Pressure in Liquids and Archimedes Force Demonstrations

Corresponding Vessels

Let's consider a system of two containers with different dimensions, connected by a tube (Figure 1). Let's predict what will happen if water is poured into either container.

From observation we know that after some time, the liquid will stop moving, reaching a point of equilibrium. From Pascal's law we know that pressure is distributed equally in all directions. Since the liquid is no longer moving, at the same level, pressure must be equal.

Recall that the formula for pressure in a liquid is:

$$P = \rho_l g h$$

Let P_1 and P_2 be the pressures at the level of the top of the connecting tube in the left and right containers respectively (Figure 1). Since these pressures must be equal we get:

$$\begin{aligned} P_1 &= P_2 \\ \cancel{\rho_l g} h_1 &= \cancel{\rho_l g} h_2 \\ h_1 &= h_2 \end{aligned}$$

This shows that the water surface in each container, will be on the same level regardless of the shape and volume of the container.

You can see a video demonstration of this phenomenon **here** (personal account) or **here** (gapps account).

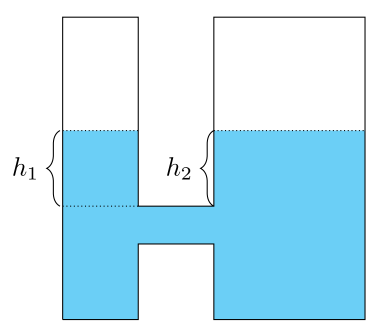


Figure 1

Density of a Block of Wood

We know that the Archimedes force acting on object is is related to the volume of its submerged part and the density of the liquid it is submerged in. If the object floats we can equate its weight to the the Archimedes force. Its weight can be expressed through the object's density and volume. It is therefore possible to determine the density of the object itself, if we know its volume, density of liquid it is submerged in, volume of submerged part.

Assume that the liquid and its density are known.

Here is an example of a procedure which can be used to determine the density of a floating object with only mass measurements. You can find a video recording of this experiment in the Physics Club Demonstrations section **here** (personal account) or **here** (gapps account).

A container was placed on a scale and filled with water. The mass of the water was recorded (m_1), and the surface level was marked on the container (Point A_1 in Figure 2(a)). The floating object (in our case a block of wood), was then added to the container. The new surface level was marked (Point A_2 in Figure 2(b)). It is easy to notice, that currently, the total volume (V_{tot2}) of the container upto point A_2 is:

$$V_{totfloat} = V_1 + V_2$$

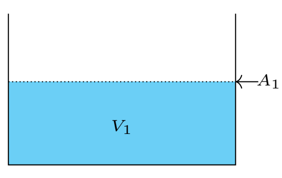
Where V_1 is the volume of water originally added, and V_2 is the volume of the submerged part of the wooden block.

The block was then pushed completely into the water. The new water level was marked (point A_3 in Figure 2(c)).

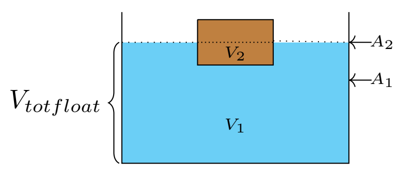
Now it is evident, that the new total volume of the container upto point A_3 is equal to:

$$V_{totsub} = V_1 + V_3$$

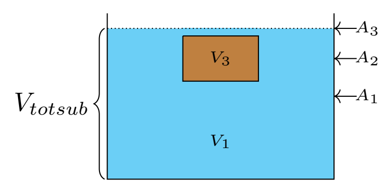
Where V_1 is the volume of water originally added, and V_3 is the volume of the entire block.



(a) Water in container. Surface level is at point A_1



(b) Block of wood floating in water. Water level at point A_2. V_2 - volume of submerged wood.



(c) Wood block completely submerged in water. V_3 - volume of block. Water level is now at point A_3.

Figure 2

It is easy to notice that the volume of the submerged part of the block and the entire block itself can easily be calculated with:

$$V_2 = V_{totfloat} - V_1 \tag{1}$$
$$V_3 = V_{totsub} - V_1 \tag{2}$$

In these equations V_1 can be directly obtained from the initial mass of water m_1 through a density calculation (recall that $V = \frac{m}{\rho}$).

Volumes $V_{totfloat}$ and V_{totsub} can be determined from the masses m_2 and m_3 of water that would fill the container upto points A_2 and A_3 respectively.

Hence:

$$V_1 = \frac{m_1}{\rho_w}$$
$$V_{totfloat} = \frac{m_2}{\rho_w}$$
$$V_{totsub} = \frac{m_3}{\rho_w}$$

Substitute this into equations (1) and (2) to get.

$$\begin{aligned} V_2 &= V_{totfloat} - V_1 \\ &= \frac{m_2}{\rho_w} - \frac{m_1}{\rho_w} \\ &= \frac{(m_2 - m_1)}{\rho_w} \end{aligned} \tag{3}$$

$$\begin{aligned} V_3 &= V_{totsub} - V_1 \\ &= \frac{m_3}{\rho_w} - \frac{m_1}{\rho_w} \\ &= \frac{(m_3 - m_1)}{\rho_w} \end{aligned} \tag{4}$$

Now that we know the volume of the submerged part of the wooden block, we can calculate the Archimedes force acting on it. Since it is floating (stationary), we also know that the force pushing up (Archimedes force) must be equal to its weight F_g (Figure 3). We get:

$F_a = F_g$

$\rho_w g V_2 = F_g$

Recall that $F_g = m_o g$, where m_o is the mass of the object. Therefore:

$\rho_w g V_2 = m_o g$

We can express m_o in terms of the density of the object as:

$m_o = V_3 \rho_o$

Substitute this into equation above to get:

$\rho_w \cancel{g} V_2 = V_3 \rho_o \cancel{g}$
$\rho_w V_2 = \rho_o V_3$

Solve for ρ_o and substitute equations (3) and (4) to get:

$$\begin{aligned}
\rho_o &= \frac{\rho_w V_2}{V_3} \\
&= \frac{\rho_w \frac{(m_2 - m_1)}{\cancel{\rho_w}}}{\frac{(m_3 - m_1)}{\cancel{\rho_w}}} \\
&= \rho_w \frac{m_2 - m_1}{m_3 - m_1} \qquad (5)
\end{aligned}$$

The units are correct (as kg over kg in the fraction cancel out, and only leave units of density).

Data Analysis

In the video experiment linked above it was determined that:

$m_1 = 1.220$ kg
$m_2 = 1.287$ kg
$m_3 = 1.355$ kg

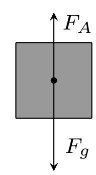


Figure 3: Forces acting on floating object

Assume the density of water to be $\rho_w = 1000\ \frac{\text{kg}}{\text{m}^3}$ and calculate the density of the wooden block with equation (5).

$$\begin{aligned}
\rho_o &= \rho_w \frac{m_2 - m_1}{m_3 - m_1} \\
&= 1000 \cdot \frac{1.287 - 1.220}{1.355 - 1.220} \\
&= 496.296\ \frac{\text{kg}}{\text{m}^3}
\end{aligned}$$

We have also separately measured the mass and dimensions of the block itself. Let's verify that the answer we got is correct with this information:

$m_o = 70$ g $= 0.070$ kg
$l = 73.5$ mm $= 0.0735$ m
$h = 37.5$ mm $= 0.0375$ m
$w = 51.0$ mm $= 0.050$ m

$$\begin{aligned}
\rho_o &= \frac{m_o}{V_o} \\
&= \frac{m_o}{l \cdot w \cdot h} \\
&= \frac{1.296}{0.0735 \cdot 0.0375 \cdot 0.050} \\
&= 507.937\ \frac{\text{kg}}{\text{m}^3}
\end{aligned}$$

Both methods yield a very similar number, however the first approach did not use any length measurements and was completed entirely with only a scale.

Suggest possible errors in the experiment that could have caused these numbers to be different.