1 Dual Space Discussion

The basis for the Dual Spaces session of the reading group, will be the section 3F in the 4th edition Linear Algebra done right by Sheldon Axler. I do have some modifications to how I would like our discussion to go relative to the book which I describe below (i.e. you do not need to read/understand everything in the chapter, but need to make sure to think about stuff below).
Particularly, I would like you to ditch Axler's notation of using primes for dual maps and dual spaces and use stars instead. Also everywhere in discussion below, V is finite dimensional.

i) What is a dual space?

ii) Given a basis of a vector space define a dual basis and argue that it is indeed a basis. (Given a basis $v_1, ..., v_n$ either use $v_1^*, ..., v_n^*$ as dual basis notation or $v^1, ..., v^n$).

iii) Suppose we had a bilinear function $\omega : V \times V \to \mathbb{F}$ with the property that $\omega(v, v) \neq 0$ Consider the following map:

$$\omega(v, -) : V \to V^*$$
$$v \to \omega(v, -)$$

Prove that this is a linear isomorphism of V and V^*
Compare and contrast this isomorphism and the one we had above with the dual basis. Which one depends on choice of basis and which one doesn't?

iv) Let $T : V \to W$. What is the dual map T^*.

v) Suppose we pick bases for V and for W. What is the matrix of T^* in the respective dual bases.

vi) In Axler's 4th edition, above block 3.125 there is a little comment on another way to prove 3.125. Prove it this way and draw a picture similar to one shown[1] in (1).

vii) Prove that dim Im(T) = dim Im(T^*). Draw a similar picture to one in (1) as part of your explanation.

viii) Prove that row rank is equal to column rank.

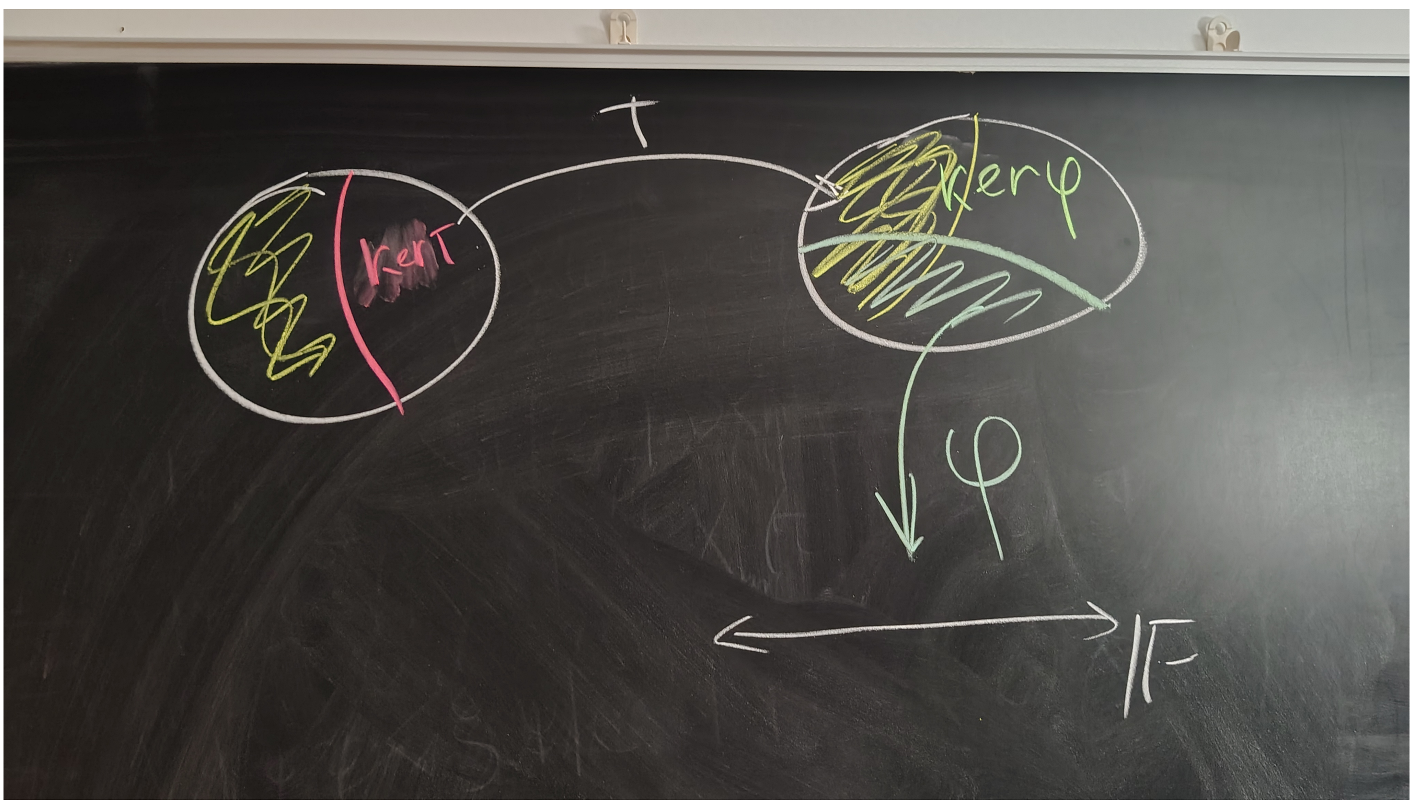

Figure 1: A picture that should be modified or improved to fit the context

[1]Make sure to draw with colours. Coloured chalk will be provided in the reading group.