Derivation of the Change of Base Formula

The Change of Base Formula states:

$$\log_a b = \frac{\log_c b}{\log_c a}$$

Proof

$$a^{\log_a b} = b \tag{1}$$

$$c^{\log_c a} = a \tag{2}$$

$$c^{\log_c b} = b \tag{3}$$

Substitute (2) and (3) into (1) to get:

$$(c^{\log_c a})^{\log_a b} = c^{\log_c b}$$

$$\sqrt[\log_c a]{(c^{\log_c a})^{\log_a b}} = \sqrt[\log_c a]{c^{\log_c b}}$$

$$c^{\log_a b} = c^{\frac{\log_c b}{\log_c a}}$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$\therefore$ QED