

### 0.0.1 At Index

The operation *atIndex* will return the Value at a specified numeric index within a Collection or an empty Collection if there is no value at the specified index.

$$\begin{array}{|l}
\hline
atIndex[Collection, \mathbb{N}] \\
idx? : \mathbb{N} \\
coll? : Collection \\
atIndex\_ : Collection \times \mathbb{N} \twoheadrightarrow V \\
\hline
\#idx? = 1 \\
coll! = atIndex(coll?, idx?) = (head(idx? \upharpoonright coll?)) \iff idx? \in coll? \\
coll! = atIndex(coll?, idx?) = \langle \rangle \iff idx? \notin coll? \\
\hline
\end{array}$$

Given the definition of the *Collection* and *V* free types

$$\begin{aligned}
Collection &::= emptyColl \mid append \langle \langle Collection \times Scalar \vee Collection \vee KV \times \mathbb{N} \rangle \rangle \\
V &::= Scalar \mid Collection \mid KV
\end{aligned}$$

The collection member  $coll?_{idx?} : V$  is implied from *append* accepting the argument of type  $Scalar \vee Collection \vee KV \equiv V$  which means each Collection member is of type *V*. Given that extraction  $(\_ \upharpoonright \_)$  returns a Collection,

$$\begin{array}{|l}
seq X : Collection \\
\hline
\_ \upharpoonright \_ : \mathbb{P} \mathbb{N}_1 \times seq X \rightarrow seq X
\end{array}$$

in order for *atIndex* to return the collection member without altering its type, the first member of *atIdx'* must be returned, not *atIdx'* itself.

$$\begin{array}{|l}
atIdx' : Collection \\
coll!, coll?_{idx?} : V \\
\hline
atIdx' = (idx? \upharpoonright coll?) \Rightarrow \langle coll?_{idx?} \rangle \\
coll! = head(atIdx') = coll?_{idx?}
\end{array}$$

The *head* call is made possible by restricting *idx?* to be a single numeric value.

$$\begin{aligned}
&idx?, idx' : \mathbb{N} \\
&\#idx? = 1 \bullet (idx? \upharpoonright coll?) = \langle coll?_{idx?} \rangle \bullet \\
&\quad (head(idx? \upharpoonright coll?)) = coll?_{idx?} \quad [\text{expected return given } idx?] \\
&\#idx' \geq 2 \bullet (idx' \upharpoonright coll?) = \langle coll?_{idx'_i} .. coll?_{idx'_j} \rangle \bullet \\
&\quad (head(idx' \upharpoonright coll?)) = coll?_{idx'_i} \quad [\text{unexpected return given } idx']
\end{aligned}$$

Additionally, if the provided  $idx? \notin coll?$  then an empty Collection will be returned given that *head* must be passed a non-empty Collection.

$$\begin{array}{|l}
head : seq_1 X \rightarrow X \\
\hline
idx? \notin coll? \Rightarrow (idx? \upharpoonright coll?) = \langle \rangle \rhd seq_1
\end{array}$$

The properties of *atIndex* are illustrated in the following examples.

$$\begin{array}{ll}
X = \langle x_0, x_1, x_2 \rangle & \\
x_0 = 0 & \\
x_1 = \textit{foo} & \\
x_2 = \langle a, b, c \rangle & \\
\textit{atIndex}(X, 0) = 0 & [\textit{head}(\langle x_0 \rangle)] \\
\textit{atIndex}(X, 1) = \textit{foo} & [\textit{head}(\langle x_1 \rangle)] \\
\textit{atIndex}(X, 2) = \langle a, b, c \rangle & [\textit{head}(\langle x_2 \rangle)] \\
\textit{atIndex}(X, 3) = \langle \rangle & [3 \notin X \Rightarrow x_3 \notin X]
\end{array}$$