Some Properties of the Basic Trigonometric Ratios

Sine of an angle

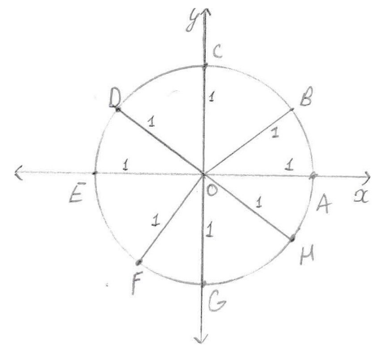

Figure 1

The sine of an angle is defined as the y coordinate of a point on the unit circle. This means, that if the point representing an angle is "above" the x-axis sine is positive; if it is below, sine is negative. The X axis is a 180° angle AOE. This means that sine is positive for all angles between 0 and 180 and negative for all angles between 180 °and 360 °.

Cosine of an angle

The cosine of an angle is defined as the x coordinate of a point on the unit circle. This means that if the point representing an angle is to the "right" of the y-axis, cosine is positive; if it is to the left of the y-axis. Hence, cosine is positive for all angles between $-90°$ and $90°$ and negative for all angles between $90°$ and $270°$.

Also please note that sine and cosine of an angle are always between or equal to -1 and 1. This is true because the hypotenuse of all the triangles made by points on the unit circle is always 1 and it is the longest side in those triangles.

Tangent of an angle

From here we get that the tangent of an angle is positive if it is either above the x-axis and to the right of the y-axis or below the x-axis and to the left y-axis.

Why?

Why is this way of phrasing it important if you can just identify what is positive in which quadrants (E.G. the horrible garbage CAST rule)?. Most of the time, visualizing the angle on a unit circle is a much faster way to figure out the sign of a trigonometric ratio compared to memorizing and going through all of the possible cases of a rule each time you need to figure out if a certain ratio is positive or negative. It is also much easier to imagine an angle on a unit circle rather than the graph of the sine, cosine or tangent functions.

Now let's consider the following two formulas:

Formula 1 *For all values of t the following two statements are true:*

$$\boldsymbol{\sin(-t) = -\sin t}$$
$$\boldsymbol{\cos(-t) = \cos t}$$

Explanation

If some number t has a corresponding point M on the unit circle, then there is some point P which will correspond to a number $-t$ (Figure 2). P is symmetrical to m relative to the horizontal diameter (x-axis) of the unit circle. This means that such points have the same x - coordinate hence: $\cos(-t) = \cos t$. Such points would also have y-coordinates of different signs. Hence: $\sin(-t) = -\sin t$

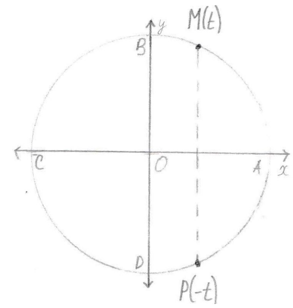

Figure 2

Reduction formulas

Formula 2 *For any value of t the following statements are true:*

$$\boldsymbol{\sin(t + 2\pi k) = \sin t}$$
$$\boldsymbol{\cos(t + 2\pi k) = \cos t}$$

This is evident since the same point corresponds to numbers t and $t + 2\pi k$, $k \in \mathbb{Z}$.

(Look at Unit Circle and Radians for a better explanation). Please note that from here onward, anywhere k is used in the following form :$2\pi k$; $\frac{\pi}{2}k$ etc. it is assumed that $k \in \mathbb{Z}$

Formula 3 *For any value of t the following statements are true:*

$$\boldsymbol{\sin(t + \pi) = -\sin t}$$
$$\boldsymbol{\cos(t + \pi) = -\cos t}$$

Explanation

If some number t corresponds to some point M on the unit circle , then some number $t + \pi$ will correspond to some point P (Figure 3). Point P will be symmetrical to point M around point O. Such points would have x and y coordinates equal in length, but of opposite signs. Hence: $\sin(t + \pi) = -\sin t$ and $\cos(t + \pi) = -\cos t$

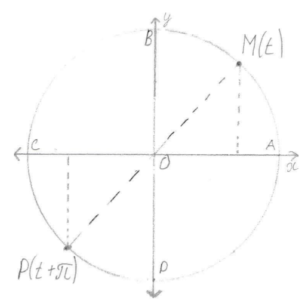

Figure 3

Formula 4 *For all values of t the following two statements are true:*

$$\boldsymbol{\sin(t + \frac{\pi}{2}) = \cos t}$$
$$\boldsymbol{\cos(t + \frac{\pi}{2}) = -\sin t}$$

Explanation

Let some number t correspond some point M. Then some number $t+\frac{\pi}{2}$ corresponds to some point P (Figure 4). It is important to note that if point M is in quadrant 1, then point P is in quadrant 2; if point M is in quadrant 2, then point P is in quadrant 3 etc. Arches AM and BP are equal $\Rightarrow \triangle OKM = \triangle OLP \Rightarrow OK = OL$; $MK - PL$. Using these equations and the note mentioned above we make two conclusions:

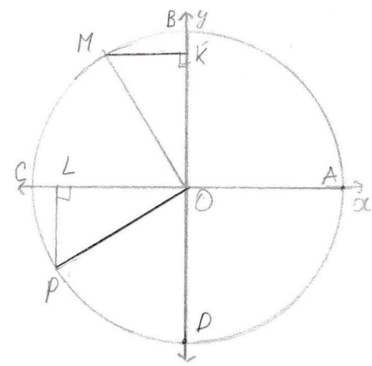

Figure 4

1) The y-coordinate of point P has the same sign and value of the x-coordinate of point M. Hence:
$\sin(t+\frac{\pi}{2}) = \cos t$

2) The x-coordinate of point$P$ has the same value but opposite sign of the y-coordinate of point M. Hence:
$\cos(t+\frac{\pi}{2}) = -\sin t$

Formula 5 *For all values of t the following statement is true:*

$$\boldsymbol{\tan(-t) = -\tan t}$$

Derivation

$\tan(-t) = \dfrac{\sin(-t)}{\cos(-t)}$

Using Formula 1 we get:

$$\begin{aligned}\frac{\sin(-t)}{\cos(-t)} &= \frac{-\sin t}{\cos t}\\ &= -\tan t\end{aligned}$$

$\therefore$ QED