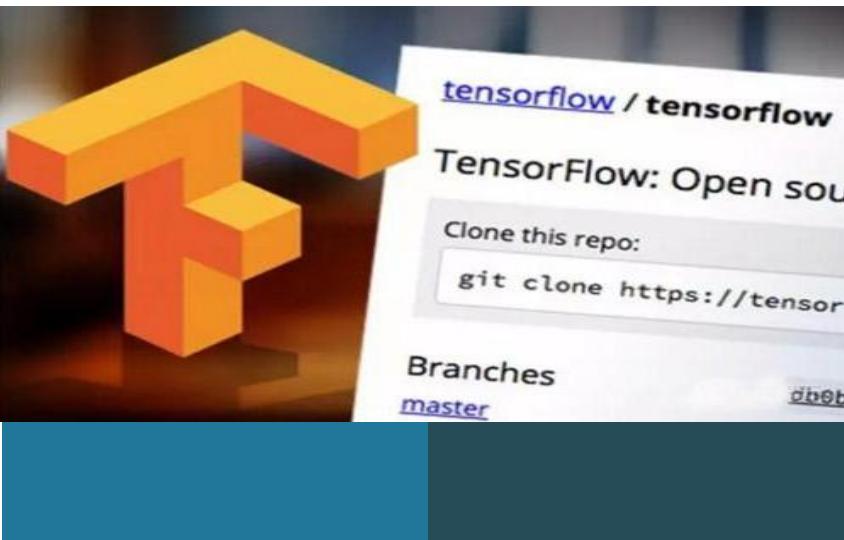




11 分类问题

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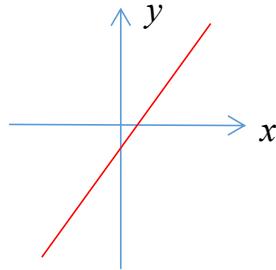
11.1 逻辑回归

线性回归

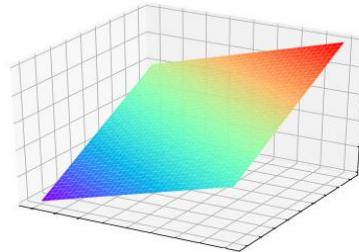
将自变量和因变量之间的关系，用**线性模型**来表示

根据已知的样本数据，对**未来的**、或者**未知**的数据进行**估计**

$$y = wx + b$$



$$y = w_1x_1 + w_2x_2 + b$$

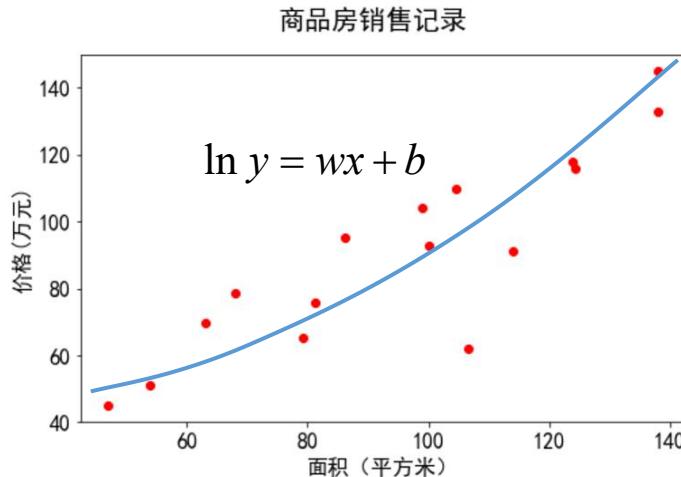


$$y = w_1x_1 + \dots + w_mx_m + b$$

超平面 (Hyperplane)



■ 广义线性回归



对数线性回归 (log-linear regression)

$$\ln y = wx + b \quad y = e^{wx+b}$$

$$Y = wx + b$$

$$g(y) = wx + b \quad y = h(wx + b)$$

$$y = g^{-1}(wx + b)$$

广义线性模型 (generalized linear model)

$g(\cdot)$: 联系函数 (link function)

任何单调可微函数

高维模型 $Y = g^{-1}(W^T X)$

$$W = (w_0, w_1, \dots, w_m)^T$$

$$X = (x^0, x^1, \dots, x^m)^T$$

$$x^0 = 1$$



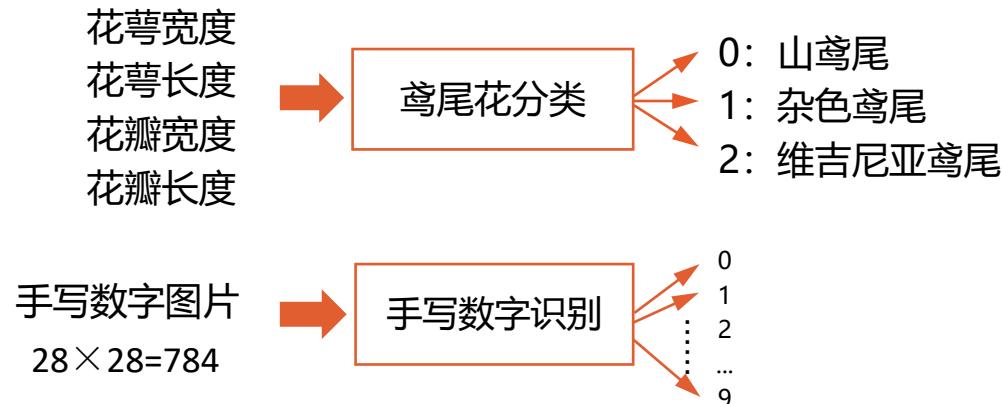
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分类问题: 垃圾邮件识别、图片分类、疾病判断

分类器: 能够自动对输入的数据进行分类

输入: **特征**, 输出: **离散值**

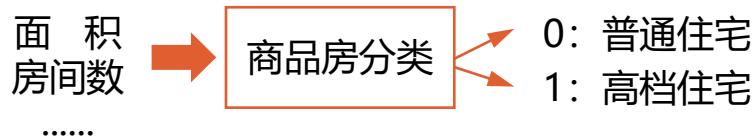


实现分类器

准备训练样本

训练分类器

对新样本分类



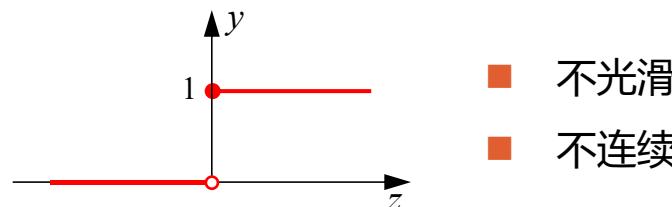
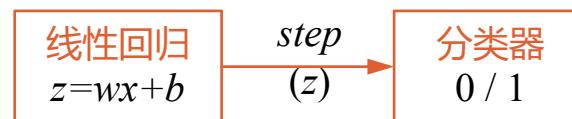
单位阶跃函数 (unit-step function)

$$y = \begin{cases} 0, & z < 0; \\ 1, & z \geq 0; \end{cases}$$

$$z = wx + b$$

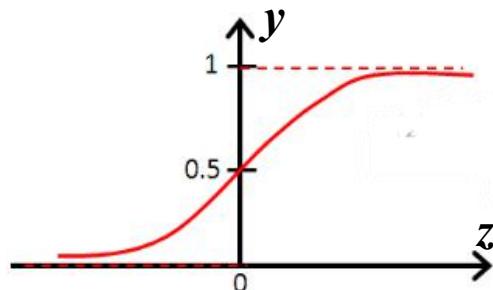
$$y = \text{step}(z) = \begin{cases} 0, & z - 1000000 < 0 \\ 1, & z - 1000000 \geq 0 \end{cases}$$

二分类问题: 1 / 0 —— 正例和反例



对数几率函数 (logistic function)

$$y = \frac{1}{1+e^{-z}} \quad \rightarrow \quad \ln \frac{y}{1-y} = z$$



- 单调上升，连续，光滑
- 任意阶可导

对数几率回归/逻辑回归 (logistic regression)

$$y = \frac{1}{1+e^{-(wx+b)}}$$

Sigmoid函数

$$y = g^{-1}(z) = \underline{\sigma(z)} = \sigma(wx + b)$$

$$\sigma(z) = \frac{1}{1+e^{-z}} = \frac{1}{1+e^{-(wx+b)}}$$

多元模型

$$y = \frac{1}{1+e^{-(W^T X)}}$$

$$W = (w_0, w_1, \dots, w_m)^T$$

$$X = (x^0, x^1, \dots, x^m)^T$$

$$x^0 = 1$$



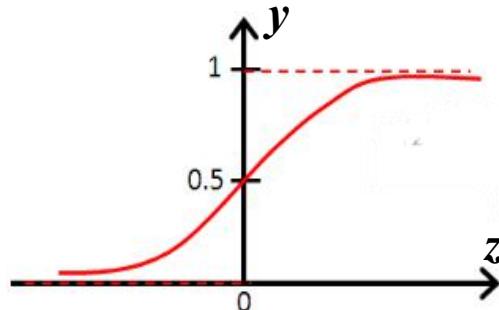
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逻辑回归

$$y = g^{-1}(z) = \sigma(z) = \sigma(wx + b)$$

$$\sigma(z) = \frac{1}{1+e^{-z}} = \frac{1}{1+e^{-(wx+b)}}$$



平方损失函数

$$Loss = \frac{1}{2} \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \frac{1}{2} \sum_{i=1}^n (y_i - \sigma(wx_i + b))^2 = \sum_{i=1}^n (y_i - \frac{1}{1+e^{-(wx_i+b)}})^2 \quad \text{非凸函数}$$

$$w^{(k+1)} = w^{(k)} - \eta \frac{\partial Loss}{\partial w}$$

$$b^{(k+1)} = b^{(k)} - \eta \frac{\partial Loss}{\partial b}$$

$$\frac{\partial Loss}{\partial w} = \sum_{i=1}^n (y_i - \sigma(wx_i + b))(-\sigma'(wx_i + b))x_i$$

$$\frac{\partial Loss}{\partial b} = \sum_{i=1}^n (y_i - \sigma(wx_i + b))(-\sigma'(wx_i + b))$$



11.1 逻辑回归

■ 交叉熵损失函数

$$Loss = - \sum_{i=1}^n [y_i \ln \hat{y}_i + (1 - y_i) \ln(1 - \hat{y}_i)]$$

y_i	第 <i>i</i> 个样本的标记
\hat{y}_i	$\hat{y}_i = \sigma(wx_i + b)$

~~$\times Loss = - \sum_{i=1}^n [\hat{y}_i \ln y_i + (1 - \hat{y}_i) \ln(1 - y_i)]$~~

平均交叉熵损失函数

$$Loss = - \frac{1}{n} \sum_{i=1}^n [y_i \ln \hat{y}_i + (1 - y_i) \ln(1 - \hat{y}_i)]$$



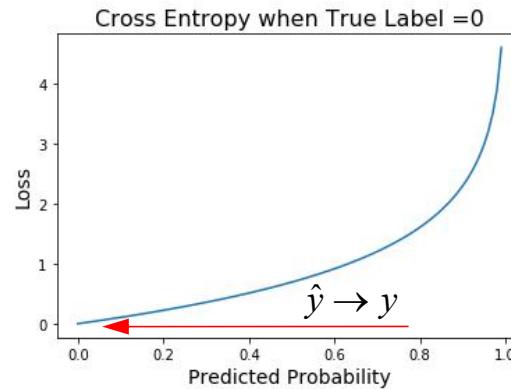
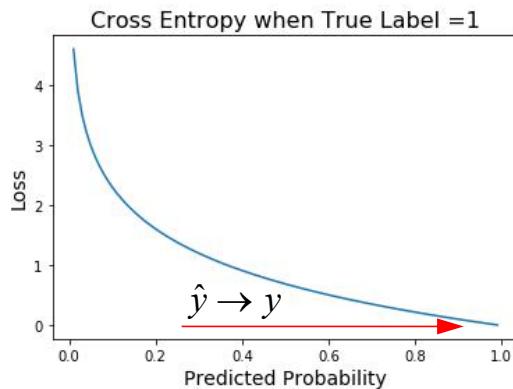
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11.1 逻辑回归

交叉熵损失函数 :概率分布之间的误差

$$Loss = - \sum_{i=1}^n [y_i \ln \hat{y}_i + (1 - y_i) \ln(1 - \hat{y}_i)]$$



■ 无需对 σ 函数求导

$$\frac{\partial Loss}{\partial w} = \frac{1}{n} \sum_{i=1}^n x_i (\hat{y}_i - y_i)$$

$$\frac{\partial Loss}{\partial b} = \frac{1}{n} \sum_{i=1}^n (\hat{y}_i - y)$$

■ 凸函数



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:

11.1 逻辑回归

样本	标记	预测值	结果判断
样本1	0	0.1	正确
样本2	0	0.2	正确
样本3	1	0.8	正确
样本4	1	0.99	正确

准确率 (accuracy): $\frac{\text{正确分类的样本数}}{\text{总样本数}} = 100\%$

交叉熵损失: $Loss = -\frac{1}{n} \sum_{i=1}^n [y_i \ln \hat{y}_i + (1-y_i) \ln(1-\hat{y}_i)]$

样本1: $-(0 \times \ln 0.1 + 1 \times \ln 0.9) = -\ln 0.9 = 0.1053\dots$

样本2: $-(0 \times \ln 0.2 + 1 \times \ln 0.8) = -\ln 0.8 = 0.2231\dots$

样本3: $-(1 \times \ln 0.8 + 0 \times \ln 0.2) = -\ln 0.8 = 0.2231\dots$

样本4: $-(1 \times \ln 0.99 + 0 \times \ln 0.2) = -\ln 0.99 = 0.0100\dots$

交叉熵损失: 0.5616... 平均损失: 0.1404...



11.1 逻辑回归

模型A

样本	标记	预测值	结果判断
样本1	0	0.1	正确
样本2	0	0.2	正确
样本3	1	0.8	正确
样本4	1	0.49	错误

准确率：75%

交叉熵损失：

$$\begin{aligned} -(0 \times \ln 0.1 + 1 \times \ln 0.9) &= -\ln 0.9 = 0.1053... \\ -(0 \times \ln 0.2 + 1 \times \ln 0.8) &= -\ln 0.8 = 0.2231... \\ -(1 \times \ln 0.8 + 0 \times \ln 0.2) &= -\ln 0.8 = 0.2231... \\ -(1 \times \ln 0.49 + 0 \times \ln 0.51) &= -\ln 0.49 = 0.7133... \end{aligned}$$

平均损失：**0.3162...**

模型B

样本	标记	预测值	结果判断
样本1	0	0.49	正确
样本2	0	0.45	正确
样本3	1	0.51	正确
样本4	1	0.1	错误

准确率：75%

交叉熵损失：

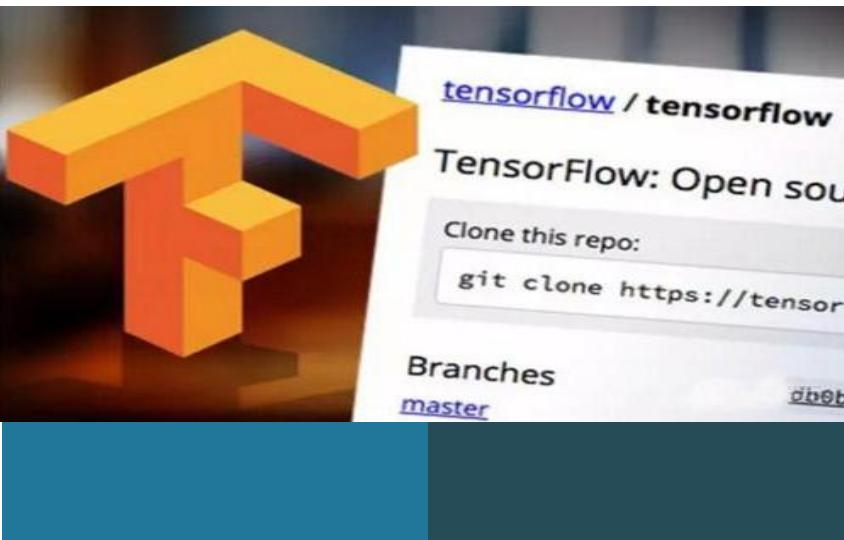
$$\begin{aligned} -(0 \times \ln 0.49 + 1 \times \ln 0.51) &= -\ln 0.51 = 0.6733... \\ -(0 \times \ln 0.45 + 1 \times \ln 0.55) &= -\ln 0.55 = 0.5978... \\ -(1 \times \ln 0.51 + 0 \times \ln 0.49) &= -\ln 0.51 = 0.6733... \\ -(1 \times \ln 0.1 + 0 \times \ln 0.9) &= -\ln 0.1 = 2.3025... \end{aligned}$$

平均损失：**1.0617...**



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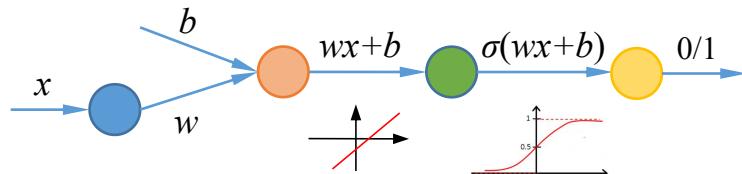


11.2 实例：实现一元逻辑回归

■ 逻辑回归

□ 一元逻辑回归

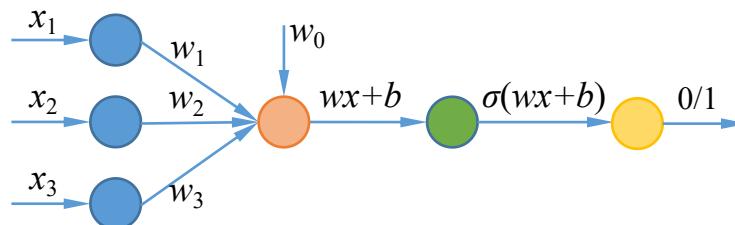
$$y = \sigma(wx + b) = \frac{1}{1 + e^{-(wx+b)}}$$



准确率 = $\frac{\text{正确分类的样本数}}{\text{总样本数}}$

□ 多元逻辑回归

$$y = \sigma(W^T X) = \frac{1}{1 + e^{-W^T X}}$$



交叉熵损失: $Loss = -\frac{1}{n} \sum_{i=1}^n [y_i \ln \hat{y}_i + (1 - y_i) \ln(1 - \hat{y}_i)]$



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□ sigmoid()函数

$$y = \frac{1}{1 + e^{-(wx+b)}}$$

```
In [1]: import tensorflow as tf
print("TensorFlow version:", tf.__version__)

TensorFlow version: 2.0.0

In [2]: import numpy as np

In [3]: x=np.array([1., 2., 3., 4.])

In [4]: w = tf.Variable(1.)
b = tf.Variable(1.)

In [5]: 1/(1+tf.exp(-(w*x+b)))

Out[5]: <tf.Tensor: id=25, shape=(4,), dtype=float32, numpy=array([0.880797, 0.95257413, 0.98201376, 0.9933072], dtype=float32)>
```



11.2 实例：实现一元逻辑回归

口 交叉熵损失函数

$$Loss = -\sum_{i=1}^n [y_i \ln \hat{y}_i + (1 - y_i) \ln(1 - \hat{y}_i)]$$

```
In [6]: y=np.array([0, 0, 1, 1])  
pred=np.array([0.1, 0.2, 0.8, 0.49])
```

```
In [7]: 1-y
```

```
Out[7]: array([1, 1, 0, 0])
```

```
In [8]: 1-pred
```

```
Out[8]: array([0.9, 0.8, 0.2, 0.51])
```

```
In [9]: -tf.reduce_sum(y*tf.math.log(pred)+(1-y)*tf.math.log(1-pred))
```

```
Out[9]: <tf.Tensor: id=37, shape=(), dtype=float64, numpy=1.2649975061637104>
```

```
In [10]: -tf.reduce_mean(y*tf.math.log(pred)+(1-y)*tf.math.log(1-pred))
```

```
Out[10]: <tf.Tensor: id=49, shape=(), dtype=float64, numpy=0.3162493765409276>
```



11.2 实例：实现一元逻辑回归

口 准确率

$$= \frac{\text{正确分类的样本数}}{\text{总样本数}}$$

```
In [11]: y=np.array([0, 0, 1, 1])
pred=np.array([0.1, 0.2, 0.8, 0.49])

In [12]: tf.round(pred)

Out[12]: <tf.Tensor: id=51, shape=(4,), dtype=float64, numpy=array([0., 0., 1., 0.])>

In [13]: tf.equal(tf.round(pred), y)

Out[13]: <tf.Tensor: id=55, shape=(4,), dtype=bool, numpy=array([ True,  True,  True, False])>

In [14]: tf.cast(tf.equal(tf.round(pred), y), tf.int8)

Out[14]: <tf.Tensor: id=60, shape=(4,), dtype=int8, numpy=array([1, 1, 1, 0], dtype=int8)>

In [15]: tf.reduce_mean(tf.cast(tf.equal(tf.round(pred), y), tf.float32))

Out[15]: <tf.Tensor: id=67, shape=(), dtype=float32, numpy=0.75>

In [16]: tf.round(0.5)

Out[16]: <tf.Tensor: id=69, shape=(), dtype=float32, numpy=0.0>

In [17]: tf.round(0.500001)

Out[17]: <tf.Tensor: id=71, shape=(), dtype=float32, numpy=1.0>
```



11.2 实例：实现一元逻辑回归

where (condition, a, b)

```
In [18]: pred=np.array([0.1, 0.2, 0.8, 0.49])
```

```
In [19]: tf.where(pred<0.5, 0, 1)
```

```
Out[19]: <tf.Tensor: id=75, shape=(4,), dtype=int32, numpy=array([0, 0, 1, 0])>
```

```
In [20]: pred<0.5
```

```
Out[20]: array([ True,  True, False,  True])
```

```
In [21]: tf.where(pred<0.4, 0, 1)
```

```
Out[21]: <tf.Tensor: id=79, shape=(4,), dtype=int32, numpy=array([0, 0, 1, 1])>
```



11.2 实例：实现一元逻辑回归

```
In [22]: pred=np.array([0.1, 0.2, 0.8, 0.49])  
  
a=np.array([1, 2, 3, 4])  
b=np.array([10, 20, 30, 40])
```

```
In [23]: tf.where(pred<0.5, a, b)
```

参数a,b是数组

```
Out[23]: <tf.Tensor: id=83, shape=(4,), dtype=int32, numpy=array([ 1,  2, 30,  4])>
```

```
In [24]: tf.where(pred>=0.5)
```

参数a,b缺省

```
Out[24]: <tf.Tensor: id=85, shape=(1, 1), dtype=int64, numpy=array([[2]], dtype=int64)>
```

```
In [25]: y=np.array([0, 0, 1, 1])  
pred=np.array([0.1, 0.2, 0.8, 0.49])
```

计算准确率

```
In [26]: tf.reduce_mean(tf.cast(tf.equal(tf.where(pred<0.5, 0, 1), y), tf.float32))
```

```
Out[26]: <tf.Tensor: id=94, shape=(), dtype=float32, numpy=0.75>
```



■ 房屋销售记录

序号	面积 (平方米)	类型	序号	面积 (平方米)	类型
1	137.97	1	9	106.69	0
2	104.50	1	10	<u>140.05</u>	1
3	100.00	0	11	53.75	0
4	<u>126.32</u>	1	12	46.91	0
5	79.20	0	13	68.00	0
6	99.00	1	14	63.02	0
7	124.00	1	15	81.26	0
8	114.00	0	16	86.21	0

0: 普通住宅
1: 高档住宅



11.2 实例：实现一元逻辑回归

□ 加载数据

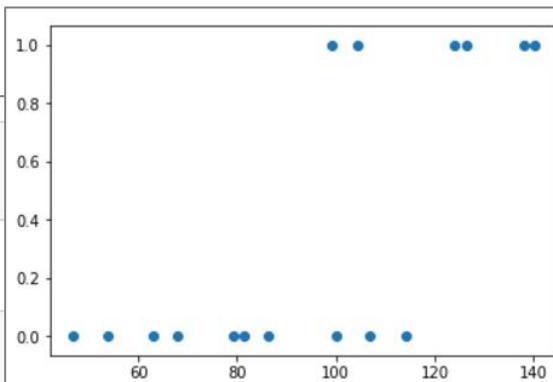
```
In [1]: import tensorflow as tf  
print("TensorFlow version:", tf.__version__)
```

TensorFlow version: 2.0.0

```
In [2]: import numpy as np  
import matplotlib.pyplot as plt
```

```
In [3]: x = np.array([137.97, 104.50, 100.00, 126.32, 79.20, 99.00, 124.00, 114.00,  
                 106.69, 140.05, 53.75, 46.91, 68.00, 63.02, 81.26, 86.21])  
y = np.array([1, 1, 0, 1, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0])
```

```
In [4]: plt.scatter(x, y)
```

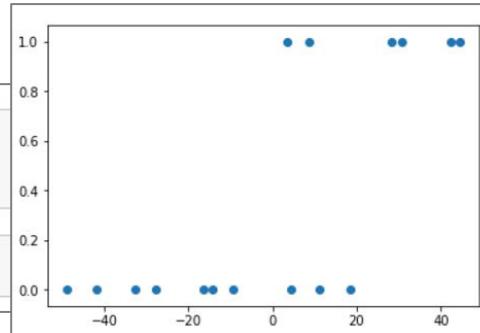


11.2 实例：实现一元逻辑回归

□ 数据处理

```
In [5]: x_train=x-np.mean(x)  
y_train=y
```

```
In [6]: plt.scatter(x_train,y_train)
```



□ 设置超参数

```
In [7]: learn_rate = 0.005  
iter=5  
  
display_step=1
```

□ 设置模型变量初始值

```
In [8]: np.random.seed(612)  
w = tf.Variable(np.random.randn())  
b = tf.Variable(np.random.randn())
```



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11.2 实例：实现一元逻辑回归

训练模型

```
In [9]: cross_train=[]
acc_train=[]

for i in range(0,iter+1):

    with tf.GradientTape() as tape:
        pred_train = 1/(1+tf.exp(-(w*x_train+b)))
        Loss_train = -tf.reduce_mean(y_train*tf.math.log(pred_train)+(1-y_train)*tf.math.log(1-pred_train))
        Accuracy_train = tf.reduce_mean(tf.cast(tf.equal(tf.where(pred_train<0.5, 0, 1), y_train),tf.float32))

        cross_train.append(Loss_train)
        acc_train.append(Accuracy_train)

        dL_dw, dL_db = tape.gradient(Loss_train, [w,b])

        w.assign_sub(learn_rate*dL_dw)
        b.assign_sub(learn_rate*dL_db)

    if i % display_step == 0:
        print("i: %i, Train Loss: %f, Accuracy: %f" %(i, Loss_train,Accuracy_train))
```

i: 0, Train Loss: 0.852807, Accuracy: 0.625000	10/16
i: 1, Train Loss: 0.400259, Accuracy: 0.875000	14/16
i: 2, Train Loss: 0.341504, Accuracy: 0.812500	13/16
i: 3, Train Loss: 0.322571, Accuracy: 0.812500	
i: 4, Train Loss: 0.313972, Accuracy: 0.812500	
i: 5, Train Loss: 0.309411, Accuracy: 0.812500	

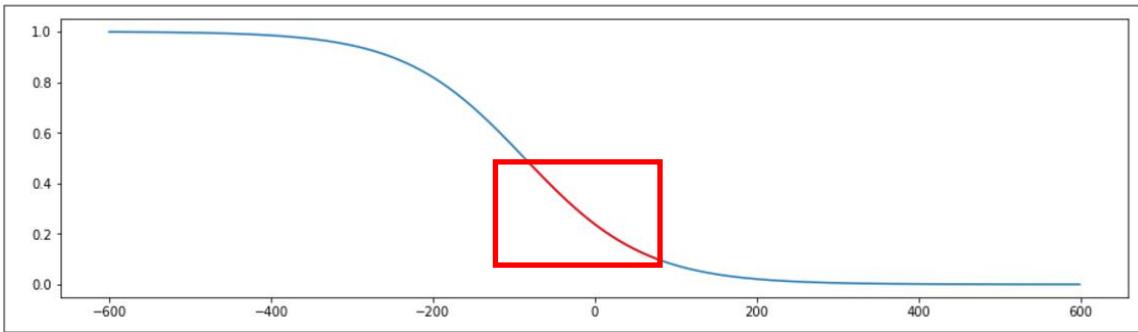
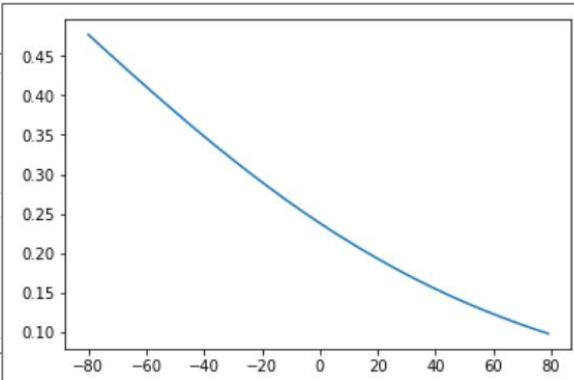


11.2 实例：实现一元逻辑回归

使用初始权值时的sigmoid函数

```
In [8]: np.random.seed(612)  
w = tf.Variable(np.random.randn())  
b = tf.Variable(np.random.randn())
```

```
In [9]: x_=range(-80, 80)  
y_=1/(1+tf.exp(-(w*x_+b)))  
plt.plot(x_, y_)
```



11.2 实例：实现一元逻辑回归

```
In [10]: plt.scatter(x_train, y_train)
plt.plot(x_, y_, color="red", linewidth=3)

cross_train=[]
acc_train=[]

for i in range(0,iter+1):

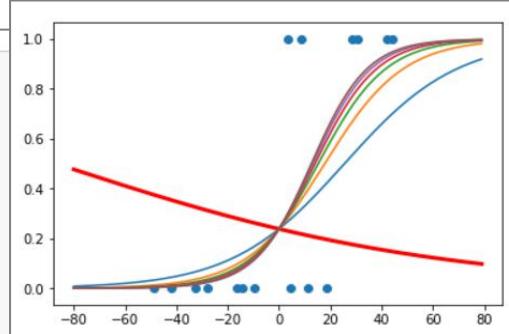
    with tf.GradientTape() as tape:
        pred_train = 1/(1+tf.exp(-(w*x_train+b)))
        Loss_train = -tf.reduce_mean(y_train*tf.math.log(pred_train)+(1-y_train)*tf.math.log(1-pred_train))
        Accuracy_train = tf.reduce_mean(tf.cast(tf.equal(tf.where(pred_train<0.5, 0, 1), y_train),tf.float32))

    cross_train.append(Loss_train)
    acc_train.append(Accuracy_train)

    dL_dw, dL_db = tape.gradient(Loss_train, [w, b])

    w.assign_sub(learn_rate*dL_dw)
    b.assign_sub(learn_rate*dL_db)

    if i % display_step == 0:
        print("i: %i, Train Loss: %f, Accuracy: %f" %(i, Loss_train, Accuracy_train))
        y_=1/(1+tf.exp(-(w*x_+b)))
        plt.plot(x_, y_)
```



11.2 实例：实现一元逻辑回归

```
In [11]: x_test=[128.15, 45.00, 141.43, 106.27, 99.00, 53.84, 85.36, 70.00, 162.00, 114.60]
```

```
In [12]: pred_test=1/(1+tf.exp(-(w*(x_test-np.mean(x))+b)))
```

```
In [13]: y_test=tf.where(pred_test<0.5, 0, 1)
```

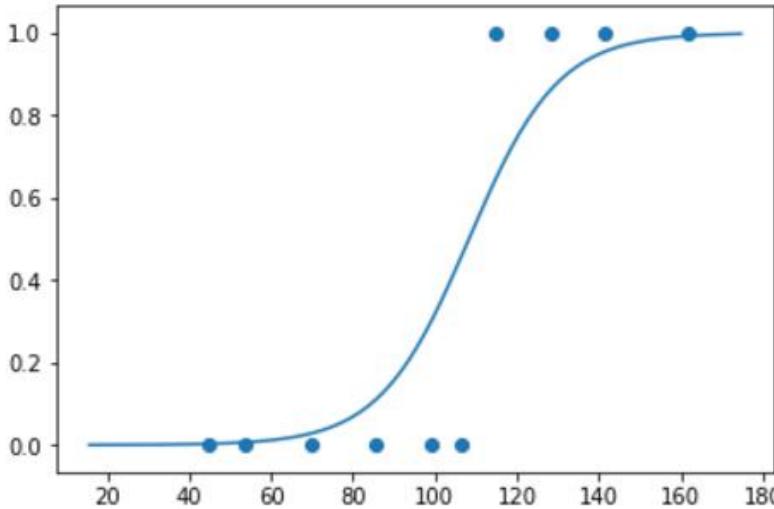
```
In [14]: for i in range(len(x_test)):
    print(x_test[i], "\t", pred_test[i].numpy(), "\t", y_test[i].numpy(), "\t")
```

128.15	0.8610252	1
45.0	0.0029561974	0
141.43	0.9545566	1
106.27	0.45318928	0
99.0	0.2981362	0
53.84	0.00663888	0
85.36	0.108105935	0
70.0	0.028681064	0
162.0	0.9928677	1
114.6	0.6406205	1



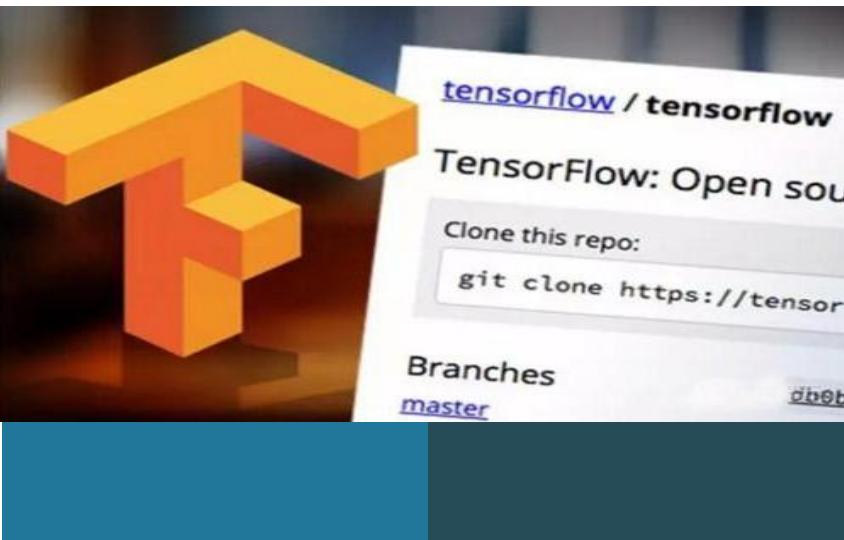
11.2 实例：实现一元逻辑回归

```
In [15]: plt.scatter(x_test,y_test)  
  
x_=np.array(range(-80,80))  
y_=1/(1+tf.exp(-w*x_+b)))  
plt.plot(x_+np.mean(x), y_)  
plt.show()
```



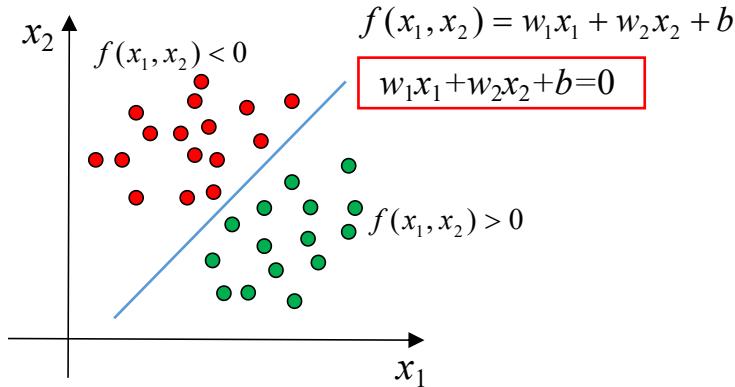
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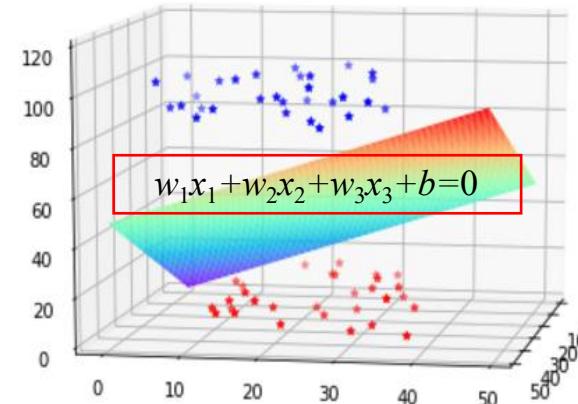
11.3 线性分类器

■ 线性分类器 (Linear Classifier)

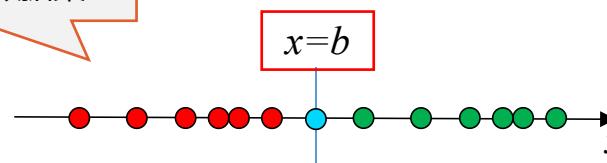


二维空间中的数据集

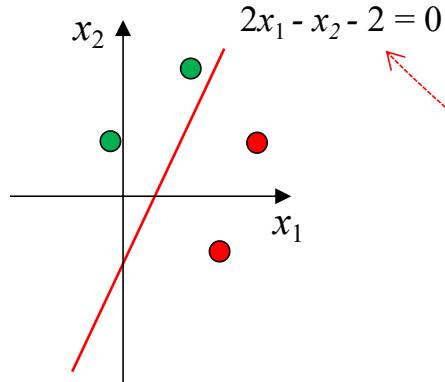
决策边界 m维空间：超平面 $W^T X = 0$



一维空间中的数据集



□ 线性分类器实例



分类器

$$y = \text{step}(z) = \text{step}(2x_1 - x_2 - 2)$$

$$\begin{aligned} z &\geq 0, & y &= 1 \\ z &< 0, & y &= 0 \end{aligned}$$

$$y = \sigma(z) = \sigma(2x_1 - x_2 - 2)$$

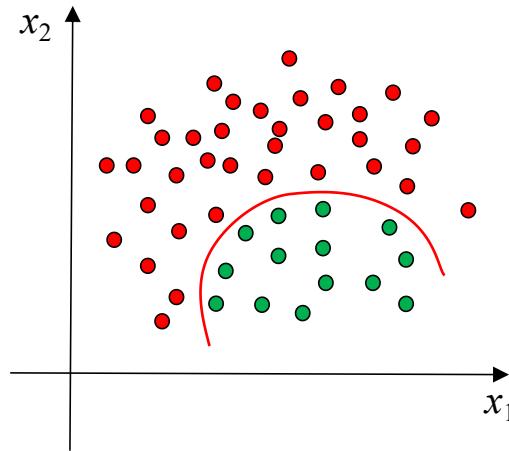
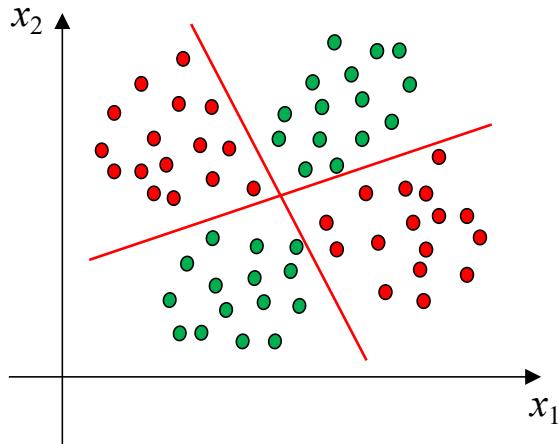
$$y = \sigma(z) = \sigma(w_1 x_1 + w_2 x_2 + b)$$

逻辑回归

(x_1, x_2)	$z = 2x_1 - x_2 - 2$	y
(1, 3)	-3	0
(3, 1)	3	1
(-1, 1)	-5	0
(2, -2)	4	1



■ 线性不可分

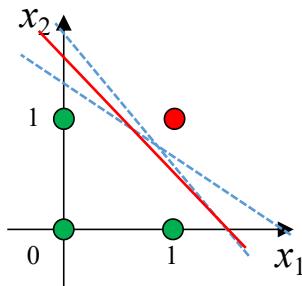


11.3 线性分类器

■ 逻辑运算

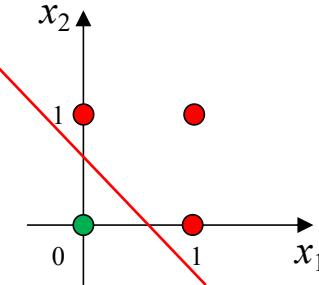
□ 与运算

(x_1, x_2)	y
(0, 0)	0
(0, 1)	0
(1, 0)	0
(1, 1)	1



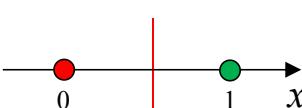
□ 或运算

(x_1, x_2)	y
(0, 0)	0
(0, 1)	1
(1, 0)	1
(1, 1)	1



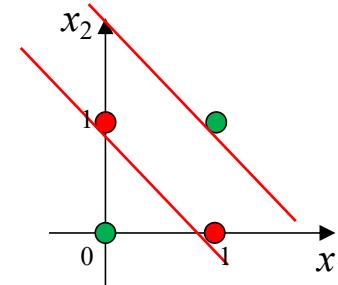
□ 非运算

x	y
0	1
1	0



□ 异或运算

(x_1, x_2)	y
(0, 0)	0
(0, 1)	1
(1, 0)	1
(1, 1)	0



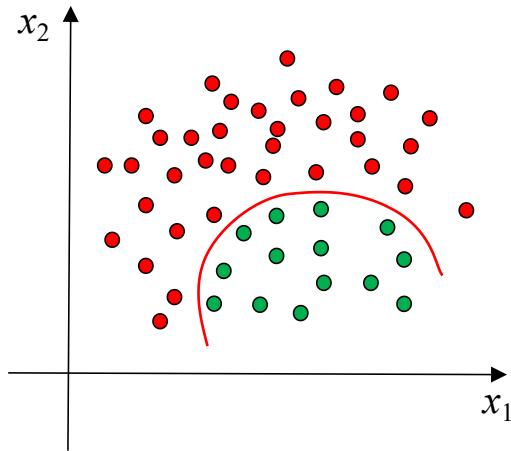
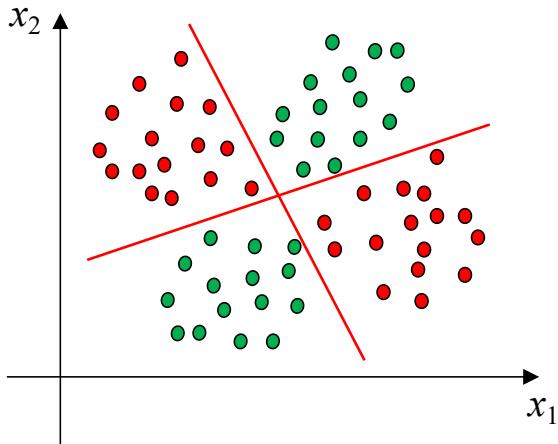


11.4 实例：实现多元逻辑回归



11.4.1 实现多元逻辑回归

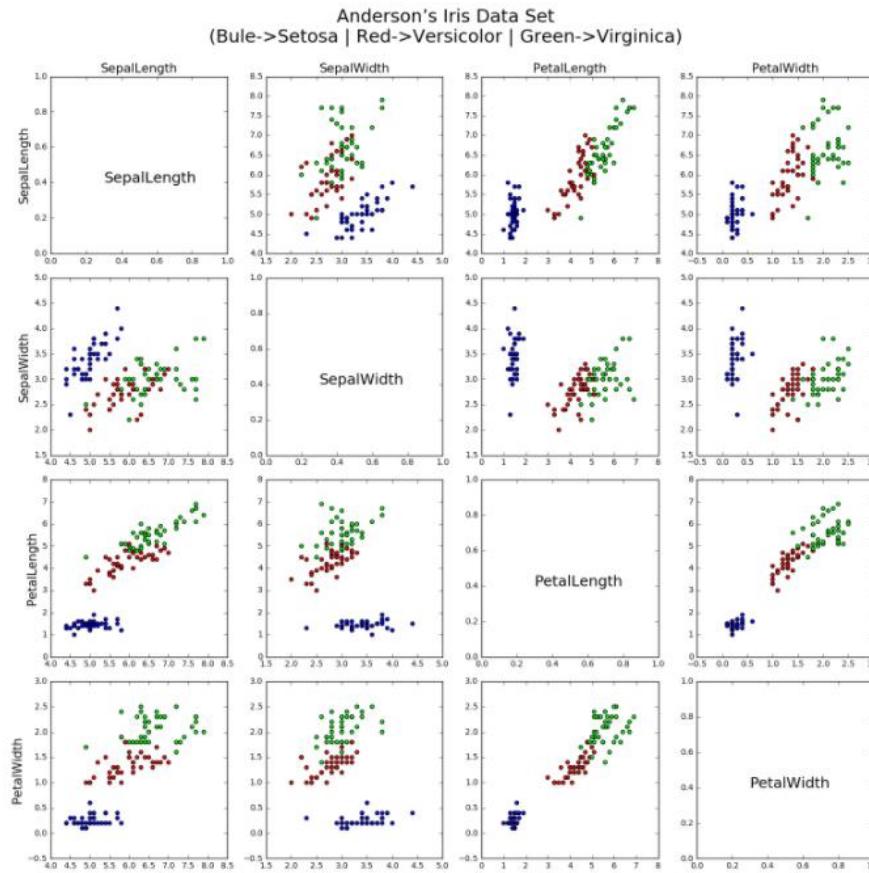
■ 线性不可分



11.4.1 实现多元逻辑回归

Iris数据集

- 150个样本
- 4个属性
 - 花萼长度 (Sepal Length)
 - 花萼宽度 (Sepal Width)
 - 花瓣长度 (Petal Length)
 - 花瓣宽度 (Petal Width)
- 1个标签
 - 山鸢尾 (Setosa)
 - 变色鸢尾 (Versicolour)
 - 维吉尼亚鸢尾 (Virginica)



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11.4.1 实现多元逻辑回归

加载数据

```
In [1]: import tensorflow as tf  
print("TensorFlow version:", tf.__version__)
```

TensorFlow version: 2.0.0

```
In [2]: import pandas as pd  
import numpy as np  
import matplotlib as mpl  
import matplotlib.pyplot as plt
```

```
In [3]: TRAIN_URL = "http://download.tensorflow.org/data/iris_training.csv"  
train_path = tf.keras.utils.get_file(TRAIN_URL.split('/')[-1], TRAIN_URL)
```

```
In [4]: df_iris = pd.read_csv(train_path, header=0)
```



11.4.1 实现多元逻辑回归

处理数据

- 转化为NumPy数组
- 提取属性和标签
- 提取山鸢尾和变色鸢尾

```
In [5]: iris=np.array(df_iris)
```

```
In [6]: iris.shape
```

```
Out[6]: (120, 5)
```

```
In [7]: train_x=iris[:,0:2]  
train_y=iris[:,4]
```

```
In [8]: train_x.shape, train_y.shape
```

```
Out[8]: ((120, 2), (120,))
```

```
In [9]: x_train = train_x[train_y < 2]  
y_train = train_y[train_y < 2]
```

```
In [10]: x_train.shape, y_train.shape
```

```
Out[10]: ((78, 2), (78,))
```

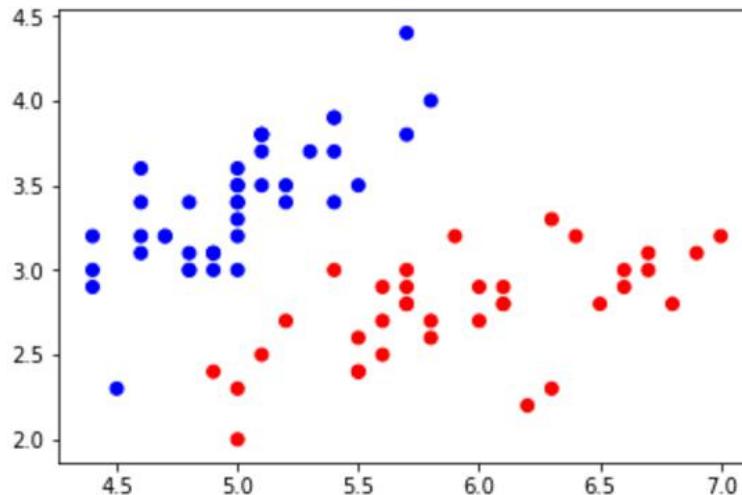
```
In [11]: num=len(x_train)
```



■ 处理数据

□ 可视化样本

```
In [12]: cm_pt = mpl.colors.ListedColormap(["blue", "red"])
plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, cmap=cm_pt)
plt.show()
```

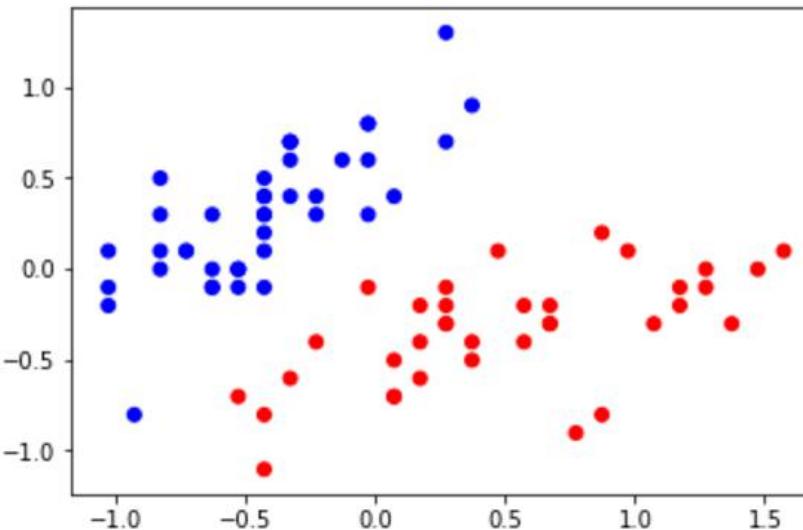


■ 处理数据

□ 属性中心化

```
In [13]: x_train=x_train-np.mean(x_train, axis=0)
```

```
In [14]: plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, cmap=cm_pt)  
plt.show()
```



■ 处理数据

□ 生成多元模型的属性矩阵和标签列向量

```
In [15]: x0_train = np.ones(num).reshape(-1, 1)
```

```
In [16]: X = tf.cast(tf.concat((x0_train, x_train), axis = 1), tf.float32)  
Y = tf.cast(y_train.reshape(-1, 1), tf.float32)
```

```
In [17]: X.shape, Y.shape
```

```
Out[17]: (TensorShape([78, 3]), TensorShape([78, 1]))
```



■ 设置超参数

```
In [18]: learn_rate=0.2  
       iter=120  
  
       display_step=30
```

■ 设置模型参数初始值

```
In [19]: np.random.seed(612)  
W=tf.Variable(np.random.randn(3, 1), dtype=tf.float32)
```



11.4.1 实现多元逻辑回归

训练模型

```
In [20]: ce=[]
acc=[]

for i in range(0, iter+1):
    with tf.GradientTape() as tape:
        PRED =1/(1+tf.exp(-tf.matmul(X,W)))
        Loss =-tf.reduce_mean(Y*tf.math.log(PRED)+(1-Y)*tf.math.log(1-PRED))

    accuracy = tf.reduce_mean(tf.cast(tf.equal(tf.where(PRED.numpy()<0.5, 0., 1.), Y), tf.float32))
    ce.append(Loss)
    acc.append(accuracy)

    dL_dW= tape.gradient(Loss, W)
    W.assign_sub(learn_rate*dL_dW)

    if i % display_step == 0:
        print("i: %i, Acc:%f, Loss: %f" % (i, accuracy, Loss))
```

```
i: 0, Acc:0.230769, Loss: 0.994269
i: 30, Acc:0.961538, Loss: 0.481892
i: 60, Acc:0.987179, Loss: 0.319128
i: 90, Acc:0.987179, Loss: 0.246626
i: 120, Acc:1.000000, Loss: 0.204982
```



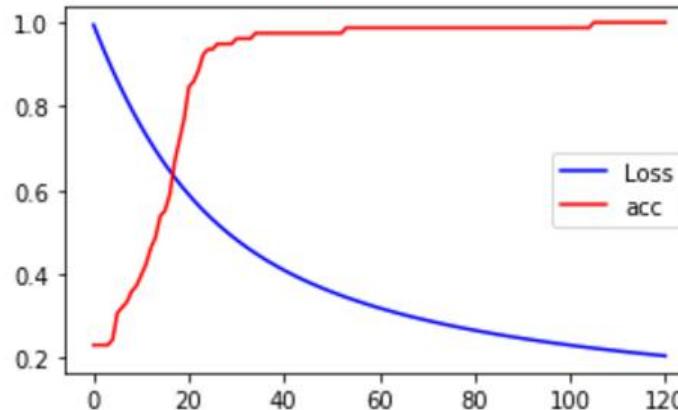
可视化

□ 绘制损失和准确率变化曲线

In [21]:

```
plt.figure(figsize=(5, 3))
plt.plot(ce, color="blue", label="Loss")
plt.plot(acc, color="red", label="acc")
plt.legend()
plt.show()
```

```
i: 0, Acc:0.230769, Loss: 0.994269
i: 30, Acc:0.961538, Loss: 0.481892
i: 60, Acc:0.987179, Loss: 0.319128
i: 90, Acc:0.987179, Loss: 0.246626
i: 120, Acc:1.000000, Loss: 0.204982
```



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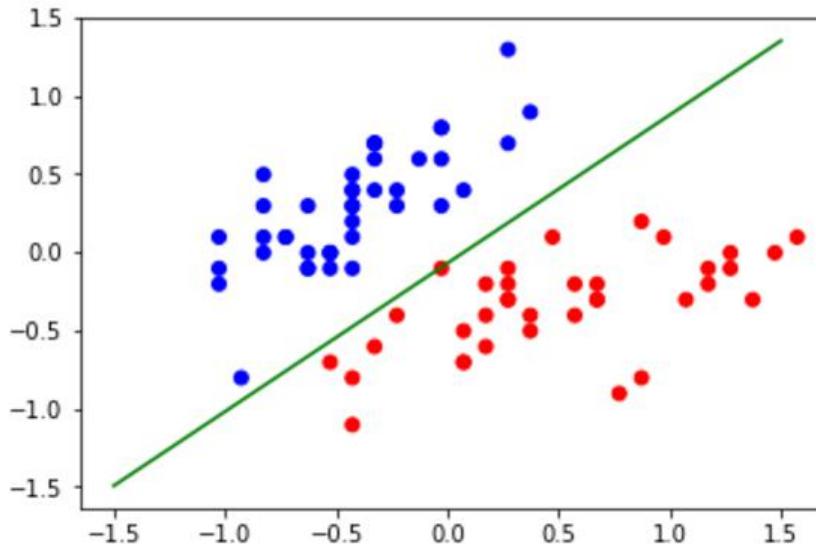
可视化

绘制决策边界

$$w_1x_1 + w_2x_2 + w_0 = 0$$

$$x_2 = -\frac{w_1x_1 + w_0}{w_2}$$

```
In [22]: plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, cmap=cm_pt)
x_=[-1.5, 1.5]
y_=-(W[1]*x_+W[0])/W[2]
plt.plot(x_, y_, color="g")
plt.show()
```



■ 可视化

□ 在训练过程中绘制决策边界

```
In [19]: np.random.seed(612)  
W= tf.Variable(np.random.randn(3, 1), dtype=tf.float32)
```

```
In [20]: cm_pt = mpl.colors.ListedColormap(["blue", "red"])
```

```
In [21]: x_=[-1.5, 1.5]  
y_=(W[0]+W[1]*x_)/W[2]
```



11.4.1 实现多元逻辑回归

训练模型

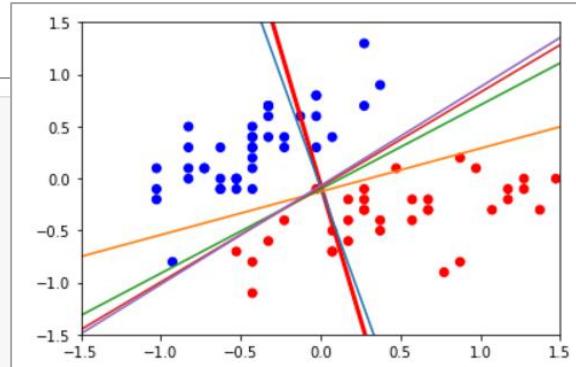
```
In [22]: plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, cmap=cm_pt)
plt.plot(x_, y_, color="red", linewidth=3)
plt.xlim([-1.5, 1.5])
plt.ylim([-1.5, 1.5])
```

```
ce=[]
acc=[]
for i in range(0, iter+1):
    with tf.GradientTape() as tape:
        PRED = 1/(1+tf.exp(-tf.matmul(X,W)))
        Loss = -tf.reduce_mean(Y*tf.math.log(PRED)+(1-Y)*tf.math.log(1-PRED))

    accuracy = tf.reduce_mean(tf.cast(tf.equal(tf.where(PRED.numpy()<0.5, 0., 1.), Y), tf.float32))
    ce.append(Loss)
    acc.append(accuracy)

    dL_dW= tape.gradient(Loss, W)
    W.assign_sub(learn_rate*dL_dW)

    if i % display_step == 0:
        print("i: %i, Acc:%f, Loss: %f" % (i,accuracy,Loss))
        y_ = -(W[0]+W[1]*x_)/W[2]
        plt.plot(x_, y_)
```



```
i: 0, Acc:0.230769, Loss: 0.994269
i: 30, Acc:0.961538, Loss: 0.481892
i: 60, Acc:0.987179, Loss: 0.319128
i: 90, Acc:0.987179, Loss: 0.246626
i: 120, Acc:1.000000, Loss: 0.204982
```



11.4.1 实现多元逻辑回归

■ 使用测试集

□ 加载数据集

```
In [1]: import tensorflow as tf  
print("TensorFlow version:", tf.__version__)
```

TensorFlow version: 2.0.0

```
In [2]: import pandas as pd  
import numpy as np  
import matplotlib as mpl  
import matplotlib.pyplot as plt
```

```
In [3]: TRAIN_URL = "http://download.tensorflow.org/data/iris_training.csv"  
train_path = tf.keras.utils.get_file(TRAIN_URL.split('/')[-1], TRAIN_URL)  
  
TEST_URL = "http://download.tensorflow.org/data/iris_test.csv"  
test_path = tf.keras.utils.get_file(TEST_URL.split('/')[-1], TEST_URL)
```



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11.4.1 实现多元逻辑回归

□ 数据处理

```
In [4]: df_iris_train = pd.read_csv(train_path, header=0)  
df_iris_test = pd.read_csv(test_path, header=0)
```

```
In [5]: iris_train=np.array(df_iris_train)  
iris_test=np.array(df_iris_test)
```

```
In [6]: iris_train.shape, iris_test.shape
```

```
Out[6]: ((120, 5), (30, 5))
```



11.4.1 实现多元逻辑回归

□ 数据处理

```
In [7]: train_x=iris_train[:, 0:2]  
train_y=iris_train[:, 4]
```

```
test_x=iris_test[:, 0:2]  
test_y=iris_test[:, 4]
```

```
In [8]: train_x. shape, train_y. shape
```

```
Out[8]: ((120, 2), (120,))
```

```
In [9]: test_x. shape, test_y. shape
```

```
Out[9]: ((30, 2), (30,))
```



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11.4.1 实现多元逻辑回归

□ 数据处理

```
In [10]: x_train = train_x[train_y < 2]
y_train = train_y[train_y < 2]
```

```
In [11]: x_train.shape, y_train.shape
```

Out[11]: ((78, 2), (78,))

```
In [12]: x_test = test_x[test_y < 2]
y_test = test_y[test_y < 2]
```

```
In [13]: x_test.shape, y_test.shape
```

Out[13]: ((22, 2), (22,))

```
In [14]: num_train=len(x_train)
num_test=len(x_test)
```

```
In [15]: num_train, num_test
```

Out[15]: (78, 22)

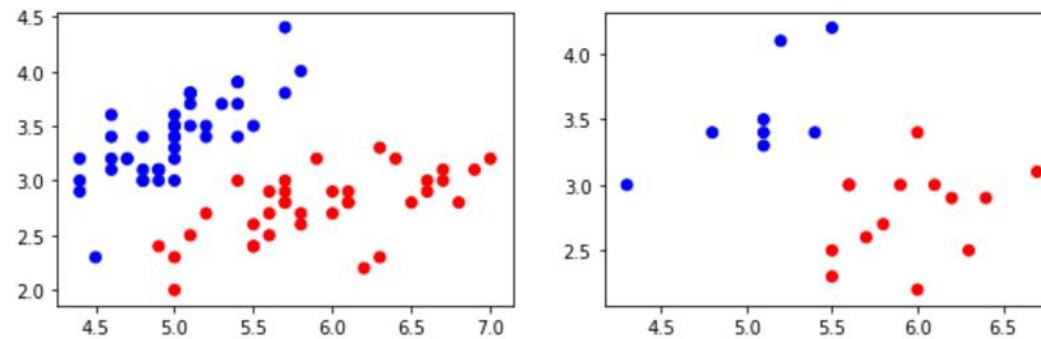
11.4.1 实现多元逻辑回归

```
In [16]: plt.figure(figsize=(10, 3))
cm_pt = mpl.colors.ListedColormap(["blue", "red"])

plt.subplot(121)
plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, cmap=cm_pt)

plt.subplot(122)
plt.scatter(x_test[:, 0], x_test[:, 1], c=y_test, cmap=cm_pt)

plt.show()
```



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11.4.1 实现多元逻辑回归

□ 数据处理——中心化

```
In [17]: print(np.mean(x_train, axis=0))  
print(np.mean(x_test, axis=0))
```

```
[5.42692308 3.1025641 ]  
[5.62727273 3.06363636]
```

```
In [18]: x_train=x_train-np.mean(x_train, axis=0)  
x_test=x_test-np.mean(x_test, axis=0)
```



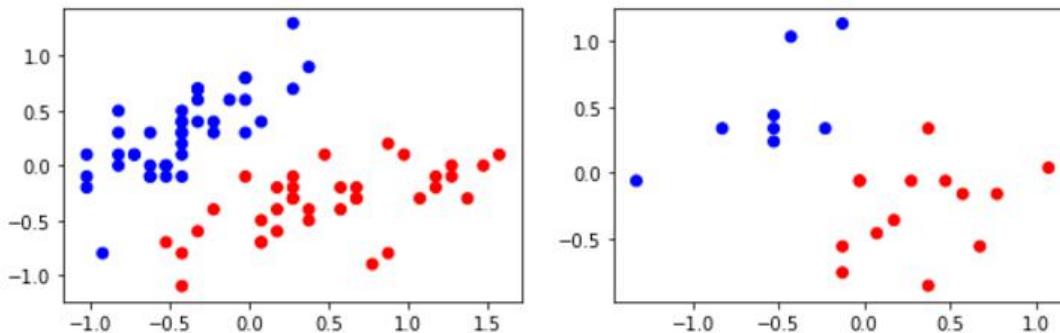
11.4.1 实现多元逻辑回归

```
In [19]: plt.figure(figsize=(10, 3))

plt.subplot(121)
plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, cmap=cm_pt)

plt.subplot(122)
plt.scatter(x_test[:, 0], x_test[:, 1], c=y_test, cmap=cm_pt)

plt.show()
```



11.4.1 实现多元逻辑回归

```
In [20]: x0_train = np.ones(num_train).reshape(-1, 1)
X_train = tf.cast(tf.concat((x0_train, x_train), axis = 1), dtype=tf.float32)
Y_train = tf.cast(y_train.reshape(-1, 1), dtype=tf.float32)
```

```
In [21]: X_train.shape, Y_train.shape
```

```
Out[21]: (TensorShape([78, 3]), TensorShape([78, 1]))
```

```
In [22]: x0_test = np.ones(num_test).reshape(-1, 1)
X_test = tf.cast(tf.concat((x0_test, x_test), axis = 1), dtype=tf.float32)
Y_test = tf.cast(y_test.reshape(-1, 1), dtype=tf.float32)
```

```
In [23]: X_test.shape, Y_test.shape
```

```
Out[23]: (TensorShape([22, 3]), TensorShape([22, 1]))
```



11.4.1 实现多元逻辑回归

□ 设置超参数、设置模型参数初识值

```
In [24]: learn_rate=0.2  
       iter=120  
  
       display_step=30
```

```
In [25]: np.random.seed(612)  
W=tf.Variable(np.random.randn(3, 1), dtype=tf.float32)
```



11.4.1 实现多元逻辑回归

In [26]:

```
ce_train=[]  
ce_test=[]  
acc_train=[]  
acc_test=[]  
  
for i in range(0, iter+1):  
    with tf.GradientTape() as tape:  
        PRED_train = 1/(1+tf.exp(-tf.matmul(X_train, W)))  
        Loss_train = -tf.reduce_mean(Y_train*tf.math.log(PRED_train)+(1-Y_train)*tf.math.log(1-PRED_train))  
        PRED_test = 1/(1+tf.exp(-tf.matmul(X_test, W)))  
        Loss_test = -tf.reduce_mean(Y_test*tf.math.log(PRED_test)+(1-Y_test)*tf.math.log(1-PRED_test))  
  
        accuracy_train = tf.reduce_mean(tf.cast(tf.equal(tf.where(PRED_train.numpy() < 0.5, 0., 1.), Y_train), tf.float32))  
        accuracy_test = tf.reduce_mean(tf.cast(tf.equal(tf.where(PRED_test.numpy() < 0.5, 0., 1.), Y_test), tf.float32))  
  
    ce_train.append(Loss_train)  
    ce_test.append(Loss_test)  
    acc_train.append(accuracy_train)  
    acc_test.append(accuracy_test)  
  
    dL_dW= tape.gradient(Loss_train, W)  
    W.assign_sub(learn_rate*dL_dW)  
  
    if i % display_step == 0:  
        print("i: {}, TrainAcc:{}f, TrainLoss: {}f, TestAcc:{}f, TestLoss: {}f" % (i, accuracy_train, Loss_train, accuracy_test, Loss_test))
```



11.4 .1 实现多元逻辑回归

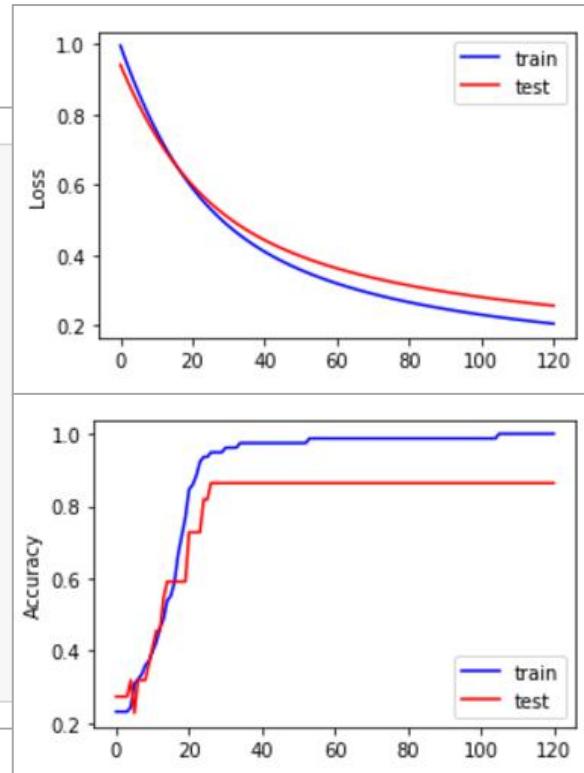
□ 可视化

```
In [27]: plt.figure(figsize=(10, 3))

plt.subplot(121)
plt.plot(ce_train, color="blue", label="train")
plt.plot(ce_test, color="red", label="test")
plt.ylabel("Loss")
plt.legend()

plt.subplot(122)
plt.plot(acc_train, color="blue", label="train")
plt.plot(acc_test, color="red", label="test")
plt.ylabel("Accuracy")

plt.legend()
plt.show()
```

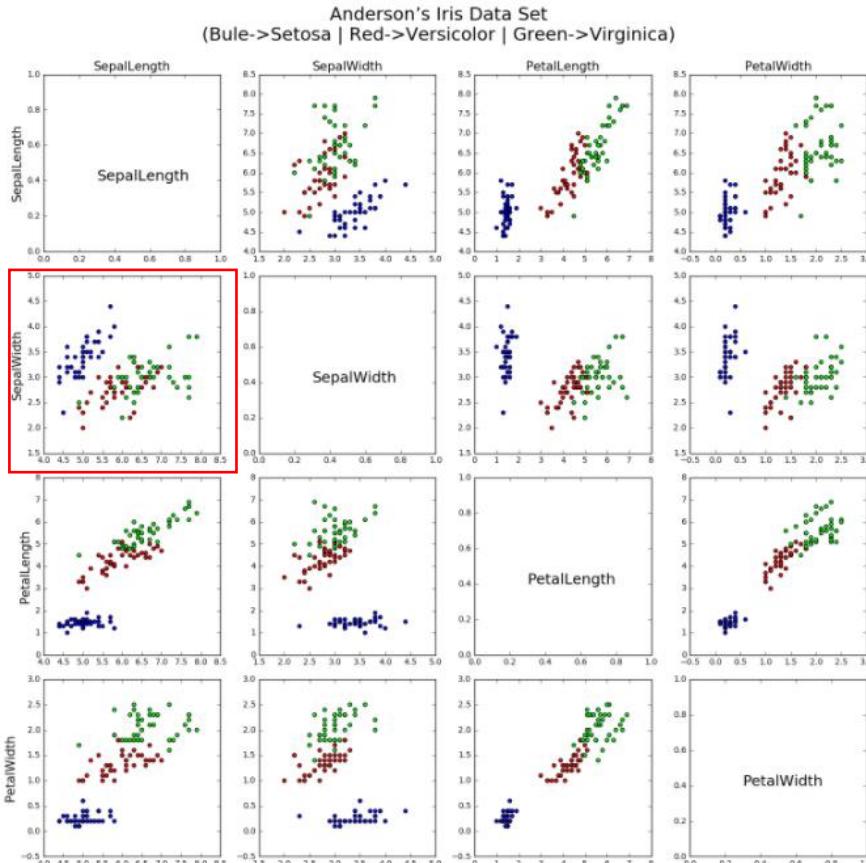


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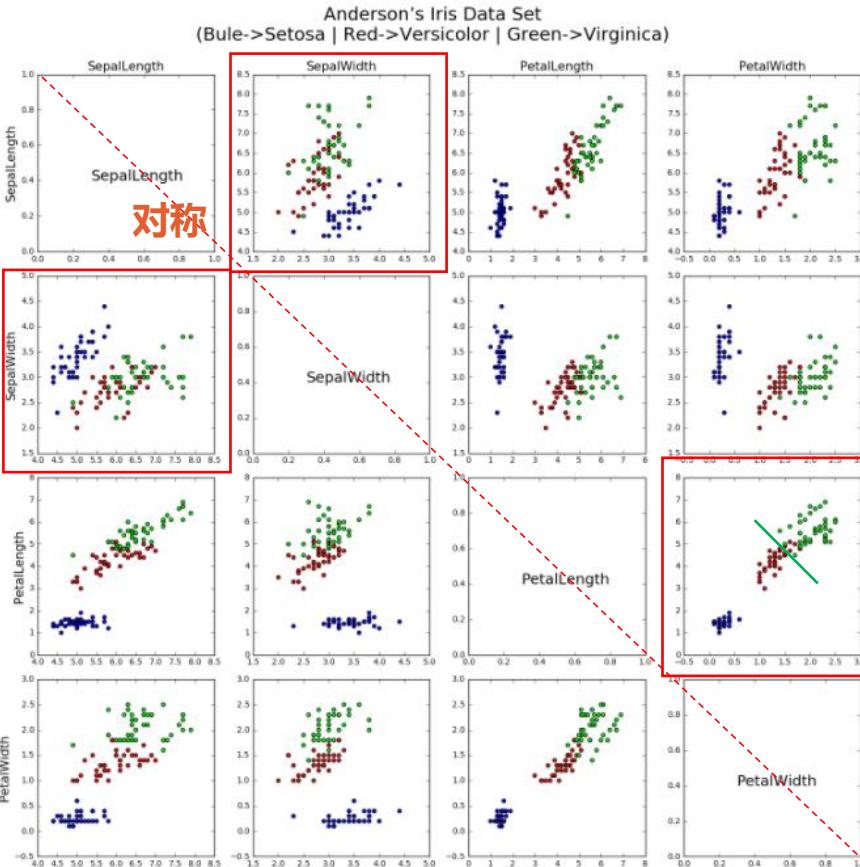
11.4 .1 实现多元逻辑回归

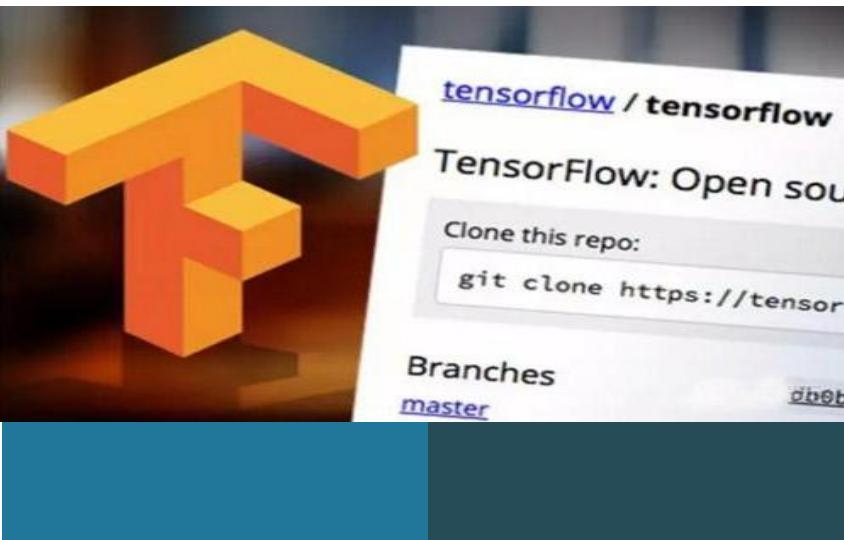
例程中的散点图



11.4.1 实现多元逻辑回归

例程中的散点图

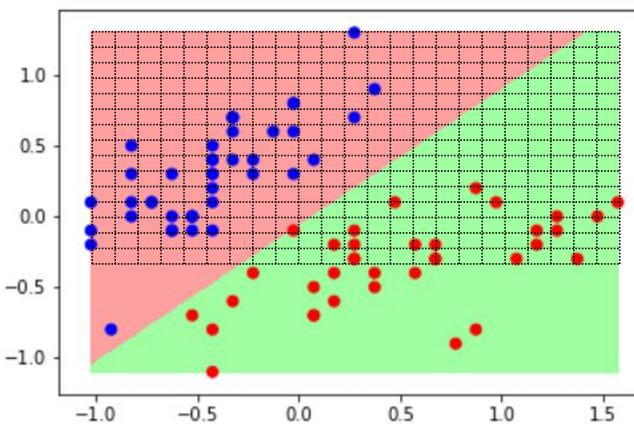
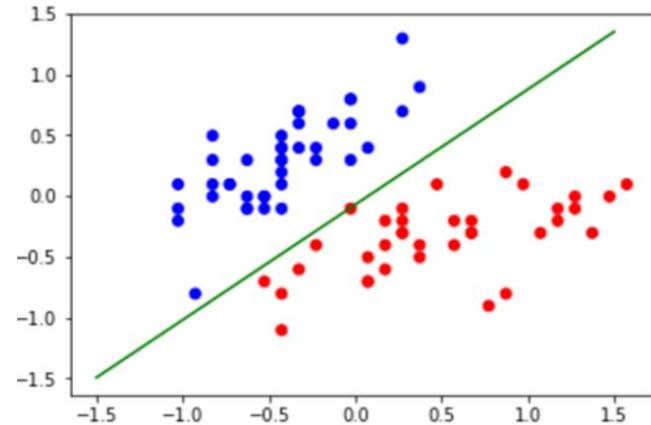




11.4.2 绘制分类图

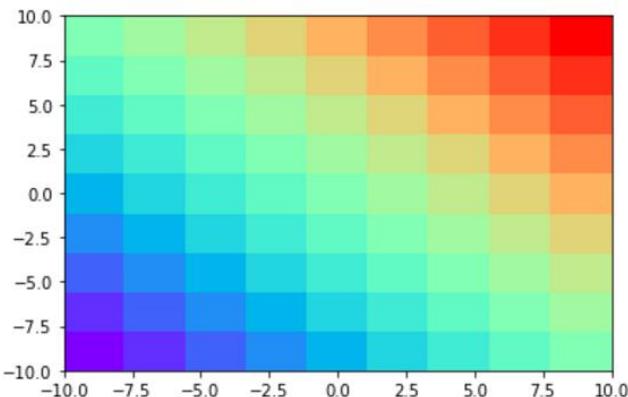
11.4.2 绘制分类图

- 线性分类器
- 决策边界



11.4.2 绘制分类图

生成网格坐标矩阵: `np.meshgrid()`
 填充网格: `plt.pcolormesh()`



```
In [1]: import tensorflow as tf
print("TensorFlow version:", tf.__version__)
```

TensorFlow version: 2.0.0

```
In [2]: import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
```

```
In [3]: n = 10
x = np.linspace(-10, 10, n)
y = np.linspace(-10, 10, n)
X, Y = np.meshgrid(x, y) meshgrid()详见9.6小节
Z = X+Y
plt.pcolormesh(X, Y, Z, cmap="rainbow")
plt.show()
```



11.4.2 绘制分类图

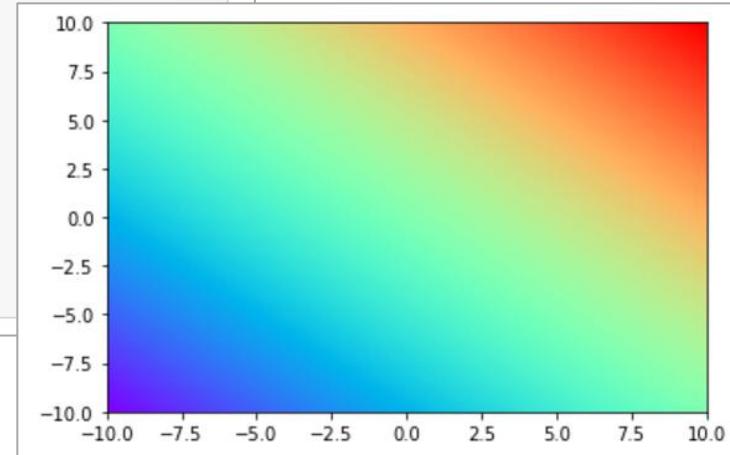
In [4]: n = 200

```
x = np.linspace(-10, 10, n)
y = np.linspace(-10, 10, n)

X, Y = np.meshgrid(x, y)
Z = X+Y

plt.pcolormesh(X, Y, Z, cmap="rainbow")

plt.show()
```



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11.4.2 绘制分类图

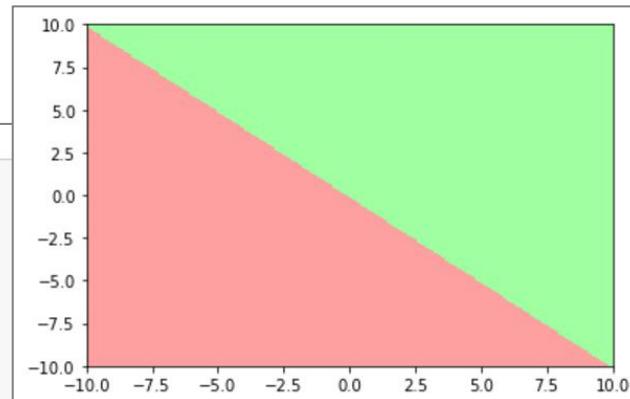
```
In [5]: n = 200
```

```
x = np.linspace(-10, 10, n)
y = np.linspace(-10, 10, n)

X, Y = np.meshgrid(x, y)
Z = X+Y
```

```
cm_bg = mpl.colors.ListedColormap(["#FFA0AO", "#A0FFA0"])
plt.pcolormesh(X, Y, Z, cmap=cm_bg)

plt.show()
```



11.4.2 绘制分类图

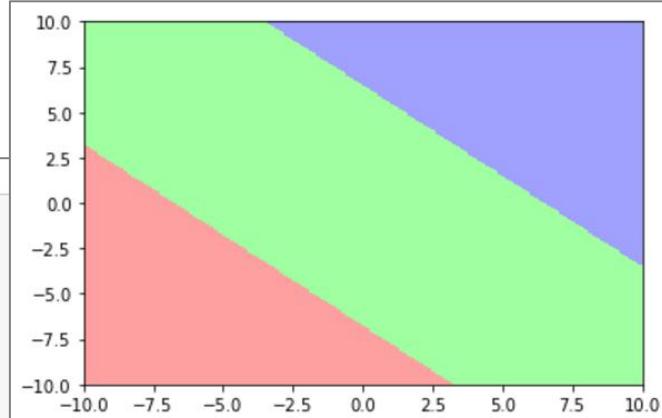
```
In [6]: n = 200
```

```
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)
```

```
X, Y = np.meshgrid(x, y)  
Z = X+Y
```

```
cm_bg = mpl.colors.ListedColormap(["#FFAOAO", "#A0FFAO", "#AOAOFF"])  
plt.pcolormesh(X, Y, Z, cmap=cm_bg)
```

```
plt.show()
```



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11.4.2 绘制分类图

绘制轮廓线: plt.contour()

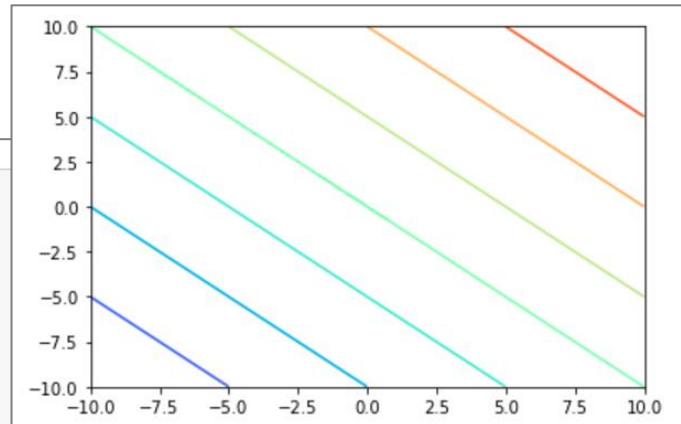
In [7]: n = 200

```
x = np.linspace(-10, 10, n)
y = np.linspace(-10, 10, n)
```

```
X, Y = np.meshgrid(x, y)
Z = X+Y
```

```
plt.contour(X, Y, Z, cmap="rainbow")
```

```
plt.show()
```



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11.4.2 绘制分类图

绘制轮廓线: plt.contour()

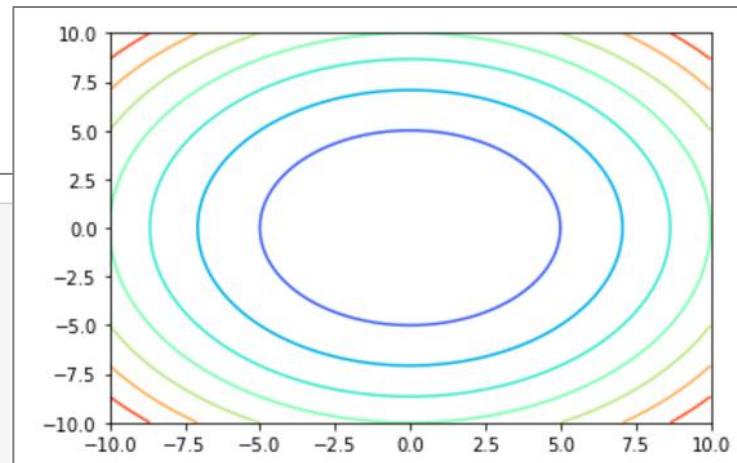
In [8]: n = 200

```
x = np.linspace(-10, 10, n)
y = np.linspace(-10, 10, n)
```

```
X, Y = np.meshgrid(x, y)
Z = X**2+Y**2
```

```
plt.contour(X, Y, Z, cmap="rainbow")
```

```
plt.show()
```



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11.4.2 绘制分类图

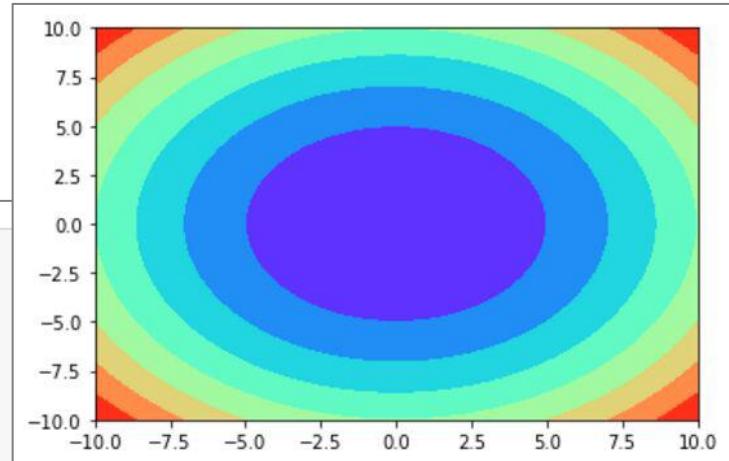
填充分区: `plt.contourf()`

In [9]:

```
n = 200  
  
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)  
  
X, Y = np.meshgrid(x, y)  
Z = X**2+Y**2
```

`plt.contourf(X, Y, Z, cmap="rainbow")`

`plt.show()`



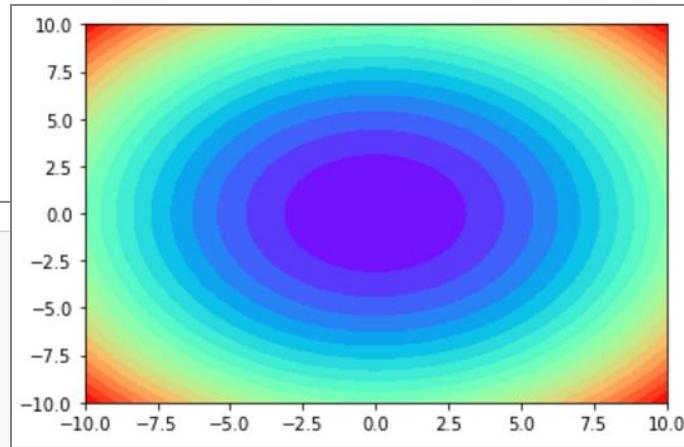
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11.4.2 绘制分类图

填充分区: plt.contourf()

```
In [10]: n = 200  
  
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)  
  
X, Y = np.meshgrid(x, y)  
Z = X**2+Y**2  
  
plt.contourf(X, Y, Z, 20, cmap="rainbow")  
  
plt.show()
```



11.4.2 绘制分类图

生成网格坐标矩阵: `np.meshgrid()`

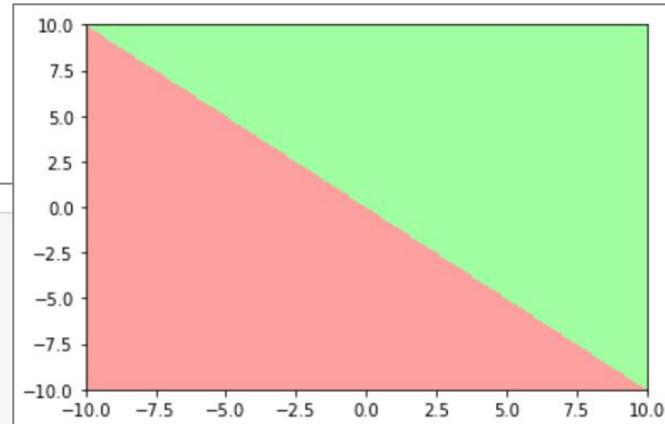
绘制分类图: `pcolormesh()/plt.contourf()`

```
In [11]: n = 200
x = np.linspace(-10, 10, n)
y = np.linspace(-10, 10, n)

X, Y = np.meshgrid(x, y)
Z = X+Y

cm_bg = mpl.colors.ListedColormap(["#FFAOAO", "#AOFFAO"])
Z=tf.where(Z<0, 0, 1)
plt.pcolormesh(X, Y, Z, cmap=cm_bg)

plt.show()
```



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11.4.2 绘制分类图

生成网格坐标矩阵: `np.meshgrid()`

绘制分类图: `pcolormesh()/plt.contourf()`

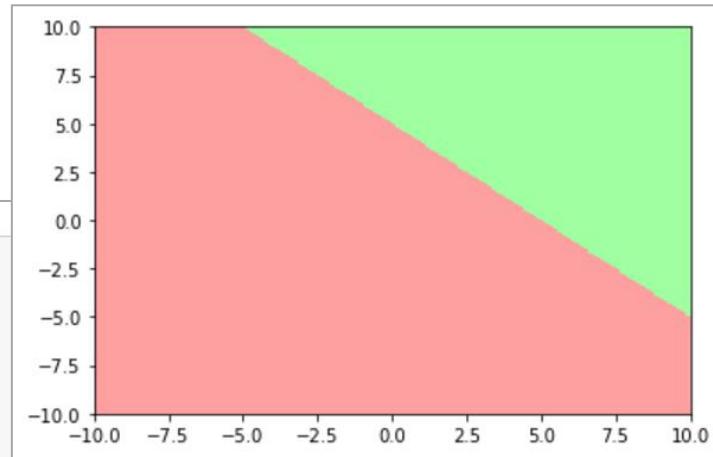
```
In [12]: n = 200
```

```
x = np.linspace(-10, 10, n)
y = np.linspace(-10, 10, n)
```

```
X, Y = np.meshgrid(x, y)
Z = X+Y
```

```
cm_bg = mpl.colors.ListedColormap(["#FFA0AO", "#A0FFAO"])
Z= tf.where(Z<5, 0, 1)
plt.pcolormesh(X, Y, Z, cmap=cm_bg)
```

```
plt.show()
```



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11.4.2 绘制分类图

■ 根据鸢尾花分类模型，绘制分类图

In [29]:

```
M=300
x1_min, x2_min = x_train.min(axis=0)
x1_max, x2_max = x_train.max(axis=0)
t1 = np.linspace(x1_min, x1_max, M)
t2 = np.linspace(x2_min, x2_max, M)
m1, m2 = np.meshgrid(t1, t2)
```

In [30]:

```
m0=np.ones(M*M)
X_mesh = tf.cast(np.stack((m0,m1.reshape(-1),m2.reshape(-1)), axis=1), dtype=tf.float32)
Y_mesh =tf.cast(1/(1+tf.exp(-tf.matmul(X_mesh,W))), dtype=tf.float32)
Y_mesh=tf.where(Y_mesh<0.5, 0, 1)
```

In [31]:

```
n=tf.reshape(Y_mesh, m1.shape)
```

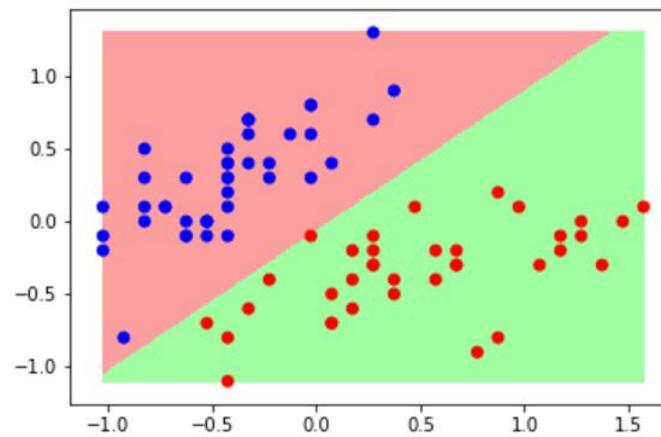


11.4.2 绘制分类图

```
In [32]: cm_pt = mpl.colors.ListedColormap(["blue", "red"])
cm_bg = mpl.colors.ListedColormap(["#FFAOAO", "#AOFFAO"])

plt.pcolormesh(m1, m2, n, cmap=cm_bg)
plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, cmap=cm_pt)

plt.show()
```

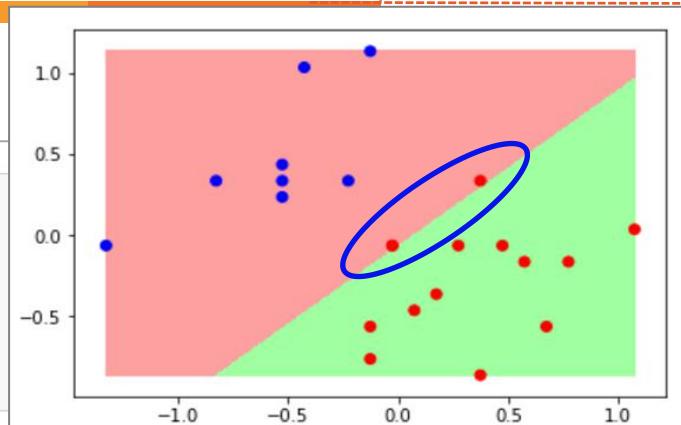


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11.4.2 绘制分类图

0.863636=19/22



```
In [33]: M=300
x1_min, x2_min = x_test.min(axis=0)
x1_max, x2_max = x_test.max(axis=0)
t1 = np.linspace(x1_min, x1_max, M)
t2 = np.linspace(x2_min, x2_max, M)
m1,m2 = np.meshgrid(t1, t2)
```

```
In [34]: m0=np.ones(M*M)
X_mesh = tf.cast(np.stack((m0,m1.reshape(-1),m2.reshape(-1)), axis=1), dtype=tf.float32)
Y_mesh =tf.cast(1/(1+tf.exp(-tf.matmul(X_mesh, W))), dtype=tf.float32)
Y_mesh=tf.where(Y_mesh<0.5, 0, 1)
```

```
In [35]: n=tf.reshape(Y_mesh, m1.shape)
```

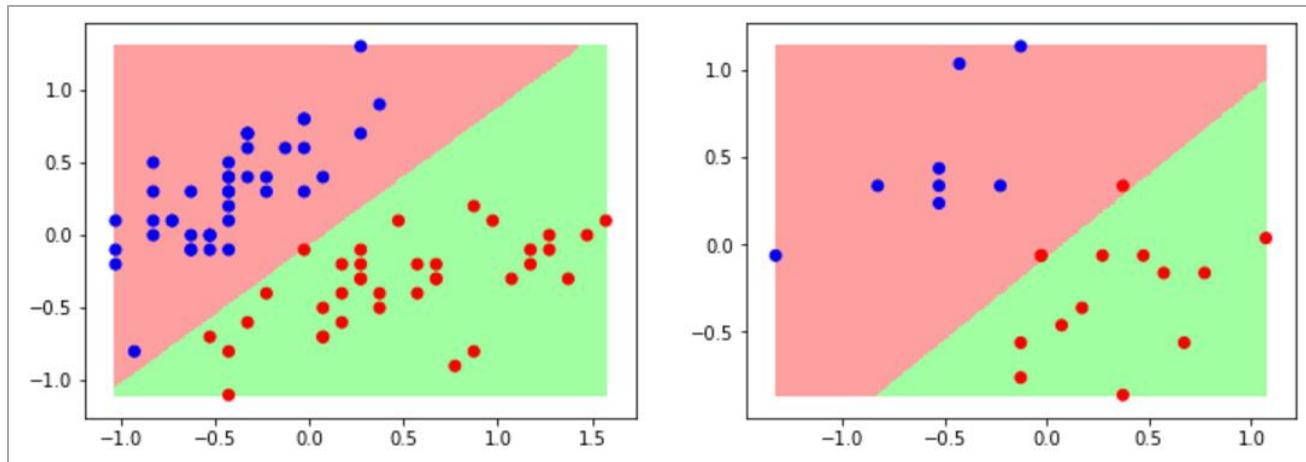
```
In [36]: plt.pcolormesh(m1, m2, n, cmap=cm_bg)
plt.scatter(x_test[:, 0], x_test[:, 1], c=y_test, cmap=cm_pt)
plt.show()
```

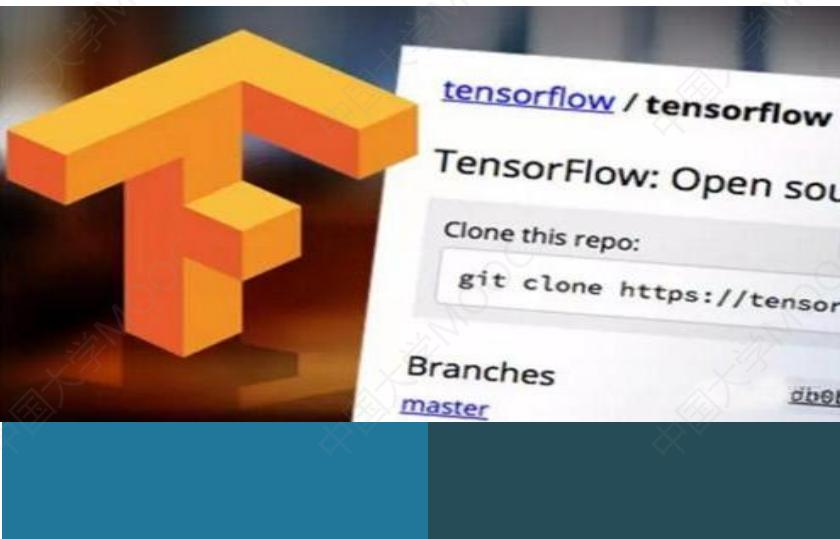
13	5.5	2.5	4	1.3	1
14	5.6	2.8	4.9	2	2
15	5.5	4.2	1.4	0.2	0
16	5.5	2.3	4	1.3	1
17	5.6	3	4.1	1.3	1
18	5.6	3	4.5	1.5	1
19	5.7	2.6	3.5	1	1
20	5.8	2.7	3.9	1.2	1
21	5.7	2.5	5	2	2
22	5.9	3	4.2	1.5	1



11.4.2 绘制分类图

- 分别绘制训练集和测试集的分类图



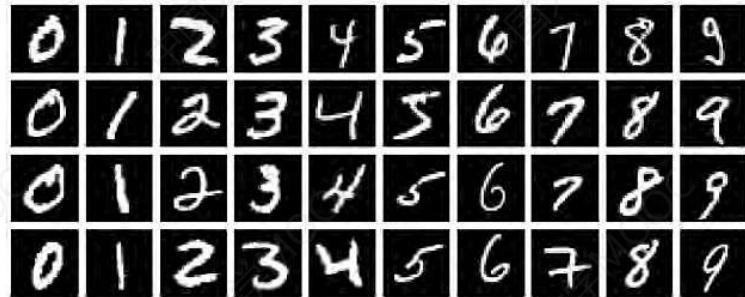


11.5 多分类问题

11.5 多分类问题

逻辑回归: 二分类问题

多分类问题: 把输入样本划分为多个类别



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11.5 多分类问题

■ 自然顺序码

- 0——山鸢尾 (Setosa)
- 1——变色鸢尾 (Versicolour)
- 2——维吉尼亚鸢尾 (Virginica)

■ 独冷编码 (one cold)

- (0,1,1)——山鸢尾 (Setosa)
- (1,0,1)——变色鸢尾 (Versicolour)
- (1,1,0)——维吉尼亚鸢尾 (Virginica)

■ 独热编码 (One-Hot Encoding)

- 使**非偏序关系**的数据，**取值不具有偏序性**
- 到原点等距
- (1,0,0)——山鸢尾 (Setosa)
- (0,1,0)——变色鸢尾 (Versicolour)
- (0,0,1)——维吉尼亚鸢尾 (Virginica)

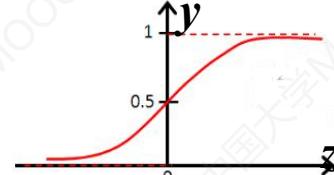
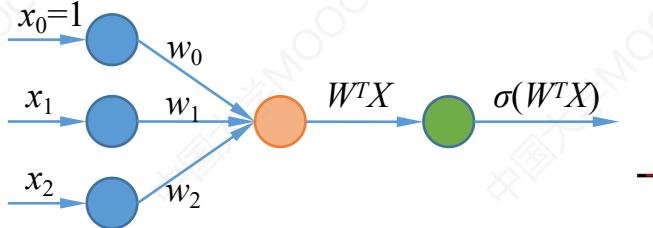
■ 应用

离散的特征
多分类问题中的类别标签

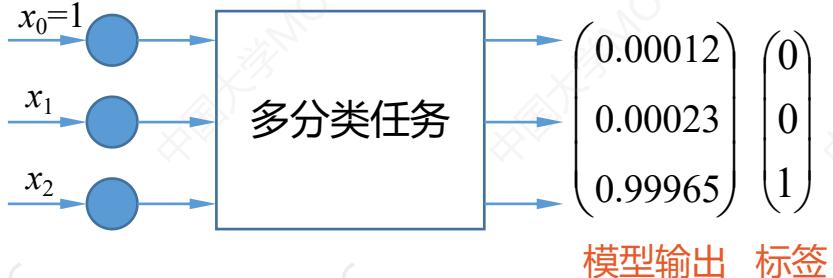


11.5 多分类问题

二分类



多分类



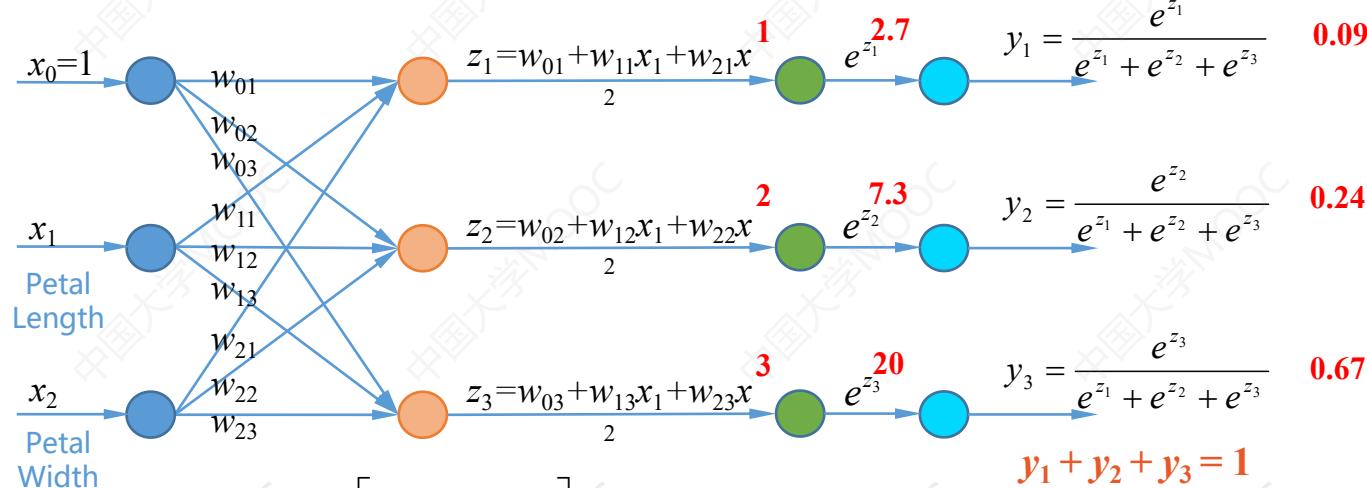
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11.5 多分类问题

■ softmax()函数 $Y = \text{softmax}(W^T X)$

广义线性回归, 实现多分类

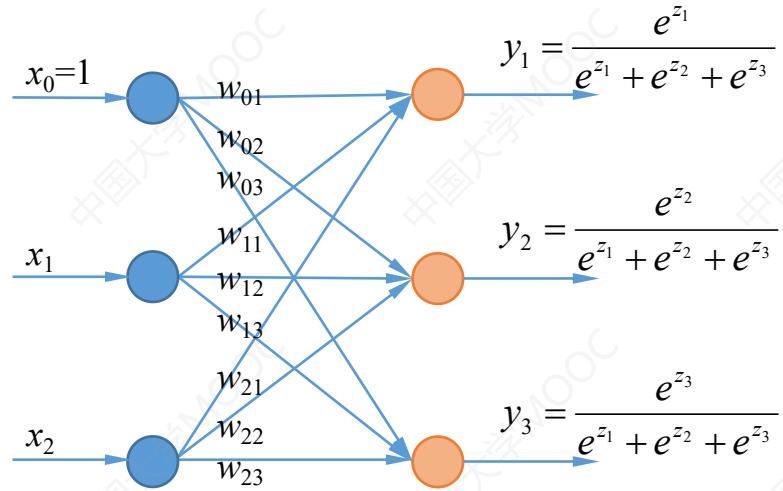
例: 使用属性花瓣长度和花瓣宽度, 构造分类器, 能够识别3种类型的鸢尾花



$$W = \begin{bmatrix} w_{01} & w_{02} & w_{03} \\ w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{bmatrix}$$



11.5 多分类问题

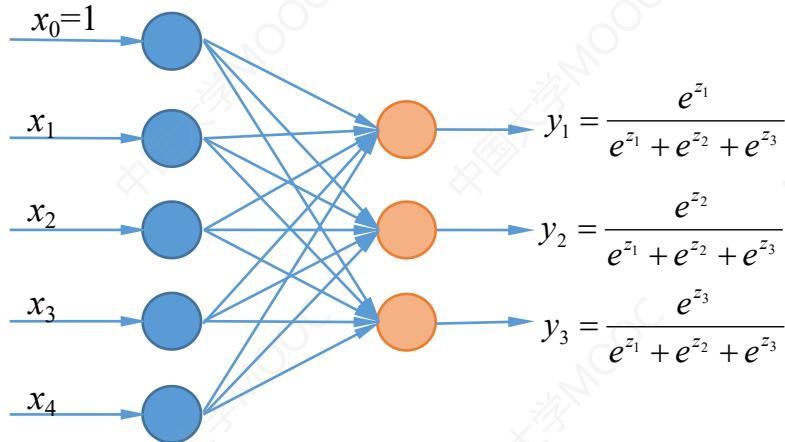


$$W = \begin{bmatrix} w_{01} & w_{02} & w_{03} \\ w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{bmatrix}$$



11.5 多分类问题

Softmax函数是Logistic函数在**多分类**问题上的推广



$$W = \begin{bmatrix} w_{01} & w_{02} & w_{03} \\ w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \\ w_{41} & w_{42} & w_{43} \end{bmatrix}$$

$$z_1 = w_{01} + w_{11}x_1 + w_{21}x_2 + w_{31}x_3 + w_{41}x_4$$

$$z_2 = w_{02} + w_{12}x_1 + w_{22}x_2 + w_{32}x_3 + w_{42}x_4$$

$$z_3 = w_{03} + w_{13}x_1 + w_{23}x_2 + w_{33}x_3 + w_{43}x_4$$

$$y_k = \frac{e^{z_k}}{\sum_{p=1}^C e^{z_p}}$$

k : 输出的索引

z_k : 第 k 个输出接收到的所有输入的线性组合

C : 类别总数



11.5 多分类问题

■ 二元交叉熵损失函数 (BCE)

$$Loss = - \sum_{i=1}^n [y_i \ln \hat{y}_i + (1 - y_i) \ln(1 - \hat{y}_i)]$$

■ 多分类交叉熵损失函数 (CCE)

$$Loss = - \sum_{i=1}^n \sum_{p=1}^C y_{i,p} \ln(\hat{y}_{i,p})$$

i : 样本的索引

n : 样本的总数

$y_{i,p}$: 第 i 个样本属于第 p 类的标记值, 0/1

$\hat{y}_{i,p}$: 第 i 个样本属于第 p 类的预测概率

C : 类别总数

$$y_k = \frac{e^{z_k}}{\sum_{p=1}^C e^{z_p}}$$

$$Loss = - \sum_{i=1}^n \sum_{p=1}^C y_{i,p} \ln(\hat{y}_{i,p})$$



11.5 多分类问题

	样本	标记	预测值	结果判断
模型A	样本1	0	0.3	正确
		0	0.3	
		1	0.4	
	样本2	0	0.3	正确
		1	0.4	
		0	0.3	
模型B	样本3	1	0.1	错误
		0	0.2	
		0	0.7	
	样本1	0	0.1	正确
		0	0.2	
		1	0.7	
	样本2	0	0.1	正确
		1	0.7	
		0	0.2	
	样本3	1	0.3	错误
		0	0.4	
		0	0.3	

准确率：

$$\text{模型A} = \text{模型B} = 2/3 = 66.7\%$$

交叉熵损失：

模型A

$$\text{样本1: } -(0 \times \ln 0.3 + 0 \times \ln 0.3 + 1 \times \ln 0.4) = -\ln 0.4 = 0.9162...$$

$$\text{样本2: } -(0 \times \ln 0.3 + 1 \times \ln 0.4 + 0 \times \ln 0.3) = -\ln 0.4 = 0.9612...$$

$$\text{样本3: } -(1 \times \ln 0.1 + 0 \times \ln 0.2 + 0 \times \ln 0.7) = -\ln 0.1 = 2.3025...$$

$$\text{交叉熵损失和: } 5.9677...$$

模型B

$$\text{样本1: } -(0 \times \ln 0.1 + 0 \times \ln 0.2 + 1 \times \ln 0.7) = -\ln 0.7 = 0.3566...$$

$$\text{样本2: } -(0 \times \ln 0.1 + 1 \times \ln 0.7 + 0 \times \ln 0.2) = -\ln 0.7 = 0.3566...$$

$$\text{样本3: } -(1 \times \ln 0.3 + 0 \times \ln 0.4 + 0 \times \ln 0.3) = -\ln 0.3 = 1.2039...$$

$$\text{交叉熵损失和: } 1.9173...$$



■ 互斥的多分类问题：每个样本只能够属于一个类别

鸢尾花的识别

手写数字识别

■ 非互斥的多分类问题：一个样本可以同时属于多个类别

构建多个一对多的逻辑回归

标签：

彩色图片

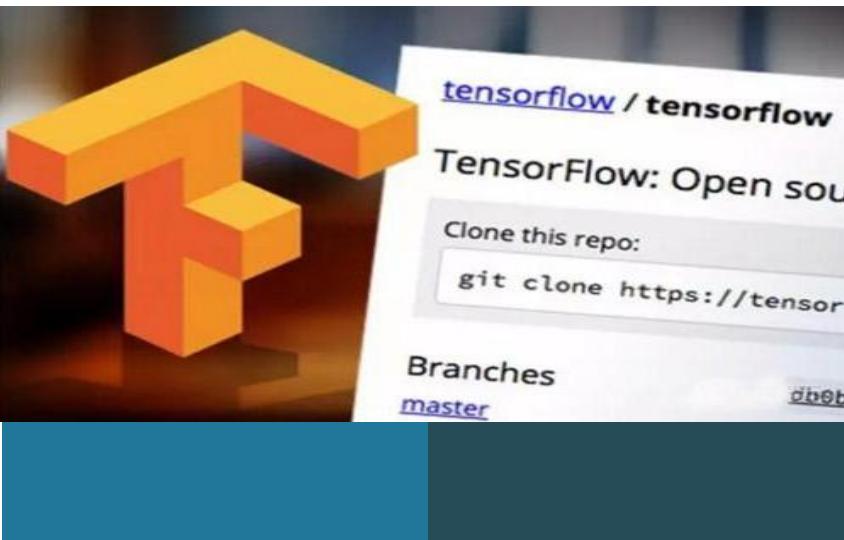
包括人物的图片

包括汽车的图片

户外图片

室内图片





11.6 实例：实现多分类问题

11.6 实例：实现多分类问题

■ 独热编码

一维数组/张量

编码深度

one_hot (indices, depth)

```
In [1]: import tensorflow as tf  
print("TensorFlow version:", tf.__version__)
```

TensorFlow version: 2.0.0

```
In [2]: import numpy as np
```

```
In [3]: a=[0, 2, 3, 5]  
b=tf.one_hot(a, 6)  
b
```

```
Out[3]: <tf.Tensor: id=4, shape=(4, 6), dtype=float32, numpy=  
array([[1., 0., 0., 0., 0., 0.], 0  
       [0., 0., 1., 0., 0., 0.], 2  
       [0., 0., 0., 1., 0., 0.], 3  
       [0., 0., 0., 0., 0., 1.]], dtype=float32)>
```



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11.6 实例：实现多分类问题

■ 准确率

```
In [4]: pred=np.array([[0.1, 0.2, 0.7],  
[0.1, 0.7, 0.2],  
[0.3, 0.4, 0.3]])  
y=np.array([2, 1, 0])  
y_onehot=np.array([[0, 0, 1],  
[0, 1, 0],  
[1, 0, 0]])
```

预测值
标记
标记独热编码

```
In [5]: tf.argmax(pred, axis=1)
```

预测值中的最大数索引

```
Out[5]: <tf.Tensor: id=7, shape=(3,), dtype=int64, numpy=array([2, 1, 1], dtype=int64)>
```

```
In [6]: tf.equal(tf.argmax(pred, axis=1), y)
```

判读预测值是否与样本标记相同

```
Out[6]: <tf.Tensor: id=12, shape=(3,), dtype=bool, numpy=array([ True,  True, False])>
```

```
In [7]: tf.cast(tf.equal(tf.argmax(pred, axis=1), y), tf.float32)
```

将布尔值转化为数字

```
Out[7]: <tf.Tensor: id=18, shape=(3,), dtype=float32, numpy=array([1., 1., 0.], dtype=float32)>
```

```
In [8]: tf.reduce_mean(tf.cast(tf.equal(tf.argmax(pred, axis=1), y), tf.float32))
```

```
Out[8]: <tf.Tensor: id=26, shape=(), dtype=float32, numpy=0.6666667>
```

准确率

11.6 实例：实现多分类问题

■ 交叉熵损失函数

$$Loss = - \sum_{i=1}^n \sum_{p=1}^C y_{i,p} \ln(\hat{y}_{i,p})$$

In [9]: `-y_onehot*tf.math.log(pred)`

Out[9]: <tf.Tensor: id=30, shape=(3, 3), dtype=float64, numpy=
array([[-0. , -0. , 0.35667494], 样本1
[-0. , 0.35667494, -0.], 样本2
[1.2039728, -0. , -0.]])>样本3

In [10]: `-tf.reduce_sum(y_onehot*tf.math.log(pred))` 所有样本交叉熵之和

Out[10]: <tf.Tensor: id=37, shape=(), dtype=float64, numpy=1.917322692203401>

In [11]: `-tf.reduce_sum(y_onehot*tf.math.log(pred))/len(pred)` 平均交叉熵损失

Out[11]: <tf.Tensor: id=46, shape=(), dtype=float64, numpy=0.6391075640678003>



11.6 实例：实现多分类问题

例：使用**花瓣长度**、**花瓣宽度**将**三种**鳶尾花区分开

□ 加载数据

```
In [1]: import tensorflow as tf  
print("TensorFlow version:", tf.__version__)
```

TensorFlow version: 2.0.0

```
In [2]: import pandas as pd  
import numpy as np  
import matplotlib as mpl  
import matplotlib.pyplot as plt
```

```
In [3]: TRAIN_URL = "http://download.tensorflow.org/data/iris_training.csv"  
train_path = tf.keras.utils.get_file(TRAIN_URL.split('/')[-1], TRAIN_URL)
```

```
In [4]: df_iris_train = pd.read_csv(train_path, header=0)
```



11.6 实例：实现多分类问题

□ 处理数据

```
In [5]: iris_train=np.array(df_iris_train)
```

```
In [6]: iris_train.shape
```

```
Out[6]: (120, 5)
```

```
In [7]: x_train=iris_train[:, 2:4] 提取花瓣长度、花瓣宽度属性  
y_train=iris_train[:, 4]
```

```
In [8]: x_train.shape, y_train.shape
```

```
Out[8]: ((120, 2), (120,))
```

```
In [9]: num_train=len(x_train)
```



□ 处理数据

```
In [10]: x0_train = np.ones(num_train).reshape(-1, 1)
X_train= tf.cast(tf.concat([x0_train, x_train], axis=1),tf.float32)
Y_train= tf.one_hot(tf.constant(y_train, dtype=tf.int32), 3)

In [11]: X_train.shape, Y_train.shape
Out[11]: (TensorShape([120, 3]), TensorShape([120, 3]))
```

□ 设置超参数、设置模型参数初始值

```
In [12]: learn_rate = 0.2
iter = 500
display_step =100

In [13]: np.random.seed(612)
W = tf.Variable(np.random.randn(3, 3), dtype=tf.float32)
```



11.6 实例：实现多分类问题

□ 训练模型

```
In [14]: acc=[]
cce=[]

for i in range(0,iter+1):
    with tf.GradientTape() as tape:

        PRED_train=tf.nn.softmax(tf.matmul(X_train,W))
        Loss_train=-tf.reduce_sum(Y_train*tf.math.log(PRED_train))/num_train

        accuracy=tf.reduce_mean(tf.cast(tf.equal(tf.argmax(PRED_train.numpy(),axis=1),y_train),tf.float32))

        acc.append(accuracy)
        cce.append(Loss_train)

        dL_dW = tape.gradient(Loss_train,W)
        W.assign_sub(learn_rate*dL_dW)

    if i % display_step == 0:
        print("i: %i, Acc: %f, Loss: %f" % (i, accuracy, Loss_train))
```

```
i: 0, Acc: 0.350000, Loss: 4.510763
i: 100, Acc: 0.808333, Loss: 0.503537
i: 200, Acc: 0.883333, Loss: 0.402912
i: 300, Acc: 0.891667, Loss: 0.352650
i: 400, Acc: 0.941667, Loss: 0.319778
i: 500, Acc: 0.941667, Loss: 0.295599
```



11.6 实例：实现多分类问题

□ 训练结果

```
In [15]: PRED_train.shape
```

```
Out[15]: TensorShape([120, 3]) 属于每种类别的概率
```

```
In [16]: tf.reduce_sum(PRED_train, axis=1)
```

概率之和为1

```
Out[16]: <tf.Tensor: id=24068, shape=(120,), dtype=float32, numpy=
array([1.          , 1.          , 1.          , 0.9999999 , 1.          ,
       0.99999994, 1.          , 1.          , 1.          , 1.          ,
       1.          , 1.0000001 , 1.          , 1.          , 1.          ,
       1.          , 0.99999994, 0.99999994, 1.          , 1.          ,
       1.          , 1.          , 1.          , 0.9999999 , 1.          ,
       0.99999994, 0.99999994, 1.          , 1.          , 1.          ,
       1.          , 0.99999994, 1.          , 1.          , 1.          ,
       1.          , 1.          , 1.          , 1.          , 1.0000001 ,
       1.          , 1.          , 1.          , 1.          , 1.          ,
       1.          , 1.          , 1.          , 1.          , 1.          ,
       1.0000001 , 1.          , 1.          , 1.          , 1.          ,
       1.          , 1.          , 1.          , 1.          , 1.          ,
```



□ 训练结果

转换为自然顺序码

In [17]: `tf.argmax(PRED_train.numpy(), axis=1)`

Out[17]: <tf.Tensor: id=24071, shape=(120,), dtype=int64, numpy=
array([2, 1, 2, 0, 0, 0, 2, 1, 0, 1, 1, 0, 0, 2, 2, 2, 2, 2, 2, 0, 2, 2,
0, 1, 1, 0, 1, 2, 1, 2, 1, 1, 1, 2, 2, 2, 2, 2, 0, 0, 2, 2, 2, 0,
0, 1, 0, 2, 0, 2, 0, 1, 1, 0, 1, 2, 2, 2, 2, 1, 1, 2, 2, 2, 1, 2,
0, 2, 2, 0, 0, 1, 0, 2, 2, 0, 1, 1, 1, 2, 0, 1, 1, 1, 2, 0, 1, 1,
2, 0, 2, 1, 0, 0, 2, 0, 0, 2, 2, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0,
2, 1, 0, 2, 0, 1, 1, 0, 0, 1], dtype=int64)>



□ 绘制分类图

```
In [18]: M=500
        x1_min, x2_min = x_train.min(axis=0)
        x1_max, x2_max = x_train.max(axis=0)
        t1 = np.linspace(x1_min, x1_max, M)
        t2 = np.linspace(x2_min, x2_max, M)
        m1,m2 = np.meshgrid(t1, t2)
```

```
In [19]: m0=np.ones(M*M)
        X_ = tf.cast(np.stack((m0,m1.reshape(-1),m2.reshape(-1))), axis=1),tf.float32)
        Y_ =tf.nn.softmax(tf.matmul(X_,W))
```

```
In [20]: Y_=tf.argmax(Y_.numpy(),axis=1) 转换为自然顺序码，决定网格颜色
```



11.6 实例：实现多分类问题

□ 绘制分类图

```
In [21]: n=tf.reshape(Y_, m1.shape) 和m1形状相同
```

```
In [22]: n
```

```
Out[22]: <tf.Tensor: id=24081, shape=(500, 500), dtype=int64, numpy=
array([[0, 0, 0, ..., 1, 1, 1],
       [0, 0, 0, ..., 1, 1, 1],
       [0, 0, 0, ..., 1, 1, 1],
       ...,
       [2, 2, 2, ..., 2, 2, 2],
       [2, 2, 2, ..., 2, 2, 2],
       [2, 2, 2, ..., 2, 2, 2]], dtype=int64)>
```



11.6 实例：实现多分类问题

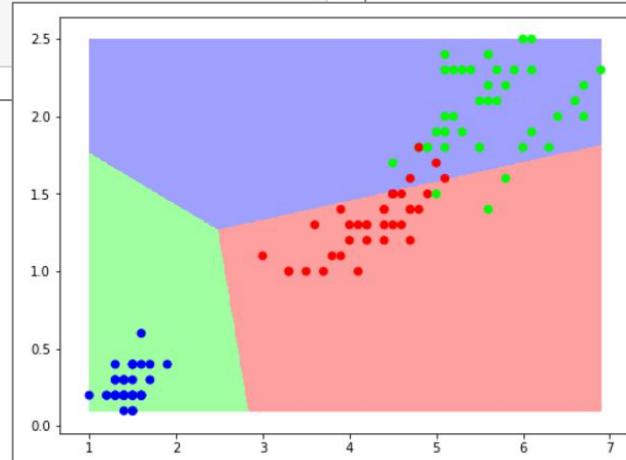
□ 绘制分类图

```
In [25]: plt.figure(figsize=(8, 6))

cm_bg = mpl.colors.ListedColormap(['#A0FFA0', '#FFA0A0', '#AOA0FF'])

plt.pcolormesh(m1, m2, n, cmap=cm_bg)
plt.scatter(x_train[:, 0], x_train[:, 1], c=y_train, cmap="brg")

plt.show()
```



兰州交通大学

计算机科学与技术学院

第11讲 分类问题

