



## BLEKINGE INSTITUTE OF TECHNOLOGY

Written test in (subject): ET2596 Simulation

Date: 2019.01.19

Name: \_\_\_\_\_

Civic number: \_\_\_\_\_

Number of sheets handed in: \_\_\_\_\_

Mark the question(s) you have answered by putting a ring around the relevant number(s)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

### Instructions

A student who cannot produce valid ID will not be permitted to take the examination.

No examination scripts will be accepted by the proctor during the first hour of the examination.

(Students arriving late will thus be permitted to take part in the examination).

Write your name and civic number on each sheet of paper you hand in.

Examination results are posted by e-mail no later than 10 working days after the date of the examination. Exceptions to this rule can occur. In this case, students will be informed by the teacher responsible for the course/program or by the examiner.

All blank answer sheets are to be handed in to the proctor.

(To be filled in by the proctor)

ID presented:

\_\_\_\_\_  
Proctor's sign.

Student union fee paid:

\_\_\_\_\_  
Proctor's sign.

Student union fee not paid:

\_\_\_\_\_  
Proctor's sign.

(To be filled in by the teacher )

Number of credits gained: \_\_\_\_\_ Grade: \_\_\_\_\_ ECTS: \_\_\_\_\_ Examiner's sign: \_\_\_\_\_

(To be filled in and signed by the student, after the correction of the examination)

I hereby sign my examination script. I am aware that by signing for my script, after correction, I waive my right to contest the examiner's comments and the credits or grade awarded.

Date \_\_\_\_\_ Signature: \_\_\_\_\_

# Re-Exam In

*Simulation*  
*(2019.01.19) ET2596*

Wednesday: 11:00 to 16;00

Lecturer: Siamak Khatibi

**Allowed items on exam: Open book**

The exam includes 5 problems (100 credit points); where for grade in ECTS you should obtain as following:

F (0-32), FX (33-49),  
E (50-57), D (58-64),  
C (65-74), B (75-82),  
A (83-100)

Good Luck

PS: You can write your explanation texts on the paper or digitally however your Matlab or C codes should be delivered digitally.

Please do not forget to write which questions you have answered by checking under their numbers on examination folder.

## Q1

In this problem we have a single server queueing model. Customers arrive at random time intervals to a service station. One customer at a time is served while others form a first-in-first-out type of queue. Inter-arrival time of customers as well as service times follow the exponential distribution with mean values of  $\lambda$  and  $\mu$ , respectively.

a)- Implement the model to simulate the delay in queue for the first  $n$  customers arriving at the service station.

Notice, that a very simple recursion can be used: Let  $D_i$  be the delay in queue for the  $i^{\text{th}}$  customer,  $S_i$  the service time and  $T_i$  the inter-arrival time between  $i+1^{\text{st}}$  and  $i^{\text{th}}$  customers. It holds:

$$D_i = (D_i + S_i - T_i) = \max\{D_i + S_i - T_i, 0\} \quad (12p)$$

b)- Use your model to estimate the average delay in queue when  $n=100$ ,  $\lambda=1$  and  $\mu=0.9$ .

(2p)

c)- Repeat the simulation to obtain multiple observations (e.g. 100 times) on the average delay and plot your result. In addition, construct a confidence interval for the estimate based on Student's t-distribution as described in the lectures. (6p)

## Q2

Consider the following queueing model which is coded in “Q2.m” file.

Customers arrive according to a (stationary) Poisson process at rate 1 per minute. Server 1 provides service whose duration is exponentially distributed with a mean of 0.7 minutes. Upon exiting server 1, customers leave the system with probability  $p$  and go to server 2 with probability  $1-p$ . At server 2, the duration of service is exponentially distributed with a mean 0.9 minutes.

The service capacity for both servers is 1. The queueing discipline at both servers is first in first out (FIFO). The system is initially empty and idle. It runs until 100 customers have been completely served. The total queueing delay for a customer that must visit both servers is, naturally, the sum of delays at both places. The performance measure of interest is the expected average total waiting time in both queues.

a) Suppose there are 2 configurations of the system with  $p$  being either 0.3 or 0.8. Make 10 replications of the simulation with both values of  $p$  by first using independent random numbers. Then repeat the experiment using common random numbers (CRN) for both inter-arrival times and service times across the systems. Compare the estimated means and variances of the difference between the performance measures. Comment your result. Hint: Choose the “rstate” input parameter in Q2.m in a way that be able to control the RNGs (i.e. to find common or independent generated random numbers).

(12 p)

b) For  $p=0.3$ , make five pairs of runs using both independent sampling and antithetic variates within each pair. Compare the estimated means and variances of the performance measures. Comment your result. (8 p)

### **Q3**

In this problem we consider implementing the acceptance/rejection method for generating random variates from the half-normal distribution:

$$f_{|z|}(x) = \frac{2}{\sqrt{2\pi}} e^{-x^2/2}$$

using, as alternative sampling distribution, the exponential distribution with the density:

$$g(x) = e^{-x}$$

Generate a sample of 1000 points with your code (Q3.m) and observe the average number of variates X that is needed to produce one accepted random variate Y (you need to modify the code to get the average number of variates X).

Compare this number to the value of c in the code. What is your observation? What does this imply from the viewpoints of selecting the alternative sampling distribution g and the efficiency of the algorithm. (20 p)

### **Q4A**

An electrical system consists of two parallel components. The probability that each individual component will fail is 0.125. The system fails if both components fail.

a)- Estimate the probability of system failure using parallel components by simulating the failure of each component for 1000 times. (8 p)

b)- What will happen if the two components are arranged in series (i.e. after 1000 simulations)? (2 p)

### **Q4B**

a)- Generate a sample of 50 random numbers with the generator (i.e. the “rand” function) and use the chi-square test (the chi-square test is coded in Q4b.m file) to assess whether the numbers are uniformly distributed. (2 p)

b)- Repeat the tests for 100 independent sequences of random numbers and define the proportion of cases in which the hypothesis of uniformity was rejected. (8 p)

### **Q5**

Please use function “Q5.m” which is simulating a M/D/1 queueing system with Poisson arrivals of intensity  $\lambda = 0.95$  and deterministic service times  $S=1$ .

Estimate average of the time a customer spends (ST) in the system for 100 and 1000 batches if  $T_{\max}=1200$ .

a) Plot ST, its histogram, and suggest a distribution for ST data from observing its histogram (Comment/argue your suggestion), (4 p)

- b) Calculate the mean of ST with 95% confidence interval (i.e. mean $\pm$  half confidence interval) ;use the “icdf” function in Matlab,  
(6 p)
- c) Calculate the autocorrelation (lag-1) for ST and based on it comment your result in b,  
(4 p)
- d) Comment/argue about your results (i.e. the calculation of mean value of ST in relation to the confidence interval and autocorrelation values) from 100 and 1000 batches.  
(6 p)