1 Review

Definition 1 *Let $X(x)$ be a vector field on $\mathbb{R}^n$. The flow along X is a curve $\varphi : \mathbb{R} \to \mathbb{R}^n$ such that:*

$$\dot{\varphi}(t) = X(\varphi(t)) \tag{1}$$

2 Suggested Exercise

I) **Embedded Circle and ODEs**

   i) Consider the function

$$f : \mathbb{R}^2 \to R$$
$$f(x, y) = x^2 + y^2 - 1$$

   Compute the gradient

   ii) Define a smooth vector field $X(x)$ such that $\langle X(x), \mathrm{grad}(f)_x \rangle = 0 \quad \forall x$ (i.e. it is always perpendicular to the gradient).

   iii) Plot the vector field X in python or julia.

   iv) Write down the differential equation for the flow of X.

   v) Find a solution given some initial condition.

   vi) Use this to give the unit circle S^1 a manifold structure.