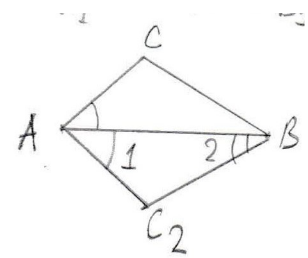
Third Sign of Similarity of Triangles

Theorem

If three sides of one triangle are proportional to three sides of another triangle, then these triangles are similar.



Proof

Let the sides of triangles ABC and $A_1B_1C_1$ be proportional.

$$\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CA}{C_1A_1} \qquad (1)$$

Let's prove that $\triangle ABC \sim A_1B_1C_1$.
Considering the Second Sign of Similarity of Triangles, we just need to prove that $\angle A = \angle A_1$.

Consider the triangles ABC_2, where $\angle 1 = \angle A_1$, $\angle 2 = \angle B_1$. Triangles ABC_2 and $A_1B_1C_1$ are similar due to the First Sign of Similarity of Triangles. Hence:

$$\frac{AB}{A_1B_1} = \frac{BC_2}{B_1C_1} = \frac{C_2A}{C_1A_1} \qquad (2)$$

Equating (1) and (2) we get: $BC = BC_2$, $CA = C_2A$.
Triangles ABC and ABC_2 are equal due to Side Side Side. Hence: $\angle A = \angle 1$.
Since $\angle 1 = \angle A_1$, then $\angle A = \angle A_1$

$\therefore QED$