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# ATTITUDE STABILIZATION OF A SATELLITE BY MAGNETIC COILS

VLADIMIR A. BUSHENKOV†

Departamento de Matemática, Universidade de Évora, Apartado 94, 7000 Évora, Portugal

MICHAEL YU. OVCHINNIKOV

Keldysh Institute of Applied Mathematics, 4 Miusskaya Sq., 125047 Moscow, Russia

and

GEORGI V. SMIRNOV

Centro de Matemática Aplicada, Departamento de Matemática Aplicada, Faculdade de Ciências, Universidade do Porto, Rua das Taipas 135, 4050 Porto, Portugal

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**Abstract**—Stabilization problem for a satellite is considered. The only measurement is of the geomagnetic field in the satellite coordinates. The control is carried out by a magnetic moment of current coils (magnetorquers) mounted on the satellite body. The stabilizer constructed in this work solves the problems of magnetic and gravitational stabilization. Qualitative analysis and results of numerical simulation are presented. The results of simulation show that the proposed stabilization system is reliable, and has an appropriate accuracy and does not need powerful sources of energy, and therefore can be used for attitude control of small satellites. © 2002 Published by Elsevier Science Ltd.

## 1. INTRODUCTION‡

The achievements of modern electronics, computer technology, material science, optics etc. give a possibility to create small satellites capable to solve problems usually considered as problems for a big and expensive spacecraft. During the last time small satellites became widely widespread because of their relative simplicity resulting in an attractive short period of design and in low cost.

Attitude control system is one of the main auxiliary systems. Its reliability and cost is of importance for the reliability and cost of the whole satellite. The attitude control of small satellites is usually fulfilled by passive or semi-passive systems. These attitude control systems use the satellite interaction with the gravitational and magnetic field of the planet, atmospheric drag and the gyroscopic properties of spinning bodies. The properties of all these phenomena are well-known. The general approaches to obtain main parameters of passive attitude control systems are presented in [1], for example. These satellite orientation methods are sufficient if the satellite does not require complex reorientation maneuvers during the flight. Otherwise, active attitude control systems, like three-rotor flywheel systems, three-magnetorquers systems, or their combinations, with actuators enable to vary control torque, have to be used. From the simplicity point of view the most reliable and light attitude control systems are active magnetic ones. At the beginning of the Space Era the magnetic attitude control systems were mostly used for desaturation of gyroscopic actuators and to control the orientation of the satellite spin-axis and the spin-velocity [2–4]. The next stage concerned with the application of magnetorquers in hybrid attitude control systems where they were used in combination with another actuators like flywheel [5,6]. Combination with a gravitational boom was used, for instance, in the microsatellites UoSat [7]. During the last years several attitude control systems with magnetorquers only

†Corresponding author.
*E-mail address:* bushen@dmat.uevora.pt (V. A. Bushenkov).
‡See Nomenclature in Appendix at the end of the paper.

were developed. An attitude control system using magnetorquers as momentum damping devices and magnetometers as sensors is mentioned in [8]. The system is constructed using standard control techniques applied to the second-order differential equations describing the satellite attitude dynamics under the assumption of small angles. Another attitude control system for an isoinertial spacecraft was proposed in [9]. It does not need the small angle assumption but uses a complete attitude information, that is, a complicated filtering procedure or additional sensors are required.

In this paper the problem of attitude control by magnetorquers is considered. It is assumed that the satellite moves along a circular orbit and that the current value of the geomagnetic field with respect to the satellite axes is known due to three-axis magnetometer measurements. The position of the satellite mass center is also assumed to be known. The magnetic torque is created by interaction of the magnetic field generated by coils mounted onto the satellite body with the geomagnetic field. The magnitudes and polarities of currents in the coils are control parameters. The stabilization system described below solves the problem of magnetic and gravitational stabilization. As our numerical simulation shows, it is reliable, has an appropriate accuracy and does not need powerful sources of energy, and therefore can be used for attitude control of small satellites.

The paper is organized as follows. In Section 2 the statement of the problem is given. Section 3 is devoted to the main ideas of the stabilizer design. The problems of magnetic and gravitational stabilization are considered in Section 4. The stabilization algorithm and results of numerical simulation are presented in Section 5. Section 6 contains the conclusions.

## 2. STATEMENT OF THE PROBLEM

Consider a satellite moving along a circular orbit. Introduce two Cartesian reference systems $OX_1X_2X_3$ and $ox_1x_2x_3$. The system $OX_1X_2X_3$ is the body reference system. The origin of the system coincides with the satellite mass center and the axes are directed along the main axes of the inertia ellipsoid. The system $ox_1x_2x_3$ is the orbital reference system. The point $o$ coincides with the center of the Earth. The axis $ox_3$ is directed along the radius vector of the satellite mass center. The axis $ox_2$ is perpendicular to the orbital plane.

Coordinates $x$ and $X$ of a vector in the systems $ox_1x_2x_3$ and $OX_1X_2X_3$, respectively, satisfy the relation $x = BX$, where $B$ is an orthogonal matrix with $\det B = 1$. Denote by $b_i$, $i = 1,2,3$, the rows of the matrix $B$. Let $\Omega_0$ be the vector of angular velocity in the body axes. The rotation of the satellite is described by the following equations:

$$\dot{b}_i = b_i \times (\Omega_0 - \omega b_2), \quad i = 1,2,3, \tag{1}$$

$$J\dot{\Omega}_0 = -\Omega_0 \times (J\Omega_0) + 3\omega b_3 \times (Jb_3) + U_0 \times F, \tag{2}$$

where

- the tensor of inertia $J$ is a diagonal matrix with the elements $J_1, J_2, J_3$ ($J_3 < J_1 \leq J_2$),
- the vector $F$ of the geomagnetic field in the body axes is given by $F = B^{\mathrm{T}} f$, $f = \mu_0 \mu_m r^{-3}(\cos u \sin i, \cos i, -2 \sin u \sin i)$, (the field of a central 'right' dipole),
- $i$ is the orbital inclination, that is, the angle between the equatorial and orbital planes,
- $\omega$ the $(\mu_g/r^3)^{1/2}$ is the angular velocity of the orbital motion,
- $r$ is the radius of the orbit,
- $\mu_{\mathrm{m}}$ the $8.06 \times 10^{22}$ A m$^2$ is the Earth magnetic dipole moment,
- $\mu_{\mathrm{g}}$ the $3.986 \times 10^{14}$ m$^3$/c$^2$ is the Earth gravitational parameter,
- $\mu_0$ the $4\pi \times 10^{-7}$ H/m is the permanence of vacuum,
- the latitude argument $u = u(t)$ is given by $u = \omega_\pi + \omega(t - t_0)$, $\omega_\pi$ and $t_0$ are parameters,
- the vector $U_0$ has the components $U_0^i = I^i wS$, $i = 1,2,3$, where $I^i$ stands for the current in the $i$th coil, $w$ is the number of turns, and $S$ is the area of a loop.

The currents $I^i$, $i = 1,2,3$, are control parameters satisfying the constraints $|I^i| \leqslant I_{\max}$. The problem is to find control laws $I^i = I^i(F)$ such that the matrix $B(t)$ tends to a given matrix $\hat{B}(t)$. If $\hat{B}^{\mathrm{T}}(t) f(t) \equiv V$, a constant vector, we get the magnetic stabilization problem, that is, the aim of stabilization is the coincidence of the vectors $F(t)$ and $V$. If $\hat{B}(t) \equiv I$, where $I$ is the identity matrix, we have the gravitational stabilization problem.

### 3. AUXILIARY STABILIZATION PROBLEM

The analysis of the system in the general case is rather involved. For this reason, we shall consider a simplified model of the geomagnetic field and a spherically symmetric satellite to explain the main ideas of the stabilizer design.

If the satellite is dynamically symmetric, i.e. $J = cI$, where $c$ is a constant, then introducing notations $\Omega = \Omega_0 - \omega b_2$ and $U = U_0/c$ from eqns (1) and (2) we get

$$\dot{b}_i = b_i \times \Omega, \quad i = 1,2,3, \tag{3}$$

$$\dot{\Omega} = U \times F - (\omega/c) b_2 \times \Omega. \tag{4}$$

For simplicity assume that the field motion is described by the following system:

$$\dot{f} = f \times v, \tag{5}$$

$$\dot{v} = 0. \tag{6}$$

Moreover we assume that $|f| = 1$, $\langle f, v\rangle = 0$, and $|v| = \omega$. In the frame of the satellite control problems this model appeared in [10]. Since $|U| \gg \omega|\Omega|$ the eqns (3) and (4) can be written in a very simple form

$$\dot{b}_i = b_i \times \Omega, \quad i = 1,2,3, \tag{7}$$

$$\dot{\Omega} = U \times F, \tag{8}$$

$$f = BF, \tag{9}$$

where $f$ is the magnetic field and $U$ is a control we have in our disposal. We shall consider the following stabilization problem. Let $q(t)$ be a unit vector. Assume that its motion is governed by the differential equation

$$\dot{q} = q \times s, \tag{10}$$

where $s = s(t)$ is a known vector. Put $Q = B^* q$ and $S = B^* s$. The aim is to find $U = U(t, B, \Omega)$ such that the vector $Q(t)$ tends to a given motion $\hat{Q}(t)$, i.e. $\lim_{t\to\infty} |Q(t) - \hat{Q}(t)| = 0$. For simplicity the function $\hat{Q}(t)$ is assumed to be periodic.

To solve the stabilization problem we have to derive the equations describing the vector $Q(t)$ dynamics. Differentiating $Q$, from eqns (7) and (10) we get $\dot{Q} = Q \times (\Omega + S)$. This can be rewritten as

$$\dot{Q} = Q \times P, \tag{11}$$

where $P = Q \times ((\Omega + S) \times Q)$ is the projection of the vector $\Omega + S$ along the vector $Q$. Differentiating $P$, from eqns (8) and (11) we obtain

$$\begin{aligned}\dot{P} &= \frac{\mathrm{d}}{\mathrm{d}t}[Q \times ((\Omega + S) \times Q)] \\ &= (Q \times P) \times ((\Omega + S) \times Q) \\ &\quad + Q \times ((\Omega + S) \times (Q \times P)) \\ &\quad + Q \times ((U \times F + \dot{S}) \times Q).\end{aligned}$$

Since

$$\begin{aligned}&(Q \times P) \times ((\Omega + S) \times Q) \\ &\quad = (Q \times P) \times ((Q \times ((\Omega + S) \times Q)) \times Q) \\ &\quad = (Q \times P) \times (P \times Q) = 0,\end{aligned}$$

we have $\dot{P} = W \times Q$, where

$$\begin{aligned}W &= Q \times [((Q \times P) \times (\Omega + S) + Q \times (U \times F) \\ &\quad + Q \times \dot{S}) \times Q].\end{aligned} \tag{12}$$

So, it suffices to solve the following auxiliary control problem. The control system is

$$\dot{Q} = Q \times P, \tag{13}$$

$$\dot{P} = W \times Q, \tag{14}$$

where the vector $W$ is a control. The aim is to find $W = W(t, Q, P)$ such that the solution $(Q(t), P(t))$ to eqns (13) and (14) with the initial conditions $Q(0)$ and $P(0)$ satisfying

$$|Q(0)| = 1 \quad \text{and} \quad \langle Q(0), P(0)\rangle = 0, \tag{15}$$

satisfies $\lim_{t\to\infty} |(Q, P) - (\hat{Q}, \dot{\hat{Q}} \times \hat{Q})| = 0$, where $\hat{Q}(t)$ is a given vector function with $|\hat{Q}(t)| \equiv 1$. First note that

$$\langle Q(t), P(t)\rangle \equiv 0 \tag{16}$$

for any solution of eqns (13) and (14) with the initial condition satisfying eqn (15). Indeed, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle Q, P\rangle = \langle Q \times P, P\rangle + \langle Q, W \times Q\rangle = 0.$$

Note also that, since $|\hat{Q}| \equiv 1$, we have $\langle \dot{\hat{Q}}, \hat{Q} \rangle \equiv 0$. Put

$$\hat{P} = \hat{P}(t,Q) = \dot{\hat{Q}} \times \hat{Q} - Q\langle \dot{\hat{Q}} \times \hat{Q}, Q\rangle + \gamma \hat{Q} \times Q \tag{17}$$

and

$$\check{P} = \check{P}(t,Q,P) = \ddot{\hat{Q}} \times \hat{Q} - Q \times P\langle \dot{\hat{Q}} \times \hat{Q}, Q\rangle - Q\langle \ddot{\hat{Q}} \times \hat{Q}, Q\rangle - Q\langle \dot{\hat{Q}} \times \hat{Q}, Q \times P\rangle + \gamma \dot{\hat{Q}} \times Q + \gamma \hat{Q} \times (Q \times P), \tag{18}$$

where $\gamma > 0$. Obviously $\dot{\hat{P}} = \check{P}$. We define $W$ by

$$W(t,Q,P) = (\alpha(P - \hat{P}) - \check{P}) \times Q, \tag{19}$$

where $\alpha > 0$. Substituting eqn (19) for $W$ in eqns (13) and (14), we get

$$\frac{\mathrm{d}}{\mathrm{d}t} Q = Q \times \hat{P} + Q \times (P - \hat{P}), \tag{20}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(P - \hat{P}) = -\alpha(P - \hat{P}) - Q\langle \check{P}, Q\rangle. \tag{21}$$

From eqns (16) and (17) we see that $\langle Q, P - \hat{P}\rangle \equiv 0$. Multiplying eqn (21) by $(P - \hat{P})$, we obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}|P - \hat{P}|^2 = -\alpha|P - \hat{P}|^2.$$

Hence

$$|P(t) - \hat{P}(t)| = |P(0) - \hat{P}(0)| \exp(-\alpha t). \tag{22}$$

Multiplying eqn (20) by $\hat{Q}$ and adding $\langle \dot{\hat{Q}}, Q\rangle$ to both sides of the obtained equality, after simple calculations we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \hat{Q}, Q\rangle = \gamma(1 - \langle \hat{Q}, Q\rangle^2) + \langle \hat{Q} \times Q, P - \hat{P}\rangle. \tag{23}$$

From eqn (22) we see that $|P(t) - \hat{P}(t)| \to 0$ as $t \to \infty$. Therefore if $\langle \hat{Q}, Q\rangle \nrightarrow -1$, then eqn (23) takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \hat{Q}, Q\rangle \approx \gamma(1 - \langle \hat{Q}, Q\rangle^2),$$

whenever $t$ is big enough. Thus we see that $\langle \hat{Q}, Q\rangle \to 1$.

A detailed analysis of this stabilization problem is given in [11], where it is shown that any solution $(Q,P)$ to eqns (13) and (14) with eqn (19) and the initial condition satisfying eqn (15) tends either to $(\hat{Q}, \dot{\hat{Q}} \times \hat{Q})$ or to $(-\hat{Q}, \dot{\hat{Q}} \times \hat{Q})$; the solution $(\hat{Q}, \dot{\hat{Q}} \times \hat{Q})$ is asymptotically stable and the solution $(-\hat{Q}, \dot{\hat{Q}} \times \hat{Q})$ is unstable; system eqns (13) and (14) with eqns (19) and (15) has a two-dimensional manifold of initial conditions such that any trajectory $(Q(t), P(t))$ starting at the manifold does not tend to $(\hat{Q}, \dot{\hat{Q}} \times \hat{Q})$; any stabilizer for system eqns (13) and (14) has a two-dimensional manifold of initial conditions such that each trajectory $(Q(t), P(t))$ starting at the manifold does not tend to $(\hat{Q}, \dot{\hat{Q}} \times \hat{Q})$. This implies that the stabilizer constructed above has the largest possible region of attraction.

## 4. APPLICATION TO THE SATELLITE STABILIZATION PROBLEM

In this section we apply the developed techniques to solve some stabilization problems. Let $q(t) = f(t)$. Recall that $Q = B^* q = F = B^* f$. The aim is to find $U = U(t, F, \dot{F})$ such that $\lim_{t\to\infty} |Q(t) - \hat{Q}(t)| = 0$, where $\hat{Q}(t)$ is a given vector function. The stabilization algorithm introduced above can be applied to solve the problem if the equation

$$Q \times [((Q \times P) \times (\Omega + S) + Q \times (U \times F) + Q \times \dot{S}) \times Q] = (\alpha(P - \hat{P}) - \check{P}) \times Q, \tag{24}$$

where $\hat{P}$ and $\check{P}$ are given by eqns (17) and (18), respectively, can be solved with respect to $U$ (see eqns (12) and (19)). Since $|P||\Omega + S| \ll |U|$ and $|\dot{S}| \ll |U|$, eqn (24) can be simplified and we get

$$U - F\langle F, U\rangle = G \times F, \tag{25}$$

where $G = \alpha(P - \hat{P}) - \check{P}$. Obviously $U = G \times F$ solves this equation.

### 4.1. Magnetic stabilization

Set $\hat{Q}(t) \equiv V = (\text{const})$. This problem is known as the problem of magnetic stabilization, that is, one of the satellite's axes (the axis $V$) has to track the geomagnetic field vector. Denote by $N$ the vector $v$ in the satellite axes: $N = B^* v$. We have $S = N$

and

$$P = F \times ((\Omega + N) \times F) = \dot{F} \times F,$$

$$\hat{P} = \gamma F \times ((V \times N) \times F) = \gamma V \times F,$$

$$\check{P} = \gamma V \times (F \times (\dot{F} \times F)) = \gamma V \times \dot{F}.$$

The control

$$U = (\alpha(\dot{F} \times F - \gamma V \times F) - \gamma V \times \dot{F}) \times F$$

stabilizes the satellite to a motion with $F(t) \approx V$, i.e. to a regime of "almost" magnetic stabilization.

### 4.2. *Gravitational stabilization*

Consider the gravitational stabilization problem, i.e. the matrix $B$ has to coincide with the unitary matrix $I$. This implies, for example, that we must have $F(t) \equiv f(t)$. Set $\hat{Q}(t) = f(t)$. We have

$$P = F \times ((\Omega + N) \times F) = \dot{F} \times F,$$

$$\hat{P} = \dot{f} \times f - F\langle \dot{f} \times f, F\rangle + \gamma f \times F,$$

$$\check{P} = \ddot{f} \times f - F \times (\dot{F} \times F)\langle \dot{f} \times f, F\rangle$$
$$+ \gamma \dot{f} \times F + \gamma f \times (F \times (\dot{F} \times F)) + \psi(t)F,$$

where $\psi(t)$ is a function. The control

$$\begin{aligned} U = {} & (\alpha(\dot{F} \times F - \dot{f} \times f + F\langle \dot{f} \times f, F\rangle - \gamma f \times F) \\ & - \ddot{f} \times f + F \times (\dot{F} \times F)\langle \dot{f} \times f, F\rangle \\ & - \gamma \dot{f} \times F - \gamma f \times (F \times (\dot{F} \times F))) \times F \end{aligned}$$

stabilizes the satellite to a motion with $F(t) \approx f(t)$. Show that if the satellite moves in a neighborhood of the gravitational mode, then the equality $F(t) = f(t)$ implies $B = I$. To this end we show that $N = v$. Since $B$ is an orthogonal matrix the equalities $f = Bf$ and $v = Bv$ are equivalent with $B = I$. First note that $F(t) = f(t)$ implies $U = 0$ (we omit the calculations). Then, from eqn (11) we get

$$\dot{f} = f \times (\Omega + N). \tag{26}$$

Equations (5) and (7) imply

$$\dot{N} = N \times \Omega. \tag{27}$$

Since $U = 0$, eqn (8) takes the form

$$\dot{\Omega} = 0. \tag{28}$$

Moreover we have

$$\langle v, f\rangle = \langle N, f\rangle = 0. \tag{29}$$

Combining eqns (26), (5), and (29), we get

$$\Omega + N = v + f\langle \Omega, f\rangle. \tag{30}$$

Differentiating this equality and invoking eqs (26), (27), and (28), we obtain

$$\begin{aligned} N \times \Omega &= \dot{f}\langle \Omega, f\rangle + f\langle \Omega, \dot{f}\rangle \\ &= f \times v\langle \Omega, f\rangle + f\langle \Omega, f \times N\rangle. \end{aligned} \tag{31}$$

Consider the case $\langle \Omega, f\rangle = 0$. From eqn (30) we have $\Omega + N = v$. Therefore eqns (28) and (6) imply $\dot{N} = 0$. From eqn (27) we obtain $\Omega = \mu N$. Combining this with equality (26), we get $\dot{f} = f \times N(1+\mu)$. Now eqn (5) implies $v = (1+\mu)N$. Since $v = BN$, we see that $|v| = |N|$. Therefore either $\mu = 0$ and $N = v$ or $\mu = -2$ and $N = -v$. Now consider the case $\langle \Omega, f\rangle \neq 0$. Multiplying eqn (31) by $N$, we get

$$0 = \langle f \times v, N\rangle\langle \Omega, f\rangle.$$

Therefore

$$0 = \langle f \times v, N\rangle.$$

This implies $f \perp v \times N$. Invoking eqn (29) we conclude that $v || N$. Thus either $N = v$ or $N = -v$. From the above consideration we see that if the satellite moves in a neighborhood of the gravitational mode, the condition $F \approx f$ is equivalent with $N \approx v$ and therefore with $B \approx I$.

## 5. STABILIZATION ALGORITHM AND NUMERICAL SIMULATION

From the previous consideration, we see that the following stabilization algorithm provides the satellite with the gravitational stabilization mode. First the satellite stabilizes to the magnetic stabilization mode. Then when the satellite passes the magnetic pole where $B$ is rather close to $I$ the gravitational stabilizer starts its work and drives the satellite to the final gravitational stabilization mode. Our considerations concerned the simplified model (the Zajak model of the geomagnetic field and spherically symmetric satellite). The study of the general situation is more complicated but the hart of the matter is the same and the qualitative behavior of the system is very similar to the model situation considered here. Motivated by the above results and by our computational experiments we propose the following stabilization algorithm:

```
            START;
MagnStab:   PassNumber_Magn:=0;   StabilTime_Magn:=0;
            WHILE (PassNumber_Magn ≤ PLimit_Magn) BEGIN
                PassNumber_Magn:=PassNumber_Magn + 1;
                REPEAT (in each Δt seconds until the end of pass) BEGIN
                    CALCULATION of U_0 with Q̂(t) = (0, 0, 1);
                END
                IF (|Q(t) − Q̂(t)| ≤ δ_Magn always during the pass) THEN
                    StabilTime_Magn:=StabilTime_Magn + 1;
                ELSE StabilTime_Magn:=0;
                IF (StabilTime_Magn ≥ T_Magn) THEN GOTO GravStab;
            END
            TURN the control OFF;
            WAIT ONE PASS;
            GOTO MagnStab;
GravStab:   PassNumber_Grav:=0;  StabilTime_Grav:=0;
            WHILE (PassNumber_Grav ≤ PLimit_Grav) BEGIN
                PassNumber_Grav:=PassNumber_Grav + 1;
                REPEAT (in each Δt seconds until the end of pass) BEGIN
                    IF (the control is turned ON) THEN
                        CALCULATION of U_0 with Q̂(t) = f(t)/|f(t)|;
                    ELSE U_0:=0;
                    IF (|Q(t) − Q̂(t)|) > δ_Grav) THEN BEGIN
                        TURN the control ON; StabilTime_Grav:=0;
                    END;
                END;
                IF (|Q(t) − Q̂(t)| ≤ δ_Grav always during the pass) THEN
                    StabilTime_Grav:=StabilTime_Grav + 1;
                ELSE StabilTime_Grav:=0;
                IF (StabilTime_Grav ≥ T_Grav) THEN TURN the control OFF;
                IF (StabilTime_Grav ≥ T_Grav + ΔT_Grav) THEN STOP;
            END;
            GOTO MagnStab;
```

with the parameters $\Delta t = 5$, $\mathrm{PLimit}_{\mathrm{Magn}} = 30$, $\delta_{\mathrm{Magn}} = 0.3$, $T_{\mathrm{Magn}} = 2$, $\mathrm{PLimit}_{\mathrm{Grav}} = 35$, $\delta_{\mathrm{Grav}} = 0.1$, $T_{\mathrm{Grav}} = 10$, $\Delta T_{\mathrm{Grav}} = 3$.

The control $U_0$ is calculated by

$$U_0 = |F|^{-1}(J(Q \times W)) \times Q,$$

$$\text{with saturation } |U_0^i| \leqslant 12.5,\ i = 1, 2, 3,$$

where

$$W = (\alpha(P - \hat{P}) - \check{P}),$$

$$P = Q' \times Q,$$

$$\hat{P} = \hat{Q}' \times \hat{Q} + \gamma \hat{Q} \times Q$$

$$\check{P} = \hat{Q}'' \times \hat{Q} - Q \times P\langle \hat{Q}' \times \hat{Q}, Q\rangle + \gamma \hat{Q}' \times Q + \gamma \hat{Q} \times (Q \times P),$$

$$Q' = \frac{Q(t) - Q(t-\tau)}{\tau},$$

$$\hat{Q}' = \frac{\hat{Q}(t + h/2) - \hat{Q}(t - h/2)}{h},$$

$$\hat{Q}'' = 4\frac{\hat{Q}(t + h/2) - 2\hat{Q}(t) + \hat{Q}(t - h/2)}{h^2},$$

$$h = 0.1 \quad \text{and} \quad \tau = 5.$$

Sometimes the magnetic and gravitational stabilizers drive the satellite not to the desired modes but to complex rotations. We observed this rare phenomenon in our numerical simulations. For this reason the algorithm is rather involved. It analyzes the situation and, if necessary, turns off the control, waits, and begins all over again. For example, if the magnetic stabilization mode is not reached after $\mathrm{PLimit}_{\mathrm{Magn}}$ passes, the control turns off, and the stabilizer begins its work again only after one

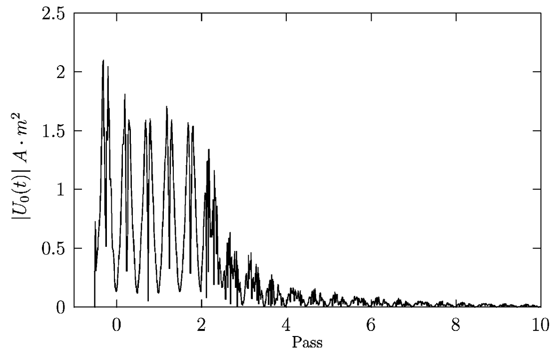


Fig. 1. The norm of $U_0(t)$.

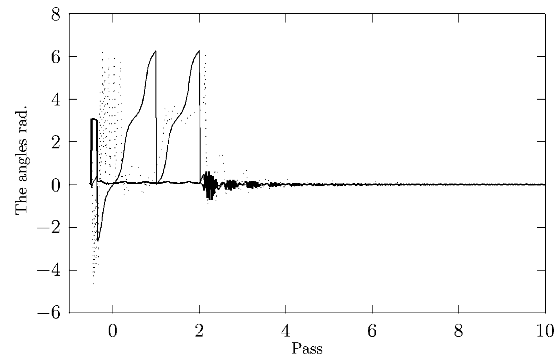


Fig. 2. The Euler angles.

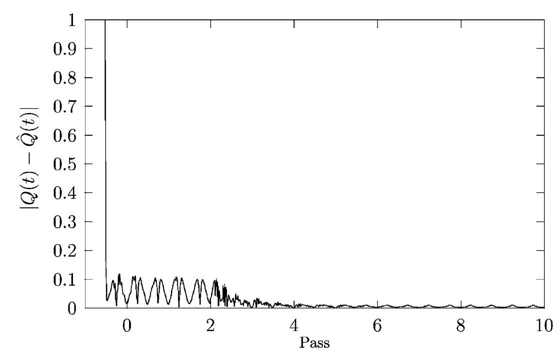


Fig. 3. The norm of the difference $Q(t) - \hat{Q}(t)$.

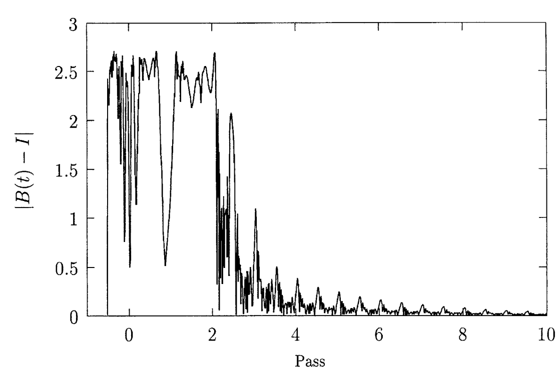


Fig. 4. The norm of the matrix $B(t) - I$.

pass. Since the failure of magnetic stabilization occurs very seldom, this strategy after all leads to the magnetic stabilization mode. The same concerns the gravitational stabilization. When the observed geomagnetic field $F(t)$ coincides with $f(t)$ during $T_{\mathrm{Grav}}$ passes the control turns off. If $B(t) = I$, then $F(t)$ will coincide with $f(t)$ for all $t$, since the gravitational stabilization mode is a stable equilibrium position of the satellite angular motion. If $B(t) \neq I$, the difference $F(t) - f(t)$ becomes different from zero. This means that the satellite is stabilized to a rotation about the vector of the geomagnetic field. If this is the case the control turns on and the stabilization begins again. The rotation about the vector of the geomagnetic field is a motion with a very small basin of attraction. Therefore usually after a few attempts it is possible to stabilize the satellite to the gravitational mode. If the gravitational mode is not achieved after $\mathrm{PLimit}_{\mathrm{Grav}}$ passes the stabilization starts from the very beginning.

To test the algorithm we considered a satellite with the following parameters:

- the tensor of inertia $J = \mathrm{diag}\,(1.7, 1.8, 1.4)$,
- the orbital inclination $i = \pi/2$,
- the radius of the orbit $r = 7.4 \times 10^6$ m,
- the parameter $t_0 = 0$,
- the parameter $\omega_\pi = 0$,
- the number of turns in the coil $w = 100$,
- the area of the loop $S = 0.25\ \mathrm{m}^2$,
- the maximal value of the current $I_{\max} = 0.5$ A.
- the stabilizer parameter $\alpha = 0.05$,
- the stabilizer parameter $\gamma = 0.05$.

System (1) and (2) was solved by the fourth-order Adams–Bashforth method with the step $h = 0.1$. A typical behavior of the stabilization process main characteristics can be seen in Figs. 1–4. One thousand numerical experiment were fulfilled. In all experiments we put $B(0) = I$. The initial angular velocity $\Omega_0(0)$ was a random vector uniformly distributed in the cube with the side 0.5 centered at the origin. The results are presented in Table 1. Table 1 shows the number of experiments where the gravitational stabilization mode was achieved after $n$ passes from the beginning of the flight.

Table 1. Results of the experiments

| Passes | 16 | 17 | 27 | 28 | 29 | 38 | 53 |
|---|---|---|---|---|---|---|---|
| Experiments | 835 | 876 | 952 | 965 | 980 | 998 | 1000 |

#### 6. CONCLUSION

A system of attitude stabilization has been proposed. A three-axis magnetometer is used as a sensor. The attitude control is fulfilled by three magnetorquers mounted on the satellite body. The control has been obtained as a function of the satellite mass center position and the magnetometer data. The stabilization system solves the problem of magnetic and gravitational stabilization. We studied only stability with respect to initial perturbations, however the stabilization system also counteracts small torque perturbations and errors in the magnetic field measurements and the orbit knowledge because the asymptotic stability automatically guarantees the stability with respect to perturbations (Malkin's theorem). Numerical simulation shows that, it is reliable, has an appropriate accuracy and does not need powerful sources of energy, and therefore can be used for attitude control of small satellites.

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## APPENDIX

*Nomenclature*

| | |
|---|---|
| $X$ | Coordinates of a vector in the body reference system |
| $x$ | Coordinates of a vector in the orbital reference system |
| $\|a\|$ | Norm of a vector $a$ |
| $\langle a, b\rangle$ | Scalar product of vectors $a$ and $b$ |
| $a \times b$ | Vector product of vectors $a$ and $b$ |