Brief Papers

Periodic Attitude Control Techniques for Small Satellites With Magnetic Actuators

Marco Lovera, Eliana De Marchi, and Sergio Bittanti

***Abstract*—The problems of attitude stabilization and disturbance torque attenuation for a small spacecraft using magnetic actuators is considered and a solution to the problem in terms of optimal periodic control is proposed, based on an optimal estimation and compensation scheme for external disturbances for linear time-periodic systems.**

***Index Terms*—Attitude control, estimation, geomagnetism, optimal control, periodic control, space vehicle control.**

I. Introduction

THE FEASIBILITY of periodic techniques for the attitude control of small satellites using magnetic actuators is becoming a topic of increasing interest in the literature (see, e.g., the recent works [1]–[6]). This is due to the combination of two factors: the (equally recent) interest in small satellites, for which magnetic torquers represent the most practical form of actuation and the fact that while a number of magnetic control laws have been derived in the past (see, e.g., [7]–[9]), the advantages of the application of modern optimal control theory to this problem were still to be explored. The operation of magnetic actuators is based on the interaction with the geomagnetic field ([10], [11]). The major consequences of this are that an accurate knowledge of the magnetic field is necessary in order to control the spacecraft and that the torques which can be applied to the spacecraft for attitude control purposes are constrained to lie in the plane orthogonal to the magnetic field vector. These issues represent a constraint on the control law. In particular, three-axis magnetic stabilization is only possible if the variability of the magnetic field along the considered orbit is sufficient to guarantee the stabilizability of the spacecraft.

The MITA (Italian Advanced Technology Minisatellite) bus is a low cost platform for small low earth orbit (LEO) missions which is being developed by Carlo Gavazzi Space S.p.A. (Milano, Italia), under an Italian Space Agency contract. The technology demonstration model has been successfully launched in July 2000. The approach taken in the design of the baseline attitude control laws for the first MITA mission is the following: first design the controller without taking into account the constraint given by the magnetic actuators; subsequently implement the actuator constraint by projecting the computed torques onto the subspace of feasible torques (see [8]). While this solution has the clear advantage of simplicity, both from the design and implementation points of view, a control design technique capable of taking directly into account such a constraint in the design phase would be very useful.

To this purpose, a joint research activity involving Carlo Gavazzi Space and the Politecnico di Milano was started, in order to explore the feasibility of a number of control design approaches, among which periodic linear quadratic optimal control seems a promising route. In addition to attitude stabilization, it is very important to consider also the problem of attenuating the effect of external disturbance torques on the pointing and stability performance of the spacecraft. To this purpose, some recently developed disturbance rejection techniques for periodic systems (see [12]) can be used to further improve the performance of the spacecraft attitude control system. The paper provides a description of the above stabilization and disturbance attenuation techniques together with the results obtained during the design and simulation of specific algorithms for the MITA bus in polar orbit. First the design and implementation of a conventional infinite horizon optimal periodic controller for asymptotic stabilization will be described; then the application to the spacecraft of the disturbance estimation and rejection techniques for periodic systems subject to periodic disturbances will be demonstrated.

This paper is organized as follows: in Section II a description of the dynamics of a generic spacecraft will be presented, and a linearized dynamic model derived; Section III offers a summary of the relevant results in optimal periodic control theory and a presentation of the disturbance attenuation technique proposed in this paper. Finally, in Section IV some simulation results are presented by means of which the performance of the periodic optimal controllers can be assessed.

II. Spacecraft Attitude Dynamics and Kinematics

A. Dynamic and Kinematic Equations

The equations of angular dynamics ([10], [11]) can be expressed in vector form as $dh(t)/dt = T(t)$ where h is the overall angular momentum of the spacecraft and T is the sum of the external torques (disturbance and control ones) acting

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on the satellite. The derivative of h is here expressed in an inertial reference frame; considering instead a body reference frame, rotating with angular rate ω, the Euler's equations become $\dot{h}(t) = -\omega(t) \wedge h(t) + T(t)$. In this formulation the vector h shall include also the contribution of rotating parts of the satellite such as the momentum wheel. Concerning the attitude, a parametric expression can be obtained with different methods: the quaternion has been chosen here because of its numerical advantages and avoidance of singularities. As is well known (see, e.g., [10]) the attitude matrix can be expressed as a function of the quaternion vector $q \in \mathbb{R}^4$; the time evolution of the attitude parameters as a function of the body angular rate can be represented in the following way (kinematic equations): $\dot{q}(t) = (1/2)W(\omega(t))q(t)$ where W is the skew-symmetric matrix function of ω defined as

$$W(\omega) = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}. \tag{1}$$

B. Disturbance Torques

External disturbance torques occur naturally and have different sources, such as gravity gradient, aerodynamics, solar radiation and residual magnetic dipoles (see [13] for details). Among internal torques, it is possible to distinguish those that occur as disturbances (e.g., satellite or payload moving parts) and those that provide controllable torques to store momentum (reaction or momentum wheels). The size of the external disturbance torques is a function of the mass properties (position of the center of mass and inertia matrix), orbit characteristics and mission lifetime. Regardless of the physical mechanism giving rise to them (i.e., magnetic, aerodynamic, solar, gravity gradient), they can be separated into a secular component (i.e., the part with nonzero mean around each orbit) and a cyclic component (i.e., the zero mean, periodic part, with a period given by the orbit period).

C. Linearized Dynamics

We assume for the spacecraft a momentum bias configuration (i.e., one momentum wheel, aligned with the body z axis, with moment of inertia J and angular velocity ν) and split the overall external torque T in three components, namely the gravity gradient torque $T_{\rm gg}$, which will be included in the linearized dynamics, the control torque $T_{\rm contr}$ and the disturbance torque $T_{\rm dist}$. Introducing now the state vector $x(t) = [q(t)' \quad \omega(t)']'$ and considering small displacements from the nominal values of the vector part of the attitude quaternion $q_1 = q_2 = q_3 = 0$, and small deviations of the body rates from the nominal ones $\omega_x = \omega_y = 0$, $\omega_z = -\Omega$ (Ω being the angular frequency associated with the orbit period), one can linearize the attitude dynamics, and define the local linear dynamics for the system as

$$\dot{\delta x}(t) = A\delta x(t) + \begin{bmatrix} 0 \\ I^{-1} \end{bmatrix} [T_{\rm contr}(t) + T_{\rm dist}(t)] \tag{2}$$

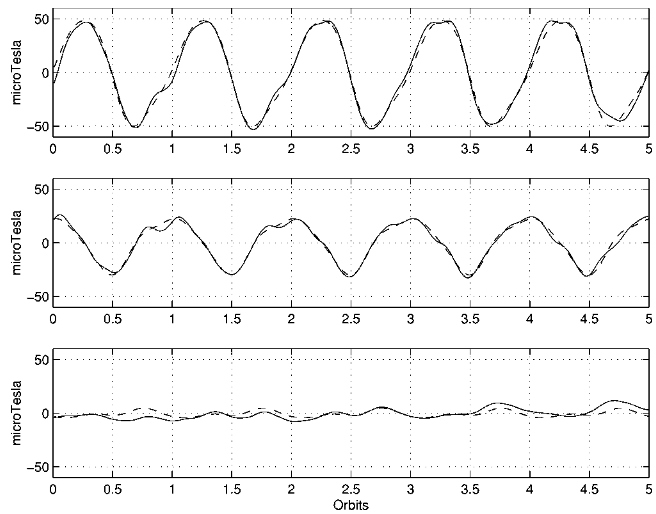

Fig. 1. Periodic approximation of the geomagnetic field in Pitch–Roll–Yaw coordinates, 87° inclination orbit, 450 km altitude.

where $I = \mathrm{diag}\{I_{xx}, I_{yy}, I_{zz}\}$ is the spacecraft inertia matrix

$$A = \begin{bmatrix} 0 & -\Omega & 0 & 0.5 & 0 & 0 \\ \Omega & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & W_x & 0 \\ 0 & -6k_y\Omega^2 & 0 & W_y & 0 & 0 \\ 0 & 0 & -6k_z\Omega^2 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

and $k_x = (I_{yy} - I_{zz})/I_{xx}$, $k_y = (I_{zz} - I_{xx})/I_{yy}$, $k_z = (I_{xx} - I_{yy})/I_{zz}$, $W_x = -k_x\Omega - K_x\nu$, $W_y = -k_y\Omega + K_y\nu$, $K_x = J/I_{xx}$, $K_y = J/I_{yy}$.

The control torques generated by the magnetic coils are given by the expression

$$T_{\rm contr}(t) = m(t) \wedge b(t) = B(b(t))m(t)$$

where

$$B(b(t)) = \begin{bmatrix} 0 & b_z(t) & -b_y(t) \\ -b_z(t) & 0 & b_x(t) \\ b_y(t) & -b_x(t) & 0 \end{bmatrix} \tag{4}$$

is a matrix the elements of which are constituted by instantaneous measurements of the magnetic field vector $b(t) \in \mathbb{R}^3$ and $m(t) \in \mathbb{R}^3$ is the vector of the coils' magnetic dipoles.

The overall linearized model therefore takes the form

$$\dot{\delta x}(t) = A\delta x(t) + \begin{bmatrix} 0 \\ I^{-1} \end{bmatrix} [B(b(t))m(t) + T_{dist}(t)]. \tag{5}$$

Clearly, if the time variation of the magnetic field is periodic, this model can be seen as a linear time-periodic one.

D. Periodic Approximation of the Magnetic Field

A time history of the international geomagnetic reference field (IGRF) model for the earth's magnetic field ([10]) along five orbits in pitch–roll–yaw coordinates for a spacecraft in polar orbit (87° inclination, 450 km altitude) is shown in Fig. 1 (solid line).

As can be seen, $b_x(t)$, $b_y(t)$ have a very regular and almost periodic behavior, while the $b_z(t)$ component is much less regular; indeed, when the spacecraft is in the nominal attitude, the x

and y body axes lie in the orbit plane while the z axis is normal to it. As a consequence, the x and y magnetometers sense only the variation of the magnetic field due to the orbital motion of the spacecraft while the z axis sensor is affected by the variation of b due to the rotation of the earth (period of 24 h).

A periodic approximant of the magnetic field can derived, by least squares fitting of the output of the IGRF model to a simplified periodic structure such as $b(t) = b_o + b_{1c}\cos(\Omega t) + b_{1s}\sin(\Omega t) + b_{2c}\cos(2\Omega t) + b_{2s}\sin(2\Omega t)$. Using data from five orbits, the results of the least squares fit are summarized in Fig. 1, where the time histories of the IGRF and approximate periodic magnetic field models are given.

Combining the linearized dynamics derived in Section I with the periodic approximation of the magnetic field obtained herein, one gets a complete periodic model of the local dynamics of the spacecraft.

III. Periodic Optimal Control

Given a T-periodic, linear, continuous-time system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$ where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$, the infinite horizon optimal periodic control problem can be stated as follows: to determine the linear time periodic state feedback control law which minimizes the following performance objective:

$$J = \int_{t_0}^{\infty} [x(t)'Q(t)x(t) + u(t)'R(t)u(t)]\,dt \tag{6}$$

where $Q(t)$ and $R(t)$ are T-periodic matrices such that $Q(t) \geq 0$ and $R(t) > 0$. Assuming that the uncontrollable and unobservable parts of the considered system are asymptotically stable, the solution exists and is given by ([14]) $u(t) = -R(t)^{-1}B(t)'\overline{P}(t)x(t)$ where $\overline{P}(t) = \lim_{t_f\to\infty} P(t, t_f)$, $P(t, t_f)$ being the solution of the following ordinary differential equation (ODE) in matrix form (periodic Riccati equation or PRE):

$$\begin{aligned} -\dot{P}(t) = {} & P(t)A(t) + A(t)'P(t) - P(t)B(t)R(t)^{-1}B(t)'P(t) \\ & + Q(t) \end{aligned} \tag{7}$$

with the boundary condition $P(t_f, t_f) = 0$, $t_f > t_0$, t_f finite. For the present study, the solution technique based on the quasi-linearization method has been adopted, which is outlined in the following.

A. Quasi-Linearization of the PRE

Consider the operator

$$\begin{aligned} Ric\colon P(\cdot) \to {} & \dot{P}(\cdot) + P(\cdot)A(\cdot) + A(\cdot)'P(\cdot) \\ & - P(\cdot)B(\cdot)R(\cdot)^{-1}B(\cdot)'P(\cdot) + Q(\cdot). \end{aligned} \tag{8}$$

A symmetric, periodic and positive semidefinite (SPPS) solution of the PRE satisfies $Ric(P(\cdot)) = 0$. Assuming that $P_i(\cdot)$ is a SPPS matrix which approximates a solution of the PRE, Newton's method consists in solving iteratively a first-order approximation of the PRE, computed in correspondence of $P_i(\cdot)$: $Ric(P_i(\cdot)) + dRic(P_i(\cdot), \Delta P_i(\cdot)) = 0$, where $dRic(\cdot,\cdot)$ is the differential of Ric computed at $P_i(\cdot)$ along $\Delta P_i(\cdot)$. From $\Delta P_i(\cdot)$, the new approximation is obtained as $P_{i+1}(\cdot) = P_i(\cdot) + \Delta P_i(\cdot)$.

It can be shown that $P_{i+1}(\cdot)$ can be computed from $P_i(\cdot)$ by solving the following Lyapunov equation:

$$\begin{aligned} -\dot{P}_{i+1}(\cdot) = {} & P_{i+1}(\cdot)A_i(\cdot) + A_i(\cdot)'P_{i+1}(\cdot) - K_i(\cdot)R(\cdot)K_i(\cdot) \\ & + Q(\cdot) \end{aligned} \tag{9}$$

where $A_i(\cdot) = A(\cdot) - B(\cdot)K_i(\cdot)$ and $K_i(\cdot) = R(\cdot)^{-1}B(\cdot)'P_i(\cdot)$. $K_i(\cdot)$ is the controller computed from the solution $P_i(\cdot)$, which is necessary for the computation of the periodic generator and the solution of the subsequent $(i+1)$th iteration and $A_i(\cdot)$ is the closed-loop matrix associated with controller $K_i(\cdot)$, which is also required in order to obtain $P_{i+1}(\cdot)$. Assuming that $\forall i$ the above Lyapunov equation has a periodic, symmetric solution (see [15] for more details on the solvability of periodic Lyapunov equations), then if the sequence of the solutions $P_i(\cdot), i = 1, 2, \ldots$ converges, one has $\lim_{i\to\infty}, P_i(t) = P_\infty(t)$ and it can be verified that $P_\infty(\cdot)$ is a SPPS solution of the PRE. Therefore by means of an appropriate initialization it is possible to ensure the global quadratic convergence of this method, which therefore can be used as a numerical method for the solution of the PRE; starting from a stabilizing gain $K_0(\cdot)$, and exploiting the properties of the Newton iteration, one can obtain a satisfactory convergence after a few (say four or five) iterations.

Remark 1: A critical issue in magnetic control is related to the stabilizability of the spacecraft dynamics. This clearly depends on both the spacecraft architecture and on the selected orbit. For example, for a circular orbit the variability of the magnetic field is maximum along a polar orbit and minimum along an equatorial one. Only numerical investigations have been carried out so far concerning stabilizability, while no analytical results are available, given the complexity of the geomagnetic models.

B. Optimal Periodic Disturbance Rejection for Periodic Systems

Optimal periodic control theory can provide stabilizing controllers for periodic systems: a different problem, which frequently arises in the practice of control engineering, however, calls for the attenuation of the effect of disturbances on the output of the considered system. In particular, when considering the spacecraft control problem at hand, it is of great importance not only to guarantee the closed-loop stability of the control system, but also to obtain a satisfactory rejection of the effect of external disturbance torques on the spacecraft attitude. An interesting feature of the disturbance torque attenuation problem is the fact that the external torques can be modeled as periodic inputs. The disturbance modeling and frequency shaping extensions of classical LQ control theory were developed to deal with the case of time-invariant systems affected by external disturbances with known dynamics (see, e.g., [16] for a review of the disturbance attenuation problem). Such techniques were known to suffer from structural difficulties in the case of periodic disturbances, as nonasymptotically stable, uncontrollable modes are introduced in the augmented plant used for controller design. This difficulty was solved in [17] and the technique proposed therein was subsequently extended to deal with the H_∞ synthesis problem in [18]. More recently, in [12] an extension of the above methods to the case of periodic systems was proposed, which is based on the application to periodic systems of the LQ disturbance modeling approach, taking into account the notion of periodic frequency response developed in [19]. This technique considers a linear, time periodic system of the type

$$\begin{aligned}\dot{x}(t) &= A(t)x(t) + B(t)[u(t) + d(t)] + v(t)\\ y(t) &= C(t)x(t) + D(t)[u(t) + d(t)] + w(t)\end{aligned} \tag{10}$$

affected by the periodic input disturbance d. The problem of designing a control system so as to attenuate the effect of d on the output of the system can be easily cast into an optimal control framework by considering the performance index

$$J = E\left\{ \lim_{\substack{t_0\to-\infty\\ t_f\to+\infty}} \frac{1}{t_f - t_0}\int_{t_0}^{t_f} \{\hat{x}(t)'Q\hat{x}(t) + \overline{u}(t)'R\overline{u}(t)\}\, dt \right\} \tag{11}$$

where $\overline{u}(t) = u(t) + d(t)$.

The introduction of the fictitious input signal $\overline{u}(t)$ makes it possible to turn the problem into a conventional periodic LQG control problem. Subsequently, the control action $u(t)$ is the sum of two terms: the compensation for the disturbance, given by the estimate of the external disturbance, and the state feedback, the gain of which is given by the solution of the above periodic optimal control problem. In [12], two different solutions to the problem of estimating the input disturbance were proposed, based on generalizations of the above fundamental results in periodic control theory. The basic idea is to apply a Kalman filter to an augmented model, formed by appending to the system dynamics a model of the external periodic disturbance. The filter therefore provides simultaneously estimates of the state vector and of the external disturbance; the latter can be used for direct feedforward in the control law. Notice that by this approach one can easily deal with disturbance models with poles on the imaginary axis.

Remark 2: The above formulation for the disturbance attenuation problem is clearly based on a matching assumption, i.e., disturbances acting on the system through the same channel as the control inputs are considered. The effect of this assumption on the attitude control problem will become clearer looking at the simulation results given in Section IV.

C. Estimation of the State and of the Input Disturbance

Consider the periodic disturbance $d(t)$ acting at the input of the system. It is composed by harmonics at frequencies $k\Omega$, k integer, so that it can be represented by means of a Fourier series expansion $d(t) = \sum_k d^{(k)}(t)$ where $d^{(k)}(\cdot)$ is the sinusoidal contribution at $k\Omega$. In order to obtain a dynamic model for the signal, we will describe each of the $d^{(k)}(t)$s as the signal generated by a time-invariant second-order filter with poles located at $\pm jk\Omega$ fed by white noise. Denoting by $\xi^{(k)}(\cdot)$ the state vector of such filter and by $z^{(k)}(\cdot)$ the white noise vector, the disturbance model for harmonic $k\Omega$ is

$$\begin{aligned}\dot{\xi}^{(k)}(t) &= W^{(k)}\xi^{(k)}(t) + z^{(k)}(t)\\ d^{(k)}(t) &= h\xi^{(k)}(t)\end{aligned} \tag{12}$$

where

$$W^{(k)} = \begin{bmatrix} 0 & -(k\Omega)^2 \\ 1 & 0 \end{bmatrix}, \qquad h = [1 \quad 0]. \tag{13}$$

For practical purposes the disturbance is in fact approximated by a finite number—say r—of sinusoids, the global disturbance model is then given by

$$\begin{aligned}\dot{\xi}(t) &= W\xi(t) + z(t)\\ d(t) &= H\xi(t)\end{aligned} \tag{14}$$

where $\xi(t) = [\,\xi^{(0)}(t) \quad \xi^{(1)}(t)' \quad \xi^{(2)}(t)' \quad \cdots \quad \xi^{(r)}(t)'\,]'$ and $z(t) = [\,z^{(0)}(t) \quad z^{(1)}(t)' \quad z^{(2)}(t)' \quad \cdots \quad z^{(r)}(t)'\,]'$ and matrices W and H are partitioned as follows:

$$\begin{aligned} W &= \text{blockdiag}[0,\ W^{(1)},\ W^{(2)},\ \ldots,\ W^{(r)}]\\ H &= [\,1 \quad h \quad h \quad \ldots \quad h\,].\end{aligned}$$

By adding the state $\xi(t)$ of the disturbance model to the state $x(t)$ of the original system, one obtains the augmented state $x_e(t) = [\,x(t)' \quad \xi(t)'\,]'$ which is governed by

$$\begin{aligned}\dot{x}_e(t) &= A_e(t)x_e(t) + B_e(t)u(t) + v_e(t)\\ y(t) &= C_e(t)x_e(t) + D_e(t)u(t) + w_e(t)\\ d(t) &= H_e x_e(t)\end{aligned} \tag{15}$$

where

$$\begin{aligned} A_e(t) &= \begin{bmatrix} A(t) & B(t)H \\ 0 & W \end{bmatrix}, \quad B_e(t) = \begin{bmatrix} B(t) \\ 0 \end{bmatrix}\\ C_e(t) &= [\,C(t) \quad D(t)H\,], \quad D_e(t) = D(t),\\ H_e &= [\,0\cdots 0 \quad H\,].\end{aligned}$$

The noises are given by $v_e(t) = [\,v(t)' \quad z(t)'\,]'$ and $w_e(t) = w(t)$, with intensities

$$V_e = \begin{bmatrix} V & 0 \\ 0 & Z \end{bmatrix} \geq 0, \quad W_e > 0.$$

With a Kalman filter it is then possible to find the estimate $\hat{x}_e(\cdot)$ of the augmented state $x_e(\cdot)$ from the measurements of $[u(\cdot), y(\cdot)]$

$$\begin{aligned}\dot{\hat{x}}_e(t) &= A_e(t)\hat{x}_e(t) + B_e(t)u(t) + L(t)e(t)\\ e(t) &= y(t) - [C_e(t)\hat{x}_e(t) + D_e(t)u(t)].\end{aligned} \tag{16}$$

$L(t)$ is the Kalman filter gain given by $L(t) = \Pi(t)C_e(t)'W_e^{-1}$, $\Pi(t)$ being the SPPS solution of the PRE associated with the filtering problem

$$\begin{aligned}\dot{\Pi}(t) = {} & A_e(t)\Pi(t) + \Pi(t)A_e(t)' + V_e\\ & - \Pi(t)C_e(t)'W_e^{-1}C_e(t)\Pi(t).\end{aligned} \tag{17}$$

In conclusion, the estimate of the *input disturbance* $d(\cdot)$ can be obtained as $\hat{d}(t) = H_e\hat{x}_e(t)$ or equivalently as $\hat{d}(t) = H\dot{\hat{\xi}}(t)$.

The above control problem can be solved by means of the available periodic control theory outlined in Section II.

Finally, the actual control input $u(t)$ can be derived from $\overline{u}(t)$ by recalling that $u(t) = \overline{u}(t) - \hat{d}(t)$.

Remark 3: The above results on the optimal control and periodic disturbance attenuation for periodic systems have been recently extended to the case of the H_∞ synthesis problem (see [3], where the results of [18] for time invariant systems are extended to the case of periodic systems) and applied to the attitude control of a small satellite.

IV. Simulation Results

MITA is a three-axis stabilized satellite carrying on board NINA, a silicon spectrometer for charged particles developed by INFN (Istituto Nazionale di Fisica Nucleare). The selected orbit for the MITA mission is a circular one, with an altitude of 450 km and an inclination of 87.3°, provided by a COSMOS launcher. The moments of inertia are $I_{xx} = 36$, $I_{yy} = 17$, $I_{zz} = 26$, $I_{xy} = 1.5$, $I_{xz} = I_{yz} = 0$ kg m^2 and the attitude control system shall ensure a three axis stabilization with the NINA detector always pointing in the opposite direction of the earth (zenith). For the purpose of the present study, the attitude control system is composed by the following sensors: one star sensor, one triaxial magnetometer (redundant), and five coarse sun sensors (redundant). The attitude actuators are one momentum wheel, mounted along the z body axis, and three magnetic coils (redundant).

In this section, some simulation results obtained by the application of the above described periodic control techniques to the dynamics of the MITA spacecraft are presented. In particular, the effect of aerodynamic torque and of a magnetic residual dipole disturbance will be taken into account and the performance of the optimal periodic controller, both in the simple and the disturbance modeling cases, will be presented and discussed and the benefits of the proposed periodic disturbance modeling technique highlighted. Because of the very small attitude perturbations involved in this application, the spacecraft dynamics has been simulated by means of the linearized model described in Section II-C, starting from an initial condition of 1° rotation and zero angular rate along all axes from the target attitude.

A first set of simulations was carried out in order to evaluate the effect of an external disturbance torque on the designed periodic controller. In particular, the presence of an external torque due to a residual magnetic dipole of the spacecraft of an intensity of 1 Am^2 along each body axis was assumed together with a disturbance torque along the pitch axis of 10^{-4} Nm. The result of such simulations is shown in Figs. 2 and 3 from which the residual oscillation of the attitude angles due to the presence of a periodic disturbance is clearly visible.

In order to improve the performance of the control law, the disturbance estimation and compensation scheme described in the previous sections has been implemented and simulated. Care has to be taken in the modeling of the input disturbance, in order to avoid an excessive increase in the dimensions of the Kalman filter state vector. For the purpose of the present study, and considering the fact that the orbit period is considerably longer than the desired filter time response, the three components of the disturbance have been simply modeled as uncertain constants. This leads to a ninth-order Kalman filter (including as state variables the attitude angles, the body rates and the three components of the disturbance). The estimated disturbance is then compensated directly by adding a feedforward component to the control action computed by the stabilizing optimal periodic controller. The results obtained by means of this technique are shown in Figs. 4 and 5. As can be seen, the improvement in the controller performance is considerable, however, the complete rejection of the disturbance torques cannot be achieved as the matching condition is not entirely satisfied in this case.

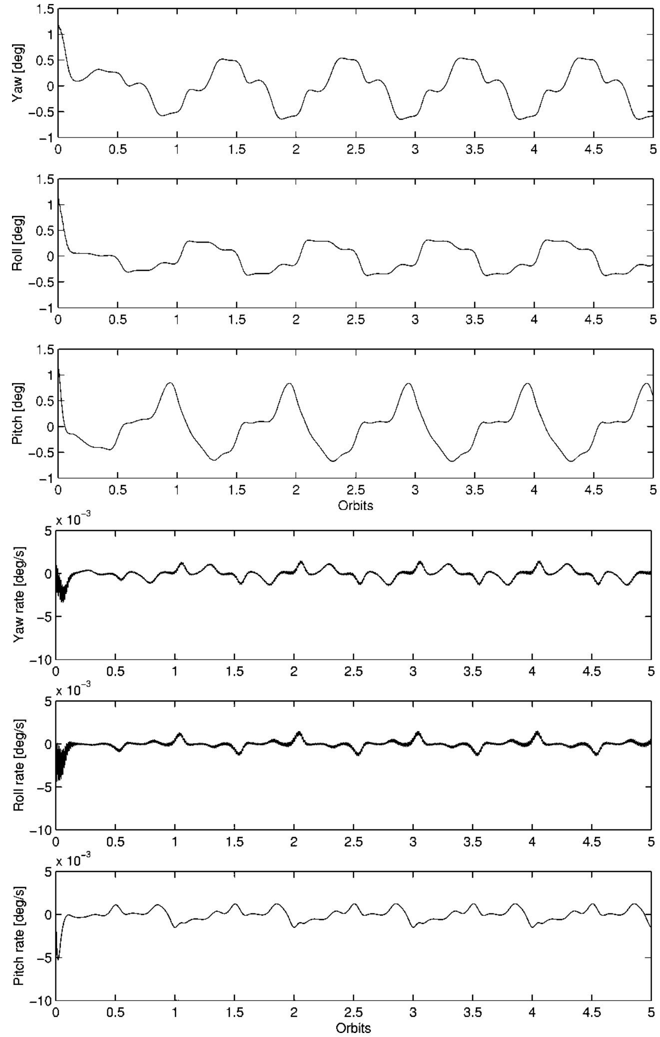

Fig. 2. Attitude angles and rates: simulation with disturbance torques.

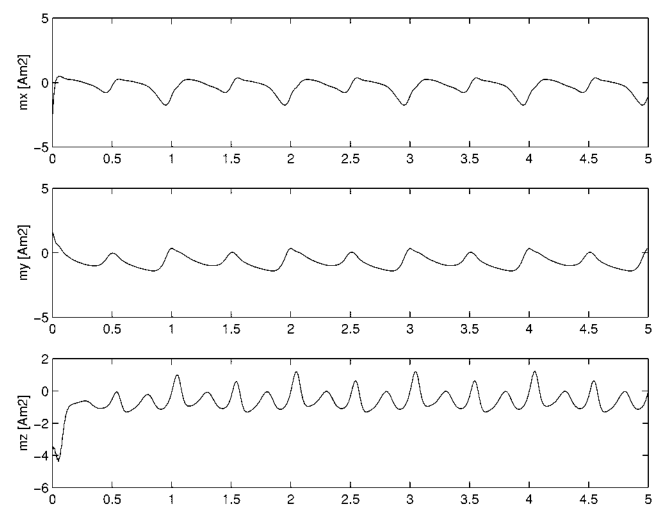

Fig. 3. Magnetic moments: simulation with disturbance torques.

V. Concluding Remarks

The attitude control problem for a small spacecraft using magnetic actuators has been considered and analyzed in the

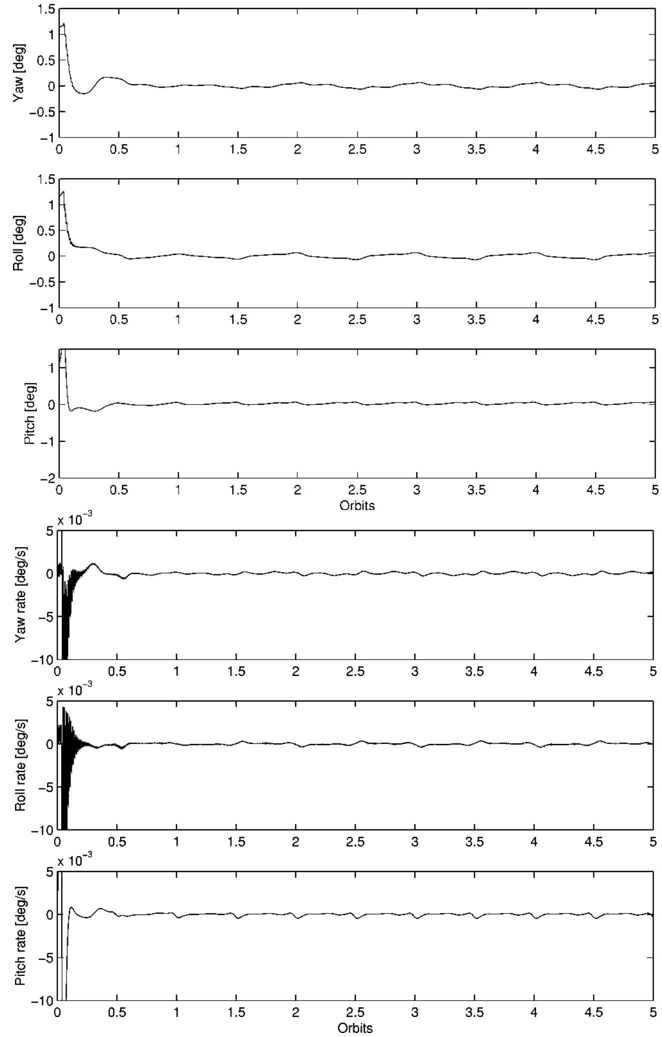

Fig. 4. Attitude angles and rates: simulation with magnetic disturbance compensation.

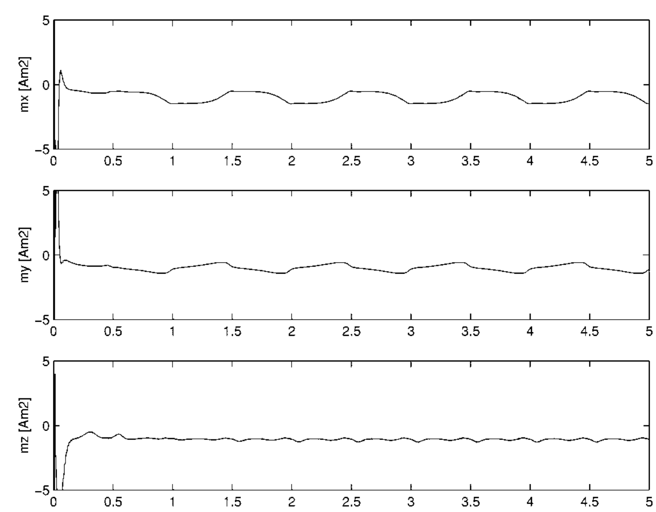

Fig. 5. Magnetic moments: simulation with magnetic disturbance compensation.

framework of periodic control theory. A solution in terms of classical LQ periodic optimal control has been proposed and extensions thereof, aiming at achieving efficient rejection of periodic disturbance torques have been presented and discussed. Simulation results show that good performance can be obtained by means of this approach.

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