

1. We have just talked about conserved quantities. We noticed that symmetry under time translations, corresponds to conservation of energy and symmetry under spatial translations corresponds to conservation of momentum. There is however one more very natural symmetry we can consider: rotations. We would expect, that if we were to rotate our entire world, the equations of motion should be invariant under that transformation.
 - (a) Consider a 2 d system. We can write coordinates in that system in terms of r and ϕ (where r is radius and ϕ is angle with the positive x -axis.). Write down the kinetic energy of a particle in these coordinates.
 - (b) Now suppose the system which is invariant under rotations around the origin. What variables can the potential V depend on then?
 - (c) Write down the Lagrangian (you can keep V as some unknown function).
 - (d) Write down the Euler Lagrange equations.
 - (e) Show that the equation for ϕ gives us a conserved quantity. Call it M and write down an equation for it.
2. In all the examples I showed you, I talked about the Euler Lagrange equations of a particle in 1 dimensional system. Of course, you can have more than 1 coordinate and therefore more than 1 symmetry. You are not always guaranteed however, to have as many conserved quantities as coordinates (you may have more or less).
 - Consider a particle of mass m , falling freely in 3-dimensions in the usual gravitational field, where the force is $F = -g$ in the downwards direction. Write down the Lagrangian.
 - Write down the Euler-Lagrange equation for x, y and z . Write which ones give us a conserved quantity and what the conserved quantity is.
 - Think about why this is true.
3. Read pages 1-17 of "Linear Algebra Done Right"
 - (a) Take notes. When I see you next time, I will ask you to tell me about what you have read using mostly your notes. So they should summarize what you understand from the chapter.
 - (b) Remember, this is not something you can read quickly. You need time to digest. Try and read 2-3 pages a day.
 - (c) In 1.A do exercises: 1 & 10
 - (d) in 1.B do exercises 3 & 4