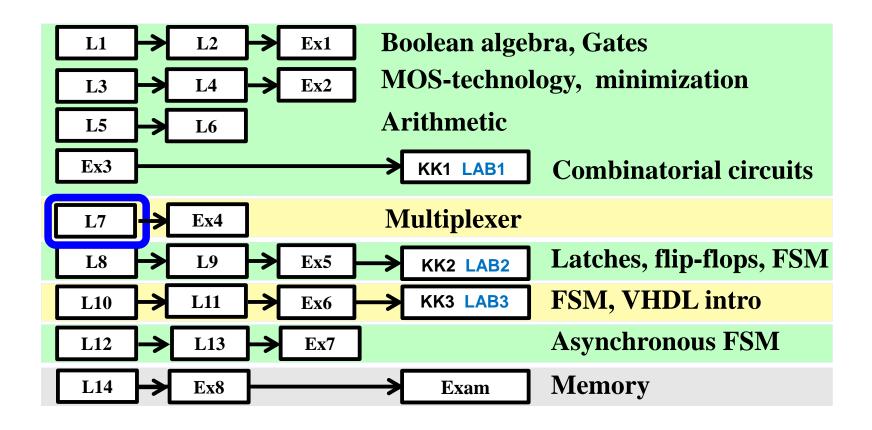
IE1204 Digital Design



L7: Combinational circuits, Introduction to VHDL

Masoumeh (Azin) Ebrahimi
KTH/ICT
mebr@kth.se

IE1204 Digital Design

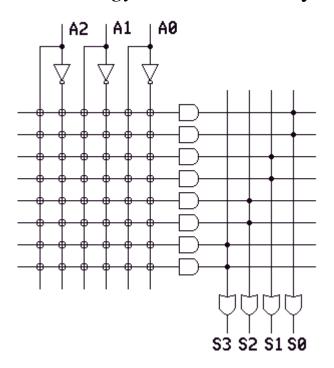


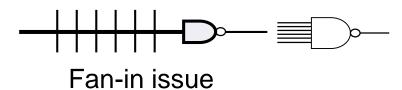
This lecture

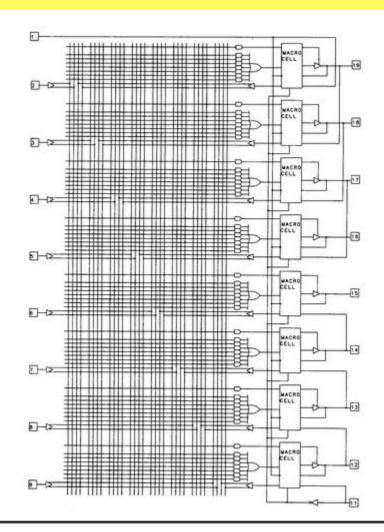
• BV 318-339, 60-65, 280-291,341-365

PLD (eg. PAL)

Technology: AND-OR array



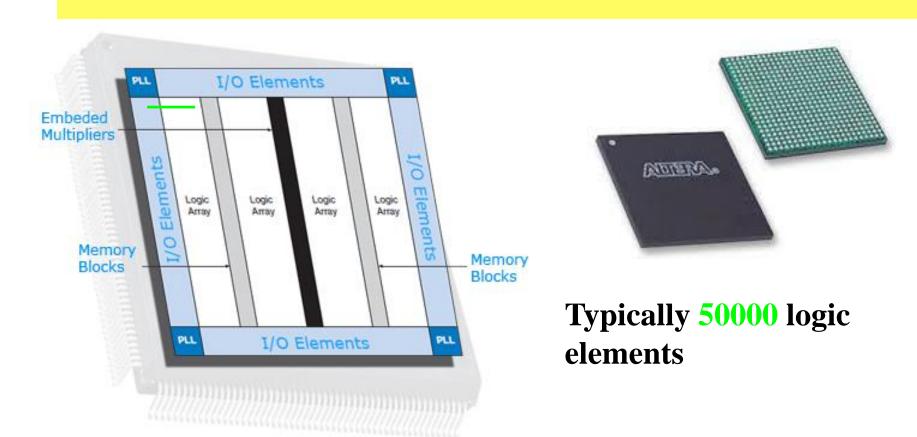




Large programmable circuits

Therefore in order to be able to build large programmable circuits in CMOS technology there is a need for other techniques that are not based on gates with many inputs!

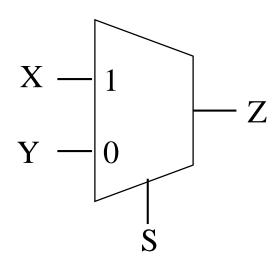
FPGA (eg. Cyclone II)



Technology: MUX tree

The multiplexer (MUX)

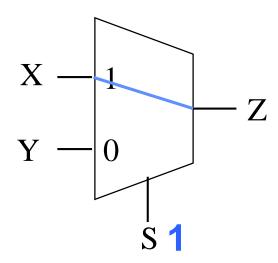
- The multiplexer can select which input you are going to connect to the output
- "If S then X, else Y"



$$Z = SX + \overline{S}Y$$

The multiplexer (MUX)

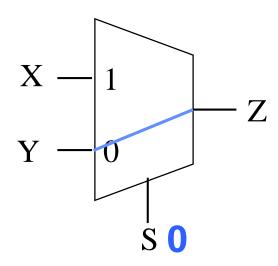
- The multiplexer can select which input you are going to connect to the output
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$$Z = SX + \overline{S}Y$$

The multiplexer (MUX)

- The multiplexer can select which input you are going to connect to the output
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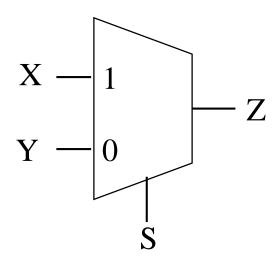


$$Z = SX + \overline{S}Y$$

Implementation of functions using MUXes

How can the following functions be implemented with a 2:1 multiplexer?

- $Z = \overline{B}$ (INV)
- Z = AB (AND)
- Z = A + B (OR)
- $Z = A \oplus B (XOR)$



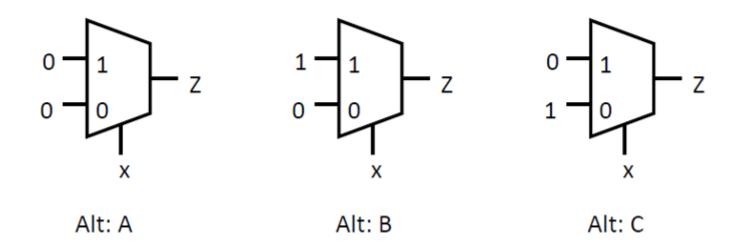
$$Z = SX + \overline{S}Y$$

Quickie Question ...

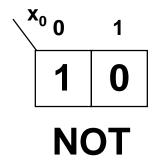


 How to connect the inputs of the MUX in order to implement an inverter?

Desired function: $z = \overline{x}$

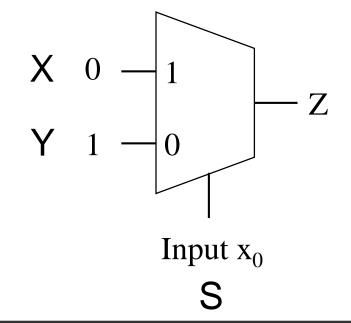


Inverter implemented with a MUX



$$Z = S \cdot X + \overline{S} \cdot Y =$$

$$= x_0 \cdot 0 + \overline{x_0} \cdot 1 = \overline{x_0} \quad NOT$$

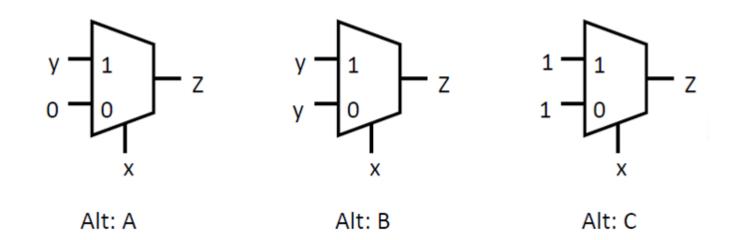


Quickie Question ...

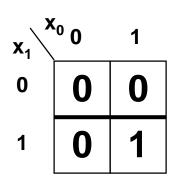


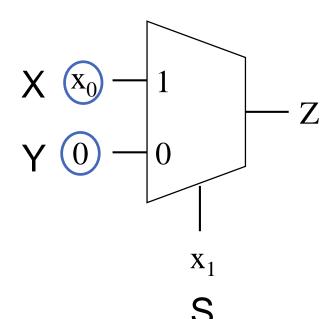
 How to connect the inputs of the MUX in order to implement an AND gate?

Desired function: z = xy



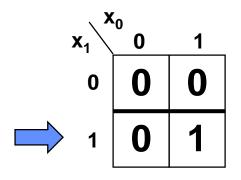
AND implemented with a MUX

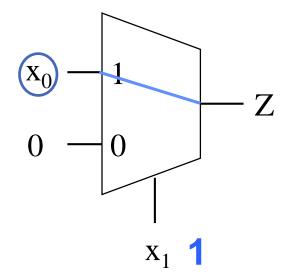




$$Z = SX + \overline{S}Y = x_1 \cdot x_0 + \overline{x_1} \cdot 0 = x_1 \cdot x_0$$

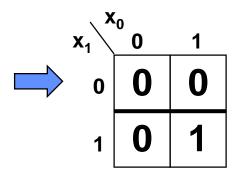
AND implemented with a MUX

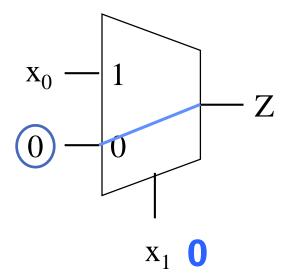




$$Z = SX + \overline{S}Y = x_1 \cdot x_0 + \overline{x_1} \cdot 0 = x_1 \cdot x_0$$

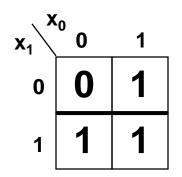
AND implemented with a MUX





$$Z = SX + \overline{S}Y = x_1 \cdot x_0 + \overline{x_1} \cdot 0 = x_1 \cdot x_0$$

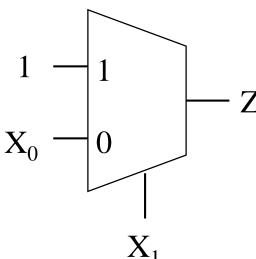
OR implemented with a Mux



$$Z = x_1 x_0 + \overline{x}_1 x_0 + x_1 \overline{x}_0 =$$

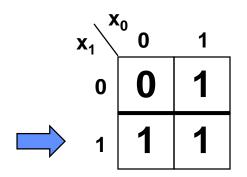
$$= \{ SX + \overline{S}Y \} = x_1 (x_0 + \overline{x}_0) + \overline{x}_1 \cdot x_0 =$$

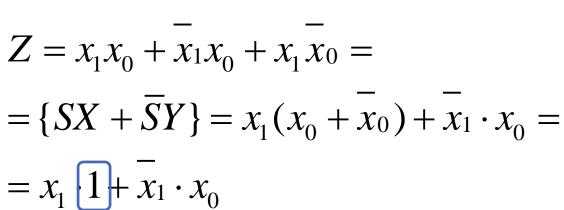
$$= x_1 \cdot 1 + \overline{x}_1 \cdot x_0$$

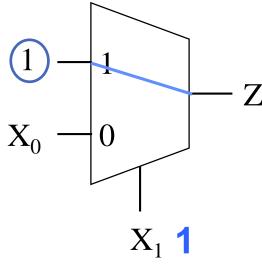




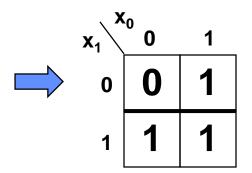
OR implemented with a Mux

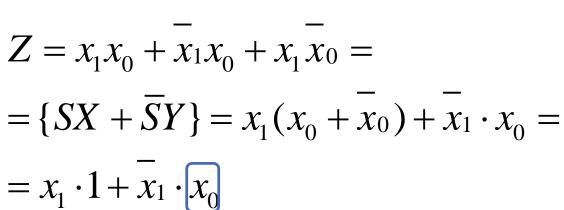


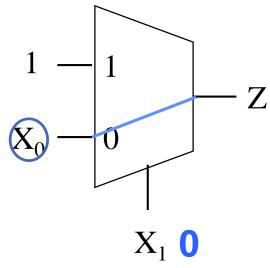




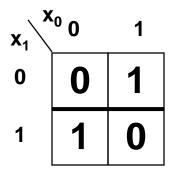
OR implemented with a Mux





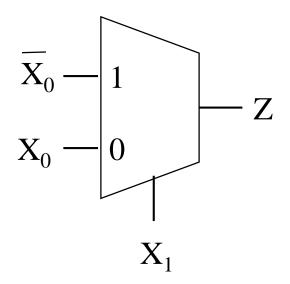


XOR implemented with a Mux

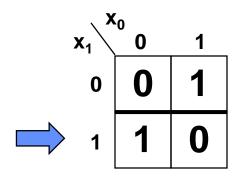


$$Z = SX + \overline{S}Y =$$

$$= x_1 \cdot \overline{x_0} + \overline{x_1} \cdot x_0 = x_1 \oplus x_0$$

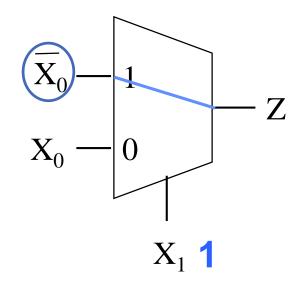


XOR implemented with a Mux

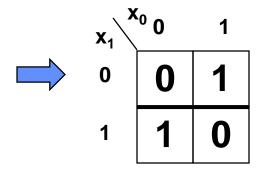


$$Z = SX + \overline{S}Y =$$

$$= x_1 \cdot \overline{x_0} + \overline{x_1} \cdot x_0 = x_1 \oplus x_0$$

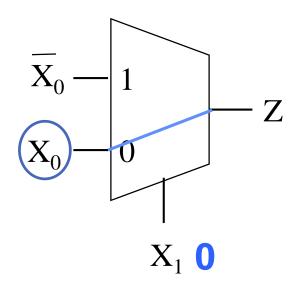


XOR implemented with a Mux

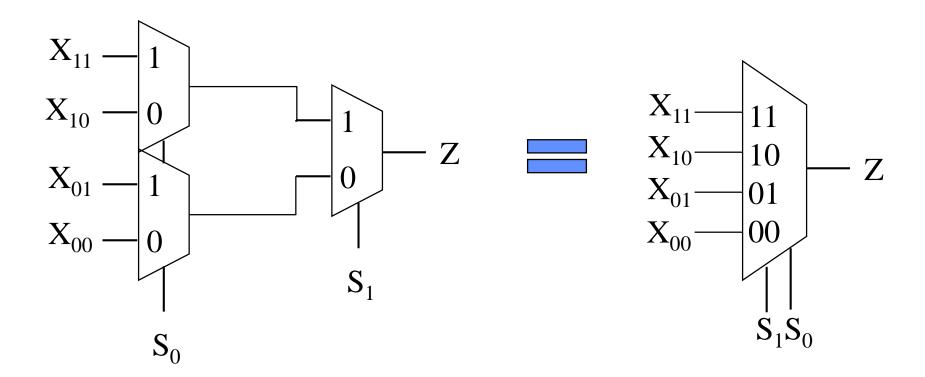


$$Z = SX + \overline{S}Y =$$

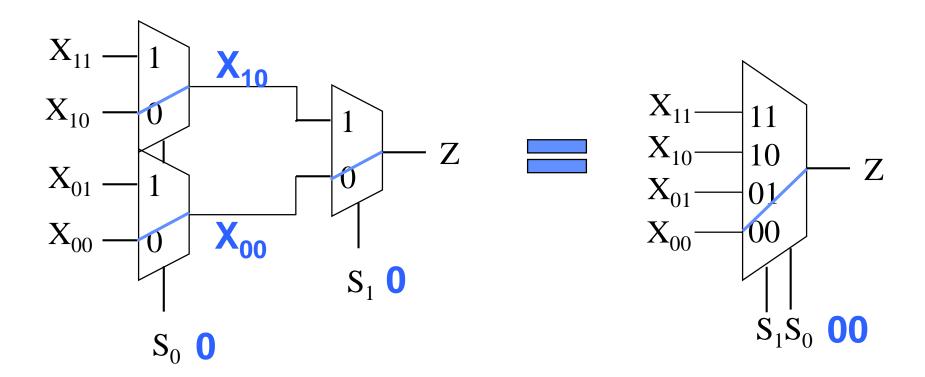
$$= x_1 \cdot \overline{x_0} + \overline{x_1} \cdot \overline{x_0} = x_1 \oplus x_0$$



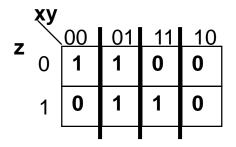
Hierarchy of MUXes



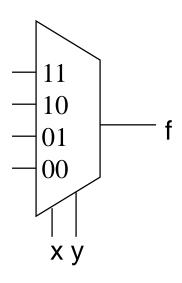
Hierarchy of MUXes



Implementation of larger functions with MUXes



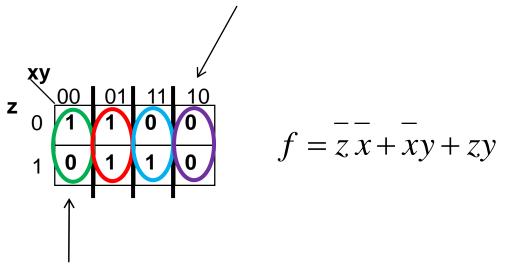
$$f = \overline{z} \, \overline{x} + \overline{x} y + z y$$

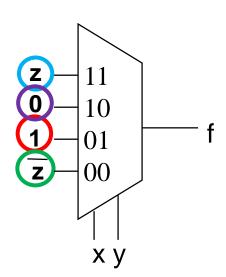




Implementation of larger functions with MUXes

Choose any of the inputs as address inputs ...



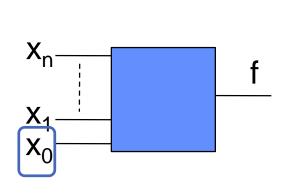


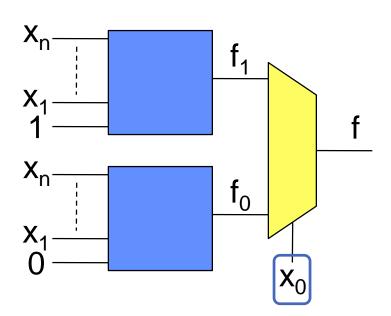
... And minimize/implement function for each input. Draw new Karnaugh diagrams if necessary.

An (n + 1)-input function can be implemented with a MUX that has n select inputs!

Mapping into MUXes: Shannon decomposition (BV 6.1.2)

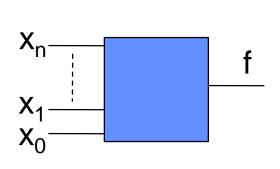
- Any Boolean function $f(x_n, ..., x_1, x_0)$ can be partitioned as $f(x_n, ..., x_1, x_0) = x_0 f_1(x_n, ..., x_1, 1) + \overline{x_0} f_0(x_n, ..., x_1, 0)$
- The function can then be implemented with a multiplexer

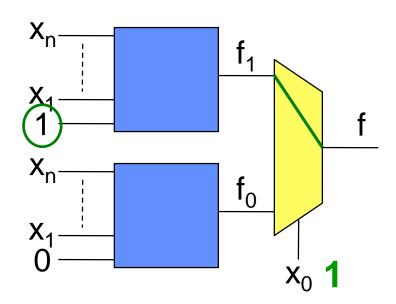




Mapping into MUXes: Shannon decomposition (BV 6.1.2)

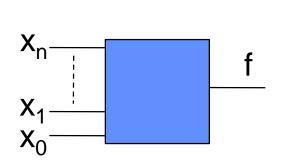
- Any Boolean function $f(x_n, ..., x_1, x_0)$ can be partitioned as $f(x_n, ..., x_1, x_0) = x_0 f_1(x_n, ..., x_1, 1) + \overline{x_0} f_0(x_n, ..., x_1, 0)$
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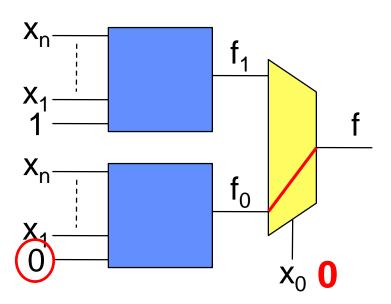




Mapping into MUXes: Shannon decomposition (BV 6.1.2)

- Any Boolean function $f(x_n, ..., x_1, x_0)$ can be partitioned as $f(x_n, ..., x_1, x_0) = x_0 f(x_n, ..., x_1, 1) + x_0 f(x_n, ..., x_1, 0)$
- The function can then be implemented with a multiplexer





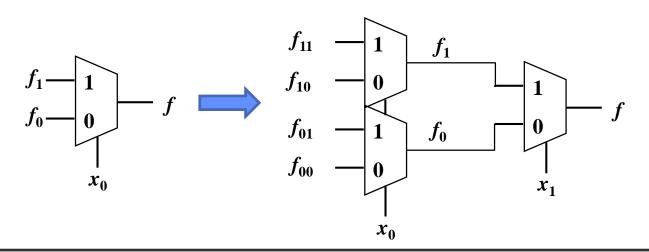
Mapping to MUXes: Shannon decomposition

 Any Boolean function f (x_n, ..., x₁, x₀) can be decomposed (recursively) as

$$f(x_{n},...,x_{1},x_{0}) = x_{0} f_{1}(x_{n},...,x_{1},1) + \overline{x}_{0} f_{0}(x_{n},...,x_{1},0)$$

$$= x_{1}x_{0} f_{11}(x_{n},...,x_{2},1,1) + x_{1}\overline{x}_{0} f_{10}(x_{n},...,x_{2},1,0)$$

$$+ \overline{x}_{1}x_{0} f_{01}(x_{n},...,x_{2},0,1) + \overline{x}_{1}\overline{x}_{0} f_{00}(x_{n},...,x_{2},0,0)$$



Proof

Left-hand side Right-hand side $f(x_n,...,x_1,x_0) = \underbrace{x_0 \cdot f(x_n,...,x_1,1)}_{\text{Left term}} + \underbrace{x_0 \cdot f(x_n,...,x_1,0)}_{\text{Right term}}$

Right-hand side:

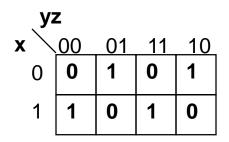
- If $x_0 = 1$ then the right term is zero. Then f is equal to the left term.
- If $x_0 = 0$, the left term is zero. Then f is equal to the right term.

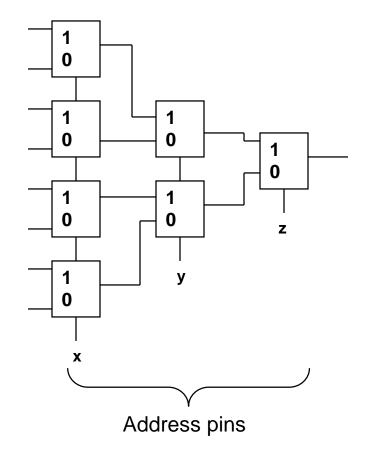
Left-hand side:

- if $x_0 = 1$, then f is equal to f $(x_n, ..., x_1, 1)$ (= left term on the right-hand side)
- if $x_0 = 0$ then f is equal to f $(x_n, ..., x_1, 0)$ (= right term in the right-hand side)
- Left-hand side = Right-hand side

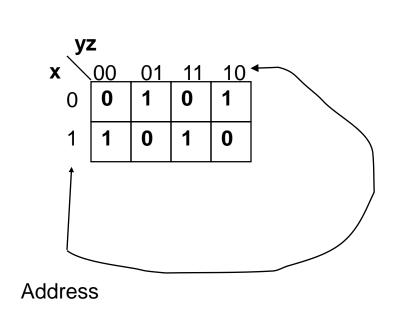
Mux circuits



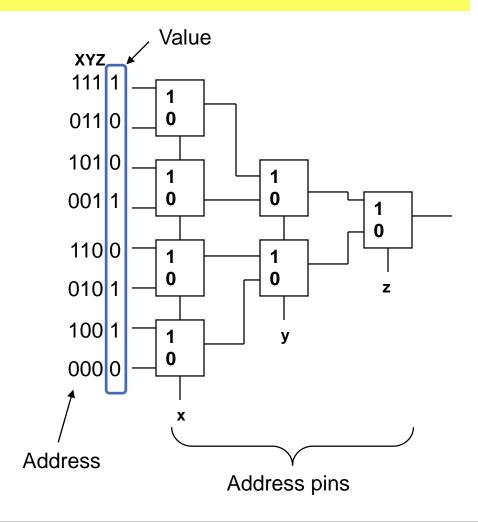




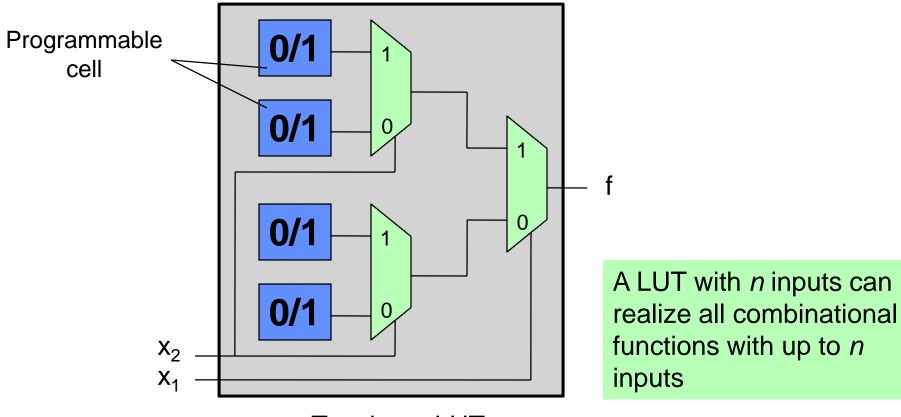
Mux circuits



But this is a memory (ROM, RAM ...)

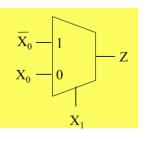


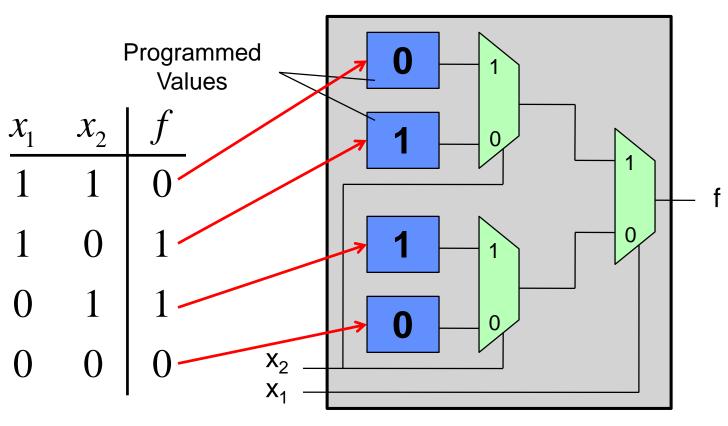
Look-up tables (LUT)



Two-input LUT

Example: XOR gate

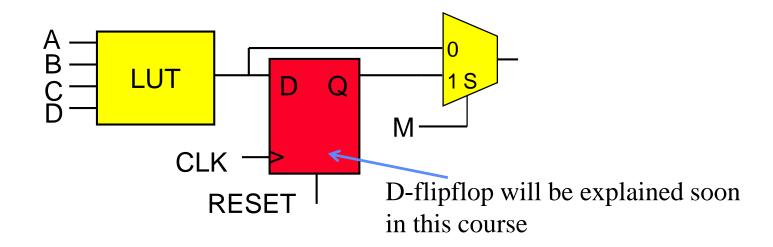




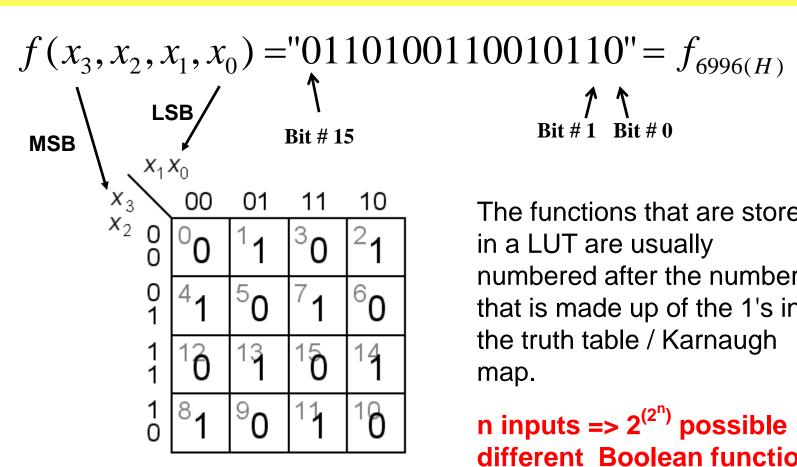
Two-input LUT

A simple FPGA cell

 The simplest FPGA cell consists of a single table (e.g. Look-Up-Table - LUT), a D flipflop and a bypass MUX.



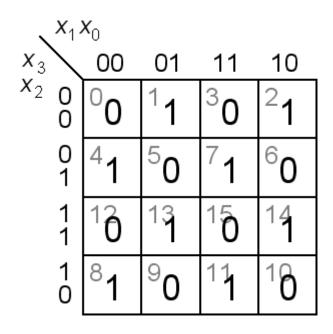
One way to identify functions...



The functions that are stored in a LUT are usually numbered after the number that is made up of the 1's in the truth table / Karnaugh map.

n inputs => 2^(2ⁿ) possible different Boolean functions

LUT function number



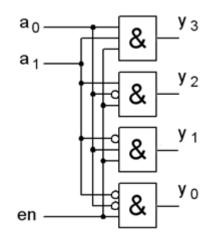
Odd parity

$$f_{6996} = x_3 \oplus x_2 \oplus x_1 \oplus x_0$$

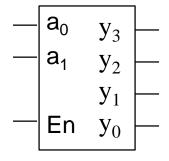
With a LUT, all functions are realized in the same way, so all of them have the same cost

Decoder

- Mostly used as address decoders
- Only one output is active when the 'enable' (En) signal is active
- The active output is selected by a₁a₀



En	a_1	a_0	y ₀	y ₁	y ₂	y ₃
1	0	0	(1)	0	0	0
1	0	1	0	(1)	0	0
1	1	0	0	0	(1)	0
1	1	1	0	0	0	(1)
0	-	_	0	0	0	0



2-to-4 decoder

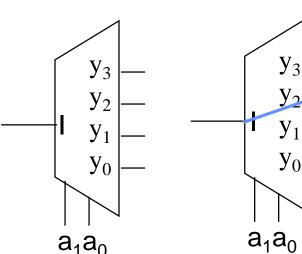
Demultiplexer

 The demultiplexer has basically the same function as the decoder, but it is drawn differently

The input I is connected to a selected

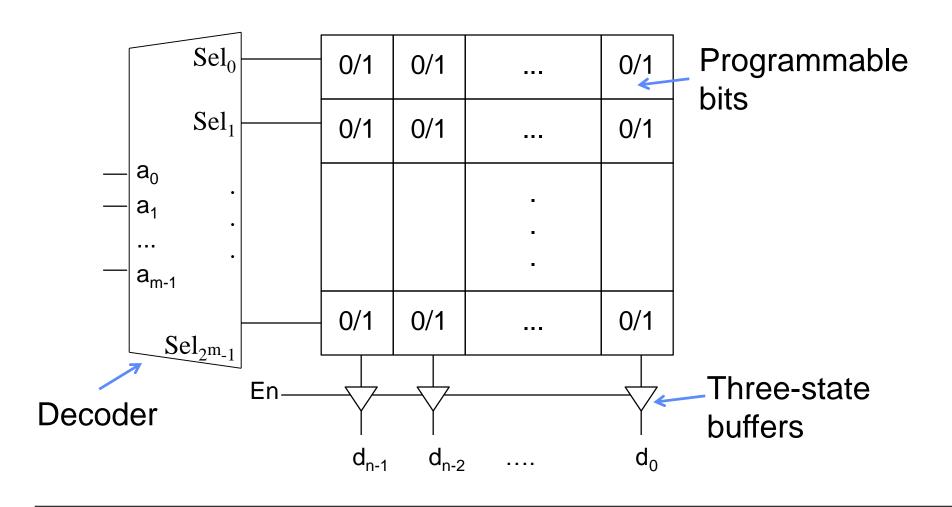
output

I	a ₁	a_0	y ₀	y ₁	y ₂	y ₃
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1
0	_	-	0	0	0	0



Demultiplexor

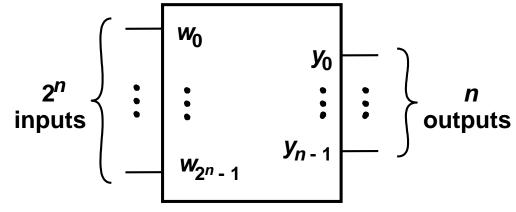
Read-Only Memory



Encoders

- Encoder has the opposite function to a decoder, i.e. it translates 2^N bit input into an N-bit code.
 - The information is greatly reduced

\mathbf{w}_0	W_1	W ₂	W ₃	y ₁	y ₀
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1



Priority Encoder

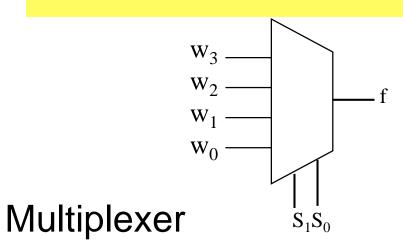
- A Priority Encoder gives back the address of the input with the lowest (or highest) indices that are set to 1 (or 0 depending on what you are looking for)
- If all inputs are 0, the output z = 0, else z = 1

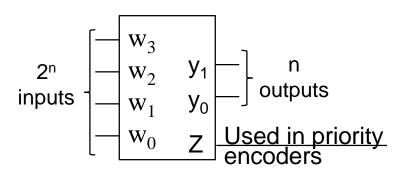
\mathbf{w}_0	w_1	W ₂	W_3	Z	y ₁	y ₀
1	-	-	-	1	0	0
0	1	-	-	1	0	1
0	0	1	-	1	1	0
0	0	0	1	1	1	1
0	0	0	0	0	-	-

The output is well-defined even if several inputs are active at the same time.

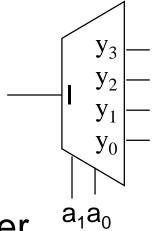
Overview

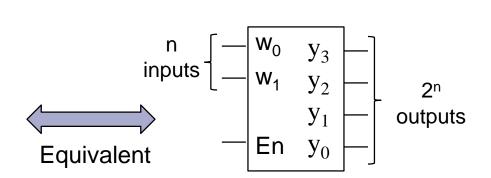






Encoder



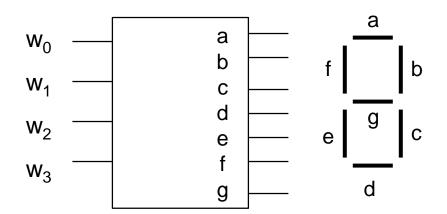


Demultiplexer

Decoder

Code converters

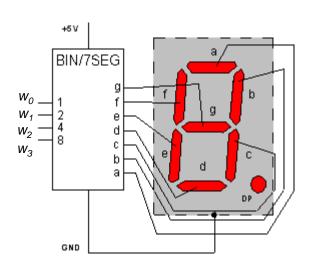
- A code converter translates from one code to another. Typical examples are:
 - Binary to BCD (Binary-Coded Decimal)
 - Binary to Gray code
 - 7-4-2-1 code
 - BCD to seven-segment decoder





A variant of the 7-4-2-1 code is used today to store the bar code

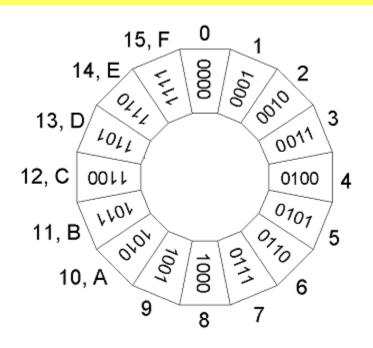
BCD-to-seven segment decoder



₩ ₃	W_2	w ₁	W_0	а	b	С	d	е	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
				•						

- BCD-to-7 segment decoder consists of 7 different combinatorial circuits, one for each segment
- To get optimal circuits, all 7 functions have to be minimized simultaneously so that common logic is shared

Disadvantage of binary codes



Binary code, adjacent code words:

1-2 double change

3-4 triple change

5-6 double change

7-8 quadruple change!

9-A double change

B-C quadruple change!

D-E double change

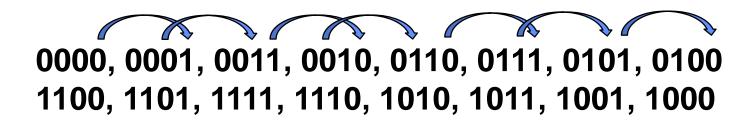
F-0 quadruple change!

Can two bits change at exactly the same time?

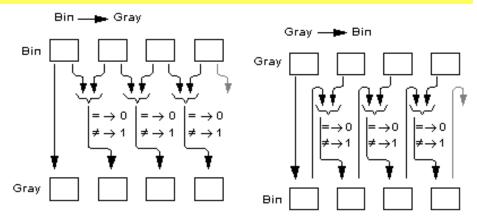
- For safe data registration use Gray code
- For data processing use binary code

Gray code

- By changing the order of the codewords in a binary code, one construct codes in which no more than one bit is changing at a time
- Such codes are called Gray codes



Conversion between binary and Gray



Binary \rightarrow Gray:

If Binary bit b_n and bit b_{n-1} are *different*, the Gray code bit g_{n-1} is "1", else "0".

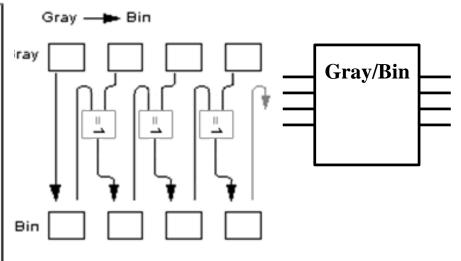
Gray \rightarrow Binary (most common transformation direction): If Binary bit b_n and Gray code bit g_{n-1} are *different* the Binary bit b_{n-1} is "1", else "0".

Logic for Gray to Binary conversion

XOR-gate is "1" if its inputs are *different*!

4 bit code converter Gray code to Binary code

	Binär-kod	Gray-kod		Binär-kod	Gray-kod
0	0000	0000	8	1000	1100
1	0001	0001	9	1001	1101
2	0010	0011	10	1010	1111
3	0011	0010	11	1011	1110
4	0100	0110	12	1100	1010
5	0101	0111	13	1101	1011
6	0110	0101	14	1110	1001
7	0111	0100	15	1111	1000



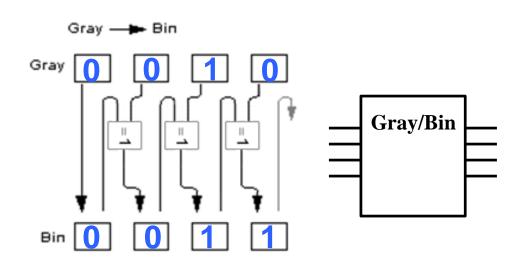
Tabell med Binärkod och Graykod.

Logic for Gray to Binary conversion

XOR-gate is "1" if its inputs are *different*!

	Binär-kod	Gray-kod		Binär-kod	Gray-kod
0	0000	0000	8	1000	1100
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2	0010	0011	10	1010	1111
3	0011	0010	11	1011	1110
4	0100	0110	12	1100	1010
5	0101	0111	13	1101	1011
6	0110	0101	14	1110	1001
7	0111	0100	15	1111	1000

4 bit code converter Gray code to Binary code



Tabell med Binärkod och Graykod.

Introduction to VHDL

- VHDL is a language used to specify the hardware
 - HDL VHSIC Hardware Description Language
 - VHSIC Very High Speed Integrated Circuit
 - Used mostly in Europe
- Verilog is another language used to specify the hardware
 - Used mostly in the United States

Why VHDL?

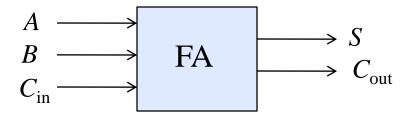
- VHDL is used to
 - Verifies that you have connected right by simulating the circuit
 - describes the large structures in a simple way and then generates the circuit by synthesis
 - allows for structured descriptions of a circuit

VHDL increases the level of abstraction!

Types of VHDL code

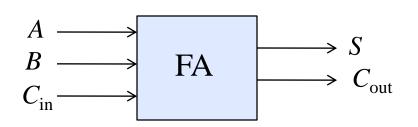
- There are two types of VHDL code
 - VHDL for synthesis: The code is used as an input to a synthesis tool which converts it into an implementation (for example FPGA or ASIC)
 - VHDL modeling and simulation code is used to describe a system at an early stage. Since the code can be simulated so you can check whether the intended functionality is correct

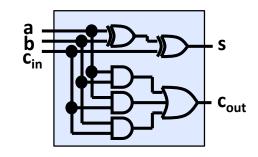
Entity (FA)



The *entity* describes the ports to the outside of the circuit. The circuit as a block.

Architecture (FA)





```
ARCHITECTURE behave OF fulladder IS

BEGIN

S <= A xor B xor Cin;

Cout <= (A and B) or (A and Cin) or (B and Cin);

END behave;
```

Architecture describes the function inside the circuit.

VHDL port

- PORT declaration establishes interface between the component and the outside world
- A port declaration contains three things:
 - The <u>name</u> of the port
 - The *direction* of the port
 - Port's <u>datatype</u>
- Example:

```
ENTITY test IS
    PORT ( name : direction data_type);
END test;
```

The most common data types

- Scalars (single-variable signals)
 - Bit ("0", "1")
 - Std_logic ('U', '0', '1', 'X', 'Z', 'L', 'H', 'W', '-')
 - Integer
 - Real
 - Time

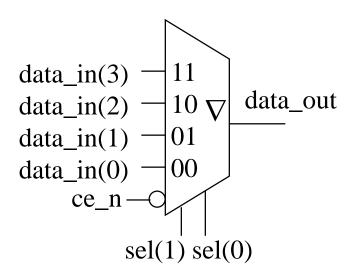
- Vectors (many-variable signals)
 - BIT_VECTOR vector of bits
 - STD_LOGIC_VECTOR vector of std_logic

VHDL Example: 4/1 MUX

```
data_in(3)
                                                       data out
                                        data_in(2)
                                                  |10 \nabla|
                                        data_in(1) = 01
LIBRARY ieee;
                                        data_in(0) = 00
USE ieee.std logic 1164.ALL;
                                           ce_n-
                                               sel(1) sel(0)
ENTITY Multiplexer 41 IS
PORT (ce n: IN std logic; -- Chip Enable (active low)
     data in: IN std logic vector(3 DOWNTO 0);
     sel: IN std logic vector(1 DOWNTO 0);
     data out: OUT std logic); -- TriState Output
END ENTITY Multiplexer 41;
```

VHDL Example: 4/1 MUX (cont.)

```
ARCHITECTURE RTL OF Multiplexer 41 IS
BEGIN
  PROCESS (ce n, data in, sel)
  BEGIN
    IF ce n = '1' THEN
      data out <= 'Z';</pre>
    ELSE
      CASE sel is
        WHEN "00"=> data out <= data in(0);
        WHEN "01"=> data out <= data in(1);
        WHEN "10"=> data out <= data in(2);
        WHEN "11"=> data out <= data in(3);
        WHEN OTHERS => null;
      END CASE;
    END IF;
  END PROCESS;
END ARCHITECTURE RTL;
```



More on VHDL

- The study material on synthesis shows a number of VHDL constructs and the resulting hardware
- The following slides contain extra material The book gives many examples and detailed explanations of VHDL

Synthesis tool Quartus

- The course textbook contains a CD with the synthesis tool, Quartus
- You will use Quartus in Lab 3

Summary

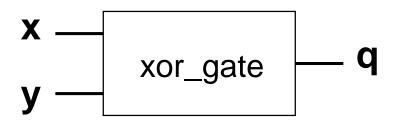
- Implementation of functions with MUXes
 - Shannon decomposition
- Look-up tables, ROM
- Decoder, encoder, code converters
- Introduction to VHDL
- Next lecture: BV pp. 383-418, 469-471

VHDL (Not part of the exam)



Entity (XOR)

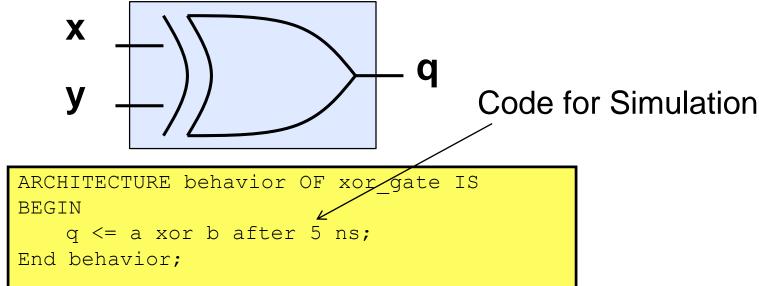
- An entity describes a component's interface with the outside world
- PORT-declaration indicates if it is an input or an output
- An Entity is a symbol of a component.



Use English names for variable names in the code!

Architecture (XOR)

- An architecture describes the operation of a component
- An entity can have many architectures, but only one can be active at a time
- An architecture corresponds to the component diagram or behavior



Signal declaration

<u>Signal-declaration</u> is used inside architectures to declare internal (local) signals:

signal a, b, c, d: bit;

signal a, b, sum: bit_vector (31 downto 0);

Signal-assignment is used to describe the behavior:

sum <= a + b; signal assignment without delay

Sequential vs Parallel Code

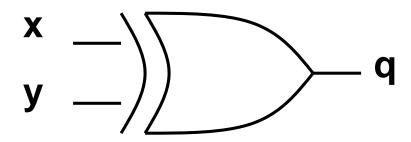
- There are two types of code execution in VHDL: sequential and parallel
 - Sequential code describes the hardware from a "programmer's" point of view and it is executed in the order which is defined
 - The parallel code is executed regardless of the order.
 It is asynchronous.

VHDL description styles

- Sequential (Behavioral)
 - similar to how to write desktop applications
- Data Flow (RTL)
 - Concurrent assignments
- Structural
 - similar to how to connect components

Sequential style

XOR gate



```
Process (x, y)
begin
    if (x/= y) then
        q <= '1';
    else
        q <= '0';
    end if;
end process;</pre>
```

Data flow style

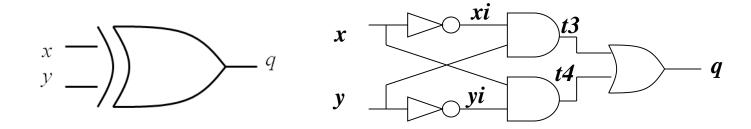
XOR gate
$$\begin{pmatrix} \mathbf{x} & - \\ \mathbf{y} & - \end{pmatrix}$$

```
q <= a xor b;

- Or in behavioral dataflow style

q <= '1' When a /= b else "0";</pre>
```

Structural style



```
u1: not_gate port map (x,xi);
u2: not_gate port map (y,yi);
u3: and_gate port map (xi,y,t3);
u4: and_gate port map (yi,x,t4);
u5: or_gate port map (t3,t4,q);
```

Structural style Component declaration

A component must be declared before it can be used

```
ARCHITECTURE Test OF test_entity

COMPONENT and_gate

Port (in1, in2: IN BIT;

out1: BIT OUT);

END COMPONENT;
... more statements...
```

• It is necessary, unless it is not in a library somewhere

Structural style Component instantiation

 Component instantiation ring connects the component interface with the signals in the architecture

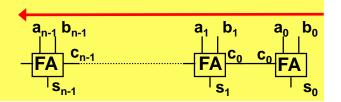
```
ARCHITECTURE Test OF test_entity

COMPONENT and_gate

Port (in1, in2: IN BIT;
out1: BIT OUT);
END COMPONENT;
SIGNAL S1, S2, S3: BIT;

BEGIN
Gate1: and_gate PORT MAP (S1, S2, S3);
END test;
```

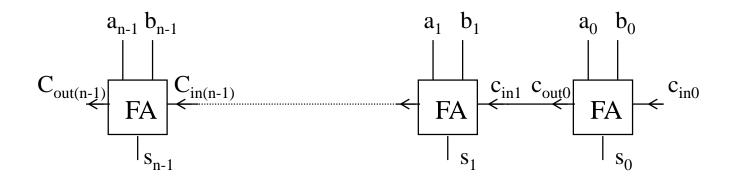
Generate



Generate-statement couples many similar elements

```
ENTITY adder IS
   GENERIC (N: integer)
   PORT (a, b: IN bit vector (N-1 downto 0);
        sum: OUT bit vector (N-1 downto 0));
END adder;
ARCHITECTURE OF structural adder IS
 COMPONENT full adder
   PORT (a, b, cin: IN bit; cout, s: OUT bit);
 END COMPONENT;
  signal c: bit vector (N-2 downto 0);
BEGIN
 GO: for i in 1 to N-2 Generate
   U0: full adder PORT MAP (a (i), b (i), c (i-1), c (i), p (i));
 end Generate; - G0
 U0: full adder PORT MAP (a(0), b(0), '0', c(0), p(0));
 UN: full adder PORT MAP (a(n-1),b(n-1),c(n-2),OPEN, s(n-1);
END structural;
```

Generate n-bit adder



Five lines of code generates the ripple-carry n-bit adder from Lecture 5!

The test bench stimuli 1

The ENTITY is empty!

```
ENTITY testbench IS END testbench;
ARCHITECTURE xor stimuli 1 of testbench IS
   COMPONENT xor gate
                                    The circuit under test is
     PORT(x, y: IN bit; q:OUT bit);
                                     used as a component of the
   END COMPONENT;
    signal x, y, u1:bit;
                                     test bench program
BEGIN
    x \le not(x) after 10 ns;
                                          Here are the test
    y \le not(y) after 20 ns;
   U1:xor gate PORT MAP (x,y,ut1);
                                           signals generated
END example;
```

T=20ns
f=50MHz
X 10ns
10ns

The test bench stimuli 2

```
ENTITY testbench IS END testbench;
ARCHITECTURE xor stimuli 2 of testbench IS
   COMPONENT xor gate
     PORT(x,y:IN bit;q:OUT bit);
   END COMPONENT;
    signal x,u1,u2,u3:bit; -- Endast en in-signal
   for U1:xor gate use entity work.xor gate (behave);
   for U2:xor gate use entity work.xor gate(data flow);
   for U3:xor gate use entity work.xor gate(structural);
BEGIN
   x \le not(x) after 10 ns;
   U1:xor gate PORT MAP (x,x,ut1);
   U2:xor gate PORT MAP (x,x,ut2);
   U3:xor gate PORT MAP (x, x, ut3);
END example;
```

Test bench

A test bench can mark when the desired events occur during the execution.



Or mark when unwanted events occur



The result of a run with a test bench can be saved in a file, as a proof that everything is ok - or as a troubleshooting aid if it did not go well.