1 Map

The *map* operation accepts the following arguments

- Operation or Primitive
- A Collection

and returns a collection of $Operation(x_n) \ \vee Primitive(x_n)$ respectively

1.1 Formal Definition

Given an input Collection X and a Operation o or Primitive p where

$$X =< x_i..x_n..x_j >$$

and that collection consists of one or more members x_n within the range $i..j$

$$i \leq n \leq j \Rightarrow i \prec n \prec j \iff i \neq n \neq j$$

then

$$map(o,\ X) = Y \ \wedge \ map(p,\ X) = Y'$$

such that

$$Y =< o(x_i)..o(x_n)..o(x_j) >$$

and

$$Y' =< p(x_i)..p(x_n)..p(x_j) >$$

which establishes both Y and Y' are a Collection where each member y_n or y'_n is the result of passing x_n to o or p respectively.

In otherwords

$$o(x_i) \mapsto y_i \wedge o(x_n) \mapsto y_n \wedge o(x_j) \mapsto y_j$$

$$p(x_i) \mapsto y'_i \wedge p(x_n) \mapsto y'_n \wedge p(x_j) \mapsto y'_j$$

which implies both collections Y and Y' have the same ordering as collection X

$$i_{o(x)} = i_y \ \wedge \ n_{o(x)} = n_y \ \wedge \ j_{o(x)} = j_y$$

$$i_{p(x)} = i_{y'} \ \wedge \ n_{p(x)} = n_{y'} \ \wedge \ j_{p(x)} = j_{y'}$$

When X contains non-distinct values, o and p are unaffected.

$$o(x_n) = y_n$$

$$o(x_{n'}) = y_{n'}$$

$$o(x_{n'+1}) = y_{n'}$$

$$\iff$$

$$x_{n'} \equiv x_{n'+1} \wedge x_{n'} \not\equiv x_n$$
$$\Rightarrow$$
$$x_n \not\equiv x_{n'+1}$$
$$\Rightarrow$$
$$o(x_{n'}) = o(x_{n'+1}) \neq o(x_n)$$

Because p is just a composition of o's, the same property holds for primitives

$$p(x_n) = y'_n$$
$$p(x_{n'}) = y'_{n'}$$
$$p(x_{n'+1}) = y'_{n'}$$
$$\iff$$
$$x_{n'} \equiv x_{n'+1} \wedge x_{n'} \not\equiv x_n$$
$$\Rightarrow$$
$$x_n \not\equiv x_{n'+1}$$
$$\Rightarrow$$
$$p(x_{n'}) = p(x_{n'+1}) \neq p(x_n)$$