Annual Performance Review 2024: Mark Simmons

by
Mark Simmons

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College of Natural Sciences and Mathematics,

University of Houston.

Chair of Committee: Loic Cappanera

Committee Member: TBD

Committee Member: TBD

Committee Member: TBD

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ABSTRACT

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Chapter 1

Introduction

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1.1 Context and motivations

Modeling three-phase flow has important application in petroleum engineering, and environmental science. Accurately capturing physical outcomes from fluids such as oil, gas, and water in an underground reservoir is crucial for the oil and gas industry. The following discussion and analysis of numerical solutions provide a robust model to solve these coupled non-linear partial differential equations in a way that accurately describes physical phenomenon we can observe.

1.2 APR Outline

This APR includes 4 chapters. Chapter 1 provides a brief overview of the model problem and equations on the 3-phase 3-component black oil model. Chapter 2 gives information and detail surrounding time discretization and a new stabilization method proof. Chapter 3 will give details surrounding space discretization. Chapter 4 will give convergence results for a test on manufactured solutions and simulation results on a 2-D domain. Finally, a conclusion including a summary of the APR and details surrounding future work.

Chapter 2

Model problem and Equations

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Chapter 3 Title

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3.1 Chapter 3 - Section 1

3.1.1 Quarter of Five Spot Problem

The domain is $\Omega = [0,100]^2 \backslash \{[0,5]^2 \cup [95,100]^2\}\, m^2$, i.e., it is the square $[0,100]^2\ m^2$ where two corners have been cut out; see following figure. This is me adding stuff to check that everything is working well. Let me run it again and see if it still takes that long to run. It doesn't! I think we are good to go!

The boundary conditions are

$$\text{Inflow on } \Gamma_1 : p_\ell = 3 \times 10^5\ Pa,\ s_a = 0.7$$

$$\text{Outflow on } \Gamma_4 : p_\ell = 10^5\, Pa,\ \lambda_a \kappa \nabla p_c \cdot \mathbf{n} = 0$$

$$\text{No-Flow on } \Gamma_2 \cup \Gamma_3 \cup \Gamma_5 \cup \Gamma_6 : \lambda_\ell \kappa \nabla p_\ell \cdot \mathbf{n} = 0,\ \lambda_a \kappa \nabla p_c \cdot \mathbf{n} = 0.$$

The initial conditions are $s_a = 0.2$ and $p_\ell = 10^5\ Pa$. The rest of the flow properties are given by:

$$\phi = 0.2, \quad \mu_\ell = 2\times 10^{-3}\, Pa\cdot s, \quad \mu_a = 5\times 10^{-4}\, Pa\cdot s.$$

The relative permeability for each phase and capillary pressure follow a Brooks-Corey model:

$$\kappa_\ell = (1-s_a)^2(1-s_a^{5/3}), \quad \kappa_a = s_a^{11/3}, \quad p_c = 5\times 10^3 s_a^{-1/3}$$

We consider a heterogeneous medium where $\kappa = 5\times 10^{-8}\, m^2$ everywhere, except for in the square $[25, 50]\times[25, 50]$ where κ is 1,000 times lower.

3.1.2 Numerical Results

Δt	DOF's	p_ℓ		S_a		S_v	
		L^2 error	Rate	L^2 error	Rate	L^2 error	Rate
0.25	64	2.20E-2	-	8.81E-3	-	3.58E-2	-
0.125	256	5.70E-3	1.95	4.41E-3	0.99	1.15E-2	1.64
0.0625	1024	1.31E-3	2.12	2.24E-3	0.98	5.55E-3	1.06
0.03125	4096	4.43E-4	1.56	1.12E-3	0.96	2.56E-3	1.11
0.015625	16384	1.67E-4	1.41	5.80E-4	0.99	1.26E-3	1.03

Table 3.1: rates of convergence for $\mathbb{Q}_1 - \mathbb{RT}_1-$ BDF1

Δt	DOF's	p_ℓ		S_a		S_v	
		L^2 error	Rate	L^2 error	Rate	L^2 error	Rate
0.25	64	2.11E-2	-	8.44E-3	-	4.08E-2	-
0.0625	256	5.55E-3	1.94	2.20E-3	1.94	5.08E-3	3.01
0.015625	1024	1.38E-3	1.99	5.55E-4	1.98	1.17E-3	2.12
0.003906	4096	3.44E-4	2.01	1.39E-4	2.00	2.89E-4	2.02
0.000976	16384	8.71E-5	1.98	3.47E-5	2.00	7.21E-5	2.00

Table 3.2: rates of convergence for $\mathbb{Q}_1 - \mathbb{RT}_1-$ BDF1 where $\Delta t = h^2$

Δt	DOF's	p_ℓ		S_a		S_v	
		L^2 error	Rate	L^2 error	Rate	L^2 error	Rate
0.25	144	3.94E-3	-	8.62E-3	-	3.43E-2	-
0.125	576	3.14E-3	0.32	4.41E-3	0.97	1.12E-2	1.55
0.0625	2304	6.22E-4	2.34	2.25E-3	0.97	5.54E-3	1.08
0.03125	9216	3.21E-4	0.96	1.15E-3	0.94	2.57E-3	1.11
0.015625	36864	1.49E-4	1.10	5.81E-5	0.99	1.26E-3	1.03

Table 3.3: rates of convergence for $\mathbb{Q}_2 - \mathbb{RT}_2-$ BDF1

Δt	DOF's	p_ℓ		S_a		S_v	
		L^2 error	Rate	L^2 error	Rate	L^2 error	Rate
0.25	144	3.61E-3	-	8.62E-3	-	3.43E-2	-
0.0625	576	7.33E-4	2.30	4.41E-3	1.83	1.12E-2	2.96
0.015625	2304	1.81E-4	2.02	2.25E-3	1.96	5.54E-3	2.11
0.00390625	9216	4.35E-5	2.06	1.15E-3	1.99	2.57E-3	2.02
0.000976	36384	1.08E-5	2.01	3.76E-5	2.00	7.16E-5	2.01

Table 3.4: rates of convergence for $\mathbb{Q}_2 - \mathbb{RT}_2-$ BDF1 where $\Delta t = h^2$

Δt	DOF's	p_ℓ		S_a		S_v	
		L^2 error	Rate	L^2 error	Rate	L^2 error	Rate
0.25	64	3.61E-3	-	8.62E-3	-	3.43E-2	-
0.0625	256	7.33E-4	2.30	4.41E-3	1.83	1.12E-2	2.96
0.015625	1024	1.81E-4	2.02	2.25E-3	1.96	5.54E-3	2.11
0.00390625	4096	4.35E-5	2.06	1.15E-3	1.99	2.57E-3	2.02
0.000976	16384	1.08E-5	2.01	3.76E-5	2.00	7.16E-5	2.01

Table 3.5: rates of convergence for $\mathbb{Q}_1 - \mathbb{RT}_1-$ BDF1 STAB METHOD

Δt	DOF's	p_ℓ		S_a		S_v	
		L^2 error	Rate	L^2 error	Rate	L^2 error	Rate
0.25	64	2.38E-2	-	9.23E-3	-	2.39E-2	-
0.0625	256	5.52E-3	2.11	2.37E-3	1.96	5.27E-3	2.18
0.015625	1024	1.39E-3	1.99	6.04E-4	1.97	1.19E-3	2.15
0.00390625	4096	3.50E-4	1.99	1.51E-4	2.00	2.91E-4	2.03
0.000976	16384	8.79E-5	2.00	3.78E-5	2.00	7.25E-5	2.00

Table 3.6: rates of convergence for $\mathbb{Q}_1 - \mathbb{RT}_1-$ BDF1 where $\Delta t = h^2$ STAB METHOD

Δt	DOF's	p_ℓ		S_a		S_v	
		L^2 error	Rate	L^2 error	Rate	L^2 error	Rate
0.25	144	4.72E-3	-	9.11E-3	-	2.22E-2	-
0.0625	576	1.88E-3	1.33	4.69E-3	0.96	1.27E-2	0.80
0.015625	2304	9.48E-4	0.99	2.43E-3	0.94	5.31E-3	1.26
0.00390625	9216	4.40E-4	1.11	1.24E-3	0.97	2.50E-3	1.09
0.000976	36384	2.08E-4	1.08	6.27E-4	0.99	1.12E-3	1.04

Table 3.7: rates of convergence for $\mathbb{Q}_2 - \mathbb{RT}_2-$ BDF1 STAB METHOD

Δt	DOF's	p_ℓ		S_a		S_v	
		L^2 error	Rate	L^2 error	Rate	L^2 error	Rate
0.25	144	3.61E-3	-	8.62E-3	-	3.43E-2	-
0.0625	576	7.33E-4	2.30	4.41E-3	1.83	1.12E-2	2.96
0.015625	2304	1.81E-4	2.02	2.25E-3	1.96	5.54E-3	2.11
0.00390625	9216	4.35E-5	2.06	1.15E-3	1.99	2.57E-3	2.02
0.000976	36384	1.08E-5	2.01	3.76E-5	2.00	7.16E-5	2.01

Table 3.8: rates of convergence for $\mathbb{Q}_2 - \mathbb{RT}_2-$ BDF1 where $\Delta t = h^2$ STAB METHOD

Δt	DOF's	p_ℓ		S_a		S_v	
		L^2 error	Rate	L^2 error	Rate	L^2 error	Rate
0.25	144	4.72E-3	-	9.11E-3	-	2.22E-2	-
0.0625	576	4.42E-4	3.42	1.12E-3	2.87	2.51E-3	3.15
0.015625	2304	5.03E-5	3.13	1.58E-4	2.98	2.97E-4	3.08
0.00390625	9216	6.29E-6	3.00	1.98E-5	3.00	3.68E-5	3.01

Table 3.9: rates of convergence for $\mathbb{Q}_2 - \mathbb{RT}_2-$ BDF1 where $\Delta t = h^3$ STAB METHOD

Chapter 4

Conclusion

4.1 Outcome

4.2 Outlook