Midsegment of a Trapezoid

Definition

The midsegment of a trapezoid is the line segment which connects the midpoints of the non-parallel sides.

Properties of the Midsegment of a Trapezoid

Theorem 1 *The midsegment of the trapezoid is parallel to the bases of the trapezoid and is equal to half of their sum.*

Proof

Let MN be the midsegment of trapezoid (Figure 1). Let's prove that $MN \parallel AD$ and $MN = \frac{AD+BC}{2}$.

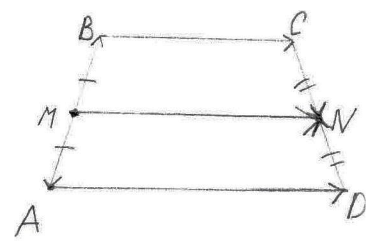

Figure 1

Using the properties of vector addition we get:

$\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BC} + \overrightarrow{CN}$

But we also can get that:

$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AD} + \overrightarrow{DN}$

Let's add these two equations to get:

$2\overrightarrow{MN} = (\overrightarrow{MB} + \overrightarrow{MA}) + (\overrightarrow{BC} + \overrightarrow{AD}) + (\overrightarrow{CN} + \overrightarrow{DN})$

Points M and N are the mid points of the non-parallel sides. Hence:

$\overrightarrow{MB} + \overrightarrow{MA} = 0$
$\overrightarrow{CN} + \overrightarrow{DN} = 0$

Substitute this into equation above:

$2\overrightarrow{MN} = \overrightarrow{BC} + \overrightarrow{AD}$
$\overrightarrow{MN} = \frac{1}{2}(\overrightarrow{BC} + \overrightarrow{AD})$ Since vectors $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are co-directional vectors, then $\overrightarrow{MN}$ and $\overrightarrow{AD}$ are also co-directional vectors. Hence the length of vector $(\overrightarrow{AD} + \overrightarrow{BC}) = AD + BC$. From here we get that: $MN \parallel AD$ and $MN = \frac{AD+BC}{2}$

$\therefore$ QED