Distance from a Point to a Plane

Given the coordinates of a point and the equation of the plane, find the distance from that point to the plane.

Solution

Let $M_0\left(x_0; \quad y_0; \quad z_0\right)$ be the given point and $ax + by + cz + d = 0$ be the equation of the given plane α. $M_1\left(x_1; \quad y_1; \quad z_1\right)$ is the projection of point M_0 on plane α. Since point M_1 is in plane α then its coordinates satisfy the equation of this plane:

$ax_1 + by_1 + cz_1 + d = 0$

Vector $\overrightarrow{M_0M_1}$(if $\overrightarrow{M_0M_1} \neq 0$), as well as vector $\vec{n}\left(a; \quad b; \quad c\right)$ is perpendicular to plane α, and hence: $\overrightarrow{M_0M_1} \parallel \vec{n}$. This means that there is some number k such that $\overrightarrow{M_0M_1} = k\vec{n}$. Let's write this down using coordinates:

$x_1 - x_0 = ka$
$y_1 - y_0 = kb$
$z_1 - z_0 = kc$

Now let's also note that the length l we are trying to find is the length of vector $M_0\vec{M}_1$ which is equal to :

$\sqrt{(x_1 - x_0^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$

Using the three equations from above we get:

$l = |k|\sqrt{a^2 + b^2 + c^2}$

Let's now express coordinates of point M_1 from those three equations and substitute them into the equation of plane α.

$a(ka + x_0) + b(kb + y_0) + c(kc + z_0) + d = 0$

Solve for k to get:

$k = -\dfrac{ax_0 + by_0 + cz_0 + d}{a^2 + b^2 + c^2}$

Substitute the line above into the equation for length l and get:

$l = \dfrac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$