# Task 4:

## Master Theorem

$$T(n) = aT(n/b) + f(n^c)$$
$$\lg_b a \Rightarrow \lg_2 3 \approx 1,58$$

Comparing $f(n) = \Theta(n)$ and $\Theta(n^{\lg_2 3})$

$$n = O(n^{\lg_2 3})$$

since $n^{\lg_2 3}$ grows faster then $n$ we can apply case 1

$f(n) = O(n^c)$ $c=1$ $\lg_2 3 \approx 1,58$, Time complexity is dominated by $\lg_2 3$ :

$$\underline{\underline{T(n) = \Theta(n^{\lg_2 3})}}$$

## Recursion free method:

Expanding $T(n) = 3T(n/2) + \Theta(n)$ where each problem is being split into 3 subproblems at size $n/2$ and adding the cost of $\Theta(n)$, we can generalize at level $i$ as:

$$3^i \cdot \Theta(n/2^i) = \Theta(3^i \cdot n/2^i)$$

Recursion ends at $n/2^i = 1 \rightarrow 2^i = 1 \rightarrow i = \lg_2 n$

Total work:

$$\sum_{i=0}^{\lg_2 n} \Theta(3^i \cdot n/2^i) = \Theta\left(n \sum_{i=0}^{\lg_2 n} (3/2)^i\right)$$

$$S = \sum_{i=0}^{\lg_2 n} (3/2)^i \qquad S_m = \frac{r^{m+1} - 1}{r - 1}$$

$$S = \frac{(3/2)^{\lg_2 n + 1} - 1}{(3/2) - 1} \rightarrow S \approx \Theta((3/2)^{\lg_2 n})$$

$$(3/2)^{\lg_2 n} = n^{\lg_2 (3/2)}$$

$$\lg_2(3/2) = \lg_2 3 - \lg_2 2 = \lg_2 3 - 1$$

$$n^{\lg_2 3 - 1} = \frac{n^{\lg_2 3}}{n} = n^{\lg_2 3}$$

Simplifies as its the dominant term

$n^{\lg_2 3}$ is the dominant term confirming the Master theorem