Conservation of Momentum

Let's consider two objects colliding and try to describe this process mathematically. Recall Newton's third law:

Newton's Third Law. *Whenever one body exerts a force on a second body,the first body experiences a force that is equal in magnitude and opposite in the direction to the force it exerts. (For every action, there is an equal and opposite reaction).*

During a collision, object 1 will exert some force $\vec{F}_1$ on object 2 (Figure 3). From Newton's third law, we know that object (2) will exert some force $\vec{F}_2$ on object (1) such that:

$$\vec{F}_1 = -\vec{F}_2 \tag{1}$$

Recall Newton's Second law:

$$\vec{F} = m\vec{a} \tag{2}$$

Using the definition of acceleration rewrite it as:

$$\begin{aligned} F &= m\frac{\Delta\vec{v}}{\Delta t} \\ &= m\frac{\vec{v}_a - \vec{v}_b}{\Delta t} \end{aligned} \tag{3}$$

Where:

$\vec{v}_b$ and $\vec{v}_a$ are the velocities of the object before and after some force F was applied.
Δt - time interval over which force was applied (collision happens).

*Note that we assume the object's mass does not change after the collision. If a piece were to "break off", we would have to consider the change in velocity of that piece separately.

In our case of a collision between two objects, let:

$\vec{v}_{1b}$ and $\vec{v}_{2b}$ be the velocities of objects 1 and 2 respectively before the collision.
$\vec{v}_{1a}$ and v_{2a} be the velocities of objects 1 and 2 respectively after the collision

Substitute these into equation (3) to get:

$$F_1 = m_1\frac{\vec{v}_{2a} - \vec{v}_{2b}}{\Delta t}$$

$$F_2 = m_2\frac{\vec{v}_{1a} - \vec{v}_{1b}}{\Delta t}$$

Where:

$\vec{F}_1 \left(\vec{F}_2\right)$ - force object 1 (2) applies on object 2 (1) during collision.

Substitute these equations above into Newtons Third Law equation (1)and rearrange to get:
$F_1 = -F_2$

$m_1 \dfrac{\vec{v}_{1a} - \vec{v}_{1b}}{\Delta t} = -m_2 \dfrac{\vec{v}_{2a} - \vec{v}_{2b}}{\Delta t}$

$m_1(\vec{v}_{1a} - \vec{v}_{1b}) = -m_2(\vec{v}_{2a} - \vec{v}_{2b})$

$$m_1\vec{v}_{1b} + m_2\vec{v}_{2b} = m_1\vec{v}_{1a} + m_2\vec{v}_{2a} \tag{4}$$

Recall that we defined the momentum of an object as the product of its velocity and mass: $p = mv$. Equation (4) can then be written in terms of momentum.

$$\vec{p}_{1b} + \vec{p}_{2b} = \vec{p}_{1a} + \vec{p}_{2a} \tag{5}$$

Where:
p_{1b} and p_{2b} are the momenta of objects (1) and (2) respectively before the collision.
p_{1a} and p_{2a} are the momenta of objects (1) and (2) respectively after the collision.

This is know as the law of conservation of momentum, since we can see that the momenta of objects before the collision is the same as the momenta of objects after collision.

Experiment

Let's verify this "theory" to be true experimentally for a 1 dimensional case.

We will use the following setup.
Initially object (2) is at rest ($\vec{v}_{2b} = 0$). Object 1 has some initial speed $\vec{v}_{1b}$. It then collides into object (2), giving it some speed $\vec{v}_{2a}$. Object 1, continue with some new speed $\vec{v}_{1a}$.

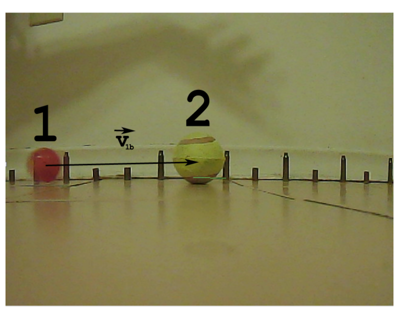

Figure 1: Object 1 collides into object 2 at initial velocity v_{1b}

From equation (4) we can use the sum of the momenta before the collision, to predict the sum of momenta after the collision.

We recorded that:

$m_1 = 0.052$ kg
$m_2 = 0.056$ kg

In order to measure velocities before and after the collision, we set up markers spread 5 cm = 0.05 m apart.

Before the collision, object 1 passed 2 of these markers (0.1 m) in $\Delta t = 0.034$ s. Then:

$$\begin{aligned} v_{1b} &= \frac{0.1}{0.034} \\ &= 2.94118\,\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

Recall the definition of momentum and get:

$$\begin{aligned} p_{1b} &= m_1 v_1 \\ &= 0.052 \cdot 2.94118 \\ &= 0.152941\,\mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

Object 2 was initially at rest. Therefore:

$p_{2b} = 0$

Therefore from equation 4 we expect that:
$p_{1b} = p_{1a} + p_{2a}$
$p_{1a} + p_{2a} = 0.155882\,\mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}}$
This is our theoretical value.

After the collision, object (1) passed 1 marker(0.05 m) in 0.216 s and object (2) passed 2 markers (0.1 m) in 0.042 s. Then:

$$\begin{aligned} v_{1a} &= \frac{0.05}{0.216} \\ &= 0.231481\,\frac{\mathrm{m}}{\mathrm{s}} \\ p_{1a} &= 0.052 \cdot 0.23 \\ &= 0.012037\,\mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

And:

$$\begin{aligned} v_{2a} &= \frac{0.1}{0.042} \\ &= 2.38095\,\frac{\mathrm{m}}{\mathrm{s}} \\ p_{2a} &= 0.056 \cdot 2.38 \\ &= 0.133333\,\mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

Our actual sum of momenta after the collision is therefore:

$$
\begin{aligned}
p_{1a} + p_{2a} &= 0.012037 + 0.133333 \\
&= 0.14537\,\mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}
$$

Recall that our predicted theoretical value was:

$p_{1b} = 0.152941\,\mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}}$

Let's compare this predicted value to our experimental, actual value and calculate the percentage difference:

$$
\begin{aligned}
E &= \left|\frac{p_{theor} - p_{exp}}{p_{theor}}\right| \\
&= \left|\frac{0.152941 - 0.14537}{0.152941}\right| \\
&= 0.0495014 \\
&\approx 5\%
\end{aligned}
$$

The percentage difference between our theoretical and actual values is less than 5 %. This is a very small measure of inaccuracy proving our mathematical model to be correct.