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IFAC PapersOnLine 56-2 (2023) 4502–4507

Robust Attitude Estimation Using Magnetic and Inertial Sensors

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Abstract: Lightweight and low-cost magnetic and inertial sensors are commonly used for attitude estimation in a wide range of applications, from motion tracking to autonomous navigation. However, the inherent and external sensor errors and the accelerations due to the actual motion of the platform highly affect the estimation accuracy. This study proposes a robust attitude estimation algorithm that compensates the sensor errors and the external accelerations at the algorithm level. The attitude estimation filter, which is structured as a Multiplicative Extended Kalman Filter (MEKF), is modified to compensate for both the long-term and short-term measurement uncertainties. Modification is applied mainly by estimating and compensating the bias and external accelerations for long-term uncertainties and tuning the measurement noise covariance matrix for short-term uncertainties. Simulations test the proposed algorithm, and the results are compared with benchmark algorithms.

Keywords: sensor data fusion; guidance, navigation and control of vehicles; attitude estimation; magnetic and inertial sensors

1. INTRODUCTION

Orientation estimation has been a significant problem for years in various applications such as robotics, navigation, aerospace, motion tracking, positioning, and localization of objects Nazarahari and Rouhani (2021). Novel application domains, which can be extended from sports, gait analysis, rehabilitation monitoring, and robotics to autonomous vehicles and aerial vehicles, have been drawing attention recently Laidig and Seel (2023). The availability of compact, lightweight, and cost-efficient micro-electromechanical systems-based (MEMS-based) IMUs, specifically widened the application domains. In all these applications, inertial measurement units (IMUs) and magnetometers are used to estimate motion variables, such as orientations, velocities, and positions, either in real-time or via post-processing of the recorded data.

The required sensor layout to estimate the orientation appropriately in all three axes is usually a set of magnetic, angular rate, and gravity (MARG) sensors, commonly referred to as Magnetic and inertial measurement units (MIMUs). MIMUs measure angular rate, specific force (also called proper acceleration), and magnetic field strength, each as a time-dependent 3D vector in an intrinsic sensor coordinate system. Those measurements are processed to determine the motion parameters of interest, e.g., the orientation of an object to which the sensor is attached, the object's velocity or position, or other application-specific motion parameters Caruso et al. (2021a,b). Various algorithms for this problem have been developed for the last fifty years and are divided into two major groups, Kalman filters (KFs) and complementary filters (CFs), respectively. Starting with the work of Rudolf E. Kalman, Kalman (1960), various algorithms have been proposed using this recursive method for estimating the orientation Dai and Jing (2021); Sun et al. (2020); Vitali et al. (2021); Valenti et al. (2016). On the other hand, simpler CF methodology, which has a less complex structure due to the frequency-based solution scheme, has become another strategy developed for the orientation estimation problem Madgwick (2010); Madgwick et al. (2020); Kok and Schön (2019)

A major challenge when using MIMUs for orientation estimation is compensating the external accelerations due to the object's actual motion. The accelerometer output will indicate gravity when the object is stationary or moving with a constant velocity. Effects due to the external acceleration will be of a time-varying nature with both short and long-term variations. Various robust algorithms are proposed to address this problem Kok and Schön (2019); Vitali et al. (2021); Sun et al. (2020). Another issue for MIMU-based orientation estimation is sensor errors. Specifically, the magnetometers will be subject to different disturbances that will deteriorate the measurements Kok et al. (2017). Depending on the application, the specific type of disturbance may vary. Still, in any case, again, the estimation algorithm must cope with both short-term and long-term variations. Unless these disturbances are compensated, the estimated attitude will be unreliable.

This paper proposed a robust multiplicative extended Kalman filter (RMEKF) algorithm for estimating the full three-axis attitude using MIMUs. The attitude is represented using the error-quaternion in the filtering algorithm, and the multiplicative approach is used whenever the full quaternion vector is updated Markley and Crassidis (2014). Following our earlier study Candan and Soken (2021b), the long-term variations in the external acceler-

10.1016/j.ifacol.2023.10.941

ation and the magnetometer biases are compensated by estimating these terms and correcting the measurements. As to the short-term variations, the measurement noise covariance matrix of the filter is tuned with a newly proposed method based on covariance matching. The algorithm is tested via simulations for a drone flight in different scenarios. Comparisons with existing benchmark methods show the proposed algorithm can provide accurate attitude estimation results even in the most challenging cases in terms of external accelerations and magnetic disturbances. Further evaluations will aim at testing the algorithms using real data.

2. PRELIMINARIES

2.1 Attitude Representation

There are three common methods for representing the attitude of a system. These are Euler angles, direction cosine matrix (DCM), and quaternions. In this work, quaternion representation is preferred since the Euler angles has the singularity problem for specific values of the second Euler angle, and the DCM representation has too many redundant parameters to represent the three-axis attitude. Quaternions are used for attitude representation in most of the designed attitude filters for aerospace vehicles. They have no singularity, and kinematics can be described without requiring trigonometric functions. The quaternion set is defined as,

$$q = \left[\, \boldsymbol{q_{1:3}^T} \; q_4 \,\right]^T = \left[\, q_1 \; q_2 \; q_3 \; q_4 \,\right]^T \ , \tag{1}$$

with

$$\boldsymbol{q_{1:3}} = \hat{\boldsymbol{e}} \sin(\vartheta/2)\, ; \quad q_4 = \cos(\vartheta/2) \ , \tag{2}$$

where $\hat{\boldsymbol{e}}$ is the axis of and ϑ is the angle of rotation. The kinematic equation expressed in quaternions is given by

$$\dot{\boldsymbol{q}} = \frac{1}{2}\Omega(\boldsymbol{\omega})\,\boldsymbol{q} \ , \tag{3}$$

where $\Omega(\boldsymbol{\omega})$ is a 4×4 skew-symmetric matrix of angular velocity $\boldsymbol{\omega} = [\, \omega_1 \; \omega_2 \; \omega_3 \,]^T$ as

$$\Omega(\boldsymbol{\omega}) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} = \begin{bmatrix} -[\boldsymbol{\omega}_\times] & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^T & 0 \end{bmatrix} . \tag{4}$$

where the matrix $[\boldsymbol{\omega}_\times]$ is a 3×3 skew-symmetric matrix as $\boldsymbol{a}\times\boldsymbol{b} = [\boldsymbol{a}_\times]\boldsymbol{b}$, with

$$[\boldsymbol{a}_\times] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} . \tag{5}$$

2.2 Sensor Models

Measurement signals from the gyroscope ($\mathbf{y}_G$), accelerometer ($\mathbf{y}_A$) and the magnetometer ($\mathbf{y}_M$) are modelled respectively as following,

$$\mathbf{y_G} = {}^S\boldsymbol{\omega} + \boldsymbol{\beta}_G + \boldsymbol{n}_G, \tag{6}$$

$$\mathbf{y_A} = {}^S\mathbf{a} + {}^S\mathbf{g} + \boldsymbol{n}_A, \tag{7}$$

$$\mathbf{y_M} = {}^S\mathbf{m} + \boldsymbol{\beta}_M + \boldsymbol{n}_M. \tag{8}$$

Note that, left superscript "S" represents the sensor frame. ${}^S\mathbf{a}$ and ${}^S\boldsymbol{\omega}$ are the actual external acceleration and angular rates, and $\boldsymbol{\beta}_G$ is the gyroscope bias respectively. ${}^S\mathbf{g}$ is the gravity vector, $\boldsymbol{n}_A$ and $\boldsymbol{n}_G$ are the sensor noises assumed to be uncorrelated, zero-mean white Gaussian noise characterized by $E[\boldsymbol{n}_G\boldsymbol{n}_G^T] = \sigma_G^2\mathbf{I}_3$ and $E[\boldsymbol{n}_A\boldsymbol{n}_A^T] = \sigma_A^2\mathbf{I}_3$ where σ_G^2 and σ_A^2 are the variance of gyroscope and accelerometer noise assumed to be split same along all axes. Hereafter, left superscript "S" will be omitted for convenience. Moreover, ${}^S\mathbf{m}$ is the actual magnetic field vector in the sensor frame. $\beta_{\mathbf{M}} = [\, \beta_{M_x} \; \beta_{M_y} \; \beta_{M_z} \,]^T$ denotes the magnetic bias vector. $\boldsymbol{n}_M$ is the magnetometer noise assumed to be uncorrelated, zero-mean white Gaussian noise characterized by $E[\boldsymbol{n}_M\boldsymbol{n}_M^T] = \sigma_M^2\mathbf{I}_3$ where σ_M^2 is the variance of magnetometer noise assumed to be split same along all axes.

2.3 Multiplicative Extended Kalman Filter (MEKF)

Attitude filtering algorithms cannot be implemented straightforwardly when the attitude is represented with quaternions, though they are commonly used. The reason is the quaternion norm constraint given as $\boldsymbol{q^T q} = 1$. An unbiased estimator cannot satisfy this constraint, and if we enforce the filter to produce norm-constrained quaternions, for instance, by normalizing the filter estimates, the estimates may be biased, and the filter covariance matrix may be ill-conditioned. One method to overcome this problem is using a three-component local attitude error in the filter and updating the global quaternion estimate by quaternion multiplication, which brings out the Multiplicative EKF (MEKF) (Markley and Crassidis, 2014). In this paper, the rotation error vector $\boldsymbol{\delta\vartheta}$ is used as the local state of EKF and derived by small-angle approximation as

$$2\boldsymbol{\delta q_{1:3}} \approx [\, \delta\phi \; \delta\theta \; \delta\psi \,]^T \equiv \boldsymbol{\delta\vartheta} \ , \tag{9}$$

where error quaternion is defined as

$$\boldsymbol{\delta q} = \boldsymbol{q} \otimes \hat{\boldsymbol{q}}^{-1} = \begin{bmatrix} \hat{q}_4\boldsymbol{q_{1:3}} + q_4\hat{\boldsymbol{q}}_{\boldsymbol{1:3}} - \boldsymbol{q_{1:3}} \times \hat{\boldsymbol{q}}_{\boldsymbol{1:3}} \\ q_4\hat{q}_4 - \boldsymbol{q_{1:3}}^T\hat{\boldsymbol{q}}_{\boldsymbol{1:3}} \end{bmatrix} . \tag{10}$$

After, every time step k the global quaternion is updated by using error quaternion by

$$\hat{\boldsymbol{q}}_{\boldsymbol{k+1/k+1}} = \boldsymbol{\delta}\hat{\boldsymbol{q}}_{\boldsymbol{k+1/k+1}} \otimes \hat{\boldsymbol{q}}_{\boldsymbol{k+1/k}} \ , \tag{11}$$

for $\boldsymbol{\delta q} = [\boldsymbol{\delta q_{1:3}^T} \;\; 1]^T$. This step should be followed by normalization for the global quaternion estimate.

2.4 MEKF Process Model

A gyro-based attitude filtering algorithm is preferred if gyros are onboard the platform and providing the attitude rate measurements. While using the gyro-based filtering, there is a closed-form solution to the state transition matrix in discrete time and an approximation for discrete process noise covariance (Markley and Crassidis, 2014). These factors make the algorithm computationally non-demanding. In the gyro-based attitude filter, rather than the angular velocity, the gyro bias terms are estimated, and the estimated bias vector corrects the gyro measurement.

The bias is propagated by discrete form of $\dot{\boldsymbol{\beta}}(t) = \mathbf{0}$ and the gyro measurements are corrected as

$$\hat{\boldsymbol{\omega}}_{\boldsymbol{k+1/k}} = \tilde{\boldsymbol{\omega}}_{\boldsymbol{k+1}} - \hat{\boldsymbol{\beta}}_{\boldsymbol{k/k}} \ , \tag{12}$$

before propagating the states. The rotation vector error and error bias are used as local error states in the filter as $\boldsymbol{\Delta}\hat{\boldsymbol{x}}(t) = \left[\ \boldsymbol{\delta}\hat{\boldsymbol{\vartheta}}^{\boldsymbol{T}}\ \ \boldsymbol{\delta}\hat{\boldsymbol{\beta}}^{\boldsymbol{T}}\ \right]^T$ and whereas the global quaternion estimate is obtained with a multiplicative approach, the global bias estimate is obtained by summing the error states with the predictions. At each recursive step, the error states are set to zero. The error state transition matrix is given by

$$F = \begin{bmatrix} -[\hat{\boldsymbol{\omega}}_{\times}] & -I_3 \\ 0_3 & 0_3 \end{bmatrix} . \tag{13}$$

2.5 MEKF Measurement Model

In the proposed method, the measurement model is governed by the following equation as,

$$\mathbf{z}_t = \mathbf{H}\mathbf{x}_t + \mathbf{v}_t, \tag{14}$$

where $\mathbf{z}$ is the measurement vector, $\mathbf{H}$ is the output matrix, relating the system states to the outputs and $\mathbf{v}$ is the noise vector for the measurement model, assumed to be zero mean white Gaussian. Measurement models for different type of sensors can be stated in line with the studies in the literature Esit et al. (2021). The linearized model for each vector measurement with respect to the rotation vector error is shown by,

$$\mathbf{H}_k = \begin{bmatrix} A\left(\hat{\boldsymbol{q}}_{\boldsymbol{k+1/k}}\right){}^{N}\mathbf{a}\ _{\times} \\ A\left(\hat{\boldsymbol{q}}_{\boldsymbol{k+1/k}}\right){}^{N}\mathbf{m}\ _{\times} \end{bmatrix} , \tag{15}$$

where A is the DCM matrix that is transforming any vector from local to body frame. ${}^{N}\mathbf{a}$ and ${}^{N}\mathbf{m}$ are the reference vectors in the navigation frame for the acceleration and magnetic field, respectively. In the end, $\mathbf{z}_t$, and $\mathbf{v}_t$ can be stated as,

$$\mathbf{z}_t = \begin{bmatrix} \mathbf{y}_{\mathbf{A}} \\ \mathbf{y}_{\mathbf{M}} \end{bmatrix} . \tag{16}$$

$$\mathbf{v}_t = \boldsymbol{\varepsilon}_t + \begin{bmatrix} \mathbf{n}_{\mathbf{A}} & 0_3 \\ 0_3 & \mathbf{n}_{\mathbf{M}} \end{bmatrix} . \tag{17}$$

where $\boldsymbol{\varepsilon}_t$ and, $\mathbf{n}_A$ and $\mathbf{n}_M$ are uncorrelated noise terms which bring about a measurement noise covariance matrix as follows,

$$\mathbf{M}_t = E[\mathbf{v}_t\mathbf{v}_t^T] = \begin{bmatrix} \boldsymbol{\Sigma}_{acc} & 0_3 \\ 0_3 & \boldsymbol{\Sigma}_{mag} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Sigma}_A & 0_3 \\ 0_3 & \boldsymbol{\Sigma}_M \end{bmatrix} . \tag{18}$$

Here $\boldsymbol{\Sigma}_A$, and $\boldsymbol{\Sigma}_M$ are the covariance matrices for the accelerometer and magnetometer measurement noise and $\boldsymbol{\Sigma}_{acc}$, and $\boldsymbol{\Sigma}_{mag}$ are the covariance matrices for the external acceleration and magnetic disturbance error. $\boldsymbol{\Sigma}_A$ and $\boldsymbol{\Sigma}_M$ are straightforward to set as $\sigma_A^2\mathbf{I}_3$ and $\sigma_M^2\mathbf{I}_3$ respectively, assuming that the variance of accelerometer and magnetometer noise, σ_A^2 and σ_M^2, are distributed equally to all axes for the same sensors. However, time-varying components, $\boldsymbol{\Sigma}_{acc}$ and $\boldsymbol{\Sigma}_{mag}$, cannot be analytically obtained since the actual external acceleration and magnetic disturbances when the measurements are sampled at each step are unknown. In Lee et al. (2012), approximations are suggested only for the external acceleration covariance matrix. Still, they are insufficient at compensating the effects of the external acceleration as reported by Candan (2022). The following chapter is to introduce the proposed method to overcome this problem.

3. PROPOSED METHOD

In this chapter, the problem of tuning the measurement noise covariance matrix is to be covered, and an adaptive tuning approach is to be proposed. Past studies show that the adaptive architecture can provide efficient attitude estimation quality within the platforms, including accelerometer-gyroscope duo Candan and Soken (2021a,b). The theoretical background behind both the proposed algorithms is executing an innovation-based adaptive tuning for the measurement noise covariance matrix. Unlike quaternion-based adaptive KF methodologies in the literature Park et al. (2020); Li et al. (2022); Odry et al. (2020), the proposed method does not execute the attitude estimation procedure via heuristic, fuzzy-logic, Markov models or outlier detection even though some of the methods use covariance matching techniques but for unscented KF framework Chiella et al. (2019). In line with the idea in Soken and Hajiyev (2016); Hajiyev and Soken (2020, 2013); Soken and Sakai (2020), the method relies on a comparison of the theoretical and real values of the innovation covariance. Therefore, when external acceleration and magnetic disturbances exist, the filter can adapt itself to this disturbed environment. Innovation in MEKF structure is defined as,

$$\mathbf{e}_t = \mathbf{z}_t - \mathbf{H}\mathbf{x}_t^-, \tag{19}$$

where $\mathbf{e}_t$ is the innovation sequence and $\mathbf{x}_t^-$ is the predicted state vector. If there exist mismatches between the process and measurement models, due to the unaccounted external accelerations, MEKF gain changes with varying innovation covariance that can be represented as,

$$\hat{\mathbf{C}}_{e_t} = \mathbf{H}\mathbf{P}_t^-\mathbf{H}^T + \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_{acc} & 0_3 \\ 0_3 & \hat{\boldsymbol{\Sigma}}_{mag} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Sigma}_A & 0_3 \\ 0_3 & \boldsymbol{\Sigma}_M \end{bmatrix} , \tag{20}$$

and MEKF gain becomes,

$$\mathbf{K}_t = \mathbf{P}_t^-\mathbf{H}^T(\mathbf{H}\mathbf{P}_t^-\mathbf{H}^T + \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_{acc} & 0_3 \\ 0_3 & \hat{\boldsymbol{\Sigma}}_{mag} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Sigma}_A & 0_3 \\ 0_3 & \boldsymbol{\Sigma}_M \end{bmatrix}) , \tag{21}$$

where $\mathbf{P}_t^-$ is the predicted covariance matrix during the filtering process and $\hat{\boldsymbol{\Sigma}}_{acc}$, $\hat{\boldsymbol{\Sigma}}_{mag}$ are the estimate for the actual $\boldsymbol{\Sigma}_{acc}$ and $\boldsymbol{\Sigma}_{mag}$ matrices given in (18). The real innovation covariance $\hat{\mathbf{C}}_{e_t}$ is to be presented individually for the proposed approach. It is the fact that, in steady-state, if the real value of error for an optimally running KF exceeds the theoretical error as,

$$tr(\mathbf{e}_t\mathbf{e}_t^T) \geq tr(\mathbf{H}\mathbf{P}_t^-\mathbf{H}^T + \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_{acc} & 0_3 \\ 0_3 & \hat{\boldsymbol{\Sigma}}_{mag} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Sigma}_A & 0_3 \\ 0_3 & \boldsymbol{\Sigma}_M \end{bmatrix}) , \tag{22}$$

there is external acceleration and/or magnetic disturbance and the filtering process must be executed adaptive. In (22), $tr(.)$ denotes that the trace operation over the matrix. Only the real innovation covariance at time t is considered to be able to detect the instantaneous variations in the external acceleration and magnetic disturbances. As reported by the past studies, Candan and Soken (2021b), it is the best to use multiple tuning factor (MTF) strategy in order to tune the measurement noise covariance matrix.

3.1 Multiple Tuning Factor (MTF) Strategy

MTF methodology is to use a matrix structure with multiple factors to encounter any disturbances even if the external acceleration is sensed in one direction or in all directions. In this case, by defining the innovation covariance as,

$$\hat{\mathbf{C}}_{e_t} = \frac{1}{\mu} \sum_{j=k-\mu+1}^{k} \mathbf{e}_j \mathbf{e}_j^T, \tag{23}$$

(20) can be rewritten to give the condition for estimating the $\hat{\mathbf{\Sigma}}_{acc}$ and $\hat{\mathbf{\Sigma}}_{mag}$ matrices as,

$$\frac{1}{\mu} \sum_{j=k-\mu+1}^{k} \mathbf{e}_j \mathbf{e}_j^T = \mathbf{H}\mathbf{P}_t^{-}\mathbf{H}^T + \begin{bmatrix} \boldsymbol{S}_A & 0_3 \\ 0_3 & \boldsymbol{S}_M \end{bmatrix} + \begin{bmatrix} \mathbf{\Sigma}_A & 0_3 \\ 0_3 & \mathbf{\Sigma}_M \end{bmatrix}. \tag{24}$$

Here $\mathbf{S}_t$ is the tuning matrix composed from MTFs, which are $\mathbf{S}_A$ for external acceleration and $\mathbf{S}_M$ for magnetic disturbances. μ defines the size of the moving window for innovation covariance calculation. By rearranging we can have the tuning matrix as,

$$\mathbf{S}_t = \frac{1}{\mu} \sum_{j=k-\mu+1}^{k} \mathbf{e}_j \mathbf{e}_j^T - \mathbf{H}_t\mathbf{P}_t^{-}\mathbf{H}_t^T - \begin{bmatrix} \mathbf{\Sigma}_A & 0_3 \\ 0_3 & \mathbf{\Sigma}_M \end{bmatrix}. \tag{25}$$

Therefore, RMEKF with MTF strategy is able to perform the optimal correction since multiple factors are used meaning that the element/s of $\mathbf{S}_t$,which correspond to the axes with external acceleration and magnetic disturbances, adapt itself if the inequality condition mentioned in (22) is met. This provides efficient tuning procedure for the measurement noise covariance matrix. Thus the final estimate for $\hat{\mathbf{\Sigma}}_{acc}$ and $\hat{\mathbf{\Sigma}}_{mag}$ can be obtained as,

$$\hat{\mathbf{S}}_t = diag(s_1, s_2, s_3, s_4, s_5, s_6), \tag{26}$$

$$s_i = max\left\{0, \mathbf{S}_{t,ii}\right\}, i = 1, 2, 3, 4, 5, 6. \tag{27}$$

Here, $\mathbf{S}_{t,ii}$ represents the i^th diagonal element of the matrix $\mathbf{S}_t$. As mentioned, if there are external acceleration or/and magnetic disturbances within the environment, $\mathbf{S}_t$ changes the measurement noise covariance matrix and in all other cases, diagonal values are zero. Following section of the paper is to evaluate the effectiveness of the proposed methods, and compare their results with those of a number of benchmark algorithms within the simulation environment.

4. SIMULATION RESULTS

A number of simulations are made in order to test the performance of the proposed methodology under the different dynamic scenarios. Simulations with IMU system for a dynamic platform are developed within MATLAB environment where the sensor signals are generated at a sampling rate of 100 Hz. The noise variance for gyroscope was set as $0.1°/\sqrt{Hz}$, for accelerometer, it is set equal to $1mg/\sqrt{Hz}$, and for the magnetometer, it is set equal to $0.1\mu T/\sqrt{Hz}$ in order to match the commercial, low-cost MEMS IMU characteristics. In order to create adequate simulation condition, along with specified IMU characteristics, external acceleration acted as disturbance and gyro bias are also added to simulate different scenarios.

4.1 Reference Methods

As mentioned earlier, there are two common strategies in filtering methods for attitude estimation problems, Kalman filtering and complementary filtering, respectively. Four different benchmark methods are selected to compare the proposed methods with the algorithms available in the literature. Two of these methods are from complementary filtering, while the remaining two are based on Kalman filtering. Following this, the reader can find brief explanations about each method chosen for the performance comparison task.

Mahony's Filter: Mahony presents a complementary filtering (CF) method, which is a Proportional-Integral (PI) compensation approach to the attitude estimation problem via fusing the gyro and accelerometer measurements Mahony et al. (2008).

Madgwick's Filter: Madgwick treats the attitude estimation as a minimization problem and proposes a CF method, which depends on the gradient decent strategy that uses the steepest decent algorithm to solve the problem recursively Madgwick (2010).

Dai's Filter: Dai proposed a lightweight quaternion-based extended Kalman Filter (LEKF) for the orientation estimation with MARG sensors. In this filter, the process model is governed by employing the quaternion kinematic equation, while a simplified measurement model is used to create the lightweight system model for Kalman filtering Dai and Jing (2021).

Guo's Filter: Guo designed a quaternion-based Kalman filter scheme where the quaternion kinematic equation is employed as the process model. The measurement model of attitude quaternion is formed from accelerometer and magnetometer measurements, which is claimed to be the fastest among other benchmark methodologies Guo et al. (2017).

4.2 Results and Discussion

The proposed methods' performance is verified using five different simulated tests. For all evaluations, the process noise covariance matrix and $\mathbf{\Sigma}_A$, $\mathbf{\Sigma}_M$ parts of the measurement noise covariance matrix are set in accordance with the sensor specifications. Long-term variations in the magnetic disturbances and the external accelerations are compensated with a similar method as in Candan and Soken (2021b), such that

$$\mathbf{z}_t = \begin{bmatrix} \mathbf{y_A} - c_a\hat{\mathbf{a}} \\ \mathbf{y_M} - c_m\hat{\mathbf{m}} \end{bmatrix}, \tag{28}$$

where, $\hat{\mathbf{a}}$ and $\hat{\mathbf{m}}$ are estimated external acceleration and magnetic disturbance terms in the body frame and c_a and c_m are cutoff frequency constants which are both selected as 0.9.

In RMEKF with MTF, the moving window size, μ, is kept minimum (e.g., $\mu = 1$ such that $\hat{\boldsymbol{C}}_{e_t} = \boldsymbol{e}_t\boldsymbol{e}_t^T$) for handling the sudden variations in the external acceleration in a better way. Table 1 and 2 present the orientation estimation results of the reference methods and the proposed method for the different simulation conditions. Results are given as root mean square error (RMSE) values that are calculated based on the true orientation data, in other words, by taking the error between the simulation results and actual measurements.

Clearly, the performance of the proposed method is superior to all the benchmark algorithms. The reason behind this superior performance in terms of attitude estimation accuracy is basically the capability of the filter for representing the measurement noise covariances more accurately whenever the platform is subject to external accelerations or/and magnetic disturbances during the motion.

Better construction of the measurement noise covariance matrix via using multiple factors in case of external acceleration or/and magnetic variations and the accurate compensation of these disturbances improve the orientation estimation performance during the filtering process.

Table 1. RMSE values for the roll and pitch angles estimation

Methods	Roll RMSE (°)	Pitch RMSE (°)
Mahony's Filter	2.7788	1.3716
Madgwick's Filter	0.9289	1.2413
Dai's Filter	1.5490	1.4928
Guo's Filter	1.5018	1.4778
RMEKF	**0.8572**	**0.5196**

Table 2. RMSE values for yaw angle estimation

Methods	Yaw RMSE (°)
Mahony's Filter	2.4513
Madgwick's Filter	0.6422
Dai's Filter	2.6189
Guo's Filter	1.5513
RMEKF	**0.5803**

Fig. 1 visually demonstrates the estimation accuracy of the RMEKF with MTF throughout the whole duration of the fifth test. Results are compared with those of Madgwick's filter, one of the most accurate benchmark algorithms for the current test. The proposed algorithm can sustain lower estimation error during the whole flight course of the MAV.

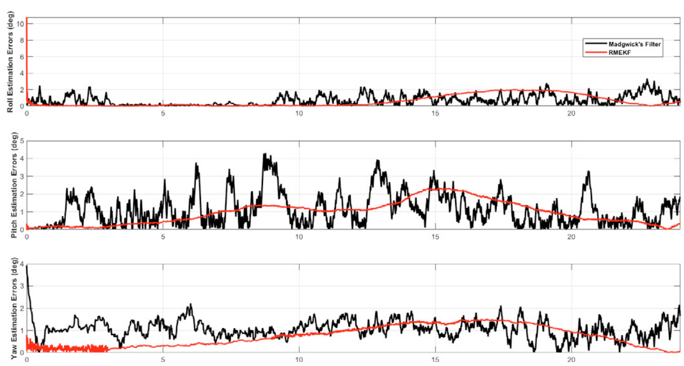

Fig. 1. Orientation estimation errors from the simulation

Fig. 2 visualizes the MTFs and the sensed external accelerations in all three sensor axes in the fifth test case. External acceleration is computed by subtracting the gravity vector in the body frame, which is obtained using the true orientation from the accelerometer measurements. As seen, whenever there is an external acceleration, the MTFs increases to compensate it and when there is no sensed sudden changes in the external acceleration (as seen for a short period in y axis) the tuning factor becomes zero.

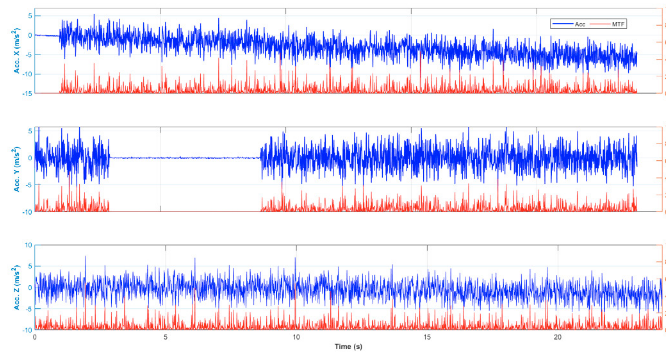

Fig. 2. External acceleration profile during the motion in the simulation and response of tuning factors

According to the results from Table 2 and Fig. 1, CFs show poor performance when prolonged external accelerations and magnetic disturbances are experienced as a result of the agile maneuvers of the platform. Actually, this is a well-known drawback of the CFs compared to the KFs and has been demonstrated in several studies in the literature Javed et al. (2020).

5. CONCLUSION

In this study, a novel robust multiplicative extended Kalman filter (RMEKF) algorithm using a multiple tuning factor (MTF) strategy is proposed for estimating the attitude using the measurements of the inertial and magnetic sensors. The performance of the proposed method in terms of the estimation quality is evaluated using the simulation environment and compared with the methods selected from the current literature. The main contribution of the paper is to introduce a robust approach for tuning the measurement noise covariance matrix in the filtering structure and compensating both the external accelerations and magnetic disturbances efficiently in an adaptive way. Results extracted via simulated tests show that the RMEKF with MTFs shows superior performance against all competitors, especially against its KF-based competitors. RMEKF, with the MTF approach, is able to overcome the external accelerations in different axes individually and embodies a more effective way of executing adaptive filtering. The algorithm incorporates a straightforward adaptation method, which has almost no extra computational demand and is easy to apply. Further investigations will focus on validating the efficiency and performance of the proposed method by using real-world datasets. In addition to this extension, it is also considered to extend the scope of this study by enhancing the measurement model of the filter via using some of the attitude determination methods such as TRIAD and quaternion estimator (QUEST).

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