Second Sign of Congruency of Triangles

Theorem 1 *If one side and angles adjacent to it of one triangle, are equal to a side and angles adjacent to it of another triangle, then these triangles are equal.*

Proof

Consider triangles $\triangle ABC$ and $\triangle A_1B_1C_1$ where: $AB = A_1B_1$, $\angle A = \angle A_1$, $\angle B = \angle B_1$ (Figure 1). Let's prove that: $\triangle ABC = \triangle A_1B_1C_1$.

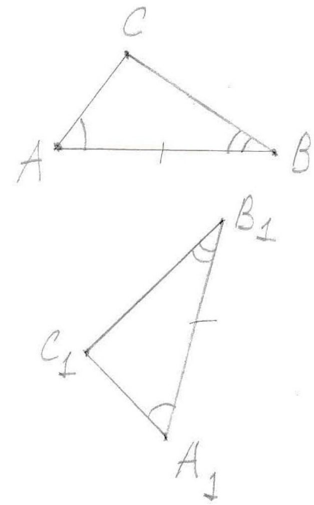

Figure 1

Put triangle ABC on top of triangle $A_1B_1C_1$ such that vertex A is on vertex A_1, side AB aligns with its equal counterpart A_1B_1 and vertices C and C_1 end up on the same side from AB (Axioms #8, #9, #10).

Since $\angle A = \angle A_1$ and $\angle B = \angle B_1$ then side AC will lie on ray A_1C_1. Side BC will lie on ray B_1C_1 (Axiom #10). Hence, C being a common point of sides AC and BC, will belong to both rays A_1C_1 and B_1C_1. This means that vertex C must overlay vertex C_1 (Axiom #3). This would mean that sides AC and A_1C_1 and sides BC and B_1C_1 will overlay each other (Axiom #7).

Triangles ABC and $A_1B_1C_1$ will completely overlay each other.
Hence $\triangle ABC = \triangle A_1B_1C_1$

$\therefore$ QED