# Task 3:

Show that for any real constants a and b where $b > 0$, $(n+a)^b = \Theta(n^b)$:

Definition of $\Theta(g(n))$

$f(n) = \Theta(g(n))$ if the positive constants $c_1, c_2$ and $n_0$ exists, such that

$$c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0$$

Upper bound: $(n+a)^b = O(n^b)$

Lower bound: $(n+a)^b = \Omega(n^b)$

$(n+a)^b = O(n^b)$:

$$(n+a)^b = n^b\left(1+\frac{a}{n}\right)^b \qquad \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^b = 1 \rightarrow \left(1+\frac{a}{n}\right)^b \leq c$$

Therefore for a large enough n:

$$(n+a)^b = n^b \cdot \left(1+\frac{a}{n}\right)^b \leq c \cdot n^b$$

$$(n+a)^b = O(n^b)$$

$(n+a)^b = \Omega(n^b)$:

$(n+a)^b = n^b\left(1+\frac{a}{n}\right)^b$ when n approaches $\infty$ $\frac{a}{n} \to 0$ for this we can substitute $\frac{a}{n}$ with x and apply taylor expansion

$(1+x)^b \approx 1+bx$ putting back $\frac{a}{n}$ we get $1+\frac{ba}{n}$

$$\left(1+\frac{a}{n}\right)^b \geq 1+\frac{ba}{n}$$

$$(n+a)^b = n^b\left(1+\frac{a}{n}\right)^b \geq n^b\left(1+\frac{ba}{n}\right)$$

$$(n+a)^b \geq n^b\left(1+\frac{ba}{n}\right)$$

$$(n+a)^b \geq n^b + ban^{b-1}$$

For large n value, $n^b$ dominates $n^{b-1}$

$(n+a)^b \geq c \cdot n^b$ for some $c > 0$ and a large enough n