

### 0.0.1 Associate

The operation *associate* establishes a relationship between  $k?$  and  $v?$  at the top level of  $m!$ .

$$\begin{array}{c}
 \text{Associate}[KV, K, V] \text{-----} \\
 m?, m!, m' : KV \\
 k? : K \\
 v? : V \\
 \text{associate}_- : KV \times K \times V \mapsto KV \\
 \hline
 m! = \text{associate}(m?, k?, v?) \bullet \\
 \text{let } m' == m? \triangleleft k? \Rightarrow \\
 \quad (\text{dom } m' = \text{dom } (m? \setminus k?)) \wedge \\
 \quad (m? \setminus m' = k? \iff k? \in m?) \wedge \\
 \quad (m? \setminus m' = \emptyset \iff k? \notin m? \Rightarrow m? = m') \\
 = \langle\langle k? \mapsto v? \rangle\rangle \cup m'
 \end{array}$$

This implies that any existing mapping at  $k? \in m?$  will be overwritten by *associate* but an existing mapping is not a precondition.

$$\begin{array}{c}
 (k?, m?_{k?}) \in m? \vee (k?, m?_{k?}) \notin m? \\
 (k?, m?_{k?}) \notin m! \\
 (k?, v?) \in m! \\
 \hline
 m! = \text{associate}(m?, k?, v?)
 \end{array}$$

*associate* does not alter any other mappings within  $m?$  and this property is illustrated by the definition of local variable  $m'$

$$\begin{array}{c}
 m' : KV \mid m' = m? \triangleleft k? \Rightarrow m' \triangleleft (m? \setminus k?) \\
 \hline
 \text{dom } m? = \{k_i : K \mid 0.. \# m? \bullet k_i \in m? \wedge 0 \leq i \leq \# m?\} \\
 \text{dom } m' = \{k'_i : K \mid 0.. \# m' \bullet k'_i \in m? \wedge k'_i \neq k? \wedge 0 \leq i \leq \# m'\} \\
 \text{dom } m' = \text{dom } m? \iff k? \notin m? \Rightarrow \forall k_i \in m? \mid k_i \neq k? \\
 \# m' = \# m? \iff k? \notin m? \\
 \# m' = \# m? - 1 \iff k? \in m?
 \end{array}$$

and its usage within the definition of *associate*.

$$\begin{array}{l}
 m! = m? \cup \langle\langle k? \mapsto v? \rangle\rangle \Rightarrow k? \notin m? \\
 m! = m' \cup \langle\langle k? \mapsto v? \rangle\rangle \Rightarrow m' \neq m? \wedge k? \in m?
 \end{array}$$

The following examples demonstrate the intended functionality of *associate*.

$$\begin{array}{l}
 M = \langle\langle k_0 v_{k_0}, k_1 v_{k_1} \rangle\rangle \\
 k_0 = abc \wedge v_{k_0} = 123 \qquad [k_0 v_{k_0} = abc \mapsto 123] \\
 k_1 = def \wedge v_{k_1} = xyz \mapsto 456 \qquad [k_1 v_{k_1} = def \mapsto xyz \mapsto 456] \\
 \text{associate}(M, baz, foo) = \langle\langle abc \mapsto 123, def \mapsto xyz \mapsto 456, baz \mapsto foo \rangle\rangle \\
 \text{associate}(M, abc, 321) = \langle\langle abc \mapsto 321, def \mapsto xyz \mapsto 456 \rangle\rangle
 \end{array}$$