0.1 Theories in Physics

Definition 1 *A theory is a set of statements derived from some initial guesses (initial premises).*

In the process of "establishing" a new theory a set of initial guesses are made. These are usually some "basic" truths about the world around us (E.G. you can always draw a straight line between two points). To verify this theory, the consequences of these initial premises are calculated; " if these guesses are true, what else would then be true?". To verify the theory, these secondary statements (consequences of a theory) are then compared to data obtained from experiment.

If these do not match, than the theory is wrong and its initial premises (the entire theory) must be changed. If they do, then the theory can be used to predict (model) this kind of process.

Let's look at an example:

0.2 Projectile Motion and Inclined Plane Experiment

0.2.1 Deriving the Model (consequences)

Consider a system where an object enters free fall with some initial velocity v_0. Let's use Newton's laws (initial premises) to build a theory that would effectively model this process and design an experiment to verify it.

First let's state our premises:

We will assume Newton's Second law:

$$\vec{F} = m\vec{a} \tag{1}$$

Where:

$\vec{F}$ - net force acting on object
m - mass of the object
$\vec{a}$ - the acceleration of the object

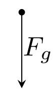
Also assume the gravitational force to be:

$$\vec{F}_g = mg \tag{2}$$

Where:

$\vec{F}_g$ - magnitude of force due to gravity acting on object
m - mass of object
g - constant of approximately 9.81 $\frac{\mathrm{m}}{\mathrm{s}^2}$

We have now have a set of premises from which to build the theory. Let's now derive its consequences.



Draw a free body diagram to find the net force.
There is only one force acting on the object. It only has a vertical (y axis component). Let's assume up to be positive and down to be negative. In that case:

$\vec{F}_{net} = \vec{F}_g$
$\vec{F}_g = m\vec{a}$
$F_y = ma_y = -mg$
$F_x = ma_x = 0$

From equations above get that:

$a_y = -g$
$a_x = 0$

Recall the following kinematic equations:

$$s_n = \frac{1}{2}a_n t^2 + v_0 t + s_{0n} \tag{3}$$

$$v_n = a_n t + v_{0n} \tag{4}$$

Where:
s_n - n coordinate of position of object at time t
n coordinate of position of object at time $t = 0$
v_n - n coordinate of velocity vector of object at time t.
v_{0n} - n coordinate of object's initial velocity at $t = 0$
a_n - n coordinate of acceleration vector of object at time t

Substitute the acceleration in a system of free fall into equations (3) and (4) to get:

$$\begin{cases} s_y = -\frac{1}{2}gt^2 + v_{0y}t + s_{0y} & \text{(5a)} \\ s_x = v_{0x}t + s_{0x} & \text{(5b)} \end{cases}$$

The relationships above are the "consequences" of our initial assumptions (1) and (2). Now, this must be compared with experiment.

0.2.2 Experiment

We now have to design an experiment which would be representative of the system described by equations (3) to verify our theory.

To use all of the relationships in (3), we must include motion in two dimensions (we can then verify equations both for x and y coordinates).

We should also be able to measure/calculate all possible variables directly from experiment. A mechanism must be designed to control the initial velocity $\vec{v}_0$.

This can be achieved with an inclined plane. If the ball begins at rest, it will accelerate and using the same kinematic equation (4) we can predict its velocity. Draw free body diagram. Recall that the object moves down on the ramp and from Figure 1 and equation (4) get the

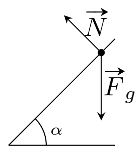


Figure 1: Introduce reference frame perpendicular to the inclined plane

following:

$F_x = mg\sin\alpha$
$F_y = 0$

Assuming that initially the object is at rest, substitute this into equations (1) and (4) to get:

$a_{rx} = g\sin\alpha$
$a_{ry} = 0$

$v_{rx} = g\sin\alpha t_r$
$v_{ry} = 0$

Where r is used to specify movement along the ramp.

From equations above get that:

$|\vec{v}_r| = g\sin\alpha\, t_r$

As the ball enters free fall after the ramp, $\vec{v}_0$ in equations (5) is equal to $\vec{v}_r$.

Combine both the ramp and free fall phase into one diagram.
Using figure (2), find the coordinates of the velocity vector $\vec{v}_0$ to be:

$v_{0x} = g\sin\alpha\cos\alpha\, t_r$
$v_{0y} = g\sin^2\alpha\, t_r$

Substitute this into equations (5) and rewrite to get:

$$\begin{cases} s_y = -\frac{1}{2}gt^2 + (g\sin^2\alpha\, t_r)t + 0 & \text{(6a)} \\ s_x = g\sin\alpha\cos\alpha\, t_r\, t + 0 & \text{(6b)} \end{cases}$$

Now, the equations above are expressed entirely in terms of things we can measure directly from experiment.

$s_{y/x}$ - position of object during free fall (measurable via camera) at time t after reaching

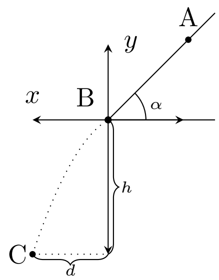


Figure 2: Object starts at rest at point A and rolls down ramp in left direction. At point B ramp ends and object enters free fall. At point C the object hits the ground. Reference frame introduced at point B.

point B (measurable via stopwatch or camera).
t_r - time it takes object to go from point A to B (roll down ramp). Measurable with camera.
α - angle ramp makes with ground (measurable with protractor).

We can now measure all values in equation (6) in experiment. We can substitute measured values in these formulas to predict other values "theoretically" and compare them to other experimental counterparts.

Let's look at an example:

The object reaches point C at some time t_f after falling off the ramp. Point C has coordinates $(d, -h)$.

We can then say that at t_f $s_y = -h$ and $s_x = d$ (left is positive).

Then rewrite equations 6 one more time:

$$\begin{cases} -h = -\frac{1}{2}gt_f^2 + (g\sin^2\alpha t_r)t_f & \text{(7a)} \\ d = g\sin\alpha\cos\alpha\, t_r t_f & \text{(7b)} \end{cases}$$

We can select any two variables to "predict" using the other ones.

Let's look at an example where we predict d and t_f and compare to experimental values. First we must express d and t_f in terms of h, α and t_r.

Using equation 7, we can find t_f:

$$\frac{1}{2}gt_f^2 - (g\sin^2\alpha\, t_r)t_f = h$$

Multiply both sides by $\frac{2}{g}$

$$t_f^2 - \frac{2}{\cancel{g}}(\cancel{g}\sin^2\alpha t_r)t_f = \frac{2h}{g}$$

Add 0 to the left hand side of the equation.

$$t_f^2 - 2(\sin^2\alpha\, t_r)t_f + \sin^4\alpha t_r^2 - \sin^4\alpha t_r^2 = \frac{2h}{g}$$

With some luck you may notice that:

$$\begin{aligned}(t_f - \sin^2\alpha\, t_r)^2 &= (t_f - \sin^2\alpha\, t_r)(t_f - \sin^2\alpha\, t_r)\\ &= t_f \cdot t_f - \sin^2\alpha t_r t_f - \sin^2\alpha t_r t_f + \sin^4\alpha t_r^2\\ &= t_f^2 - 2(\sin^2\alpha\, t_r)t_f + \sin^4\alpha t_r^2\end{aligned}$$

Substitute this into equation from earlier to get:

$$(t_f - \sin^2\alpha t_r)^2 - \sin^4\alpha t_r^2 = \frac{2h}{g}$$

$$(t_f - \sin^2\alpha t_r)^2 = \frac{2h}{g} + \sin^4\alpha t_r^2$$

Take the square root of both sides (remember that there are two possible answers):

$$t_f - \sin^2\alpha t_r \pm \sqrt{\frac{2h}{g} + \sin^4\alpha t_r^2}$$

$$t_f = \sin^2\alpha t_r \pm \sqrt{\frac{2h}{g} + \sin^4\alpha t_r^2}$$

We know that our solution for time must be a positive number. We will therefore dismiss one of the roots and state that:

$$t_f = \sin^2\alpha t_r + \sqrt{\frac{2h}{g} + \sin^4\alpha t_r^2} \tag{8}$$

We now have two ways of determining t_f. Directly by measuring it in experiment, or predicting it with other values in equation (8).
The experimental value is what is "actually happening". Equation (8) is a consequence of our theory. We can then compare the two values to evaluate the theory.

0.2.3 Experiment and Evaluation

Construct a setup equivalent to Figure (2):

Using logger pro, introduce reference frame, measure h and indicate points when ball reaches points A, B and C.

Get that:

$h = 0.4429$ m
$t_A = 6.83333$ s
$t_B = 7.33333$ s
$t_C = 7.6$ s
$\alpha = 30°$

Recall that: $g = 9.81$

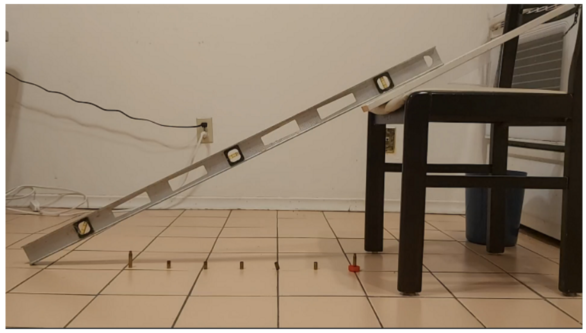


Figure 3: Bullet casings used for scale set 0.1 meters apart

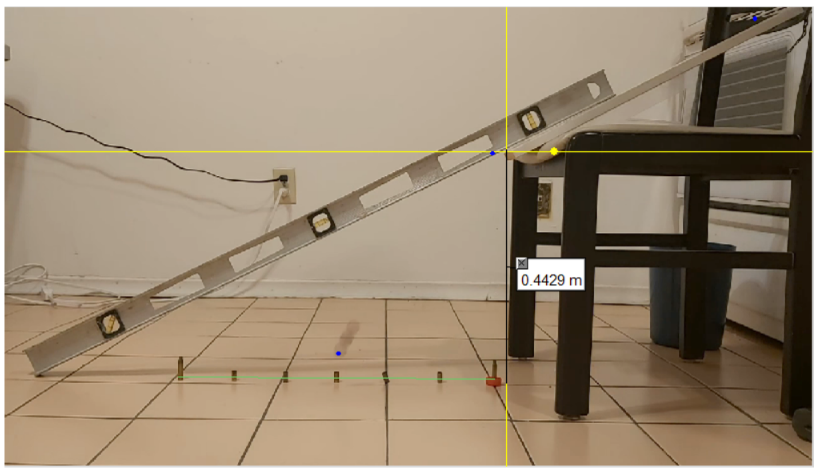


Figure 4

From times above calculate t_f and t_r:

$$
\begin{aligned}
t_r &= t_B - t_A \\
&= 7.33 - 6.83 \\
&= 0.50 \text{ s}
\end{aligned}
$$

$$
\begin{aligned}
t_f &= t_C - t_B \\
&= 7.66 - 7.33 \\
&= 0.26 \text{ s}
\end{aligned}
$$

Using values for h, g, α and t_r and equation (8) predict what t_f should be:

$$\begin{aligned} t_{fexp} &= \sin^2 \alpha t_r + \sqrt{\frac{2h}{g} + \sin^4 \alpha t_r^2} \\ &= \sin^2 30° \cdot 0.5 + \sqrt{\frac{2 \cdot 0.4429}{9.81} + \sin^4 30° \cdot 0.5^2} \\ &= 0.44\text{s} \end{aligned}$$

$t_{fexp} = 0.44$s does not match the experimental $t_f = 0.26$s. This suggests that our theory is incorrect and does not model our experiment. Two things are possible: either our theory (premises) are wrong or we messed up in experiment. Since similar experiments were done for hundreds of years, proving the aforementioned assumptions to be correct we know something If we were the first to ever do this experiment, we would however conclude that we are in fact incorrect. This shows the importance of extensive experimentation as part of establishing any scientific theory.

0.2.4 Extension

1. Using (7b) and t_{fexp} calculate the predicted distance d the ball will be from the chair when it hits the ground.
2. Assuming the bullet casings are 0.1 meters apart from Figure 4 find d experimental.
3. Find Percentage error of our experiment using the following formula

$$\text{Error} = \frac{|x_{exp} - x|}{x_{exp}}$$

Where:
Error - percentage error
x_{exp} - expected value
x - experimental value