0.0.1 At Depth

The Primitive *atDepth* will return the Value at a specified depth of indices within a passed in Collection. The following helper Operation *getFirstIndex* is introduced to establish navigation into a nested Collection given a Collection of Indices.

$$
\begin{array}{|l}
\hline
GetFirstIndex[Collection, Collection] \\
coll?\,, idxs? : Collection \\
v! : V \\
getFirstIndex\_ : Collection \times Collection \pfun V \\
\hline
v! = getFirstIndex(coll?\,, idxs?\,) \bullet v! = atIndex(coll?\,, head(idxs?\,)) \\
\hline
\end{array}
$$

This allows for the navigation into a nested Collection to be defined as $\langle getFirstIndex\_, recur\_\rangle^{\#\, idxs?}$ which represents a step down into *coll*? for each member of *idxs*?. If there is not a value at some specified index or navigation can't continue despite what is being dictated by *idxs*?, the empty sequence $\langle\rangle$ will be returned

$$
\begin{array}{|l}
\hline
AtDepth[Collection, Collection] \\
Recur, GetFirstIndex \\
coll?\,, idxs? : Collection \\
v! : V \\
atDepth\_ : Collection \times Collection \pfun V \\
\hline
atDepth = \langle getFirstIndex\_, recur\_\rangle^{\#\, idxs?} \\
v! = atDepth(coll?\,, idxs?\,) \bullet \\
\quad \forall n : i \upto j \in \dom idxs? \bullet j = first(last(idxs?\,)) \Rightarrow first(j, idxs?_j\,) \mid \exists_1\, v_n \bullet \\
\qquad let \quad idxs_n == tail(idxs?\,)^{n-i} \\
\qquad\qquad v_i == getFirstIndex(coll?\,, idxs_n) \\
\qquad\qquad v_n == recur(v_i, idxs_n, getFirstIndex\_)^j \\
\qquad\qquad v_j == atIndex(v_n, last(idxs?\,)) \iff n = j - 1 \\
v! = v_j \bullet v_j : \seq_1 \iff (v_{n-1} = \seq_1 \land \\
\qquad\qquad\qquad\qquad head(idxs_n) \mapsto v_n \in v_{n-i}) \lor \\
\qquad v_j : \seq \iff v_{n-1} = \langle\rangle \Rightarrow atIndex(v_{n-2}, head(idxs_{n-1})) = \langle\rangle \\
\hline
\end{array}
$$

The following examples demonstrate the properties of *atDepth* described above.

$$
\begin{array}{l}
X = \langle x_0, x_1, x_2\rangle \\
\quad x_0 = 0 \\
\quad x_1 = foo \\
\quad x_2 = \langle a, b, c\rangle \\
atDepth(X, \langle 1\rangle) = foo \\
atDepth(X, \langle 1, 0\rangle) = f \Rightarrow foo = \langle f, o, o\rangle \\
atDepth(X, \langle 2, 0\rangle) = a \\
atDepth(X, \langle 2, 5\rangle) = \langle\rangle
\end{array}
$$