First Sign of Similarity of Triangles

Theorem

If two angles of one triangle are equal to two angles of another triangle, then these two triangles are similar.

Proof

Recall:
Two triangles are considered similar if their respective angles are equal and the sides of one triangle are proportional to the respective sides of the other.

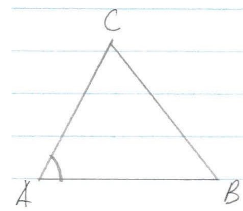

Figure 1

Let $\triangle ABC$ and $\triangle A_1B_1C_1$ be two triangles where: $\angle A = \angle A_1, \angle B = \angle B_1$.
Lets prove that $\triangle ABC \sim \triangle A_1B_1C_1$.

Using the theorem of the sum of angles of a triangle:

$\angle C = 180° - \angle A - \angle B$
$\angle C_1 = 180° - \angle A_1 - \angle B_1$
$\Rightarrow \angle C = \angle C_1$

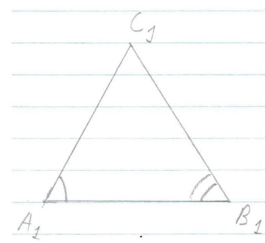

Figure 2

Now let's prove that the sides of $\triangle ABC$ are proportional to sides of $\triangle A_1B_1C_1$

Recall theorem 1 (Look within the Similar Angles folder (The Very Useful Theorem))

Since $\angle A = \angle A_1$ and $\angle C = \angle C_1$ then:

$\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{AB \cdot AC}{A_1B_1 \cdot A_1C_1}$

$\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{CA \cdot CB}{C_1A_1 \cdot C_1B_1}$

From these proportionality statements we get that:

$\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$

Likewise using $\angle A = \angle A_1$ and $\angle B = \angle B_1$ we get:

$\frac{BC}{B_1C_1} = \frac{CA}{C_1A_1}$

Sides of triangle ABC are proportional to the respective sides of triangle $A_1B_1C_1$.
$\therefore$ QED