Task 3.

Show that $\frac{n^2}{\lg n} = o(n^2)$

Definition of $o(n)$:

$$f(n) = o(g(n)) \text{ if } \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$$

Proof:

for $\frac{n^2}{\lg n} = o(n^2) \Rightarrow \lim_{n\to\infty} \frac{\frac{n^2}{\lg n}}{n^2} = 0$

$$\lim_{n\to\infty} \frac{\frac{n^2}{\lg n}}{n^2} = 0 \Rightarrow \lim_{n\to\infty} \frac{n^2}{\lg n} \cdot \frac{1}{n^2} \Rightarrow \lim_{n\to\infty} \frac{1}{\lg n} = 0$$

When the limit $\lim_{n\to\infty} \frac{1}{\lg n} = 0$ as $n \to \infty$
the answer becomes 0 and therefor:

$$\frac{n^2}{\lg n} = o(n^2)$$

Show that $n^2 \neq o(n^2)$

Following the same definition

$$f(n) = o(g(n)) \text{ if } \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$$

If $n^2 = o(n^2)$ then $\lim_{n\to} \frac{n^2}{n^2} = 0$, however $\lim_{n\to} \frac{n^2}{n^2} = 1$
Therefor $n^2 = o(n^2)$ cannot be true, therefor $n^2 \neq o(n^2)$