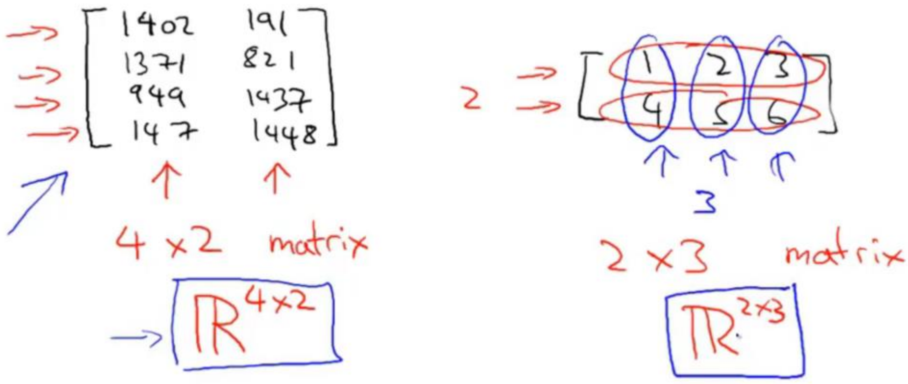
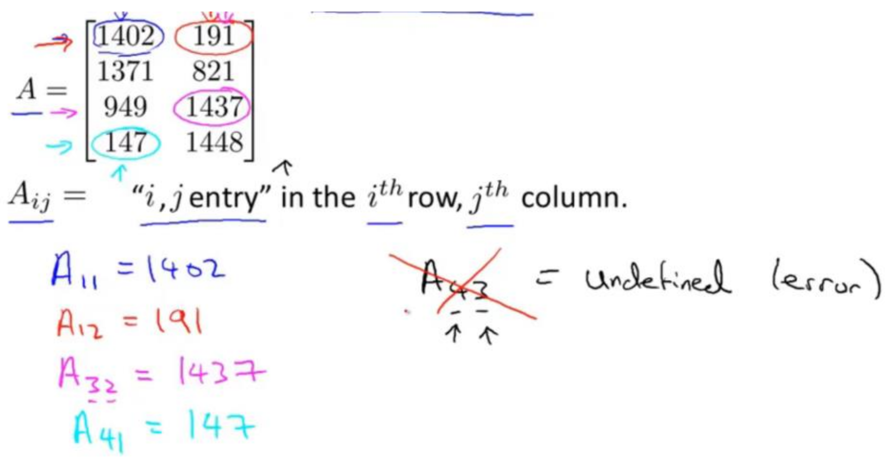
Linear Algebra review

1. Matrices and vectors

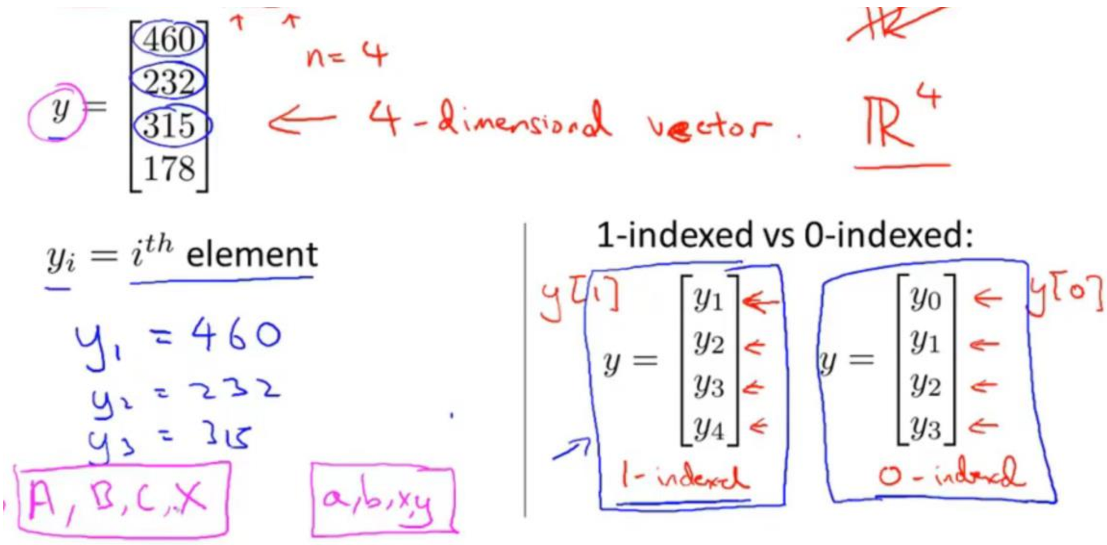
Matrix: Rectangular array of numbers
Dimension of matrix: number of rows x number of columns



Matrix elements(entries of matrix)



Vector: An n x 1 matrix.



2. Addition and scalar multiplication

Matrix addition:

$$\underbrace{\begin{bmatrix}1&0\\2&5\\3&1\end{bmatrix}}_{3\times 2\ \text{matrix}} + \underbrace{\begin{bmatrix}4&0.5\\2&5\\0&1\end{bmatrix}}_{3\times 2} = \underbrace{\begin{bmatrix}5&0.5\\4&10\\3&2\end{bmatrix}}_{3\times 2}$$

$$\cancel{\underbrace{\begin{bmatrix}1&0\\2&5\\3&1\end{bmatrix}}_{3\times 2} + \underbrace{\begin{bmatrix}4&0.5\\2&5\end{bmatrix}}_{2\times 2}} = \text{error}$$

Scalar multiplication:

real number

$$3 \times \underbrace{\begin{bmatrix}1&0\\2&5\\3&1\end{bmatrix}}_{3\times 2} = \underbrace{\boxed{\begin{bmatrix}3&0\\6&15\\9&3\end{bmatrix}}}_{3\times 2} = \begin{bmatrix}1&0\\2&5\\3&1\end{bmatrix} \times 3$$

$$\boxed{\begin{bmatrix}4&0\\6&3\end{bmatrix}/4} = \frac{1}{4}\begin{bmatrix}4&0\\6&3\end{bmatrix} = \begin{bmatrix}1&0\\ \frac{3}{2}&\frac{3}{4}\end{bmatrix}$$

Combination of operands:

$$\boxed{3 \times \begin{bmatrix}1\\4\\2\end{bmatrix}} + \begin{bmatrix}0\\0\\5\end{bmatrix} - \boxed{\begin{bmatrix}3\\0\\2\end{bmatrix}/3}$$

Scalar multiplication

Scalar division

$$= \begin{bmatrix}3\\12\\6\end{bmatrix} + \begin{bmatrix}0\\0\\5\end{bmatrix} - \begin{bmatrix}1\\0\\ \frac{2}{3}\end{bmatrix}$$

matrix subtraction / vector subtraction

matrix addition / vector addition

$$= \begin{bmatrix}2\\12\\10\frac{1}{3}\end{bmatrix}$$

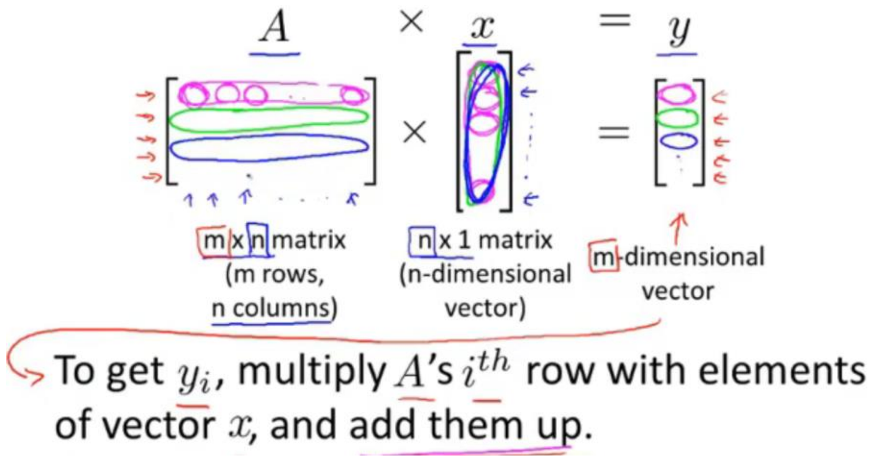
3×1 matrix

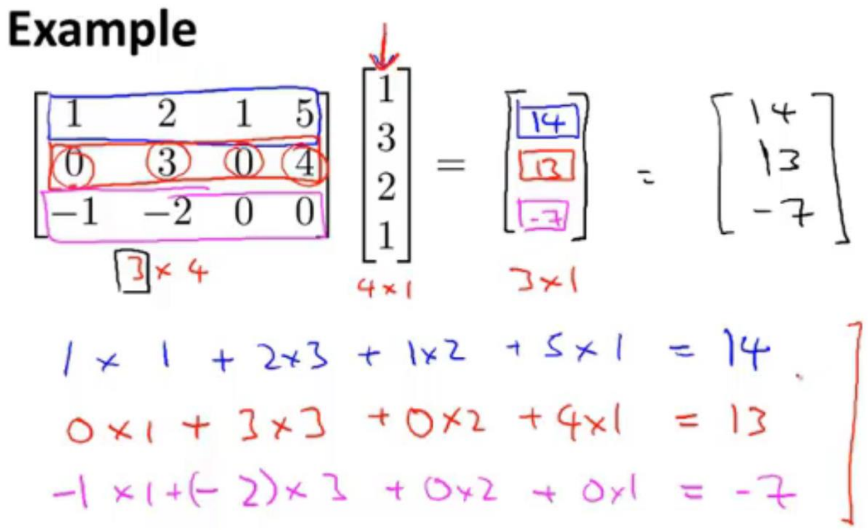
3-dimensional vector

Andrew Ng

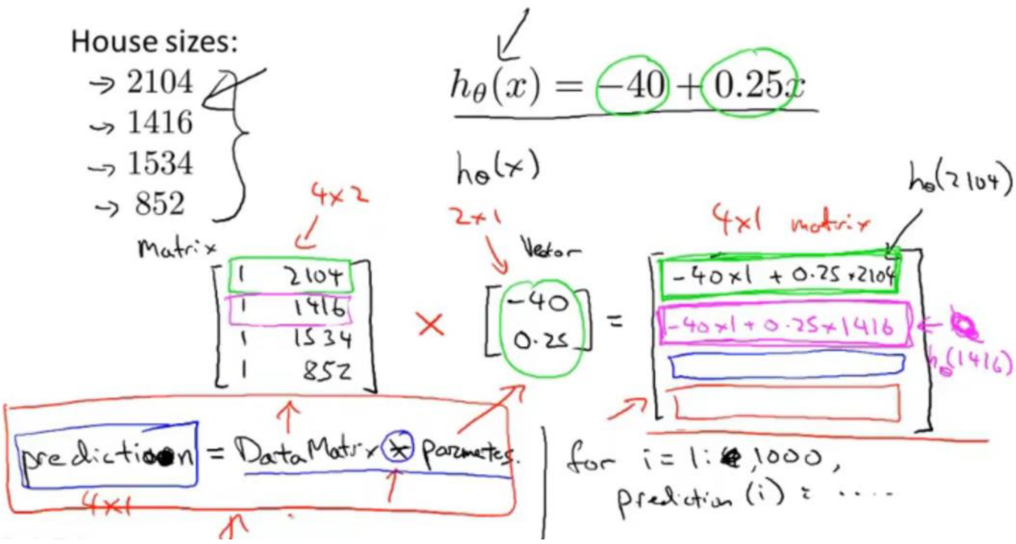
3. Matrix-vector multiplication

Details:

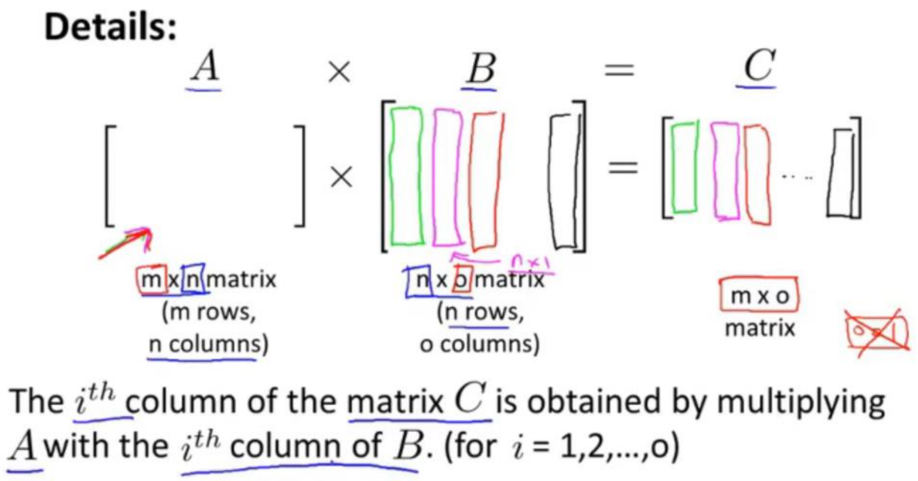


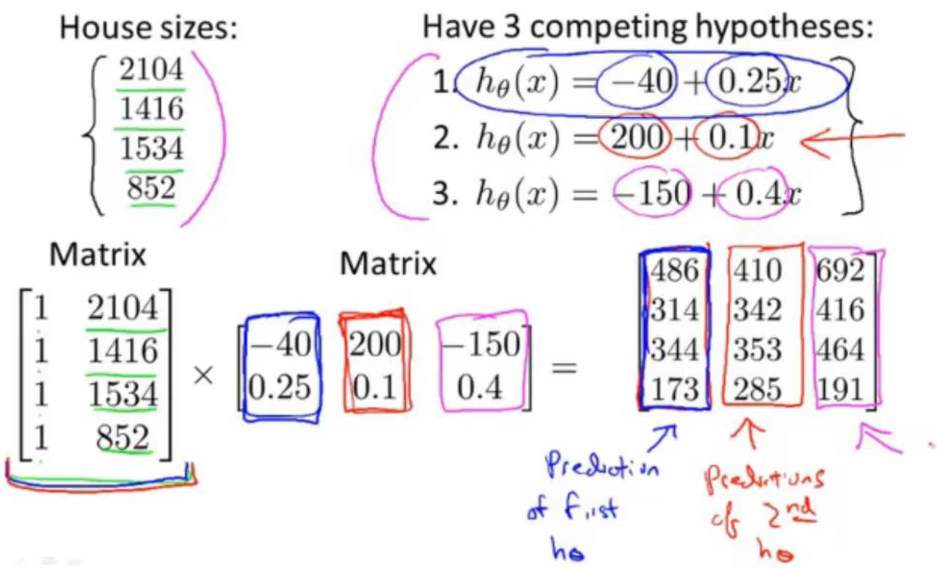


A neat trick:



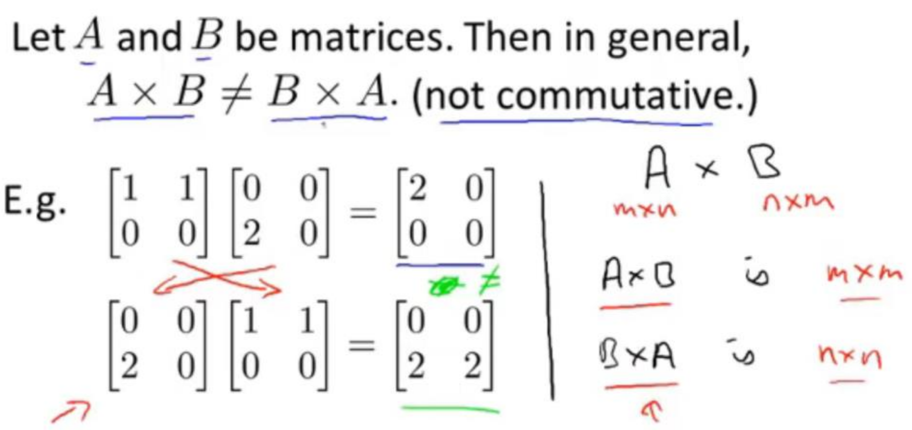
4. Matrix-matrix multiplication



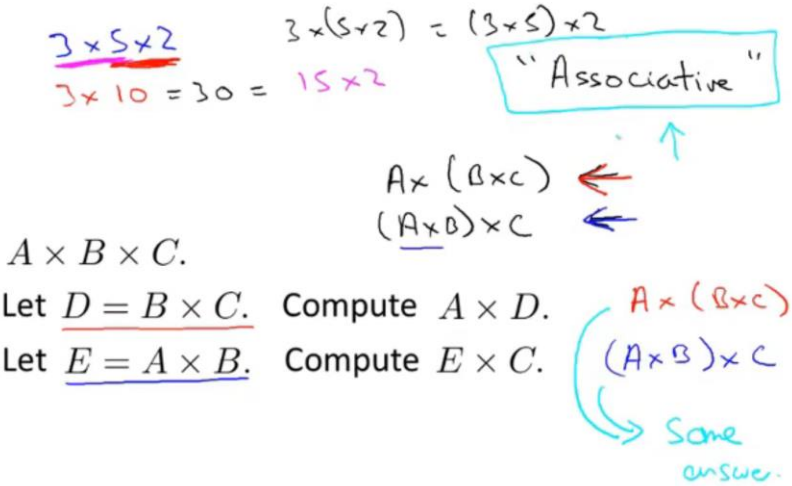


5. Matrix multiplication properties

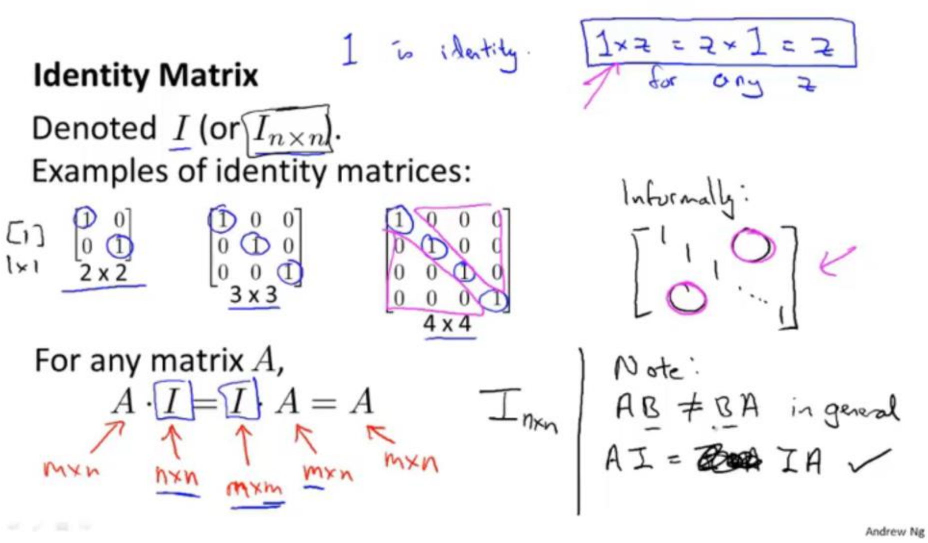
not commutative:



associative:



Identity Matrix:



6. Inverse and transpose

It always must be a square matrix.

I = "identity" $3\boxed{(3^{-1})} = 1$ $12 \times (\underline{12^{-1}}) = 1$

$\frac{1}{3}$ $\frac{1}{12}$

Not all numbers have an inverse. $0\,(0^{-1})$ undefined

Matrix inverse: square matrix (#rows = #columns) A^{-1}

If A is an m x m matrix, and if it has an inverse,

$\rightarrow$ $A(A^{-1}) = A^{-1}A = I.$

E.g. $\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}}_{A^{-1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$

2x2 $A^{-1}A$

Matrices that don't have an inverse are "singular" or "degenerate".

Matrix Transpose

Example: $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix}$ $B = A^T = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$

2x3   3x2

Let A be an m x n matrix, and let $B = A^T$.

Then B is an n x m matrix, and

$$B_{ij} = A_{ji}.$$

$B_{12} = A_{21} = 2$

$B_{32} = 9$ $A_{23} = 9.$