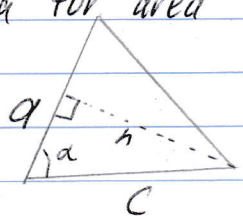
Compound & double angle formulas.

Compound angle formula for sin function.

First we need a bit of supplementary information.
Consider an arbitrary triangle with two given side lengths a and c and a given angle α between these two sides.
We need to come up with a formula for area using given information.

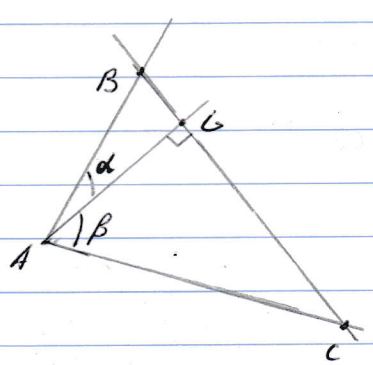
$S = \frac{1}{2} b \cdot h$



$h = c \cdot \sin\alpha$

$b = a \Rightarrow S = \frac{1}{2} a \cdot c \cdot \sin\alpha$

Now let's derive a compound angle formula for the function of sin
Consider the following drawing:



$S_{ABC} = S_{ABG} + S_{AGC}$

Using formula from above

$S_{ABC} = \frac{1}{2} \cdot AB \cdot AC \cdot \sin(\alpha+\beta)$

$S_{ABG} = \frac{1}{2} \cdot AB \cdot AG \cdot \sin\alpha$

$S_{AGC} = \frac{1}{2} \cdot AG \cdot AC \cdot \sin\beta$

$\frac{1}{2} AB \cdot AC \cdot \sin(\alpha+\beta) = \frac{1}{2} \cdot AB \cdot AG \cdot \sin\alpha + \frac{1}{2} \cdot AG \cdot AC \cdot \sin\beta$

Divide both sides by $AB \cdot AC$

$\sin(\alpha+\beta) = \frac{AB \cdot AG}{AB \cdot AC} \cdot \sin\alpha + \frac{AG \cdot AC}{AB \cdot AC} \cdot \sin\beta$

$\sin(\alpha+\beta) = \frac{AG}{AC} \cdot \sin\alpha + \frac{AG}{AB} \cdot \sin\beta$

$\sin(\alpha+\beta) = \cos\beta \cdot \sin\alpha + \cos\alpha \cdot \sin\beta$