BLEKINGE INSTITUTE OF TECHNOLOGY

Written test in (subject): ET2596 Simulation

Date: 2019.03.28

Name: _____

Civic number: _____

Number of sheets handed in:_____

Mark the question(s) you have answered by putting a ring around the relevant number(s)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Instructions
A student who cannot produce valid ID will not be permitted to take the examination.
No examination scripts will be accepted by the proctor during the first hour of the examination. (Students arriving late will thus be permitted to take part in the examination).
Write your name and civic number on each sheet of paper you hand in.
Examination results are posted by e-mail no later than 10 working days after the date of the examination. Exceptions to this rule can occur. In this case, students will be informed by the teacher responsible for the course/program or by the examiner.
All blank answer sheets are to be handed in to the proctor.

(To be filled in by the proctor)

ID presented: _____
Proctor's sign.

Student union fee paid: _____
Proctor's sign.

Student union fee not paid: _____
Proctor's sign.

(To be filled in by the teacher)

Number of credits gained:_____ Grade:_____ ECTS:_____Examiner´s sign:_____

(To be filled in and signed by the student, after the correction of the examination)

I hereby sign my examination script. I am aware that by signing for my script, after correction, I waive my right to contest the examiner's comments and the credits or grade awarded.

Date_____ Signature:_____

Re-Exam In

Simulation
(2019.03.28) ET2596

Wednesday: 09:00 to 14:00

Lecturer: Siamak Khatibi

Allowed items on exam: Open book

The exam includes 5 problems (100 credit points); where for grade in ECTS you should obtain as following:

F (0-32), FX (33-49),
E (50-57), D (58-64),
C (65-74), B (75-82),
A (83-100)

Good Luck

PS: Each question is answered by a mfile using Matlab (i.e. Q1sol.m, Q2sol.m, …). In the header of each mfile you write your name and personal number. In each mfile beside your code for solution of the respective problem you should write your comments/arguments. You can write/draw on the paper as complementary material to your respective mfile (please mention in mfile you have such complementary).

You should zip all your digital materials (your mfiles related to each question, other used functions which are not standard Matlab functions, mat file, …) and upload it to the Canvas under module “exam”, in “Download your answer to exam 2019-03-28”. You deliver your complementary materials to the invigilator of exam.

Q1A

Consider the joint probability density function for random variables **X** and **Y** that is uniform on a circle of radius 10 in the X-Y plane; i.e.,

$$f_{XY}(x,y) = \begin{cases} k & x^2 + y^2 < 100 \\ 0 & else \end{cases}$$

a)- Find k. (5p)

b)- Calculate Pr($x^2 + y^2 < 5$). (5p)

Q1B

Let **X** and **Y** be Gaussian independent random variables with mean 0 and variance 4. Let $\mathbf{Z} = \sqrt{\mathbf{X}^2 + \mathbf{Y}^2}$.

a)- Find an expression for the probability density function for **Z**. (5p)

b)- Calculate Pr(**Z**<1). (5p)

Q2

a)- Consider a Gaussian white random process X(t) that is N(0, $\sigma = 9$) for $2 < t < 5$, and 0 elsewhere. Plot 3 sample functions from this random process, on the range $t \in (0, 6)$. (5p)

b)- Now consider another Gaussian white random process Y(t) that is N(0, $\sigma = 7$) for $t<2$ and $t> 5$, and 0 elsewhere. Plot 3 sample functions from this random process, on the range $t \in (0, 6)$. (3p)

c)- Combine the two random processes of X(t) and Y(t) in (a) and (b) and Plot 3 sample functions from this random process, on the range $t \in (0, 6)$. Show the histograms of your 3 samples. Are these 3 histograms the same? Argue/comment your answer. (8p)

d)- Is the result from (c) an iid signal? Can you suggest a distribution model for (c)? Argue/comment your answer. (4p)

Q3

In this problem you should use "Q3.m". The file includes 3 systems of M/D/1, M/G/1 and M/M/1. From customer point of view which system is preferred. Argue about your choice. Hint: Run several times the code. Choose different parameters which can be significant between systems. Think yourself as a customer, which parameter becomes important to you. (20 p)

Q4A

In simulation run, we discussed about warm-up in the lectures. We argued that without warm-up procedure the result will be non-stationary. Now we would like to simulate a non-stationary sequence of random numbers which has Gaussian characteristic.
You can simulate this sequence in many ways. Please show one of these ways. (10 p)

Q4B

For a simulation, we need to generate random variate numbers (RVN). The cumulative distribution F is known and is according the following code:

```
x=0:(pi/100): pi/2;
F=cos(x);
```

Generate 2000 random variate number and show the histogram of the numbers. (10 p)

Q5A

We would like to estimate $\theta = \int_0^1 e^{2x}$ by simulation. We can show some of statistical properties by

```
disp([mean(X) std(X) 2*std(X)/sqrt(N)])
```

Now use 2 methods of the variance reduction techniques (Common Random Numbers and Antithetic Variates) to reduce the variance of the estimation.

Hint: Use half of N to generate two new random numbers. (12 p)

Q5B

In Q5A we calculate 3 relative mean values (i.e. the mean of estimation and then the mean values by the two methods). Make a 100 batch of the three mean calculations then calculate their means by 95% and 99% confidence interval. (8p)