First Sign of Congruency of Triangles

Theorem 1 *If two sides and the angle between these sides of one triangle are equal to two sides and the angle between them of another triangle, then these triangles are congruent.*

Proof

Consider two triangles ABC and $A_1B_1C_1$, where $AB = A_1B_1$, $AC = A_1C_1$ and angles A and A_1 are equal.

Since $\angle A = \angle A_1$, then we can overlay triangle ABC onto triangle $A_1B_1C_1$ such that vertices A and A_1 coincide and sides AB and AC will be on top of rays A_1B_1 and A_1C1 respectively. Since $AB = A_1B_1$ and $AC = A_1C1$, then side AB will coincide with side A_1B1 and side AC will coincide with side A_1C_1 (Axioms 8,10). Points B and B_1 and C and C_1 will coincide. This means that sides BC and B_1C_1 will coincide (Axiom 7).

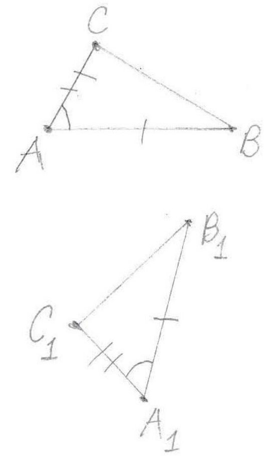

Figure 1

Now we need to prove that all points inside of the triangle ABC will overlay all points of triangle $A_1B_1C_1$. Let's take a random point M inside triangle ABC. Draw a line segment PQ through point M such that its ends are on sides AB and AC of $\triangle ABC$. Since side AB overlays side A_1B_1 then point P will overlay some point P_1 on side A_1B_1. In the same fashion, point Q will overlay some point Q_1 on side A_1C_1. By axiom #7, line segment PQ will overlay some line segment P_1Q_1, and hence point M of line segment PQ will overlay some point M_1 of line segment P_1Q_1.

The same way we can prove that any interior point of triangle $A_1B_1C_1$ will overlay with any interior point of ABC meaning, that all interior points of both triangles overlay only with points of the other triangle. Therefore triangles ABC and $A_1B_1C_1$ will completely overlay each other, meaning that they are congruent.

$\therefore$ QED

* Note: The proof that all interior points of both triangles overlay each other is identical for all other signs of congruency of triangles.