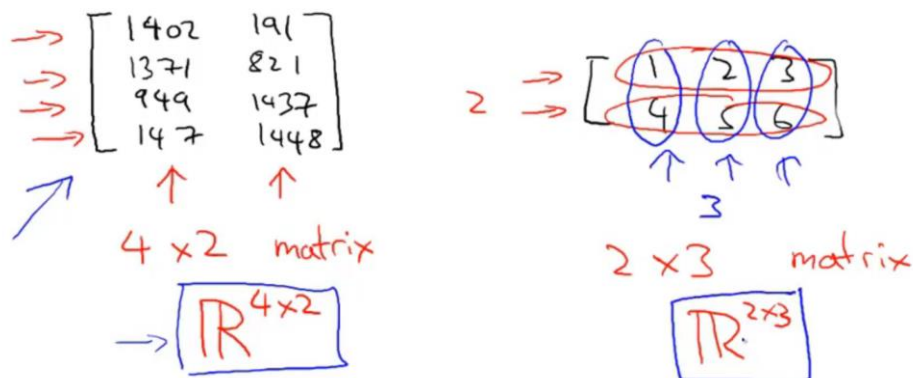


Linear Algebra review

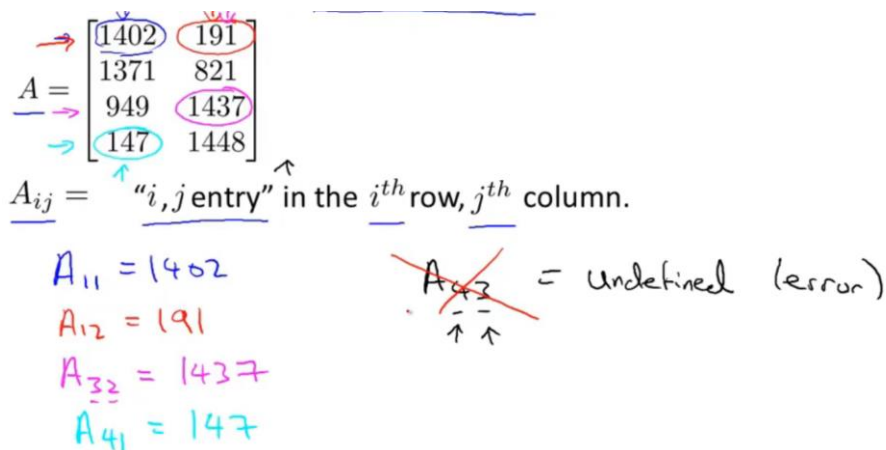
1. Matrices and vectors

Matrix: Rectangular array of numbers

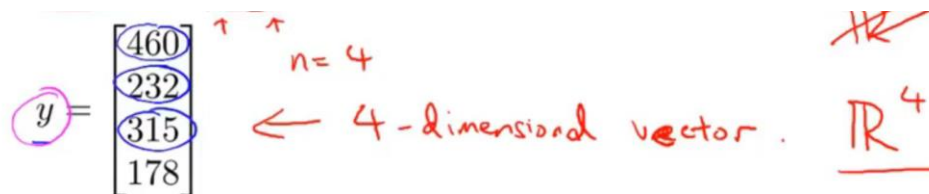
Dimension of matrix: number of rows x number of columns



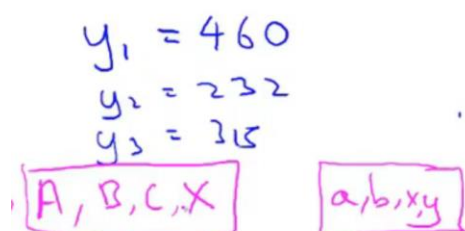
Matrix elements (entries of matrix)



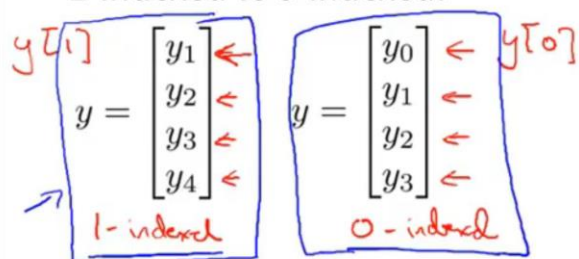
Vector: An $n \times 1$ matrix.



$y_i = i^{th}$ element



1-indexed vs 0-indexed:



2. Addition and scalar multiplication

Matrix addition:

$$\begin{array}{c}
 \downarrow \downarrow \\
 \rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix} \\
 \text{3x2 matrix} \quad \text{3x2} \quad \text{3x2}
 \end{array}$$

$$\begin{array}{c}
 \rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \end{bmatrix} = \text{error} \\
 \text{3x2} \quad \text{2x2}
 \end{array}$$

Scalar multiplication:

real number

$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} \times 3$$

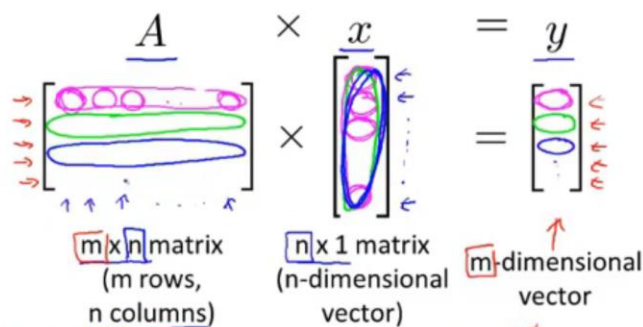
$$\begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} / 4 = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & \frac{3}{4} \end{bmatrix}$$

Combination of operands:

$$\begin{array}{c}
 \rightarrow 3 \times \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} / 3 \\
 \text{Scalar multiplication} \quad \text{Scalar division} \\
 = \begin{bmatrix} 3 \\ 12 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ \frac{2}{3} \end{bmatrix} \\
 \text{matrix subtraction / vector subtraction} \\
 = \begin{bmatrix} 2 \\ 12 \\ 10\frac{1}{3} \end{bmatrix} \\
 \text{matrix addition / vector addition} \\
 \text{3x1 matrix} \\
 \text{3-dimensional vector}
 \end{array}$$

3. Matrix-vector multiplication

Details:



To get y_i , multiply A 's i^{th} row with elements of vector x , and add them up.

Example

$$\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 4 \\ -1 & -2 & 0 & 0 \end{bmatrix}_{3 \times 4} \times \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}_{4 \times 1} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix}$$

$$\left. \begin{aligned} 1 \times 1 + 2 \times 3 + 1 \times 2 + 5 \times 1 &= 14 \\ 0 \times 1 + 3 \times 3 + 0 \times 2 + 4 \times 1 &= 13 \\ -1 \times 1 + (-2) \times 3 + 0 \times 2 + 0 \times 1 &= -7 \end{aligned} \right\}$$

A neat trick:

House sizes:

- 2104
- 1416
- 1534
- 852

Matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix}_{4 \times 2} \times \begin{bmatrix} -40 \\ 0.25 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} -40 \times 1 + 0.25 \times 2104 \\ -40 \times 1 + 0.25 \times 1416 \\ \vdots \\ -40 \times 1 + 0.25 \times 852 \end{bmatrix}_{4 \times 1}$$

$h_\theta(x) = -40 + 0.25x$
 $h_\theta(x)$
 $h_\theta(2104)$
 $h_\theta(1416)$

prediction = DataMatrix \times Parameters

for $i = 1, \dots, 1000$,
prediction(i) = ...

4. Matrix-matrix multiplication

Details:

$$\begin{array}{c} \underline{A} \quad \times \quad \underline{B} \quad = \quad \underline{C} \\ \left[\begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right] \times \left[\begin{array}{c|c|c} \vdots & \vdots & \vdots \\ \hline \vdots & \vdots & \vdots \end{array} \right] = \left[\begin{array}{c|c|c} \vdots & \vdots & \vdots \\ \hline \vdots & \vdots & \vdots \end{array} \right] \\ \text{m} \times \text{n} \text{ matrix} \quad \text{n} \times \text{o} \text{ matrix} \quad \text{m} \times \text{o} \text{ matrix} \\ \text{(m rows, n columns)} \quad \text{(n rows, o columns)} \end{array}$$

The i^{th} column of the matrix C is obtained by multiplying A with the i^{th} column of B . (for $i = 1, 2, \dots, o$)

House sizes:

$$\begin{Bmatrix} 2104 \\ 1416 \\ 1534 \\ 852 \end{Bmatrix}$$

Have 3 competing hypotheses:

$$\begin{array}{l} 1. h_{\theta}(x) = -40 + 0.25x \\ 2. h_{\theta}(x) = 200 + 0.1x \\ 3. h_{\theta}(x) = -150 + 0.4x \end{array}$$

$$\begin{array}{c} \text{Matrix} \\ \begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \end{array} \times \begin{array}{c} \text{Matrix} \\ \begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix} \end{array} = \begin{array}{c} \begin{bmatrix} 486 & 410 & 692 \\ 314 & 342 & 416 \\ 344 & 353 & 464 \\ 173 & 285 & 191 \end{bmatrix} \end{array}$$

Prediction of first h_{θ}
Predictions of 2nd h_{θ}

5. Matrix multiplication properties

not commutative:

Let A and B be matrices. Then in general,
 $A \times B \neq B \times A$. (not commutative.)

E.g. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ | $\begin{matrix} A \times B \\ m \times n \quad n \times m \end{matrix}$
 $\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$ | $\begin{matrix} A \times B \text{ is } m \times m \\ B \times A \text{ is } n \times n \end{matrix}$

associative:

$3 \times 5 \times 2$ $3 \times (5 \times 2) = (3 \times 5) \times 2$
 $3 \times 10 = 30 = 15 \times 2$ "Associative"

$A \times (B \times C)$ \leftarrow
 $(A \times B) \times C$ \leftarrow
 $A \times B \times C$.
 Let $D = B \times C$. Compute $A \times D$. $A \times (B \times C)$
 Let $E = A \times B$. Compute $E \times C$. $(A \times B) \times C$
 Same answer.

Identity Matrix:

Identity Matrix 1 is identity. $1 \times z = z \times 1 = z$ for any z
 Denoted I (or $I_{n \times n}$).
 Examples of identity matrices:
 $\begin{bmatrix} 1 \end{bmatrix}_{1 \times 1}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4}$
 Informally: $\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$
 For any matrix A ,
 $A \cdot I = I \cdot A = A$
 $\begin{matrix} m \times n & n \times n & n \times m & m \times n & m \times n \end{matrix}$
 $I_{n \times n}$ | Note:
 $AB \neq BA$ in general
 $AI = IA$ ✓

Andrew Ng

6. Inverse and transpose

It always must be a square matrix.

1 = "identity."

$$3 \underbrace{(3^{-1})}_{\frac{1}{3}} = 1$$

$$12 \times \underbrace{(12^{-1})}_{\frac{1}{12}} = 1$$

$$0 \underbrace{(0^{-1})}_{\text{undefined}}$$

Not all numbers have an inverse.

Matrix inverse:

square matrix
(# rows = # columns)

A^{-1}

If A is an $m \times m$ matrix, and if it has an inverse,

$$\rightarrow \underline{A(A^{-1}) = A^{-1}A = I.}$$

e.g. $\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}}_A \underbrace{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}}_{A^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{A^{-1}A} = I_{2 \times 2}$

Matrices that don't have an inverse are "singular" or "degenerate".

Matrix Transpose

Example:

$$\underline{A} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix}$$

2×3

$$\underline{B} = \underline{A}^T = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$$

3×2

Let A be an $m \times n$ matrix, and let $B = A^T$.

Then B is an $n \times m$ matrix, and

$$\underline{B_{ij}} = \underline{A_{ji}}.$$

$$B_{12} = A_{21} = 2$$

$$B_{32} = 9 \quad A_{23} = 9.$$