of cross section on centre of mass energy
- Various physical parameters can be extracted by fitting this theoretical distribution to the observed data



Breit Wigner distribution of the cross section for $e^+e^- \rightarrow q\bar{q}$ process ⁷

⁷Thomson, *Modern Particle Physics*.

The LEP Experiment

- LEP built at CERN; started operating in 1989
- One of the major goals: make high precision measurements of Z^0 boson properties
- Produced e^+e^- collisions at \sqrt{s} close to Z^0 resonance
- Recorded about 17 million $e^+e^- \rightarrow Z^0$ events (1989-95) ⁸
- Collisions at four different points in the circular collider \rightarrow four detectors:
 - ALEPH (Apparatus for LEP Physics)
 - DELPHI (DEtector with Lepton, Photon and Hadron Identification)
 - L3 (Third LEP experiment)
 - OPAL (Omni-Purpose Apparatus for LEP)

⁸Thomson, *Modern Particle Physics*.

OPAL Detector and its Components

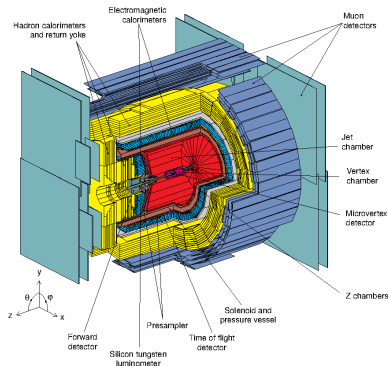
Starting from origin, as e^+e^- collision products fly outwards, various detector components are encountered. These are discussed here in brief:

- **Vertex detector:**

- Surrounds the central beam pipe
- Key role: locating vertices of short-lived decay products
- Improves momentum resolution

- **Jet chamber:**

- Has good spatial and track resolution
- Records events associated with jets
- Also determines dE/dx of charged particles



Cross sectional view of the OPAL detector⁹

⁹Das OPAL Experiment. <https://www-static.etp.physik.uni-muenchen.de/fp-versuch/node7.html>.

OPAL Detector and its Components

- **z chambers:**

- Locates z coordinates of decay particles
- Helpful in improving the resolutions of the polar angle

- **Solenoid:**

- Surrounds central detector
- Generates a uniform magnetic field of 0.435 T along beam dirn.
- \vec{B} field \rightarrow charged particles have helical path \rightarrow momenta can be measured

- **ECAL:**

- γ , e^- and e^+ $\xrightarrow[\text{Pair production}]{\text{Bremsstrahlung}}$ deposit all energy in ECAL
- Hadrons \rightarrow lose some energy (don't stop completely)
- Lead glass used (high Z; entire EM shower contained in small region of space)

OPAL Detector and its Components

- **HCAL:**

- Just like ECAL; detects mesons and baryons
- Hadronic energy loss : EM shower + strong interaction with nuclei
- Numerous decay products
- HCAL occupies more detector volume (significant distance between two consecutive nuclear interactions)
- Decays more complicated \rightarrow large uncertainty in determining energy loss \rightarrow worse energy resolution compared to ECAL

- **Muon detector:**

- Only remaining particles: muons
- Four layers of muon detectors outside HCAL
- Provides a coverage of almost the entire solid angle of 4π
- Has a barrel region and two endcap regions

Analysis

- Analysis is divided into two parts - Analysis of event displays and Statistical analysis of Z^0 decays.
- The main objective of the first part is to understand how different channels affect different observed quantities.
- We use this knowledge to come up with cut criteria - a filter to separate out different channels.
- The main object of the second part is to use the knowledge from first part and use it to analyse large sets of real world data.

Analysis of Event Displays

- We analyse event displays of pure events - events of specific channel.
- Every event display contains different parameters - number of charged tracks, momentum of all charged tracks, total energy in the EM calorimeter and the total energy in the hadronic calorimeter.
- We then plot histograms of different parameters try to come up with a cut criteria.
- Since we already know the behaviour of our detectors to different kinds of particles, we have a rough idea of what to expect.

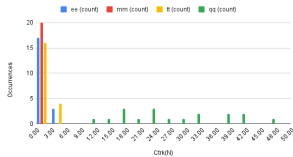
- e^-e^+ -channel: We expect two charged tracks, SumP will have the value of centre of mass energy and all the energy is expected to be deposited in the EM calorimeter and no signal is expected in the hadronic calorimeter.
- $\mu^-\mu^+$ -channel: Again, two charged tracks, SumP will have the value of centre of mass energy, but no energy deposited in the calorimeters. Instead, signals in the muon chamber.
- $\tau^-\tau^+$ -channel: Short life of τ results in interesting outcomes. We expect to see a large number of charged tracks due to the decay of τ . The dominant decay products are μ and π , which can then be detected. This channel can be classified by the number of charge tracks, called prongs.
- $q\bar{q}$ -channel: Since quarks instantly hadronise, we again have interesting outcomes. The produced hadrons decay further and can be detected. This results in the so-called *jets*, which are much bigger clusters of charged tracks. SumP value is expected to be less than the centre of mass energy, since neutrinos are produced which then carry away some of the energy.

- We plot histograms of all the quantities to develop cuts.
- The bin sizes are chosen so that we are able to see the best possible separation of the different channels.
- These cuts are tested on the mixed samples. The mixed samples were classified and checked visually if it matched the channel as determined by the cuts.

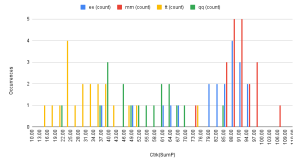
Channel	Ctrk(N)	Ctrk(Sump)	Ecal(SumE)	Hcal(SumE)
e^-e^+	(0,6)		≥ 80	< 1
$\mu^-\mu^+$	< 6	≥ 75	≤ 10	
$\tau^-\tau^+$	< 6	≤ 75	≤ 60	
$q\bar{q}$	≥ 8		[36,79]	

Table: Cuts determined from event display analysis

Ctcr(N) Histogram (pure data)



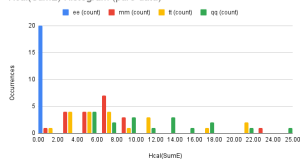
Ctcr(SumP) Histogram (pure data)



Ecal(SumE) Histogram (pure data)



Hcal(SumE) Histogram (pure data)



Histograms of $Ctcr(N)$, $Ctcr(Sump)$, $Ecal(SumE)$ and $Hcal(SumE)$ for the four channels, generated from *pure* samples.







Statistical Analysis of Z^0 Decays

- A simple analysis of event displays is not possible when we have a large number of events. We resort to statistical analysis methods. We use the software, *ROOT* for this purpose.
- *ROOT* works with *.root* files, which contain all the information in a tree-like structure, called *ntuple*.
- The *ntuple* contains various parameters - run number, event number, number of charged tracks, total scalar sum of track momenta, total energy in the EM calorimeter, total energy in the hadronic calorimeter, LEP beam energy, polar angle between beam axis and thrust axis, polar angle between incoming positron and outgoing positive particle.
- We first use Monte Carlo events to refine our cuts. Then we work with the actual data to calculate different physical quantities.

Refining the cuts

- For the e^-e^+ -channel, we would like to exclude t-channel events. This is because t-channel is possible only in this mode and we omit this for the sake of consistency.
- This is achieved by limiting ourselves to smaller angles, since t-channel dominates at large angles. Therefore we take $\cos\theta < 0.5$.
- But this means that we are excluding some of the s-channel events in this region. The correction factor to account for this, 1.5829 in our case, is taken from theory.
- For the $\mu^-\mu^+$ -channel, we observed a lot of events with scalar momenta exactly equal to zero. These are not physical and we ignore them by modifying our cuts.
- For the above two channels, we also modified our cuts to ignore angles very close to the beam axis, since the resolution of our detector is far from perfect in that region.
- The other two channels did not require any additional cut, but we did need $\cos\theta$ to take only the values between -1 and 1 to exclude unphysical events.

References

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