0.0.1 At Index

The operation $atIndex$ will return the Value at a specified numeric index within a Collection or an empty Collection if there is no value at the specified index.

$$\begin{array}{|l}
\underline{atIndex[Collection, \mathbb{N}]} \\
idx? : \mathbb{N} \\
coll? : Collection \\
atIndex : Collection \times \mathbb{N} \rightarrow V \\
\hline
\# idx? = 1 \\
coll! = atIndex(coll?, idx?) = (head\,(\,idx? \restriction coll?\,)) \iff idx? \in coll? \\
coll! = atIndex(coll?, idx?) = \langle\rangle \iff idx? \notin coll?
\end{array}$$

Given the definition of the *Collection* and V free types

$$Collection ::= emptyColl \,|\, append\langle\!\langle Collection \times Scalar \vee Collection \vee KV \times \mathbb{N}\rangle\!\rangle$$
$$V ::= Scalar \,|\, Collection \,|\, KV$$

The collection member $coll?_{idx?} : V$ is implied from *append* accepting the argument of type $Scalar \vee Collection \vee KV \equiv V$ which means each Collection member is of type V. Given that extraction ($\_ \restriction \_$) returns a Collection,

$$\begin{array}{|l}
seq\,X : Collection \\
\hline
\_ \restriction \_ : \mathbb{P}\,\mathbb{N}_1 \times seq\,X \rightarrow seq\,X
\end{array}$$

in order for $atIndex$ to return the collection member without altering its type, the first member of $atIdx'$ must be returned, not $atIdx'$ itself.

$$\begin{array}{|l}
atIdx' : Collection \\
coll!, coll?_{idx?} : V \\
\hline
atIdx' = (\,idx? \restriction coll?\,) \Rightarrow \langle coll?_{idx?} \rangle \\
coll! = head(atIdx') = coll?_{idx?}
\end{array}$$

The *head* call is made possible by restricting $idx?$ to be a single numeric value.

$$idx?, idx' : \mathbb{N}$$
$$\# idx? = 1 \bullet (\,idx? \restriction coll?\,) = \langle coll?_{idx?} \rangle \bullet$$
$$(head(\,idx? \restriction coll?\,)) = coll?_{idx?} \qquad [\text{expected return given } idx?]$$
$$\# idx' \geq 2 \bullet (\,idx' \restriction coll?\,) = \langle coll?_{idx'_i} \,..\, coll?_{idx'_j} \rangle \bullet$$
$$(head(\,idx' \restriction coll?\,)) = coll?_{idx'_i} \quad [\text{unexpected return given } idx']$$

Additionally, if the provided $idx? \notin coll?$ then an empty Collection will be returned given that *head* must be passed a non-empty Collection.

$$\begin{array}{|l}
head : seq_1 X \rightarrow X \\
\hline
idx? \notin coll? \Rightarrow (\,idx? \restriction coll?\,) = \langle\rangle \neg seq_1
\end{array}$$

The properties of $atIndex$ are illustrated in the following examples.

$$X = \langle x_0, x_1, x_2 \rangle$$
$$x_0 = 0$$
$$x_1 = foo$$
$$x_2 = \langle a, b, c \rangle$$

$atIndex(X, 0) = 0$ $[head\,(\langle\, x_0\, \rangle)]$

$atIndex(X, 1) = foo$ $[head\,(\langle\, x_1\, \rangle)]$

$atIndex(X, 2) = \langle a, b, c \rangle$ $[head\,(\langle\, x_2\, \rangle)]$

$atIndex(X, 3) = \langle\rangle$ $[3 \notin X \Rightarrow x_3 \notin X]$