

# 1 Discussion of Inner Product Spaces

Read Sections 6A and some of 6B in Linear Algebra Done right 4th edition and answer the following questions.

- I) What is an inner product? Discuss usual definitions on  $\mathbb{R}^n$ ,  $\mathbb{C}^n$  and continuous functions on a compact interval. State some basic properties.
- II) Search up the definition of the norm without inner products? Why is that the natural definition? Search up the definition of a metric. How do we get a metric from a norm? (Do this very quickly (under 5 minutes) during the presentation, and defer all details to the end of the session).
- III) Explain how we define a norm using an inner product. Explain why we wanted conjugate symmetry and positivity for inner products. What information does the inner product contain, that the norm does not?
- IV) Define orthogonality of a vector. Define an orthonormal basis. Prove the properties in 6.30. Explain the gram Schmidt procedure (it may be helpful to first discuss the result in 6.13). Argue that every finite dimensional inner product space, has an orthonormal basis.
- V) Go back, prove the Cauchy-Bunyakovsky-Schwarz inequality and triangle inequality. Conclude that the thing we defined to be the norm, is indeed a norm.