Data Analytics and Visualization Efficiency Framework for xAPI and the Total Learning Architecture: DAVE Learning Analytics Algorithms

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Introduction

This report introduces the initial learning analytics algorithms, **timeline of learner success** and **which assessment questions are the most difficult** of the DAVE Framework. In doing so, it establishes a set of style guidelines for the reporting of algorithms and associated visualization templates necessary to work with the DAVE Framework. This document will be updated to include the learning analytics questions defined within the 2018 TLA Data Requirements document in addition to other learning analytics algorithms which have yet to be defined. Updates may also address refinement of these algorithms and this document should be understood to be an example of algorithm presentation and not the final state of any defined algorithm.

The structure of this documents is as follows:

1. A formal specification for xAPI written in Z and referenced within the formal specifications of learning analytics algorithms
2. An algorithm definition which will consist of:
   (a) an introduction for the algorithm
   (b) the structure of the ideal input data
   (c) how to retrieve input data from an LRS
   (d) the statement parameters which the algorithm will utilize
   (e) notices regarding data collected during the 2018 pilot test of the TLA
   (f) a summary of the algorithm
   (g) the formal specification of the algorithm
   (h) pseudocode representation of the algorithm
   (i) JSONSchema for the output of the algorithm
   (j) a description of the associated visualization
   (k) a prototype of the visualization
   (l) a collection of suggestions describing how the algorithm could be adapted to improve the quality of the visualization prototype

1 xAPI Formal Specification

The current formal specification only defines xAPI statements abstractly within the context of Z. A concrete definition for xAPI statements is outside the scope of this document.

1.1 Basic Types

$IFI ::= mbox \mid mbox\_sha1sum \mid openid \mid account$

- Type unique to Agents and Groups, The concrete definition of the listed values is outside the scope of this specification

$OBJECTTYPE ::= Agent \mid Group \mid SubStatement \mid StatementRef \mid Activity$

- A type which can be present in all activities as defined by the xAPI specification

$INTERACTIONTYPE ::= true{-}false \mid choice \mid fill{-}in \mid long{-}fill{-}in \mid matching \mid performance \mid sequencing \mid likert \mid numeric \mid other$

- A type which represents the possible interactionTypes as defined within the xAPI specification

$INTERACTIONCOMPONENT ::= choices \mid scale \mid source \mid target \mid steps$

- A type which represents the possible interaction components as defined within the xAPI specification
- the concrete definition of the listed values is outside the scope of this specification

$CONTEXTTYPES ::= parent \mid grouping \mid category \mid other$

- A type which represents the possible context types as defined within the xAPI specification

$[STATEMENT]$

- Basic type for an xAPI data point

$[AGENT, GROUP]$

- Basic types for Agents and collections of Agents

1.2 Id Schema

Id

$id : \mathbb{F}_1 \#1$

- the schema Id introduces the component id which is a non-empty, finite set of 1 value

1.3 Schemas for Agents, Groups and Actors

$$
\begin{array}{|l}
\hline
\textit{Agent} \\
agent : AGENT \\
objectType : OBJECTTYPE \\
name : \mathbb{F}_1 \#1 \\
ifi : IFI \\
\hline
objectType = Agent \\
agent = \{ifi\} \cup \mathbb{P}\{name, objectType\} \\
\hline
\end{array}
$$

- The schema *Agent* introduces the component *agent* which is a set consisting of an ifi and optionally an *objectType* and/or *name*

$$
\begin{array}{|l}
\hline
\textit{Member} \\
Agent \\
member : \mathbb{F}_1 \\
\hline
member = \{a : AGENT \mid \forall a : a_0..a_n \bullet a = agent\} \\
\hline
\end{array}
$$

- The schema $Member$ introduces the component *member* which is a set of objects a, where for every a within $a_0..a_n$, a is an *agent*

$$
\begin{array}{|l}
\hline
\textit{Group} \\
Member \\
group : GROUP \\
objectType : OBJECTTYPE \\
ifi : IFI \\
name : \mathbb{F}_1 \#1 \\
\hline
objectType = Group \\
group = \{objectType, name, member\} \vee \{objectType, member\} \vee \\
\qquad\qquad \{objectType, ifi\} \cup \mathbb{P}\{name, member\} \\
\hline
\end{array}
$$

- The schema *Group* introduces the component *group* which is of type *GROUP* and is a set of either *objectType* and *member* with optionaly *name* or *objectType* and ifi with optionally *name* and/or *member*

$$
\begin{array}{|l}
\hline
\textit{Actor} \\
Agent \\
Group \\
actor : AGENT \vee GROUP \\
\hline
actor = agent \vee group \\
\hline
\end{array}
$$

- The schema *Actor* introduces the component *actor* which is either an *agent* or *group*

1.4 Verb Schema

$$
\begin{array}{|l}
\textit{Verb} \\
\hline
Id \\
display, verb : \mathbb{F}_1 \\
\hline
verb = \{id, display\} \lor \{id\} \\
\hline
\end{array}
$$

- The schema $Verb$ introduces the component $verb$ which is a set that consists of either id and the non-empty, finite set $display$ or just id

1.5 Object Schema

$$
\begin{array}{|l}
\textit{Extensions} \\
\hline
extensions, extensionVal : \mathbb{F}_1 \\
extensionId : \mathbb{F}_1 \, \#1 \\
\hline
extensions = \{e : (extensionId, extensionVal) \ | \forall e : e_i..e_j \bullet \\
\qquad (extensionId_i, extensionVal_i) \lor (extensionId_i, extensionVal_j) \land \\
\qquad (extensionId_j, extensionVal_i) \lor (extensionId_j, extensionVal_j) \land \\
\qquad extensionId_i \neq extensionId_j\} \\
\hline
\end{array}
$$

- The schema $Extensions$ introduces the component $extensions$ which is a non-empty, finite set that consists of ordered pairs of $extensionId$ and $extensionVal$. Different $extensionId$s can have the same $extensionVal$ but there can not be two identical $extensionId$ values
- $extensionId$ is a non-empty, finite set with one value
- $extensionVal$ is a non-empty, finite set

$$
\begin{array}{|l}
\textit{InteractionActivity} \\
\hline
interactionType : INTERACTIONTYPE \\
correctResponsePattern : \mathrm{seq}_1 \\
interactionComponent : INTERACTIONCOMPONENT \\
\hline
interactionActivity = \{interactionType, correctReponsePattern, interactionComponent\} \lor \\
\qquad \{interactionType, correctResponsePattern\} \\
\hline
\end{array}
$$

- The schema $InteractionActivity$ introduces the component $interactionActivity$ which is a set of either $interactionType$ and $correctResponsePattern$ or $interactionType$ and $correctResponsePattern$ and $interactionComponent$

$$
\begin{array}{|l}
\hline
\textit{Definition} \\
\textit{InteractionActivity} \\
\textit{Extensions} \\
definition, name, description : \mathbb{F}_1 \\
type, moreInfo : \mathbb{F}_1 \ \#1 \\
\hline
definition = \mathbb{P}_1\{name, description, type, moreInfo, extensions, interactionActivity\} \\
\hline
\end{array}
$$

- The schema *Definition* introduces the component *definition* which is the non-empty, finite power set of *name*, *description*, *type*, *moreInfo* and *extensions*

$$
\begin{array}{|l}
\hline
\textit{Object} \\
\textit{Id} \\
\textit{Definition} \\
\textit{Agent} \\
\textit{Group} \\
\textit{Statement} \\
objectTypeA, objectTypeS, objectTypeSub, objectType : OBJECTTYPE \\
substatement : STATEMENT \\
object : \mathbb{F}_1 \\
\hline
substatement = statement \\
objectTypeA = Activity \\
objectTypeS = StatementRef \\
objectTypeSub = SubStatement \\
objectType = objectTypeA \lor objectTypeS \\
object = \{id\} \lor \{id, objectType\} \lor \{id, objectTypeA, definition\} \\
\qquad \lor\{id, definition\} \lor \{agent\} \lor \{group\} \lor \{objectTypeSub, substatement\} \\
\qquad \lor\{id, objectTypeA\} \\
\hline
\end{array}
$$

- The schema *Object* introduces the component *object* which is a non-empty, finite set of either *id*, *id* and *objectType*, *id* and *objectTypeA*, *id* and *objectTypeA* and *definition*, *agent*, *group*, or *substatement*
- The schema *Statement* and the corresponding component *statement* will be defined later on in this specification

1.6 Result Schema

$$
\begin{array}{|l}
\hline
\multicolumn{1}{l}{\mathit{Score}} \\
score : \mathbb{F}_1 \\
scaled, min, max, raw : \mathbb{Z} \\
\hline
scaled = \{ n : \mathbb{Z} \mid -1.0 \leq n \leq 1.0 \} \\
min = n < max \\
max = n > min \\
raw = \{ n : \mathbb{Z} \mid min \leq n \leq max \} \\
score = \mathbb{P}_1 \{ scaled, raw, min, max \} \\
\hline
\end{array}
$$

- The schema *Score* introduces the component *score* which is the non-empty powerset of *min*, *max*, *raw* and *scaled*

$$
\begin{array}{|l}
\hline
\multicolumn{1}{l}{\mathit{Result}} \\
Score \\
Extensions \\
success, completion, response, duration : \mathbb{F}_1 \#1 \\
result : \mathbb{F}_1 \\
\hline
success = \{ true \} \lor \{ false \} \\
completion = \{ true \} \lor \{ false \} \\
result = \mathbb{P}_1 \{ score, success, completion, response, duration, extensions \} \\
\hline
\end{array}
$$

- The schema *Result* introduces the component *result* which is the non-empty power set of *score*, *success*, *completion*, *response*, *duration* and *extensions*

1.7 Context Schema

$$
\begin{array}{|l}
\hline
\multicolumn{1}{l}{\mathit{Instructor}} \\
Agent \\
Group \\
instructor : AGENT \lor GROUP \\
\hline
instructor = agent \lor group \\
\hline
\end{array}
$$

- The schema *Instructor* introduces the component *instructor* which can be ether an *agent* or a *group*

$$
\begin{array}{|l}
\hline
\multicolumn{1}{l}{\mathit{Team}} \\
Group \\
team : GROUP \\
\hline
team = group \\
\hline
\end{array}
$$

- The schema $Team$ introduces the component $team$ which is a $group$

$$
\begin{array}{l}
\textbf{Context} \\
\hline
Instructor \\
Team \\
Object \\
Extensions \\
registration, revision, platform, language : \mathbb{F}_1 \, \#1 \\
parentT, groupingT, categoryT, otherT : CONTEXTTYPES \\
contextActivities, statement : \mathbb{F}_1 \\
\hline
statement = object \setminus (id, objectType, agent, group, definition) \\
parentT = parent \\
groupingT = grouping \\
categoryT = category \\
otherT = other \\
contextActivity = \{ca : object \setminus (agent, group, objectType, objectTypeSub, substatement)\} \\
contextActivityParent = (parentT, contextActivity) \\
contextActivityCategory = (categoryT, contextActivity) \\
contextActivityGrouping = (groupingT, contextActivity) \\
contextActivityOther = (otherT, contextActivity) \\
contextActivities = \mathbb{P}_1\{contextActivityParent, contextActivityCategory, \\
\qquad contextActivityGrouping, contextActivityOther\} \\
context = \mathbb{P}_1\{registration, instructor, team, contextActivities, revision, \\
\qquad platform, language, statement, extensions\} \\
\hline
\end{array}
$$

- The schema $Context$ introduces the component $context$ which is the non-empty powerset of $registration$, $instructor$, $team$, $contextActivities$, $revision$, $platform$, $language$, $statement$ and $extensions$

- The notation $object \setminus agent$ represents the component $object$ except for its subcomponent $agent$

1.8 Timestamp and Stored Schema

$$
\begin{array}{l}
\textbf{Timestamp} \\
\hline
timestamp : \mathbb{F}_1 \, \#1 \\
\hline
\end{array}
$$

$$
\begin{array}{l}
\textbf{Stored} \\
\hline
stored : \mathbb{F}_1 \, \#1 \\
\hline
\end{array}
$$

- The schema $Timestamp$ and $stored$ introduce the components $timestamp$ and $stored$ respectively. Each are non-empty, finite sets containing one value

1.9 Attachements Schema

$$
\begin{array}{|l}
\multicolumn{1}{l}{\textit{Attachments}} \\
\hline
display, description, attachment, attachments : \mathbb{F}_1 \\
usageType, sha2, fileUrl, contexntType : \mathbb{F}_1 \, \#1 \\
length : \mathbb{N} \\
\hline
attachment = \{usageType, display, contentType, length, sha2\} \cup \mathbb{P}\{description, fileUrl\} \\
attachments = \{a : attachment\} \\
\hline
\end{array}
$$

- The schema *Attachements* introduces the componenet *attachements* which is a non-empty, finite set of the component *attachement*
- The component *attachment* is a non-empty, finite set of the componenets $usageType$, $display$, $contentType$, $length$, $sha2$ with optionally $description$ and/or $fileUrl$

1.10 Statement and Statements Schema

$$
\begin{array}{|l}
\multicolumn{1}{l}{\textit{Statement}} \\
\hline
Id \\
Actor \\
Verb \\
Object \\
Result \\
Context \\
Timestamp \\
Stored \\
Attachements \\
statement : STATEMENT \\
\hline
statement = \{actor, verb, object, stored\} \cup \\
\qquad\qquad \mathbb{P}\{\mathrm{id}, result, context, timestamp, attachments\} \\
\hline
\end{array}
$$

- The schema *Statement* introduces the component *statement* which consists of the components *actor*, *verb*, *object* and *stored* and the optional components *id*, *result*, *context*, *timestamp*, and/or *attachments*
- The schema *Statement* allows for subcomponent of *statement* to refrenced via the . (selection) operator

$$
\begin{array}{|l}
\multicolumn{1}{l}{\textit{Statements}} \\
\hline
\textit{Statement} \\
\textit{IsoToUnix} \\
\textit{statements} : \mathbb{F}_1 \\
\hline
\textit{statements} = \{ s : \textit{statement} \mid \forall s : s_i .. s_j \bullet i = 0 \land i = j \lor i \neq j \\
\qquad \bullet \textit{convert}(s_i.\textit{timestamp}) \leq \textit{convert}(s_j.\textit{timestamp}) \} \\
\hline
\end{array}
$$

- The schema *Statements* introduces the component *statements* which is a non-empty, finite set of the component *statement* which are in chronological order.

2 Timeline Of Learner Success

As learners engage in activities supported by a learning ecosystem, they will build up a history of learning experiences. When the digital resources of that learning ecosystem adhere to a framework dedicated to supporting and understanding the learner, such as the Total Learning Architecture (TLA), it becomes possible to retell their learning story through data and data visualization. One important aspect of that story is the' learners history of success.

2.1 Ideal Statements

In order to accurately portray a learner's timeline of success, there are a few base requirements of the data produced by a Learning Record Provider (LRP). They are as follows:

- the learner must be uniquely and consistently identified across all statements
- learning activities which evaluate a learner's understanding of material must report if the learner was successful or not
  - the grade earned by the learner must be reported
  - the minimum and maximum possible grade must be reported
- The learning activities must be uniquely and consistently identified across all statements
- The time at which a learner completed a learning activity must be recorded
  - The timestamp should contain an appropriate level of specificity.
  - ie. Year, Month, Day, Hour, Minute, Second, Timezone

2.2 Input Data Retrieval

How to query an LRS via a GET request to the Statements Resource via curl. The following section contains the appropriate parameters with example values as well as the curl command necessary for making the request. [1] [2] [3]

```
Agent = "agent={"account":
                  {"homePage": "https://example.homepage",
                   "name": 123456}}"

Since = "since=2018-07-20T12:08:47Z"

Until = "until=2018-07-21T12:08:47Z"

Base = "https://example.endpoint/statements?"

endpoint = Base + Agent + "&" + Since + "&" + Until

Auth = Hash generated from basic auth

S = curl -X GET -H "Authorization: Auth"
         -H "Content-Type: application/json"
         -H "X-Experience-API-Version: 1.0.3"
         Endpoint
```

2.3 Statement Parameters to Utilize

The statement parameter locations here are written in JSONPath. This notation is also compatable with the xAPI Z notation due to the defined hierarchy of components. Within the Z specifications, a variable name will be used instead of the \$

- $\$.timestamp$
- $\$.result.success$
- $\$.result.score.raw$
- $\$.result.score.min$
- $\$.result.score.max$

[1] S is the set of all statements parsed from the statements array within the HTTP response to the Curl request. It may be possible that multiple Curl requests are needed to retrieve all query results. If multiple requests are necessary, S is the result of concatenating the result of each request into a single set

[2] Querying an LRS will not be defined within the following Z specifications but the results of the query will be utilized

[3] If you want to query across the entire history of a LRS, omit Since and Until from the endpoint and remove the associated & symbols.

- $\$.verb.id$

2.4 2018 Pilot TLA Statement Problems

The initial pilot test data supports this algorithm. This section may require updates pending future data review following iterations of the TLA testing.

2.5 Summary

1. Query an LRS via a GET request to the statements endpoint using the parameters agent, since and until
2. Filter the results to the set of statements where:
   - $\$.verb.id$ is one of:
     - http://adlnet.gov/expapi/verbs/passed
     - https://w3id.org/xapi/dod-isd/verbs/answered
     - http://adlnet.gov/expapi/verbs/completed
   - $\$.result.success$ is true
3. process the filtered data
   - extract $\$.timestamp$
   - extract the score values from $\$.result.score.raw$, $\$.result.score.min$ and $\$.result.score.max$ and convert them to the scale 0..100
   - create a pair of [$\$.timestamp$, #]

2.6 Formal Specification

2.6.1 Basic Types

$COMPLETION :==$
$\{http://adlnet.gov/expapi/verbs/passed\} \mid$
$\{https://w3id.org/xapi/dod-isd/verbs/answered\} \mid$
$\{http://adlnet.gov/expapi/verbs/completed\}$

$SUCCESS :== \{true\}$

2.6.2 System State

$$
\begin{array}{|l}
\multicolumn{1}{l}{\textit{TimelineLearnerSuccess}} \\
\hline
Statements \\
S_{all} : \mathbb{F}_1 \\
S_{completion}, S_{success}, S_{processed} : \mathbb{F} \\
\hline
S_{all} = statements \\
S_{completion} \subseteq S_{all} \\
S_{success} \subseteq S_{completion} \\
S_{processed} = \{pair : (statement.timestamp, \mathbb{N})\} \\
\hline
\end{array}
$$

- The set S_{all} is a non-empty, finite set and is the component *statements*
- The sets $S_{completion}$ and $S_{success}$ are both finite sets
- the set $S_{completion}$ is a subset of S_{all} which may contain every value within S_{all}
- the set $S_{success}$ is a subset of $S_{completion}$ which may contain every value within $S_{completion}$
- the set $S_{processed}$ is a finite set of pairs where each contains a *statement.timestamp* and a natural number

2.6.3 Initial System State

$$
\begin{array}{|l}
\multicolumn{1}{l}{\textit{InitTimelineLearnerSuccess}} \\
\hline
TimelineLearnerSuccess \\
\hline
S_{all} \neq \emptyset \\
S_{completion} = \emptyset \\
S_{success} = \emptyset \\
S_{processed} = \emptyset \\
\hline
\end{array}
$$

- The set S_{all} is a non-empty set
- The sets $S_{completion}, S_{success}$ and $S_{processed}$ are all initially empty

2.6.4 Filter for Completion

$$
\begin{array}{|l}
\multicolumn{1}{l}{\textit{Completion}} \\
\hline
Statement \\
completion : STATEMENT \twoheadrightarrow \mathbb{F} \\
s? : STATEMENT \\
s! : \mathbb{F} \\
\hline
s? = statement \\
s! = completion(s?) \\
completion(s?) = \textbf{if } s?.verb.id : COMPLETION \\
\qquad\qquad\qquad\quad \textbf{then } s! = s? \\
\qquad\qquad\qquad\quad \textbf{else } s! = \emptyset \\
\hline
\end{array}
$$

- The schema *Completion* inroduces the function *completion* which takes in the variable $s?$ and returns the variable $s!$
- The variable $s?$ is the component *statement*
- $s!$ is equal to $s?$ if $\$.verb.id$ is of the type $COMPLETION$ otherwise $s!$ is an empty set

$$
\begin{array}{|l}
\hline
FilterForCompletion \\
\Delta TimelineLearnerSuccess \\
Completion \\
completions : \mathbb{F} \\
\hline
completions \subseteq S_{all} \\
completions' = \{s : STATEMENT \mid completion(s) \neq \emptyset\} \\
S'_{completion} = S_{completion} \cup completions' \\
\hline
\end{array}
$$

- the set *completions* is a subset of S_{all} which may contain every value within S_{all}
- The set $completions'$ is the set of all statements s where the result of $completion(s)$ is not an empty set
- the updated set $S'_{completion}$ is the union of the previous state of set $S_{completion}$ and the set $completions'$

2.6.5 Filter for Success

$$
\begin{array}{|l}
\hline
Success \\
Statement \\
success : STATEMENT \nrightarrow \mathbb{F} \\
s? : STATEMENT \\
s! : \mathbb{F} \\
\hline
s? = statement \\
s! = success(s?) \\
success(s?) = \textbf{if } s?.result.success : SUCCESS \\
\qquad\qquad\qquad\quad \textbf{then } s! = s? \\
\qquad\qquad\qquad\quad \textbf{else } s! = \emptyset \\
\hline
\end{array}
$$

- the schema *Success* introduces the function *success* which takes in the variable $s?$ and returns the variable $s!$
- the variable $s?$ is the component *statement*
- $s!$ is equal to $s?$ if $\$.result.success$ is of the type $SUCCESS$ otherwise $s!$ is an empty set

$$
\begin{array}{|l}
\multicolumn{1}{l}{\underline{\quad FilterForSuccess \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}} \\
\Delta TimelineLearnerSuccess \\
Success \\
successes : \mathbb{F} \\
\hline
successes \subseteq S_{completion} \\
successes' = \{s : STATEMENT \mid success(s) \neq \emptyset\} \\
S'_{success} = S_{success} \cup successes' \\
\hline
\end{array}
$$

- the set *successes* is a subset of $S_{completion}$ which may contain every value within $S_{completion}$
- The set $successes'$ contains elements s of type $STATEMENT$ where $success(s)$ is not an empty set
- The updated set $S'_{success}$ is the union of the previous state of $S_{success}$ and $successes'$

2.6.6 Processes Results

$$
\begin{array}{|l}
\multicolumn{1}{l}{\underline{\quad Scale \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}} \\
scaled! : \mathbb{N} \\
raw?, min?, max? : \mathbb{Z} \\
scale : \mathbb{Z} \to \mathbb{N} \\
\hline
scaled! = scale(raw?, min?, max?) \\
scale(raw?, min?, max?) = \\
\qquad\qquad (raw? * ((0.0 - 100.0)\ div\ (min? - max?))) + \\
\qquad\qquad (0.0 - (min? * ((0.0 - 100.0) div (min? - max?)))) \\
\hline
\end{array}
$$

- The schema *Scale* introduces the function *scale* which takes 3 arguments, $raw?$, $min?$ and $max?$. The function converts $raw?$ from the range $min?..max?$ to 0.0..100.0

$$
\begin{array}{l}
\textit{ProcessStatements} \\
\hline
\Delta \textit{TimelineLearnerSuccess} \\
\textit{Scale} \\
\textit{FilterStatements} \\
\textit{processed} : \mathbb{F} \\
\hline
\textit{processed} \subseteq S_{success} \\
\textit{processed}' = \{p : (\mathbb{F}_1 \# 1, \mathbb{N}) \mid \\
\quad \textbf{let } \{\textit{processed}_i .. \textit{processed}_j\} == \{s_i .. s_j\} \bullet \\
\quad \forall s : s_i .. s_j \bullet \exists p_i .. p_j \bullet \\
\quad \textit{first}\, p_i = s_i.\textit{timestamp} \land \\
\quad \textit{second}\, p_i = \textit{scale}(s_i.\textit{result.score.raw}, \\
\qquad s_i.\textit{result.score.min}, \\
\qquad s_i.\textit{result.score.max}) \land \\
\quad \textit{first}\, p_j = s_j.\textit{timestamp} \land \\
\quad \textit{second}\, p_j = \textit{scale}(s_j.\textit{result.score.raw}, \\
\qquad s_j.\textit{result.score.min}, \\
\qquad s_j.\textit{result.score.max})\} \\
S'_{processed} = S_{processed} \cup \textit{processed}'
\end{array}
$$

- The operation *ProcessStatements* introduces the variable *processed* which is a subset of $S_{success}$ which may contain every value within $S_{success}$
- $S_{success}$ is the result of the operation *FilterStatements*
- The operation defines the variable $\textit{processsed}'$ which is a set of objects p which are ordered pairs of (1) a finite set containing one value and (2) a single positive number.
- The first component of every object p, is the timestamp from the associated *statement* within *processed* ie. *s.timestamp*
- The second component of every object p is the result of the function *scale*. The score values contained within the associated *statement* s are the arugments passed to *scale*. ie $\textit{scale}(s.\textit{result.score.raw}, s.\textit{result.score.min}, s.\textit{result.score.max})$
- The result of the operation *ProcessStatements* is to updated the set $S_{processed}$ with the values contained within $\textit{processed}'$

2.6.7 Sequence of Operations

$\textit{FilterStatements} \mathrel{\widehat{=}} \textit{FilterForCompletion} \mathbin{⨾} \textit{FilterForSuccess}$

- The schema *FilterStatements* is the sequential composition of operation schemas *FilterForCompletion* and *FilterForSuccess*
- *FilterForCompletion* happens before *FilterForSuccess*

$\textit{ProcessedStatements} \mathrel{\widehat{=}} \textit{FilterStatements} \mathbin{⨾} \textit{ProcessStatements}$

- The schema *ProcessedStatements* is the sequential composition of operation schemas *FilterStatements* and *ProcessStatements*
- *FilterStatements* happens before *ProcessStatements*

2.6.8 Return

$$
\begin{array}{|l}
\textit{Return} \\
\hline
\Xi TimelineLearnerSuccess \\
ProcessedStatements \\
S_{processed}! : \mathbb{F} \\
\hline
S_{processed}! = S_{processed} \\
\hline
\end{array}
$$

- The returned variable $S_{processed}!$ is equal to the current state of variable $S_{processed}$ after the operations *FilterForCompletion*, *FilterForSuccess* and *ProcessStatements*

2.7 Pseudocode

Algorithm 1: Timeline of Learner Success

```
Input: S_all
Result: coll
init coll := []
while S_all is not empty do
    for each statement s in S_all
    if s.verb.id = COMPLETION then
        add s to S_completion
        remove s from S_all
        recur
    else
        remove s from S_all
        recur
    end
end
while S_completion is not empty do
    for each statement sc in S_completion
    if sc.result.success = SUCCESS then
        add sc to S_success
        remove sc from S_completion
        recur
    else
        remove sc from S_completion
        recur
    end
end
while S_success is not empty do
    for each statement ss in S_success
         let ss.result.score.raw = raw?
             ss.result.score.max = max?
             ss.result.score.min = min?
             scaled = scale(raw?, min?, max?)
          concat coll [ss.timestamp, scaled]
          remove ss from S_success
          recur
end
```

- The Z schemas are used within this pseudocode
- The return value coll is an array of arrays, each containing a *statement.timestamp* and a scaled score.

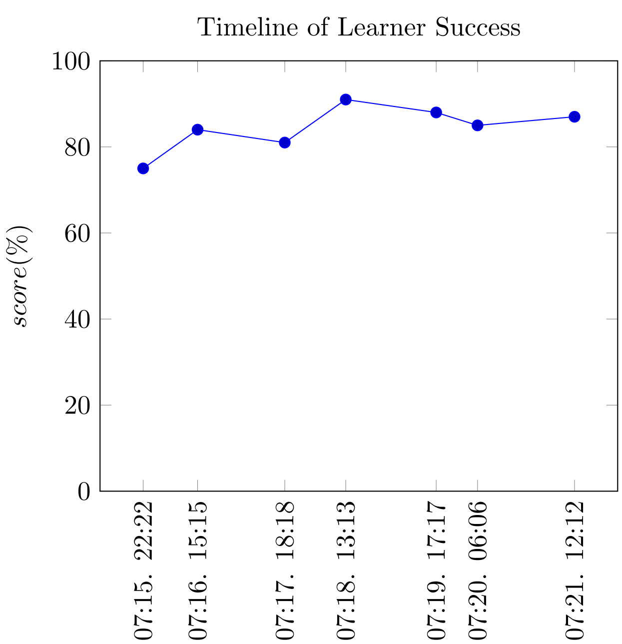
2.8 JSON Schema

```
{"type":"array",
   "items":{"type":"array",
      "items":[{"type":"string"}, {"type":"number"}]}}
```

2.9 Visualization Description

The **Timeline of Learner Success** visualization will be a line chart where the domain is time and the range is score on a scale of 0.0 to 100.0. Every subarray will be a point on the chart. The domain of the graph should be in chronological order.

2.10 Visualization prototype



2.11 Prototype Improvement Suggestions

Additional features may be implemented on top of this base specification but they would require adding aditional values to each subarray returned by the algorithm. These additional values can be retrieved via (1) performing metadata lookup within or independently of the algorithm (2) by utilizing additional xAPI statement paramters and/or (3) by performing additional computations. The following examples assume the metadata is contained within each statement available to the algorithm.

- A tooltip containing the name of an activity when hovering over a specific point on the chart
  - this would require utilizing $\$.object.definition.name$
- A tooltip containing the device on which the activity was experienced
  - this would require utilizing $\$.context.platform$
- A tooltip containing the instructor associated with a particular data point
  - this would require utilizing $\$.context.instructor$

3 Which Assessment Questions are the Most Difficult

As learners engage in activities supported by a learning ecosystem, they will experience learning content as well as assessment content. Assessments are designed to measure the effectiveness of learning content and help assess knowledge gained. It is possible that certain assessment questions do not accurately represent the concepts contained within learning content and this may be indicated by a majority of learners getting the question wrong. It is also possible that the question accurately represents the learning content but is very difficult. The following algorithm will identify these types of questions but will not be able to deduce why learners answer them incorrectly.

3.1 Ideal Statements

In order to accurately determine which assessment questions are the most dificult, there are a few requirements of the data produced by a LRP. They are as follows:

- statements describing a learner answering a question must report if the learner got the question correct or incorrect via $\$.result.success$
- if it is possible to get partial credit on a question, the amount of credit should be reported within the statement
  - the credit earned by the learner should be reported within $\$.result.score.raw$
  - the minimum and maximum possible credit amount should be reported within $\$.result.score.min$ and $\$.result.score.max$ respectively
- If it is possible to get partial credit on a question, it must still be reported if the learner reached the threshold of success via $\$.result.success$
- Statements describing a learner answering a question should contain activities of the type *cmi.interaction*

- activities must be uniquely and consistently identified across all statements
- Statements describing a learner answering a question should[4] use the verb *http://adlnet.gov/expapi/verbs/answered*

3.2 Input Data Retrieval

How to query an LRS via a GET request to the Statements Resource via curl. The following section contains the appropriate parameters with example values as well as the curl command necessary for making the request. [5] [6] [7]

```
Verb = "verb=http://adlnet.gov/expapi/verbs/answered"

Since = "since=2018-07-20T12:08:47Z"

Until = "until=2018-07-21T12:08:47Z"

Base = "https://example.endpoint/statements?"

endpoint = Base + Verb + "&" + Since + "&" + Until

Auth = Hash generated from basic auth

S = curl -X GET -H "Authorization: Auth"
         -H "Content-Type: application/json"
         -H "X-Experience-API-Version: 1.0.3"
         Endpoint
```

3.3 Statement Parameters to Utilize

The statement parameter locations here are written in JSONPath. This notation is also compatable with the xAPI Z notation due to the defined hierarchy of components. Within the Z specifications, a variable name will be used instead of the $

- $.*result.success*
- $.*object.id*

[4] it is possible to use another verb but if another is used, that will need to be accounted for in data retrieval

[5] See footnote 1.

[6] See footnote 2.

[7] See footnote 3.

3.4 2018 Pilot TLA Statement Problems

The initial pilot test data supports this algorithm. Given that the offical 2018 pilot test is scheduled to take place on July 27th, 2018, this section may require updates pending future data review.

3.5 Summary

1. Query an LRS via a GET request to the statements endpoint using the parameters verb, since and until
2. Filter the results to the set of statements where:
   - $\$.result.success$ is false
3. process the filtered data
   - group by $\$.object.id$
   - determine the count of each group
   - create a collection of pairs = [$\$.object.id$, #]

3.6 Formal Specification

3.6.1 Basic Types

$INCORRECT ::= \{false\}$

3.6.2 System State

$$
\begin{array}{|l}
\hline
MostDifficultAssessmentQuestions \\
Statements \\
S_{all} : \mathbb{F}_1 \\
S_{incorrect}, S_{grouped}, S_{processed} : \mathbb{F} \\
\hline
S_{all} = statements \\
S_{incorrect} \subseteq S_{all} \\
S_{grouped} = \{groups : \mathrm{seq}_1\, statement\} \\
S_{processed} = \{pair : (id, \mathbb{N})\} \\
\hline
\end{array}
$$

- The set S_{all} is a non-empty, finite set and is the component *statements*
- The sets $S_{incorrect}$, $S_{grouped}$ and $S_{processed}$ are all finite sets
- the set $S_{incorrect}$ is a subset of S_{all} which may contain every value within S_{all}
- the set $S_{grouped}$ is a finite set of objects *groups* which are non-empty, finite sequences of the component *statement*
- the set $S_{processed}$ is a finite set of pairs where each contains the component *id* and a natural number

3.6.3 Initial System State

$$
\begin{array}{|l}
\hline
\textit{InitMostDifficultAssessmentQuestions} \\
\hline
MostDifficultAssessmentQuestions \\
\hline
S_{all} \neq \emptyset \\
S_{incorrect} = \emptyset \\
S_{grouped} = \emptyset \\
S_{processed} = \emptyset \\
\hline
\end{array}
$$

- The set S_{all} is a non-empty set
- The sets $S_{incorrect}$, $S_{grouped}$ and $S_{processed}$ are all initially empty

3.6.4 Filter for Incorrect

$$
\begin{array}{|l}
\hline
\textit{Incorrect} \\
\hline
Statement \\
incorrect : STATEMENT \pfun \mathbb{F} \\
s? : STATEMENT \\
s! : \mathbb{F} \\
\hline
s? = statement \\
s! = incorrect(s?) \\
incorrect(s?) = \textbf{if } s?.result.success : INCORRECT \\
\qquad\qquad\qquad\quad \textbf{then } s! = s? \\
\qquad\qquad\qquad\quad \textbf{else } s! = \emptyset \\
\hline
\end{array}
$$

- the schema $Incorrect$ introduces the function $incorrect$ which takes in the variable $s?$ and returns the variable $s!$
- the variable $s?$ is the component $statement$
- $s!$ is equal to $s?$ if $\$.result.success$ is of the type $INCORRECT$ otherwise $s!$ is an empty set

$$
\begin{array}{|l}
\hline
\textit{FilterForIncorrect} \\
\hline
\Delta MostDifficultAssessmentQuestions \\
Incorrect \\
incorrects : \mathbb{F} \\
\hline
incorrects \subseteq S_{all} \\
incorrects' = \{s : STATEMENT \mid incorrect(s) \neq \emptyset\} \\
S'_{incorrect} = S_{incorrect} \cup incorrects' \\
\hline
\end{array}
$$

- the set $incorrects$ is a subset of S_{all} which may contain every value within S_{all}

- The set $incorrects'$ contains elements s of type $STATEMENT$ where $incorrect(s)$ is not an empty set
- The updated set $S'_{incorrect}$ is the union of the previous state of $S_{incorrect}$ and $incorrects'$

3.6.5 Processes Results

$$
\begin{array}{l}
\textbf{schema } GroupByActivityId \\
\quad Statements \\
\quad g? : \mathbb{F} \\
\quad g! : \mathbb{F} \\
\quad group : \mathbb{F} \to \mathbb{F} \\
\hline
\quad g? = statements \Rightarrow \{g : statement\} \\
\quad g! = group(g?) \\
\quad g! = \{groups : \mathrm{seq}_1\ statement \mid \\
\qquad \textbf{let } \mathrm{seq}_1\ statement_i..statement_j == \mathrm{seq}_1\ s_i..s_j \bullet \\
\qquad \forall s : s_i..s_j \bullet s_i.object.id = s_j.object.id\}
\end{array}
$$

- The schema $GroupByActivityId$ introduces the function $group$ which has the input of $g?$ and the output of $g!$
- The input variable $g?$ is the component $statements$ which implies its a set of objects g which are each a $statement$
- the output variable $g!$ is a set of objects $groups$ which are each a non-empty, finite sequence of $statement$ where each member of the sequence $s_i..s_j$ has the same $\$.object.id$

$$
\begin{array}{l}
\textbf{schema } CountPerGroup \\
\quad Statement \\
\quad c? : \mathrm{seq}_1\ statement \\
\quad c! : \mathbb{N} \\
\quad count : \mathrm{seq}_1\ statement \to \mathbb{N} \\
\hline
\quad c! = count(c?) \\
\quad c! \geq 1 \\
\quad count(c?) = \forall c? : c?_i..c?_j \bullet i, j : \mathbb{N} \land i = 0 \land i = j \lor i \neq j \bullet \\
\qquad\qquad\qquad \exists_1\, c! : \mathbb{N} \bullet c! = j + 1
\end{array}
$$

- The schema $CountPerGroup$ introduces the function $count$ which has the input of $c?$ and the output of $c!$
- The input variable $c?$ is a non-empty, finite sequence in which each element is a $statement$

- The function *count* reads: for all elements within the sequence $c?_i..c?_j$, i and j are natural numbers, i is equal to zero and may or may not be equal to j such that there exits a number $c!$ which is equal to $j + 1$

$$
\begin{array}{|l}
\hline
\textit{AggregateQuestionStatements} \\
\Delta MostDifficultAssessmentQuestions \\
FilterForIncorrect \\
GroupByActivityId \\
CountPerGroup \\
grouped, processed : \mathbb{F} \\
\hline
grouped = \emptyset \\
grouped' = group(S_{incorrect}) \\
S'_{grouped} = S_{grouped} \cup grouped' \\
processed \subseteq S'_{grouped} \\
processed' = \{p : (id, \mathbb{N}) \mid \\
\qquad\qquad \mathbf{let}\ \{processed_i..processed_j\} == \{g_i..g_j\} \bullet \\
\qquad\qquad \forall g : g_i..g_j \bullet \exists\ p_i..p_j \bullet \\
\qquad\qquad first\, p_i = head\, g_i.object.id \wedge second\, p_i = count(g_i) \\
\qquad\qquad first\, p_j = headg_j.object.id \wedge second\, p_j = count(g_j)\} \\
S'_{processed} = S_{processed} \cup processed' \\
\hline
\end{array}
$$

- The schema *AggregateQuestionStatements* introduces the variables *grouped* and *processed*
- *grouped* starts as an empty set but then becomes $grouped'$ which is the output of applying the function *group* to the set of statements $S_{incorrect}$ created by the opperation $FilterForIncorrect$
- $grouped'$ is a set of sequences. The elements of those sequences are statements which all have the same *statement.object.id*
- The set $S_{grouped}$ is updated to the set $S'_{grouped}$ which is the union of $S_{grouped}$ and $grouped'$
- the variable *processed* is a subset of $S'_{grouped}$ which can contain every value within $S'_{grouped}$
- the variable *processed* is updated to be the variable $processed'$ which is a set of objects p which are ordered pairs of the component *id* and a natural number. p is defined as:
  - for all sequences $g_i..g_j$ within the set *processed*, there exists ordered pairs $p_i..p_j$ such that:
    - the first element of p_i is equal to the *object.id* of the first statement within the sequence g_i.

- The second element of p_i is equal to the value returned when g_i is passed to the function *count*.
- The first element of p_j is equal to the *object.id* of the first statement within the sequence g_j.
- the second element of p_j is equal to the value returned when g_j is passed to the function *count*

- The set $S'_{processed}$ is the union of the sets $S_{processed}$ and $processed'$

3.6.6 Sequence of Operations

$ProcessedQuestions \mathrel{\widehat=} FilterForIncorrect \mathbin{⨟} AggregateQuestionStatements$

- The schema *ProcessedQuestions* is the sequential composition of operation schemas *FilterForIncorrect* and *AggregateQuestionStatements*
- *FilterForIncorrect* happens before *AggregateQuestionStatements*

3.6.7 Return

$$
\begin{array}{l}
\textbf{ReturnAggregate} \\
\hline
\Xi MostDifficultAssessmentQuestions \\
ProcessedQuestions \\
S_{processed}! : \mathbb{F} \\
\hline
S_{processed}! = S_{processed}
\end{array}
$$

- The returned variable $S_{processed}!$ is equal to the current state of variable $S_{processed}$ after the operations *FilterForIncorrect* and *AggregateQuestionStatements*

3.7 Pseudocode

Algorithm 2: Most Difficult Assessment Questions

```
Input: S_all, display-n
Result: display
init id-to-count := []
init display := []
while S_all is not empty do
    for each statement s in S_all
    if s.result.success = INCORRECT then
        add s to S_incorrect
        remove s from S_all
        recur
    else
        remove s from S_all
        recur
    end
end
while S_incorrect is not empty do
    for each statement si in S_incorrect
        let si.object.id = id
    if id-to-count is empty then
        concat id-to-count [id , 1]
        remove si from S_incorrect
        recur;
    else
        if id-to-count contains [id , #] then
            add one to #
            remove si from S_incorrect
            recur
        else
            concat id-to-count [id , 1]
            remove si from S_incorrect
            recur
        end
    end
end
Sort id-to-count by second value of each subarray (#)
take the first display-n subarrays from id-to-count and concat them into
  display
```

- The Z schemas are used within this pseudocode
- The return value display is an array of length display-n, where each element of display is an array of [*statement.object.id*, #] where # representing the number of times *statement.object.id* appeared within $S_{incorrect}$

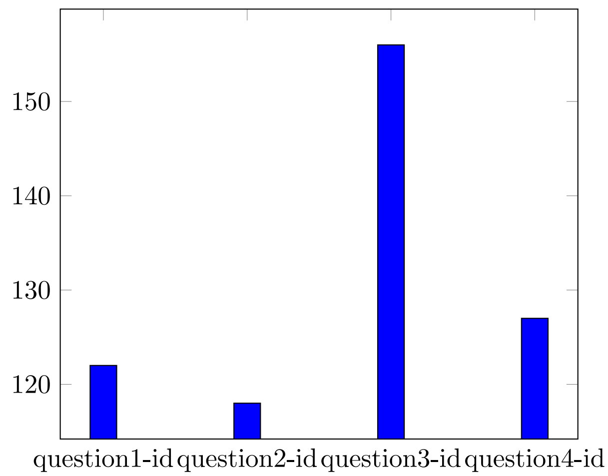
3.8 JSON Schema

```
{"type":"array",
   "items":{"type":"array",
      "items":[{"type":"string"}, {"type":"number"}]}}
```

3.9 Visualization Description

The **Most Difficult Assessment Questions** visualization will be a bar chart where the domain consists of *statement.object.id* and the range is a number greater than or equal to 1. Every subarray within the array display will be a grouping within the bar chart. The pseudocode specifies an input paramter display-n which controls the length of the array display and therefor the number of groups contained within the visualization.

3.10 Visualization prototype



3.11 Prototype Improvement Suggestions

Additional features may be implemented on top of this base specification but they would require adding aditional values to each subarray returned by the algorithm. These additional values can be retrieved via (1) performing metadata lookup within or independently of the algorithm (2) by utilizing additional xAPI statement paramters and/or (3) by performing additional computations. The following examples assume the metadata is contained within each statement available to the algorithm.

- Use the name of the activity for the x-axis label instead of its id.
  - $\$.object.definition.name$

- grouping of statements should still happen by $\$.object.id$ to ensure an accurate count

- a tooltip containing contextual information about the question such as:
  - The question text
    - $\$.object.definition.description$
  - Interaction Type
    - $\$.object.definition$ which contains interaction properties
  - Answer choices
    - $\$.object.definition$ which contains interaction properties
  - Correct answer
    - $\$.object.definition$ which contains interaction properties
  - Most popular incorrect answer
    - This would require an extra step of processing and all statements would need to utilize interaction properties within $\$.object.definition$
  - average partial credit earned (if applicable)
    - $\$.result.score.scaled$
    - The one potential issue with using scaled score is the calculation of scaled is not stricly defined by the xAPI specification but is instead up to the authors of the LRP. This results in the inability to reliably compare scaled scores across LRPs.
    - if $\$.result.score.raw$, $\$.result.score.min$ and $\$.result.score.max$ are reported for all questions, it becomes possible to reliably compare scores across questions and LRPs.
  - average number of re-attempts
    - this would require additional steps of processing so that $\$.actor$ is considered as well
    - due to the problem of actor unification, ie the same person being identified differently across statements, this metric may not be accurate.
  - average time spent on the question
    - $\$.result.duration$
    - this would require additional steps of processing to extract the duration and average it.

- a tooltip containing contextual information about the course and/or assessment the question was within
  - the instructor for the course
    - $\$.context.instructor$

– competency associated with the question and/or course

  * $\$.context.contextActivities$

– metadata about the learning content associated with the question such as average time spent engaging with associated content before attempting the question.

– this would require additional steps of processing to retrieve metadata about the content and its usage.

  * $\$.context.contextActivities$

4 Rate of Completions

As learners engage in activities supported by a learning ecosystem, they will build up a history of learning experiences. When the digital resources of that learning ecosystem adhere to a framework dedicated to supporting and understanding the learner, such as the Total Learning Architecture (TLA), it becomes possible to retell their learning story through data and data visualization. One important aspect of that story is the rate of completion [8] of the various digital resources within the learning ecosystem.

4.1 Ideal Statements

In order to accurately portray the rates of completion, there are a few base requirements of the data produced by a Learning Record Provider (LRP). They are as follows:

- statements describing a learner completing an activity should [9] use the verb $http://adlnet.gov/expapi/verbs/completed$
- statements describing a learner completing an activity should report if the learner was successful or not via $\$.result.success$
- statement describing a learner completing a scored activity should report the learners score via $\$.result.score.raw$, $\$.result.score.min$ and $\$.result.score.max$
- activites must be uniquely and consistently identified across all statements
- The time at which a learner completed a learning activity must be recorded

[8] Completion can be defined by the presence of the verb completed or by the presence of $\$.result.completion$ set equal to true. In this algorithm, completion is defined by the presence of the verb completed regardless of $\$.result.completion$. This decision affects how statements are retrieved and filtered. In the case where completion is defined by $\$.result.completion$, the query to the LRS would not include the verb paramater and there would need to be a filtering process which looks for the presence of $\$.result.completion$ = true

[9] it is possible to use another verb but if another is used, that will need to be accounted for in data retrieval

    - The timestamp should contain an appropriate level of specificity.
    - ie. Year, Month, Day, Hour, Minute, Second, Timezone

- statements describing a learner completing an activity should report the amount of time taken to complete the activity via $.*result.duration*

4.2 Input Data Retrieval

How to query an LRS via a GET request to the Statements Resource via curl. The following section contains the appropriate parameters with example values as well as the curl command necessary for making the request. [10] [11] [12]

```
Verb = "verb=http://adlnet.gov/expapi/verbs/completed"

Since = "since=2018-07-20T12:08:47Z"

Until = "until=2018-07-21T12:08:47Z"

Base = "https://example.endpoint/statements?"

endpoint = Base + Verb + "&" + Since + "&" + Until

Auth = Hash generated from basic auth

S = curl -X GET -H "Authorization: Auth"
         -H "Content-Type: application/json"
         -H "X-Experience-API-Version: 1.0.3"
         Endpoint
```

4.3 Statement Parameters to Utilize

The statement parameter locations here are written in JSONPath. This notation is also compatable with the xAPI Z notation due to the defined hierarchy of components. Within the Z specifications, a variable name will be used instead of the $

- $.*timestamp*
- $.*object.id*

4.4 2018 Pilot TLA Statement Problems

The initial pilot test data supports the core requirements of this algorithm but completion statements only reports completion scores via $.*result.scaled*

[10] See footnote 1.
[11] See footnote 2.
[12] See footnote 3.

instead of $\$.result.score.raw$, $\$.result.score.min$ and $\$.result.score.max$. [13] Given that the offical 2018 pilot test is scheduled to take place on July 27th, 2018, this section may require updates pending future data review.

4.5 Summary

1. Query an LRS via a GET request to the statemetns endpoint using the paramters verb, since and until.
2. group statements by their $\$.object.id$
3. select time range unit for use within rate calculation. Will default to per day.
4. group statements with the same $\$.object.id$ by the unit of time by utilizing $\$.timestamp$
5. calculate rate by taking the count of statements within unit time groups and dividing by the number of unit time groups.

4.6 Formal Specification

4.6.1 Basic Types

$TIMEUNIT ::= \{second\}|\{minute\}|\{hour\}|\{day\}|\{week\}|\{month\}|\{year\}$

4.6.2 System State

$$
\begin{array}{l}
\textit{RateOfCompletion} \\
\hline
Statements \\
S_{completions} : \mathbb{F}_1 \\
S_{grouped}, S_{timeunit}, S_{processed} : \mathbb{F} \\
\hline
S_{completions} = statements \\
S_{grouped} = \{byId : \mathrm{seq}_1\ statement\} \\
S_{withRate} = \{byGroup : (\mathrm{seq}_1\ statement, \mathbb{N})\} \\
S_{processed} = \{rate : (id, \mathbb{N}, TIMEUNIT)\} \\
\hline
\end{array}
$$

- The set $S_{completions}$ is a non-empty, finite set and is the component *statements* which contains the results of the query to the LRS.
- The sets $S_{grouped}$, $S_{withRate}$ and $S_{processed}$ are all finite sets
- the set $S_{grouped}$ is a finite set of objects *byId* which are non-empty, finite sequences of the component *statement*

[13] The one potential issue with using scaled score is the calculation of scaled is not stricly defined by the xAPI specification but is instead up to the authors of the LRP. This results in the inability to reliably compare scaled scores across LRPs. if $\$.result.score.raw$, $\$.result.score.min$ and $\$.result.score.max$ are reported for all questions, it becomes possible to reliably compare scores across LRPs by generating a scaled score in a consistent way.

- the set $S_{withRate}$ is a finite set of objects *byGroup* which are ordered pairs of non-empty, finite sequences of the component *statement* and a natural number

- the set $S_{processed}$ is a finite set of objects *rate* where each contains the component *id*, a natural number and the type $TIMEUNIT$

4.6.3 Initial System State

$$
\begin{array}{|l}
\hline
\textit{InitRateOfCompletion} \\
\textit{RateOfCompletion} \\
\hline
S_{completions} \neq \emptyset \\
S_{grouped} = \emptyset \\
S_{withRate} = \emptyset \\
S_{processed} = \emptyset \\
TIMEUNIT = \{day\} \\
\hline
\end{array}
$$

- The set $S_{completions}$ is a non-empty set which contains the results of the GET request(s) to the LRS

- The sets $S_{grouped}$, $S_{withRate}$ and $S_{processed}$ are all initially empty

- the basic type $TIMEUNIT$ is set to $\{day\}$

4.6.4 Calculate Rate

$$
\begin{array}{|l}
\hline
\textit{IsoToUnix} \\
convert : \mathbb{F}_1 \rightarrow \mathbb{N}\#1 \\
c? : \mathbb{F}_1 \\
c! : \mathbb{N}\#1 \\
\hline
c! = convert(c?) \\
\hline
\end{array}
$$

- The schema $IsoToUnix$ introduces the function *convert* which takes in a finit set of one thing (a timestamp) and converts it to a single natural number.

- the purpose of this function is to convert an ISO 8601 timestamp to the Unix epoch. The concrete definition of the conversion is outside the scope of this document

  - The Unix epoch is the number of seconds that have elapsed since January 1, 1970 (midnight UTC/GMT), not counting leap seconds.

$$
\begin{array}{l}
\textit{CalcRateByUnit} \\
\hline
\textit{Statement} \\
\textit{IsoToUnix} \\
\textit{CountPerGroup} \\
unit? : TIMEUNIT \\
s?\,, s! : \mathbb{F} \\
r : \mathbb{N} \\
rate : (\mathbb{F}, TIMEUNIT) \rightarrow \mathbb{F} \\
\hline
unit? = \ \{second\} \Rightarrow 1 \vee \{minute\} \Rightarrow 60 \vee \{hour\} \Rightarrow 3600 \vee \\
\qquad\qquad \{day\} \Rightarrow 86400 \vee \{week\} \Rightarrow 604800 \vee \\
\qquad\qquad \{month\} \Rightarrow 2629743 \vee \{year\} \Rightarrow 31556926 \\
s? = \{g : seq_1 statement\} \\
s! = rate(s?\,, unit?\,) \\
s! = \{s : (g, r) \mid \forall g_n : g_i..g_j \bullet i \leq n \leq j \bullet \exists\, s_n : (g_n, r_n) \bullet \\
\qquad r_n = count(g_n) \div ((convert(last\, g_n.timestamp) - convert(head\, g_n.timestamp)) \div unit?\,)\}
\end{array}
$$

- The schema *CalcRateByUnit* introduces the function *rate* where the input $s?$ is a set of objects g which are each a non-empty, finite sequence of statements and the input $unit?$ represents a unit of time.
- for every g_n within the range $g_i..g_j$, there exists an associated object s_n which is an orderd pair of (g_n, r_n) where r_n is equal to the number of items within g_n divided by the number of $unit?$s within the time range of $last\, g_n.timestamp - head\, g_n.timestamp$
- the output of the function *rate* is $s!$, the set of all s_n

4.6.5 Processes Results

$$
\begin{array}{|l}
\hline
\textit{AggergateCompletionStatements} \\
\Delta \textit{RateOfCompletion} \\
\textit{GroupByActivityId} \\
\textit{CalcRateByUnit} \\
\textit{grouped}, \textit{processed}, \textit{withRate} : \mathbb{F} \\
r : \mathbb{N} \\
T? : \textit{TIMEUNIT} \\
\hline
T? = \{\textit{day}\} \\
\textit{grouped} = \emptyset \\
\textit{grouped}' = \textit{group}(S_{\textit{completions}}) \\
S'_{\textit{grouped}} = S_{\textit{grouped}} \cup \textit{grouped}' \\
\textit{withRate} \subseteq S'_{\textit{grouped}} \\
\textit{withRate}' = \textit{rate}(\textit{withRate}, T?) \\
S'_{\textit{withRate}} = \textit{withRate}' \cup S_{\textit{withRate}} \\
\textit{processed} \subseteq S'_{\textit{withRate}} \\
\textit{processed}' = \{p : (\textit{id}, r, T?) \mid \\
\qquad\qquad \textbf{let } \{\textit{processed}_i .. \textit{processed}_j\} == \{b_i .. b_j\} \bullet \\
\qquad\qquad \forall b_n : b_i .. b_j \bullet i \leq n \leq j \bullet \exists\, p_n : (\textit{id}_n, r_n, T?) \bullet \\
\qquad\qquad \textit{id}_n = (\textit{head}\,(\textit{first}\, b_n)).\textit{object}.\textit{id} \land \\
\qquad\qquad r_n = (\textit{second}\, b_n)\} \\
S'_{\textit{processed}} = \textit{processed}' \cup S_{\textit{processed}} \\
\hline
\end{array}
$$

4.6.6 Return

4.7 Pseudocode

4.8 JSON Schema

4.9 Visualization Description

4.10 Visualization prototype

4.11 Prototype Improvement Suggestions