BLEKINGE INSTITUTE OF TECHNOLOGY

Written test in (subject): ET2596 Simulation

Date: 2021.05.31

Name: ____

Civic number: ____________________

Number of sheets handed in:________

Mark the question(s) you have answered by putting a ring around the relevant number(s)

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Instructions
A student who cannot produce valid ID will not be permitted to take the examination.
No examination scripts will be accepted by the proctor during the first hour of the examination. (Students arriving late will thus be permitted to take part in the examination).
Write your name and civic number on each sheet of paper you hand in.
Examination results are posted by e-mail no later than 10 working days after the date of the examination. Exceptions to this rule can occur. In this case, students will be informed by the teacher responsible for the course/program or by the examiner.
All blank answer sheets are to be handed in to the proctor.

(To be filled in by the proctor)

ID presented: ________
Proctor's sign.

Student union fee paid: ________
Proctor's sign.

Student union fee not paid: ________
Proctor's sign.

(To be filled in by the teacher)
Number of credits gained:_____Grade:_____ECTS:_____Examiner´s sign:_______

(To be filled in and signed by the student, after the correction of the examination)
I hereby sign my examination script. I am aware that by signing for my script, after correction, I waive my right to contest the examiner's comments and the credits or grade awarded.

Date______________ Signature:______________________________________

Re-Exam In

Simulation
(2020.05.31) ET2596

Tuesday: 9:00 to 15:00

Lecturer: Siamak Khatibi

Allowed items on exam: Open book

The exam includes 5 problems (100 credit points); where for grade in ECTS you should obtain as following:

F (0-32), FX (33-49),
E (50-57), D (58-64),
C (65-74), B (75-82),
A (83-100)

Digital solutions: Please use Matlab and write your solutions as mfiles (i.e., Q1sol.m, Q2sol.m, …). In the header of each mfile you should write your name and personal number. In each mfile beside your code you can write your comments/arguments.
You should zip all your digital materials (e.g., your mfiles related to each question, other used functions which are not standard Matlab functions, mat file, …) and upload it to the Canvas under module "exam", in "Upload your result to re-exam 2021-05-31".

Paper solutions: You can write/draw on the paper as complementary material to your respective mfile (please mention in mfile you have such complementary). You can also digitalize your written work in any form it is convenient for you (e.g., photo, scan, …).

References and Plagiarism: Please mention any used or inspired reference (e.g., link, paper, book, assignment, notes, …). Otherwise, your solution will be marked as copy plagiarism and reported to discipline committee of BTH.

Q1

In this problem, please use the function "Q1sim.m" which is simulating a M/M/1 system. One of the output parameters of the function is "ro" representing the offered load, $\rho_{offered}$. The offered load shows the average amount of services to average number of arrivals until measured simulation time (i.e., the actual load in the specific time).

a)- Simulate the M/M/1 system when ρ is 75% and simulation time is 1500 minutes. Plot the $\rho_{offered}$. (3p)

b)- Find a way (i.e., generate your own algorithm) which can identify how long and when $\rho_{offered}$ of the system is around the designed load (e.g., in question a) we had ρ which was 75%) with a deviation not more than 3% of designed load. Test your algorithm with the simulation parameters in a). (6p)

c)- Now assume we call the simulation time before and after reaching to designed load as setup_time and system_time respectively. Repeat the simulation in a) for 500 times and for each simulation calculate the setup_time according to your algorithm from b). Use the function "sort" and plot the sorted data of setup_times for your 500 simulations. Comment/argue the obtained result in your plot. (6p)

d)- Repeat the simulation in c) when ρ is 15% and 95%. Plot the sorted data of setup_times for your 500 simulations for all three ρ values. Comment/argue the obtained results in your plot. (5p)

Q2

In this problem we continue to use the function "Q1sim.m" which is simulating a M/M/1 system. We also use your answer for question Q1-b) which can help us to find setup_time and system_time.

a)- Simulate the M/M/1 system when ρ is 75% and simulation time is 1500 minutes. Find the sojourn time and number of customers for the whole simulation time and plot the sorted of these data. (3p)

b)- Now find the sojourn time and number of customers during the system_time for your simulation in a). Plot the sorted data of these data. How long is the system_time duration in your simulation? Compare the result from a) and b) and comment/argue about them. (5p)

c)- Repeat the simulation as a) for 50 times and calculate mean and standard error of number of customers in the system with and without consideration of system_time. Plot your results (as sorted data) in one figure and from that comment/argue about your results. How would these results affect the confidential interval calculation? (7p)

d)- Show the statistical pattern of your results of mean in c) without and with consideration of system_time in the same figure. Show in another figure the statistical pattern of your results of standard error in c) without and with consideration of system_time. Comment/argue the figures. Are they confirming what you conclude from the result in c)? (5p)

Q3

In this problem we intend to use our results and argumentation from previous problems and investigate the performance of two methods: Batch-Means and Replication-Deletion methods. Still, you can use the function "Q1sim.m" which is a M/M/1 system. Let us configure the system with 65% loading (i.e., the ρ=0.65) and system_time will be calculated for a designed load deviation not more than 0.5%.

a)- Calculate the 50%, 60%, 75% confidence intervals of the number of customers in the system and the sojourn time of customers in the system with Batch-Means method. Consider that if you intend to have a certain number of batches (e.g., 41), you cannot predefine the batch size and need to find an appropriate simulation which its setup_time (i.e., the batch size) can correspondent to your simulation endtime (i.e, the endtime is multiplication of batch_size and number_of_batches).

Hint: repeat the simulation with predefined loading, number_of_batches and endtime and find the appropriate batch size (i.e., your batch size should be big enough to obtain meaningful statistical values). (10p)

b)- Calculate the 50%, 60%, 75% confidence intervals of the number of customers in the system and the sojourn time of customers in the system with Replication-Deletion method. Consider that if you intend to have a certain number of batches (e.g., 40), you need to find simulations which have a system_time bigger or equal to setup_time. (7p)

c)- Argue/comments your results from a) and b) to compare the performance of the two methods while you are obtaining different levels of confidence intervals. (4p)

Q4

Generate 10000 samples of random variates utilizing the acceptance rejection method for the following probability density function,

$$f(x) = \begin{cases} \dfrac{2(x-a)}{(b-a)(c-a)}, & if\ a \le x \le c \\ \dfrac{2(b-x)}{(b-a)(b-c)}, & if\ c \le x \le b \\ 0, & otherwise \end{cases}$$

where a=0 and $b=10$ and c, the mode, can be 1, 2, …, 9.

a)- Generate 30000 samples of random variates for each case of c value, the mode. Use the "sort" function and plot the sorted of generated samples. Show all these plots in the same figure. Also show in another figure nine histograms of samples together.

(15p)

b)- Explain/argue the role of the mode with consideration of your results from a). For which typical applications can we use such random variates in our simulation?

(5p)

Q5A

Assume we are interested to estimate the following integral by simulation (i.e., a Monte Carlo approach).

$$\partial = \int_0^1 x e^{x^2} dx$$

This is achieved by calculation $\partial = E\left[Ue^{U^2}\right]$where U has an uniform distribution as U(0,1).

a)- Simulate for 10000 points and calculate the ∂. Also calculate the variance of your simulated result.

(4p)

b)- Use antithetic variates method and repeat the simulation in a). Compare the two simulation results in relation to variance.

(6p)

Q5B

We can generate antithetic normal random variates without using the inverse transform function method. The reason is if our aim is to generate random variates with a normal distribution, i.e. X is $N(\mu, \sigma^2)$, then by defining a new random variate as Y= 2μ-X, its distribution still is $N(\mu, \sigma^2)$. However, the X and Y are negatively correlated which makes it possible to use Y for antithetic variates application. Now assume we are interested to estimate the following mathematical term,

$$\partial = X^3 + 3X + 2.$$

This is achieved by calculation $\partial = E[X^3 + 3X + 2]$ where X has a normal distribution, $N(\mu, \sigma^2)$.

a)- Simulate for 10000 points and calculate the ∂. Also calculate the variance of your simulated result.

(4p)

b)- Use antithetic normal random variates method and repeat the simulation in a). Compare the two simulation results in term of variance.

(6p)