Parallel Vectors

Theorem 1 *If some vector has coordinates* (a, b) *and another vector has coordinates* $(k \cdot a, k \cdot b)$*, then these vectors are colinear.*

Proof # 1

Consider $\vec{m} = (a, b)$ and $\vec{n} = (k \cdot a, k \cdot b)$. Let α be the angle that $\vec{m}$ makes with the x-axis, and β be the angle that $\vec{n}$ makes with the x-axis. Then:

Basic Trigonometric Ratios of α:
$\sin \alpha = \frac{a}{\sqrt{a^2+b^2}}$
$\cos \alpha = \frac{b}{\sqrt{a^2+b^2}}$

Basic Trigonometric Ratios of β:
$\sin \beta = \frac{k \cdot a}{\sqrt{(k \cdot a)^2+(k \cdot b)^2}} = \frac{k \cdot a}{k \cdot \sqrt{a^2+b^2}} = \frac{a}{\sqrt{a^2+b^2}} = \sin \alpha$
$\cos \beta = \frac{k \cdot b}{\sqrt{(k \cdot a)^2+(k \cdot b)^2}} = \frac{k \cdot b}{k \cdot \sqrt{a^2+b^2}} = \frac{b}{\sqrt{a^2+b^2}} = \cos \alpha$

Since the trigonometric ratios of both angles are the same, and by definition both vectors start at the origin. This means that the two vectors are on the same line.
$\therefore$ QED

Proof # 2

Consider $\vec{m} = (a, b)$ and $\vec{n} = (k \cdot a, k \cdot b)$. Since both vectors start at the origin, then they intersect at the origin and therefore must make an angle with each other. If the vectors are colinear, then they either make an angle of 0° or 180°. This means that cosine of the angle between the two vectors will be either equal to 1 or -1.

Using the two dot product formulas we get:
$\cos \alpha = \frac{a \cdot k \cdot a + b \cdot k \cdot b}{\sqrt{a^2+b^2} \cdot \sqrt{(k \cdot a)^2+(k \cdot b)^2}} = \frac{k \cdot (a^2+b^2)}{|k| \cdot \sqrt{a^2+b^2} \cdot \sqrt{a^2+b^2}} = \frac{k \cdot (a^2+b^2)}{|k| \cdot (a^2+b^2)} = \pm 1$

$\therefore$ QED