# How to derive a state diagram?

State diagrams are not only used in Digital Design, but are also used in programming.

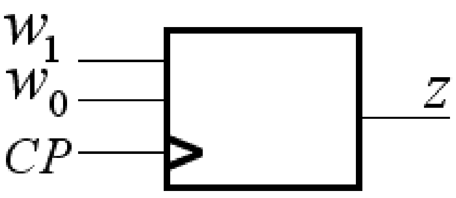
State charts are included in the diagram standard UML.

# Task

A synchronous Moore machine has two inputs $w_1w_0$ and one output $z$.

- For the input sequence $w_1w_0$ 10, 11 the output will be $z = 0$.
- For the input sequence $w_1w_0$ 01, 11 the output will be $z = 1$.
- For $w_1w_0$ 01, 10 the output will **change value**.
- For other input sequences the output will **keep** its **value**.

Derive the automato **state table**, minimize it as far as possible. Then draw the automato **state diagram**.



# How many states?

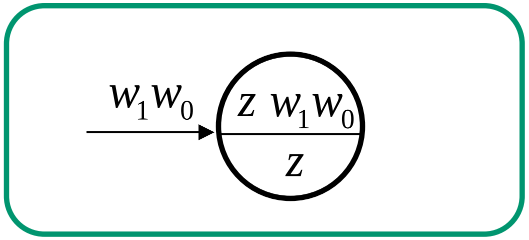
All the described sequences are no longer than two steps. We therefore does not need (this time) to "handle" longer sequences than so.
Input $w_1 w_0$ can have four different values, 00 01 10 and 11.
Start by Start by assuming that one may need to go to two different states from each input value, one with $z = 0$ and one with $z = 1$ (because the output for some conditions should ”toggle”). This gives us 8 states.

One need not to be afraid of using too many states, because we can later reduce the number with state minimization.

# "Temporary" state numbers

It will be easier to reason the way to the state table if state symbols (or state numbers) reflects the "characteristics" of the states. Eg "z $w_1 w_0$". **Temporary state numbers** are here derived from output value in states and from the input combination that leads to the state.

If you just name the states *a* … *h* it may be a little harder to follow the transitions in the table.

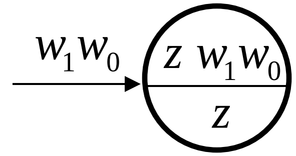


# State: ”output input”

$state^{+}(w_1 w_0)$

| $z\,w_1 w_0$ | | $w_1 w_0$ = 00 | 01 | 11 | 10 | $z$ |
|---|---|---|---|---|---|---|
| 0 00 | $a$ | $0\,00 : a$ | $-\,01 : -$ | $-\,11 : -$ | $-\,10 : -$ | 0 |
| 0 01 | $b$ | $-\,00 : -$ | $0\,01 : b$ | $-\,11 : -$ | $-\,10 : -$ | 0 |
| 0 10 | $c$ | $-\,00 : -$ | $-\,01 : -$ | $-\,11 : -$ | $0\,10 : c$ | 0 |
| 0 11 | $d$ | $-\,00 : -$ | $-\,01 : -$ | $0\,11 : d$ | $-\,10 : -$ | 0 |
| 1 00 | $e$ | $1\,00 : e$ | $-\,01 : -$ | $-\,11 : -$ | $-\,10 : -$ | 1 |
| 1 01 | $f$ | $-\,00 : -$ | $1\,01 : f$ | $-\,11 : -$ | $-\,10 : -$ | 1 |
| 1 10 | $g$ | $-\,00 : -$ | $-\,01 : -$ | $-\,11 : -$ | $1\,10 : g$ | 1 |
| 1 11 | $h$ | $-\,00 : -$ | $-\,01 : -$ | $1\,11 : h$ | $-\,10 : -$ | 1 |

**First:**



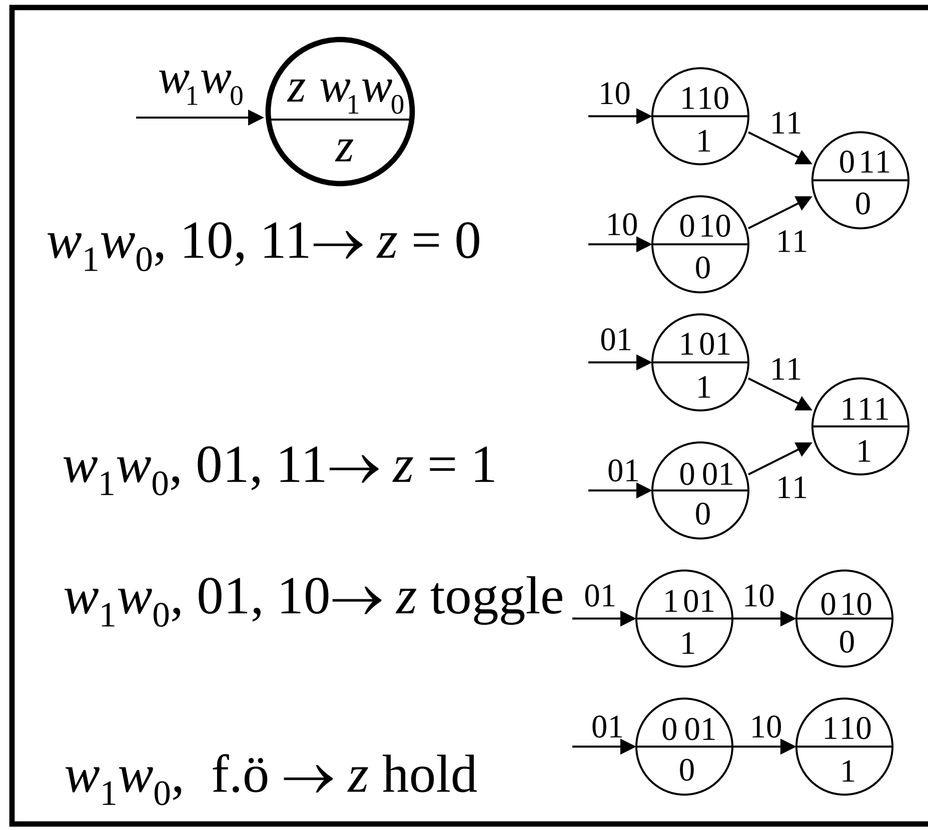

Every input leads to one of the two states that has this input in it’s state number (eg. 00 *a* or *e*, 01 *b* or *f*). For this input one stays in the state.

# The description

$state^+(w_1w_0)$

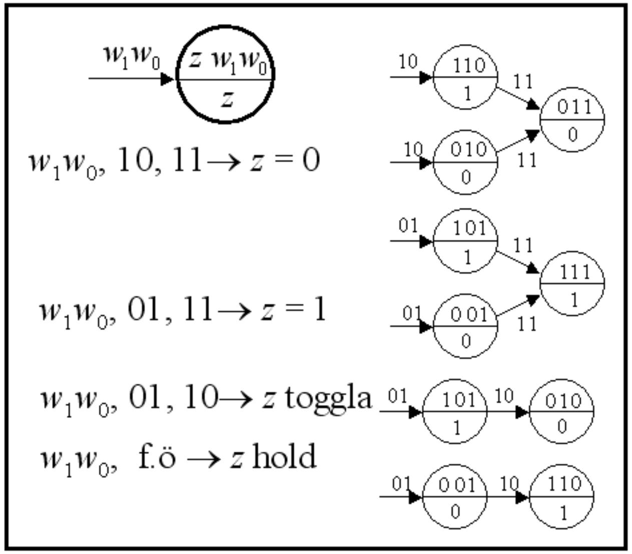
| $z\,w_1w_0$ | | $w_1w_0$ = 00 | 01 | 11 | 10 | $z$ |
|---|---|---|---|---|---|---|
| 0 00 | *a* | *a* | | | | 0 |
| 0 01 | *b* | | *b* | | | 0 |
| 0 10 | *c* | | | | *c* | 0 |
| 0 11 | *d* | | | *d* | | 0 |
| 1 00 | *e* | *e* | | | | 1 |
| 1 01 | *f* | | *f* | | | 1 |
| 1 10 | *g* | | | | *g* | 1 |
| 1 11 | *h* | | | *h* | | 1 |

*The state table is now underway.*




*The state transitions from the description.*

# The description – state transitions




$state^{+}(w_1w_0)$

| $z\ w_1w_0$ | | $w_1w_0$ 00 | 01 | 11 | 10 | $z$ |
|---|---|---|---|---|---|---|
| 0 00 | $a$ | $000:a$ | $-01:-$ | $-11:-$ | $-10:-$ | 0 |
| 0 01 | $b$ | $-00:-$ | $001:b$ | $111:h$ | $110:g$ | 0 |
| 0 10 | $c$ | $-00:-$ | $-01:-$ | $011:d$ | $010:c$ | 0 |
| 0 11 | $d$ | $-00:-$ | $-01:-$ | $011:d$ | $-10:-$ | 0 |
| 1 00 | $e$ | $100:e$ | $-01:-$ | $-11:-$ | $-10:-$ | 1 |
| 1 01 | $f$ | $-00:-$ | $101:f$ | $111:h$ | $010:c$ | 1 |
| 1 10 | $g$ | $-00:-$ | $-01:-$ | $011:d$ | $110:g$ | 1 |
| 1 11 | $h$ | $-00:-$ | $-01:-$ | $111:h$ | $-10:-$ | 1 |

*Then you enter the state transitions as described.*

# Other state transitions

$$state^{+}(w_1 w_0)$$

| $z\ w_1 w_0$ | | $w_1 w_0$ 00 | 01 | 11 | 10 | $z$ |
|---|---|---|---|---|---|---|
| 0 00 | $a$ | $0\,00 : a$ | $0\,01 : b$ | $0\,11 : d$ | $0\,10 : c$ | 0 |
| 0 01 | $b$ | $0\,00 : a$ | $0\,01 : b$ | $1\,11 : h$ | $1\,10 : g$ | 0 |
| 0 10 | $c$ | $0\,00 : a$ | $0\,01 : b$ | $0\,11 : d$ | $0\,10 : c$ | 0 |
| 0 11 | $d$ | $0\,00 : a$ | $0\,01 : b$ | $0\,11 : d$ | $0\,10 : c$ | 0 |
| 1 00 | $e$ | $1\,00 : e$ | $1\,01 : f$ | $1\,11 : h$ | $1\,10 : g$ | 1 |
| 1 01 | $f$ | $1\,00 : e$ | $1\,01 : f$ | $1\,11 : h$ | $0\,10 : c$ | 1 |
| 1 10 | $g$ | $1\,00 : e$ | $1\,01 : f$ | $0\,11 : d$ | $1\,10 : g$ | 1 |
| 1 11 | $h$ | $1\,00 : e$ | $1\,01 : f$ | $1\,11 : h$ | $1\,10 : g$ | 1 |

*The other state transitions leads to the state with the input number that has the same output – so no change in output.*

# State table

$state^+(w_1 w_0)$

| $w_1 w_0$ | 00 | 01 | 11 | 10 | $z$ |
|---|---|---|---|---|---|
| $a$ | $a$ | $b$ | $d$ | $c$ | 0 |
| $b$ | $a$ | $b$ | $h$ | $g$ | 0 |
| $c$ | $a$ | $b$ | $d$ | $c$ | 0 |
| $d$ | $a$ | $b$ | $d$ | $c$ | 0 |
| $e$ | $e$ | $f$ | $h$ | $g$ | 1 |
| $f$ | $e$ | $f$ | $h$ | $c$ | 1 |
| $g$ | $e$ | $f$ | $d$ | $g$ | 1 |
| $h$ | $e$ | $f$ | $h$ | $g$ | 1 |

Now you can remove the temporary numbering of the states and present the state table with letter designations.

One sees immediately that a number of the states are equivalent, so the process of state minimization will follow.

# State minimization

$state^{+}(w_1w_0)$

| | 00 | 01 | 11 | 10 | $z$ |
|---|---|---|---|---|---|
| $a$ | $a$ | $b$ | $d$ | $c$ | 0 |
| $b$ | $a$ | $b$ | $h$ | $g$ | 0 |
| $c$ | $a$ | $b$ | $d$ | $c$ | 0 |
| $d$ | $a$ | $b$ | $d$ | $c$ | 0 |
| $e$ | $e$ | $f$ | $h$ | $g$ | 1 |
| $f$ | $e$ | $f$ | $h$ | $c$ | 1 |
| $g$ | $e$ | $f$ | $d$ | $g$ | 1 |
| $h$ | $e$ | $f$ | $h$ | $g$ | 1 |

$$P_0 = (a,b,c,d,e,f,g,h)$$
$$P_1 = (a,b,c,d)(e,f,g,h)$$
$$P_2 = (a,c,d)(b)(e,h)(f)(g)$$
$$P_3 = P_2 \quad a,b,e,f,g$$

Designation of the new states.

$state^{+}(w_1w_0)$

| | 00 | 01 | 11 | 10 | $z$ |
|---|---|---|---|---|---|
| $a$ | $a$ | $b$ | $a$ | $a$ | 0 |
| $b$ | $a$ | $b$ | $e$ | $g$ | 0 |
| $e$ | $e$ | $f$ | $e$ | $g$ | 1 |
| $f$ | $e$ | $f$ | $e$ | $a$ | 1 |
| $g$ | $e$ | $f$ | $a$ | $g$ | 1 |

# Minimized state diagram

$state^{+}(w_1 w_0)$

| $w_1 w_0$ | 00 | 01 | 11 | 10 | $z$ |
|---|---|---|---|---|---|
| $a$ | $a$ | $b$ | $a$ | $a$ | 0 |
| $b$ | $a$ | $b$ | $e$ | $g$ | 0 |
| $e$ | $e$ | $f$ | $e$ | $g$ | 1 |
| $f$ | $e$ | $f$ | $e$ | $a$ | 1 |
| $g$ | $e$ | $f$ | $a$ | $g$ | 1 |