Problem set -5

Q1

Consider the simulation of a queueing system with m parallel servers. You can use the provided "Q1.m" file your simulation. Customers arrive at exponentially distributed inter-arrival times with rate λ. The service rate of server i is μ_i. Each server has its own queue with FIFO discipline.
An arriving customer will always select the server with least customers present. If ties exist, the customer selects the server randomly.
Simulate the system to estimate the expected difference in the average queueing delay of customers for the following cases:

1- λ=1, m=2, μ_1=0.6, μ_2=0.6

2- λ=1, m=2, μ_1=0.3, μ_2=0.9

Terminate the simulation in both designs, when 100 customers have left the system. First, define the difference in queueing delays by performing 100 independent replications for the two designs. Then, apply common random numbers to see, whether you can improve the accuracy of your estimate without increasing the amount of simulation replications.

Q2

Consider the following queuing model which is coded in "Q2.m" file.
Customers arrive according to a (stationary) Poisson process at rate 1 per minute. Server 1 provides service whose duration is exponentially distributed with a mean of 0.7 minutes. Upon exiting server 1, customers leave the system with probability p and go to server 2 with probability 1-p. At server 2, the duration of service is exponentially distributed with a mean 0.9 minutes.
The service capacity for both servers is 1. The queuing discipline at both servers is first in first out (FIFO). The system is initially empty and idle. It runs until 100 customers have been completely served. The total queuing delay for a customer that must visit both servers is, naturally, the sum of delays at both places. The performance measure of interest is the expected average total waiting time in both queues.

a) Suppose there are 2 configurations of the system with p being either 0.3 or 0.8. Make 10 replications of the simulation with both values of p by first using independent random numbers. Then repeat the experiment using common random numbers (CRN) for both inter-arrival times and service times across the systems. Compare the estimated means and variances of the difference between the performance measures. Comment your result. Hint: Choose the "rstate" input parameter in Q2.m in a way that be able to control the RNGs (i.e. to find common or independent generated random numbers).

b) For p=0.3, make five pairs of runs using both independent sampling and antithetic variates within each pair. Compare the estimated means and variances of the performance measures. Comment your result.

Q3

In the Excel file of Q3.xls you can find the result data which were generated by a simulation software for a M/M/1 system. In this problem we are interested to investigate how the number of batches (or, in general, the number of independent data points) affects the confidence interval for the point estimate value (in this case, the mean of que delay).
In the simulation we had mean interarrival time of 4, mean service time of 3.5 and 200 customers. The file contains several worksheets (Batches = 10, Batches = 100, Batches =1000, Batches =2000 and normalness). In each Runs worksheet of 10, 100 and 1000 the confidence intervals for 80%, 90% and 95% for the related data in the sheet are calculated. However, in the Batches=2000 worksheet we try to determine the number of batches necessary for a given confidence interval.
Note: Some of the cells in the spreadsheet have comments (designated by the small red triangle in the upper-right corner). Read these comments -- some of the comments are important.

Now, from the results in the data sheets

a) Argue (by facts) how the confidence intervals (CIs) behaves in relation to the confidence levels.

b) Argue (by facts) how the confidence intervals (CIs) behaves in relation to the same confidence level.

c) Argue (by facts), i.e. using lag-1 autocorrelation, if we are successful to obtain a given confidence interval and determine the number of batches.

d) Argue (by facts) what we are doing in the last worksheet (the normalness).

e) Why we are using t-student distribution in calculation of CIs. Does it affect our CI results? If yes show how much?

Q4

In this problem we try to experience the input modeling problem which affects a M/M/1 system. Assume we are observing a small bank in Karlskrona which its queuing system can be modeled as a M/M/1. Let's assume the modeling is working smoothly with a loading of 70%. As a matter of fact, such a small bank use to have a special door which facilitate the arrival of bank customers. If the door has two parts (i.e. a customer can open one of the two door parts to approach the bank) then effect of each part can be thought as a random variable (RV) such as $X1$ and $X2$ with the same probability of a normal distribution; $N(0, 0.025)$. Assuming each part of door is independent of the other one, then to show the joint distribution of RVs $X1$ and $X2$ we can write

$$Z = X1 + j\,X2$$

where Z is a RV and has a complex normal distribution. By this way we have guaranteed that $X1$ (the real) and $X2$ (the imaginary) are independent. Now assume what affects the bank system is the amplitude of such complex distribution (i.e. the amplitude of Z is our new arrival distribution instead of the exponential one for the arrival).

a) – Generate enough random data which can represent the amplitude of Z as RV.

b) - Estimate the pattern of the outcome of a).

c) - Use the generated data in a) and simulate a M/M/1 system, assuming still we have a Markovian system with the disturbances. Here you can use the Q4sim.m file if you have no better idea.