

Second Sign of Similarity of Triangles

Theorem

If two sides of one triangle are proportional to two sides of another triangle, and the angles between these sides are equal, then these two triangles are similar.

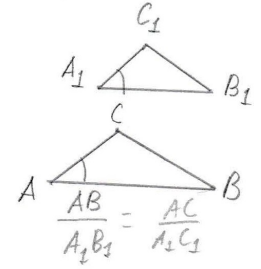


Figure 1

Proof

Consider two triangles ABC and $A_1B_1C_1$, where $\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1}$ and $\angle A = \angle A_1$ (Figure 1).

Let's prove that $\triangle ABC \sim \triangle A_1B_1C_1$. Considering the First Sign of Similarity of Triangles, it is enough to prove that $\angle B = \angle B_1$.

Let's look at triangle ABC_2 , where $\angle 1 = \angle A_1$, $\angle 2 = \angle B_1$ (Figure 2).

Triangles ABC_2 and $A_1B_1C_1$ are similar by the First Sign of Similarity of Triangles hence:
 $\frac{AB}{A_1B_1} = \frac{AC_2}{A_1C_1}$.

On the other hand, we are given that: $\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1}$.

From these two equations we get: $AC = AC_2$

Triangles ABC and ABC_2 are equal due to Side Angle Side (AB - common side, since $\angle A = \angle A_1$ and $\angle 1 = \angle A_1$).

We get that: $\angle B = \angle 2$. Since $\angle 2 = \angle B_1$ then: $\angle B = \angle B_1$

$\therefore QED$

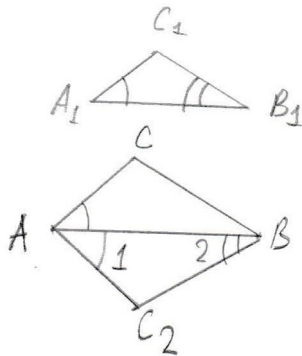


Figure 2