Midsegment of a Triangle and the Point of Intersection of a Triangle's Medians

The **midsegment of a triangle** is the line segment which connects the midpoints of two of the triangle's sides.

Theorem 1 *The midsegment of a triangle is parallel to one of its sides and is equal to half of that side.*

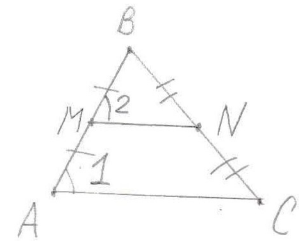

Figure 1

Proof

Let MN be the midsegment of triangle ABC Figure 1). Let's prove that $mn \parallel BC$ and that $MN = \frac{1}{2}BC$.

Triangles BMN and BAC are similar due to the second sign of similarity of triangles ($\angle B$ - common angle, $\frac{BM}{BA} = \frac{BN}{BC} = \frac{1}{2}$).Hence:
$\angle 1 = \angle 2$ and $\frac{MN}{AC} = \frac{1}{2}$.
From the fact that $\angle 1 = \angle 2$ we get that: $MN \parallel AC$ (Explain why yourself).
From the second equation we get that $MN = \frac{1}{2}AC$.
$\therefore$ QED

Using this theorem, let's prove the following statement:

Statement 1 *The medians of a triangle intersect each other in one point, which divides each median into two line segments which relate as 2:1, starting from the vertex.*

Proof

Consider an arbitrary Triangle ABC (Figure 2). Let's label the point of intersection of medians AA and BB_1 as O and draw the midsegment A_1B_1.

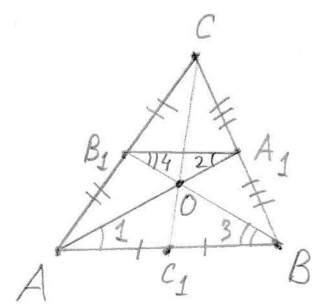

Figure 2

Line segment A_1B_1 is parallel to AB. This means that: $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ (alternate angles). This means that $\triangle AOB \sim \triangle A_1OB_1$. Hence: $\frac{AO}{A_1O} = \frac{BO}{B_1O} = \frac{AB}{A_1B_1}$.
But from the previous theorem we know: $AB = 2A_1B_1$. Now using these statements:

$$\frac{AB}{A_1B_1} = \frac{AO}{A_1O}$$

$$\frac{2A_1B_1}{A_1B_1} = \frac{AO}{A_1O}$$

$AO = 2A_1O$
The same way we can show that : $BO = 2B_1O$.
We proved that the medians AA_1 and BB_1 splits each median $2:1$.

We can prove the same way, that the point of intersection of medians BB_1 and CC_1 splits them $2:1$. But since the point of intersection of medians BB_1 and AA_1 splits BB_1 $2:1$ too, that means that the point of intersection of BB_1 and CC_1 and the point of intersection of

AA_1 and BB_1 is the same point.

All three medians of triangle ABC intersect each other at point O and split themselves into line segments which relate $2:1$, going from the vertex.