0.0.1 Associate At

The Primitive *associateAt* establishes a relationship between $k?_j$ and $v?$ at the nesting $k?_i \,..k?_{j-1}$ within Map $m!$

$$
\begin{array}{|l}
k? = \langle k?_i \,..\, k?_j \rangle \\
(k?_j, m?_{k?}) \in m? \lor (k?_j, m?_{k?}) \notin m? \\
(k?_j, m?_{k?}) \notin m! \iff m?_{k?} \neq v? \\
(k?_j, v?) \in m! \\
\hline
m! = associateAt(m?, k?, v?)
\end{array}
$$

This implies that any existing mapping at $k?_j \in m?$ will be overwritten by *associateAt* but an existing mapping is not a precondition. The following helper Operation *getFirstKey* is introduced to establish navigation into a nested Map given a Collection of Keys.

$$
\begin{array}{|l}
\hline
GetFirstKey[KV, Collection] \\
m? : KV \\
k? : Collection \\
v! : V \\
getFirstKey\_ : KV \times Collection \twoheadrightarrow V \\
\hline
v! = getFirstKey(m?, k?) \bullet v! = atKey(m?, head(k?)) \\
\hline
\end{array}
$$

This allows for the navigation into a nested Map to be defined as $\langle getFirstKey\_, recur\_ \rangle^{\# k? - 1}$ which represents a step down for each $k \in (k? \setminus k?_j)$. Once at $k?_{j-1}$, the mapped value v_{j-1} has $(k?_j, v?)$ added to it. This update is localized within $m?$ and all other mappings within $m?$ are left alone.

$$
\begin{array}{|l}
\hline
AssociateAt[KV, Collection, V] \\
GetFirstKey, Recur \\
m?, m! : KV \\
k? : Collection \bullet \forall k?_n \in k?_{\langle i \,..\, j \rangle} \mid k_n : K \\
v? : V \\
associateAt\_ : KV \times Collection \times V \rightarrowtail KV \\
\hline
associateAt = \langle \langle atFirstKey\_, recur\_ \rangle^{\# k? - 1}, \langle associate\_ \rangle^{\# k} \rangle \\
m! = associateAt(m?, k?, v?) \bullet \\
\quad \forall n : i \,..\, j - 1 \in \operatorname{dom} k? \bullet j = first(last(k?)) \Rightarrow first(j, k?_j) \mid \exists_1 v_n \bullet \\
\qquad let \quad c_n == tail(k?)^{n-i} \\
\qquad\qquad v_i == getFirstKey(m?, c_n) \Rightarrow \\
\qquad\qquad\quad c_n = k? \iff n = i \bullet v_i = atKey(m?, head(k?)) \\
\qquad\qquad v_n = recur(v_i, c_n, getFirstKey\_)^{j-1} \\
\qquad\qquad v_{j-1} == getFirstKey(v_n, (j - 1 \upharpoonright k?)) \iff n = j - 2 \\
\qquad\qquad v_j == associate(v_{j-1}, last(k?), v?) \Rightarrow \langle\!\langle k?_j \mapsto v? \rangle\!\rangle \cup v_{j-1} \lhd k?_j \\
\quad = (v_j \cup v_{n-succ(1)} \lhd k?_{n-succ(0)})^{j-1} \bullet n \leq j - 1 \Rightarrow \\
\qquad v_{j-4} \cup (v_{j-3} \cup (v_j \cup v_{j-2} \lhd k?_{j-1}) \lhd k?_{j-2}) \lhd k?_{j-3} \bullet \\
m! = associate(m?, k?_i, associate(v_i, k?_n, associate(v_n, k?_{j-1}, associate(v_{j-1}, k?_j, v?)))) \\
\hline
\end{array}
$$

In the schema above, the localization of the change and the retention of the other mappings is indicated via

$$\begin{aligned}&(v_j \cup v_{n-succ(1)} \lhd k?_{n-succ(0)}\,)^{j-1} \bullet n \leq j-1 \Rightarrow\\ &\qquad v_{j-4} \cup (v_{j-3} \cup (v_j \cup v_{j-2} \lhd k?_{j-1}\,) \lhd k?_{j-2}\,) \lhd k?_{j-3}\end{aligned}$$

and is the reason why $\langle associate\,\_\rangle^{\#\,k}$ is included in the definition of $associateAt$ and is equivalent to

$$associate(m?\,, k?_i\,, associate(v_i, k?_n\,, associate(v_n, k?_{j-1}\,, associate(v_{j-1}, k?_j\,, v?\,))))$$

which gracefully walks into and back out of a KV regardless of $k?_n \in m?_{k?_{n-1}}$.

$$\begin{aligned}&M = \langle\!\langle k_i \mapsto v_i,\ k_n \mapsto \langle\!\langle k_{ni} \mapsto v_{ni},\ k_{nj} \mapsto v_{nj}\rangle\!\rangle\rangle\!\rangle\\ &\quad associateAt(M, \langle k_n, k_{nn}\rangle, v?\,) = \langle\!\langle k_i \mapsto v_i,\ k_n \mapsto \langle\!\langle k_{ni} \mapsto v_{ni},\ k_{nj} \mapsto v_{nj},\ k_{nn} \mapsto v?\,\rangle\!\rangle\rangle\!\rangle\\ &\quad associateAt(M, \langle k_j, k_{ji}\rangle, v?\,) = \langle\!\langle k_i \mapsto v_i,\ k_n \mapsto \langle\!\langle k_{ni} \mapsto v_{ni},\ k_{nj} \mapsto v_{nj}\rangle\!\rangle, k_j \mapsto \langle\!\langle k_{ji} \mapsto v?\,\rangle\!\rangle\rangle\!\rangle\\ &\quad associateAt(M, \langle k_i\rangle, v?\,) = \langle\!\langle k_i \mapsto v?\,,\ k_n \mapsto \langle\!\langle k_{ni} \mapsto v_{ni},\ k_{nj} \mapsto v_{nj}\rangle\!\rangle\rangle\!\rangle\\ &\quad associateAt(M, \langle k_i, k_{ii}\rangle, v?\,) = \langle\!\langle k_i \mapsto \langle\!\langle k_{ii} \mapsto v?\,\rangle\!\rangle,\ k_n \mapsto \langle\!\langle k_{ni} \mapsto v_{ni},\ k_{nj} \mapsto v_{nj}\rangle\!\rangle\rangle\!\rangle\end{aligned}$$

The last example demonstrates what happens when the value at some key is not a Map.

$$\begin{aligned}&atKey(v_i, k_{ii}) = \emptyset \iff k_{ii} \notin \operatorname{dom} v_i \bullet\\ &\quad associateAt(M, \langle k_i, k_{ii}\rangle, v?\,) \Rightarrow\\ &\qquad associate(M, k_i, associate(\emptyset, k_{ii}, v?\,))\end{aligned}$$