0.0.1 At Index

The operation $atIndex$ will return the Value at a specified numeric index within a Collection or an empty Collection if there is no value at the specified index.

$$
\begin{array}{|l}
atIndex[Collection, \mathbb{N}] \\
idx? : \mathbb{N} \\
coll? : Collection \\
atIndex : Collection \times \mathbb{N} \nrightarrow V \\
\hline
\# idx? = 1 \\
coll! = atIndex(coll?, idx?) = (head(idx? \upharpoonright coll?)) \iff idx? \in coll? \\
coll! = atIndex(coll?, idx?) = \langle\rangle \iff idx? \notin coll? \\
\end{array}
$$

Given the definition of the $Collection$ and V free types

$$Collection :== emptyColl \mid append\langle\!\langle Collection \times Scalar \lor Collection \lor KV \times \mathbb{N} \rangle\!\rangle$$
$$V ::= Scalar \mid Collection \mid KV$$

The collection member $coll?_{idx?} : V$ is implied from $append$ accepting the argument of type $Scalar \lor Collection \lor KV \equiv V$ which means each Collection member is of type V. Given that extraction $(\_ \upharpoonright \_)$ returns a Collection,

$$
\begin{array}{|l}
\operatorname{seq} X : Collection \\
\hline
\_ \upharpoonright \_ : \mathbb{P}\, \mathbb{N}_1 \times \operatorname{seq} X \to \operatorname{seq} X \\
\end{array}
$$

in order for $atIndex$ to return the collection member without altering its type, the first member of $atIdx'$ must be returned, not $atIdx'$ itself.

$$
\begin{array}{|l}
atIdx' : Collection \\
coll!, coll?_{idx?} : V \\
\hline
atIdx' = (idx? \upharpoonright coll?) \Rightarrow \langle coll?_{idx?} \rangle \\
coll! = head(atIdx') = coll?_{idx?} \\
\end{array}
$$

The $head$ call is made possible by restricting $idx?$ to be a single numeric value.

$$
\begin{array}{l}
idx?, idx' : \mathbb{N} \\
\quad \# idx? = 1 \bullet (idx? \upharpoonright coll?) = \langle coll?_{idx?} \rangle \bullet \\
\qquad (head(idx? \upharpoonright coll?)) = coll?_{idx?} \qquad \text{[expected return given } idx?\text{]} \\
\quad \# idx' \geq 2 \bullet (idx' \upharpoonright coll?) = \langle coll?_{idx'_i} \mathinner{..} coll?_{idx'_j} \rangle \bullet \\
\qquad (head(idx' \upharpoonright coll?)) = coll?_{idx'_i} \quad \text{[unexpected return given } idx'\text{]}
\end{array}
$$

Additionally, if the provided $idx? \notin coll?$ then an empty Collection will be returned given that $head$ must be passed a non-empty Collection.

$$
\begin{array}{|l}
head : \operatorname{seq}_1 X \to X \\
\hline
idx? \notin coll? \Rightarrow (idx? \upharpoonright coll?) = \langle\rangle \neg \operatorname{seq}_1 \\
\end{array}
$$

The properties of $atIndex$ are illustrated in the following examples.

$$
\begin{aligned}
&X = \langle x_0, x_1, x_2 \rangle \\
&\qquad x_0 = 0 \\
&\qquad x_1 = foo \\
&\qquad x_2 = \langle a, b, c \rangle \\
&atIndex(X, 0) = 0 && [head\,(\langle\, x_0 \,\rangle)] \\
&atIndex(X, 1) = foo && [head\,(\langle\, x_1 \,\rangle)] \\
&atIndex(X, 2) = \langle a, b, c \rangle && [head\,(\langle\, x_2 \,\rangle)] \\
&atIndex(X, 3) = \langle\rangle && [3 \notin X \Rightarrow x_3 \notin X]
\end{aligned}
$$