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A GRADIENT DESCENT SPARSE ADAPTIVE MATCHING PURSUIT ALGORITHM BASED ON COMPRESSIVE SENSING

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Abstract:

Aiming at the problem of original signal reconstruction based on one-dimensional (1D) compressive sensing (CS), a gradient descent sparse adaptive matching pursuit (GDSAMP) algorithm is proposed for 1D sparse signal. By setting the augmented lagrange function, the process of signal reconstruction is transformed to the unconstrained optimization problem. The iteration procedure of algorithm includes three steps: the gradient descent searching based on total deviations of one-order and two-order, the adaptive cutting on sparse coefficients and the improvement of least square projection. Based on the above steps, an algorithm framework is designed for recognizing and locating sub-pattern signal on large signal sets. Experimental results show that the GDSAMP algorithm has better efficiency on reconstructing original signal. At the same time, it can quickly locate the matching interval of sub-pattern signal on large signal sets. The researching results can be used in signal retrieval, voice recognition, image recognition and computer vision, .. etc.

Keywords:

Compressive sensing; Signal reconstruction; Pattern recognition; Adaptive matching pursuit

1. Introduction

The CS theory[1-3] proposed by Donoho et al. can be used to conduct compressive sampling on sparse signal by much lower frequency than Nyquist. If the measurement matrix satisfies with Restricted Isometry Property (RIP)[4-5], the original signal can be reconstructed accurately by fewer measurements. In recent years, the CS theory has been widely applied in radar imaging, signal sparse representation, multimedia coding, face recognition, image analysis and visual computing. The CS theory mainly involves three core issues: (1) The signal sparse representation, to design the basises or over-complete dictionary[6-8] with the capability of sparse representation. (2) The compressive measurement on sparse signal, to design the measurement matrix[9] which satisfies with incoherent or RIP. (3) The reconstruction of original signal to design fast signal reconstruction algorithm. On signal sparse representation and measurement matrix designing, there have been some better solutions. On signal reconstruction, it is always a hot topic in CS theory[10]. Currently, some mature signal reconstruction algorithms have been proposed. They are divided into two categories: one category is based on the minimized l^1 norm algorithm, including Basis Pursuit (BP) algorithm and Linear Programming (LP) algorithm. These algorithms have better reconstruction performance but have higher computation complexity. The other category is the greedy algorithm based on minimized l^0 norm, including Orthogonal Matching Pursuit (OMP) algorithm[11-12], Subspace Pursuit (SP) algorithm[13] and Iterative Hard Threshold (IHT) algorithm[14]. In these algorithms, the iteration is conducted by searching the original sparse signal supporting set and residual estimation but the cropping on supporting set is slow. In order to reduce the iteration times, a Sparse Adaptive Matching Pursuit (SAMP) algorithm is proposed by Thong T. Do[15] et al. Even if the sparse value K is unknown, it will have a better reconstruction result by adjusting step length to implement adaptive iteration but it is also easy to cause the over-iteration problem that supporting sets is bigger than sparse values. The algorithm is improved by author in [17] and a Meticulous Sparse Adaptive Matching Pursuit (MSAMP) algorithm[16] is proposed. When applied in image retrieval, it has a better result but the expansion efficiency on supporting set depends on the residual accuracy. A compressive measurement and reconstruction algorithm for two-dimensional (2D) image was proposed in Ref. [17]. It is designed based on gradient descent iteration but its reconstruction only makes use of the Total Variation of 2D image. When cropping sparse coefficients, hard threshold is applied but not considering the sparsity of original signal.

In summary, the research on efficient signal reconstruction algorithm is still a key problem in CS theory. This paper is based on the MSAMP algorithm. By gradient descent and adaptive iteration with sparse signal, a Gradient Descent Sparse Adaptive Matching Pursuit algorithm (GDSAMP) is proposed for 1D sparse signal. By setting the

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augmented lagrange function, the process of signal reconstruction is transformed to the unconstrained optimization problem. Three sub options of the unconstrained optimization objective function are processed respectively. Based on the above algorithm, a method is proposed for recognizing and locating the sub-pattern signal on large signal sets.

2. CS theory and GDSAMP algorithm

2.1. 1D CS theory

For $x \in R^N$, if there are only K components be non-zero, then x is called as K- sparse signal. In general, x is expressed as following:

$$x = \sum_{i=1}^{N} \alpha_i \psi_i = \Psi \alpha \qquad (1)$$

where Ψ_i (i=1,2,…,N) are the basises of sparse representation, Ψ is the matrix constituted by $\{\Psi_i\}_{i=1}^{N}$. For $a \in R^N$, if there are only K components be non-zero, then x is called sparse signal. By selecting measurement matrix Φ where $\Phi \in R^{M\times N}$ ($M<<N$) , we conduct compressive measurement on the sparse signal and get the measurement $y \in R^M$.

$$y = \Phi x = \Phi\Psi a = \Theta a \qquad (2)$$

where $\Theta = \Phi\Psi$ is called sensing matrix. (2) is an underdetermined equation. If matrix Θ satisfies with certain RIP condition, the reconstruction on signal x is equivalent to the following optimization problem[5]:

$$\min_{x\in R^N} \| \Psi^T x \|_0 \quad s.t \;\; y = \Phi x \qquad (3)$$

(3) is a non-convex NP optimization problem. When the RIP coefficients δ_k of matrix Θ satisfies with $\delta_{2k} \le \sqrt{2} - 1$, (3) is equivalent to the following optimization problem:

$$\min_{x\in R^N} \| \Psi^T x \|_1 \quad s.t \; y = \Phi x \qquad (4)$$

To solve (4), many algorithms have been proposed to reconstruct the original signal including OMP[13], SAMP[16], MSAPM[17]. However, these algorithms need to compute the constraint conditions repeatedly during iteration for the iteration time and accuracy eager to be improved.

2.2. GDSAMP reconstruction algorithm for 1D signal

To solve the conditional constraint optimization problem (3), a new idea is proposed in [18]. The constraint optimization problem is transformed to a non-constraint optimization problem by the augmented lagrange function and the original signal is reconstructed by iteration. The algorithm is only used for 2D signal and the selection for penalty function only considers the one-order derivation of signal but not considers the sparsity of signal in cropping. For the above defects, by improving the algorithm in [18], a GDSAMP algorithm is proposed for 1D sparse signal.

1) The K times Total Variation of one-order and two-order deviation

To solve the constraint optimization problem, the key for augmented lagrange function is to establish a suitable penalty function. By solving the non-constraint optimization problem on it, the original constraint optimization problem can be solved.

Assuming $x \in R^N$ and defining d interval of one-order deviation operator:

$$T_1(x,d) = \frac{1}{d}\sqrt{\sum_{i=1}^{N}(x_i - x_{i-d})^2}$$

where supposing x_{1-d}=x_1.

$T_1(x, d)$ reflects the self-similarity of signal x with d interval and reflects the one-order derivation features of each point of signal.

Defining d interval of two-order deviation operator:

$$T_2(x,d) = \frac{1}{d^2}\sqrt{\sum_{i=1}^{N}(2x_i - x_{i-d} - x_{i+d})^2}$$

where supposing x_{1-d}=x_1, x_{N+d}=x_N.

$T_2(x, d)$ reflects the two-order derivation features of each point of signal x with d interval.

Thus, the K times Total Variation of one-order and two-order deviation is as following:

$$T(x,K) = \sum_{d=1}^{K}(T_1(x,d) + T_2(x,d))$$

2) The designing for augmented lagrange function

For optimization problem (3), the augmented lagrange function $f(x)$ is introduced as following:

$$f(x) = \| \Psi^T x \|_0 + \lambda T(x,K) + C\|y\text{-}\Phi x\|_2^2$$

where λ, C are the penalty parameters and C reflects the self-similarity and change intensity about the signal. When the selected threshold parameters are enough large, the (3) can be transformed to the following unconditional constraint optimization problem:

$$\min_x f(x) = \min_x (\| \Psi^T x \|_0 + \lambda T(x,K) + C\|y\text{-}\Phi x\|_2^2) \qquad (5)$$

When solving (5) by gradient descent iteration method, it needs to compute the gradient of function $f(x)$. To compute the partial derivation of $\|\Psi^T x\|_0$ for the first item of (5) is difficult, it is better to consider the three items of function $f(x)$ when designing iteration algorithm. (1) The first item $\min\|\Psi^T x\|_0$ is used to restrict the sparsity of expansion coefficients during iteration. (2) The second item is the Total Variation of one-order and two-order deviation, and the gradient vector $\frac{\partial T(x, K)}{\partial x_i}, i = 1, ...$ is the iterative searching direction. (3) The third item $\min\|y - \Phi x\|_2^2$ is processed as the least square projection during iteration.

3) The cropping for sparse expansion coefficients

Introducing iterative segment counter during iteration, when cropping the sparse coefficients combining with iterative segment counter, dynamically setting the size of the cropping sets by adaptive method in case that the sparse value is unknown. It can avoid subjectivity when cropping sparse coefficients and improve the accuracy of the algorithm.

4) The iteration of GDSAMP algorithm is designed as following:

Algorithm 1: GDSAMP algorithm

Input: Gauss normalized random matrix Φ, step length s, measurement y.

Output: Approximation value x for the original signal, the sparse value sp.

Initial condition: Setting $x_0=\Phi^+y$, large integer N_L, iterative threshold N_S, k=1, I=s, small constant δ, large constant λ, C.

*Method***:**

(1) Compute the gradient of total deviation $T(x, K)$:

$$\frac{\partial T_1(x^{k-1},d)}{\partial(x^{k-1})}|_i = \frac{(x_i^{k-1} - x_{i-d}^{k-1})/d}{\sqrt{\sum_{i=1}^{N}(x_i^{k-1} - x_{i-d}^{k-1})^2 + \delta}}$$

$$\frac{\partial T_2(x^{k-1},d)}{\partial(x^{k-1})}|_i = \frac{2(2x_i^{k-1} - x_{i-d}^{k-1} - x_{i+d}^{k-1})/d^2}{\sqrt{\sum_{i=1}^{N}(2x_i^{k-1} - x_{i-d}^{k-1} - x_{i+d}^{k-1})^2 + \delta}}$$

$$\frac{\partial T(x^{(k-1)},K)}{\partial x^{(k-1)}}|_i = \sum_{d=1}^{K}\left(\frac{\partial T_1(x^{(k-1)},d)}{\partial x^{(k-1)}}|_i + \frac{\partial T_2(x^{(k-1)},d)}{\partial x^{(k-1)}}|_i\right)$$

where i=1,2,…,N. The small constant δ guarantees that the denominator is nonzero during computation.

(2) Compute the new solution along the gradient direction and get the following components:

$$\bar{x}_i^k = x_i^{k-1} - \lambda\sum_{d=1}^{K}\left(\frac{\partial T_1(x^{k-1},d)}{\partial x}|_i + \frac{\partial T_2(x^{k-1},d)}{\partial x}|_i\right)$$

where i=1,2,…,N.

(3) Expand the sparse coefficients of $\bar{x}^k$ and crop the coefficients adaptively:

$$\alpha = f_{adp}(\Psi^T \cdot \bar{x}^k, I)$$

where $f_{adp}(v, I)$ is the adaptive cropping function. About I of components for the maximum absolute values in $v \in R^N$ are reserved, and other components are zero.

(4) Recover the new signal: $\tilde{x}^k = \Psi \cdot \alpha$

(5) Least squares projection:

$$x^k = \tilde{x}^k + \Phi^+ \cdot (y - \Phi \cdot \tilde{x}^k)$$

(6) Compute the latest residual of the *kth* iteration:

$$r^k = y - \Phi \cdot x^k$$

(7)Segment converting self-adaptively, k=k+1. If $\|r^k\|_2 \geq \|r^{k-1}\|_2$, then $I = I+s$.

(8) If $\|x^k-x^{k-1}\| \leq \varepsilon$ or $k>N_S$, then stop iteration, output $x=x^k$ and sp=MAX(I,(k-N_s)×N_L), where Φ^+ represents a generalized inverse matrix:$\Phi^+=\Phi^T(\Phi \bullet \Phi^T)^{-1}$.

When reconstructing original signal by GDSAMP algorithm, it also estimates the sparse value. When the times of segment iteration reaches to certain threshold, it considers that the sparsity of the original signal is poor. Directly setting a larger value N_L for the sparse value sp and marking the recovery signal as x', the sparse value as sp: (x',sp)=GDSAMP (Φ, y, s).

3. Sub-pattern recognition and location on large signal sets

Defining $\hat{x} \in R^L$ is a 1D large signal set, components $(\hat{x}_1, \hat{x}_2, ..., \hat{x}_L)$ represent the signal sequences collected online, which also called analog signal set. $x=(x_1,x_2,…,x_N) \in R^N$ (N<<L) is a known signal which is called sub-pattern signal. It needs to judge whether $\hat{x} \in R^L$ includes sub-pattern signal $x \in R^N$ and the position occurs which is called signal pattern recognition problem. It is abstracted from signal retrieval, audio retrieval, image recognition, computer vision and other relative problems, the research of which can be applied in all kinds of fields [18-19].

As shown in Figure 1, for a large signal set $\hat{x}=(\hat{x}_1, \hat{x}_2, ..., \hat{x}_L) \in R^L$, starting from $\hat{x}_p$ and defining window selecting function $win(\hat{x}, p)$ as following:

$$win(\hat{x}, p) = \bar{x} = (\hat{x}_p, \hat{x}_{p+1}, ..., \hat{x}_{p+N-1})$$

where N is the length of the signal observing window. The signal components of the window constitute matrix $\bar{x}$, $\bar{x}_i = \hat{x}_{p+i-1}$, ($i$=1,2,…,$N$).

According to (2), conducting compressive measurement on the sub-pattern signal $x \in R^N$ and the observing window signal $\bar{x} \in R^N$ and getting the difference Δy among compressive measurements as following:

$$\Delta y = \Phi\bar{x} - \Phi x = \Phi(\bar{x} - x) = \Phi \cdot \Delta x$$

If the sub-pattern signal is similar with the observing window signal, then Δx is a sparse signal. When reconstructing Δx from Δy by GDSAMP algorithm by computing (Δx,sp)=GDSAMP(Φ,Δy,s), it not only can estimate the size of Δx but also can compute the sparsity of

Δx.

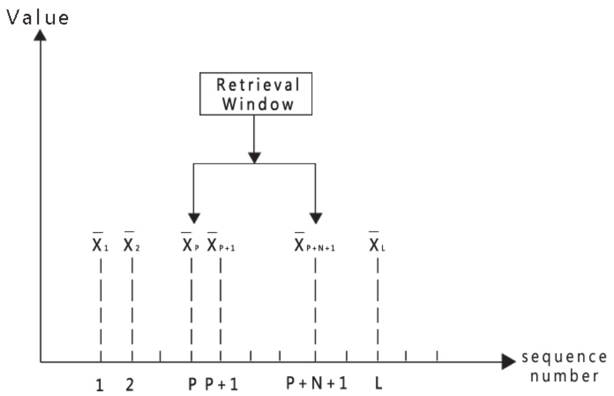

FIGURE 1. The signal sequence and observing window

In order to evaluate the similarity between x and $\bar{x}$, proposing the following similarity computing index:

$$\mathrm{SIM}(x,\bar{x}) = \|\Delta x\|_2^2 \cdot \log(1+sp) \qquad (6)$$

It indicates that by merging Δx and sparsity sp, it can measure the similarity well among signals.

Using CS theory, combining the GDSAMP algorithm proposed in section 2, a sub-pattern signal recognition algorithm framework is introduced as following:

Algorithm 2: Sub-pattern signal recognition algorithm:

Input: Gauss random matrix Φ, sub-pattern signal x, large signal set $\hat{x}$, step s.

Output: The matching interval [a, b] for sub-pattern signal.

Initial value: Searching initial step L_S, k=1, searching interval a_1=1, b_1=L.

Method:

(1)Compute the compressive measurement of sub-pattern signal: $y = \Phi x$

(2)Select observing window vector according to step d:

$$\tilde{x}^i = win(\hat{x}, h_i) = (\hat{x}_{h_i}, \hat{x}_{h_i+1}, \ldots, \hat{x}_{h_i+N-1}),$$

$$h_i = a_k + (i-1) \times L_s, i = 1,2,\ldots,(b_k - N)/L_s$$

(3) Compute CS measurements:

$$y^i = \Phi \tilde{x}^i \qquad i = 1,2,\ldots,(b_k - N)/L_s$$

(4) Compute the difference by GDSAMP algorithm:

$$\Delta y^i = y - y^i \qquad i = 1,2,\ldots,(b_k - N)/d$$

$$(\Delta x^i, sp^i) = GDSAMP(\Phi, \Delta y^i, s)$$

(5) Compute the top priority problem: By the similarity computing method in (6), solving the following optimization problem:

$$i_0 = \arg\min_i (\|\Delta x^i\|_2^2 \cdot \log(1+sp^i))$$

(6) If $\|\Delta x^{i_0}\|_2^2 \cdot \log(1+sp^{i_0}) \le \varepsilon$ or L_S==1, then stop iteration and output sub-pattern matching interval [a, b] where $a = h_{i_0}$, $b = h_{i_0} + (N-1) \cdot L_s$. At the same time, output the similarity as following:

$$1 - \|\Delta x^{i_0}\|_2^2 \cdot \log(1+sp^{i_0})$$

Otherwise: k=k+1, L_S=L_S/2, $a_k = h_{i_0} - 4 \cdot L_s$, $b_k = h_{i_0} + 4 \cdot L_s + N - 1$, jump to (2).

By sub-pattern signal recognition algorithm, it can find the interval which matching with sub-pattern signal $x \in R^N$ on large signal set $\hat{x} \in R^L$. It can recognize special sub-pattern signal x in complex signal sets.

4. Simulation and experimental results analysis

The development platforms under simulation environment include Matlab2012 and Visual Studio 2008, the testing platform is Lenovo M440S PC with hardware configuration includes: CPU: Intel(R) I5-4200U4×2.4GHz, Memory: 4GBDDR3L, OS: 32bits Windows7 SP1.

Experiment 1: The reconstruction accuracy for sparse signal

In the experiment, the sparse signal X is generated randomly whose length is N=200 and the size of measurement matrix Φ is M=100. Assuming the signal reconstruction accuracy is computed by $\left(1 - \|X - X'\|_2 / \|X\|_2\right)$ when the sparsity rate K/N about signal X is selected differently, it conducts 200 experiments and get the average reconstruction accuracy. The average reconstruction accuracy for GDSAMP, SAMP and OMP algorithm are shown in figure 2. From the experimental results, it can find that when the sparsity rate of original signal is less than 0.35, the GDSAMP algorithm has better reconstruction result.

Experiment 2: Sub-pattern signal recognition on large signal set

Randomly generating 1000-dimensional large signal set, randomly generating 90-dimensional signal as sub-pattern signal, and assuming the initial step L_s=6, the similarity threshold ε=0.73. By the pattern recognition algorithm 2 proposed in this paper, the retrieval and locating results are shown in figure 3. In the interval [235, 324], it can recognize the signal interval similar with the sub-pattern signal.

Randomly generating 90-dimensional signal as sub-pattern signal, assuming the initial step L_s=8, the similarity threshold ε=0.83. By sub-pattern signal retrieving in algorithm 2, it can retrieve 3 similar signal segments and the results are shown in figure 4. The similarity for signal segment in interval [200, 289] is 1, in interval [644, 733] is 0.63694 and in interval [783, 872] is 0.63098.

Obtaining 3000-dimensional large signal set from real signal, picking 300-dimensional signal as sub-pattern signal from the real signal and assuming the initial step $L_s=8$, the similarity threshold $\varepsilon=0.83$. By sub-pattern signal retrieving in algorithm 2, the retrieval and locating results are shown in figure 5. The similarity for signal segment in the interval [1000, 1299] is 0.92123.

The experimental results show that by the proposed algorithm framework, it can recognize special sub-pattern signal in complex large signal sets. By setting different similarity thresholds, it can recognize the sub-pattern signal vaguely.

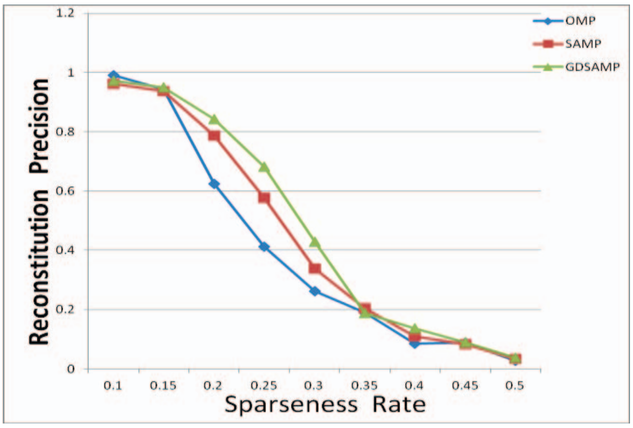

FIGURE 2. The comparison results for GDSAMP, SAMP and OMP algorithm

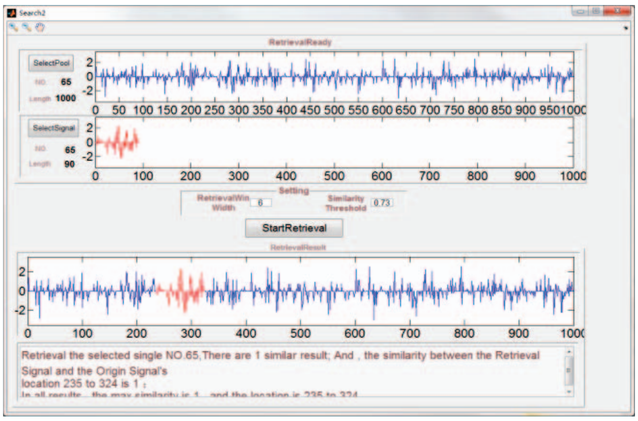

FIGURE 3. The retrieval result using random signal as sub-pattern

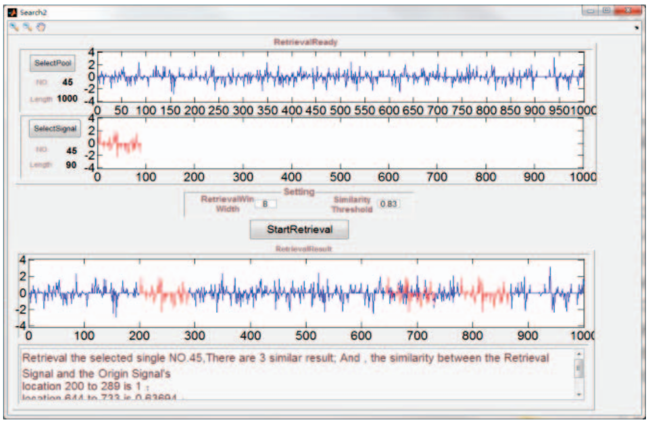

FIGURE 4. The retrieval result using random signal as sub-pattern

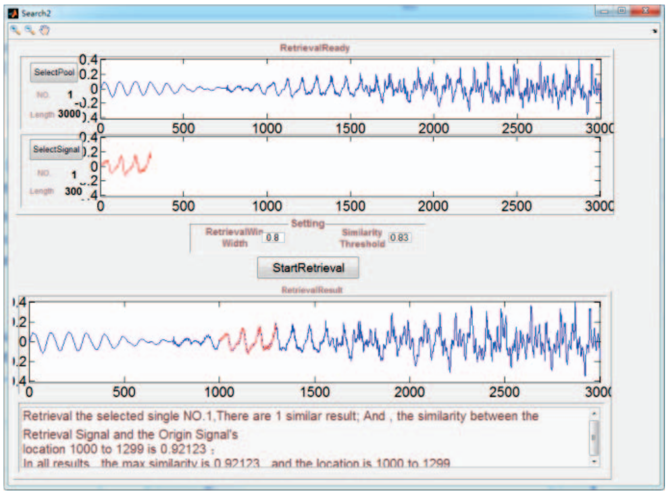

FIGURE 5. The retrieval result using real signal as sub-pattern

5. Conclusions

By introducing K times of one-order and two-order total deviation, constructing augmented lagrange function, the CS reconstruction problem for original signal can be transformed to the unconstrained optimization problem. With the segment converting idea from adaptive matching pursuit algorithm, it uses the sparse coefficients during iteration to crop adaptively and proposes the GDSAMP algorithm. It outputs the original reconstruction signal and the size of signal sparsity. From the results, it can find that the algorithm efficiency and reconstruction accuracy have been improved. By introducing GDSAMP algorithm into sub-pattern signal recognition and location on large signal sets, it obtains better performance. By further research, it will explore a new idea for CS theory in signal retrieval, image recognition and computer vision.

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References

[1] Donoho D, "Compressed sensing", IEEE Transactions on Information Theory, Vol.52, No.4, pp.1289-1306, Apr. 2006.

[2] Candes E,Wakin B, "An introduction to compressive sampling", IEEE Signal Processing Magazine, Vol.25, No.2, pp.21-30, Feb. 2008.

[3] T.Wimalajeewa, H.Chen, and P.K.Varshney, "Performance Limits of Compressive Sensing-Based Signal Classification", IEEE Trans. on Signal Processing, Vol.60, No.6, pp.2758-2770, Jan. 2012.

[4] Mahdi Cheraghchi, Venkatesan Guruswami, and Ameya Velingker, "Restricted Isometry of Fourier Matrices and List Decodability of Random Linear Codes", In Proceedings of the ACM-SIAM Symposium on Discrete Algorithms (SODA), Vol.42, No.5, pp.1888-1914, Oct. 2013.

[5] Zhiqiang Xu, "Compressed sensing: a survey" (in Chinese), Science China:Math, Vol.42, No.9, pp.865-877, Aug. 2012.

[6] M Akcakaya, V Tarokh, "A frame construction and a universal distortion bound for sparse representations", IEEE Transactions on Signal Processing, Vol.56, No.6, pp.2443-2450, Dec. 2008.

[7] Li-Wei Kang,Chao-Yung Hsu,Hung-Wei Chen et al, "Feature Based Sparse Representation for Image Similarity Assessment", IEEE Transactions on Multimedia, Vol.13, No.5, pp.1019-1030, Mar. 2011.

[8] Ukash Nakarmi, Nazanin Rahnavard, "Compressive Sensing for Binary Sparse Signals", 2012 IEEE Military Communications Conference, Vol.13, No.5, pp.1-5, Mar. 2011.

[9] Borhan M. Sanandaji, Tyrone L. Vincent, and Michael B. Wakin, "Concentration of Measure Inequalities for Toeplitz Matrices with Applications", IEEE Trans. on Signal Processing, Vol.61, No.1, pp.109-117, Sep. 2013.

[10] Taner Ince, Arif Nacaroglu, and Nurdal Watsuji, "Nonconvex compressed sensing with partially known signal support", Signal Processing, Vol.93, No.1, pp.338-344, Apr. 2013.

[11] J. Wang, S. Kwon, adn B. Shim, "Generalized orthogonal matching pursuit", IEEE Trans. Signal Process, Vol.60, No.12, 6202-6216, Dec. 2012.

[12] Entao Liu, Vladimir N. Temlyakov, "The Orthogonal Super Greedy Algorithm and Applications in Compressed Sensing", IEEE Trans. on Information Theory, Vol.58, No.4, pp.2040-2047, Jan. 2012.

[13] Wei Dai, Olgica Milenkovic, "Subspace pursuit for compressive sensing signal reconstruction", IEEE Transactions on Information Theory, Vol.55, No.5, pp. 2230-2249, May. 2009.

[14] Blumensath T, Davies M E, "Iterative hard thresholding for compressed sensing", Applied and Computational Harmonic Analysis, Vol.27, No.3, pp.265-274, Mar. 2009.

[15] Thong T. Do, Lu Gan, Nam Nguyen et al., "Sparsity adaptice matching pursuit algorithm for practical compressed sensing", 2008 42nd Asilomar Conference on Signals, Systems and Computers, pp.581-587, Oct. 2008.

[16] Yan Zhou, Fanzhi Zeng, Huimin Zhao, et al, "An image retrieval method based on meticulous sparsity adaptive matching pursuit algorithm", Chinese Journal of Electronics, Vol.42, No.12, pp.2457-2466, Jul. 2014.

[17] Gao Chen, Defang Li, and Jiashu Zhang, "Iterative gradient projection algorithm for two-dimensional compressed sensing sparse image reconstruction", Signal Processing, Vol.104, pp.15-26, Nov. 2014.

[18] Xiaona Wang, "The research of typical communication signal modulation recognition technology", University of Electronic Science and Technology, 2011.

[19] Nandi A K, Azzouz E E, "Algorithms for automatic recognition of communications", IEEE Transactions on Signal Processing, Vol.46, No.4, pp.431-436, Jun. 1998.