I completely forgot to tell you about the common way string tension is modeled. Supposed you have two points connected by a stretched string (Figure 1). We usually assume that the string then exerts a force T, which is uniform throughout the string (If you were to somehow "hold it in the middle", you would still feel a force of magnitude T). We also assume that this force is always directed "into" the string (i.e. the string can only pull and never push). This force is usually called the tension.

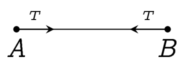

Figure 1: String connecting two points A and B.

1. Consider a mass m attached to two strings of length l_1 and l_2 respectively. These strings are attached to a flat surface at points A and B, separated by distance d.
   (a) Draw a free body diagram of the mass m.
   (b) Write down Newton's equations in the x and y components.
   (c) Suppose the system is at equilibrium. What are the tensions T_1 and T_2 in each string, and what are the angles φ and α. Check whether your answer makes sense, by setting $l_1 = l_2$.

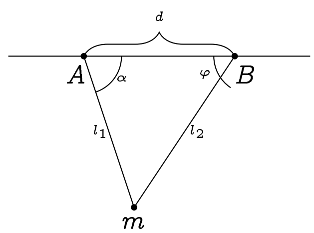

Figure 2: Suspended mass

2. Consider the diagram below. There are rigid rods of length l attached to point A and two masses m_1. Then two more rigid rods attach masses m_1 to mass m_2. These rods can "bend" at m_1. Mass m_2 can only move vertically along the axis through point A. Now suppose this whole system is rotating with a constant angular speed ω around the vertical axis through A.
   (a) Let φ be as in Figure 3. Write down the lagrangian using this generalized coordinates. (Hint: It may be difficult to directly write down the velocity in these coordinates. Note however that always $v^2 = v_x^2 + v_y^2 + v_z^2 = (\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2$. So to find the speed, you could express x, y and z in terms of φ and take some time derivatives. Note that this is a 3-d problem, as this system is rotating about the axis).
   (b) Write down the equations of motion.
   (c) Suppose the system achieves equilibrium. i.e. mass m_2 is at a fixed distance r away from A and does not move (masses m_1 keep rotating as before, but at a constant horizontal level). What is the this equilibrium angle φ? (Hint: If the system is at equilibrium, what is 0 in your equations of motions?)

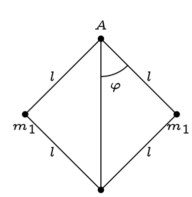

Figure 3: This equation is taken from page 22 of Landau and Liefshitz Classical Mechanics.

3. Consider a mass m suspended on a string of length l. Suppose that the mass is somehow rotated at a constant angular speed ω. The mass will stabilize and rotate at some fixed angle φ to the axis of rotation.

(a) Draw a free body diagram of the mass in the only in the plane as in Figure ??.

(b) Draw a “top view” of the trajectory of the system. Label or relevant lengths and express them in terms of l and φ.

(c) Recall the result from the previous homework, on 2-dimensional circular motion with constant angular speed. What is the speed of the mass in this case? Give l and ω, determine what the equilibrium angle φ must be.

(d) If you have free time. Write down the lagrangian as in the previous question. Write down the equations of motion and use them to determine the equilibrium angle. (You should get the same one as in the previous part of the question).

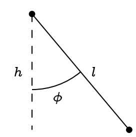

Figure 4