

# Theorem on the Parallelity of Two lines Perpendicular to a Third Line

**Theorem 1** *If two lines in the same plane are perpendicular to a third line, then these two lines are parallel.*

## Proof

Consider two distinct lines  $AA_1$  and  $BB_1$  which are perpendicular to the line  $PQ$  (Figure 1).

Suppose that  $AA_1$  and  $BB_1$  are not parallel. This would mean that they intersect at some kind of point  $M$ , forming a triangle  $\triangle PQM$ .

Since  $\angle 1 = \angle 2$ ,  $\angle 3 = \angle 4$  (they are supplementary and equal to  $90^\circ$ ) and these angles are adjacent to side  $PQ$ , then by the second sign of congruency of triangles, lines  $AA_1$  and  $BB_1$  will make another triangle on the other side of  $PQ$  and hence intersect at point  $M_1$ . This would mean that two distinct lines  $AA_1$  and  $BB_1$  pass through two same points.

This contradicts axiom #3 and hence our original supposition that these two lines intersect is incorrect. This means that  $AA_1$  and  $BB_1$  must be parallel (as that is the only other option left).

$\therefore$  QED

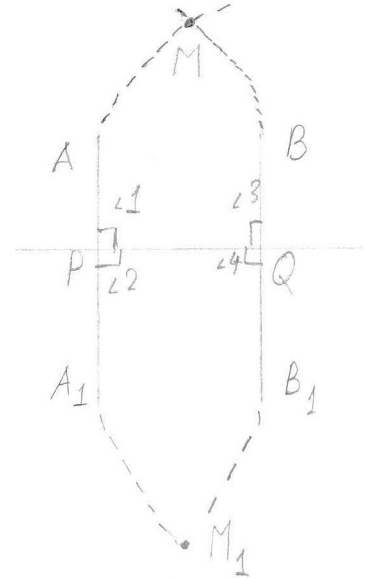


Figure 1