Theorem on the Area of a Triangle

Theorem 1 *The area of a triangle is equal to half of the product of the lengths of two of its sides times the sine of the angle between these sides.*

Proof

Let ABC be a triangle where $BC = a$, $CA = b$. Let S be the area of this triangle and α be the angle between sides AB and BC .

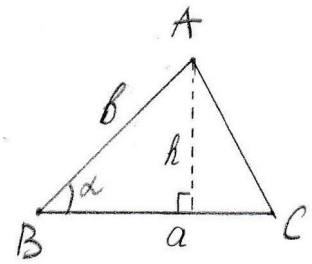

Figure 1

Let's prove that:

$\frac{1}{2} a \cdot b \cdot \sin \alpha$

Recall that:

$S = \frac{1}{2} h \cdot a$

Notice that:

$\sin \alpha = \frac{h}{b}$

$h = b \cdot \sin \alpha$

Substitute this into our formula for area from above. We get:

$S = \frac{1}{2} a \cdot b \cdot \sin \alpha$

$\therefore$ QED