Sine Law

Theorem 1 *The sides of a triangle are proportional to the sines of angles opposite of the sides*

Proof

Let $BC = a$, $AC = b$, $AB = c$, $\angle A = \alpha$, $\angle B = \beta$, $\angle C = \gamma$ in triangle ABC.
We need to prove that:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Recall the theorem on the area of the triangle. Let S be the area of $\triangle ABC$, then:

$$S = \frac{1}{2}\, a \cdot b \cdot \sin\gamma \tag{1}$$

$$S = \frac{1}{2}\, a \cdot c \cdot \sin\beta \tag{2}$$

$$S = \frac{1}{2}\, b \cdot c \cdot \sin\alpha \tag{3}$$

Equate (1) and (2). We get:

$$\frac{1}{2}\, a \cdot b \cdot \sin\gamma = \frac{1}{2}\, a \cdot c \cdot \sin\beta$$

$$b \cdot \sin\gamma = c \cdot \sin\beta$$

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Equate (2) and (3). We get:

$$\frac{1}{2}\, a \cdot c \cdot \sin\beta = \frac{1}{2}\, b \cdot c \cdot \sin\alpha$$

$$a \cdot \sin\beta = b \cdot \sin\alpha$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

Therefore we get:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$\therefore$ QED