Task 4 Divide and Conquer Analysis

when $T(n) = aT(\frac{n}{b}) + f(n)$

and R1: $T(n) = 16T(\frac{n}{4}) + n$

Then $a = 16$ and $b = 4$ and $f(n) = n$

Comparing with Case 1:

if $f(n) = O(n^{\log_b a - \varepsilon})$ for Som $\epsilon > 0$

and assming/testing $\epsilon = 1$ we get:

$$f(n) = O(\underbrace{n^{\log_4 16}}_{n^2}{}^{-\epsilon}) \Rightarrow f(n) = O(n^{2-1})$$

$$\Rightarrow f(n) = O(n)$$

This shows that Case 1 would work. then:

$$T(n) = \Theta(n^{\log_b a}) = \Theta(n^2)$$

Task 4 (or maybe should be 5)

with $T(n) = 4T(n/2) + n$

Hypostheses 1:

1. $T(n) \leq c \cdot n^2$ where $c > 0$ [upper bound]

with: $T(n/2) \leq c \cdot (n/2)^2$

replacing $T(n/2) \Longrightarrow T(n) = 4T(n/2) + n$

$$\leq 4(c(n/2)^2) + n$$

$$\leq 4(c \cdot (n^2/4)) + n$$

$*c$

$\hookrightarrow 4(c(n^2/2)) = c \cdot n^2/4$

$*4$

$\hookrightarrow 4(cn^2/4) = cn^2$

This is false for $\leftarrow T(n) \leq cn^2 + n$

1. Since $\underbrace{c \cdot n^2 \leq cn^2 + n}_{\text{false}}$

Task 4 (or 5)

Hypothesis 2:

$T(n) \geq cn^2$ where $c>0$ [lower bound]

$$T(n) = 4.(T(n/2) + 2$$

and $T(n/2) \geq c(n/2)^2$

$$\Longrightarrow T(n) \geq \underbrace{4(c(n/2)^2)} + n$$

$$T(n) \geq cn^2 + n$$

OR simply

$$\underbrace{T(n) \geq cn^2}_{\text{True}}$$

True and hypothesis 2 works

Hypothesis 3:

$T(n) \leq (cn^2 - b.n)$ where $c>0$ and $b>0$

and

$$T(n/2) = c(n/2)^2$$

$$\Longrightarrow T(n) = 4T(n/2) + n$$

$$\leq 4(c(n/2)^2 - b(n/2)) + n$$

$$= 4\left(\frac{cn^2}{4} - \frac{bn}{2}\right) + n$$

$$= cn^2 - 2bn + n$$

$$= cn^2 - (2b-1)n.$$

$$T(n) \leq cn^2 - (2b-1)n \leq (cn^2 - bn)$$

Since we can $n = 1$ for proofing:

$$c1^2 - (2b-1) \cdot 1 \leq c1^2 - b \cdot 1)$$

Removing Constant.

$$\Rightarrow -(2b-1) \leq -b$$

$$\Rightarrow 2b - 1 \geq b$$

$$\xRightarrow{+b} b \geq 1$$

meaning as long as $b \geq 1$

$$T(n) \leq cn^2 - (2b-1)n \leq cn^2 - bn$$

is valid and hypothesis 3 also works