Estimation and Learning in Aerospace 2023/2024 exam project

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Project tasks

The project deals with the dynamics of a multirotor UAV and involves the following two tasks.

Task I

- System identification of longitudinal dynamics from simulated data for a given input

Task II

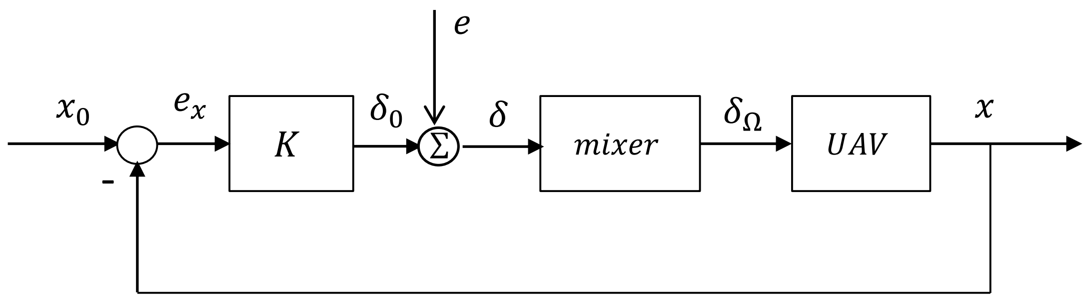
- Robust optimisation of experiment design to maximise model accuracy

A small quadrotor

Weight: 270g
Dimensions: 20 x 20 x 4 cm
Diameter (motor-to-motor distance): 16cm
Hovering time: 7’30’’

ANT-X

Identification experiments and I/O estimation data



Decoupled dynamics

- *Longitudinal dynamics*
  - Input: δ_{lon} / Outputs: $q,\ a_x$

Linearised model for longitudinal dynamics

State equation

$$\begin{bmatrix} \dot{u} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_q & -g \\ M_u & M_q & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_\delta \\ M_\delta \\ 0 \end{bmatrix} \delta_{lon}$$

u Longitudinal (body) velocity [m/s]
q Pitch rate [rad/s]
θ Pitch angle [rad]
a_x Longitudinal (body) acceleration [m/s^2]
δ_{lon} Pitch moment [normalised to -1, +1 range]

Output equation

$$\begin{bmatrix} q \\ a_x \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ X_u & X_q & 0 \end{bmatrix} \begin{bmatrix} u \\ q \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ X_\delta \end{bmatrix} \delta_{lon}$$

Model parameters: $\Theta = \begin{bmatrix} X_u \\ X_q \\ M_u \\ M_q \\ X_\delta \\ M_\delta \end{bmatrix}$

+ delay (computational time, sensor readings, …)

Linearised model for longitudinal dynamics

Model class: $M(\Theta)$

True system: $M(\Theta^*)$

$$\begin{bmatrix} \dot{u} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_q & -g \\ M_u & M_q & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_\delta \\ M_\delta \\ 0 \end{bmatrix} \delta_{lon}$$

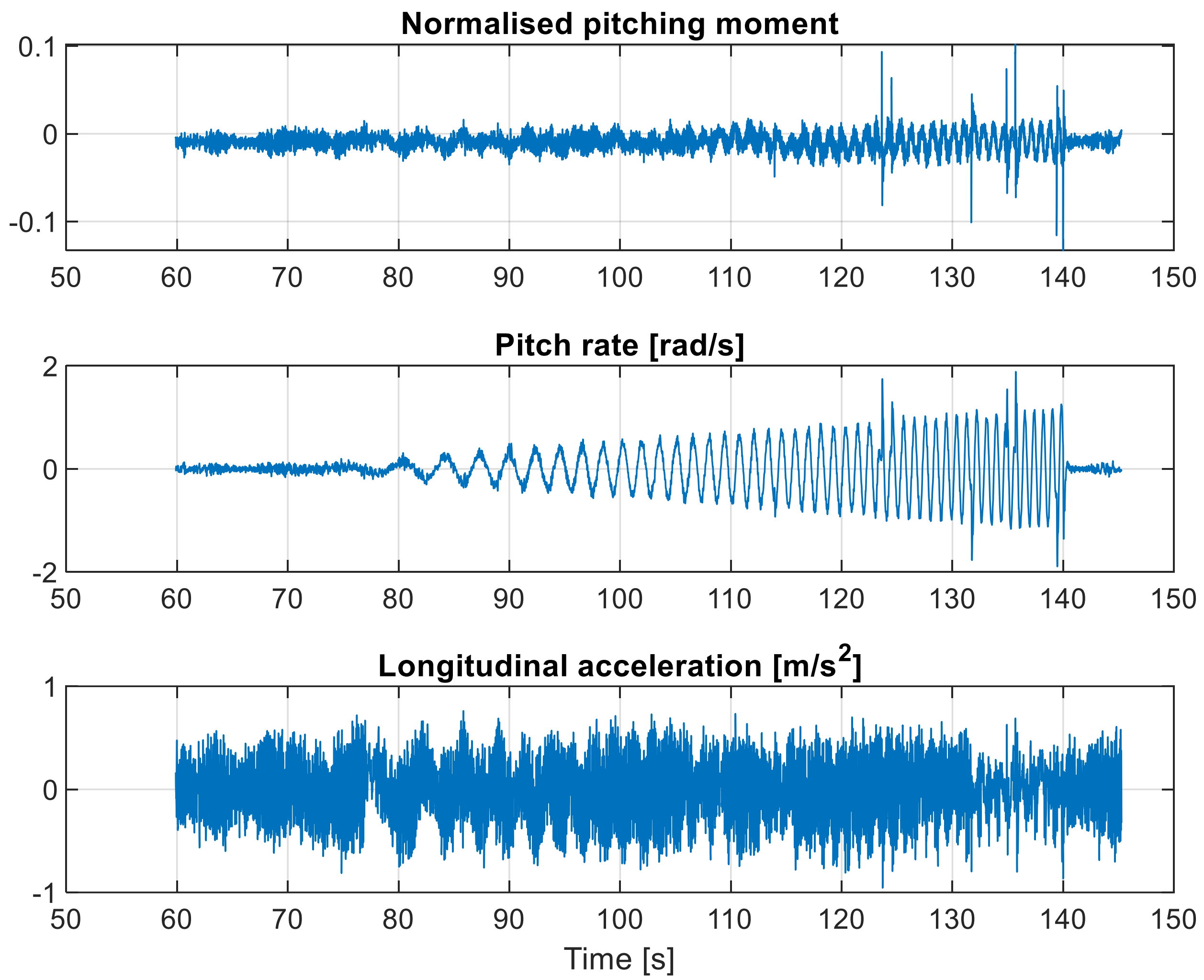
$$\begin{bmatrix} q \\ a_x \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ X_u & X_q & 0 \end{bmatrix} \begin{bmatrix} u \\ q \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ X_\delta \end{bmatrix} \delta_{lon}$$

$$\Theta = \begin{bmatrix} X_u \\ X_q \\ M_u \\ M_q \\ X_\delta \\ M_\delta \end{bmatrix}$$

Longitudinal dynamics: excitation signal

- Experiments are carried out under position and attitude feedback, in order to guarantee safe operation.

- Baseline excitation input is a long duration sweep: excite low-frequency translational dynamics



Task 1: model identification

Use the simulator and the provided baseline excitation to:

- Identify a grey-box model for the longitudinal dynamics of the multirotor (input: total pitching moment; outputs: pitch rate and longitudinal acceleration) using the response to the sweep provided in the ExcitationM.mat file

- Assess the uncertainty of the identified model

- Output of the identification procedure: $\hat{\Theta} \sim G(\bar{\Theta}, \Sigma_{\bar{\Theta}})$

Notes:

- the open loop dynamics is *unstable*
- MATLAB functions *iddata* and *greyest* can be used for the implementation

Task 2: optimisation of experiment design

- The identified model can be used to re-design the experiment so as to optimise the accuracy of the model

- Therefore, with reference to the class of inputs and the performance metric defined in the following slide, the second task of the project consists in:
    - Optimising the input class to maximise the quality of the obtainable model.
    - Evaluating the quality of the optimal obtainable model.

- Nominal experiment design:

  - Nominal model for experiment design: $M(\bar{\Theta})$ (identified model from Task1)

  - Input class: $u(t,\eta\ ) = 0{,}1\sin(2\pi f(t)t)$ with $f(t) = f_1 + \frac{f_2 - f_1}{T}t, \quad \eta = \begin{bmatrix} f_1 \\ f_2 \\ T \end{bmatrix}$

  - Performance metric:

    $$J(\eta, \bar{\Theta}) = trace(\Sigma_{\eta,\bar{\Theta}})$$

    (*i.e.*, sum of the variances of the parameter estimates computed from the dataset obtained by applying the input sequence $u(t,\eta\ )$ to the model $M(\bar{\Theta})$)

Task 2: optimisation of experiment design

- Nominal experiment design:

  - Solve: $\min_{\eta} J(\eta, \overline{\Theta})$ to find optimal input sequence $u(t, \eta^{o}_{\overline{\Theta}})$ and cost $J(\eta^{o}_{\overline{\Theta}}, \overline{\Theta})$

  - Use optimal sequence $u(t, \eta^{o}_{\overline{\Theta}})$ to find new estimate $\widehat{\Theta}^{o} \sim G(\overline{\Theta}^{o}, \Sigma_{\overline{\Theta}^{o}})$

- Note: the duration of the excitation sequence should not exceed 90 seconds.

Task 2: optimisation of experiment design

- Experience from last year’s project shows that the optimal solution is sensitive to model uncertainty.

- In view of this, this year’s task is to optimise the input sequence robustly, i.e., taking into account uncertainty information.

- It is proposed to use a scenario-based optimization approach (see following slides).

Task 2: optimisation of experiment design

Scenario-Based Optimization

Considers multiple scenarios or realizations of uncertain parameters to find a solution that performs well on average across these scenarios.

Involves generating a finite set of scenarios and solving the optimization problem for each scenario, then considering the average or aggregated performance of solutions.

Utilizes techniques like Monte Carlo simulation to cover a representative set of scenarios.

Task 2: optimisation of experiment design

Monte Carlo simulation is a computational technique used to analyze the impact of uncertainty in systems. It involves using random sampling to model and simulate the behavior of systems where the inputs or parameters are uncertain or variable.

1. System Representation:

Start with a mathematical model that describes the system or process being studied. This model includes uncertain parameters or variables.

2. Uncertain Parameters:

Determine the parameters or variables that have uncertainty associated with them, often represented by probability distributions.

Task 2: optimisation of experiment design

3. Random Sampling:

Randomly sample from the probability distributions of uncertain parameters. This generates a set of potential scenarios or realizations for these parameters.

4. Simulation Runs:

Perform multiple simulations of the model, each using different sets of sampled parameter values obtained in the random sampling step.

5. Aggregate Results:

Gather the output or response of interest from each simulation run. This could be the performance metric, behavior, or outcome of the system under those specific parameter values.

6. Analysis:

Analyze the collected results statistically, such as calculating mean, standard deviation, confidence intervals, or histograms.

Robust experiment design:

- Using the same notation for nominal model, input class and performance metric:
  - From $\widehat{\Theta} \sim G(\overline{\Theta}, \Sigma_{\overline{\Theta}})$ generate *N* random models corresponding to $\Theta_1, \Theta_2, \dots, \Theta_N$ (scenarios)
  - Solve *N* nominal experiment design problems corresponding to each scenario:

$$\min_{\eta} J(\eta, \Theta_n), n = 1, \dots, N$$

to find input sequence $u\left(t, \eta^o_{\Theta_n}\right)$ and optimal cost $J\left(\eta^o_{\Theta_n}, \Theta_n\right), , n = 1, \dots, N.$

Task 2: optimisation of experiment design

- Evaluate each solution against all scenarios:
  - $\eta^o_{\Theta_1}: J\left(\eta^o_{\Theta_1}, \Theta_n\right), n = 1, \dots, N$
  - $\eta^o_{\Theta_2}: J\left(\eta^o_{\Theta_2}, \Theta_n\right), n = 1, \dots, N$
  - …
  - $\eta^o_{\Theta_N}: J\left(\eta^o_{\Theta_N}, \Theta_n\right), n = 1, \dots, N$

  - Pick solutions which provide
    - Best average performance: $\eta^{o,av}$
    - Best worst-case performance: $\eta^{o,wc}$

Task 2: optimisation of experiment design

- Construct optimal sequences

  $u(t, \eta^{o,av})$ and $u(t, \eta^{o,wc})$

- Run the simulated experiments and compute new estimates $\widehat{\Theta}^{o,av} \sim G(\overline{\Theta}^{o,av}, \Sigma_{\overline{\Theta}^{o,av}})$ and $\widehat{\Theta}^{o,wc} \sim G(\overline{\Theta}^{o,wc}, \Sigma_{\overline{\Theta}^{o,wc}})$

- Analise the results

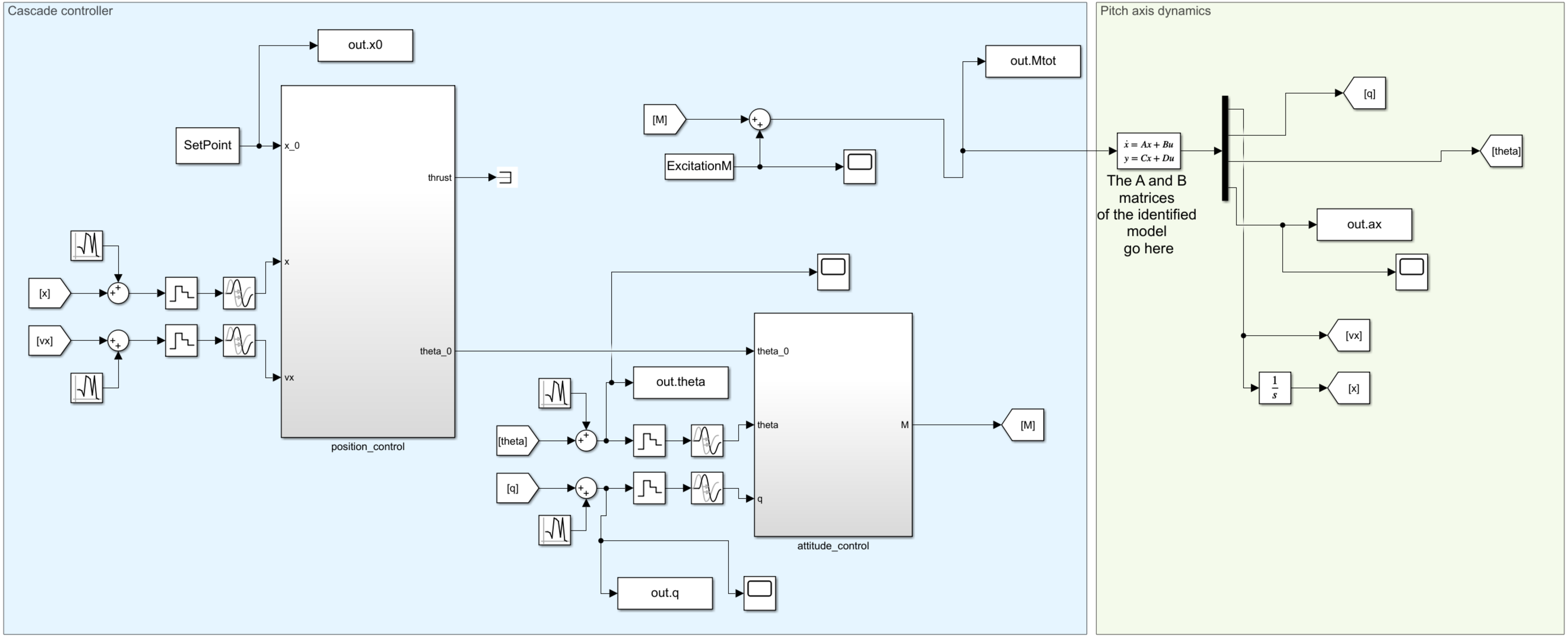
Simulator

The simulator is representative of the local linear closed-loop dynamics of the drone and combines the identified model with a complete model of the full attitude and position control system.

It includes the following files:

- main_quad_ANTX.m (which calls parameters_controller.m): set up the parameters and run the simulation
- ExcitationM.mat contains the vectors of time and normalised pitching moment corresponding to the baseline excitation
- Simulator_Single_Axis.slx the Simulink model corresponding to the closed-loop longitudinal dynamics

Simulator



Cascade controller
out.x0
SetPoint
x_0
thrust
x
vx
theta_0
position_control
[x]
[vx]
[M]
ExcitationM
out.Mtot
out.theta
[theta]
[q]
theta_0
theta
q
M
attitude_control
[M]
out.q
Pitch axis dynamics
$\dot{x} = Ax + Bu$
$y = Cx + Du$
The A and B matrices of the identified model go here
[q]
[theta]
out.ax
[vx]
$\frac{1}{s}$
[x]

Outputs

- Model identified using data generated with the provided excitation input:
    - Estimated parameters
    - Associated uncertainties
- Formulation of the robust experiment design problem
- Implementation of the solution
- Results:
    - Optimised excitation
    - Final identified model (estimated parameters and uncertainties)