$\mathbb{A}$, $\mathbb{a}$ matrix

A , a number

$$p_a = \mathrm{tr}(\mathbb{P}\mathbb{M}_a) \qquad \mathbb{P} = \sum_b p_b \mathbb{N}^b$$

$$\Rightarrow p_a = \mathrm{tr}\left(\sum_b p_b \mathbb{N}^b \mathbb{M}_a\right)$$

$$= \sum_b p_b \,\mathrm{tr}(\mathbb{N}^b \mathbb{M}_a)$$

$$\Rightarrow \mathrm{tr}(\mathbb{N}^b \mathbb{M}_a) = \delta^b{}_a$$

$$\dot{\mathbb{P}} = -i[\mathbb{H}, \mathbb{P}] + \sum_j \left[\mathbb{L}_j \mathbb{P} \mathbb{L}_j^\dagger - \frac{1}{2}(\mathbb{L}_j^\dagger \mathbb{L}_j \mathbb{P} + \mathbb{P}\mathbb{L}_j^\dagger \mathbb{L}_j)\right]$$

$$① \quad \dot{\mathbb{P}} = \frac{\partial}{\partial t} \sum_b p_b \mathbb{N}^b = \sum_b \dot{p}_b \mathbb{N}^b$$

$$② \quad -i[\mathbb{H}, \mathbb{P}] = -i\left[\mathbb{H}, \sum_b p_b \mathbb{N}^b\right] = -i\sum_b p_b [\mathbb{H}, \mathbb{N}^b]$$

$$③ \quad \mathbb{L}_j \mathbb{P} \mathbb{L}_j^\dagger - \frac{1}{2}(\mathbb{L}_j^\dagger \mathbb{L}_j \mathbb{P} + \mathbb{P}\mathbb{L}_j^\dagger \mathbb{L}_j)$$

$$= \sum_b p_b \left[\mathbb{L}_j \mathbb{N}^b \mathbb{L}_j^\dagger - \frac{1}{2}(\mathbb{L}_j^\dagger \mathbb{L}_j \mathbb{N}^b + \mathbb{N}^b \mathbb{L}_j^\dagger \mathbb{L}_j)\right]$$

$$① = ② + \sum_j ③_j$$

$$\mathrm{tr}(① \mathbb{M}_a) = \mathrm{tr}\left(\sum_b \dot{p}_b \mathbb{N}^b \mathbb{M}_a\right) = \sum_b \dot{p}_b \,\mathrm{tr}(\mathbb{N}^b \mathbb{M}_a)$$

$$= \sum_b \dot{p}_b \,\delta^b{}_a = \dot{p}_a$$

$$\mathrm{tr}(② \mathbb{M}_a) = -i\sum_b p_b \,\mathrm{tr}([\mathbb{H}, \mathbb{N}^b]\mathbb{M}_a)$$

$$= -i\sum_b p_b \,\mathrm{tr}(\mathbb{H}\mathbb{N}^b \mathbb{M}_a - \mathbb{N}^b \mathbb{H} \mathbb{M}_a)$$

$$\equiv -i\sum_b p_b A_a{}^b$$

$$\sum_j \mathrm{tr}(③_j \mathbb{M}_a) = \sum_{b,j} p_b \Big[\mathrm{tr}(\mathbb{L}_j \mathbb{N}^b \mathbb{L}_j^\dagger \mathbb{M}_a)$$

$$- \frac{1}{2}\mathrm{tr}(\mathbb{L}_j^\dagger \mathbb{L}_j \mathbb{N}^b \mathbb{M}_a)$$

$$- \frac{1}{2}\mathrm{tr}(\mathbb{N}^b \mathbb{L}_j^\dagger \mathbb{L}_j \mathbb{M}_a)\Big]$$

$$\equiv \sum_b p_b B_a{}^b$$

$$\Rightarrow \dot{p}_a = \sum_b p_b(-i A_a{}^b + B_a{}^b)$$

$$A_a{}^b = \sum_{r,s,t} (H_{rs} N^b{}_{st} M_{a\,tr} - N^b{}_{rs} H_{st} M_{a\,tr})$$

$$B_a{}^b = \sum_{j,r,s,t,u} L_{j\,rs} N^b{}_{st} L^*_{j\,ut} M_{a\,ur} - \frac{1}{2} L^*_{j\,sr} L_{j\,st} N^b{}_{tu} M_{a\,ur}$$

$$- \frac{1}{2} N^b{}_{rs} L^*_{j\,ts} L_{j\,tu} M_{a\,ur}$$