Position of a Line Relative to a Circumference

Let's consider some line p which does not cross through the center O of a circumference with radius r. Let's draw a line segment OH perpendicular to line p and name its length as d. That would be the distance from the center of this circumference to line p.
Let's now consider the possible position line p can be in relative to the circumference depending on the relationship between d and r. There are three possible cases.

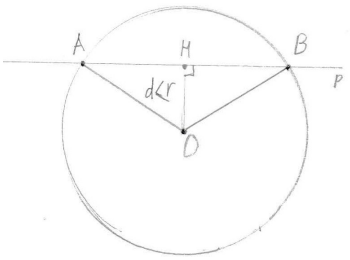

Figure 1

1) $d < r$. (Figure 1) On line p, let's draw two line segments HA and HB of length $\sqrt{r^2 - d^2}$. Using Pythagoras theorem:

$OA = \sqrt{OH^2 + HA^2} = \sqrt{d^2 + (r^2 - d^2} = r$
$OB = \sqrt{OH^2 + HB^2} = \sqrt{d^2 + (r^2 - d^2} = r$

This means that points A and B lie on the circumference and belong to both the circumference and line p. Let's prove that line p does not have any other common points with the circumference.
Let's assume that they do have at least one other shared point C. Then the median OD of side AC in the equilateral triangle OAC is the height of this triangle. Hence: $OD \perp p$. Line segments OD and OH do not coincide as the midpoint of AC is not the same point as point H - the midpoint of AB. From this we get that two different perpendiculars were drawn from point O to line p which is not possible.

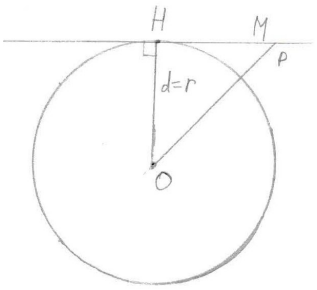

Figure 2

If the length from the center of a circumference to a line is less than the radius of the circumference, then the line and circumference share two points.

2) $d = r$. IN this case, $OH = r$. This means that point H is on the circumference and is a common point of line p and the circumference (Figure 2). Line p does not share any other points with the circumference as for any point M on line p, different from point H, $OM > OH = r$, and therefore point M does not lie on the circumference.

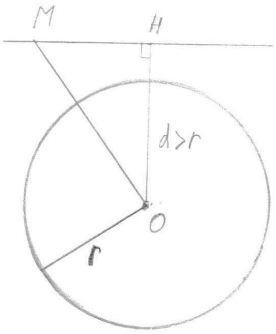

Figure 3

If the length from the center of a circumference to a line is equal to the radius of the circumference, then the circumference and line share one point.

3) $d > r$ In this case $OH > r$ and that means that for any point M on line P: $OM \geq OH > r$. That means that point M is not on the circumference.

If a the distance from the center of a circumference to a line is larger than the radius of the circumference, then the line and circumference do not have any common points.