HW07

BY ZIHAN ZHOU

1 WILD

Case $r \cong$ Wild:

to show :

$\forall$cs: char list, k: char list $\rightarrow$ bool $\{$match wild cs $k \hookrightarrow$true$|\exists p$, s, $p \circ s \cong$ cs $\wedge\, p \in L(\_) \wedge k\, s \hookrightarrow$ true$\}$

```
match Wild cs k =
    (case cs of
      nil => false
      | c'::cs' => k cs')
```

Since match Wild cs k $\cong$ true

then cs is for sure not nil.

so cs must has form *c'::cs'*, and *k cs'*$\hookrightarrow$ true

let p = [c'], s=cs',

then cs=p@s, and p$\in$L(_), and k s $\hookrightarrow$true

2 BOTH

Case r $\sim$= Both(r1, r2): To show:

$\forall$cs: char list, k: (char list $\rightarrow$ bool)$\{$match(Both(r_1, r_2)) cs$'$ $k \hookrightarrow$true$|\exists p$, s: char list, $p \circ s \cong$ cs$' \wedge p \in L(r_1 \cap r_2) \wedge k\, s \hookrightarrow$true$\}$

two IHs:

IH1: $\forall$cs$_1$: char list, k_1: (char list $\rightarrow$ bool)$\{$match(r_1) cs$_1$ $k_1 \hookrightarrow$true$|\exists p_1$, s_1: char list, $p_1 \circ s_1 \cong$ cs$_1 \wedge p_1 \in L(r_1) \wedge k_1\, s_1 \hookrightarrow$true$\}$

IH2: $\forall$cs$_2$: char list, k_2: (char list $\rightarrow$ bool)$\{$match(r_2) cs$_2$ $k_2 \hookrightarrow$true$|\exists p_2$, s_2: char list, $p_2 \circ s_2 \cong$ cs$_2 \wedge p_2 \in L(r_2) \wedge k_2\, s_2 \hookrightarrow$true$\}$

```
 | match (Both(r1,r2)) cs k =
match r1 cs (fn u1=>
 match r2 cs (fn u2=>
   u1=u2 andalso k(u1) andalso k(u2)))
```

we know

1.match (Both(r1,r2)) cs k $\hookrightarrow$true

so

match r1 cs (fn u1=>

match r2 cs (fn u2=>

u1=u2 andalso k(u1) andalso k(u2))) $\hookrightarrow$true

by IH1:

let $\text{cs}_1 = \text{cs}$,

let $k_1 = (\text{fn u1=>}$

match r2 cs (fn u2=>

u1=u2 andalso k(u1) andalso k(u2)))

then we have

2.$\text{cs}_1 = p_1 \circ u_1$

3.$p_1 \in L(r1)$

4.(fn u1=>

match r2 cs (fn u2=>

u1=u2 andalso k(u1) andalso k(u2))) $u1 \hookrightarrow \text{true}$

5. match r2 cs (fn u2=>

u1=u2 andalso k(u1) andalso k(u2))$\hookrightarrow$true

apply IH2 on 5.

let $\text{cs}_2 = \text{cs}$

let $k_2 = (\text{fn u2=>}$

u1=u2 andalso k(u1) andalso k(u2))

then we have

6.$\text{cs}_2 = p_2 \circ u_2$

7.$p_2 \in L(r2)$

8.(fn u2=>

u1=u2 andalso k(u1) andalso k(u2)) u2 $\hookrightarrow$true

9.u1=u2 andalso k(u1) andalso k(u2) $\hookrightarrow$true

by lemma 1

because u1 =u2 and p1@u1=p2@u2=cs ,so p1=p2

So we find

p=p1=p2

$p \in L(r1) \cap L(r2) \Rightarrow p \in L(r1 \cap r2)$

u=u1=u2

cs$'$=cs=p@u

k u= true

QED