### 0.0.1 Associate At

The Primitive $associateAt$ establishes a relationship between $k?_j$ and $v?$ at the nesting $k?_i \mathrel{..} k?_{j-1}$ within $m!$

$$
\begin{array}{|l}
k? = \langle k?_i \mathrel{..} k?_j \rangle \\
(k?_j, m?_{k?_j}) \in m? \lor (k?_j, m?_{k?_j}) \notin m? \\
(k?_j, m?_{k?_j}) \notin m! \\
(k?_j, v?) \in m! \\
\hline
m! = associateAt(m?, k?, v?)
\end{array}
$$

This implies that any existing mapping at $k?_j \in m?$ will be overwritten by $associateAt$ but an existing mapping is not a precondition. The following helper Operation $getFirstKey$ is introduced to help with the recursive nature of $associateAt$.

$$
\begin{array}{|l}
\hline
GetFirstKey[KV, Collection] \\
m? : KV \\
k? : Collection \\
v! : V \\
getFirstKey\_ : KV \times Collection \twoheadrightarrow V \\
\hline
v! = getFirstKey(m?, k?) \bullet v! = atKey(m?, head(k?)) \\
\hline
\end{array}
$$

This allows for the recursive aspect of $associateAt$ to be defined as $\langle getFirstKey\_, recur\_\rangle^{\# k?-1}$

$$
\begin{array}{|l}
\hline
AssociateAt[KV, Collection, V] \\
GetFirstKey, Recur \\
m?, m! : KV \\
k? : Collection \bullet \forall k?_n \in k?_{\langle i \mathrel{..} j \rangle} \mid k_n : K \\
v? : V \\
associateAt\_ : KV \times Collection \times V \rightarrowtail KV \\
\hline
associateAt = \langle\langle atFirstKey \;\; recur\_\rangle^{\# k?-1}, associate\_\rangle \\
m! = associateAt(m?, k?, v?) \bullet \\
\quad \forall n : i \mathrel{..} j-1 \in k? \bullet j = first(last(k?)) \Rightarrow first(j, k?_j) \mid \exists_1 v_n \bullet \\
\qquad let \;\; c_n == tail(k?)^{n-i} \\
\qquad\qquad v_i == getFirstKey(m?, c_n) \Rightarrow \\
\qquad\qquad\qquad c_n = k? \iff n = i \bullet v_i = atKey(m?, head(k?)) \\
\qquad\qquad v_n = recur(v_i, c_n, getFirstKey\_)^{j-1} \\
\qquad\qquad v_{j-1} == getFirstKey(v_n, (j-1 \upharpoonright k?)) \iff n = j-2 \\
\qquad\qquad v_j == associate(v_{j-1}, last(k?), v?) \Rightarrow \langle\!\langle k?_j \mapsto v? \rangle\!\rangle \cup v_{j-1} \lhd k?_j \\
\quad = (v_j \cup v_{n-succ(1)} \lhd k?_{n-succ(0)})^{j-1} \bullet n \leq j-1 \Rightarrow \\
\qquad v_{j-4} \cup (v_{j-3} \cup (v_j \cup v_{j-2} \lhd k?_{j-1}) \lhd k?_{j-2}) \lhd k?_{j-3} \\
\hline
\end{array}
$$

No other mappings which exist at the various levels of depth will be altered