0.0.1 Associate

The operation *associate* establishes a relationship between $k?$ and $v?$ at the top level of $m!$.

$$
\begin{array}{|l}
\multicolumn{1}{l}{Associate[KV, K, V]} \\
m?\,, m!\,, m' : KV \\
k? : K \\
v? : V \\
associate : KV \times K \times V \rightarrowtail KV \\
\hline
m! = associate(m?\,, k?\,, v?\,) \bullet \\
\quad let\ m' == m? \mathbin{⩤} k? \Rightarrow \\
\qquad\qquad (\mathrm{dom}\ m' = \mathrm{dom}\,(\,m? \setminus k?\,)) \land \\
\qquad\qquad (\,m? \setminus m' = k? \iff k? \in m?\,) \land \\
\qquad\qquad (\,m? \setminus m' = \emptyset \iff k? \notin m? \Rightarrow m? = m') \\
\quad = \langle\!\langle k? \mapsto v?\,\rangle\!\rangle \cup m' \\
\hline
\end{array}
$$

This implies that any existing mapping at $k? \in m?$ will be overwritten by *associate* but an existing mapping is not a precondition.

$$
\begin{array}{|l}
(k?\,, m?_{k?}\,) \in m? \lor (k?\,, m?_{k?}\,) \notin m? \\
(k?\,, m?_{k?}\,) \notin m! \\
(k?\,, v?\,) \in m! \\
\hline
m! = associate(m?\,, k?\,, v?\,)
\end{array}
$$

associate does not alter any other mappings within $m?$ and this property is illustrated by the definition of local variable m'

$$
\begin{array}{|l}
m' : KV \mid m' = m? \mathbin{⩤} k? \Rightarrow m' \lhd (m? \setminus k?\,) \\
\hline
\mathrm{dom}\ m? = \{\, k_i : K \mid 0 \mathrel{..} \#\,m? \bullet k_i \in m? \land 0 \leq i \leq \#\,m?\,\} \\
\mathrm{dom}\ m' = \{\, k_i' : K \mid 0 \mathrel{..} \#\,m' \bullet k_i' \in m? \land k_i' \neq k? \land 0 \leq i \leq \#\,m'\} \\
\mathrm{dom}\ m' = \mathrm{dom}\ m? \iff k? \notin m? \Rightarrow \forall k_i \in m? \mid k_i \neq k? \\
\#\,m' = \#\,m? \iff k? \notin m? \\
\#\,m' = \#\,m? - 1 \iff k? \in m?
\end{array}
$$

and its usage within the definition of *associate*.

$$
\begin{array}{l}
m! = m? \cup \langle\!\langle\, k? \mapsto v?\,\rangle\!\rangle \Rightarrow k? \notin m? \\
m! = m' \cup \langle\!\langle\, k? \mapsto v?\,\rangle\!\rangle \Rightarrow m' \neq m? \land k? \in m?
\end{array}
$$

The following examples demonstrate the intended functionality of *associate*.

$$
\begin{array}{ll}
M = \langle\!\langle k_0 v_{k_0}, k_1 v_{k_1} \rangle\!\rangle & \\
\quad k_0 = abc \ \land v_{k_0} = 123 & [k_0 v_{k_0} = abc \mapsto 123] \\
\quad k_1 = def \ \land v_{k_1} = xyz \mapsto 456 & [k_1 v_{k_1} = def \mapsto xyz \mapsto 456] \\
associate(M, baz, foo) = \langle\!\langle abc \mapsto 123,\ def \mapsto xyz \mapsto 456,\ baz \mapsto foo \rangle\!\rangle & \\
associate(M, abc, 321) = \langle\!\langle abc \mapsto 321,\ def \mapsto xyz \mapsto 456 \rangle\!\rangle &
\end{array}
$$