Double Slit Experiment

This handouts looks at the theoretical predictions for the famous Double Slit Experiment, which in the early 1800s demonstrated wave-like properties of light.

Setup

Consider a solid wall with two small slits separated by separated d, placed distance L away from a screen that can detect light (Figure 1).

A wave of light (with wavelength λ) comes in and hits the wall from the left, passing through the two slits. Assuming the slits are very small, we can claim that they act as point sources, projecting light on the screen on the right (Figure 1).

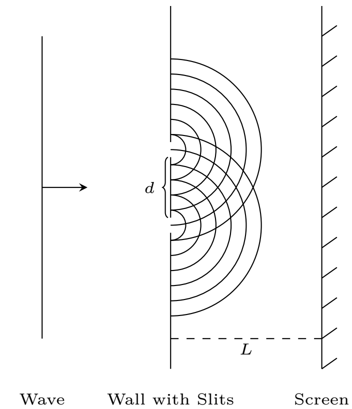

Figure 1: Wave comes in from the left, hitting two slits. Assuming the two slits are relatively small, they now act as “in phase” point sources with same wavelength as initial wave.

Model

Let’s study the light pattern that will be observed on the screen as a result of these two slits.

Recall that waves can interfere (see the overlaps of waves in Figure 1) and cancel each other out. We therefore expect to see some combination of dark and light spots on the screen.

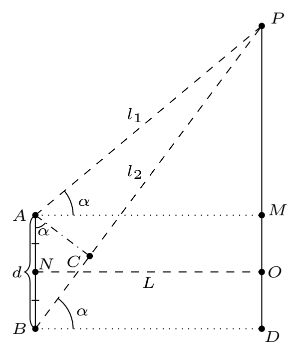

Figure 2: Light sources (slits) are located at A and B. Let P be a point of maximum constructive interference. Angles labeled using far-field approximation.

Now using the diagram above, let’s find distances OP (See Figure 2) at which a bright spot will be observed. First however, we will need some supplementary information.

Recall that a “bright spot” is caused by the two wave constructively interfering (a crest meets a crest).

Since the waves coming out of the two slits have the same speed and wavelength (as they are caused by the same initial wave), we get that for a bright spot, the difference between the paths traveled by the two waves must be an integer number of wavelengths long. So we get:

$$PB - PA = l_2 - l_2 = n \cdot \lambda \quad n \in \mathbb{Z} \tag{1}$$

To simplify the problem, we will use the "far-field" approximation, assuming that the screen PD is very far away from our slits at A and B. We can then assume that AP and BP are roughly parallel (convince yourself of this, by looking at Figure 2, and mentally moving PD to the right.

Since we are assuming PA and PB to be parallel and they both intersect AB, we can claim that:

$\angle NBP = \angle MAP$

Now, let's draw a perpendicular from point A to segment PB (AC on Figure 2).

So by construction: $\angle PCA = 90^{\circ}$

Notice from Figure (2), that the path difference $PB - PA = l_2 - l_1$ is equal to BC.

Let's find BC.

Since we assume PA and PB are parallel, then also get that $\angle PAC = 90^{\circ}$

Note that:

$\angle BAC + \angle PCA = \angle PAM + \angle BAM$

$\angle BAC + 90^{\circ} = \angle PAM + 90^{\circ}$

$\angle BAC = \angle PAM = \alpha$

Using basic trigonometry we get that:

$BC = AB \cdot \sin \alpha = d \cdot \sin \alpha$

We can plug this into equation (1) to get:

$$\begin{gathered} d \cdot \sin \alpha = n\lambda \\ \sin \alpha = \frac{n\lambda}{d} \end{gathered} \tag{2}$$

Now that we have this extra bit of information, let's solve for OP.

We can look at the triangle POD (find it on Figure 2) and using the same far field approximation, we can guess that:

$\tan \alpha = \dfrac{OP}{L}$

Assuming α is small, we can also guess that $\tan \alpha \approx \sin \alpha$ and get:

$\sin \alpha = \dfrac{OP}{L}$

Substitute equation (2) into above, and solve for OP to get:

$\dfrac{n\lambda}{d} = \dfrac{OP}{L}$

$$OP = \frac{\lambda L}{d} \cdot n \tag{3}$$

Where:

OP - distance on screen from center between slits to a light spot.
λ - wavelength
L - distance from slits to screen
d - separation between slits
n - some integer (representing path difference in wavelengths)

We therefore expect that light spots will be evenly spaced from each other,with adjacent ones separated by distance $\frac{\lambda L}{d}$.

This fact was confirmed by experiment, proving light has wave-like properties (you can search up many images of this effect online).