0.0.1 Append

The operation *append* will return a Collection with a Value added at a specified numeric Index.

$$
\begin{array}{|l}
\multicolumn{1}{l}{Append[Collection, V, \mathbb{N}]} \\
\hline
coll?\,, coll! : Collection \\
v? : V \\
idx? : \mathbb{N} \\
append\_ : Collection \times V \times \mathbb{N} \rightarrowtail Collection \\
\hline
\#\, idx? = 1 \\
coll! = append(coll?\,, v?\,, idx?\,) \bullet \\
\quad\quad let\ coll' == front(\{\, i : \mathbb{N} \mid i \in 0 \mathinner{..} idx?\,\} \upharpoonright coll?\,) \frown v? \\
\quad\quad\quad coll'' == \{\, j : \mathbb{N} \mid j \in idx? \mathinner{..} \#\, coll?\,\} \upharpoonright coll? \\
\quad\quad = coll' \frown coll'' \Rightarrow \\
\quad\quad\quad (front(coll') \frown v? \frown coll'') \wedge \\
\quad\quad\quad (\, v? \mapsto idx? \in coll!\,) \wedge \\
\quad\quad\quad (\, \#\, coll! = \#\, coll? + 1) \\
\hline
\end{array}
$$

append results in the composition of $coll'$ and $coll''$ such that

$$coll! = coll' \frown coll'' \ \wedge\ idx? \mapsto v? \in coll!$$

- $coll'$ is the items in $coll?$ up to and including $idx?$ but the value at $idx?$ is replaced with $v?$ such that $idx? \mapsto coll?_{idx?} \notin coll'$
- $coll''$ is the items in $coll?$ from $idx?$ to $\#\, coll? \Rightarrow coll?_{idx?} \in coll''$

The following example illustrates these properties.

$$
\begin{aligned}
&X = \langle x_0, x_1, x_2 \rangle \\
&\quad x_0 = 0 \\
&\quad x_1 = foo \\
&\quad x_2 = \langle a, b, c \rangle \\
&\quad v? = bar \\
&append(X, v?\,, 0) = \langle bar, 0, foo, \langle a, b, c \rangle \rangle \\
&append(X, v?\,, 1) = \langle 0, bar, foo, \langle a, b, c \rangle \rangle \\
&append(X, v?\,, 2) = \langle 0, foo, bar, \langle a, b, c \rangle \rangle \\
&append(X, v?\,, 3) = \langle 0, foo, \langle a, b, c \rangle, bar \rangle \\
&append(X, v?\,, 4) = append(X, v?\,, 3) \iff 3 \notin \operatorname{dom} X
\end{aligned}
$$