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Review Article

A survey on active magnetic attitude control algorithms for small satellites

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ABSTRACT

Control algorithms for the active magnetic attitude control systems are covered in the survey. Three different situations in the magnetic system implementation are considered. First, angular velocity damping is covered. Second part is devoted to the combined operation of the active magnetic system with other actuators and with the help of some passive stabilization concepts. Magnetic control torque is restricted in its direction, as it cannot be implemented along the geomagnetic induction vector. This restriction may be lifted by enhancing the active magnetic attitude control with other actuators and concepts. This comes at the cost of restrictions on the available attitude modes of the satellites. Namely, passive gravity-gradient stabilization provides the nadir pointing; bias momentum satellites are restricted by the wheel axis pointing along the orbital normal; spin stabilized satellites acquire only one axis attitude. Finally, solely three-axis magnetic attitude control is considered. Different approaches to the control torque restriction problem are covered, with distinction between the local feedback laws and optimization methods. The survey does not cover passive magnetic control concepts and auxiliary role in the reaction wheels momentum unload. Comparison of the covered algorithms is provided to highlight the authors' opinion on the algorithms modern implementation and further research directions.

1. Introduction

Magnetic attitude control systems have been used since the beginning of space era. The first satellite with the passive magnetic control was Transit 1B [1], launched April 13, 1960. The first active magnetic control flown on Tiros II [2], launched November 23, 1960. Attitude reconstruction using the geomagnetic field measurements was first performed with Sputnik-3 [3], launched May 15, 1958.

Passive systems were extremely popular in the first decades of space exploration. They include the permanent magnet, hysteresis rods and different dampers. Permanent magnet allows the satellite to stabilize roughly along the local geomagnetic induction vector. Hysteresis rods as well as passive viscous dampers provide the angular velocity damping. These systems are simple and reliable; they do not require any onboard computation effort and consume no energy. These features were very important in the beginning, with limited experience in space exploration and low satellite capabilities. These systems are still popular, especially for miniaturized satellites, if the satellite demands in the time-response and pointing accuracy are low.

Active magnetic control systems become preferable in most situations with the advancement of microelectronics and overall satellite components miniaturization. They are especially attractive for small satellites which, in turn, received increased interest and capabilities due to the very same reasons. Small satellites are relatively low cost and fast to build. This makes them extremely valuable as a technology demonstration and educational missions, delivering new technologies and ideas into space as fast as possible. Small satellites have soft demands on the attitude accuracy quite often, and magnetic control system is a natural choice for these missions.

The magnetic attitude control system is composed of magnetorquers (three in most cases) mounted perpendicular to each other. The magnetorquer (Fig. 1) is essentially a number of wires, elongated with the magnetizing core or flat air core coil, that produces the dipole moment as the current is applied to the circuit. The dipole moment $\mathbf{m}$ interacts with the external magnetic field (governed largely by the geomagnetic field) with induction $\mathbf{b}$ leading to the control torque $\mathbf{M} = \mathbf{m} \times \mathbf{b}$. The induction $\mathbf{b}$ is expressed in the satellite reference frame.

The cross product implies that the torque cannot be applied along the geomagnetic induction vector (Fig. 2). This gives rise to the controllability problem: any required control torque $\mathbf{M}_{ref}$ cannot be applied, only its projection on the plane orthogonal to $\mathbf{B}$ is available. However, the geomagnetic induction vector changes its direction as the satellite moves along the obit. The uncontrollable direction changes and all directions become available with time. This intuitive reasoning is properly justified in [4,5]. These papers prove the controllability in case of periodic geomagnetic field vector change and strongly accessible

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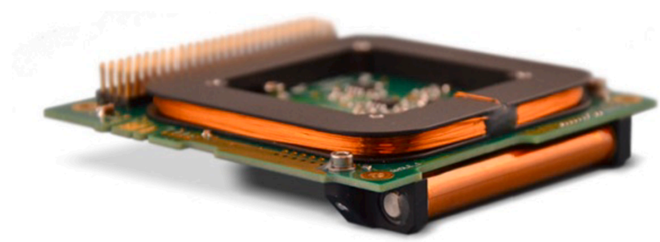

Fig. 1. ISIS magnetorquer board.

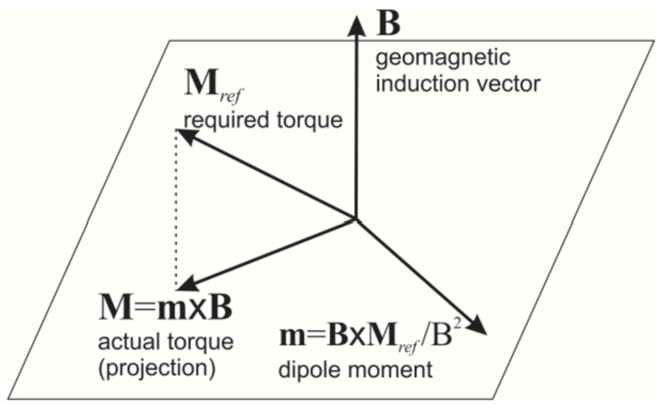

Fig. 2. Magnetic control geometry.

otherwise. The assumption on the periodic nature of the geomagnetic field is reasonable, as the field is often modelled with the dipole approximation. The most popular approach is to use the tilted dipole model, with the dipole aiming at the magnetic poles. The induction vector change is governed by the satellite motion along the orbit and Earth rotation. The latter is slow relative to the orbital rotation on the low Earth orbit. Neglecting the slow Earth rotation, the direct dipole model is constructed. The dipole is antiparallel to the Earth rotation axis. The induction vector changes periodically with the period twice of the circular orbital one. Discussion on different models applicability for the satellite angular motion analysis can be found in [6,7] with quadrupole and higher order models covered in [8]. In [9] the controllability of the linearized system is proven in case the satellite inertia tensor does not satisfy two specific conditions, one of them stating the satellite is not axisymmetrical. Paper [10] proposes the proof and control strategy for small divergence from the necessary attitude.

There are three main approaches to overcome the restriction in the control. First, other actuators and mechanical concepts may be used to enhance the magnetic control and, essentially, provide the control authority along the induction vector. Second, the control may be constructed with specific optimization approaches that directly account for the control restriction. Finally, only the available control part is provided by the control system. This is the projection on the plane perpendicular to the geomagnetic induction vector. In [11] an interesting projection approach fact is noted and utilized. As there are only two independent directions in the inertial space to provide the torque along, only two magnetorquers are necessary. This implies that one magnetorquer failure does not render the satellite uncontrollable. This may not be the case for the specific orbit geometry of near polar and near equatorial orbits due to the geomagnetic field vector rotation. Clear interpretation lies in the cross product nature of the control torque. Two magnetorquers provide control authority along all three satellite axes, although with limited variability.

This survey covers algorithms for the active magnetic attitude control system, with missions out of the scope. Passive magnetic systems, as well as auxiliary magnetic system role in the reaction wheels momentum management, are not covered. The focus lies on the works where the magnetic control system plays key role, or at least provides an important benefit to the control system. The engineering aspects of the magnetic control onboard implementation are covered very briefly, and mainly with regards to the control construction.

The paper is organized in three main sections. First, the angular velocity damping is considered. This is an important factor in the mission success, as it is necessary to provide fast Sun acquisition, antenna pointing and transition to subsequent control regimes.

Second part is devoted to the operation of the active magnetic system with other actuators and mechanical concepts. This section starts with the spin stabilized satellites. Fast rotating satellite acquires the gyro properties and tends to maintain the spin axis direction in the inertial space. Disturbance torques slowly change the spinning direction and increase the nutation amplitude, with the effect of the gravitational and magnetic torques on the spin stabilization carefully studied in [12]. Spin stabilized satellite is fully controllable by the magnetic control system. This concept, very popular in the beginning of space era, is still actively used. Next, the bias momentum satellite is covered. The satellite is equipped with a flywheel with constant rotation rate. As a result, the satellite again resembles the gyro. The flywheel axis tends to align with the orbital normal, with the magnetic control providing the asymptotic stability and control authority along the flywheel axis. Next important passive stabilization concept utilizes the gravitational torque, with the magnetic control system providing additional stability properties. Finally, other concepts (reaction wheel, thrusters, one axis control, aerodynamic drag and some exotic approaches) are briefly covered.

Third part is devoted to the fully magnetic three-axis attitude. This is the most challenging task from the control construction and implementation point of view. The feedback law is considered first. This approach is straightforward and suitable for the onboard implementation but fully susceptible to the control restriction. The control is implemented as a projection, and careful control gains adjustment and disturbance consideration are necessary. Sliding control provides some room for the improvement. Sliding surface construction offers the tool for the control restriction constraint leveraging. The path of the satellite angular maneuvers may be constructed to soften the restriction, trying to adopt the path that can be controlled by the magnetic system. This is fully utilized with the optimization approaches. Offering great tools in the specific magnetic control problem, these approaches are hard to grasp for the onboard implementation and often require significant computational effort.

This paper division is based on the specific magnetic control implementation issues. However, some control concepts are utilized with little differences in a number of situations. Linear quadratic regulator is a sound example, as it is very popular control concept. As a result, interconnection arises between the sections. Still, this division was preferred over the control concept-based due to the practical attitude of this survey. Moreover, a lot of popular control laws, especially early ones, are based on the engineering insight rather than on the established results in control theory. Finally, chronological division was discarded as the least informative one.

Survey is centered on recent papers, especially published in the past twenty years. With the review on the magnetic attitude control concepts published in 2001 [13]; brief survey on the magnetic control published in 2005 [14]; and comprehensive study on the advances of the beginning of space exploration published in 1983 [15], it is not necessary to cover fully earlier research. Moreover, the most interesting problem of the solely magnetic three-axis control was proposed relatively recently [16]. The survey covers most important and influential early results, and the majority of papers on the magnetic control published in this century. The most important and interesting papers are covered in detail. Each section has a number of works mentioned in the end without details. These are solid papers that lack any new control schemes and major new results, but provide some additional and important insight on the problem. Some papers are omitted due to the simple fact that the magnetic control law itself is not present, even in a hint of a control strategy (as is often with optimization approaches). Offering good simulation results, these papers do not contribute anything to the control of real satellites due to the unknown used law. Some conference proceedings are not covered due to the lack of online representation. However, this automatically makes them less influential and important. Conference proceedings are also omitted if proper journal paper finalizing the conference report or multiple conference papers exist.

2. Angular velocity damping

Angular velocity damping is an essential process for the satellite success. Excessive velocity may arise during the separation from the launch vehicle, orbital correction thruster firings, and in emergency situations. Damping may be performed with any attitude control actuators. However, this requires either fuel for the thruster or saturates reaction wheels. At the same time, the accuracy necessary in the damping process is often low, as it is only a transient process before the actual attitude stabilization. Magnetorquers, requiring no fuel and witnessing no saturation, are a natural choice for the angular velocity damping.

Paper [17] introduced the underactuation problem. Proposing no rigorous solution, this work pointed out that manipulation with the satellite angular momentum is easier than with the satellite body attitude. This fact was used with great success in the Bdot control construction. This control was first publicly published in [18]. However, it was first mentioned in the report [19] prepared by the authors of [18]. The authorship is prescribed to the GSFC engineer Seymor Kant. The control is constructed on the basis of the negative kinetic energy derivative. The actual control is

$$\mathbf{m} = k\boldsymbol{\omega} \times \mathbf{b} \tag{1.1}$$

with the satellite angular velocity in the inertial space. This control is transformed into the well-known Bdot law according to the relation between the time derivative of the geomagnetic induction vector in the bound reference frame $d\mathbf{b}/dt$ and its derivative in the inertial frame $d\mathbf{B}/dt$

$$\frac{d\mathbf{b}}{dt} = \frac{d\mathbf{B}}{dt} - \boldsymbol{\omega} \times \mathbf{b}. \tag{1.2}$$

The first term in the right side is the natural change of the geomagnetic induction vector with time. The field changes almost periodically with approximately double orbital velocity. The second term is due to the satellite rotating around its center of mass. Angular velocity damping is always performed for the fast rotating satellite, so the second term prevails over the first one. Omitting $d\mathbf{B}/dt$ in (1.3) the control (1.2) becomes

$$\mathbf{m} = -k\frac{d\mathbf{b}}{dt} = -k\dot{\mathbf{b}} \tag{1.3}$$

As the time derivative is often denoted by a dot the algorithm name becomes clear. Control (1.4) is very simple and offers great advantages for the initial detumbling. It does not require rigorous sensor filtering and is always implemented in the form

$$\mathbf{m} = -k\frac{\mathbf{b}_{k+1} - \mathbf{b}_k}{t_{k+1} - t_k} \tag{1.4}$$

Here two consequent raw measurements of the induction vector in the satellite reference frame at two consequent moments of time are used. Note that higher order approximations of the derivative may be used. However, as the overall algorithm accuracy is low, this is hardly worth it.

As mentioned above, the term of the natural change of the geomagnetic field is omitted. Satellite velocity damping by the law (1.4) is restricted by the omitted term. So the satellite ends tumbling with roughly double angular velocity. The actual value is a bit less [20]. Control (1.2) provides great accuracy as it is asymptotically exponentially reduces the angular velocity to zero [21,22] with the time-response depending on the orbit inclination [21,23]. The expected accuracy may even force the residual magnetic dipole moment of the satellite estimation [24] and compensation, especially if the residual dipole is large and has unknown direction [25].

Control (1.2) may utilize the relative angular velocity for the motion of the satellite in the orbital reference frame. The control is constructed on the basis of the Lyapunov function, with the Jacobi first integral for the motion in the gravitational field as the candidate function. Paper [26] revisited the stability problem of the control (1.2). Practical control gain adjustment technique is proposed. Paper [27] enhanced control (1.2) substituting scalar control gain with a positive-defined matrix. This provides more room for the control gain selection optimization. Asymptotic stability of the passive gravitational attitude with enhanced control is proven (with remarks suggested in [28]).

Paper [29] provides a rigorous example of numerical simulation of different Bdot modifications with engineering aspects taken into account. Intuitive approach to the damping control construction is presented in [30]. Based on the idea of simple velocity damping for each channel ($M_i \sim -\omega_i$), corresponding magnetic control that tries to provide the supposed scheme is constructed. Asymptotic stability of the necessary attitude is proven through the Lyapunov function and applicability is tested in the hardware-in-the-loop tests.

Table 1summarizes main pros and cons of different angular velocity damping algorithms.

Two first rows represent almost the same asymptotically damping algorithm. Control gain enhancement proposed in [27] slightly enhances control (1.2). However, it does not provide major overhaul and basic control algorithm may be used in most situations. Main pros of this algorithm over the Bdot are the accuracy of the resulting damping and the ability to stabilize in the orbital (local vertical) reference frame. Control (1.4) can hardly be proposed instead of (1.5) due to the extreme simplicity of the latter. Control proposed in [30] does not significantly outclass Bdot. Overall, Bdot control is best suited for the coarse initial detumbling, while control (1.2) should be used for the accurate stabilization process, for example with other stabilization devices or concepts (gravity gradient, flywheel, etc).

Further investigation of the angular velocity damping algorithms has relatively low priority in authors' opinion. This problem is very well established. It requires some knowledge level maintaining research but it is hardly opened for significant breakthroughs. Simplicity and reliability of the Bdot law, as well as the accuracy of the asymptotic damping (1.2), and their flight heritage present a natural barrier for any

Table 1
Angular velocity damping algorithms.

Algorithm/paper	Simplicity	Time response	Accuracy	Flight heritage	Inertial damping	Orbital damping
(1.2)	Moderately high	Moderately high	High	High	+	+
[27]	Moderately high	High	High	Low	+	+
(1.4)	Moderately high	Moderately high	Mediocre	Low	+	–
(1.5)	High	Moderately high	Low	High	+	–
[30]	High	Moderately high	Low	Low	+	+

new developments.

Some other recent papers on the subject are [31–40].

3. Magnetic attitude control with other actuators and mechanical concepts

Implementation of the magnetic attitude control system combined with other actuators and especially passive stabilization concepts is very tempting. This approach maintains relatively low cost, mass and power consumption and provides high performance at the cost of restricted attitude patterns. However, as the major patterns (local vertical, Sun pointing) are available, approaches considered below are preferable over less reliable and poorly verified solely magnetic attitude control in most situations.

3.1. Spin stabilization

Fast rotating satellite acquires gyro properties. Spinning around the maximum moment of inertia is stable, and the axisymmetrical satellite maintains its attitude in the inertial space provided there are no disturbances. External disturbances and errors in the attitude command and determination induce nutation motion. As the satellite rotation rate is large, even small error in the angular velocity produces considerable disturbing torque due to the coupling term $\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega}$ in the equations of motion. Control system is necessary to counteract to this term and reorient the spinning axis in the necessary direction. Magnetic control system handles these tasks, as the necessary control torque may luckily be implemented perpendicular to the geomagnetic induction vector. This provides some crucial attitude regimes like Sun pointing of the solar panels, inertial star pointing or sky sweeping for space telescopes.

One of the most popular schemes was proposed in [41]. The control is divided into three steps. First, nutation damping is implemented along the third axis of the satellite (assumed to be the axis of symmetry), then the satellite is spun around this axis, and, finally, the spin axis is reoriented in the inertial space. Bang-bang control and corresponding switching functions were provided. This technique is still popular. For example, it was used in [42] with extensive numerical simulation. The control here is

$$m_j = \begin{cases} m_0, & \mathbf{h}\cdot(\mathbf{e}_j \times \mathbf{b}) > 0 \\ -m_0, & \mathbf{h}\cdot(\mathbf{e}_j \times \mathbf{b}) < 0 \end{cases} \tag{2.7}$$

Here m_j is the dipole moment along the satellite j axis with the unit vector $\mathbf{e}_j$, $\mathbf{h}$ is the angular momentum vector of the satellite. As the spin stabilization utilizes the large angular momentum properties, it is naturally present in most relevant control schemes. Assuming that the third axis is the spin axis, taking $j = 1$ or 2 provides the spin rate control in (2.7). Taking $j = 3$ this provides the precession control which was proposed in [43] for coarse precession logic.

Following this general scheme, three simple control dipole moments were introduced in [44–46]. First, the nutation damping algorithm is

$$m_3 = -k\frac{d\mathbf{b}}{dt}\mathbf{e}_3. \tag{2.8}$$

This is the Bdot control implemented with one coil. It does not affect the spin rate about the spin axis, as it is necessary to spin the satellite around this axis. Comprehensive analysis of that control law is performed in [47]. In [48] another approach is proposed. Based on the Lyapunov function, the following law was introduced for the asymmetric satellite,

$$m_3 = m_{30}\,\mathrm{sign}[(C - A)b_2\omega_1 - (C - B)b_1\omega_2].$$

If necessary, spinning algorithm is implemented,

$$\mathbf{m} = k(b_2, \ -b_1 \ \ 0) \tag{2.9}$$

Clearly, from the cross product this produces spinning torque along the third axis. Disturbing torque components about two first axes are suppressed with the nutation damping algorithm. Popular spinning algorithm was proposed in [49]. Called "Y-Thomson" and very often "Y-Thompson" spin, the control for the second axis (or y axis in $x - y - z$ reference frame notation) rotation is

$$m_1 = k(\omega_2 - \omega_{2ref})\mathrm{sign}(b_3) \tag{2.10}$$

with ω_{2ref} being the required spin rate. Finally, spin axis reorientation algorithm is

$$m_3 = k\Delta\mathbf{h}\cdot(\mathbf{e}_3 \times \mathbf{b}) \tag{2.11}$$

where $\Delta\mathbf{h} = \mathbf{h}_{ref} - \mathbf{h}$ is the difference between the necessary $\mathbf{h}_{ref}$ and current $\mathbf{h}$ angular momentum vectors. Satellite dynamics is studied with the averaging method to assess the algorithms performance dependence on the orbit inclination in [44–46].

Proposed control essentially utilizes the difference between the necessary and current attitude of the angular momentum, not the satellite body. This is generalized in [50] with attitude errors

$$\Delta_1 = \mathbf{H}_{ref} - \mathbf{h} - \mathbf{J}\boldsymbol{\omega}, \quad \Delta_2 = \mathbf{h}_{ref} - \mathbf{h} - \mathbf{J}\boldsymbol{\omega}. \tag{2.12}$$

The first error utilizes the necessary angular momentum direction in the inertial space $\mathbf{H}_{ref}$. It should drive the spin axis to orient along the necessary direction. The second error utilizes the angular momentum in the body frame and should drive the satellite angular velocity to the desired spin value.

Paper [51] extends the angular velocity damping approach and analysis method proposed in [26] on the spin acquisition problem. The control to drive the satellite angular momentum $\mathbf{h}$ to the necessary value $\mathbf{h}_{ref}$ is

$$\mathbf{M} = k(\mathbf{E} - \mathbf{b}\mathbf{b}^T)\Delta\mathbf{h}.$$

This is the projection of the error in the angular momentum vector on the plane perpendicular to the geomagnetic induction vector. Global exponential stability for that control is proven. Simulation provides good results with convergence times of about one orbital revolution.

Basic law (2.11) may be further simplified. In [52] the spin axis is maintained perpendicular to the Sun direction $\mathbf{S}$ with the law similar to (2.7),

$$m_3 = \begin{cases} m_0, & \mathbf{e}_3\mathbf{S} > 0 \\ -m_0, & \mathbf{e}_3\mathbf{S} < 0 \end{cases} \tag{2.13}$$

Three similar algorithms are proposed in [53]. Based on the Lyapunov function approach, the dipole moment is

$$\mathbf{m} = \mathbf{b} \times [k_1(\omega_1, \ \ \omega_2, \ \ 0) + k_2(0, \ \ 0, \ \ h_3 - h_{3ref}) + k_3\Delta\mathbf{h}]. \tag{2.14}$$

Here the nutation damping, spinning and spin axis reorientation control parts are clearly identified. Spin stabilization, especially if only some of these three control components are necessary, is often performed with one or two magnetorquers. In [53] it is explicitly shown that only one magnetorquer that is perpendicular to the spin axis is enough to stabilize the satellite in the inertial space, although with reduced time-response. The control proposed in [53] was used in [54,55]. However, as the residual dipole moment was too large to fully compensate, the control was modified for successful very high-speed rotation.

In [43,56] the following spinning algorithm was proposed and studied

$$\mathbf{m} = -k(\dot{\mathbf{b}} + \boldsymbol{\omega}_{ref} \times \mathbf{b}) \tag{2.15}$$

Lyapunov function is used to prove the asymptotic stability in the spin acquisition phase. Note the presence of the clear Bdot damping part in (2.15). Velocity-driven error may also be introduced on the basis of the Bdot construction logic, but with respect to the frame rotating with the required angular velocity [57].

In [58] control law

$$\mathbf{u} = -\mathbf{K}_1(\boldsymbol{\omega} - \boldsymbol{\omega}_{ref}) - \mathbf{K}_2|h_3|\delta\theta[\mathbf{h}' \times (\dot{\mathbf{s}} \times \mathbf{h}')] + \mathbf{K}_3\dot{\mathbf{s}} \times \mathbf{e}_3$$

is proposed and numerically studied, where $\mathbf{h}'$ is the unit angular momentum vector, $\mathbf{s}$ is the Sun direction (necessary angular momentum and third axis direction) and the error in pointing is given by $\cos\delta\theta = -s_3$. The control is implemented by the magnetorquers in a projection manner and only the speed regulating term is used if $h'_3 < 0.8$, so the satellite is first spun around the necessary axis.

Paper [59] proposed very popular law for the roll-yaw angles control

$$m_2 = k_1 b_1 \varphi - k_2 \dot{b}_2 \tag{2.16}$$

Here the pitch dipole moment depends on the roll error (which provides the precession control) and pitch speed (nutation damping). Angles ψ, φ, θ utilize the 1-2-3 rotation sequence from the orbital frame with the third axis pointed at nadir and second antiparallel to the orbit normal. Multiple time scales method is used to provide the asymptotic stability of the necessary attitude depending on the control gains. This provides an effective control gains selection procedure. The result is further validated using the Floquet theory and numerical simulation. Finally, exact approximate expressions are provided for the roll and yaw errors due to the solar radiation pressure disturbance, the pitch axis residual dipole moment and the geomagnetic induction vector diurnal rotation.

Paper [60] proposed more general control

$$m_3 = k_1 f\left[b_1(a_{23} - k_2\omega_2) - b_2(a_{13} - k_2\omega_1)\right] \tag{2.17}$$

for the third satellite axis attitude control. Here the function f satisfies the conditions $xf(x) > 0$ and $f(0) = 0$. This function provides a generalization for a feedback law and $f(x) = x$ is satisfactory choice in most situations.

Paper [61] proposes a robust scheme with control commands computed on the ground in advance, based on the geomagnetic induction vector direction prediction, and its averaged motion. In [62] Pontryagin's principle is used to generate time-optimal maneuvers of a spin stabilized satellite. This scheme is constructed with the assumptions of already spinning satellite and slow reorientation procedure. The latter is not an artificial one. The magnetic control system often produces relatively small control torque. As a result, spinning satellite with large angular momentum is reoriented slowly.

An intuitive approach to the rotation rate control is presented in [63]. Equations of motion are linearized and the control is considered to be constant on some time interval. This allows simple control formulation, i.e. apply the dipole moment if the yaw angle exceeds some threshold value and if $b_1 - \psi b_3 \approx 0$. The first condition represents the deviation of the spin axis from the necessary attitude, which may be due to the decay in the spinning rate. So, it is necessary to spin the satellite. Second condition represents the situation of the proper attitude of the magnetorquer (placed in the xy plane of the satellite) with respect to the geomagnetic induction vector. Finally, the spinning control is applied for a given time when the satellite attitude is favorable. This time Δt and the value of the applied dipole moment m_0 are derived from the approximate change in the yaw angle,

$$\Delta\psi \approx -\frac{m_0}{h_0} b_2 \Delta t$$

where h_0 is the current angular momentum vector magnitude. This provides the time of the control implementation if the magnetorquer is able to produce only constant dipole moment. It is important to note that the longer the control time interval, the less accurate is the assumption on the “ideal” satellite attitude with respect to the geomagnetic field for the optimal control implementation.

Spin stabilization is fully controllable by the magnetic control system. As a result, simple control concepts are very popular. Still, optimal control approaches are also used. Paper [64] is a good example. The linearized equations of motion are used for the LQR control construction. The dipole moment derivation utilizes the spin stabilization property. The main task in the dipole construction is to obtain proper control torque perpendicular to the spin axis. For example, let

$$m_1 = -kM_{2ref}, \quad m_2 = kM_{1ref}$$

where $\mathbf{M}_{ref}$ is the required control torque which is then implemented by the magnetorquers. As a result, the torque applied to the satellite is

$$M_1 = kb_3 M_{1ref}, \quad M_2 = kb_3 M_{2ref},$$
$$M_3 = -k(b_1 M_{1ref} + b_2 M_{2ref}).$$

The third geomagnetic induction vector component may be eliminated from these expressions by the gain scaling, or the sign of kb_3 may be maintained positive by changing the gain sign. Disturbing third component of the torque produces negligible effect due to the fast spinning satellite and provides almost no evolutionary effect on the satellite motion. In paper [65] the spin axis attitude control was considered constant during one orbit revolution due to the simplified dynamics of the fast spinning satellite and to some extent due to the passive dampers present on the satellite.

Paper [66] incorporated different control laws in one general strategy of the one-axis stabilization with the control

$$\mathbf{m} = -\mathbf{k}_1 \mathbf{b}_{ref} - \mathbf{k}_2(\dot{\mathbf{b}}_{ref} - \dot{\mathbf{b}}) - diag(\mathbf{k}_2)(\omega_3 - \omega_{3ref})(\mathbf{b} \times \mathbf{e}_3) \tag{2.18}$$

where $\mathbf{b}_{ref}$ is constructed in the following way. The goal of the attitude control system is to align $\mathbf{e}_3$ with some prescribed direction $\mathbf{e}$. Assume the satellite is already directed in that way and it is spinning about this axis. For each position on the orbit this provides a cone of possible geomagnetic field vector directions in the body reference frame. Vector $\mathbf{b}_{ref}$ is the intersection between this cone and the plane given by the actual geomagnetic field vector measured in the body frame $\mathbf{b}$ and the spin axis $\mathbf{e}_3$. The reorientation problem is slightly reformulated with this approach. Instead of driving the spin axis directly into the necessary position, some derived attitude is commanded which in the end results in the desired spin axis attitude. The gains are stated to be matrices instead of common scalars. However, paper does not present actual gains used in the simulation, as well as gains selection procedure. The latter is briefly covered by the stability analysis section, where the Lyapunov function is constructed. The analysis is carried out in the linear approximation.

A specific case of a spin stabilized satellite was provided by the passive satellite in the PRISMA mission, a pioneering formation flying effort. The target satellite was equipped with magnetic control system that maintained the solar panels direction roughly to the Sun. However, due to the specific sun-synchronous orbit, the direction to the Sun was also close to the orbit normal. This facilitated the spin stabilization scheme, bringing it close to the bias momentum satellite concept. The control proposed for that motion was [67].

$$\mathbf{M}_{ref} = -k_1 \mathbf{J}(\boldsymbol{\omega} - \boldsymbol{\omega}_{ref}),$$
$$\boldsymbol{\omega}_{ref} = \frac{\mathbf{e}_3 \times \mathbf{s}}{|\mathbf{e}_3 \times \mathbf{s}|} k_2 \Delta\theta + \omega_{spin} \frac{\mathbf{s} + \mathbf{e}_3}{|\mathbf{s} + \mathbf{e}_3|}. \tag{2.19}$$

The first component of the necessary angular velocity $\boldsymbol{\omega}_{ref}$ utilizes the third axis pointing error $\Delta\theta$ which is used to drive the spin axis in the necessary direction. The second term utilizes the required spin rate ω_{spin}, and it should be directed along both the third axis of the satellite and the Sun direction.

Two main categories may be outlined based on the analyzed information (see Table 2).

The bang-bang control does not have fundamental differences from the continuous one. Utilizing the sign function, the bang-bang approach was very popular in the beginning of space exploration. Modern onboard hardware handles continuous control well and it is preferable in most cases. Both approaches are based on the error between the current angular momentum, spin axis direction or spinning rate and their reference values. In this regard most of the outlined control laws are very similar. Laws like (2.9), (2.11), (2.12) present a good choice due to their simplicity and performance.

As with the angular velocity damping, this field of research has very strong legacy and is hardly open for improvements. Moreover, as the

Table 2
Spin stabilization control concepts.

Control concept	Examples	Accuracy	Time-response	Flight heritage	Simplicity
Bang-bang	(2.7), (2.8), (2.10), (2.13)	Moderate	Moderate	High	High
Continuous	(2.9), (2.11), (2.14), (2.15), (2.16), (2.18), (2.19)	High	Moderate	Moderate	High

spin stabilization concept itself is based on the high angular momentum, the time response of the system in terms of the spin axis reorientation is inherently low and is poorly suited for the optimization.

3.2. Bias momentum satellites

Bias momentum satellite incorporates a flywheel with a constant rotation rate. This results in gyro properties if the wheel speed and/or mass are large enough. This resembles the spin stabilization scheme. However, as the satellite itself is not rotating, this admits passive gravitational stabilization concept. The wheel axis tends to align along the orbital normal. The satellite is gravitationally stabilized in the orbital plane. Magnetic control may be used to provide any necessary rotation around the orbital normal, enhance stability properties of a passive gravitationally stabilized satellite, and provide the asymptotic stability by adding the energy dissipation to the system. Note that bias momentum allows the magnetic control even on the equatorial orbit which is always considered the worst case scenario. Angular momentum conservation eliminates the necessity in the control along the orbital normal which is unavailable for the magnetic system. This fact was utilized, for example, in [68].

Paper [18], apart from the Bdot law, utilized simple control (2.16) for the bias momentum satellites. Note that in the linear approximation this becomes

$$m_2 = k_1 b_1 \varphi - k_2 (b_3 \dot{\varphi} - b_1 \dot{\psi}) \tag{2.20}$$

In [69] magnetometer readings in (2.16) are substituted with the Earth sensor outputs passed through the low and high pass filters. Time-response of the control system is evaluated with simple formula.

In [70] similar, essentially a PD control, is proposed,

$$\begin{aligned} m_1 &= \tfrac{1}{b_2}(k_1(\psi_{ref} - \psi) + k_2\dot{\psi}), \\ m_2 &= -\tfrac{1}{b_2}(k_3(\varphi_{ref} - \varphi) + k_4\dot{\varphi}). \end{aligned} \tag{2.21}$$

This control is considered in a pair with the momentum unload control and the pitch control that is implemented with the reaction wheel. Final control utilizes seven different control gains (four for the roll-yaw magnetic control (2.21), two for similar pitch control, one for the momentum unload). These gains are selected with the multi-objective optimization problem. The cost function incorporates the transient time, errors in the attitude and angular velocity. There are predefined constraints on the dipole moment, angular momentum of a reaction wheel and attitude error. These are incorporated into the cost function as penalty functions. In [71] control gains are adjusted using the generic algorithm.

Control (2.21) enhances the stability properties already provided by the flywheel. Relying on the flywheel and gravitational torque requires only damping component from the control system. The time-response of the system, that is the time necessary to align the wheel axis with the orbital normal, is provided by a direct expression in [72] for the control (1.2). In [73,74] the dipole moment proportional to the angular velocity is proposed to drive the total angular momentum vector of the satellite along the wheel axis. Paper [75] compared different wheel start up methods. Namely, there are two general approaches. The satellite is either stabilized first (feedback law was used) and then the wheel is slowly started up while roughly maintaining the attitude, or the wheel is started during the general attitude acquisition phase with simple Bdot law (1.4) to cancel resulting satellite velocity. As expected, the second method provides better time-response while allowing for the easier control logic also.

The main task of the magnetic control system of a bias momentum satellite is to provide the torque along the wheel axis. An interesting approach was proposed in [76]. The linearized equations of motion are augmented with the integral roll angle error. It serves the purpose of easing the control construction for configurations on the orbit when the control system exhibits problems with the disturbance rejection. The control is a state feedback with LQR parameter tuning procedure. Simple in-plane reorientation control is proposed in [72].

$$\mathbf{M} = (0,\, k\sin(\alpha_{ref} - \alpha),\, -k\sin(\alpha_{ref} - \alpha)\frac{b_2}{b_3})$$

for the satellite with the flywheel mounted along the second body axis. This provides the pendulum equation for the in-plane motion (with vanishing amplitude as the control (1.2) is added), with disturbing gravitational torque. Simple equilibrium position solution is given that represents the balance between the control and gravitational torques.

In [77] control (2.20) was modified to the form

$$\mathbf{m} = -k_1(\mathbf{b} - \mathbf{B}) - k_2(\dot{\mathbf{b}} - \dot{\mathbf{B}}) \tag{2.22}$$

earlier proposed for the gravitational stabilization [78]. This control, as well as LQR with the steady-state Riccati equation solution, provided good performance aboard the Gurwin satellite. The LQR approach was earlier proposed for the magnetic control in [79]. Here the polar orbiting satellite was considered with the simplified approach to the disturbance modelling. The periodic torques with orbital and double orbital velocity frequencies are constructed basing on the real analytical models. Also, although the simplified dipole model is used for the control construction to obtain periodic linear system, inclined dipole is utilized in the numerical simulation (as well as a near polar orbit instead of strictly polar). Practically interesting discrete LQR implementation is discussed in [80].

Paper [81] deals with a large number of proposed earlier controllers for the stabilization of bias momentum satellites. The generalization of (2.16) and other simple feedback laws is presented as

$$m_2 = k_1 h_{wheel}(b_1\varphi + k_2 b_3 \psi) - k_3(b_3\dot{\varphi} - k_4 b_1 \dot{\psi}) - k_5(b_3\varphi - k_6 b_1 \psi) \tag{2.23}$$

with six control parameters to be tuned. Note that taking $k_2 = k_5 = 0$, $k_4 = 1$ provides control (2.20). Parameters $k_5 = 0$, $k_2 = k_4 = 1$ provide the law analogous to (2.17) for the bias momentum satellite. In [82] the problem of nutation damping and precession control is generalized for the slightly elliptical orbit. The next generalization level is provided by adding the integral part to this essentially PD control, as was done in [83] for the cancellation for strong, although easily predictable disturbances. Control (2.23) was revisited in [84]. Its performance was compared with the optimization technique for the feedback control

$$m_2 = \frac{1}{b_1^2 + b_3^2}[b_3, \quad -b_1]\cdot(\mathbf{K}\Delta)$$

where Δ is the attitude error measure given by the linear combination of the attitude angles. The gain matrix $\mathbf{K}$ is derived from the LQR-type discrete cost function optimization.

Further optimization efforts are proposed with H_2 control in [85]. H_∞ control is proposed in [86] for the linearized equations of motion. As correctly noted in the paper, further simplification is quite popular.

Namely, the geomagnetic field vector is replaced with its average value. As a result, linear time invariant system of equations arises with well-established control theory. This approach is enhanced, the geomagnetic field is considered to be the sum of its constant averaged value and periodic "disturbing" part. The resulting control shows good performance in the disturbance rejection. Note that the reduction to the linear time invariant system may be performed using general techniques for the stability analysis, as shown in [87]. Disturbances are treated in H_∞ control construction as a decomposition by the frequencies in [88]. The disturbing part can be constructed in advance on the basis of the actual local geomagnetic field. Field depends on the orbit of the satellite and on the position on this orbit (given by time). In [89] the varying part of the equations of motion is represented as a parameter-dependent matrix to transform the linear time-varying control problem to the linear parameter varying problem. The resulting feedback law (to be implemented by the magnetorquers in the projection fashion) is

$$\mathbf{u} = -[\mathbf{B}_\times^T(\rho)\mathbf{X}^{-1}(\rho) + \mathbf{C}(\rho)]\mathbf{x}.$$

Here matrix $\mathbf{C}$ is used to assess the performance of the control, and it was selected as an identity one in [89], $\mathbf{X}$ is a function to be obtained from the linear matrix inequality (the control resembles the Lyapunov function approach in this respect, any appropriate function should be found, not strictly the one and only possible), and $\mathbf{B}_\times$ is a vector product matrix of the geomagnetic induction vector discussed above. Varying parameters that substitute the change with time here are the components of this matrix. Clearly, there are 6 parameters due to this matrix symmetry. However, as the orbit considered is near polar, one of the diagonal parameters is close to zero (induction vector along the orbital normal is zero in the direct dipole model for the polar orbit). Next, sum of two other elements is close to zero, so there remain only four effective parameters. Function $\mathbf{X}$ is presented in the basis of two most important parameters (diagonal elements in $\mathbf{B}_u$), $\mathbf{X} = \mathbf{X}_0 + \mathbf{X}_1\rho_1 + \mathbf{X}_2\rho_2$.

Sliding control is proposed in [90] for the polar orbiting satellite. Using the linearized equations of motion, pitch control is separated from the roll-yaw one. The latter is constructed through the sliding surface comprising of the roll and yaw angles and their derivatives,

$$s = k_1\varphi + k_2\psi + k_3\dot{\varphi} + k_4\dot{\psi}$$

Gains k_i are time varying due to the dependence on the geomagnetic field induction vector. The control is a practically important switching one,

$$m_2 = \begin{cases} m_0, & s(t) > \varepsilon \\ 0, & -\varepsilon \le s(t) \le \varepsilon \\ -m_0, & s(t) < -\varepsilon \end{cases}$$

Here ε is a thickness of the sliding surface boundary layer that is the deviation from the necessary transient trajectory when the control is activated.

Most of the considered algorithms are essentially the output feedback laws. In [91] the robust adaptive enhancement is provided with the control gains adjusted for different control steps to accommodate the uncertainty in the satellite parameters, especially inertia tensor.

Main conclusion on the bias momentum satellites resembles the one for the spin stabilized satellites. This is due to the fact that the high angular momentum is utilized in both control approaches. However, as the high level of agility remains for the rotation around the flywheel axis, optimization approaches are more suitable for the bias momentum satellites. As a result, optimization is already extensively applied in this field of research. This is an interesting and important area for further investigation, for example, in a fast response disaster monitoring with low cost constellations of small satellites. Attitude accuracy is also worth studying. The flywheel provides very strong stability properties that prevail over the stabilization errors induced by the magnetic control.

3.3. Gravitational stabilization

Passive gravitational stabilization is one of the most simple and reliable concepts. However, small satellites often have very limited control capability provided by the gravitational torque. This is due to the low mass and small dimensions, and, as a result, small difference in the moments of inertia. Gravitational boom is often used to ensure the one axis attitude with the satellite pointing along the radius vector. Rotation around this axis is less stable and should be controlled by the magnetic control system.

Paper [78] proposed the control (2.22) similar to the state feedback and inspired by (2.16). This control provides good results in damping the angular velocity of the satellite. Substituting the geomagnetic induction vector with three Euler angles (and their derivatives), the authors also acquire the possibility to control the rotation around the local vertical in the linear approximation. Control (2.22), although clear in its main idea, is not a precise one. Considering its simplicity, it is tempting to use raw magnetometer readings "as is" rather than to perform rigorous attitude determination techniques. This was outlined in [92]. Control (2.22) is used there for the attitude acquisition phase of a gravity-gradient satellite before the boom deployment. The only task for the magnetic system is to ensure approximate attitude for the boom deployment in a proper direction. The threshold is applied on the differences in (2.22) to prevent frequent switching near the necessary attitude due to the measurements noise and the attitude oscillation itself. Station keeping mode utilizes more accurate simple feedback law

$$\mathbf{M}_{ref} = -(k_q\mathbf{q} + k_\omega\boldsymbol{\omega}), \tag{2.24}$$

as well as a filtering of magnetometer and Earth sensor readings to provide the satellite attitude represented as a quaternion vector part $\mathbf{q}$. Paper [93] provides the exponential stability analysis of the control (2.24) for the gravitationally stabilized satellite. Stability is proven for sufficiently small velocities, and full state feedback is substituted with damping control otherwise. Numerical simulation is provided for the satellite with the gravity-gradient boom on the near polar orbit. Paper [94] proposes the damping scheme for the gravitationally stabilized satellite. The initial control is a state feedback. It is implemented by two magnetorquers out of three. These two coils are selected to provide the torque closest to the initial one for each control step. In [95] control (2.24) parameters are adjusted with the optimization technique and pole placement for the considered spacecraft configuration.

Paper [96] utilizes classic sliding control to enhance the gravity-gradient attitude of a satellite with deployable boom. In [97] fuzzy logic controller is proposed and compared with the LQR (steady state on a sample period) to yield better accuracy results.

In [98] the boom deployment process is discussed. The Bdot law in conjunction with the constant dipole is used as it is a common way to implement the process. It is shown that the satellite constant dipole vector aligns with the geomagnetic field, which, itself, is almost vertical in the polar regions. This fact is used to deploy a boom in the necessary direction. This fact, however, is a bit more complicated. The satellite with the constant dipole moment and some sort of damping acquires periodic motion in the vicinity of the geomagnetic field vector. Moreover, a resonance may occur depending on the magnet value and satellite inertia [99]. Another simple strategy, that is also discussed in [98], is to destabilize the satellite after the boom deployment in case it acquires inverted attitude, and to repeat the process until necessary position is acquired. Another concept was proposed in [100]. Here the satellite is spinning using control (2.10). The satellite acquires the gyro properties aligning its angular velocity vector with the orbital normal (damping is implemented simultaneously). Controlling the necessary spin rate and current attitude, the boom is deployed when the satellite is in favorable position (the boom axis is close to the local vertical).

3.4. Other approaches

Here other augmenting actuators and concepts are very briefly covered.

3.4.1. Thrusters

The easiest approach is to complement magnetic control with thrusters. However, even this scheme has its own peculiarities. Simple additional actuation along the geomagnetic field may be challenging due to the hardware restrictions on the thruster activation threshold and its discrete values, especially if it has on/off architecture. In this case the control allocation problem may require an optimization approach with constraints [101]. A passivity-based hybrid control was considered in [102] and an LQR-based optimal control allocation in [103].

3.4.2. One axis control

One axis control is close to the spin stabilization scheme. However, large spin rate is not required and the satellite is not a gyro. In [104] simple one-axis control is proposed,

$$\mathbf{m} = \mathbf{K}(\mathbf{e}_3 \times \mathbf{b}).$$

This control is implemented with x and y axes magnetorquers, so $\mathbf{K} = diag(k, k, 0)$. It is also augmented with the Bdot law provided by the same two magnetorquers. In [105] the control is

$$\mathbf{M}_{ref} = k_1\delta\mathbf{e}_3 + k_2\mathbf{e}_1 \times \mathbf{s}_{\max}.$$

Here δ is the normalized difference in the current output of two opposite solar panels, and the first term serves the purpose to reduce that difference. The $\mathbf{s}_{\max}$ vector is the normal to the solar panel/sensor that gives the highest current output, so the second term should maximize the output from the panels by facing the $\mathbf{e}_1$ vector to the Sun (this attitude is preferable due to the satellite geometry). In [106,107] control

$$\mathbf{m} = k\cos\alpha(\boldsymbol{\omega} \times \mathbf{s})$$

is constructed and analyzed. The control utilizes only Sun sensor readings $\mathbf{s}$ and the angle α between the geomagnetic induction vector and the Sun direction, with the latter computed using the onboard models. This control drives the satellite angular momentum vector to orient along the Sun direction, and the maximum moment of inertia along the angular momentum. As a result the satellite is spinning around the Sun direction with direct formula for the speed estimate given.

3.4.3. Reaction wheels

Although momentum wheel with constant rotation rate is a classical addition to the magnetic control, reaction wheel can be used also. The control torque may be written as [108].

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & -b_2 \\ -b_3 & 1 & -b_1 \\ b_2 & 0 & 0 \end{pmatrix}\begin{pmatrix} m_1 \\ \dot{h}_{wheel} \\ m_3 \end{pmatrix}.$$

Here the "traces" of the skew-symmetric matrix of the cross product are clear. Two magnetorquers produce two control torque components, while the third one is commanded with the reaction wheel (despite some control authority from magnetorquers as well). The control commanded to magnetorquers is (2.24).

Clear control torque redistribution scheme is provided in [109] with different number of reaction wheels. In [110] magnetorquers and one wheel are used to provide control (2.24). Even two wheels require and addition to the control system. An example is presented in [111] with the real experience of saving a mission after the reaction wheels failure. In [112] the conditions for the controllability of the bias momentum satellite with wheel speed control and for the dual spin satellite with a bias are provided in case of the magnetorquers saturation limit.

In [113] the generalization of the errors in the attitude is performed in the form (2.12). Second error drives the satellite angular velocity to zero and the wheel speed to the necessary value, as opposed to the spin stabilization scheme with the satellite itself spinning. This control is augmented with the wheel speed correction control. Suppression of vibrations in solar panels and other elongated flexible structures by the combination of the magnetic control system and a reaction wheel is covered in [114].

3.4.4. Aerodynamic drag

Aerodynamic drag is a common solution to enhance the controllability of magnetically actuated small satellites. As the latter are often placed on low Earth orbits, aerodynamic torque may have great impact on the dynamics of the satellite. However, this requires proper mass distribution and even additional structure parts. This is a challenging solution for small satellites, especially CubeSats. In [115] the control is a modification to (2.22) given by

$$\mathbf{m} = k_1[\mathbf{b}_{ref} + k_2(\dot{\mathbf{b}}_{ref} - \dot{\mathbf{b}})]$$

Here the necessary direction of the geomagnetic induction vector in the bound reference frame is used, as well as a damping term. Clearly this control can provide only one-axis stabilization with rotation around the induction vector. However, the aerodynamic torque provides strong stabilization along two spacecraft axes. So, the magnetic control should stabilize only one axis. Moreover, as the drag torque is strong for a specially designed satellite, the disturbing components of the magnetic torque are easily cancelled.

In [116] an example is provided, with magnetic control utilizing the LQR. However, for the passively controlled satellite the use of passive dampers, especially hysteresis rods, or at least simple damping algorithm, is preferable.

These briefly covered techniques have strong drawbacks. Thrusters and reaction wheels spoil the simplicity of the magnetic attitude control system; one axis attitude is very close to a more reliable (due to the flight heritage and strong stability properties) spin stabilization scheme, aerodynamic drag requires geometrical and dynamical properties that are hard to achieve for small satellites. Among other notable although exotic concepts are the charged satellite shell providing the Lorentz force [117–120]; solar radiation pressure for the near equatorial orbit [121]; the use of the fluid ring [122]. Other recent papers on the subject of augmenting the magnetic control with other actuators and concepts are [123–155].

4. Solely magnetic three axis attitude control

Satellite angular motion has an "average" controllability with the magnetic system. This opens wide field for the research on this topic. The problem, although proposed recently, is extensively studied. It draws attention of both the attitude control specialists inclined to the engineering aspects and the control theory ones.

4.1. Simple local control

4.1.1. Feedback control

The most straightforward idea is to use the classic feedback law (2.24). This was performed in papers [156,157] with the control implemented by the magnetorquers in a projection manner. Namely, the dipole moment is

$$\mathbf{m} = -k_\omega \mathbf{B} \times \boldsymbol{\omega} - k_q \mathbf{B} \times \mathbf{q} = -k_\omega \mathbf{B} \times \boldsymbol{\omega} - k'_a \mathbf{B} \times \mathbf{S} \tag{3.26}$$

where $\mathbf{S} = (a_{23} - a_{32},\ \ a_{31} - a_{13},\ \ a_{12} - a_{21})$ provides equivalent control formulation via the direction cosines matrix elements a_{ij}. Rigorous proof is provided that the resulting control ensures the asymptotic stability, even for the satellite with flexible appendages [158] (if they have natural damping). However, this control stabilizes the satellite only with specific relations between the control gains (as opposed to the fully controlled case that provides stability for any positive gains). The most important result here is that the positional control part is smaller than the damping one. The overall value of the dipole moment should also be restricted as shown in [159] with the bifurcation analysis and in [160] with the numerical continuation of solutions on large control values. In [161] an example of the loss of controllability with the gains raising is presented, and the magnetic control is augmented with small torque from other sources to tackle this problem. The restriction on the control gains arises due to the uncontrollable direction. As the magnetic system implements only some part of the "ideal" control, it drives the satellite not to the necessary position but somewhere close instead on each control implementation step. Large control torque results in the satellite ending in a larger error after each step. The control should slowly and iteratively move the satellite closer and closer to the necessary attitude. However, as the damping part in (3.26) is simply (1.2), it provides the asymptotical angular velocity damping. As a result, the restriction on the damping part is softer than on the positional part.

Control gains selection process is crucial for the success of the control (3.26) implementation. Simple scheme, based on the analytical results and Floquet theory, is presented in [162] for the inertially pointing satellite and in [163] for stabilization in the orbital reference frame. Note that the analytical result is based on the decomposition of the motion of the satellite on the fast (angular velocity higher than the orbital one) and slow (commensurable with the orbital rate) ones. In [164] it is shown that the low angular velocity is crucial for the controllability. This restriction is lifted using averaging technique with equations of motion enforced with artificial small parameter. Moreover, in [165] the feedback control is enhanced with the errors in the inertia matrix estimation to handle even fast rotating satellite. The errors in the inertia matrix can render the system uncontrollable. Namely, due to the initial underactuation, the gains should be selected very carefully for a given satellite parameters, especially inertia moments. Even small deviations may render the satellite uncontrollable [166], while other sources of errors (attitude determination and disturbances) have mainly quantitative effect. However, if the restrictions on the inertia moments are known, the gains should become available [167]. Paper [168] provides the procedure to select the control gains based on the minimization of the quaternion vector part settling time. Magnetorquers saturation is taken into account as this procedure inevitably provides large gains. Moreover, as the settling time depends on the initial conditions also, the min-max problem is constructed. The idea is to find the worst-case scenario in terms of initial conditions and prepare control gains for that situation. In [169] this approach was extended to control (3.27). In [27] control (3.26) is enhanced the with positive-defined matrices instead of scalar gains. Asymptotic stability is provided for the polar orbit in the averaged geomagnetic field model and some specific stability results are provided under different assumptions.

In [77] algorithm (3.26) was tested onboard the Gurwin satellite and failed to provide three axis control. It was also tested onboard Oersted satellite with better success. However, the latter satellite was equipped with a gravitational boom [170]. PRISMA mission showed very close success. Its TANGO satellite successfully obtained all attitude profiles by the feedback law with gains based on the LQR optimization. Although the scheme was very marginally spin stabilized along the maximum moment of inertia, slow angular velocity brings it very close to the three axis magnetic control in the orbital reference frame (with Zenith pointing as one of the requirements). The accuracy was up to 10–20° [171,172].

The angular velocity-free control is proposed in [156] in the form

$$\begin{aligned}\dot{\boldsymbol{\delta}} &= \alpha([q_0,\ \ \mathbf{q}] - \varepsilon\lambda\boldsymbol{\delta}),\\ \mathbf{u} &= -\varepsilon^2(k_1\mathbf{q} + k_2\alpha\lambda\mathbf{W}(q_0,\mathbf{q})([q_0,\ \ \mathbf{q}] - \varepsilon\lambda\boldsymbol{\delta})).\end{aligned} \tag{3.27}$$

Here the angular velocity is substituted with the new variable $\boldsymbol{\delta}$, two additional parameters α and λ are used, and $\mathbf{W}$ is the kinematic equations in quaternions matrix. This control, however, is designed mainly for the satellites close to the spherically symmetrical ones.

In [173] applicability of the feedback law to the real engineering attitude control problem is considered. The control is always a piecewise-constant one, changing after each sensor reading processing. However, instead of simple discretization a sampled-data discrete system is used along with the guideline for the control parameters selection. The discretization is performed on the defined sampling interval with the restriction on the attitude after a number of steps. Control construction and analysis is simplified with the averaging of the right side of the discrete equations of motion. The proposed technique requires very short measurements and computational interval. If the measurements interval is commensurable with the control interval, the corresponding discretization is handled in [174]. The delay in the control command implementation is covered in [175].

Most of the results on the control gains selection are valid for the satellite with commensurable moments of inertia. GOCE satellite is a prominent example of a magnetically actuated spacecraft [176,177] with specific dynamical configuration. The satellite was very elongated with the equatorial moment of inertia about 30 times greater than the axial one. The satellite required very low microacceleration environment. Magnetic control was the main system for that satellite. However, being placed on the very low orbit, it was at a large part aerodynamically stabilized with fine adjustment and drag cancellation by the ion engines. The control implemented in GOCE is essentially (2.24) with disturbance torque compensation,

$$\mathbf{M}_{ref} = -(k_q\mathbf{q} + k_\omega\boldsymbol{\omega}) - \mathbf{M}_{dist} - \boldsymbol{\omega}\times\mathbf{J}\boldsymbol{\omega} \tag{3.28}$$

which is the implemented through the classical projection approach. Note that control (3.28) is more accurate than (2.24). This control is constructed with the Lyapunov function and should take into account disturbances, as shown in (3.28). They are often discarded as this simplifies control law which itself is very robust and capable of disturbance rejection. GOCE satellite was used in [178] as an example of low performance of the projection-based magnetic dipole construction for the satellite with uneven inertia distribution. The projection approach may be formulated as a minimization of the Euclidean norm of the difference between the requited and the implemented torques,

$$J = \|\mathbf{M}_{ref} - \mathbf{M}\|^2 \to \min \tag{3.29}$$

This approach provides degraded accuracy for the considered satellite with specific inertia properties on near-polar orbit which is shown with numerical simulation. To compensate for the specified direction in the bound reference frame (principal axis with very low inertia moment), weighted cost function is proposed,

$$J = (\mathbf{M}_{ref} - \mathbf{M})^T\mathbf{Q}(\mathbf{M}_{ref} - \mathbf{M}) \to \min$$

and the minimization is run with the constraint on the torque direction $\mathbf{M}\cdot\mathbf{b} = 0$. Using the Lagrange multiplier, the control torque becomes

$$\mathbf{M} = \mathbf{M}_{ref} - \mathbf{Q}^{-1}\mathbf{b}(\mathbf{b}^T\mathbf{Q}^{-1}\mathbf{b})^{-1}\mathbf{b}^T\mathbf{M}^T.$$

Weighing matrix $\mathbf{Q}$ is diagonal. The accuracy of stabilization of the angle of rotation around the very low inertia axis significantly increases by the proper weight selection for that axis. This also results in a slightly degraded accuracy of rotations around two other axes. This strategy is implemented to the control (3.28) also. Euclidean norm (3.29) may be subjected to other constraints. For example, in [179] a constraint $\mathbf{M}\cdot\mathbf{e}_3 = 0$ is introduced for the spinning satellite to cancel unwanted torque component along the spinning axis.

Papers [180,181] proposes the singularity robust inverse matrix for the control dipole calculation,

$$\mathbf{m} = (\mathbf{B}_\times^T\mathbf{B}_\times + k\mathbf{E})^{-1}\mathbf{B}_\times^T\mathbf{M}_{des}$$

Control (2.24) may be generalized for any error function and its derivative instead of the quaternion and the angular velocity. The most common way is to use the Euler angles directly in the linear approximation. For example, flowing control is proposed in [182].

$$\mathbf{M}_{ref} = -(k_q\mathbf{e} + k_\omega\dot{\mathbf{e}}),$$
$$\mathbf{e} = \mathbf{e}_{mag} - \mathbf{b}$$

where $\mathbf{e}_{mag}$ is a given direction in the body frame that should be aligned with the geomagnetic field vector. This control is, therefore, quite close to the magnetic stabilization using the constant magnet (the direction $\mathbf{e}_{mag}$ is fixed in the body frame) with a damping source.

Assumption of the spherically symmetrical satellite greatly facilitates control problem. This eliminates the coupling of the angular velocity components in the $\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega}$ term in the equations of motion. The dynamical equation of motion acquire the form $\dot{\mathbf{x}} = \mathbf{u}$, where $\mathbf{u}$ is, however, restricted in the direction. This simplification was used in [183,184]. The control is a modified feedback law without angular velocity information. However, full attitude information is still required, and if proper attitude determination sensors and algorithms are present on the satellite, angular velocity estimation does not pose a problem. In [185] this approach was applied to the non-spherical satellite by augmenting the averaged linearized equations of motion with the simple quaternion observer. In [186,187] an intuitive control is constructed for the spherically symmetrical satellite. The control provides either magnetic or gravitational stabilization in the simplified dipole field model. In [188] the control is constructed using two rotations in the magnetic induction-fixed reference frame.

4.1.2. Sliding control

Paper [189] briefly proposed the sliding mode control for detumbling of an underactuated spacecraft (no control authority along one of the principal axes). In [16] sliding mode control was proposed for the magnetically actuated satellite. Necessary attitude is asymptotically stable for the spherically symmetrical satellite. The sliding surface utilizes only the angular velocity information which is feasible due to the symmetry of the satellite.

In [190] classic sliding manifold is constructed,

$$\mathbf{s} = \mathbf{J}\boldsymbol{\omega} + \mathbf{K}\mathbf{q}. \tag{3.30}$$

The control vector is parallel to the sliding variable vector. This control is implemented by magnetorquer in a projection fashion. Numerical simulation provides good control performance. Gain $\mathbf{K}$is a positive defined matrix. However, in most papers it is basically a number as $\mathbf{K} = k\mathbf{E}$.

Slightly modified sliding plane is proposed in [191] without the "weighing" inertia tensor matrix for the angular velocity components. Further modification is provided in [192,193]. Integral sliding mode controller is designed. It provides two additional terms in (3.30),

$$\mathbf{s} = \boldsymbol{\omega} + \mathbf{K}_1\mathbf{q} + \mathbf{K}_2\int_{t_0}^{t}\mathbf{q}d\tau + \mathbf{J}^{-1}\int_{t_0}^{t}[\frac{\mathbf{b}\mathbf{b}^T}{b^2}\mathbf{u}]d\tau$$

The first additional term depends on the average attitude error, and the second one depends on the proximity of the required control torque vector to the plane perpendicular to the geomagnetic induction vector. This approach comes close to the idea of constructing the trajectory accessible by the magnetic control. This is the transition process trajectory. There are no means in direct disturbance rejection in the nominal attitude: if the disturbance is acting along the induction vector, it cannot be rejected. However, integral attitude error term should indirectly leverage this problem. In [193] extensive simulation is performed with different disturbance sources, and the control proposed successfully solves the problem. In [194] the goal is to construct the sliding manifold that provides directly controllable by the magnetic system trajectory. This is performed with the time-varying gain matrix $\mathbf{K}$, and an iterative sliding manifold construction approach is proposed. However, no recipe for the disturbance rejection is proposed.

Higher order sliding mode control is proposed in [195]. The sliding manifold is nonlinear and, accordingly, the control is constructed on the basis of the higher order derivatives of the manifold. Namely, the manifold is

$$\mathbf{s} = \mathbf{J}\boldsymbol{\omega} + \mathbf{K}_1\,|q_0\mathbf{q}|^\alpha\,\mathrm{sign}(q_0\mathbf{q}) + \mathbf{K}_1(q_0\mathbf{q})$$

with diagonal gain matrices and $\alpha \in (0,1)$. The control, however, is constructed on the typical projection basis. Nevertheless, the proposed control provides good results in presence of the inertia tensor uncertainty and constant and periodic disturbances. Similar sliding surface is proposed in [196] for the averaged equations of motion. The averaging is performed by introducing artificial small parameter.

In [197] sliding control is used in conjunction with the model reference adaptive control. The latter is constructed for each satellite axis (channel) separately. In [198] the dichotomous coordinate descent is used to implement the linear least squares method for the sliding control construction. The control authority is constructed, and appropriate dipole magnetic moment is found with the least squares method. This approach provides better accuracy than typical feedback control projection.

4.2. Optimization methods

Optimization is a natural tool for achieving best results in terms of some cost function and for incorporating different restrictions. The magnetic control construction is centered around a restriction, so optimization techniques were extensively applied recently.

One of the most used and reliable optimization methods is a linear quadratic regulation. The satellite motion is very often represented by the linear equations of motion in the vicinity of necessary attitude. Moreover, as the geomagnetic induction vector has almost periodic nature, this gives rise to the linear periodic systems with well-established control theory. Periodic LQR control is proposed in [199] which is able to handle the magnetorquers saturation problem. The linearized equations of motion are

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(t)\mathbf{m} + \mathbf{B}_d\mathbf{w}$$

with the time-varying and periodic (in the dipole geomagnetic filed model) matrix $\mathbf{B}$ and constant disturbance matrix $\mathbf{B}_d$. The cost function is

$$J = \frac{1}{2}\int_0^T(\mathbf{x}\mathbf{Q}\mathbf{x} + \mathbf{m}\mathbf{R}\mathbf{m})dt + \frac{1}{2}\mathbf{x}\mathbf{P}\mathbf{x}$$

with weighing matrices $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{P}$ representing the penalty for the error in the attitude over the motion in one period of the geomagnetic field vector change; for the control value and for the final attitude error. Although linear, the proposed technique stabilizes the satellite from about 30° error in attitude angles. In [200] this approach is modified to effectively cancel external disturbances under the assumption of their periodicity. In [201] the time-varying control gains are substituted with the position varying ones, utilizing the easily predictable geomagnetic field behavior and easing the burden on the onboard computer. Paper [202] proposes an algorithm for the periodic Riccati equation solving with the application to the magnetic control. LQR was extended on the nonlinear dynamical system in a classic fashion of a state-dependent Riccati equation in [203], with the latter equation solved using the approximated sequence Riccati equations, and in [204] with the extension on the attitude determination problem and actuator saturation, but with a bias momentum. The saturation effect is extensively studied in [205,206].

Performance of the finite horizon, infinite horizon and LQR controllers is compared with a numerical simulation in [207,208]. The control is constructed on the basis of the averaged over a number of orbits IGRF field. This greatly facilitates the control construction and analysis. Applicability is restricted, however, by the polar orbit assumption.

Pseudospectral method with receding horizon is used in [209] to simplify the minimization of the cost function. The latter are a classic LQR function and a feedforward cost based on the disturbances model. The proposed method reduces the necessary computation time by the order of magnitude. The satellite is considered on the elliptical orbit. This gives rise to the forcing pitch torque in the equations of motion, which is explicitly included in the control and effectively rejected. This torque may be replaced with other disturbing torques, enlarging the paper applicability.

Paper [210] proposes classical in orbital mechanics cost function that is the convergence time. The cost is also penalized with the terminal attitude and velocity,

$$J=\int_0^T dt+k_1\mathbf{q}^2+k_2\boldsymbol{\omega}^2$$

with the maneuver time T to be determined and control value constraints added. Paper utilizes well defined algorithms and software to solve the fixed-time optimization problems. However, as the time itself is to be defined, additional states are added to represent the scaling of the dynamics with respect to time, while the dynamical equations itself are given for the fixed final time. Finally, the proposed angular path is tracked by the model predictive control. This allows almost the same convergence time while effectively cancelling errors due to the bad knowledge of the attitude and disturbances, which cannot be performed with simple optimization law that does not implement the mentioned errors in the dynamical model.

In [211] the retrospective cost adaptive control is used. However, it is used to construct the required torque, not the dipole moment, which is then given by the projection approach. In [212] the Riccati equations is integrated forward in time. In [213] the linear quadratic cost function is minimized using the Markovian jump-linear system to find the optimum control sequence for a discrete dynamical model with random failures in the magnetometer readings. In [214] the time-optimal control is constructed with the pseudospectral method.

The errors in two target directions are constructed along with the simple angular velocity error in [215]. Two direction error values are used to construct two independent control values necessary to diminish them, and angular velocity error is computed according to [51], that is similar to the Bdot law. The dipole moment is obtained in a common way as a projection on the plane perpendicular to the geomagnetic induction vector. The most important contribution is the control construction approach. It is based on the integral error in the attitude and angular velocity over a number of orbits. This exploits the Earth geomagnetic field rotation factor that provides “average” controllability. The optimization utilizes the genetic algorithm approach.

H_2 control was proposed in [216]. In [217] H_∞ control is used to adjust control gains in the feedback law (3.26) for the linearized equations of motion. The performance measure is given by the variable

$$\mathbf{z}=(\mathbf{E},\ \ 0_{3\times3},\ \ \sigma\mathbf{K})\mathbf{x}$$

Here $\mathbf{x}$ is composed of the angular velocity components and three attitude error measures (vector part of the quaternion). As the feedback control is $\mathbf{u}=\mathbf{K}\mathbf{x}$, the variable σ acts like a penalty value for the control value. Note that this measure can be used to construct the linear quadratic cost function as $J=\mathbf{z}^T\mathbf{z}$. However, to tune control gains in order to reject disturbances acting on the satellite, the H_∞ norm is used for the cost function construction,

$$J(\mathbf{K})=\sup_{\mathbf{T}_{dist}\in L_2}\frac{\|\mathbf{z}\|_2}{\|\mathbf{T}_{dist}\|_2}$$

Here the response to the disturbance torque $\mathbf{T}_{dist}$ is analyzed. The minimization process also utilizes the restriction on the characteristic multipliers of the resulting linear time-varying system (clearly the asymptotic stability of the unperturbed system comes first, and disturbance rejection second). H_∞ is also considered in [218]. The control is constructed on the basis of the discrete linear time varying system directly utilizing the disturbances vector $\mathbf{w}$ and the sensor signals vector $\mathbf{y}$,

$$\begin{aligned}\mathbf{x}_{k+1}&=\mathbf{A}_k\mathbf{x}_k+\mathbf{B}_{1k}\mathbf{w}_k+\mathbf{B}_{2k}\mathbf{u}_k,\\ \mathbf{y}_{k+1}&=\mathbf{C}_{1k}\mathbf{x}_k+\mathbf{D}_{11k}\mathbf{w}_k,\\ \mathbf{z}_{k+1}&=\mathbf{C}_{2k}\mathbf{x}_k+\mathbf{D}_{21k}\mathbf{w}_k+\mathbf{D}_{21k}\mathbf{u}_k.\end{aligned}$$

Here $\mathbf{z}$ represents the error in the attitude with respect to the state vector and values of disturbances and signals from sensors. The control is constructed as a feedback law depending on the state and sensor readings. The H_∞ control performance is simulated in specific cases, namely, for the gravitationally stable and unstable, almost disc-shaped satellites. In [219] the simulation is provided for the satellite with more even inertia distribution, taking into account inertia uncertainty and different disturbance sources.

4.3. Model predictive control

An important method for the magnetic control is a model predictive control (MPC), proposed in [14]. As with the sliding control, it directly utilizes the rotation of the geomagnetic field vector to provide any necessary control direction. The dynamical model is used to predict the motion of the satellite. This information is used to construct an error function that is compared to the one for the reference trajectory. This cost function is minimized to provide the best convergence to the necessary attitude. For example, paper [220] utilizes the linearized equations of motion (this is the method requirement). They are further transformed into in the discrete form to represent control steps,

$$\mathbf{x}(k+1)=\mathbf{A}\mathbf{x}(k)+\mathbf{B}\mathbf{u}(k).$$

Iterating this equation, $k+N$ step yields

$$\mathbf{x}(k+N)=\mathbf{A}^N\mathbf{x}(k)+\mathbf{A}^{N-1}\cdot\mathbf{A}^{N-1}\mathbf{B}\cdot\mathbf{A}^{N-2}\mathbf{B}^2...[\mathbf{u}(k),\,\mathbf{u}(k+1),\,\mathbf{u}(k+2),...,\mathbf{u}(k+N-1)].$$

Here the control sequence defines the resulting satellite motion. The sequence is determined minimizing the classical cost function

$$J=\sum_{j=1}^{N}\mathbf{x}(k+j)\mathbf{Q}\mathbf{x}(k+j)+\mathbf{u}(k+j-1)\mathbf{R}\mathbf{u}(k+j-1)\tag{3.31}$$

with weights $\mathbf{Q}$ and $\mathbf{R}$ assigned to the errors in the attitude and to the control value. The control, as well as the cost function, is similar to the LQR approach. An important feature is the prediction horizon: the attitude is compared on the predefined time interval, not for the whole motion or the period of the coefficients in the linear periodic equations. The control is reconstructed at each control iteration. Control sequence construction also provides some flexibility. For example, the control may be constant over a number of steps within the prediction horizon to represent the real behavior of the control systems. The same goes to the geomagnetic field models. Most comprehensive models may be used, or the field may be considered constant on short time intervals with periodic updates to the induction vector. As an example, papers [14,88] proposed an interesting approach by fitting the IGRF data with the least squares method to obtain the periodic geomagnetic field with constant part and two periodic parts with orbital and double orbital frequencies. Disturbances may also be modelled with approximate expressions that are easier for the onboard computation. In [221] the disturbances were approximated as constant ones and as periodic ones with the orbital frequency. It is shown that the constant approximation provides acceptable accuracy while the periodic disturbance improves it only a little. This holds if the prediction horizon is significantly less than the orbital period. The onboard computation limitations requires short horizon anyway. In the following work [222] the control construction was further simplified. Pitch angle was treated separately in the linear model with the feedback control, while the roll-yaw dynamics was stabilized with MPC. In [223] computational simplification is obtained by approximation of the control effort by the Laguerre polynomials.

Further enhancement is provided by the explicit model predictive control [224]. Cost function (3.31) minimization is replaced by the quadratic programming problem with one parameters vector (both states and controls are joined into one variable). The simulation provides less accuracy and time response than (3.26) but also less power consumption due to the control value present in the minimization problem. General MPC usage in satellite control is covered by [225].

Nonlinear MPC utilizes the continuous cost function for the optimal angular velocity damping in [226]. It may be enhanced with the terminal cost (which is also subjected to constraint) to provide the cost

$$J(t)=\int_t^{t+T} f(\mathbf{x}(s),\mathbf{u}(s))ds+E_{kin}[\mathbf{x}(t+T)]$$

on the control horizon T. Classic quadratic cost function was used for f. The terminal restriction is the rotational kinetic energy of the satellite. It is shown that the constraints on the terminal angular velocity result in the considerable increase in the number of iterations needed to obtain the solution, which becomes infeasible for the onboard calculation. To ease the burden on the onboard computer, the cost reduction is utilized to stop iterations when a "good enough" performance is achieved. Namely, the restriction

$$J(\mathbf{x}(t),\mathbf{u}(t),t)\le J(\mathbf{x}(t-\delta t),\mathbf{u}(t-\delta t),t-\delta t)-\gamma\mathbf{x}^2(t)$$

is added. Here the cost function rate of change (on the time span δt) is considered together with the initial state vector norm at the time step t. Choosing the parameter γ one can adjust the necessary decrease in the cost after which the iterations are terminated and the solution is designated to be sub-optimal in a certain sense. The proposed scheme provided better performance for angular rates 5 deg/s larger than in the unconstrained nonlinear MPC. In [227] the cost function directly utilizes the property of the control (1.2) to provide the asymptotic stability for the angular velocity damping problem. The cost function is

$$J=\int_t^{t+T}[k\mathbf{g}^2+\mathbf{m}^T\mathbf{R}\mathbf{m}]ds+E_{kin}[\mathbf{x}(t+T)]$$

where $\mathbf{g}=2m_0/\pi\cdot\arctan[\mathbf{K}(\mathbf{b}\times\boldsymbol{\omega})]$. Typically quadratic penalty on the angular velocity along the path is used in the cost function. Here it is substituted with the divergence from the path provided by the control (1.2). So the control (1.2) and related satellite motion may be considered as some sort of the first approximation. Restrictions on the gains are discussed. Simulations are provided for small gains. It corresponds to the situation when the terminal kinetic energy dominates the cost function.

In [228] the MPC control is implemented for the satellite on the elliptical orbit with the relevant state-space model.

4.4. Adaptive methods

Adaptive methods may incorporate restriction in the control direction. However, due to the computational difficulties, they are rarely considered for the control of small satellites. In [229] adaptive control based on the dynamic neural units is presented. The control partly utilizes the state feedback approach (2.24) and shows better performance in a simulation (except for the large initial attitude errors that require long learning). Most important, the proposed technique effectively cancels uncertainties in the disturbances and dynamical model. Similarly, in [230] control (2.24) is used to train the network. Neural network with the time varying weights was proposed in [231]. First, approximate optimal control is formulated through the quadratic with respect to the attitude error cost function with time elapsed penalty. This cost function (without the elapsed time) is used in the neural network. The latter is trained on the random initial conditions first. Then, on each iteration of the training process, random disturbances (inertia tensor error, disturbing toques, errors in the environmental models) are added to increase the network robustness.

Fuzzy inference system is utilized in [232] to provide control gains for (3.26) that best fit for the current attitude and the geomagnetic field. Moreover, the best fuzzy controller is determined with the genetic algorithm (utilizing the crossover and mutation) that minimizes the cost function

$$J=\int_0^T(|\boldsymbol{\omega}|+5\cdot10^{-5}|\Delta\alpha|)dt+5\cdot10^{-8}t_{settle}.$$

This cost utilizes the average angular velocity and the angle-based error during the simulation period T, and t_{settle} is the last time the angle-based error was greater than 10°. The proposed scheme provided slightly better performance than the LQR in terms of the settling time. However, the attitude and angular velocity error was greater on average in the Monte Carlo simulation. In [233] the dipole moment is represented as six different variables from the vector product. Namely, the torque components are$M_i=m'_j b_k-m'_k b_j,\quad i,j,k=1,2,3$ with each m'_j appearing twice. Although there are three components, m_j may be considered independent for each torque component. So the torque along each principal axis is calculated and corresponding m'_j found. The actual dipole to be implemented is $m_j=(m'_j+m''_j)/2$.

Three axis magnetic control is the key research topic in authors opinion. Two main areas can be outlined here. First, feedback control is fully susceptible to the direction restriction problem. However, it is extremely simple both for the onboard implementation and for the analysis. This justifies large amount of interest in this field and further research on the control gain tuning and extensive simulation with possibly accurate environment and hardware models. Second group of approaches should be centered on overcoming the control direction restriction. Sliding control is an important tool. It maintains control simplicity for the onboard calculation and provides the flexibility in control construction by different sliding surfaces formulations. Model predictive control and other optimization methods require further attention. Most of the relevant research is performed from the control and optimization theory perspective. The advancements in this field should be reevaluated form the engineering point of view, making strong and successful optimization results applicable to the onboard computers. Extensive flight verification may be expected in this field, starting with the feedback law (3.26) and sliding control.

Other recent papers on the solely three axis magnetic control are [234−243].

5. Conclusion

The survey covers main control algorithms and their analysis results for the active magnetic attitude control system. Implementation with other actuators and mechanical concepts, as well as the angular velocity damping, is performed mainly with simple control concepts. These approaches are used since the beginning of space era and show excellent in-flight performance. As a result, modern control techniques penetrate into this field of research very slowly. Solely magnetic three-axis control, on the contrary, is the testing ground for some new (for the satellite attitude) control concepts, such as optimization and adaptive methods. Restriction on the control direction, that does not prevent the controllability due the change in this direction, attracts researchers to this challenging and practical problem.

The problem is far from the final solution. Solely three-axis magnetic control requires extensive in-flight validation and further research. Sliding control, model predictive control and global optimization techniques offer great tools to accommodate the control restriction in the control construction process. Simple feedback concepts, although extensively studied in a number of sound papers, still require further tuning for the onboard implementation. The well-established field of the magnetic control augmentation with other actuators should not be omitted in further research also. There is always room for the improvement. Moreover, even old and tested concepts need modern attention to maintain necessary expertise in the community.

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