Deriving integrating Factor Method

1. Recall the fundamental theorem of calculus and write down the derivative with respect to time of the following

$$\frac{d}{dt}\int_a^t f(s)ds$$

2. Now use chain rule to write down the following derivative (Your final answer should be written in terms of u,g,u' and g')

$$\frac{d}{dt}\left(u(t)\cdot e^{f(t)}\right)$$

3. Set $f(t)=\int_a^t g(s)ds$ and substitute that into the equation above.

4. Now suppose we had the following differential equation. Compare the equation you got in the previous part to the differential equation. What is missing? And how do we add it. :

$$u'+g(t)u=0$$

5. Add the thing you figured out how to add in the previous part. Write the left hand side as the derivative of something and integrate the equation and solve for u.

6. Now suppose the equation looked like this. Can we still use the same method to solve it?

$$u'+g(t)u=f(t)$$

1 Solving some differential Equations

Use integrating factor method to solve the following:

1. $u'+u=0$
2. $u'+tu=0$
3. $u'+u=1$
4. $u'+\sin(t)u=x$