0.0.1 At Index

The operation $atIndex$ will return the Value at a specified numeric index within a Collection or an empty Collection if there is no value at the specified index.

$$
\begin{array}{l}
\underline{\;AtIndex[Collection, \mathbb{N}]\;} \\
\quad idx? : \mathbb{N} \\
\quad coll? : Collection \\
\quad atIndex\,\_ : Collection \times \mathbb{N} \twoheadrightarrow V \\
\hline
\quad \#\,idx? = 1 \\
\quad coll! = atIndex(coll?, idx?) = (head\,(\,idx? \upharpoonright coll?\,)) \iff idx? \in coll? \\
\quad coll! = atIndex(coll?, idx?) = \langle\rangle \iff idx? \notin coll?
\end{array}
$$

Given the definition of the *Collection* and V free types

$$Collection ::== emptyColl \mid append\langle\!\langle Collection \times Scalar \vee Collection \vee KV \times \mathbb{N} \rangle\!\rangle$$
$$V ::= Scalar \mid Collection \mid KV$$

The collection member $coll?_{idx?} : V$ is implied from *append* accepting the argument of type $Scalar \vee Collection \vee KV \equiv V$ which means each Collection member is of type V. Given that extraction $(\,\_ \upharpoonright \_\,)$ returns a Collection,

$$
\begin{array}{l}
\quad seq\,X : Collection \\
\hline
\quad \_ \upharpoonright \_ : \mathbb{P}\,\mathbb{N}_1 \times seq\,X \to seq\,X
\end{array}
$$

in order for $atIndex$ to return the collection member without altering its type, the first member of $atIdx'$ must be returned, not $atIdx'$ itself.

$$
\begin{array}{l}
\quad atIdx' : Collection \\
\quad coll!, coll?_{idx?} : V \\
\hline
\quad atIdx' = (\,idx? \upharpoonright coll?\,) \Rightarrow \langle coll?_{idx?} \rangle \\
\quad coll! = head(atIdx') = coll?_{idx?}
\end{array}
$$

The *head* call is made possible by restricting $idx?$ to be a single numeric value.

$$idx?, idx' : \mathbb{N}$$
$$\#\,idx? = 1 \bullet (\,idx? \upharpoonright coll?\,) = \langle coll?_{idx?} \rangle \bullet$$
$$(head(\,idx? \upharpoonright coll?\,)) = coll?_{idx?} \qquad [\text{expected return given } idx?]$$
$$\#\,idx' \geq 2 \bullet (\,idx' \upharpoonright coll?\,) = \langle coll?_{idx'_i} \,..\, coll?_{idx'_j} \rangle \bullet$$
$$(head(\,idx' \upharpoonright coll?\,)) = coll?_{idx'_i} \quad [\text{unexpected return given } idx']$$

Additionally, if the provided $idx? \notin coll?$ then an empty Collection will be returned given that *head* must be passed a non-empty Collection.

$$
\begin{array}{l}
\quad head : seq_1\,X \to X \\
\hline
\quad idx? \notin coll? \Rightarrow (\,idx? \upharpoonright coll?\,) = \langle\rangle \neg seq_1
\end{array}
$$

The properties of $atIndex$ are illustrated in the following examples.

$$X = \langle x_0, x_1, x_2 \rangle$$
$$x_0 = 0$$
$$x_1 = foo$$
$$x_2 = \langle a, b, c \rangle$$

$$atIndex(X, 0) = 0 \qquad\qquad [head\,(\langle\, x_0\, \rangle)]$$
$$atIndex(X, 1) = foo \qquad\qquad [head\,(\langle\, x_1\, \rangle)]$$
$$atIndex(X, 2) = \langle a, b, c \rangle \qquad\qquad [head\,(\langle\, x_2\, \rangle)]$$
$$atIndex(X, 3) = \langle\rangle \qquad\qquad [3 \notin X \Rightarrow x_3 \notin X]$$