Other ways of representing Vectors

So far, we have only considered representing vectors via their end point coordinates. Let's try to determine another way to notate quantities with both magnitude and direction. Draw a

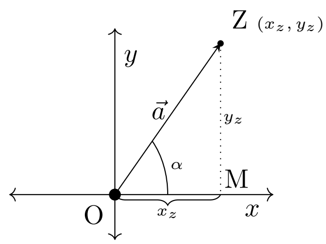

Figure 1

line from the tip of the vector, perpendicular to the x axis (Figure 1). Mark their intersection as point M. Line segments OZ,OM and ZM make a right angled triangle with side lengths $|\vec{a}|$, x_z and y_z respectively. Angle M is the right angle. Angle α is the angle the vector makes with the x axis.

Property of Triangles 1. *It is known that a right angled triangle can be defined through one side length and one of its acute angles. This means that only one possible right angled triangle can be constructed with a given side length and angle.*

This implies that with such information, we can somehow deduce a right angled triangle's side lengths.

Using the property (1), we can now introduce a new notation for record vectors through their magnitude and angle with x-axis, rather than the coordinates of their endpoint. The general convention is:

$$\vec{a} = |\vec{a}|[\text{Direction1}\,\alpha\,\text{Direction2}] \tag{1}$$

Where:

Direction 1 and direction 2, are the directions you draw angle α from and to.

For example, figure (1) represents the following vector:

$$\vec{a} = |\vec{a}|[\text{E}45°\text{N}]$$

In this case α is the angle going from the east part of the x axis to the north part of the y axis.

Now that we have defined a new notation for vectors, lets find a way to convert between a vector's coordinates and its angle direction and magnitude.

Recall that two triangles are similar (ratios of respective sides are constant) if they have equal angles. From this we can easily derive that in all triangles with equal angles, the ratios of their side lengths stay constant. For right angled triangles, these ratios have been recorded for each measurable set of angles.

In a right angled triangle with angle α (Figure 1):

$$\sin\alpha = \frac{y_z}{|\vec{a}|} \tag{2a}$$

$$\cos\alpha = \frac{x_z}{|\vec{a}|} \tag{2b}$$

$$\tan\alpha = \frac{y_z}{x_z} \tag{2c}$$

Where:

y_z is the side opposite to α.
x_z is the side adjacent to α.
$|\vec{a}|$ is the hypotenuse of the right angled triangle.

These are called the basic trigonometric ratios.

We have mentioned above that for every α there is a known sin, cos and tan value. Therefore, if in Figure (1) we know $|\vec{a}|$ and α, using a look up table/calculator and equations (2), we can always find the vector's coordinates.

The coordinates of a vector $\vec{a} = |\vec{a}|[\mathrm{E}\,\alpha\,N]$ are:

$$\vec{a} = (|\vec{a}| \cdot \cos\alpha, |\vec{a}| \cdot \sin\alpha) \tag{3}$$

*Note that the definition of the trigonometric ratios through triangles is the most basic one. Calculators use a slightly different formulation to yield coordinates in the correct quadrants for angles greater than 90 °.