Proceedings of the 45th IEEE Conference on Decision & Control
Manchester Grand Hyatt Hotel
San Diego, CA, USA, December 13-15, 2006

WeIP3.6

Magnetic attitude control of spacecraft with flexible appendages

Francesco Schiavo, Marco Lovera and Alessandro Astolfi

Abstract— **A mathematical model for the attitude dynamics of a spacecraft with flexible appendages is derived and the problem of inertial pointing for a flexible spacecraft equipped with magnetic actuators is addressed. An almost global solution to the problem is obtained by means of static rigid body attitude and rate feedback. Simulation results demonstrate the practical applicability of the proposed approach.**

I. INTRODUCTION

The attitude and orbit control subsystems (AOCS) play a fundamental role in the operation of modern spacecraft. When considering the particular case of satellites operating in Low Earth Orbits (LEO) a very commonly employed technology for attitude control actuators is based on magnetic torquers. A considerable amount of work has been dedicated in recent years to the problems of analysis and design of attitude control laws for spacecraft equipped with such actuators. In particular, in the linear case, i.e., when dealing with the design of control laws for *nominal* operation of a satellite near its equilibrium attitude, nominal and robust stability and performance have been studied, mostly using tools from periodic control theory exploiting the (quasi) periodic behavior of the system near an equilibrium (see, e.g., [1], [2], [3], [4]).

Similarly, global formulations of the magnetic attitude control problem have been extensively studied. In [5], [6], [7] the attitude regulation problem for Earth pointing spacecraft has been addressed exploiting periodicity assumptions on the system, hence resorting to standard passivity arguments to prove local asymptotic stabilisability of stable open loop equilibria. In [8] similar arguments have been used to analyse a state feedback control law for the particular case of an inertially spherical spacecraft. In [9], [10] the stabilization problems by means of full (or partial) state feedback have been analysed and almost global stability conditions have been provided for simple PD-like attitude control laws.

In all the considered studies, the spacecraft has been assumed to be characterised by rigid body dynamics. In many applications, however, flexible dynamics give rise to significant issues for control design problems. Therefore, the aim of this paper is to derive a model for the dynamics of a flexible spacecraft and to investigate the properties of the PD-like control algorithms for magnetic attitude regulation originally developed under the rigid body assumption (see [9], [10]).

F. Schiavo and M. Lovera are with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy {schiavo lovera}@elet.polimi.it

A. Astolfi is with the Electrical & Electronic Engineering Department, Imperial College London, Exhibition Road, London SW7 2AZ, UK and with the Dipartimento di Informatica, Sistemi e Produzione, Universita' di Roma Tor Vergata, Via del Politecnico 1 - 00133 Roma, Italy a.astolfi@imperial.ac.uk

The paper is organised as follows: Section II gives an overview of the modular approach to the multibody modelling of flexible bodies used in this work; in Section III the coordinate frames used and the detailed spacecraft equations of motion are introduced, while Section IV deals with the control design problem for a flexible spacecraft with magnetic actuators. Finally, in Sections V and VI a simulation case study is briefly analysed.

II. DYNAMICS OF MULTIBODY SYSTEMS WITH FLEXIBLE APPENDAGES

Consider a generic deformable body in a multibody system. The position, in local coordinates, of a point on the body has the following expression:

$$u = u_0 + u_f, \tag{1}$$

where u_0 is the "undeformed" (i.e., rigid) position vector and u_f is the deformation contribution to position (i.e., the deformation field).

The formal and mathematically sound description of the generic deformation of a body requires the deformation field to belong to an infinite dimensional functional space, requiring, in turn, an infinite number of deformation degrees of freedom.

In the model used in this paper, the deformation field is described by an approximation of the functional basis space it belongs to, by assuming that such a space has a finite dimension, say M, so that the vector u_f can be expressed by the following finite dimensional product:

$$u_f = S\,q_f, \tag{2}$$

where S is the $3 \times M$ shape functions matrix (i.e., a matrix of functions defined over the body domain and used as a basis to describe the deformation field of the body itself) and q_f is the M-dimensional vector of deformation degrees of freedom.

The position of a point on a deformable body can then be expressed in the inertial reference frame as

$$r = R + Au = R + A(u_0 + Sq_f) = R + Au_0 + ASq_f, \tag{3}$$

where R is the vector identifying the origin of the body local reference system and A is the rotation matrix for the body reference system.

The representation of a generic deformable body in the inertial reference frame requires then $6 + M$ degrees of freedom. (i.e., 6 corresponding to rigid displacements and rotations and M to deformation fields):

$$q = [q_r \;\; q_f]^T = [R \;\; \theta \;\; q_f]^T \,, \tag{4}$$

where θ represents the undeformed body orientation angles and q_r is a vector containing the 6 rigid degrees of freedom.

The equations of motion for a generic flexible body in a multibody system can be derived by applying the principle of virtual work (see [11]). Such equations can be expressed in body axes, resulting in:

$$\begin{array}{rcl} m_{RR}\ddot{R} + \widetilde{S}_t^T\alpha + S\ddot{q}_f & = & Q_v^R + Q_e^R \\ \widetilde{S}_t\ddot{R} + I_{\theta\theta}\alpha + I_{\theta f}\ddot{q}_f & = & Q_v^\theta + Q_e^\theta \\ S^T\ddot{R} + I_{\theta f}^T\alpha + m_{ff}\ddot{q}_f & = & -K_{ff}q_f + Q_v^f + Q_e^f \end{array} \tag{5}$$

where Q_e^R, Q_e^θ and Q_e^f represent, respectively, the generalized components of the active forces associated to translational, rotational and deformation coordinates, $Q_v^R = -\omega \times \omega \times S_t - 2\omega \times S\dot{q}_f$ and $Q_v^\theta = -\omega \times I_{\theta\theta}\omega - \dot{I}_{\theta\theta}\omega - \omega \times I_{\theta f}\dot{q}_f$ are the quadratic velocity vectors (due to Coriolis and centrifugal forces) associated to translational and rotational degrees of freedom, and $Q_v^f = -\int_V \rho S^T \left(\widetilde{\omega}^2 u + 2\widetilde{\omega} S\dot{q}_f\right) dV$, with $\widetilde{\omega} = [\omega \times \omega \times]$ and

$$m_{RR} = \int_V \rho dV, \quad m_{R\theta} = \int_V \rho A\left(u\times\right)^T A^T dV$$
$$m_{Rf} = \int_V \rho ASdV, \quad m_{\theta\theta} = -\int_V \rho Au \times u \times A^T dV$$
$$m_{\theta f} = \int_V \rho Au \times SdV, \quad m_{ff} = \int_V \rho S^T SdV$$

$$S = \int_V \rho SdV = A^T m_{Rf}, \quad S_t = \int_V \rho udV$$
$$\widetilde{S}_t = \int_V \rho\left(u\times\right) dV = Am_{R\theta}^T A^T$$

$$I_{\theta\theta} = \int_V \rho\left(u\times\right)^T \left(u\times\right) dV = A^T m_{\theta\theta} A$$
$$I_{\theta f} = \int_V \rho\left(u\times\right) SdV = A^T m_{\theta f}.$$

Equations (5) are valid for a general deformable body, though many of the quantities involved (e.g., the matrix K_{ff}) depend on specific body characteristics such as the shape or the material properties. Such equations can thus be specialised to any specific case by a suitable choice of the shape functions for the matrix S. In this paper, we focus on spacecraft composed of a rigid body with a long flexible boom attached, so that such a boom can be considered as a flexible *thin beam*. In detail, we assume that the boom is a 1D elastic *continuum* with constant cross-sectional properties. Furthermore, we assume that the constitutive material of the beam is homogeneous, isotropic and perfectly elastic (i.e., the elastic internal forces are conservative). The detailed equations of motion for the flexible boom, obtained with the application of the finite element method, can be found in [11].

III. Modelling multibody flexible spacecraft

A. *Coordinate frames for satellite work*

For the purpose of the present analysis, the following reference systems are adopted.

- Earth Centered Inertial reference axes (ECI). The origin of these axes is in the Earth's centre. The X-axis is parallel to the line of nodes. The Z-axis is parallel to the Earth's geographic north-south axis and pointing north. The Y-axis completes the right-handed orthogonal triad.
- Orbital Axes ($\mathbf{X}_0$, $\mathbf{Y}_0$, $\mathbf{Z}_0$). The origin of these axes is in the satellite centre of mass. The X-axis points to the Earth's centre; the Y-axis points in the direction of the orbital velocity vector. The Z-axis is normal to the satellite orbit plane.
- Satellite body axes. The origin of these axes is in the satellite centre of mass; the axes are assumed to coincide with the body's principal inertia axes.

B. *Control oriented model of the spacecraft*

Consider equations (5) and take into account only the angular degrees of freedom, to get

$$\begin{array}{rcl} I_{\theta\theta}\alpha + I_{\theta f}\ddot{q}_f & = & Q_v^\theta + Q_e^\theta \\ I_{\theta f}^T\alpha + m_{ff}\ddot{q}_f & = & -K_{ff}q_f + Q_v^f + Q_e^f. \end{array} \tag{6}$$

which, recalling the definition of Q_v^θ, can be equivalently written as

$$\begin{array}{rcl} I_{\theta\theta}\alpha + I_{\theta f}\ddot{q}_f & = & S(\omega)\left(I_{\theta\theta}\omega + I_{\theta f}\dot{q}_f\right) - \dot{I}_{\theta\theta}\omega + Q_e^\theta \\ I_{\theta f}^T\alpha + m_{ff}\ddot{q}_f & = & -K_{ff}q_f + Q_v^f + Q_e^f. \end{array} \tag{7}$$

For practical purposes a number of additional assumptions on the dynamics of the spacecraft can be introduced at this stage, namely:

- $\dot{I}_{\theta\theta} = 0$.
- $\dot{I}_{\theta f} = 0$, i.e., $I_{\theta f}$ is assumed constant, with value corresponding to the undeformed configuration of the beam.
- Q_v^f=0.
- A damping factor $C_{ff}\dot{q}_f$ is present in the equation for q_f.

Therefore, the dynamics of the flexible spacecraft is given by

$$\begin{array}{rcl} I_{\theta\theta}\dot{\omega} + I_{\theta f}^T\ddot{q}_f & = & S(\omega)\left(I_{\theta\theta}\omega + I_{\theta f}\dot{q}_f\right) + Q_e^\theta \\ I_{\theta f}^T\dot{\omega} + m_{ff}\ddot{q}_f & = & -K_{ff}q_f - C_{ff}\dot{q}_f. \end{array} \tag{8}$$

In order to write the model in state space form and to arrive at a somewhat simpler representation, we define the normalised variables

$$\eta_f = m_{ff}^{-1/2}q_f, \quad \psi_f = \Delta\omega + \dot{\eta}_f, \tag{9}$$

and the matrices

$$\Delta = m_{ff}^{-1/2}I_{\theta f}^T \tag{10}$$

$$I = I_{\theta\theta} - \Delta^T\Delta \tag{11}$$

$$K_f = m_{ff}^{-1/2}K_{ff}m_{ff}^{-1/2} \tag{12}$$

$$C_f = m_{ff}^{-1/2}C_{ff}m_{ff}^{-1/2}. \tag{13}$$

Using the above definitions, equations (8) can be written as

$$\begin{array}{rcl} I\dot{\omega} & = & S(\omega)\left(I\omega + \Delta^T\psi_f\right) + \\ & + & \Delta^T\left(K_f\eta_f + C_f\psi_f - C_f\Delta\omega\right) + Q_e^\theta \\ \dot{\eta}_f & = & \psi_f - \Delta\omega \\ \dot{\psi}_f & = & -K_f\eta_f - C_f\psi_f + C_f\Delta\omega. \end{array} \tag{14}$$

Finally, the attitude kinematics of the spacecraft can be described by means of a number of possible parameterisations. The most common one is given by the four Euler parameters (or quaternions), which leads to the following representation for the attitude kinematics

$$\dot{q} = W(\omega)q \tag{15}$$

where $q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T = \begin{bmatrix} \boldsymbol{q}^T & q_4 \end{bmatrix}^T$ is the vector of unit norm ($q^T q = 1$) Euler parameters and

$$W(\omega) = \frac{1}{2}\begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}. \tag{16}$$

It is useful to point out that equation (15) can be equivalently written as

$$\dot{q} = \tilde{W}(q)\omega \tag{17}$$

where

$$\tilde{W}(q) = \frac{1}{2}\begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix}. \tag{18}$$

Note that the attitude of inertially pointing spacecraft is usually referred to the ECI reference frame.

Then, the complete model for the attitude dynamics can be obtained by combining (17) and (14), to get

$$\begin{array}{rcl} \dot{q} & = & \tilde{W}(q)\omega \\ I\dot{\omega} & = & S(\omega)\left(I\omega + \Delta^T\psi_f\right) + \\ & + & \Delta^T\left(K_f\eta_f + C_f\psi_f - C_f\Delta\omega\right) + Q_e^\theta \\ \dot{\eta}_f & = & \psi_f - \Delta\omega \\ \dot{\psi}_f & = & -K_f\eta_f - C_f\psi_f + C_f\Delta\omega. \end{array} \tag{19}$$

Remark 1: It is interesting to point out that equations (19) are equivalent to the mathematical model used in the formulation of the attitude control problem studied in [12], which however was derived using a modal approach instead of the one considered in this work.

IV. CONTROLLER DESIGN

The problem of regulating the non linear attitude dynamics of a spacecraft with flexible appendages has been extensively studied in the literature in the case of a spacecraft equipped with three independent actuators (see, e.g., [13], [12] and the references therein). In particular, the approach commonly followed in the literature assumes that some of the flexible modes associated with the spacecraft dynamics are measurable and the attitude control system is in charge of both stabilising the spacecraft attitude and attenuating the vibrations due to the deformations of the flexible part of the satellite.

In this paper, however, we will be concerned with the case of a spacecraft with flexible appendages equipped with magnetic actuators. The problem of magnetic attitude control has been extensively studied in the literature both in the linear and in the non linear case (see, e.g., [9], [4] and the references therein), but the available literature deals exclusively with the case of rigid spacecraft. In particular, we deal with the PD-like almost globally stabilising control algorithm originally proposed in [9]. This paper aims at deriving conditions under which the above mentioned stabilising control law remains valid in the case of a flexible spacecraft.

The magnetic attitude control torques are generated by a set of three magnetic coils, aligned with the spacecraft principal inertia axes, which generate torques applied to the rigid part of the spacecraft according to the law

$$Q_e^\theta = m_{coils} \times \tilde{b}(t), \tag{20}$$

where $m_{coils} \in \mathbb{R}^3$ is the vector of magnetic dipoles for the three coils (which represent the actual control variables for the coils), $\tilde{b}(t) \in \mathbb{R}^3$ is the vector formed with the components of the Earth's magnetic field in the body frame of reference. Note that the vector $\tilde{b}(t)$ can be expressed in terms of the attitude matrix $A(q)$ (see [14] for details) and of the magnetic field vector expressed in the ECI coordinates, namely $\tilde{b}_0(t)$, as

$$\tilde{b}(t) = A(q)\tilde{b}_0(t), \tag{21}$$

and that the orthogonality of $A(q)$ implies $\|\tilde{b}(t)\| = \|\tilde{b}_0(t)\|$. The dynamics of the magnetic coils reduce to a very short electrical transient and can be neglected. The cross product in equation (20) can be expressed more simply as a matrix-vector product as

$$Q_e^\theta = S(\tilde{b}(t))m_{coils}. \tag{22}$$

Note that since $S(\tilde{b}(t))$ is structurally singular, as mentioned in the Introduction, magnetic actuators do not provide full controllability of the system at each time instant. In particular, it is easy to see that rank $\left[S(\tilde{b}(t))\right] = 2$ (since $\|\tilde{b}_0(t)\| \neq 0$ along all orbits of practical interest for magnetic control) and that the kernel of $S(\tilde{b}(t))$ is given by the vector $\tilde{b}(t)$ itself, i.e., at each time instant it is *not* possible to apply a control torque along the direction of $\tilde{b}(t)$.

If a preliminary feedback of the form

$$m_{coils} = \frac{1}{\|\tilde{b}_0(t)\|^2} S^T(\tilde{b}(t))u \tag{23}$$

is applied to the system, where $u \in \mathbb{R}^3$ is a new control vector, the overall dynamics can be written as

$$\begin{array}{rcl} \dot{q} & = & \tilde{W}(q)\omega \\ I\dot{\omega} & = & S(\omega)\left(I\omega + \Delta^T\psi_f\right) + \\ & + & \Delta^T\left(K_f\eta_f + C_f\psi_f - C_f\Delta\omega\right) + \Gamma(t)u \\ \dot{\eta}_f & = & \psi_f - \Delta\omega \\ \dot{\psi}_f & = & -K_f\eta_f - C_f\psi_f + C_f\Delta\omega. \end{array} \tag{24}$$

where $\Gamma(t) = S(b(t))S^T(b(t)) \geq 0$ and $b(t) = \frac{1}{\|\tilde{b}_0(t)\|}\tilde{b}(t) = \frac{1}{\|\tilde{b}(t)\|}\tilde{b}(t)$. Similarly, let $\Gamma_0(t) = S(b_0(t))S^T(b_0(t)) \geq 0$ and $b_0(t) = \frac{1}{\|\tilde{b}_0(t)\|}\tilde{b}_0(t)$. Note, also, that $\Gamma(t)$ can be written as $\Gamma(t) = \mathcal{I}_3 - b(t)b(t)^T$, where $\mathcal{I}_3$ is the 3×3 identity matrix and $\Gamma(t) \geq 0$.

Furthermore, we will assume in the following that the (linear) structural dynamics associated with the deflections of the flexible part of the system is asymptotically stable, i.e., matrix

$$A = \begin{bmatrix} 0 & I \\ -K_f & -C_f \end{bmatrix} \tag{25}$$

has all the eigenvalues in the open left half plane (i.e., $K_f > 0$ and $C_f > 0$), or, equivalently, given a symmetric positive definite matrix Q, there exists a symmetric positive definite matrix P such that

$$PA + A^T P = -Q. \tag{26}$$

It is clear from the above model that the dynamics of a magnetically controlled satellite turn out to be time-varying. The following preliminary result is useful in the analysis of magnetic attitude control problems as it allows to reformulate the time-varying model as a time-invariant one by using *averaging* arguments.

Lemma 1: ([9]) Consider the system (24) and assume that the considered orbit for the spacecraft satisfies the condition

$$\bar{\Gamma}_0 = \lim_{T\to\infty} \frac{1}{T}\int_0^T S(b_0(t))S^T(b_0(t))dt > 0.$$

Then, there exists $\omega_M > 0$ such that if $\|\omega\| < \omega_M$ for all $t > \bar{t}$, for some $0 < \bar{t} < \infty$, then

$$\bar{\Gamma} = \lim_{T\to\infty} \frac{1}{T}\int_0^T S(b(t))S^T(b(t))dt > 0 \tag{27}$$

along the trajectories of the system (24).

On the basis of this Lemma, a general stabilisation result for the considered flexible spacecraft with magnetic actuators can be given in the case of feedback of the rigid body state variables (attitude and rate). Without loss of generality we will assume that the equilibrium to be stabilised is given by $(\bar{q}, 0, 0, 0)$, where $\bar{q} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$.

Proposition 1: Consider the magnetically actuated spacecraft described by (24) and the control law

$$u = -(\varepsilon^2 k_p I^{-1}\boldsymbol{q} + \varepsilon k_v I\omega). \tag{28}$$

Then there exist $\varepsilon^\star > 0$, $k_p > 0$ and $k_v > 0$ such that for any $0 < \varepsilon < \varepsilon^\star$ the control law (28) ensures that $(\bar{q}, 0, 0, 0)$ is a locally exponentially stable equilibrium for the closed loop system (24)-(28). Moreover, all trajectories of (24)-(28) are such that $\boldsymbol{q} \to 0$, $\omega \to 0$, $\eta_f \to 0$ and $\psi_f \to 0$.

Proof: Introduce the coordinates transformation

$$\boldsymbol{z}_1 = q \qquad z_2 = \frac{\omega}{\varepsilon}, \qquad \boldsymbol{z}_3 = \eta_f \qquad z_4 = \frac{\psi_f}{\varepsilon} \tag{29}$$

(so that $\mathbf{z}_1 = \boldsymbol{q}$ and $z_{14} = q_4$) and let $K = \frac{K_f}{\varepsilon^2}$, $C = \frac{C_f}{\varepsilon}$. In the new coordinates, the system (24) is described by the equations

$$\begin{array}{rcl} \dot{\boldsymbol{z}}_1 &=& \varepsilon\tilde{W}(\boldsymbol{z}_1)z_2 \\ I\dot{z}_2 &=& \varepsilon\left(S(z_2)\left(Iz_2 + \Delta^T z_4\right) + \right. \\ &+& \Delta^T\left(Kz_3 + Cz_4 - C\Delta z_2\right) + \\ &+& \left.\Gamma(t)\left(-k_p I^{-1}\mathbf{z}_1 - k_v I z_2\right)\right) \\ \dot{z}_3 &=& \varepsilon\left(z_4 - \Delta z_2\right) \\ \dot{z}_4 &=& \varepsilon\left(-Kz_3 - Cz_4 + C\Delta z_2\right). \end{array} \tag{30}$$

System (30) satisfies all the hypotheses[1] for the applicability of the generalised averaging theory (see [15, Theorem 10.5]), which yields the averaged system

$$\begin{array}{rcl} \dot{\boldsymbol{z}}_1 &=& \varepsilon\tilde{W}(\boldsymbol{z}_1)z_2 \\ I\dot{z}_2 &=& \varepsilon\left(S(z_2)\left(Iz_2 + \Delta^T z_4\right) + \right. \\ &+& \Delta^T\left(Kz_3 + Cz_4 - C\Delta z_2\right) + \\ &+& \left.\bar{\Gamma}\left(-k_p I^{-1}\mathbf{z}_1 - k_v I z_2\right)\right) \\ \dot{z}_3 &=& \varepsilon\left(z_4 - \Delta z_2\right) \\ \dot{z}_4 &=& \varepsilon\left(-Kz_3 - Cz_4 + C\Delta z_2\right). \end{array} \tag{31}$$

As a result, there exists $\varepsilon^\star > 0$ such that for any $0 < \varepsilon < \varepsilon^\star$ the trajectories of system (31) are close to the trajectories of system (30). Consider now the positive definite function

$$V(z_2, z_3, z_4) = \frac{\lambda}{2}\left(Iz_2 + \Delta^T z_4\right)^T\left(Iz_2 + \Delta^T z_4\right) + + \frac{1}{2}\begin{bmatrix} z_3^T & z_4^T \end{bmatrix} P \begin{bmatrix} z_3 \\ z_4 \end{bmatrix}, \tag{32}$$

$\lambda > 0$, and its time derivative

$$\dot{V}(z_2, z_3, z_4) = \varepsilon\left(Iz_2 + \Delta^T z_4\right)^T \bar{\Gamma}\left(-k_p I^{-1}\mathbf{z}_1 - k_v I z_2\right) - \varepsilon\begin{bmatrix} z_3^T & z_4^T \end{bmatrix} Q \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} + 2\varepsilon\begin{bmatrix} z_3^T & z_4^T \end{bmatrix} PBz_2, \tag{33}$$

where $B = \begin{bmatrix} -\Delta^T & \Delta^T C^T \end{bmatrix}^T$, and note that the positive definiteness of $\bar{\Gamma}$ and the boundedness of $\boldsymbol{z}_1$ imply that

$$\dot{V} \leq -rV + d \tag{34}$$

for some constants $r > 0$ and $d > 0$. In particular, equation (34) implies that for any K along the trajectories of the closed loop system one has

$$\|z_2(t)\|^2 \leq K^2, \tag{35}$$

for all $t \geq t^\star$ and for some $0 \leq t^\star \leq \infty$. As a result, for any $K > 0$ the set

$$Z_K = \{(\boldsymbol{z}_1, z_2, z_3, z_4) : \|z_2\| < K\} \tag{36}$$

is attractive and positively invariant. Observe that K can be made arbitrarily small by a suitable choice of k_p and k_v.

We now prove that all trajectories of system (31) starting in the set (36) are such that $\mathbf{z}_1 \to 0$, $z_2 \to 0$, $z_3 \to 0$ and $z_4 \to 0$. For, consider the Lyapunov function

$$V_2(\boldsymbol{z}_1, z_2, z_3, z_4) = 2k_p(\mathbf{z}_1^T\mathbf{z}_1 + (z_{14} - 1)^2) + + (Iz_2)^T\,\bar{\Gamma}^{-1}\,(Iz_2) + \begin{bmatrix} z_3^T & z_4^T \end{bmatrix} P \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} \tag{37}$$

[1]In particular, it is easy to verify that the Jacobian of the difference between the right hand sides of (30) and (31) has zero average.

and note that its time derivative

$$\dot{V}_2(\boldsymbol{z}_1, z_2, z_3, z_4) = \varepsilon k_p \mathbf{z}_1^T z_2 + \varepsilon z_2^T I \bar{\Gamma}^{-1} S(z_2) \left(I z_2 + \right. \\ \left. + \Delta^T z_4 + \varepsilon z_2^T I \left(-k_p I^{-1} \mathbf{z}_1 - k_v I z_2\right) + \right. \\ - \varepsilon \begin{bmatrix} z_3^T & z_4^T \end{bmatrix} Q \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} + 2\varepsilon \begin{bmatrix} z_3^T & z_4^T \end{bmatrix} P B z_2, \quad (38)$$

can be made negative as a function of z_2, z_3 and z_4 for a suitable choice of k_v. As a result, $z_2 \to 0$, $z_3 \to 0$, $z_4 \to 0$ and, applying La Salle invariance principle, $\mathbf{z}_1 \to 0$.

Finally, consider the linear approximation of system (31) around the equilibrium $(\bar{q}, 0, 0, 0)$, which is given by

$$\begin{array}{rcl} \dot{\mathbf{z}}_1 & = & \frac{\varepsilon}{2} z_2 \\ I \dot{z}_2 & = & \varepsilon \left(\Delta^T \left(K z_3 + C z_4 - C \Delta z_2\right) + \right. \\ & + & \left. \bar{\Gamma} \left(-k_p I^{-1} \mathbf{z}_1 - k_v I z_2\right)\right) \\ \dot{z}_3 & = & \varepsilon \left(z_4 - \Delta z_2\right) \\ \dot{z}_4 & = & \varepsilon \left(-K z_3 - C z_4 + C \Delta z_2\right). \end{array} \quad (39)$$

It is easy to verify that

$$V_L(\boldsymbol{z}_1, z_2) = 2 k_p \mathbf{z}_1^T \mathbf{z}_1 + (I z_2)^T \bar{\Gamma}^{-1} (I z_2) + \\ + \begin{bmatrix} z_3^T & z_4^T \end{bmatrix} P \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} \quad (40)$$

is a Lyapunov function for the linear system (39), so the convergence of the trajectories of the closed loop system is locally exponential, for sufficiently large k_v. ■

By means of this result it is possible to guarantee that the considered PD-like control law ensures global asymptotic stability of the desired equilibrium attitude also in the presence of perturbations due to a flexible appendage.

V. Satellite modular model

Models for space environment, spacecraft dynamics, sensors, actuators and controller have been implemented within a modular framework with the object-oriented language Modelica [16].

For the purpose of the present study the satellite is assumed to be equipped with a star sensor providing a measurement of the inertial attitude of the body frame, a set of three gyroscopes providing a measurement of the body components of the inertial angular rate, and a triaxial magnetometer which measures the components of the geomagnetic field of the Earth, again in body axes. As for the attitude control actuators, the spacecraft has been equipped only with a set of three magnetic coils.

The space environment has been modeled through an extension of the component `World` of the Modelica MultiBody library, with the definition of new gravitational field approximations and the addition of the geomagnetic field, of the solar radiation pressure and of the air density and air velocity fields (see Section 4).

The spacecraft dynamics has been modeled by connecting two standard rigid body components, representing the satellite main body and the boom tip mass, through a 1D flexible beam, which represents the satellite boom. In addition to that, two newly developed components from [17] have been connected to the satellite main body so to account for the dynamical effects due to the satellite interaction with the space environment.

VI. Simulation results

In this Section the results obtained by simulating the above described Modelica models within the Dymola [18] environment are presented.

For the sake of simplicity the flexible spacecraft configuration described in [12] has been considered in the simulation example. The satellite operates in a polar (90^o inclination) orbit with an altitude of $450\,\mathrm{km}$ and a corresponding orbit period of about $5600\,\mathrm{s}$. The control system aims at maintaining the satellite with its body axes aligned with the inertial reference frame, i.e, in an inertial pointing mode.

In the following figures, the results obtained by simulating the spacecraft using the proposed PD-like control law are presented, starting with an initial condition characterised by a large initial angular rate. Figure 1 shows the time history of the spacecraft attitude during the simulation. As can be seen from the Figure, the spacecraft acquires the desired attitude (defined by the nominal quaternion $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$; the duration of the transient is basically dominated by the time the control system takes to "dump" the initial kinetic energy of the satellite (see Figure 2 in which the components of the angular rate of the satellite are shown). The deflections of the flexible boom, together with the corresponding velocities, are shown in Figures 3-4; as can be seen, the control law guarantees the stability of the desired attitude and of the flexible dynamics. Finally, the time history of the magnetic dipoles of the actuators are shown in Figure 5.

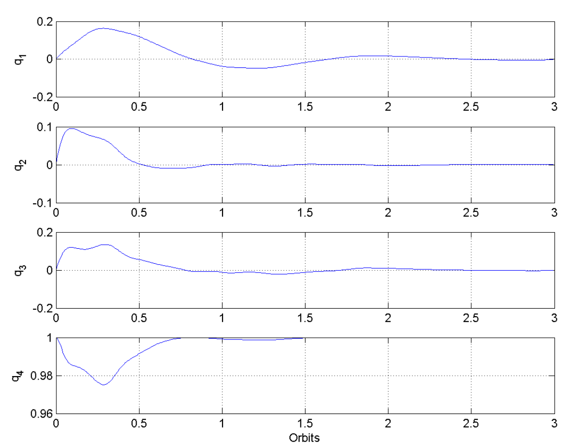

Fig. 1. Satellite attitude quaternion.

VII. Conclusions

A mathematical model for the dynamics of a flexible spacecraft has been derived and the problem of regulating its attitude using magnetic actuators has been discussed. A global solution to the problem has been proposed, based on static attitude and rate feedback. Future work will aim at extending the modeling and control approaches to more

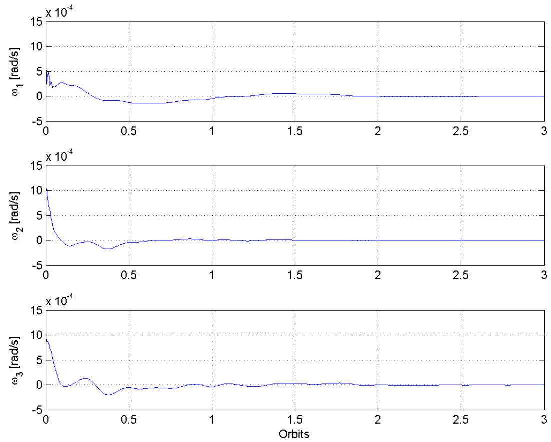

Fig. 2. Satellite angular rate components.

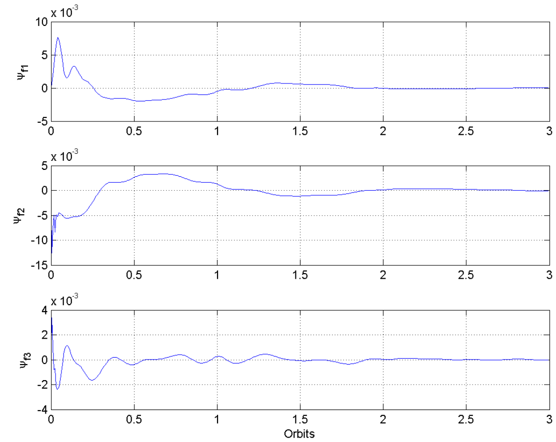

Fig. 4. Boom deflection rates.

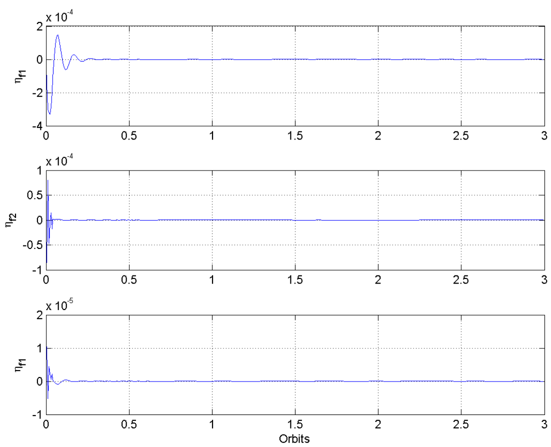

Fig. 3. Deflections of the flexible boom.

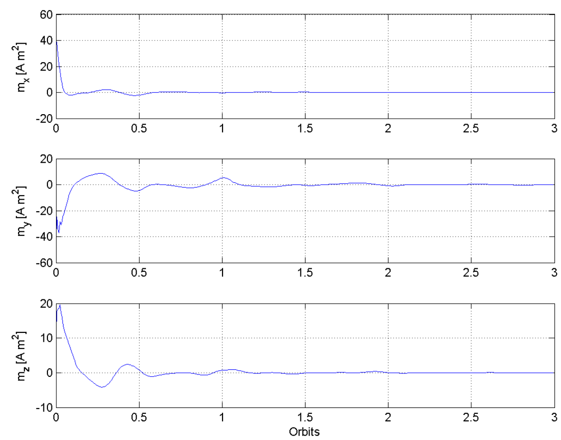

Fig. 5. Dipole moments of the magnetic actuators.

general configurations such as spacecraft with 2D flexible appendages.

REFERENCES

[1] R. Wisniewski and L. Markley, “Optimal magnetic attitude control,” in *14th IFAC World Congress, Beijing, China*, 1999.

[2] M. Lovera, E. D. Marchi, and S. Bittanti, “Periodic attitude control techniques for small satellites with magnetic actuators,” *IEEE Transactions on Control Systems Technology*, vol. 10, no. 1, pp. 90–95, 2002.

[3] M. Psiaki, “Magnetic torquer attitude control via asymptotic periodic linear quadratic regulation,” *Journal of Guidance, Control and Dynamics*, vol. 24, no. 2, pp. 386–394, 2001.

[4] E. Silani and M. Lovera, “Magnetic spacecraft attitude control: A survey and some new results,” *Control Engineering Practice, Special Section on Aerospace Control*, vol. 13, no. 3, pp. 357–371, 2005.

[5] R. Wisniewski and M. Blanke, “Fully magnetic attitude control for spacecraft subject to gravity gradient,” *Automatica*, vol. 35, no. 7, pp. 1201–1214, 1999.

[6] C. J. Damaren, “Comments on ”Fully magnetic attitude control for spacecraft subject to gravity gradient”,” *Automatica*, vol. 38, no. 12, p. 2189, 2002.

[7] C. Arduini and P. Baiocco, “Active magnetic damping attitude control for gravity gradient stabilised spacecraft,” *Journal of Guidance and Control*, vol. 20, no. 1, pp. 117–122, 1997.

[8] P. Wang and Y. Shtessel, “Satellite attitude control using only magnetic torquers,” in *AIAA Guidance, Navigation, and Control Conference and Exhibit, Boston, USA, 1998*, 1998.

[9] M. Lovera and A. Astolfi, “Spacecraft attitude control using magnetic actuators,” *Automatica*, vol. 40, no. 8, pp. 1405–1414, 2004.

[10] ——, “Global magnetic attitude control of inertially pointing spacecraft,” *Journal of Guidance, Control and Dynamics*, vol. 28, no. 5, pp. 1065–1072, 2005.

[11] F. Schiavo, L. Viganó, and G. Ferretti, “Modular modelling of flexible beams for multibody systems,” *Multibody Systems Dynamics*, 2006.

[12] S. Di Gennaro, “Output attitude tracking for flexible spacecraft,” *Automatica*, vol. 38, no. 10, pp. 1719–1726, 2002.

[13] S. Joshi, A. Kelkar, and P. Maghami, “A class of stabilising controllers for flexible multibody systems,” NASA, Technical paper 3494, 1995.

[14] J. Wertz, *Spacecraft attitude determination and control*. D. Reidel Publishing Company, 1978.

[15] H. Khalil, *Nonlinear systems*. Macmillan, 2001.

[16] S. Mattsson, H. Elmqvist, and M. Otter, “Physical system modeling with Modelica,” *Control Engineering Practice*, vol. 6, no. 4, pp. 501–510, 1998.

[17] M. Lovera, “Control-oriented modelling of spacecraft attitude and orbit dynamics,” *Mathematical and Computer Modelling of Dynamical Systems*, vol. 12, no. 1, pp. 73–88, 2006.

[18] Dynasim AB, *Dymola−Dynamic Modelling Laboratory*, Lund, Sweden, http://www.dynasim.se/dymola.htm.