Section IV

Task Curve Planning for Painting Robots—Part I: Process Modeling and Calibration

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***Abstract*—This paper reports the first phase of a project whose aim is the automatic generation of tool center trajectories for robots engaged in spray painting of arbitrary surfaces. The first phase consists of proposing a mathematical model for the paint flux field within the spray cone. We have called this quantity the paint flux field partly to emphasize that it is a vector field and partly to distinguish it from a paint flux distribution function, which describes the angular variation of the flux field within the spray cone. It is shown that this flux field can be derived from experimental measurements performed by robots, of coverage profiles of paint strips on flat plates by solving a singular integral equation. This flux field is derived both for published experimental data as well as two sets of data from experiments performed by the authors. The correctness of the model is demonstrated by using the underlying distribution function to predict coverage profiles for other experiments in which the spray gun is no longer vertical to the plane surface.**

I. Introduction

ODENSE Steel Shipyard (OSS) is a producer of advanced container ships and supertankers. The yard makes extensive use of automation, especially in the area of robot welding. It was the first shipyard in the world to produce a double-hulled supertanker (ELEO MÆRSK, 299 000 tons dw). The sections forming the double hull were entirely welded together with robots.

A modern shipyard such as OSS builds ships by decomposing the design into a hierarchical assembly network of substructures. Substructures are welded together into increasingly complex assemblies, the root of the assembly tree being the complete ship. At the lowest level, such a ship consists of over one million atomic components.

Although many of the sections to be welded are identical or similar, there are many which are unique to a particular ship. Some of these are composed of curved surfaces. For these assemblies none of the standard macroprogramming tools [1] can generate robot programmes economically. For this reason, OSS established the Lindø Center for Applied Mathematics at Odense University to collaborate on the AMROSE (Autonomous Multiple Robot Operation in Structured Environments) project. The aim of AMROSE is to use a so-called force strategy [2], a generalization of the method of artificial potential fields [3] to automatically generate trajectories directly from data in the product model PROMOS. In this context, a product model is a fusion of geometric data describing the work piece with process-specific data on welding.

AMROSE is currently in the process of transition from a research project to a production system in which one-off ship sections containing curved plates will be autonomously welded by robots using the CAD system as the primary sensor. This prototype will be in production at the yard on March 1, 1995.

An important feature of AMROSE is the separation of robot programming into intertask motion, in which we do not care how the robot brings the tool center into the vicinity of the start of the task curve, and task performance, which places additional constraints on the motion of the robot in the form of its geometrical and kinetic relation to the task curve [4]. This means that all process-specific data is encapsulated in the product model. Thus the AMROSE software system can be applied directly to other processes without alteration.

The type of ship produced by OSS can contain up to 120 tons of paint and other types of surface coatings, most of which is applied manually. There exists [5] studies of the economics of automating spray painting in the shipbuilding industry, especially related to reducing the up to 100% tolerances permitted in manual painting. Apart from the economical penalties associated with overconsumption of paint, environmental considerations make the type of project considered here one of the highest priority.

Although robots are extensively used in industrial spray painting, the range of application is restricted to situations in which the equipment is used to paint a small number of preprogrammed standard components. Such equipment cannot be used economically to paint small or even single runs in situations where the number of components is very large because of the prohibitive cost of the programming involved. Such a situation arises in the shipbuilding industry, although there are many other situations (car repair, construction) with

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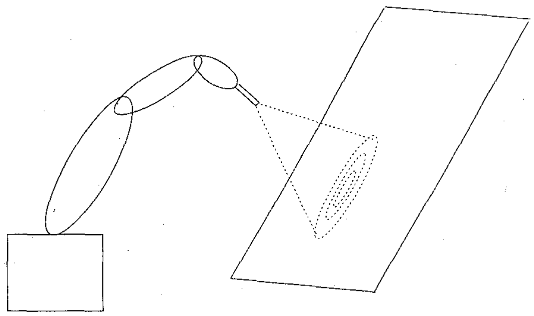

Fig. 1. Schematic diagram of a robot painting a flat plate.

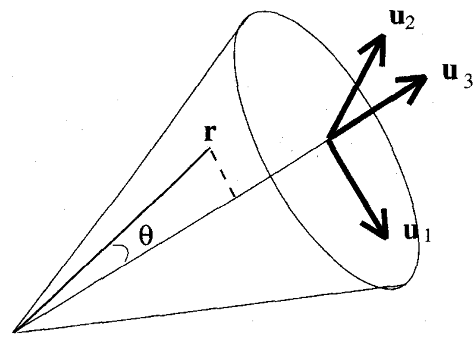

Fig. 2. Showing the geometry of the spray cone.

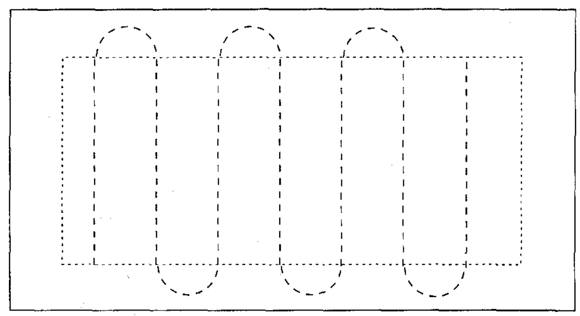

Fig. 3. Possible task curve for painting a flat plate.

the same problem that programming the robot takes the same time as performing the actual painting.

The purpose of this article is to extend the range of application to spray painting of curved surfaces. To generate a product model for this process, we need to find a suitable trajectory for the robot tool center (which is a spray gun) such that every part of the surface receives a layer of paint whose thickness and quality lies within specified tolerances. A schematic diagram of a robot painting a flat surface using a spray gun is shown in Fig. 1.

An elliptical cone containing a aerosol consisting of small paint particles suspended in air is ejected from the spray gun. The geometry of this cone is shown in Fig. 2.

The orthonormal unit vectors $\mathbf{u}_1 \cdots \mathbf{u}_3$ are fixed in the spray gun and thus denote its orientational state. When this aerosol impinges on the surface to be painted, some or all of the paint drops adhere to the surface. If we could find the steady state paint flux field $\mathbf{j}(\mathbf{r}, \mathbf{u}_1 \cdots \mathbf{u}_3)$ of paint within the cone, we could predict the coverage of paint on the surface using standard theorems from multivariable calculus and differential geometry.

To do this we need to make an accurate mathematical model for the spraying process, which can be used to calculate the surface coverage for a given motion of the tool centre. We then need to find algorithms which will enable us to find good task curves for a given surface so that the coating quality criteria are satisfied globally. A possible task curve for a robot painting a rectangular plate is shown in Fig. 3.

The main aims of this project are to provide a graphical tool which can be used to evaluate various painting strategies as well as a tool which will compute such curves automatically and optimally for a given surface to be painted.

In the remainder of this article, we review existing published work on the problem, concluding that it cannot be used in the current context. We then propose a model of the paint flux field $\mathbf{j}$ (which is a vector) involving a purely geometrical part and a paint distribution function $Q(\mu)$ which is essentially a function of the cosine μ of the angle θ subtended by the target point at the tool center, as shown in Fig. 2. We then show that this function $Q(\mu)$ can be found from coverage experiments by solving an integral equation. We then report some experiments performed using a MOTOMAN robot painting with a commercial spray gun provided by the Danish marine paint company Hempel, using two different nozzles. By fitting these profiles to a small number of suitable basis functions, we are able again to solve the integral equation for the paint distribution function $Q(\mu)$. Using these distribution functions, we are able to predict quantitatively the coverage profiles for different situations, when the height of the pistol is altered or when the pistol is no longer perpendicular to the target surface.

We conclude that a good mathematical model of the spray painting process can be used to accurately calibrate the equipment. This article thus opens the way for automatic model-based generation of tool center trajectories and thus the construction of spray painting systems with the same functionality as the AMROSE robot welding system, which is capable of welding complex, one-off structures.

II. Definition of the Painting Problem and State of the Art

As remarked in the introduction, the paint flux field $\mathbf{j}(\mathbf{r}, \mathbf{u}_1 \cdots \mathbf{u}_3)$ is the key to making a mathematical model of the painting process which can be used to predict the coverage of a given surface from knowledge of the motion of the tool center. There have been a number of attempts to do this in the literature. Klein [7] has described an off-line programming tool for painting robots. He proposes that the rate of coverage of a plane surface a perpendicular distance z from the nozzle will be a function of z and the angle θ of the form $q(z, \theta)$, assumed to have a fixed empirical form. Why this form is chosen is not explained: in any case, it looks rather different from the ones we determine directly from experiments. Duelen *et al.* [8] have made a model of electrostatic powder spraying, but we do not consider that process here, since it is intrinsically more complex to model.

In a recent publication, Persoons and van Brussel [6] made a model of the spraying process in which they assumed that the primary source of cone broadening was diffusion of paint droplets away from the axis $\mathbf{u}_3$(or z direction). Their model for the flux field (translated into our notation) was

$$\mathbf{j}(\mathbf{r}, \mathbf{u}_1 \cdots \mathbf{u}_3) = Q_0 V/(4Dz) \exp(-VR^2/(4Dz))\mathbf{u}_3 \quad (1)$$

where Q_0 is a constant, V is the speed of the aerosol, R is the distance from the spray axis $\mathbf{u}_3$, and D is the diffusion coefficient (Persoons and van Brussel used the time of flight $t = z/V$ rather than z as a coordinate).

The behavior of this model is determined by the size of the diffusion coefficient D. A guide to its order of magnitude is given by the Stokes–Einstein formula

$$D = k_B T/(6\pi\eta d) \quad (2)$$

where k_B is Boltzmann's constant, T the absolute temperature, η the viscosity of air, and d the particle radius. For particles of radius 10 μm, this value is less than $10^{-14}\,\mathrm{m}^2/\mathrm{s}$. Their distribution function is thus essentially a delta function for values of R, V, and z appropriate to the painting process. We thus conclude that diffusion within the paint cone can be neglected. Similar considerations lead us to conclude that gravity has a negligible influence on the motion of the droplets. Both these observations are confirmed by measurements. There are also further problems with their work which we discuss in connection with the problem of determining the paint flux field from measurements of the coverage of flat plates.

Assuming that the paint flux field $\mathbf{j}$ is known, we may calculate the coverage thickness $c(u, v)$ in terms of the volume of paint $c(u, v)dA$ on the surface element dA at the point $\mathbf{r}(u, v)$ of the surface ∂A as the time integral

$$\begin{aligned} c(u, v)dA = -\int_{-\infty}^{\infty} &\mathbf{j}(\mathbf{r}(u, v) - \mathbf{r}_{\mathrm{TC}}(t), \mathbf{u}_1(t), \mathbf{u}_2(t), \mathbf{u}_3(t)) \\ &\cdot \mathbf{r}_u(u, v)\, \mathbf{r}_v(u, v) du dv dt. \end{aligned} \quad (3)$$

Here $\mathbf{r}_{\mathrm{TC}}(t)$ is the position of the tool center with respect to a coordinate system fixed in the surface.

In the case where ∂A is a plane surface perpendicular to the spray axis, a suitable parametrization involves the usual Cartesian coordinates x, y in the plane. Persoons and van Brussel computed the time integral over their flux model in the case where the tool axes are held fixed in space, the height z of the tool center above the plane is held fixed and the spray is moved with constant velocity V parallel to the y-axis fixed in the target surface. The experimental situation is shown in Fig. 4.

Apart from near the ends of the strip thus painted, the coverage function, which we denote by $c(x, z)$ to emphasize that it depends on the height z of the tool center above the plane, will be independent of the distance y along the strip. For the case where the flux distribution is Gaussian, the coverage function $c(x, z)$ is also Gaussian in x. The function $c(x, z)$ calculated in this way disagrees rather violently with the parabolic profile observed by Persoons and van Brussel and shown in Fig. 5.

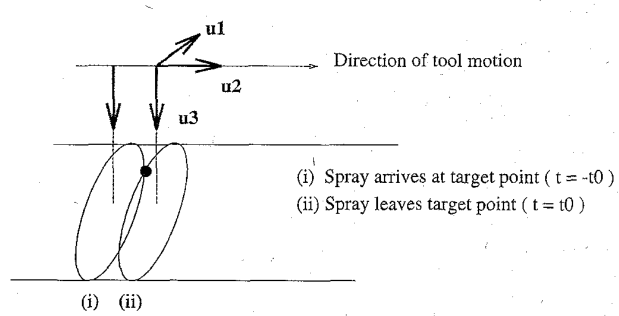

Fig. 4. Showing the movement of the spray cone and its elliptical cross section.

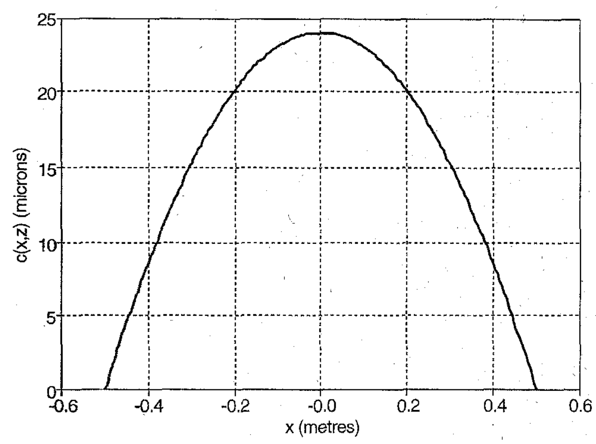

Fig. 5. Showing the parabolic profile measured by Persoons and van Brussel.

Because the functional form of the flux distribution and the coverage function are basically the same if the former is Gaussian, Persoons and van Brussel then assumed that this is generally the case, and henceforth identified the two basically different quantities. This is of course not the case as we shall see presently. Other problems with their model for the flux field are that it does not have zero divergence (as required if the total flux across any surface intersecting the cone is to be constant) and that the silhouette of the cone is not linear as it must be.

Berg and Hertling [5] made a study of robot painting in an industrial environment, partly with a view to evaluating the reliability of a commercial simulation tool, PAINTMASTER, a tool within the ROBCAD simulation package. Their conclusion was that either the algorithms used by the tool were inadequate or that the equipment was not very reproducible. It should also be mentioned that some of the coverage profiles measured by them appeared to be parabolic in shape.

III. Model for the Paint Flux Field

In this section, we make a model for the paint flux field $\mathbf{j}(\mathbf{r}, \mathbf{u}_1 \cdots \mathbf{u}_3)$. Any such model must satisfy the following conditions, which are illustrated in Fig. 6.

1) The streamlines radiate from the tool center, which ensures that the paint flux is contained within a cone whose silhouettes are straight lines,

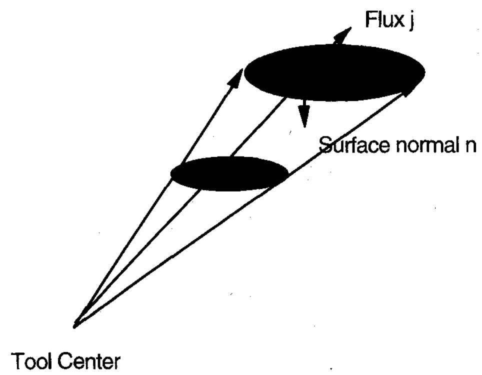

Fig. 6. Showing the structure of the paint streamlines and the zero divergence criterion for the flux distribution function.

2) If the cone is intersected by a plane perpendicular to the cone axis, the curves of constant flux in this plane are ellipses,
3) The divergence of $\mathbf{j}$ must be zero, which ensures that the total flux across any surface completely intersecting the cone is a constant, expressing conservation of paint.

Our model for the flux field which satisfies these three conditions is

$$\mathbf{j}(\mathbf{r}, \mathbf{u}_1 \cdots \mathbf{u}_3) = \alpha Q((\mathbf{r} \cdot \mathbf{u}_3)/(\mathbf{r}^T \mathrm{Ar})^{1/2})\mathbf{r}/(\mathbf{r}^T \mathrm{Ar})^{3/2} \quad (4)$$

where α is the ratio of the long (1) axis to the short (2) axis of the ellipse and $\mathbf{A}$ is the square matrix

$$\mathbf{A} = \mathbf{u}_1(t)\mathbf{u}_1(t)^T + \alpha^2 \mathbf{u}_2(t)\mathbf{u}_2(t)^T + \mathbf{u}_3(t)\mathbf{u}_3(t)^T.$$

The unknown function $Q(\mu)$ we call the flux distribution function. Notice that if the cone were circular ($\alpha = 1$), then the argument

$$\mu = (\mathbf{r} \cdot \mathbf{u}_3)/(\mathbf{r}^T \mathrm{Ar})^{1/2}$$

of Q degenerates to the cosine μ of the angle θ between $\mathbf{r}$ and the axis $\mathbf{u}_3$ of the spray cone, as shown in Fig. 2.

If we can find $Q(\mu)$, then we can find the flux at an arbitrary point $\mathbf{r}$ relative to the tool center and hence compute the coverage function $c(x, z)$.

IV. Calculation of the Flux Distribution Function

We now paint a long strip on a flat plate using a spray whose axis $\mathbf{u}_3$ is perpendicular to the plate and whose height z above the plate is kept constant. The other two axes $\mathbf{u}_1$ and $\mathbf{u}_2$ are kept fixed in space and the spray gun moves parallel to $\mathbf{u}_2$ with constant speed V. We suppose that outer limits of the spray in the direction of $\mathbf{u}_1$ subtend an angle $2\theta_1$ at the tool center. For commercially available sprays, this angle is typically 45 or 60 degrees. The angle subtended by the outer limits in the direction of $\mathbf{u}_2$ tends to be much smaller, so that the elliptical cross sections of the cone tend to be fairly eccentric. The geometry of this situation is shown in Fig. 4.

If we now consider what happens to the point at the intersection of the two ellipses in Fig. 4 as the tool moves, this is the result: Nothing will happen until the cone reaches this point (at time $-t_0$, for example). Paint will then continue to accumulate there until time t_0 when the cone will have moved on. If x is the distance of this point from the center of the strip, then t_0 is given by

$$(\alpha V t_0)^2 = (z \tan \theta_1)^2 - x^2.$$

To compute the coverage function $c(x, z)$ from the definition (3), we replace the upper and lower limits $\pm\infty$ by $\pm t_0$. After several changes of variable, we obtain

$$Vzc(x, z) = 2\zeta \int_{\mu_1}^{\zeta} \mu Q(\mu)/(\zeta^2 - \mu^2)^{1/2} d\mu \qquad (5)$$

where

$$\mu_1 = \cos\theta_1$$

and

$$\zeta = z/(x^2 + z^2)^{1/2}.$$

The quantity ζ satisfies the inequality

$$\mu_1 \le \zeta \le 1.$$

We now convert (5) into a form where we can determine $Q(\mu)$ from experimental measurements of $c(x, z)$. Since $Q(\mu)$ vanishes for $\mu < \mu_1$, it is natural to rewrite it as a function $q(u)$ of the variable

$$u = \mu^2 - \mu_1^2.$$

We change the variable of integration in (5) from μ to u to obtain

$$\zeta^{-1} Vzc(x, z) = \int_0^{\zeta^2 - \mu_1^2} \frac{q(u)}{\left(\zeta^2 - \mu_1^2 - u\right)^{1/2}} du. \qquad (6)$$

The form of this equation suggest that the left hand side be regarded as a function $C(w)$ of the variable

$$w = \zeta^2 - \mu_1^2$$

since $C(0) = 0$.

Notice that the range of both u and w is $[0, 1 - \mu_1^2]$. With this definition (6) becomes

$$C(w) = \int_0^w \frac{q(u)}{\sqrt{w - u}} du. \qquad (7)$$

This is a singular integral equation of a type first studied by the Norwegian mathematician N. H. Abel in the early part of the 19th century [9]. The solution is given by

$$q(u) = \frac{1}{\pi} \int_0^u \frac{C'(\eta) d\eta}{\sqrt{u - \eta}}. \qquad (8)$$

If $C(w)$ is a polynomial in w, that is

$$C(w) = \sum_{n=0}^{N} c_n w^{n+1} \qquad (9)$$

then the solution $q(u)$ is given by

$$q(u) = \frac{1}{\pi} \sum_{n=0}^{N} (n+1) B(n+1, 1/2) c_n u^{n+\frac{1}{2}} \qquad (10)$$

where $B(x, y)$ is the usual Beta function, so that the flux distribution function is given by

$$Q(\mu) = \frac{1}{\pi}\sum_{n=0}^{N}(n+1)B(n+1, 1/2)c_n\left(\mu^2 - \mu_1^2\right)^{n+\frac{1}{2}}. \quad (11)$$

All the above is predicated on the assumption that the cross section of the spray is elliptical. It turns out that the current formalism can also deal with the situation where the curves of constant flux are curves on which the value of

$$x^n + (\alpha y)^n$$

is constant, where n is an even integer. An analysis very similar to the above can be performed to obtain the integral equation

$$C(w) = \int_0^w \frac{q(u)du}{(w-u)^{\frac{n-1}{n}}} \quad (12)$$

where

$$w = \zeta^n - \mu_1^n$$
$$\zeta = z/(x^n + z^n)^{\frac{1}{n}}$$

and $q(u)$ is related to $Q(\mu)$ by

$$Q(\mu) = \mu^{n-2}q(u)$$

and $C(w)$ is given by

$$C(w) = \frac{1}{2}nVz\zeta^{1-n}c(x, z). \quad (13)$$

Abel's method can also be used to solve this integral equation, where the solution is

$$q(u) = \frac{\sin((n-1)\pi/n)}{\pi}\int_0^u \frac{C'(\eta)d\eta}{(u-\eta)^{\frac{1}{n}}}. \quad (14)$$

V. Experimental Evaluation

As remarked in the introduction, both Persoons and van Brussel [6] and Berg and Hertling [5] have reported measurements of experiments taken under the circumstances mentioned above. In both cases, the function $c(x, z)$ appeared to be parabolic, as shown in Fig. 5, which is a curve fitted to the data of Persoons and van Brussel. The functional form of the fitted function is thus

$$c(x, z) = K(1 - x^2/(z^2 \tan^2 \theta_1)) \quad (15)$$

where the numerical value of K is 24 micrometers (μ). If Q_0 is the total effective flux of paint, then normalizing $c(x, z)$ gives

$$K = 3Q_0/(4Vz \tan \theta_1). \quad (16)$$

With this form of $c(x, z)$ we get

$$C(w) = 3Q_0\mu_1 w/\left(4(w + \mu_1^2)^{\frac{3}{2}}(1 - \mu_1^2)^{\frac{3}{2}}\right). \quad (17)$$

Insertion into the solution (8) of the integral (7) yields

$$Q(\mu) = 3Q_0(\mu_1/\mu)^4((\mu/\mu_1)^2 - 1)^{1/2}/(2\pi\mu_1(1 - (\mu_1)^2)^{3/2}). \quad (18)$$

The reader may check that this is indeed the solution of the integral equation.

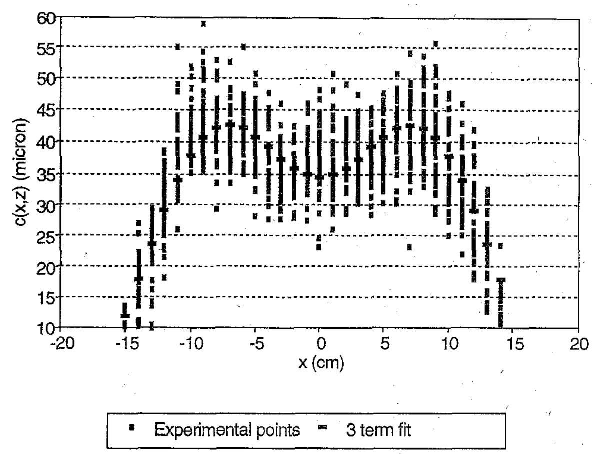

Fig. 7. Showing the experimental coverage profile for the 60-degree spray gun and the three-term fit using the basis functions.

Unfortunately, Persoons and van Brussel do not report any details of their experiments, i.e., any values for the height z, the tool speed V or the cone angle θ_1, so that their results cannot be used here.

We have instead performed a number of experiments in which a spray gun was attached to a MOTOMAN robot. Two apertures were used, in which the values of $2\theta_1$ were 45 and 60 degrees. A number of strips were painted for various values of z lying between 25 and 45 cm, where the vertical spray moved with a constant speed V of 50 cm/s parallel to the plate. Two types of paint were used.

When the paint had dried, the coverage $c(x, z)$ was measured at about 1000 points within each strip using an ELCOMETER measuring device. A typical set of such measurements is shown in Fig. 7, in this case with $V = 50$ cm/s, $\theta_1 = 30$ degrees, and $z = 30$ cm.

Note that the coverage is of the order of 50 μm, so that most of the variation in the measured coverage is due to roughness of the surface.

Also shown in Fig. 7 is the result of least square fitting with of this data with the first 3 basis functions. The value of 3 was chosen because this was the minimum number which gave the tolerable fit shown in Fig. 7. Notice that the coverage profile is definitely not parabolic, but has a minimum at the center of the spray cone. Fig. 8 shows the corresponding solution of the integral equation for $Q(\mu)$, with the surprising result that the flux of paint at the center of the spray cone is essentially zero.

The function $Q(\mu)$ was then employed to predict what would happen if the height z was reduced to 25 cm. The predicted and experimentally determined values are shown in Fig. 9.

The predicted values seem to be a little high, but the fit is indeed very sensitive to small measurement errors in for example the height. Both sets of measurements were performed with the same water based paint.

In Fig. 10, we show the results of experiments performed with the other aperture $\theta_1 = 22.5$ degrees with the same velocity V and a pistol height of 35 cm.

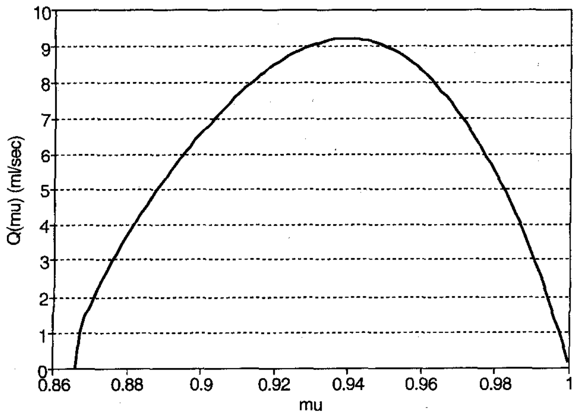

Fig. 8. Showing the flux distribution function Q for the 60-degree spray.

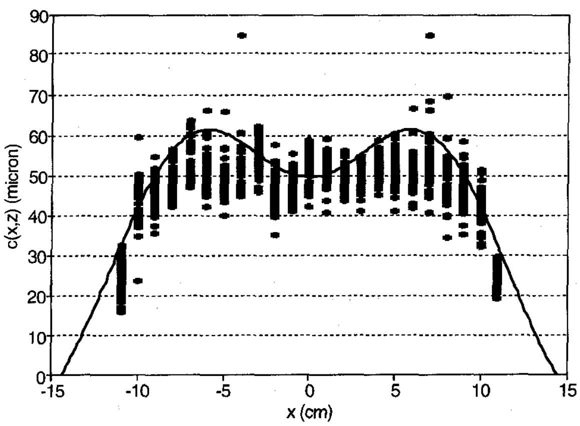

Fig. 9. Comparing the experimental coverage for the 60-degree spray at a height of 25 cm with that predicted from the flux distribution function.

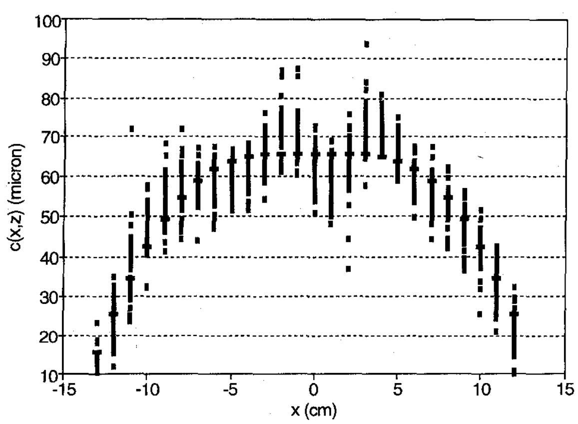

Fig. 10. Showing experimental values for the 45-degree spray cone fitted using 3 basis functions.

In this case, the paint solvent was organic. Again, the experiments can be fitted tolerably well using 3 of the basis functions, as can be seen in Fig. 10. There is now no minimum in $c(x, z)$ at the axis of the spray. The flux distribution function $Q(\mu)$ can be found as before, having the form shown in Fig. 11.

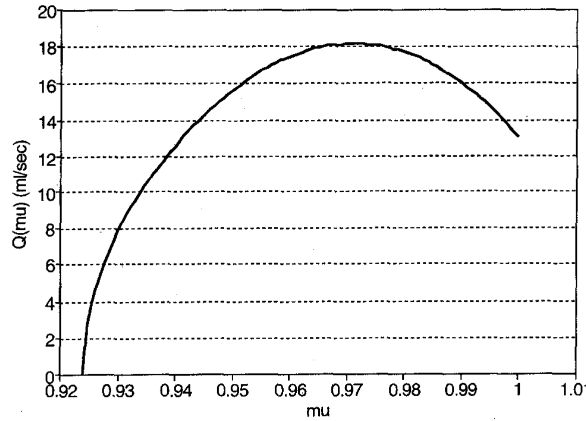

Fig. 11. Showing the flus distribution function Q for the 45-degree spray nozzle. Note that the value of Q at the center of the spray cone is higher than was the case with the 60-degree nozzle.

The maximum paint flux is still displaced from the center of the cone. We shall subject this function to a more stringent test in the next section.

VI. Calculation of Coverage Profiles

We also performed a number of experiments in which the pistol was not perpendicular to the plate, but instead rotated through an angle γ about an axis parallel to the direction of motion.

A lengthy calculation leads to the expression

$$c(x, z) = 2\zeta z/(V(z \sec\gamma + x \sin\gamma)^2) \times \int_{\mu_1}^{\zeta} \mu Q(\mu)/(\zeta^2 - \mu^2)^{1/2} d\mu \qquad (19)$$

for the coverage function under these circumstances, where ζ is now given by

$$\zeta^2 = (x \sin\gamma + z \sec\gamma)^2/(x^2 + 2xz \tan\gamma + (z \sec\gamma)^2).$$

Here x is measured from the line traced out on the target by the tool center axis. This function was evaluated for the case $V = 50$ cm/s, $\theta_1 = 22.5$ degrees, and $z = 30$ cm, using the same distribution function $Q(\mu)$ found in the previous section. The form of this function is shown in Fig. 12, along with the corresponding experiments performed under these circumstances. Even though water based paint was employed, the agreement between the model and the experiment is impressive.

VII. Conclusion and Perspectives

The conclusion of this work must be that model based painting of complex structures using robots is possible provided the model is good enough. We thus conclude discussing briefly where we go from here to implement a real shopfloor system.

The first thing which must be done is to automate the calibration process. This requires that the measurements of the surface coverage function be carried out automatically using a robot to collect the data into a portable PC. Although the formulas used to calculate the flux field seem formidable, all

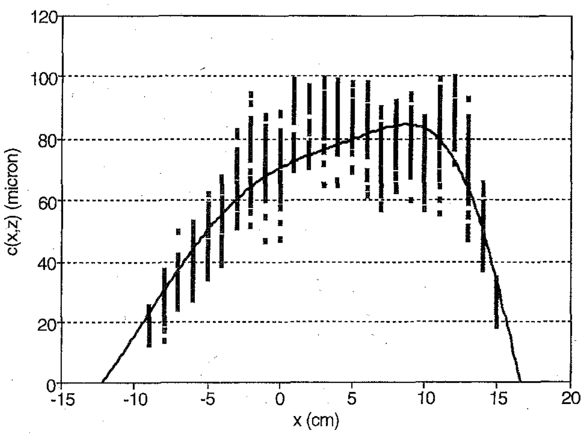

Fig. 12. Comparing the experimental coverage function for the tilted spray with values calculated using the flux distribution function.

data treatment and fitting reported here was in fact done using a commercial spreadsheet program Quattro Pro. This will also need to be automated within a piece of real software.

The next thing which needs to be done is to construct a software tool which can determine the coverage function $c(u, v)$ at a point on an arbitrary NURBS surface for a given trajectory of the tool center.

This tool will form the basis for a graphical painting simulator which can be used to investigate various strategies for painting a variety of surfaces. Knowledge gained from these investigations can then be used to design algorithms for predicting optimal (more probably, suboptimal) tool center trajectories for painting a given structure.

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