The Tangent to a Circle

The tangent to a circle is a line that shares only one point with a circle.

Theorem 1 *The tangent to a circle is perpendicular to the radius, which is drawn to the point where the tangent touches the circle.*

Proof

Let p be a tangent to a circle with center O. A is the point where p touches the circle (Figure 1). We need to prove that $OA \perp p$.

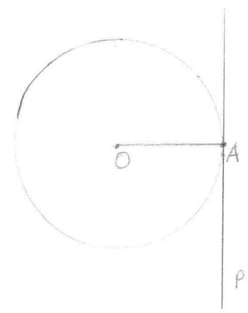

Figure 1

Let's assume that this is not true. This means that a perpendicular from point O to line p is shorter than radius $OA \Rightarrow$ the distance from point O to line p is shorter than the circle's radius. This would mean that line p intersects the circle at two points, which contradicts the original statement that p is a tangent to the circle. Hence: $OA \perp p$
$\therefore$ QED

Corollary 1 *The line segments of tangents to a circle which are drawn from the same point are equal and make equal angles with the line, which pass through hat point and the center of the circle.*

Proof

Let points C and B be the points where the tangents that pass through point A touch the circle. We need to prove that $AB = AC$ and that $\angle 3 = \angle 4$ (Figure 2).
Using Theorem 1, we know that $\angle 1 = \angle 2 = 90°$. This means that triangles ABO and ACO are right angled. They share hypotenuse OA. $OB = OC$ since both of these line segments. Given two sides in a right angled triangle, we can always solve for the third, hence $\triangle AOB = \triangle AOC$.This means that $AB = AC$ and $\angle 3 = \angle 4$
$\therefore$ QED

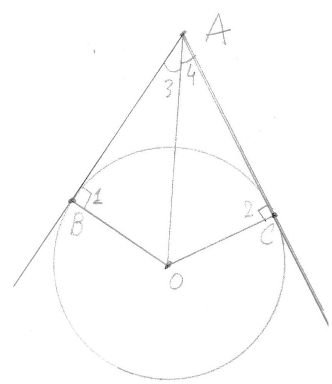

Figure 2

Theorem 2 *If a line passes through the end of a radius, which is on the edge of a circle, and is perpendicular to the radius, then that line is a tangent to the circle.*

Proof

From the theorem we get that the radius is a perpendicular to some line drawn through the center of the circle. This means that the distance from the center of the circle to that line is equal to the length of the radius. This would mean that the circle and that line share only one point, which by definitions makes that line a tangent to the circle.
$\therefore$ QED