

A Model Predictive Control Based Magnetorquer-only Attitude Control Approach for a Small Satellite

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Abstract: Magnetic attitude control is an essential topic in attitude determination and control (ADC) studies as it can be used as the main or backup attitude control method in different scenarios. The magnetic actuation system is advantageous for its low cost, reliability, and ease of implementation when the pointing accuracy requirement is not high. The major issue in using magnetorques for attitude control is that it provides controllability over a period. In this study, a magnetic attitude control algorithm is designed for a low Earth orbit sun-synchronous small satellite by using a model predictive control approach which is an advanced control technique that generates control input using the system model to predict the future behavior of the system. The control algorithm is integrated into the overall ADC algorithm and the complete algorithm is tested via simulations. The simulation results show that the three-axis attitude control can be achieved with an accuracy of below 10 deg in around 10 orbits from the tumbling status of the satellite.

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1. INTRODUCTION

The space industry has transformed in recent years. The biggest driver of this transformation is the easier access to space with cost-effective small satellites as they perform similar functions with lower costs compared to larger satellites with the advancement of technology Lal et al. (2017). In addition to the widespread use of small satellites in low Earth orbit (LEO), it has gained importance to be able to carry out operations with low-cost instruments onboard the small satellites. The selection of low-cost equipment is also an important issue in spacecraft attitude determination and control system (ADCS) design. In this case, the algorithms must be well designed to best satisfy the requirements within the capacity of the low-cost ADCS equipment.

The satellite attitude control can be performed by using attitude thrusters, magnetorquers, or devices that make use of the law of conservation of angular momentum such as reaction wheels. Among them, the most affordable device is magnetorquers when the attitude pointing accuracy requirement is low. Besides, when using other actuators as the main attitude actuator, the magnetorquers can also be used as the backup actuators in malfunction cases or low energy mode alongside their other duties such as momentum dumping, satellite detumbling, etc.

In lower Earth orbits, one of the main tasks of the magnetorquers, even if the satellite is to be stabilized using other actuators, is to

detumble the satellite after its free rotation in orbit. Bdot control algorithm which was proposed by Stickler and Alfried (1976) is the most preferred detumbling method and is straightforward to implement by using magnetorquers and magnetometers.

With the advancement of microelectronics technologies, in addition to their other uses, the use of magnetorquers for attitude control has increased for missions that do not need high pointing accuracy. Especially for small satellites, the magnetic control system can be preferable for its low-cost, lightweight, reliable, and energy-efficient nature. As magnetic actuators depend on the magnetic field strength, they are mostly used in lower Earth orbits. The biggest disadvantage of magnetic control is that use of the magnetorquers does not provide full controllability at any instant since the generated torque is orthogonal to the geomagnetic field vector. Thus, even if three-axis magnetorquers are used, the attitude control is only accomplished in two axes instantly Ovchinnikov and Roldugin (2019); Yang (2019). However, coarse three-axis attitude control is achieved in highly inclined orbits over a period as the magnetic field is changing along the orbit. Thus, the satellite attitude becomes controllable over a period by magnetic actuation Lovera et al. (2002); Bhat and Dham (2003).

The earliest applications of magnetic control were not for solely active magnetic attitude control purposes, the studies were restricted to the passively controlled satellites or combination of the other actuators/components Shigehara (1972); Goel and Rajaram (1979); Martel et al. (1988); Steyn (1994); Lagrasta and Bordin (1996). In Wiśniewski and Blanke (1999), Wisniewski

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designed a three-axis attitude controller by magnetorquers to be used in Orsted satellite with the help of gravity boom and investigated the stability issues. Solely magnetic control was addressed in Wang et al. (1998) where the authors proposed nonlinear control techniques for three-axis stabilization in LEO. Psiaki (2001) proposed a periodic linear quadratic controller for nadir-pointing satellites. In Lovera and Astolfi (2005), a global solution to the magnetic control was proposed using a full-state feedback controller and projection technique. In more recent literature, many papers dealing with sole magnetic actuation have been presented, as low-cost small satellite missions have gained more attention. In Ovchinnikov et al. (2016), solely magnetic control was proposed using the sliding manifold technique. The slowly-varying systems theory was used for magnetic control in Reyhanoglu and Hervas (2012). In Gatherer and Manchester (2019), a nominal trajectory was calculated using an optimization method, then this nominal trajectory was tracked with an LQR controller by magnetorquers. A similar approach was proposed in Okhitina et al. (2022) using particle swarm optimization. Bahu and Modenini (2021) presents a hybrid magnetic controller that combines a local H-inf controller with a global nonlinear-based controller.

In addition to the above-mentioned literature, the model predictive control (MPC) approach was also applied in several studies on magnetic control. MPC is a reasonable choice as it can produce the control input by considering the future behavior of the system. More specifically, MPC can adjust the control moment by considering how the magnetic field change in the horizon. In Krogstad et al. (2005), an explicit MPC was designed to control attitude by magnetorquers for a Norwegian student satellite. MPC was also applied in Wood et al. (2006) considering the stability analysis by Floquet theorem. In Kim et al. (2018), a linear time-varying MPC was derived and investigated for both nadir-pointing and inertial-pointing satellites. Nonlinear MPC was proposed in Kondo et al. (2021) magnetorquers. By using two magnetorquers, Alger and de Ruiter (2022) presented an LQR-based control approach for attitude stabilization.

In this study, it is desired to design a magnetic attitude control using the MPC approach. Then, this attitude control method is integrated into an entire low-cost ADC algorithm that estimates the attitude with gyroscopes, magnetometers, and Sun sensors and controls the attitude with magnetorquers. Thus, an overall ADC algorithm suitable to be implemented on a small satellite with a limited set of sensors and actuators is proposed. After that, this overall ADC algorithm is simulated in the MATLAB environment. This low-cost architecture is considered to be used as the main system in low-budget satellites, or as the backup system in case of failure of the primary sensors and actuators.

2. SPACECRAFT DYNAMICS

In this section, the equations of attitude motion are reviewed. Then, the magnetic actuator model is presented.

2.1 Satellite Attitude Motion

In this study, the quaternion is preferred as the attitude parameterization since it does not suffer from singularity issue and the attitude kinematics by quaternion is represented without using the trigonometric functions. The quaternion can be represented as

$$\mathbf{q} = [\mathbf{q}_{1:3}^T \ q_4]^T = [q_1 \ q_2 \ q_3 \ q_4]^T \quad (1)$$

with

$$\mathbf{q}_{1:3} = \bar{\mathbf{e}} \sin(\vartheta/2); \quad q_4 = \cos(\vartheta/2) \quad (2)$$

where $\bar{\mathbf{e}}$ represents the vector of rotation and ϑ is the angle of rotation.

The attitude kinematics in terms of quaternion with respect to the orbital frame can be given as Markley and Crassidis (2014)

$$\dot{\mathbf{q}} = \frac{1}{2} \Omega(\omega_o) \mathbf{q} \quad (3)$$

where ω_o the angular velocity of the satellite body frame with respect to the orbital frame and defined in the body frame, and it can be calculated by Markley and Crassidis (2014)

$$\omega_o = \omega - A \begin{bmatrix} 0 \\ -n \\ 0 \end{bmatrix} \quad (4)$$

where n represents the satellite orbital rate, A is the attitude matrix and ω is the angular velocity vector of the satellite body frame with respect to the inertial frame and defined in the body frame. The angular rate vector notations are taken as

$$\omega = [\omega_1 \ \omega_2 \ \omega_3]^T \quad (5)$$

and

$$\omega_o = [\omega_x \ \omega_y \ \omega_z]^T \quad (6)$$

Also, $\Omega(\omega)$ stands for a 4×4 skew-symmetric matrix of the angular rate vector as

$$\Omega(\omega) = \begin{bmatrix} -[\omega_\times] & \omega \\ -\omega^T & 0 \end{bmatrix} = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \quad (7)$$

where the $[\omega_\times]$ component indicates 3×3 skew-symmetric matrix of ω .

The rotational dynamics of the rigid spacecraft is given by Markley and Crassidis (2014)

$$\dot{\omega} = J^{-1} (\mathbf{L} - \omega \times (J\omega)) \quad (8)$$

where J represents the tensor of inertia of the satellite. \mathbf{L} is net torque including control and disturbance torques acting on the spacecraft.

2.2 Magnetic Actuation

Magnetic actuators (magnetorquers) are basically electrical coils that generate magnetic dipole moment. The interaction of generated magnetic moment with the magnetic field creates torque that orients the satellite. Magnetorquers are mostly used in LEOs and can be used for satellite detumbling, momentum dumping of attitude actuation wheels, residual dipole moment compensation and attitude control. The generated torque using magnetorquers can be modeled as

$$\mathbf{L}_c = \mathbf{M}_c \times \mathbf{B}_b = -[\mathbf{B}_b \times] \mathbf{M}_c \quad (9)$$

where \mathbf{B}_b represents the magnetic field vector in the satellite body frame and \mathbf{M}_c stands for the magnetic moment generated by magnetic actuators which is limited to the maximum moment value that can be produced within the capacity of magnetic actuators as shown below

$$|\mathbf{M}_c| \leq M_{\max} \quad (10)$$

With three-axis magnetic actuator system, magnetic-only control does not provide instantaneous three-axis controllability. This fact can be seen in (9), as the $[\mathbf{B}_b \times]$ term is singular, its inverse cannot be evaluated and required moment to control all

three-axis is not possible to produce. However, in near-polar orbits, three-axis attitude control can be achieved, although not very sensitive, within a period as the magnetic field periodically changes along the orbit Ovchinnikov and Roldugin (2019).

2.3 Linearized System Representation of Attitude Motion

The state vector for the attitude motion with respect to the orbital frame is formed of vector part of quaternion and angular rate shown as

$$\mathbf{x} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (11)$$

Here, the scalar part of the quaternion is not included in the state vector since this value is included implicitly by the vector part of quaternion. Thus, controlling of the vector part also adjusts the scalar part indirectly as a result of the quaternion norm constraint.

In case the magnetorquers are used as the sole attitude actuation system and gravity gradient torque is considered, the linearized attitude system in which it is assumed that the orbit is circular and linearization is applied for nadir-pointing satellite, can be written as Psiaki (2001)

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (12)$$

with

$$\mathbf{u} = \mathbf{M}_c \quad (13)$$

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \\ 8k_x n^2 & 0 & 0 & 0 & 0 & (k_x + 1)n \\ 0 & 6k_y n^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2k_z n^2 & (k_z - 1)n & 0 & 0 \end{bmatrix} \quad (14)$$

$$\mathbf{B} = \begin{bmatrix} 0_3 \\ -J^{-1}[\mathbf{B}_r \times] \end{bmatrix} \quad (15)$$

where \mathbf{B}_r is the magnetic field vector in the reference frame (orbital frame), and

$$k_x = \frac{J_3 - J_2}{J_1}, \quad k_y = \frac{J_3 - J_1}{J_2}, \quad k_z = \frac{J_2 - J_1}{J_3} \quad (16)$$

where J_i represents the principal components of tensor of inertia.

3. MPC FOR MAGNETIC TORQUER BASED ATTITUDE CONTROL

MPC is an optimization-based control technique that finds the current optimal control input by considering the behaviour of the system in a horizon. The system model and the current state measurements/estimates must be known in the MPC and this information helps to predict future states with the best control action in an open-loop control framework. At each time step, the horizon is shifted and control action is calculated with the control objectives. Thus, this approach is essentially a closed-loop control, but finds the optimal control input in an open-loop manner. Moreover, the input and output constraints can be taken into account in the optimization problem of the MPC.

In the case the system is linear, the state equations can be given by

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}_k\mathbf{u}_k \quad (17)$$

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k \quad (18)$$

in which, likewise the presented linearized attitude system, only the matrix \mathbf{B}_k is time-variant, and the matrices \mathbf{A} and \mathbf{C} are considered to be time-invariant. Here, \mathbf{x} stands for the state, \mathbf{u} is the input and \mathbf{y} is the output vectors. The subscript k value represents the discrete-time step. Note that, although the state-space representation was presented in continuous time in previous section, the system is discretized by zero-order hold discretization in order to be used in the control input calculations.

The cost function to be minimized is given by

$$\mathbf{C} = \mathbf{Y}^T \mathbf{Q} \mathbf{Y} + \mathbf{U}^T \mathbf{R} \mathbf{U} \quad (19)$$

with

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_k \\ \mathbf{y}_{k+1} \\ \vdots \\ \mathbf{y}_{k+N} \end{bmatrix}; \quad \mathbf{U} = \begin{bmatrix} \mathbf{u}_k \\ \mathbf{u}_{k+1} \\ \vdots \\ \mathbf{u}_{k+N_c} \end{bmatrix} \quad (20)$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{Q}_N \end{bmatrix}; \quad \mathbf{R} = \begin{bmatrix} \mathbf{R}_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{R}_{N_c} \end{bmatrix}$$

where \mathbf{Y} and \mathbf{U} are output and input vectors, and \mathbf{Q} and \mathbf{R} are the penalty matrices for output and input values, respectively. The values N and N_c stand for the prediction horizon and control horizon, respectively.

At time step k , future states can be determined by the known system model and the current state information as follows

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}_k\mathbf{u}_k \quad (21)$$

$$\mathbf{x}_{k+2} = \mathbf{A}^2\mathbf{x}_k + \mathbf{A}\mathbf{B}_k\mathbf{u}_k + \mathbf{B}_{k+1}\mathbf{u}_{k+1} \quad (22)$$

$$\mathbf{x}_{k+3} = \mathbf{A}^3\mathbf{x}_k + \mathbf{A}^2\mathbf{B}_k\mathbf{u}_k + \mathbf{A}\mathbf{B}_{k+1}\mathbf{u}_{k+1} + \mathbf{B}_{k+2}\mathbf{u}_{k+2} \quad (23)$$

Thus, the future state vector in the estimation horizon can be constructed by

$$\mathbf{X} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^2 \\ \vdots \\ \mathbf{A}^N \end{bmatrix} \mathbf{x}_0 + \begin{bmatrix} \mathbf{B}_0 & 0 & \cdots & 0 \\ \mathbf{A}\mathbf{B}_0 & \mathbf{B}_1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \mathbf{A}^{N_c}\mathbf{B}_0 & \mathbf{A}^{N_c-1}\mathbf{B}_1 & \cdots & \mathbf{B}_{N_c} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}^{N-1}\mathbf{B}_0 & \mathbf{A}^{N-2}\mathbf{B}_1 & \cdots & \mathbf{A}^{N-N_c-1}\mathbf{B}_{N_c} \end{bmatrix} \mathbf{U} \quad (24)$$

where the step time k is taken as the current/initial time value, $k = 0$. Then, the future output vector is determined as

$$\mathbf{Y} = \begin{bmatrix} \mathbf{C}\mathbf{A} \\ \mathbf{C}\mathbf{A}^2 \\ \vdots \\ \mathbf{C}\mathbf{A}^N \end{bmatrix} \mathbf{x}_0 + \begin{bmatrix} \mathbf{C}\mathbf{B}_0 & 0 & \cdots & 0 \\ \mathbf{C}\mathbf{A}\mathbf{B}_0 & \mathbf{B}_1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \mathbf{C}\mathbf{A}^{N_c}\mathbf{B}_0 & \mathbf{C}\mathbf{A}^{N_c-1}\mathbf{B}_1 & \cdots & \mathbf{C}\mathbf{B}_{N_c} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}\mathbf{A}^{N-1}\mathbf{B}_0 & \mathbf{C}\mathbf{A}^{N-2}\mathbf{B}_1 & \cdots & \mathbf{C}\mathbf{A}^{N-N_c-1}\mathbf{B}_{N_c} \end{bmatrix} \mathbf{U} \quad (25)$$

So, the output vector turns into

$$\mathbf{Y} = \mathbf{C}\mathbf{X} = \mathbf{S}_x\mathbf{x}_0 + \mathbf{S}_u\mathbf{U} \quad (26)$$

The cost function can be rewritten as

$$\begin{aligned}
C &= Y^T Q Y + U^T R U \\
&= (S_x x_0 + S_u U)^T Q (S_x x_0 + S_u U) + U^T R U \\
&= (S_x^T x_0 + U^T S_u^T)^T Q (S_x x_0 + S_u U) + U^T R U \quad (27) \\
&= S_x^T x_0 Q S_x x_0 + 2U^T S_u^T Q S_x x_0 + U^T (S_u^T Q S_u + R) U
\end{aligned}$$

So, the optimization problem can be formulated as

$$C = S_x^T x_0 Q S_x x_0 + 2U^T S_u^T Q S_x x_0 + U^T (S_u^T Q S_u + R) U \quad (28)$$

subject to

$$\begin{aligned}
u &\leq u_{\max} \\
-u &\leq -u_{\min}
\end{aligned} \quad (29)$$

then, the problem can be written in the quadratic form Löfberg (2001) as

$$C_{MPC} = S_x^T x_0 Q S_x x_0 + 2U^T F x_0 + \frac{1}{2} U^T H U \quad (30)$$

subject to

$$EU \leq b \quad (31)$$

with

$$H = 2 (S_u^T Q S_u + R) \quad (32)$$

$$F = S_u^T Q S_x \quad (33)$$

$$E = \begin{bmatrix} I_{Nc} \\ -I_{Nc} \end{bmatrix} \quad (34)$$

$$b = \begin{bmatrix} U_{\max} \\ -U_{\min} \end{bmatrix} \quad (35)$$

After transforming the problem to a quadratic form, the problem can be solved by quadratic programming and input variables are obtained. After all, the input value corresponding to the current state is used for control input.

4. OVERALL ATTITUDE DETERMINATION AND CONTROL ALGORITHM

4.1 The Attitude Determination and Control Algorithm

The ADCS is considered to be equipped with gyroscope, three-axis magnetometer (TAM), Sun sensor as the attitude sensors and magnetorquers as the attitude actuators.

In the attitude determination part, the gyro-based integrated QUEST/MEKF estimation technique is used to estimate attitude and rate information Esit et al. (2021). The filter basically estimates quaternion and gyro bias vectors. Then the gyroscope angular velocity measurements are corrected by the gyro bias estimate. The vector measurements from magnetometer and Sun sensor are pre-processed by QUEST which is a widely used static attitude determination method and the quaternion outputs of QUEST are fed into the filter as the measurement model. After all, the attitude determination system provides quaternion and angular rate estimates for the attitude control system in order to calculate the desired control action.

Controlling the attitude from the beginning of the mission requires first to detumble the angular velocity of the satellite. This can be accomplished by using one of the detumbling algorithms. In this study, the Bdot control technique by Stickler and Alfriend (1976) which makes use of magnetometer and magnetorquers is applied considering the satellite is tumbling in the beginning. After detumbling of the angular rate of the

satellite, the model predictive approach is applied for satellite attitude control.

Additionally, in order to calibrate the TAM measurements, the TWO-STEP algorithm proposed by Alonso and Shuster (2002) which is an attitude-independent batch calibration method is applied.

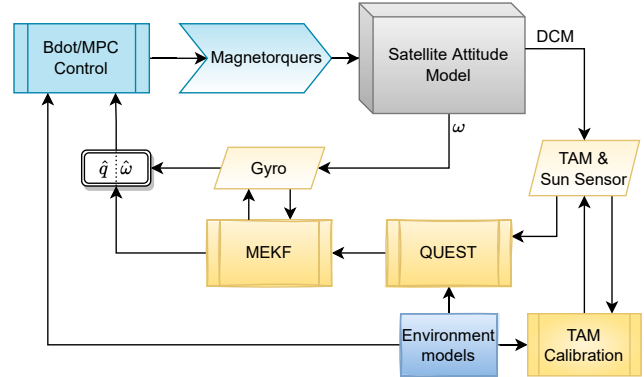


Fig. 1. Overall ADC algorithm

The overall ADCS system designed for a LEO small satellite is illustrated in Fig. 1. As seen in the figure, QUEST algorithm processes the Sun vector and calibrated TAM vector measurements to determine the quaternion and this quaternion is used as the measurement model of gyro-based integrated QUEST/MEKF. The output of the filter is the estimation of the attitude and angular rate and these values are fed into the controller. Then, the calculated control action by MPC (after detumbling) is applied to the satellite by using magnetorquers.

4.2 Simulation Results and Discussion

The complete ADCS algorithm is tested for a LEO Sun-synchronous satellite, whose orbital parameters are presented in Table 1, via MATLAB. The gravity gradient torque is applied to the spacecraft as a disturbance torque. IGRF model Alken et al. (2021) and Sun direction model Vallado and McClain (1997) are implemented in order to simulate the environment and model the TAM and Sun sensors. The dipole model Hajiyev and Soken (2020) is also used for magnetic field calculations in each step of MPC to reduce the computational burden.

Table 1. Microsatellite parameters

Tensor of inertia	$J = \begin{bmatrix} 9.8194 & -0.071 & -0.2892 \\ -0.071 & 9.7030 & -0.1011 \\ -0.2892 & -0.1011 & 9.7309 \end{bmatrix}$
Altitude	~675 km
Inclination	~98.14 deg
Orbital period	~98.12 min

The characteristics of the ADCS instruments chosen for the simulation are as follows:

- TAM (1 Hz) with zero-mean Gaussian white-noise and its standard deviation is 300 nT. The other sensor error parameters are considered to be calibrated via TWO-STEP algorithm.
- Sun sensor (1 Hz) with zero-mean Gaussian white-noise and its standard deviation is 0.002.

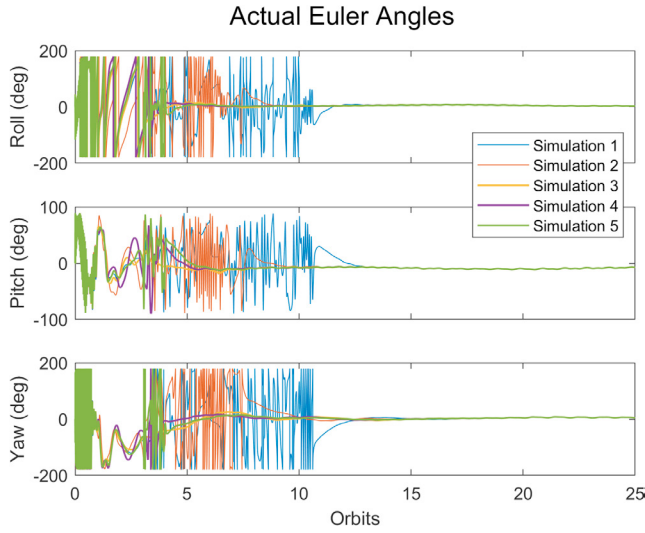


Fig. 2. Actual Euler angle values controlled by Bdot (0-3 orbits) and MPC (>3 orbits)

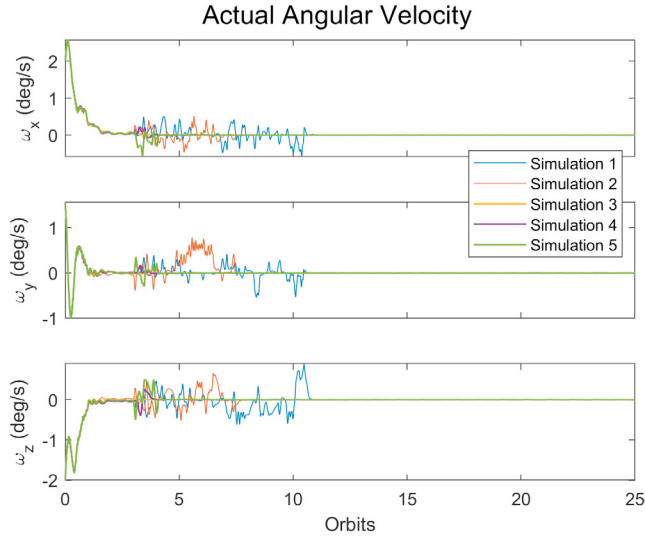


Fig. 3. Actual angular velocity values controlled by Bdot (0-3 orbits) and MPC (>3 orbits)

- Gyroscope (20 Hz) with bias and zero-mean Gaussian white-noise. The noise has the standard deviation of $\sigma_v = 1.1975 \times 10^{-5}$ rad/s. The bias is modeled as random walk process by zero-mean Gaussian white-noise with a standard deviation of $\sigma_u = 3.0834 \times 10^{-9}$ rad/s² and initialized at 8.7×10^{-4} rad/s for each axis.
- Magnetorquers with the maximum moment capacity of 5 Am².

The simulations run for 25 orbits. Besides, the simulation is repeated 5 times to show the robustness of the algorithm under the uncertain/random variables. During the first 3 orbits, the Bdot control algorithm is applied for satellite detumbling. Then, attitude is controlled by MPC.

In Table 2, the MPC tuning parameters are given. In Fig. 2, Euler angle values are given with respect to the orbital frame. It is seen that, after 10 orbital periods, the angles comes close to the desired values in all simulations. However, in the three of the simulations, the attitude converges in 6-7 orbits, that means

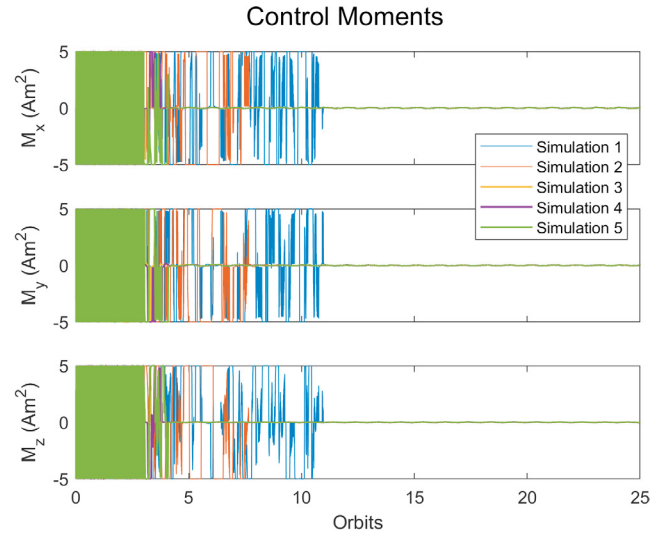


Fig. 4. Control moment values generated by magnetorquers

Table 2. MPC Parameters

State weight matrix	$Q_i = \begin{bmatrix} 8 \times 10^{-4} I_3 & 0_3 \\ 0_3 & 1.5 I_3 \end{bmatrix}$ for $i = 1 : N$
Control weight matrix	$R_i = 10^{-10} I_3$ for $i = 1 : N_c$
Sampling interval	$T_c = 0.1$ s
Prediction horizon	$N = 40$
Control horizon	$N_c = 20$

the MPC approach stabilizes the satellite in all three axes in 3-4 orbits after the satellite is detumbled in these simulations.

In Fig. 3, the angular rate values in body frame with respect to the orbital frame are shown. The initial rates at $\omega_0 = [2 \ 1.5 \ -2]^T$ deg/s are detumbled in the first 3 orbits. In Simulation 1-2, the angular rates converges to the zero and the satellite reaches to the desired attitude after 10 orbits. However, the other simulations has better convergence characteristics as also seen in Fig. 2. Besides, Fig. 4 shows the control moment values generated by magnetorquers for each simulation.

5. CONCLUSION

In this study, a magnetic attitude control algorithm was designed for a LEO satellite by using the MPC approach. The attitude motion was first linearized by taking gravity gradient torque and magnetic actuation model into account, and the state space equation is written. The input matrix in the state space is taken as time-variant as the magnetic field is changing. The MPC approach suitable for the linearized attitude motion was presented and applied for the attitude motion. This control algorithm is integrated into a complete ADC algorithm and tested via MATLAB. In the ADCS, the integrated QUEST/MEKF was used to provide attitude and angular rate estimates to the MPC. The complete algorithm was tested for a LEO microsatellite from its tumbling status via MATLAB environment. The satellite was detumbled after 3 orbital periods. After that, the MPC algorithm for attitude control was implemented to calculate control action and three-axis attitude control was achieved with an accuracy of below 10 deg in around 7 orbits at worst after the satellite had been detumbled.

Results, in general, show that the proposed complete ADC algorithm is capable of controlling the attitude of the satellite with an accuracy higher than 10deg. Considering the limitations

due to the satellite size and also the fact that only magnetorquers are used for attitude control, this accuracy level is satisfactory.

REFERENCES

- Alger, M. and de Ruiter, A. (2022). Magnetic spacecraft attitude stabilization with two torquers. *Acta Astronautica*, 192, 157–167.
- Alken, P. et al. (2021). International geomagnetic reference field: the thirteenth generation. *Earth, Planets and Space*, 73, 49.
- Alonso, R. and Shuster, M.D. (2002). Complete linear attitude-independent magnetometer calibration. *Journal of the Astronautical Sciences*, 50, 477–490.
- Bahu, A. and Modenini, D. (2021). Hybrid controller for global, robust, attitude stabilization of a magnetically actuated spacecraft. *CEAS Space Journal*, 13, 543–554.
- Bhat, S. and Dham, A. (2003). Controllability of spacecraft attitude under magnetic actuation. In *42nd IEEE International Conference on Decision and Control*. Maui, HI.
- Esit, M., Yakupoglu, S., and Soken, H.E. (2021). Attitude filtering for nanosatellites: A comparison of different approaches under model uncertainties. *Advances in Space Research*, 68(6), 2551–2564.
- Gatherer, A. and Manchester, Z. (2019). Magnetorquer-only attitude control of small satellites using trajectory optimization. In *Proceedings of AAS/AIAA Astrodynamics Specialist Conference*.
- Goel, P. and Rajaram, S. (1979). Magnetic attitude control of a momentum-biased satellite in near-equatorial orbit. *Journal of Guidance and Control*, 2(4), 334–338.
- Hajiyeve, C. and Soken, H.E. (2020). *Fault Tolerant Attitude Estimation for Small Satellites*. CRC Press, Boca Raton, FL. doi:10.1201/9781351248839.
- Kim, J., Jung, Y., and Bang, H. (2018). Linear time-varying model predictive control of magnetically actuated satellites in elliptic orbits. *Acta Astronautica*, 151, 791–804.
- Kondo, K., Yoshimura, Y., Nagasaki, S., and Hanada, T. (2021). Pulse width modulation method applied to nonlinear model predictive control on an under-actuated small satellite. In *AIAA Scitech 2021 Forum*.
- Krogstad, T.R., Gravdahl, J.T., and Tøndel, P. (2005). Explicit model predictive control of a satellite with magnetic torquers. In *Proceedings of the 2005 IEEE International Symposium on, Mediterrean Conference on Control and Automation Intelligent Control*. Limassol, Cyprus.
- Lagrasta, S. and Bordin, M. (1996). Normal mode magnetic control of leo spacecraft, with integral action. In *Guidance, Navigation, and Control Conference*. San Diego, CA.
- Lal, B., de la Rosa Blanco, E., Behrens, J.R., Corbin, B.A., Green, E.K., Picard, A.J., and Balakrishnan, A. (2017). Global trends in small satellites. Technical report, Institute for Defense Analyses.
- Lovera, M., De Marchi, E., and Bittanti, S. (2002). Periodic attitude control techniques for small satellites with magnetic actuators. *IEEE Transactions on Control Systems Technology*, 10(1), 90–95.
- Lovera, M. and Astolfi, A. (2005). Global magnetic attitude control of inertially pointing spacecraft. *Journal of Guidance, Control, and Dynamics*, 28(5), 1065–1072.
- Löfberg, J. (2001). *Linear Model Predictive Control Stability and Robustness*. Ph.D. thesis, Linköping University, Linköping, Sweden.
- Markley, L. and Crassidis, J. (2014). *Fundamentals of Spacecraft Attitude Determination and Control*. Springer, New York.
- Martel, F., Pal, P.K., and Psiaki, M.L. (1988). Active magnetic control system for gravity gradient stabilized spacecraft. In *Proceedings of 2nd Annual AIAA/USU Conference, Small Satellite*.
- Okhitina, A., Roldugin, D., and Tkachev, S. (2022). Application of the pso for the construction of a 3-axis stable magnetically actuated satellite angular motion. *Acta Astronautica*, 195, 86–97.
- Ovchinnikov, M.Y. and Roldugin, D.S. (2019). A survey on active magnetic attitude control algorithms for small satellites. *Progress in Aerospace Sciences*, 109, 100546.
- Ovchinnikov, M., Roldugin, D., Penkov, V., Tkachev, S., and Mashtakov, Y. (2016). Fully magnetic sliding mode control for acquiring three-axis attitude. *Acta Astronautica*, 121, 59–62.
- Psiaki, M.L. (2001). Magnetic torquer attitude control via asymptotic periodic linear quadratic regulation. *Journal of Guidance, Control, and Dynamics*, 24(2), 386–394.
- Reyhanoglu, M. and Hervas, J.R. (2012). Magnetic attitude control design for small satellites via slowly-varying systems theory. In *IECON 2012 - 38th Annual Conference on IEEE Industrial Electronics Society*, 2325–2330. Montreal, Quebec.
- Shigehara, M. (1972). Geomagnetic attitude control of an axisymmetric spinning satellite. *Journal of Spacecraft and Rockets*, 9(6), 391–398.
- Steyn, W.H. (1994). Comparison of low-earth-orbit satellite attitude controllers submitted to controllability constraints. *Journal of Guidance, Control, and Dynamics*, 17(4), 795–804.
- Stickler, A.C. and Alfrend, K. (1976). Elementary magnetic attitude control system. *Journal of Spacecraft and Rockets*, 13(5), 282–287.
- Vallado, D.A. and McClain, W.D. (1997). *Fundamentals of Astrodynamics and Applications*. McGraw-Hill Companies.
- Wang, P., Shtessel, Y., and Wang, Y.Q. (1998). Satellite attitude control using only magnetorquers. In *Proceedings of Thirtieth Southeastern Symposium on System Theory*, 500–504.
- Wiśniewski, R. and Blanke, M. (1999). Fully magnetic attitude control for spacecraft subject to gravity gradient. *Automatica*, 35(7), 1201–1214.
- Wood, M., Chen, W.H., and Fertin, D. (2006). Model predictive control of low earth orbiting spacecraft with magnetorquers. In *Proceedings of the IEEE International Conference on Control Applications*. Munich, Germany.
- Yang, Y. (2019). *Spacecraft Modeling, Attitude Determination, and Control Quaternion-based Approach*. CRC Press, Boca Raton, FL.